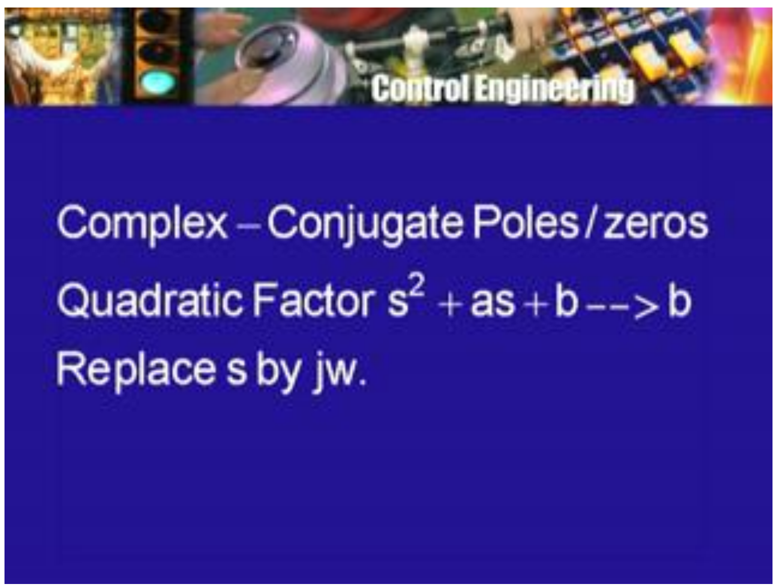
Control Engineering
Prof. S. D. Agashe
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 43

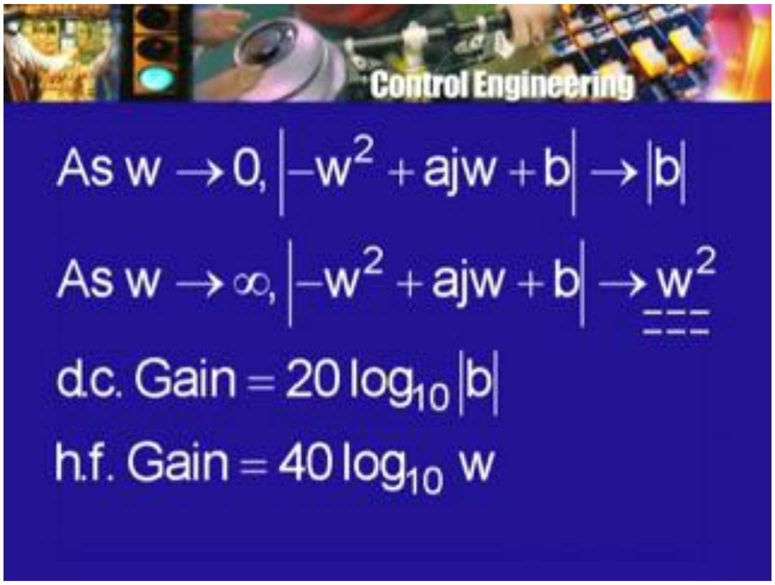
For the case of a pair of conjugate complex poles or 0s, thinking about the low frequency or DC end and the high frequency end of the frequency response, the magnitude or the db gain is not at all difficult as we saw because in one case, where we are looking at a factor like s squared plus a s plus b at the DC end or towards the low frequency end since, we are replacing s by j omega and omega is small. This is nearly equal to b and therefore nearly equal to constant as a result the low frequency asymptote is just a horizontal line that is a constant gain line. At the high frequency end because omega is large, now the terms coming from s and b are both negligible.

(Refer Slide Time: 01:40)



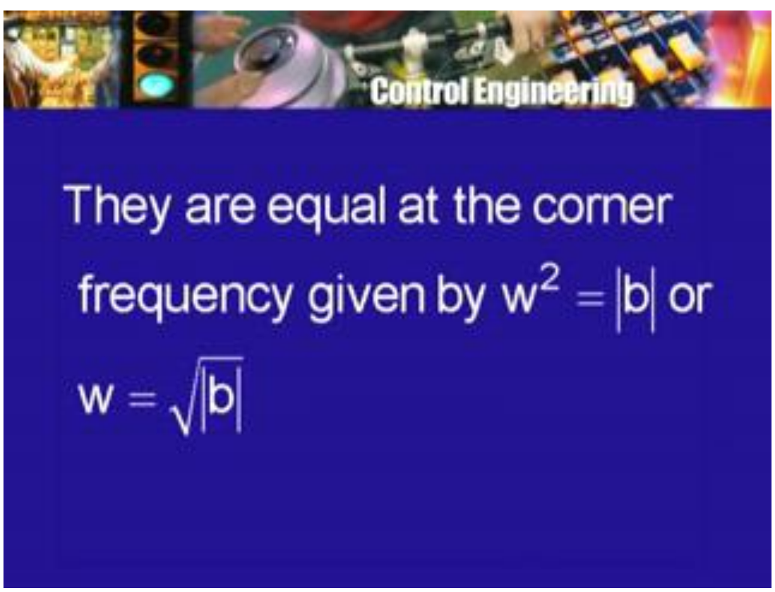
So essentially, we get the term coming from s square which is simply minus omega square when s is replaced by j omega and then, we take the modulus of this and 20 log to the base 10. So that gives us an approximation which is 40 log to the base 10 of omega and therefore on the logarithmic, semi logarithmic scale that is the omega axis marked logarithmically. We get a line with slope 40 db per decade, the slope is positive if we have a pair of 0s and the slope is negative, if we have a pair of poles. So the low frequency asymptote and the high frequency asymptote, they are quite simple, what about their point of intersection.

(Refer Slide Time: 02:13)

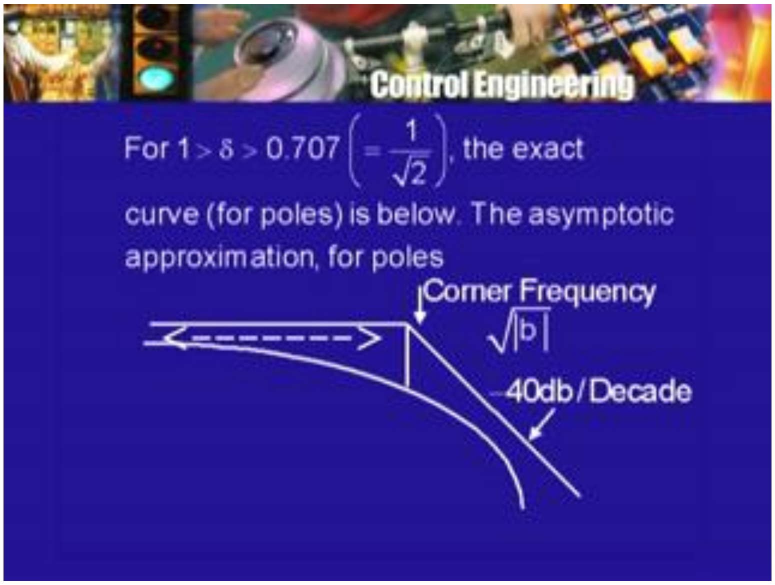


Now one can immediately see that the point of intersection will occur when the low frequency expression b equals the high frequency expression which is absolute value of omega square and so the square root of this constant term will give us the corner frequency. The corner frequency, when the 2 intersect here is the DC or the low frequency approximation, here is the high frequency approximation, where the 2 intersect the 2 have equal value and therefore we get omega square equal to b or therefore omega equal to square root of b is the corner frequency.

(Refer Slide Time: 03:07)

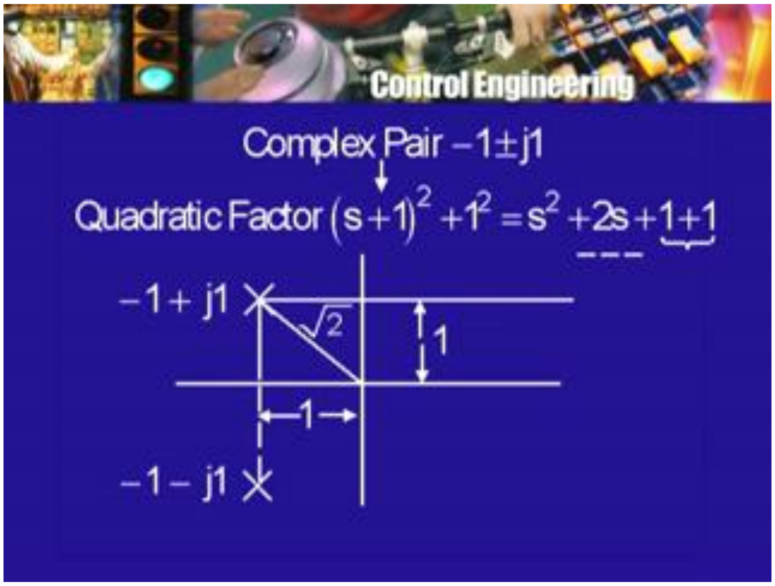


(Refer Slide Time: 03:18)



Now, if the factor in the numerator or in the denominator is written like s square plus as plus b then of course, there is no difficulty in seeing the number b and taking its square root and that is the corner frequency for the approximate Bode plot for this quadratic or the pair of conjugate complex poles of 0s is concerned. Of course, we can figure out b in terms of the location of the poles or 0s and to continue with the example that I had taken if I have minus 1 plus minus j 1 as the pair of poles or the pole pair of 0s then, corresponding to this what is the quadratic factor because of this minus I have s plus 1 and because of this j 1, I have 1 square and therefore I have s squared plus 2 s plus 1 plus 1.

