Control Engineering
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Lecture - 42

So let us see what the so-called asymptotic Bode plots look like. These are approximations to the exact frequency response and more than that it provides a quick method of sketching the frequency response and as I told you earlier, it gives you good qualitative idea of what is going to happen? It is not a very good quantitative method, it is not a substitute for exact calculations of gain and phase shift. So with this understanding let us go ahead. So suppose, we have a numerator factor which because of is being a 0, we write as s minus z and remember that I am looking at the case, when the z is a real number that is this is a real 0, the 0 may be in the left half plane or it may be in the right half plane.

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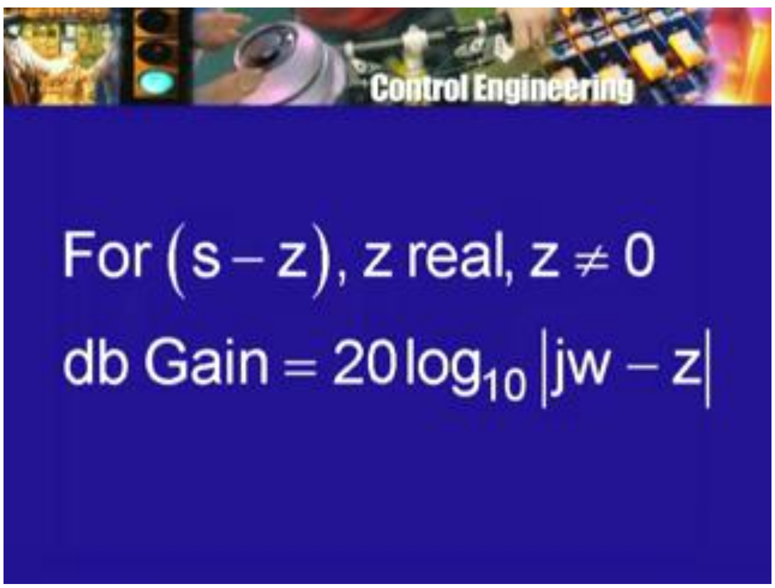

So I am assuming that z is not 0, in this case when we look at the frequency response part corresponding to this term we replace S by j omega. So I have j omega minus z and the modulus of that is the gain or the absolute gain as one might call it and 20 log to the base 10 of that is the db gain and the angle or the arguments of that is the phase shift, right. Now if we look at the gain first that is the magnitude and the db gain first then, as we saw earlier there are 2 approximations, one is as omega tends to 0. The gain approaches the low frequency gain, so called low frequency or dc gain which is given by 20 log to the base 10 of the absolute values of the real number z. Of course, depending on the value this real number z if it is less than 1 then, this will negative if it is greater than 1 it will be positive, if z just happens to be 1 for example, there is a 0 at plus 1 or minus 1 then, this will be equal to 0. So this will be some number and remember the unit for this is set to be decibels.

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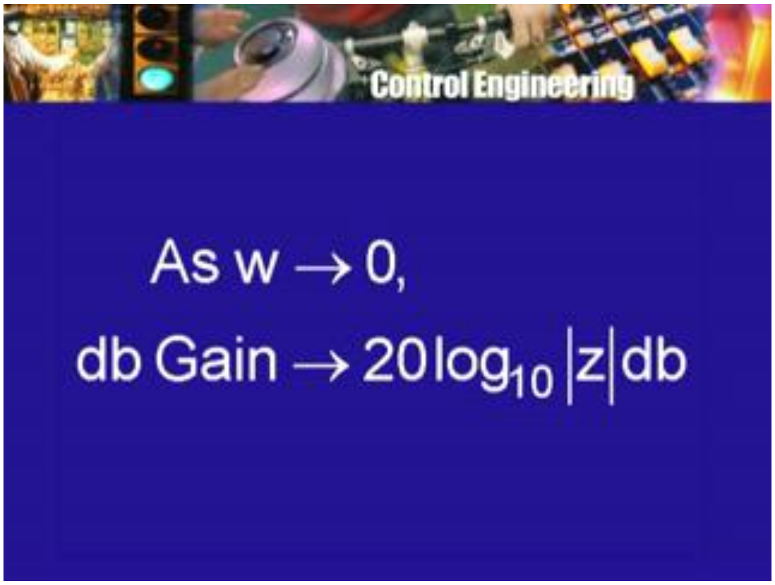

So this is the dc gain or the limit as omega tends to 0. Now on the other hand we can look at the case where omega tends to infinity and that is when from j omega minus z, we can ignore z and all we are left with is omega and therefore 20 log to the base 10 of omega is what we get as the db gain and therefore, if this is plotted against log 10 to the base of omega and I said that you can think of log 10 to the base of omega as a new variable, let us call it u then, this is simply 20 u and therefore the plot of the db gain, this is the db gain verses u will be a straight line. It will be a straight line passing through the origin of u plane or u equal to 0 and it will have a slope of 20 and remember, now we are looking at u which is logarithm to the base 10 of omega.