(Refer Slide Time: 04:09)

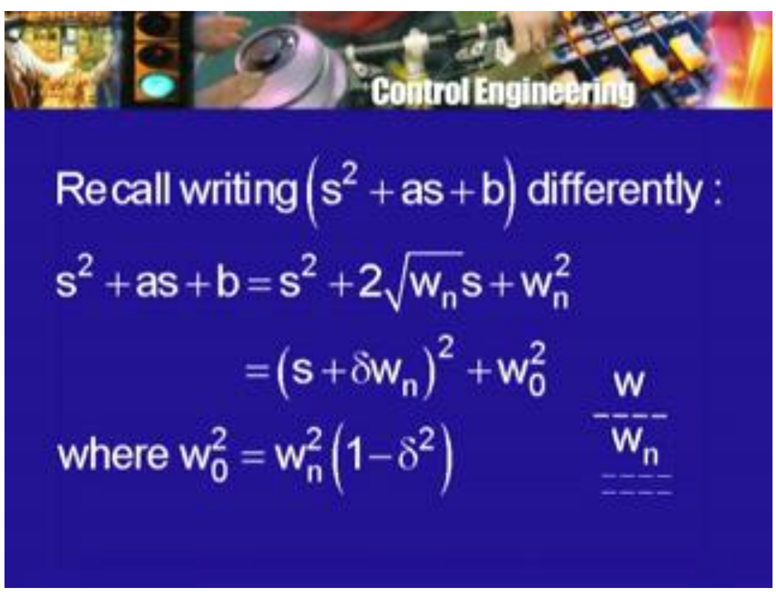


Now, what is the significance of this one and one here and you can see in the general case that if I have a pair of poles here then, this is the or corresponds to the real part of the root, this corresponds to the imaginary part of the root and what we have here is the sum of the squares of the real part and the imaginary part and what is that then, that is the square of the distance from the origin to the particular pole or the 0. So the corner frequency in this case, if we have not written the expression in the form of a quadratic already and we may not do it because we may just have the pole 0 diagram then it is not necessary to write it in the quadratic form. The distance of the complex pole or complex 0 from the origin that gives us the corner frequency.

So that is the corner frequency in this case. So the 2 ends are easily sketched as I have shown here, this one is constant this one has the slope, if it is a pole of minus 40 db per decade and the point of intersection is this corner frequency which as we have seen can be found out easily but, how good is this approximation. For the case of a simple real pole or a real 0 there is a factor just s minus z or s minus p, we saw that the approximation was quite good and we also saw that there was a slight error and one can find out what the error is the error at the corner frequency was 3 db and the error 1 decade, 1 octave below and 1 octave above the corner frequency was just about 1 db.

So we could put some 3 points and then join them by a smooth curve etcetera. Now what is the error in this case in this case unfortunately, the error may not be small and therefore one may have to look up a table or a chart, where the gain verses frequency variation of such a pair of poles or 0s or such a quadratic factor is shown and that is where some kind of a normalized or standardized form is useful and we have seen that earlier, when we talked in terms of the damping ratio.

(Refer Slide Time: 06:46)



So the quadratic factor is written as s square plus a s plus b is written as follows it is written as s squared plus 2 delta omega n s plus omega n square and this is rewritten as s plus delta omega n whole square plus some number omega 0 whole square. The quadratic factor can be rewritten in

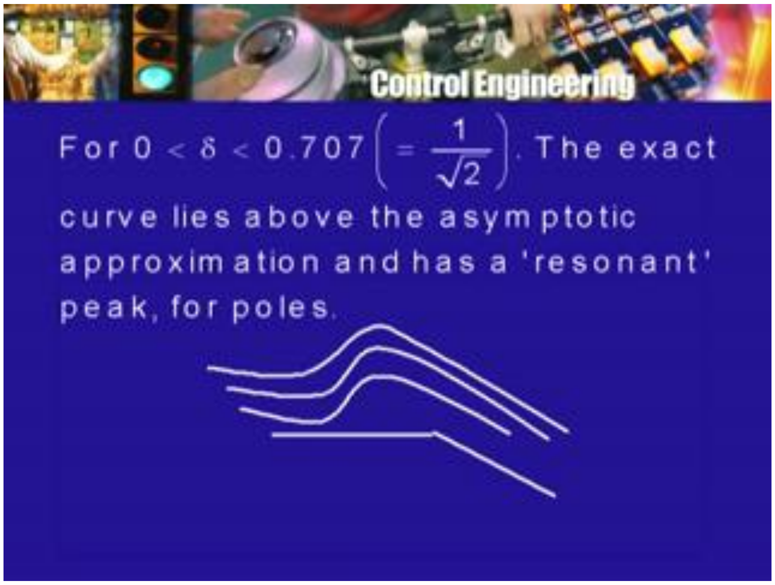
this way and then, we see that minus delta omega n is the real part and omega 0 is the imaginary part of the complex pole or 0 pair. So from this we can find out what delta is of course omega 0 can be found out in terms of omega n square,
omega 0 square is simply omega n square into 1 minus delta square.

So from this we can find out omega 0 and from this, we can find out delta the damping ratio and then, there is a normalized form of this. This is almost in the normalized form except that instead of absolute frequency omega one will look at the frequency ratio omega divided by omega n and you have such curves available in most of your textbooks or in handbooks. So one may have to refer to those to get the correction but one should have some idea of what the error is how much it is and so on.

Now the following can be proved and I would like you to try it out on your own, all you have to do is take the quadratic expression replace s by j omega and find out, what is the variation of that with respect to omega and you can use your calculus here very advantageously. For example, the following can be shown to be true when delta is greater than point 707 to be precise actually it should be 1 by root 2. So when the damping coefficient or the damping ratio is greater than 1 by root and for this case by very choice or by the very fact that the roots are complex the delta is less than 1.

Remember, long time ago we discussed this case of the un damped, the under damped, the critically damped and the over damped second order system. So when delta the damping ratio lies between 1 and point 707 that is it is under damped but the damping is greater that equal to point 707. In that case the exact variation lies in the case of a pair of poles, this is what I have shown here for a pair of poles because the slope here is minus 40 db per decade. In that case the exact curve lies below the approximate true part curve.

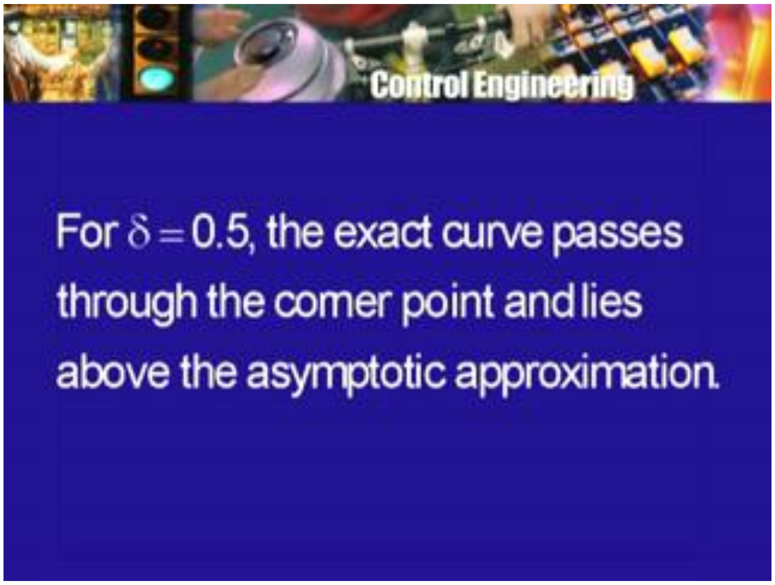
(Refer Slide Time: 09:54)



So the exact curve lies somewhat like this and what the error here is you can find out, the error here is not very large, for delta less than point 707, it is a strange behavior but that is something which does hold for delta less than point 707 and of course greater than 0 because there is some damping, the system is under damped and the damping is less than point 707. In that case the actual curve does not lie below this but goes above this. So it may for example, have a shape something like this and as you decrease the value of delta that is you make the system less and less damped there is a higher and higher peaking kind of effect taking place near about this corner frequency. At the very low frequency end there is not a big error at the high frequency end, the error is also not very big but near the corner frequency, the error can be substantially large and as delta decreases in fact this peaking goes on increasing.

Now this is something you should try out on your own, of course you can consult your textbook where the curves are given but you should you have learnt enough calculus to be able to show this that is take s square plus a s plus b replace s by j omega and then, take the modulus of that, of course you can then you have to take 20 log to the base 10 of that, that is the db gain versus frequency, what is its graph going look like. It depends on the value of delta, for the value of delta less than point 707, the curves lie above these 2 approximations for delta less than point 707, they lie below and for delta exactly equal to point 707, you can find out what happens. There is another critical value which is delta equal to point 5, for delta equal to point 5 the curve passes through this corner frequency although it lies above. As I said one will have to look up this set of curves in order to find out what the error is and made that correction and then sketch the variation of the gain with frequency.

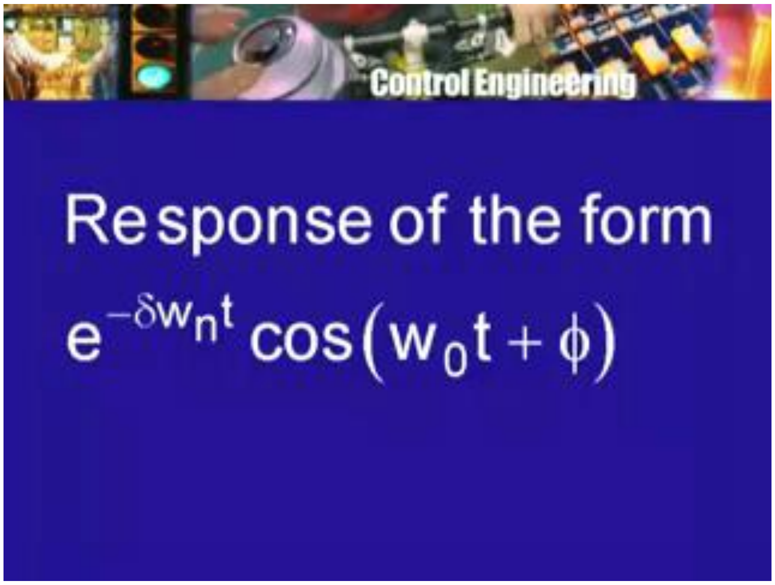
(Refer Slide Time: 11:41)



Of course, as I told you for a quick qualitative understanding or a quick qualitative assessment it is not necessary to take into account the correction and for one more reason because in practice, you will not use a value of delta which is very small and why is that so, delta is called the damping coefficient of the damping ratio and why is it called the damping coefficient of the damping ratio because when this delta is less than 1 that is you have a pair of complex poles of

0s of poles rather, the system responses has an oscillatory component, the transient response has an oscillatory component. However, the oscillations get damped out and the damping ratio or the damping coefficient or constant determines how fast the transient will die out, how fast the oscillations will die out or in other words, they are related to the time constant of the oscillations and of course, I do not have to remind you because you will notice from this s plus delta n that there is going to be e raised to minus delta omega n t multiplier for the response and therefore the time constant will be the reciprocal of 1 by delta omega n.

(Refer Slide Time: 13:00)

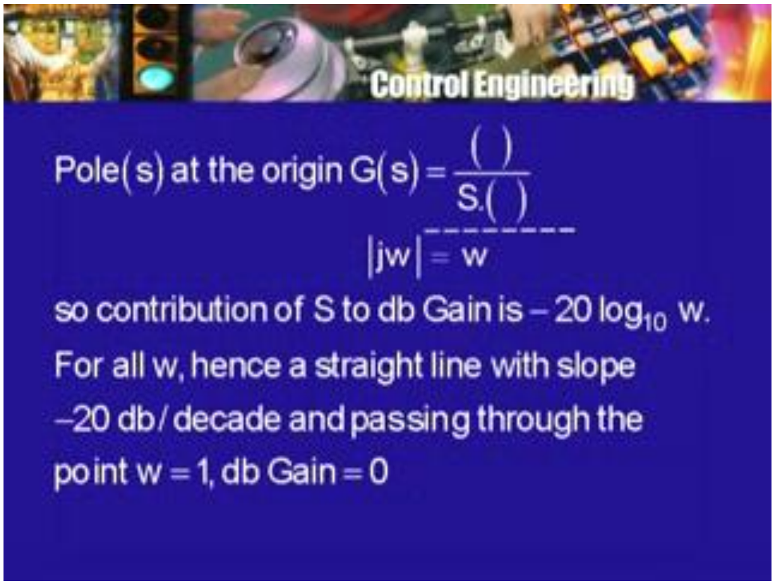


So if delta is small the time constant will be very large, the oscillations will settle down very slowly and from a practical point of view one does not want oscillations to continue for a long time because the system itself may not be operating for a very long time. If you want an operation to be over in 5 minutes, you do not want your oscillations to last for say 1 minute of it that is too a long period therefore generally, the delta is not small certainly not as small as point 5 or less than that it may even be greater than point 707 and when that is the case the error real is not very large.

So one can get a good qualitative understanding or assessment by simply ignoring this error and using the low frequency, the high frequency asymptote approximation. Now if you have a pair of complex poles or complex 0s, what you have to do is as you move along the frequency axis from the low frequency end to the high frequency end, as we did earlier at each corner frequency there is going to be a change of slope and that is what you have to remember, in the case of a pair of complex poles there will be a change of slope which will be minus 40 db per decade, for a pair of complex 0s there will be a change in slope which is going to be plus 40 db per decade, other than that there is very little difference between this and the simple real pole or real 0 case.

So then to summarize the asymptotic or the approximate bode plot that is variation or db gain verses frequency plot consists of what, consist of straight line parts. At the DC end we have a horizontal line then, wherever there is a corner frequency corresponding to a pole or a 0 either real or complex there is a change in the slope of the line. The change in the slope of the line is 20 db per decade for a real pole or a 0 and plus 20 db for a 0 minus 20 db for a pole, for a pair of complex conjugate poles and 0s, it is plus 40 for the pair of 0s minus 40 for the pair of poles and so on, there is one case which needs to be looked into and which can occur in fact it has occurred already. In the problem that we considered namely, there is a pole or very rarely a 0 at the origin. For example if there is a denominator term s then, what now interestingly this is very easy to deal with because if there is a denominator factor s then what are we going to do we are going to replace s by j omega.

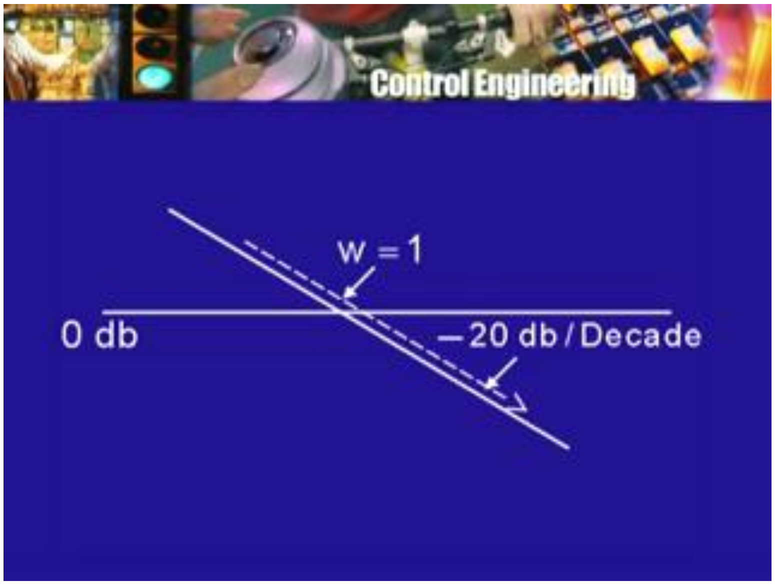
(Refer Slide Time: 15:42)



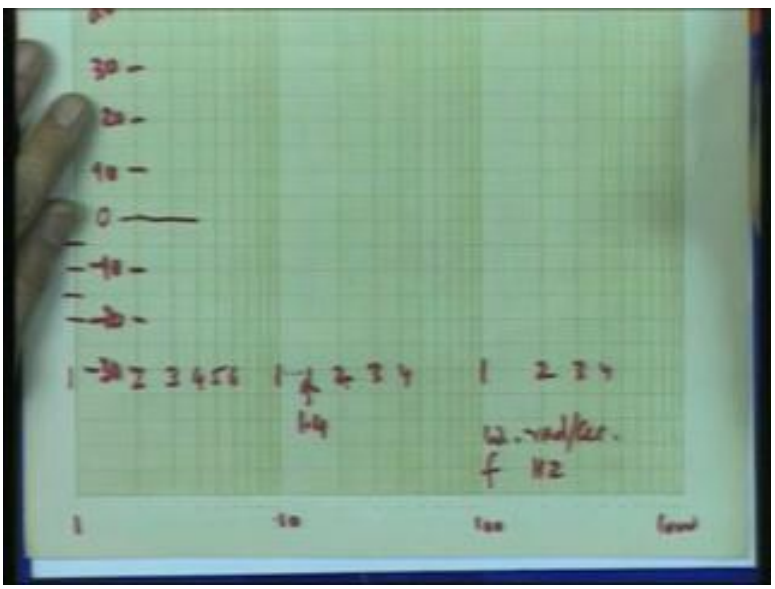
So I have j omega and then I am going to take the modulus of this and what is the modulus of this, it is just omega and 20 log to the base 10 of omega therefore the contribution of the s factor will be simply 20 log to the base 10 of omega with a minus sign, if it is a pole all right. Now what does this look like the plot of this is a straight line with a slopes minus 20 db per decade but continuing all the way into the low frequency end as well as towards the high frequency end. So in other words, there is no horizontal part for this one at low frequency the whole approximation in fact, there is no approximation this is exact it simply consists of line slope in down with a slope of minus 20 db per decade.