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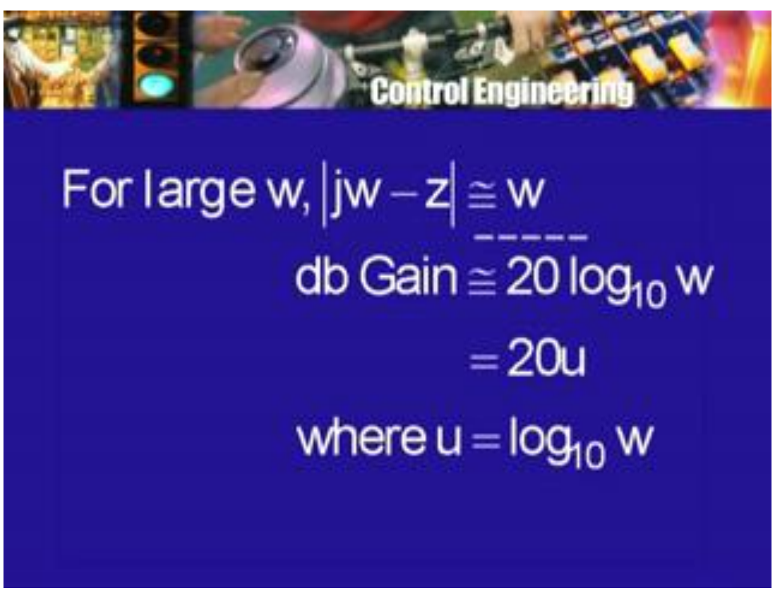

So when omega is changed by a factor of 10 that is you multiply or divide it by a factor of 10, u changes by plus 1 or minus 1 therefore the db gain will change by plus or minus 20 db, according as we increase or decrease omega and therefore this slope is referred to as 20 db per decade that is for change in frequency by a factor of 10, there will be change in the decibel gain of 20 decibels because we are looking at the 0, when the frequency increases the gain increases, when the frequency decreases, the gain decreases. For every decade change and remember, these decade change does not mean an amount of 10 hertz but it is a factor of 10. So 1 hertz to 10 hertz or 1 hertz to 10 hertz that is a difference of 9 hertz but it is a difference by a factor of or it is multiplication by a factor of 10.

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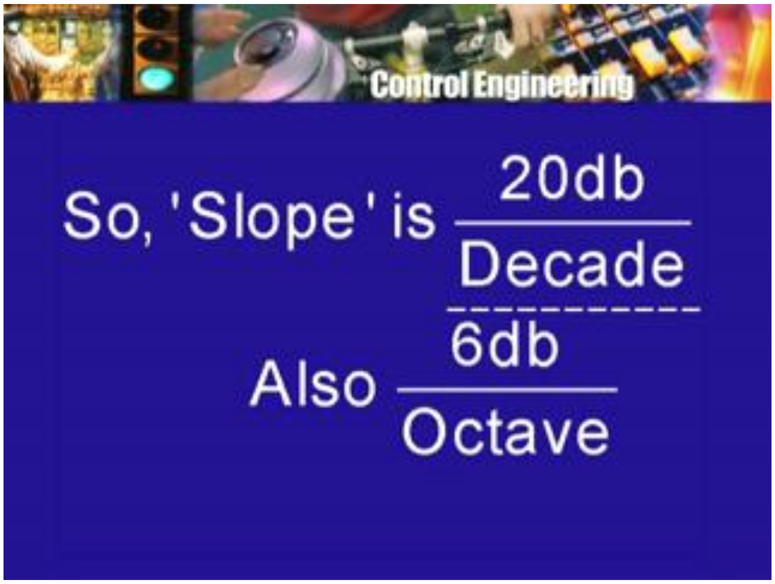