So if there is a pole at the origin then, the contribution of this is minus 20 db per decade continuing over the whole range of frequency and of course, if there is a 0 then it will be plus 20, if there are 2 poles at the origin or a second order pole at the origin s squared then this will be minus 40 db per decade and so on. So when this happens of course then, you are not going to start of with a horizontal potion but you are going to start of with a sloping portion already and therefore you have to figure out one point on the approximation. So that from there you can draw the backward going or the straight line towards the low frequency end and this is not very difficult to do and you should try it out with examples from the textbook.

(Refer Slide Time: 16:42)



(Refer Slide Time: 18:14)



Now this is as for as the frequency response, the variation of db gain with frequency is concerned and I had asked you to look up of course, you may find it in your text book but you can go to the stationary shop and obtain what is call the semi logarithmic paper. I hope you have done it, if you have not so you should go and buy this paper and then do the actual plotting for some of the problems given in your text book rather than do it the I have shown you by simply approximately drawing some lines and so on. So here I have brought this semi logarithm paper.

Now how to hold this semi logarithmic paper that is how to use it properly. I have put it this way, now is this the correct way of using it remember the convention is the omega axis or logarithm of omega to the base 10 that is the horizontal axis. The db gain axis is the vertical axis, I showed to you that the markings along the horizontal axis are not uniform because for each decade, you have 1, 2, 3, 4 etcetera which are not equally space, where as the vertical axis is simply db gain and so the markings are uniform. Now the way I am holding the paper, you can see that the horizontal markings appear to be uniform but the vertical markings are not. So I am not holding the paper correctly.

Now I will going the turn paper around now is this is the correct way of using the paper.
No, this is not and I hope you can see the cycles on this paper. Now there are 3 cycles here, so such a paper is called the 3-decade or a 3-cycle paper and I am holding it to some extent correctly that is the horizontal line is correct and the vertical line is correct but what is happening here. Now here is 1 beginning of 1 cycle here is the end of that cycle,
here is the beginning of the second cycle, here is the end of the second cycle and then from second cycle to the third cycle. So this is the 3 cycle paper that I have but look at the vertical lines as I go from here to here, the spacing is increasing instead of decreasing but that is not what is the case because as you start with log 1 which is 0 log 2 is point 3, log 3 is point 4771, log 4 is point 6, the difference between the successive logarithms goes on decreasing and so the spacing is between the vertical lines should go on decreasing and not increasing, many students commit the mistake of using this paper partly correctly but not fully correctly.

So I will turn it around now, now I am using it in the correct fashion, here is the beginning of 1 cycle and on some papers, you will see actually mark there 1, 2, 3, 4 up to 1 again of course, this 1 means not 1 but 10, if this is a 1, so if this is 1 then, this is 10, then again you will see 2, 3, 4. Now this 2 does not mean 2 but if this is 10, this means 20 then the next 1 is 1 but that is not 10 or it is the 100 and finally the last 1 is 1000. So my paper is now ready to use for omega marked as 1 radian per second, 10 radian per second, 100 radian per second, 1000 radian per second or I can use hertz cycles per second also except when I talk about corner frequency I have to be careful about the choice between omega and f, it is omega which is equal to the absolute value or the real 0 or the real pole, not f.

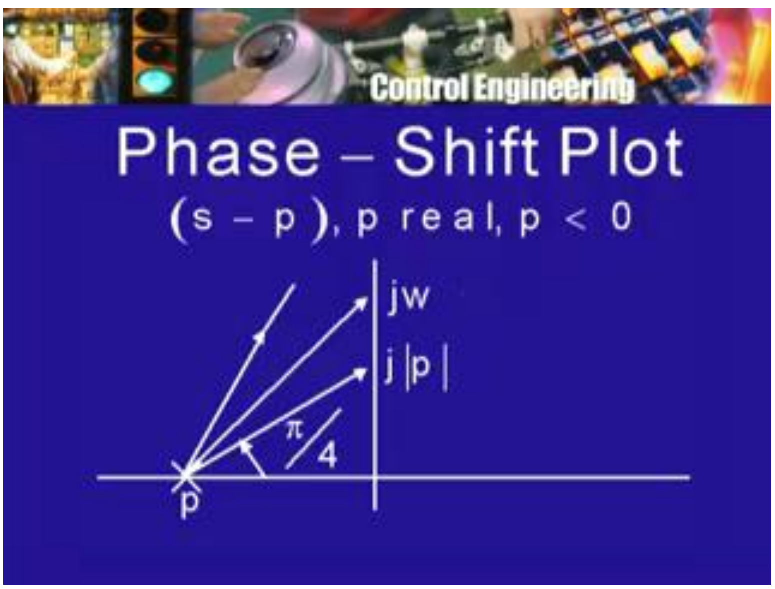
So if I keep that in mind there is no problem and now, we can see that the spacing is going on decreasing here is 1, here is 2 here is 3, 4, 5, 6 and so on. So in fact they are getting very close then the next cycle starts with the 1, this is the 2, 3, 4 and so on. Third one, 2, 3, 4 and so on and of course, there are finer divisions so one can even show say 1 point 4 as something lying between 1 and 2 there are 10 divisions and these again are not uniformly divided, I have to take 4 divisions from this end. So 1.1, 1.2, 1.3, 1.4, this is 1.4 of course, such a paper is not very cheap because it as you can see the printing of it the marking of it is not that easy but it is very useful, what about the vertical axis. The vertical scale is uniform for example here each one of these corresponds to 1 centimeter and they are all uniform.

So if I choose my 0 db line and we have seen that you have to think about it little bit and choose it appropriately, let us say here is my 0 db line then, I have to choose a scale for the db. So I have a choose let us say 1 division or 2 divisions for whatever as 10 db typically it will be of course either 1 or 2 divisions or 4 divisions and so on. So, let us say I choose 2 divisions as 10 db, so this is 0, so this is 10, this is 20, this is 30 and so on, on the negative side, this is 0, so 2 divisions below that is minus 10 db minus 20 db minus 30 db and so on. It is always better to mark along the 2 axis your variables or quantities or at least the scale value.

So here it is better to write db of course we are plotting gain so we need not write db again. On this side you can write omega if you are using marking omega and therefore omega rad per second or frequency, if you have marking frequency then frequency in hertz or cycles per second. So with this semi logarithmic paper then, using Bode's approximation one can get a fairly good idea of how the gain is going to vary with frequency over the range of interest and the range of interest is usually what, the smallest corner frequency, say about a decade below that or one 10th of that the largest corner frequency or decade above that that is 10 times that frequency beyond these 2 you need not worry because in one case the gain is nearly constant or goes up as minus 20 db per decade line for the pole at the origin.

Similarly, the higher frequency end, the gain drafts of very rapidly it almost becomes absolute gain becomes 0 as we saw earlier, when the number of poles is greater than the number of 0s, the gain goes to 0, quite rapidly. So it is not necessary to go way beyond in the omega range. So this is the way the Bode plot for the gain works, now we have to look at the other aspect namely the phase shift.

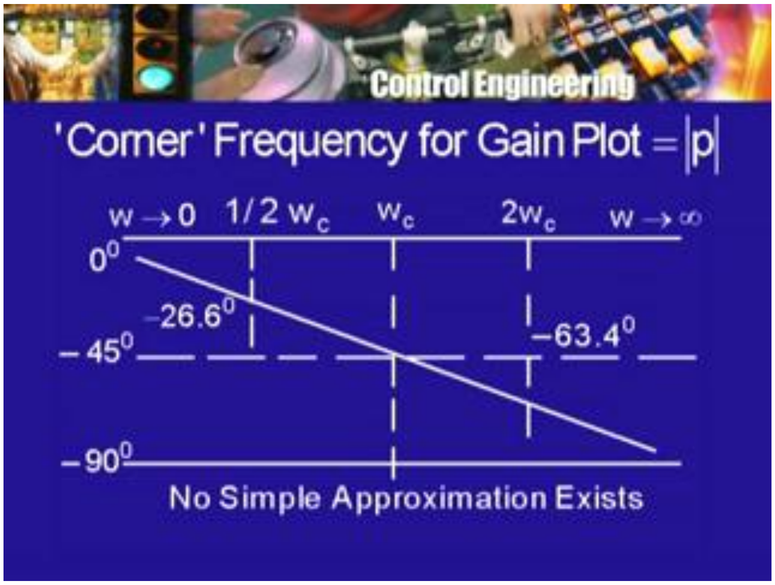
(Refer Slide Time: 24:38)



Now this is where the approximation or it is not really the approximation because one the variation of phase with frequency is such that there is no nice approximation available.
However, a few values of the phase shift can be remembered and used without much difficulty. So for that let us look at the phase shift considerations. So let us say I have a pole s minus p corresponding to the factor s minus p and p is real, once again and let us assume that p real is negative that is I have a pole in the left half of the s plane, as it is usually the case then, what about the variation of phase shift.