So, 10 hertz is said to be 1 hertz, 1 decade above 1 hertz. Similarly, 10 hertz to 100 hertz, the difference is 90 hertz but the relationship in terms of ratio is by a factor of 10. So we say that 100 hertz is 1 decade higher than 10 hertz and now, what therefore this is that the gain when the frequency is changed from 1 hertz to 10 hertz, the decibels gain change will be by 20 decibels.
When, if you change from 10 hertz to 100 hertz it will also be by the same amount namely 20 decibels. So we will speak of the slope of this line or the change in db gain as related to the frequency but frequency expressed logarithmically, log to the base 10 of omega as 20 decibels per decade and as I told you earlier, in music one talks about not decade but octave that is the doubling of frequency. So, if omega is changed by a factor of 2 not by 2 hertz but by a factor of 2 that is if omega was 1 hertz, now we consider 2 hertz or if it was 100 hertz, we consider 200 hertz. So there is change in omega factor of 2 then because log to the base 10 of 2 is how much is about point 3, therefore the change in the db gain will be about 6 db and therefore same slope is also referred to as 6 db per octave.

So 20 db per decade and 6 db per octave are the same rate of the change of decibel gain with change in frequency but change in frequency is talked about in logarithmic terms that is in terms of multiplication by a factor and not by a difference by a certain amount okay. So with this then the graph of db gain verses u will be a straight line but it will be a straight line passing through the origin corresponding to u equal to 0.

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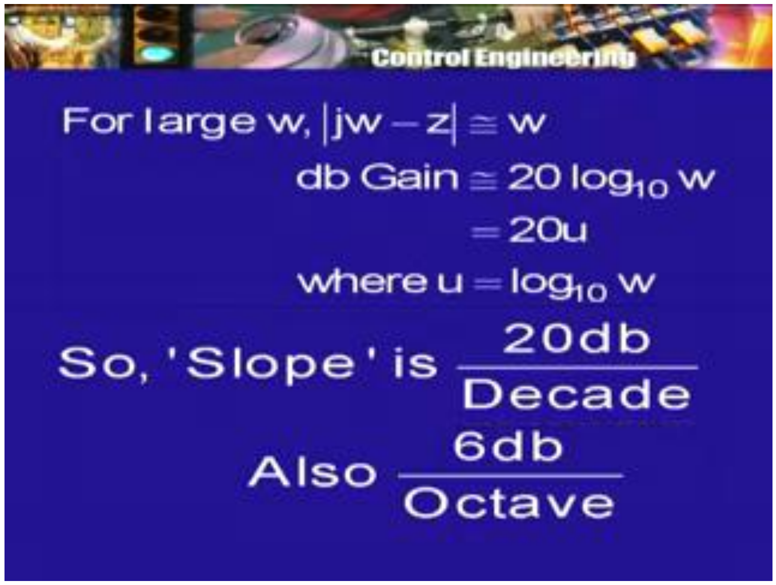

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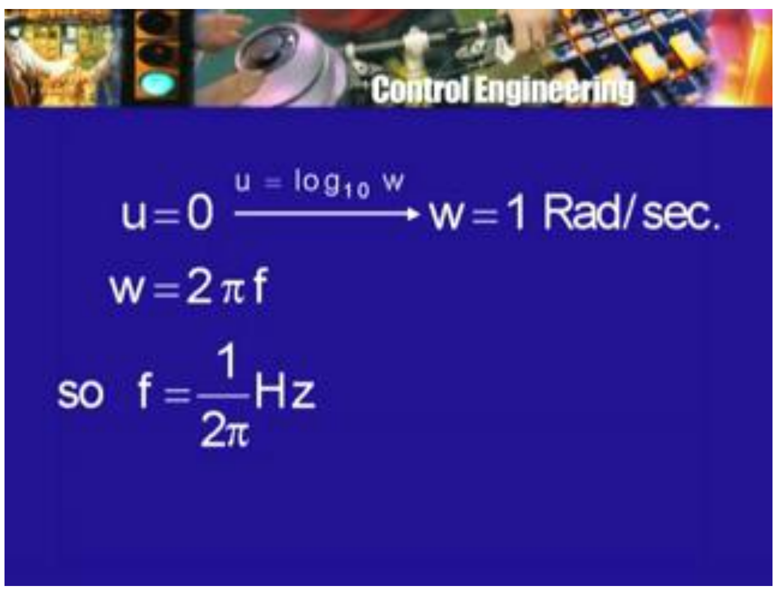

Now, unfortunately u equal to 0 or I should not say unfortunately, you have to keep in mind that u equal to 0 is not 0 frequency because u is what, u is log to base 10 of omega. So, u equal to 0 corresponds to omega equal to 1. Now depending on how you are plotting the graph whether you are plotting it in terms of hertz that is actual cycle per second, frequency as such or you are plotting in terms of the corresponding angular velocity omega which is 2 phi f and remember, we have to substitute s by omega not s by f. So if you want to put f there then you have to replace s by 2 phi f.