We have looked at this earlier, when we were talked about the root locus method and geometrically we saw that the phase shift variation can be found out here is the point j omega here is this pole at p and then, the phase corresponds to this angle and of course this is the factor in the denominator. So the phase is negative and so what is happening as omega increases, when omega is very small the phase shift is nearly 0 degrees or minus 0 degrees that is little less than 0 and a negative number when omega increases and becomes very large, this angle is nearly phi by 2 and therefore the phase shift is minus phi by 2.

(Refer Slide Time: 26:16)



So from this we know that if I were to plot and now I am not showing the scale of the omega axis as logarithmic because there is no log occurring anywhere. So I am just drawing horizontal line to show the omega scale, the phase shift is going to vary from nearly 0 degrees or a small negative value towards the low frequency end.
So omega tending to 0 to minus phi by 2 for omega tending to infinity that is the high frequency end and therefore, if I show this as my 0 degree a 0 line and the phase shift is usually expressed in degrees and not in radians then, 0 degrees line is here minus phi by 2 or of course, I should not say minus phi 2 say I am talking about degrees minus 90 degrees is the line here then, I will be here somewhere for small frequency, I will be somewhere here for a large frequency what is the variation in between.
Now one can easily see that when p is equal to omega or omega is equal to be absolute value of p that is when this line is equal in length to this line that is omega corresponds to the absolute value of the pole which is of course, the corner frequency what is this angle,
since these 2 sides are equal this angle is phi by 4 or 45 degrees and therefore the phase shift corresponding to the corner frequency is 45 degrees. So, if this is my corner frequency for that particular pole then the phase shift here will be 45 degrees and that is between 0 and 90.

So here is one more point on the curve, I am here somewhere for the low frequency and here for the high frequency end. One can try to remember, it is not very difficult but of course one can always look up a table, some intermediate values for example, if I the if this is the corner frequency. So let me call it omega c, c for corner what is the phase shift if I consider a frequency which is 1-half, the corner frequency that is an octave below the corner frequency. Of course, what is going to be the value if you look at the diagram once again now, omega is 1-half p by 2. So we get an angle which is tan inverse of 1-half and you can look it up on your calculator or from the trigonometric table and you will find out that the angle is about minus 26.6 degrees.

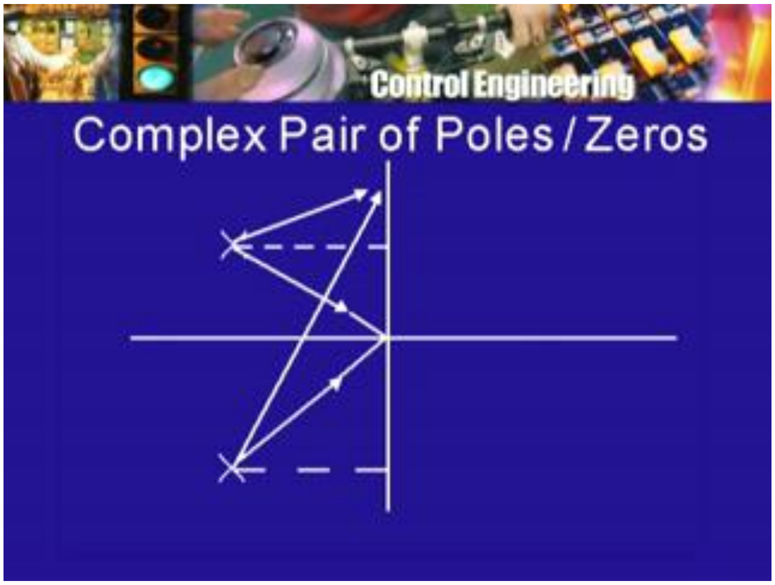
So that is going to be the phase shift of an octave below the corner frequency that is half omega c, the phase shift at a frequency or angular velocity which is half the corner frequency is minus 26.6 degrees what will be the phase and octave above the corner frequency of course, from trigonometry we know that it will be tan inverse of 2 and tan inverse of 2 will be the compliment of this angle 26.6. So what will it be it will be minus 63.4 degrees. The phase shift corresponding to a frequency and octave above the corner frequency will be 63.4 degrees, 63.4 plus 26.6 adds up to 90 degrees, what about a frequency which is a decade below or a decade above the corner frequency.

Now that will correspond to tan inverse of point 1 or tan inverse of 10, now that tan inverse of point 1 is pretty close to 0 although it is not 0 of course, it turns out to be about 5.7 degrees. Remember, when we have plotting we are not really going to be able to plot things very accurately and that is not really the idea, we have to make subsequent calculations, those calculations can be made with the help of a calculator and of course, now there are programs which will draw the Bode plots for you. So you can even get the exact Bode plots plotted with the computer program but the computer program will not be able to do the design for you, you have to use your intelligence to use the computer program.

So the all these things are useful, 1 decade below the corner frequency phase shift is minus 5.7 degrees and therefore 1 degree above, 1 decade above the corner frequency, what will be the phase shift it will be the compliment of this. So it will be minus 84.3 degrees 5.7 and 84.3, they will add up to 90 degrees right. So if I have this corner frequency omega c plotted already on the omega axis, I put down this as minus 45 degrees octave above and below I put down these 2-phase shifts a decade above and below I put down these 2 phase shift and then through them I can draw a smooth curve and that smooth curve looks somewhat like this and it is not possible to approximate it by a pair of straight line segments or even 3 straight line segments, one does not get a very good approximation that way, unlike the db gain case where you get a good approximation by means of straight line segments.

Now for each pole we will get a phase shift variation like this of course the different poles will have different corner frequencies. So these curves will be shifted horizontally along the frequency axis, for each 0 the phase shift instead of being negative, it is positive so it will vary from 0 to 90 degrees rather than from 0 to minus 90 degrees, what if you have a pair of complex poles or a pair of complex 0s then, what do you expect? You expect that at the low frequency and the phase shift will be 0 degrees at the high frequency and the phase shift will be 180 degrees in between what is going to happen one can try it out for a pair of complex poles and see, what is happening and is not difficult to visualize.

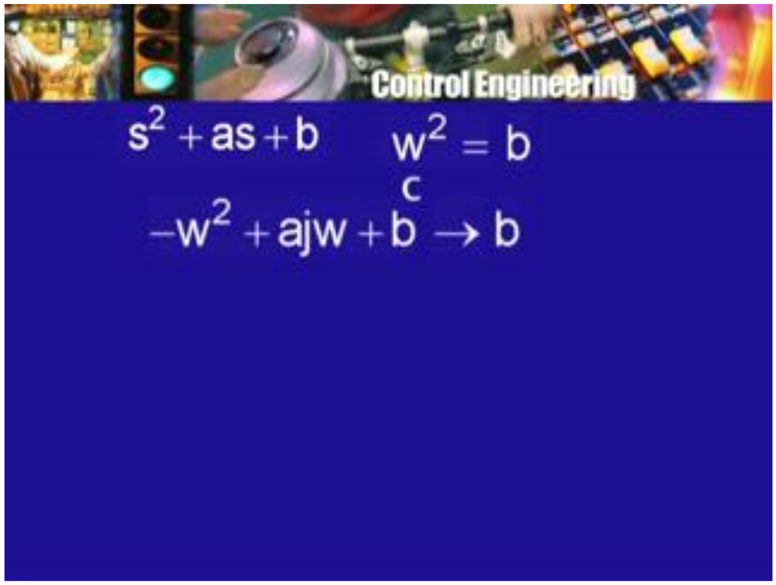
(Refer Slide Time: 32:19)



So here is the pair of complex poles I have at a value omega very close to 0, so what is happening these 2 vectors are contributing angles which are equal in magnitude but opposite in side. So the total contribution is 0 so that is why the low frequency phase shift is 0 on the other hand when the frequency is very large each one of them is going to contribute an angle which is nearly 90 degrees. So the total is 180 degrees and since, it is a pair of poles the total phase shift will approach 180 degrees, what is going to happen in between, in between of course the phase shift will go through various values like 10 degrees, 20 degrees 50, 90, 100 and 20 and so on, what will happen at the corner frequency. Suppose, I have the corner frequency if this is the corner frequency no what is the corner frequency corner frequency corresponds to the distance of the pole from the origin.

So when I have that I have to plot a point here is corresponds to the corner frequency for that then, I have to find out what will be the net phase shift and I will leave it to you to find out for yourself, what is the phase shift corresponding to the corner frequency and if you are about to be a good engineer, you can make a good guess as to what the phase shift will be at that corner frequency. Did you make a guess about the phase shift at the corner frequency for the complex pair of poles or 0s case, the answer should be minus 90 degrees for a pair of poles or plus 90 degrees for a pair of 0s and one does not have to really do any trigonometrical calculation at all because we are looking at the quadratic factor s square pus a s plus b, what did we do replace s by j omega at the low frequency end, it is only b that matters, at the high frequency end s squared gave rise to omega square and that is what mattered and at the corner frequency you had omega square equal to b.

(Refer Slide Time: 34:11)



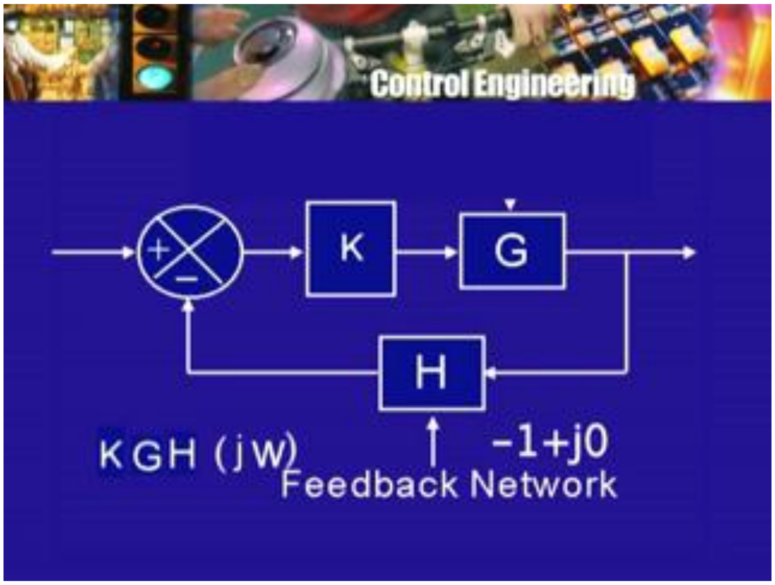
So the corner of frequency was actually determined or defined by omega square equal to b right. Now for a frequency which is neither very low nor very high what does this look like. Well, s square is minus omega square plus a j omega plus b and let me write this now as omega corner omega corner square is b. So when omega is omega c at the corner frequency, this minus omega square and b just cancel off. So the real part become 0 and the imaginary part is just this a omega j into a omega.

Now a purely imaginary in number, what is its argument or the phase corresponding to it 90 degrees. So it is as simple as that of course, if when it is in the a pair of poles, this is in the denominator so the phase shift is minus 90 degrees and if it is a pair of 0s, it is in the numerator, so the phase shift is plus 90 degrees. Now, once again in this case one can make some calculations of phase shift at frequencies which are an octave or a decade below or above the corner frequency but once again, things are going to depend on the damping ratio delta.

So in practice, one could of course make reference to a set of curves, normalized curves which are available in some handbooks but one may simply prefer to compute the phase shift as and when required rather than use any set of curves or remember, a certain set of numbers. However, 90 degrees, 45 degrees these are numbers which are not very difficult to remember and nobody, none of us should for get that tangent of 45 degrees is an angle of 45 degrees or phi by 4 radians is 1 or conversely or inversely, tan inverse of 1 is phi by 4 radians or 45 degrees. I am sure, you would not forget it for the rest of your life. Electrical engineering and particular makes very good pretty heavy use of there trigonometric functions the sine and the cosine in particular but also the tangent and the co-tangent and other functions.

So of this much one can easily remember, for the complex case 0 degrees, 180 degrees at the low frequency high frequency end, at the cornet frequency minus 90 degrees that is not very difficult to remember for a real pole or a 0, 0 and minus 90 or plus 90 at the 2 ends and at the corner frequency minus 45 degrees or plus 45 degrees, all right. So now going back to our original problem, you have a closed loop control system, where, what going back to the old diagram, here is G, here is the feedback transfer function H, the difference device, the reference input R and there is this gain K that corresponds to an amplifier of some sort and a converter of energy from one form to another or changing a small voltage at a low power such as the reference voltage into a large voltage at a large power such as the DC generator that drives the motor that is the represented by K and here, we have G and H.

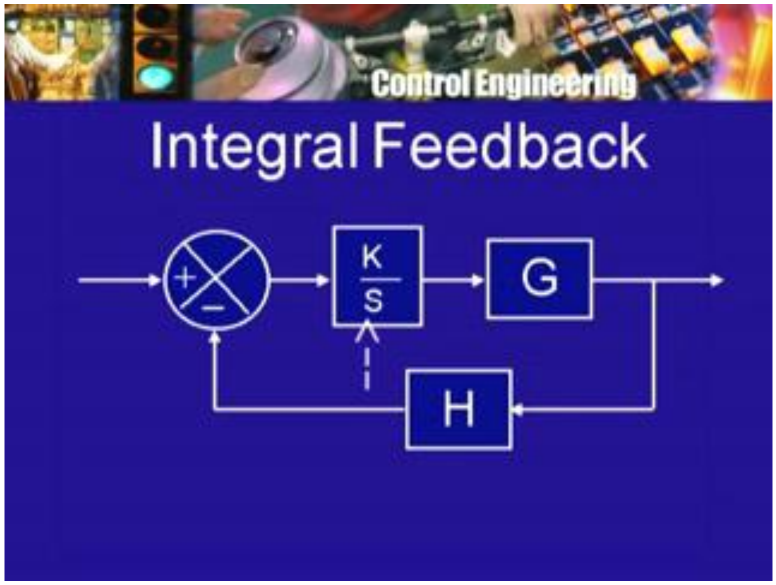
(Refer Slide Time: 37:23)



Now, as we saw in we started our started our discussion on the Nyquist criterion or the Nyquist stability method of determining stability, we are looking at G and H the product along with K and this is called the loop gain and this as a function of frequency is the polar plot or the Nyquist plot and from that we can infer as to what will be the range of values of K, if any for which the closed loop system will be stable and you remember that the point minus 1 plus j is 0 plays a important role there also the idea of the Nyquist contour be image contour and the encirclement of the point minus 1 plus j 0 by the image contour.

We also saw that from that kind of diagram, we were able to figure out that when we have proportional feedback for our motor control system, there was H was just a constant coefficient corresponding to the tacho generator G had 2 first order transfer functions, the armature circuit transfer function and the motor mechanical circuit transfer function, the system was stable for all positive values of k, there was no question of instability.

(Refer Slide Time: 39:26)



However, from the root locus method we saw that the system can become oscillatory. But when we introduce an integral element into this feedback scheme namely, this K is no longer just K but it is K divided by s that there is an integrator the difference is not just multiplied but it is integrated then, there was a difference. The system then was such that it would become unstable for values of K greater than a certain critical or marginal amount and this is what we could see from the Nyquist locus or the Nyquist plot and the Nyquist criterion.

Of course, we had seen that earlier using the root locus method. So this was another approach but now as I said, the advantage of the Nyquist method is that if you do not know the transfer function but if we can make frequency response measurements then, we can plot the Nyquist plot. Otherwise, we have to find out what the transfer function is and then calculate the values of the gain and phase shift at the various frequencies either approximately that is the Bode plot or precisely by using the calculator or program.

So the Nyquist idea can be used, when we have experimental data for the frequency response but we it may not know the parameter values of the system and therefore the transfer function of the system of the loop transfer function GH, also it suggests some methods of improving the performance of the system by thinking in terms of the polar plot and by thinking of changes in it that will take place, if we do certain things. The simplest thing to do is of course change the gain K and that was the case we studied first, for the case when we had just proportional feedback but the amount of the proportionality constant K here could be changed. The second thing that we did was not just have K here but change the transfer function 2 K by s that is introduce an additional transfer function here 1 by s that corresponding to the integrator.

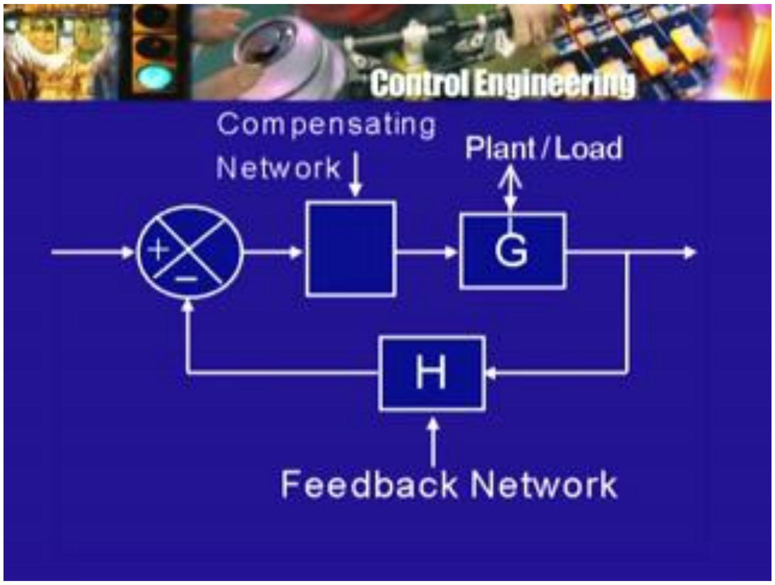
Now that resulted in certain changes and certain consequence of that such as the system could become unstable for a value of K greater than some critical value. Now of course, as I told you earlier all this work was done primarily not for control systems but for amplifiers and for amplifiers, the idea of feedback was used to stabilize the amplifiers with regard to parameter variations. This is not the kind of stability that we are talking about right now, when we talk about the Nyquist stability criterion that stability refers to the transient response that is the transient response becomes more or less oscillatory or there could be the transient response that really is not transient, it does not die out, it is just keeps on oscillating forever that is of course, the ideal case but that is the limit towards which one may go that stability which is what is studied by the Nyquist criterion is different from the stability that I have talked about right in the beginning. The purpose of feedback was to make the system less sensitive to parameter variations.