So accordingly then, one may plot radians per second or hertz that is cycles per second, so accordingly the u equal to 0, u equal to 0 will correspond to omega equal to 1 and therefore one radians per second or if you want in terms of hertz then this will correspond to 1 by 2 phi hertz. So when the angle of velocity is 1 radian per second or the corresponding frequency is 1 by 2 phi hertz, this high frequency approximation as it is called will give you a value of 0 db and therefore it will pass through the point 0 db and omega equal to 1 radians per second.

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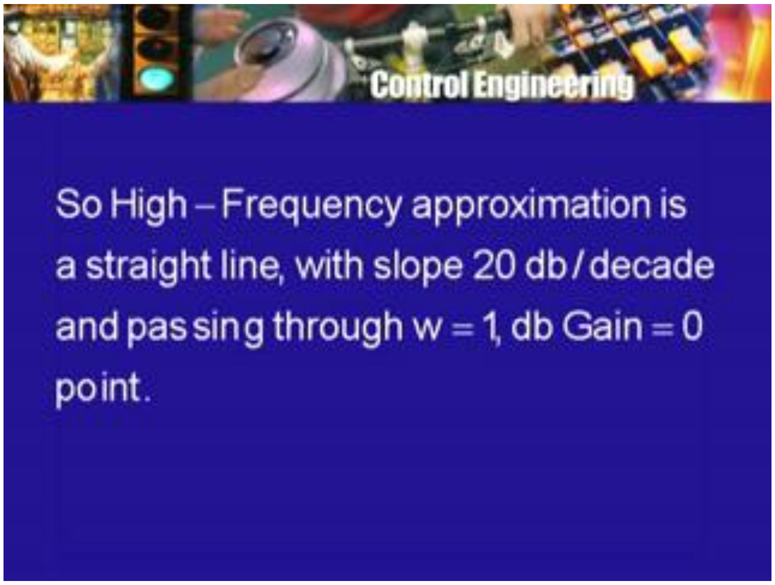

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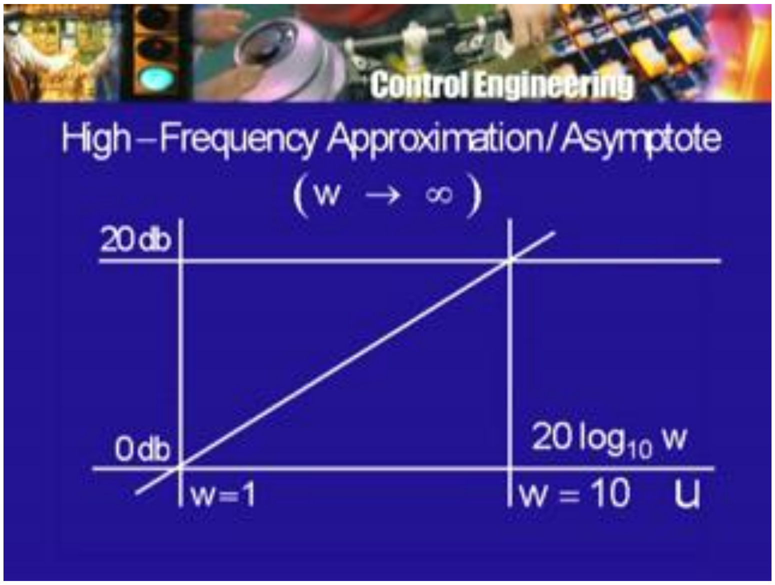

So for a moment if I just work with omega rather than f on the horizontal axis then, let see here is my horizontal axes and I am going to put here both omega as well as u although when we use the semi logarithmic paper, the graduation along the horizontal axis is such that one should think of not omega but u, if you think of equal increments equal increments equal movement along the horizontal axis correspond to equal changes in u and correspond to multiplication of omega by the same factor.