Now, when people introduced feedback into amplifiers for the purpose of making them less sensitive to parameter variations and that sense of stability was used. They though of various things which could be done because after all, you were designing amplifier and not a control system there was not anything like a DC motor here and a load which the DC motor was driving. This was not anything like a tacho generator there, which was measuring the speed which was on the same shaft and producing a voltage proportional to the speed, no everything was an amplifier or parts of an amplifier or what are called stages of an amplifier and in the 30's and 40's of course, as you know vacuum tubes had already become very common place and a large number of various kinds of vacuum tubes had been developed for various amounts of power and therefore the amplifier design thing was the problem for which this method was first applied.

Of course, it was realized that vacuum tube is not an ideal device as one would like it to be that is it simply amplifies output voltage is proportional to input voltage exactly, no matter what frequency of the input you have, no there are capacitive if x, there are of course resistive and inductive effects also as a result an amplifier has a frequency response there is a behavior with frequency which is not ideal ideally is the gain of an amplifier should be constant for all range of frequencies and the phase shift introduced by the amplifier should be best thing is to have it to be 0 degrees but if not it should vary in a particular way with respective frequency for some applications but practical amplifiers do not have that behavior because of capacitive resistive and inductive effects.

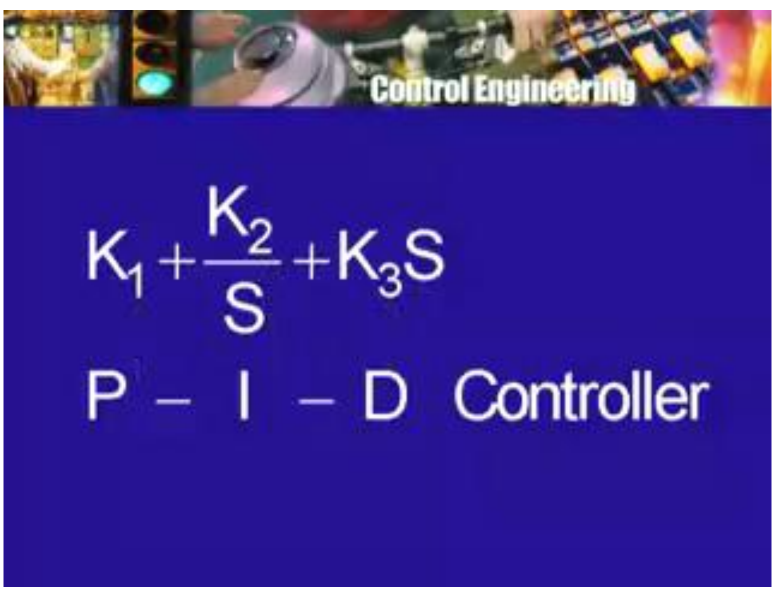
However, it was not difficult to think of introducing some modifications in the amplifier circuitry and to see, what would with the effect now that is where the idea of a compensating network was introduced and these were actually physical networks because the amplifier itself was nothing but a vacuum tube network. A network consisting of vacuum tubes resistors, capacitors, power supply and of course signal source to drive the amplifier.

(Refer Slide Time: 44:44)



Now this amounts to from the control system point of view of doing the following here is G and here is H, for a control system of course I have this block G, what was this block called we had called it a plant or a system which is being controlled H is the feedback element in the configuration and preceding this G was something which was powering this plant and that of course was that represented by the gain K in the simplest case. In place of this gain K, you could have a something which has a transfer function which is not just a constant K or it may not be just the transfer function K by s, K is a pure amplifier, K by s is an integrator amplifier.

(Refer Slide Time: 45:45)



Of course, one can equally then have K into s that is a differentiating amplifier but we could have something here which has a transfer function which is not just a constant K or K by s or K into s that it is not proportional or integral or derivative or even a sum of them, the so called PID controller but it could be something different. Now it was natural to think of electrical networks and then look at their transfer functions and then, see what can be done.

Now such compensating networks or compensators because one was looking at amplifiers at performance of amplifiers in terms of frequency response. It was natural to think of compensating networks and their frequency response that is how do the compensating networks behave as the input frequency varies and because of this several kinds of networks were identified and these are networks which are called some of them are phase lag networks that is they are networks which introduce a phase lag from input to output and of course there is a change in the amplitude or there is a gain or there could be an attenuation.

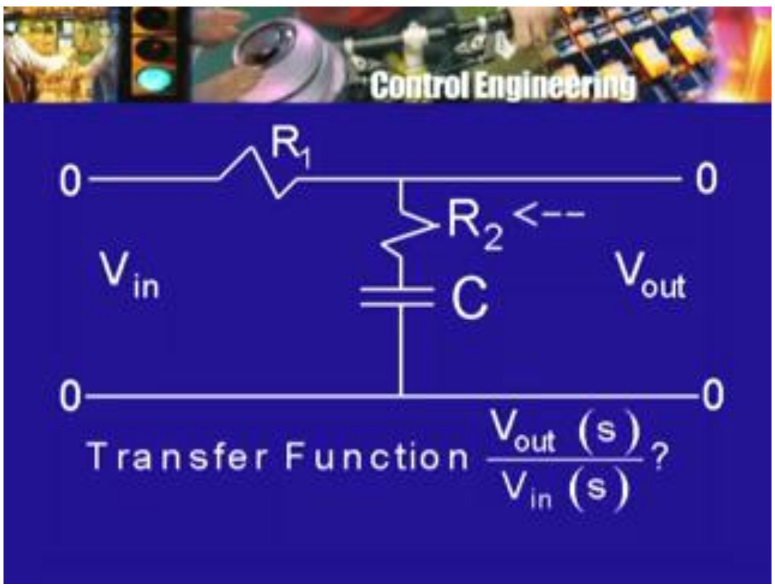
(Refer Slide Time: 46:57)

So there are phase lag compensating networks. On the other hand, the network may introduce a lead of phase that is the output phase, output signal of the amplifier will have a phase which is leading the input, phase of the input the amplifier introduces a phase lead and then of course, you can think of a combination behavior where for some part of the frequency response, there is a lag and for some other part the remaining part of the frequency response of frequency range, there is a lead and in this case therefore, one talks about a lag, lead network or one could even talk about a lead, lag network.

So when amplifier design was in its hay day, in the 1930 and 40's people tried all these kinds of networks phase lag, phase lead, lag, lead, lead, lag networks and they were actual electrical networks consisting of as for as possible resistors and capacitors, resistors alone will not give you a variation with respective frequency. So you needed capacitors in the beginning one also did not mind having inductors but then, one wanted to do without inductors, inductors being heavy etcetera. Eventually, when the transistor was introduced and that was in the late 40's one could even have an electrical network which consisted of some transistors themselves and such is today's operational amplifier.

So a compensating network may be built up not from just R and C but from RC and operational amplifiers, in fact that is what is very commonly done today and so, you may have op amp has components and that opens a whole new area of compensating network design. You can achieve a large number of transfer functions with the help of op amps and RC elements. Of course, our aim is not to go into amplifier design at all so, certainly we are not going to look at these various kinds of networks and so on but I will show you a very few and simple networks and I will leave it to you to work out there behavior. For example, here is a network here is the input pair of terminals, here is the output pair of terminals of this network and then, here is 1 resistor and then, we have a series combination of a resistor and a capacitor.