So let us say on this graph paper I have put down here a point which corresponds to omega equal to 1 and on the paper you do label it as omega equal to 1 although in terms of u, it is equal to u equal to 0 then, here is point. The next cycle or decade this is omega equal to 10 and this will correspond to u equal to 1 and let us say, the vertical scale is such that this line corresponds to 0 db and this line corresponds to 20 db then, the high frequency approximation or high frequency asymptote as it is called is a straight line passing through this point when omega equal to 1, u equal to 0 and having a slope of 20 db per decade therefore if I move from omega equal to 1 to omega equal to 10, this is change of 1 decade by a factor of 10, there should be a gain a change in db gain by an amount 20 and therefore the line will look like this.

So this will be the high frequency asymptote that is this will more and more accurate as omega increases. So the graph of the gain as a function of the frequency or a logarithm of frequency is of this kind at high frequencies, on the other hand at low frequency it tends to a constant and therefore it just a line.

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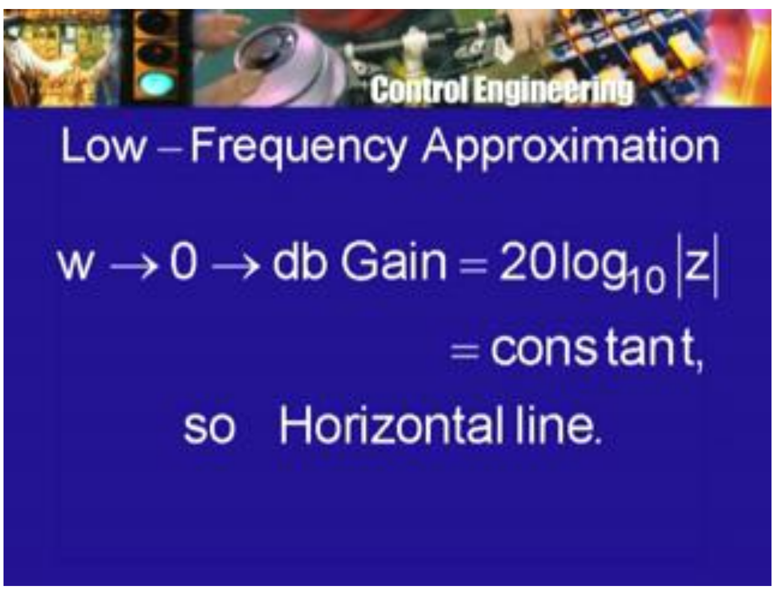

Now one can check that now I am not going to show the scales. So I am just going to draw the curves or the approximations without showing the scales. So let us say here is the low frequency approximation which is just a horizontal straight line because the db gain is consistent equal to the dc gain. Here is the high frequency approximation which is intersecting this line with this then, what is the value of omega corresponding to this point.

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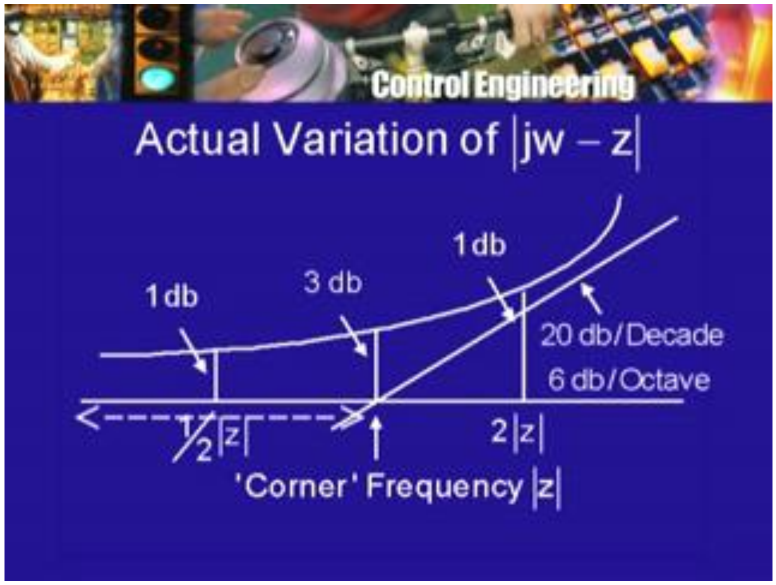

Now this is the low frequency approximation, this is the high frequency of approximation. So what you mean is as omega tends to 0 we are getting closer and closer to this curve and as omega increases to a large value, we are taking closer and closer to this curve. So what is the actual shape of the curve modulus of j omega minus z with z real, what is the actual shape of the curve. Now little bit of a calculation will show you that the following is the shape of the curve. The curve looks like this so it lies above this d c asymptote, so it lies above the high frequency asymptote and what is the point where the 2 asymptote intersect as for an obvious reason, this point or the corresponding frequency or omega is referred to as the corner frequency because if you think of this and this as an approximation, then this approximation is not given by a single straight line but is given by portion of a straight line and another straight line and there is a corner here and this is called frequency, you can show that this corner frequency is exactly the absolute value of this number z.

So where ever the 0 is, if the 0 is at minus 2, it is in a left half plane its absolute value of minus 2 therefore 2. So 2 radians per second will be the corner frequency, I am referring it to frequency but it is actually the value of omega. So if you are thinking of real frequency which is in hertz then this will have to be divided by 2 parts. So this is the corner frequency all right, it corresponds to the absolute value of z, if z is plus 1, let us say there is a 0 in the in the right half plane then absolute value of plus 1 is of course plus 1 and therefore it is still the same thing namely, 1 radians per second. Now obviously there is an error here because the actual curve is like this, the asymptotic curve we are at the corner, so what is the amount of this error.

Now again I will leave it to you and do it on your own, do not look at the book, look at this expression mod j omega minus z and put omega equal to absolute value of z, you can take a numerical example if you wish and then find out what is the difference between this value which is the dc gain and the exact value at the corner frequency. Then one can show that this is very close to 3 db, so in other wards this asymptotic approximation which is very good at low frequencies and quite good at high frequencies is in error at this corner frequency by an amount of 3 db.

So what one can do is one can draw these 2 asymptotes and then, choose a point 3 db above this corner point then, you have this point on the curve and you have these 2 asymptotes and I am assuming you are good at engineering drawing then, you should be able to draw a smooth curve of this kind. It should not intersect this asymptote, it should not intersect asymptote, it should lie above the asymptotes and it should pass through this point which is 3 db above the corner frequency.

In fact, one can get 2 more points on this curve that is if you take a frequency which is 1 octave below this frequency that is what ever this frequency is the frequency which is one half of that if you go 1 octave below then the error or the difference is just 1 db and if you go 1 octave above that is take a frequency which is double the corner frequency then, here also the error is just 1 db. So if you want to plot this approximation in such a way that it a little better then, you choose this point 3 db over the corner frequency, choose this point 1 db above a frequency which is 1 octave below the corner frequency that is one half of the corner frequency, then take a frequency which is double the corner frequency that is 1 octave above the corner frequency and go 1 db above this asymptote. Then, you will have one point here at the corner frequency 1 point here which is one decade, one octave below, another point here which is 1 octave above. Now we have 3 points to which through pass the curve in such a way that these are the 2 asymptotes and so one should be good enough at engineering drawing to draw this smooth kind of curve and this is a fairly good approximation to the actual variation of gain with frequency.

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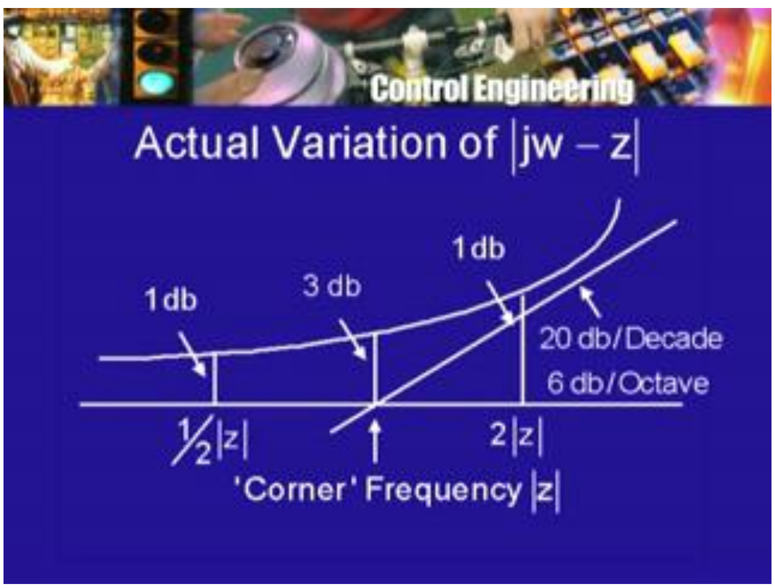