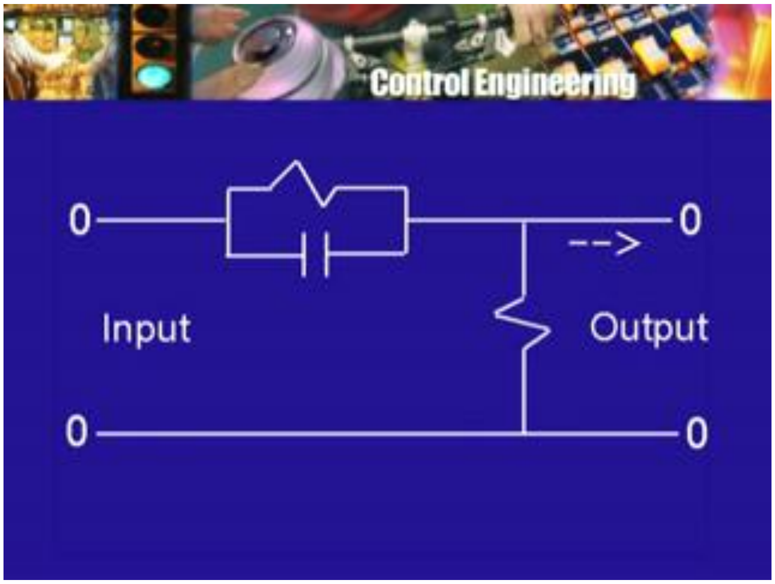
(Refer Slide Time: 49:33)



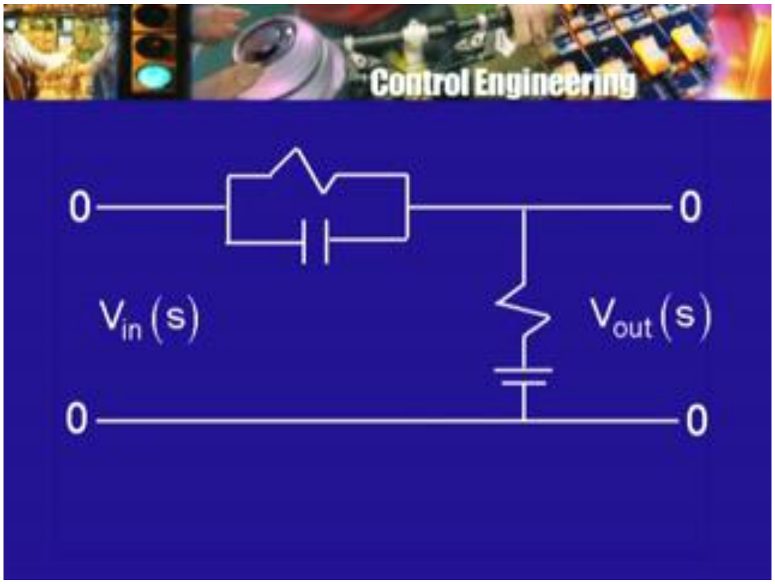
So this is the in, this is out okay and we will assume that at the output there is no loading that is there is no current going out of the output terminals. So we have simply a voltage existing here so here is v out and here is V in. Now the transfer function of this will be V out divided by V in of course, the Laplace transforms, the ratio of the Laplace transforms, the contribution which arises without considering the initial voltage or the charge on the capacitor. So if this is R 1, this is R 2 and this is C find out what distance for function is going to be V out s divided by V in s for this very simple circuit. Find out whether, it will give you lagging phase that is output phase will lag the input phase or whether it will give you a leading phase.

So this is one example of a network, another example of a network has instead of a resistor from the input terminal, a parallel combination of resistor and a capacitor and then across the output however, we simply have a resistor. So this is another network, this is in, this is out again we will assume, that there is no current flowing here find out the transfer function V out (s) divided by V in (s) and find out whether it will introduce a phase lag or a phase lead. So this is one structure, this is another structure and so a third way will to be combine these 2 structures and that is the following one. Here, we have a parallel RC combination and then, a series RC combination and do the same thing of course, the resistors need not be equal, the capacitors need not be equal.

(Refer Slide Time: 50:52)

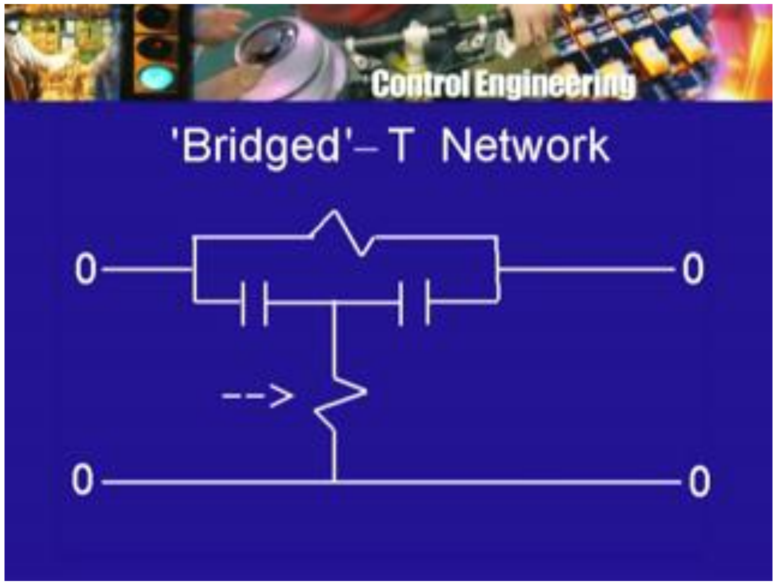


(Refer Slide Time: 51:30)

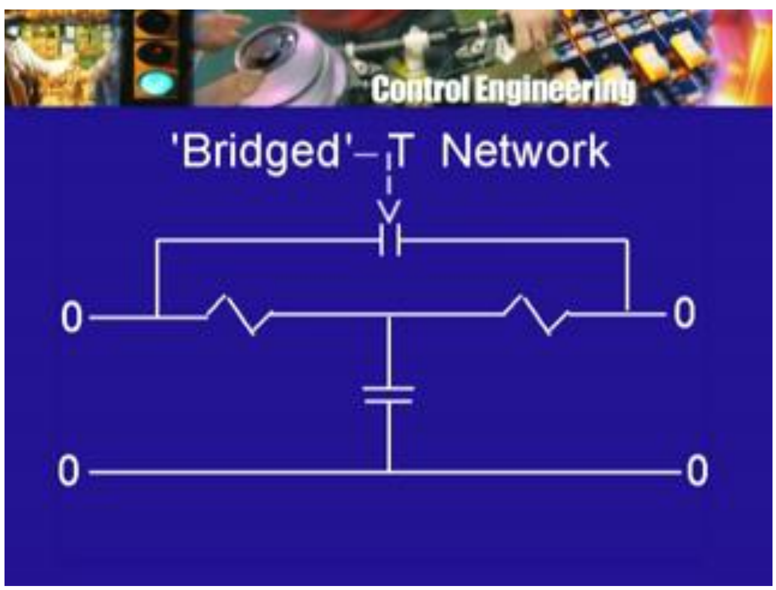


So what is the transfer function of this network V out (s), V in (s), what is the ratio, again under the assumption that there is no loading by this network of the subsequent amplifier or whatever follows it. This of course is going to be different from this case or from this case and you can see, if this is again where some kind of electrical engineering judgment you should build up. Here I should expect a first order kind of transfer function that is may be a pole or a 0 or a pair of poles and 0s but first order not second order.

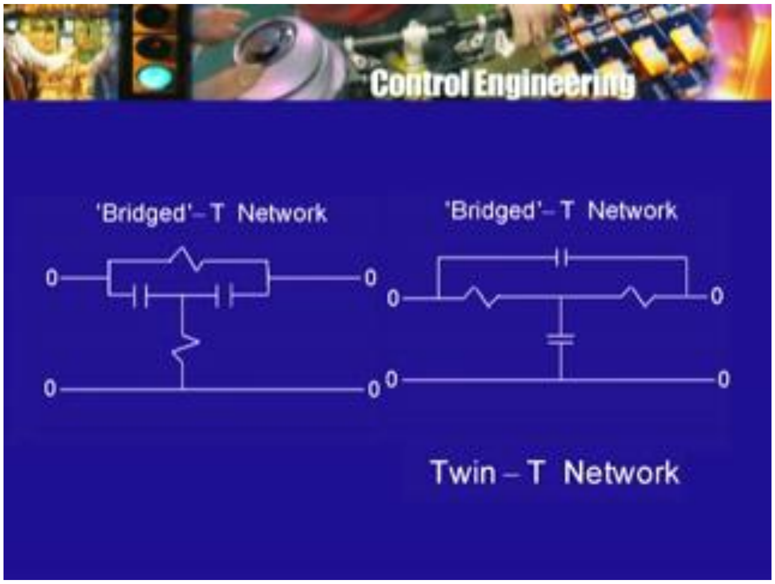
(Refer Slide Time: 52:33)



(Refer Slide Time: 52:56)



(Refer Slide Time: 53:32)



Similarly here, here because there are 2 capacitors I should expect a second order transfer function that is the numerator may be of second order or denominator may be of second order or may be both will of second order, find out which one it is and then, 2 more examples I will quickly sketch one is what is called the bridged T network and there are 2 varieties of it, here is the C, C, there is an R and there is a R here. One end is the input, the other end is the output this is called a bridged T network, this is the part of it and this is the thing that bridges it.

So this is the bridge T network, I can think of another bridge T network in which I interchange the R and C. So here is the R another R, there is a C which is bridging it and there is a C here and now again, one end is the input the other end is the output. So I have another version of the bridge T network again, I have 2 capacitors therefore I expect that the transfer function will be of second order, may be 2 poles 2, 0s but something 2 find out for yourself what it is going to be. It is possible to think of one more network which is a little more complicated which is called a twin T network, if a T network is what this looks like, this looks like a T right, this capacitor arm another capacitor arm and the resistor arm, this is a T, this is another T. Now you can connect these 2 T's in parallel, so to speak in parallel at the input terminals in parallel at the output terminals, you get a twin T network. You can find out what will be the transfer function of a twin T network.

Now, of course it will be a little more complicated it will involve more calculation and so on but you should try this out on your own before you look up any expressions which are given in your textbook.