Now, be what is idea was not just to draw such a curve but then was to see something more which I will come to very quickly. Now this was for a 0 this was for 1, 0. Now the transfer function may have several 0s. So then, what happens as we saw the reason for taking logarithm of the modulus was because the modulus was the modulus of a product of a numerators terms divided by product of denominator terms. Taking logarithm converts the product into sum converts the division into difference. So essentially what we have is if we have one 0 then, we this approximation the one 0 will correspond to one corner frequency if I have another 0, I will have another approximation with a different corner frequency and so on.

Now what happens in the case of a pole it is just a little different but of course the difference is important. So I am now looking at a factor like s minus p which however is in the denominator and so corresponding, I have in the denominator j omega minus p, where p is the location of the pole and the pole may be in the left half plane or in the right half plane.

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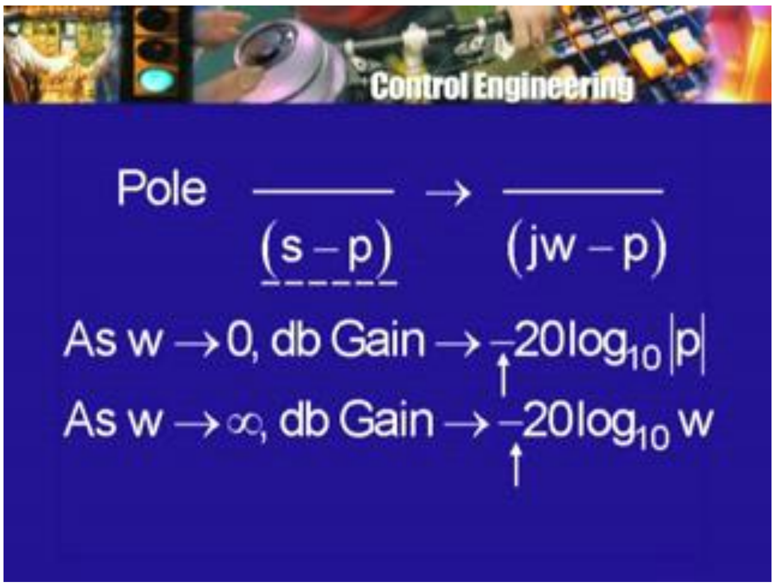

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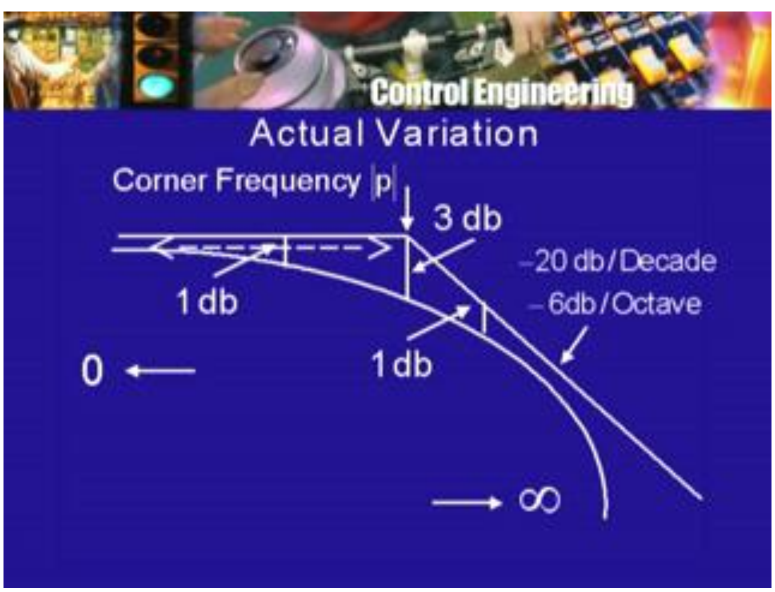

I am assuming the p is not 0 once again that is there is no imaginary axis pole or in imaginary axis 0 right. Then, what happens the only difference is because now this is in the denominator, when I take 20 log to the base 10, I get a minus sign. So once again we can have the low frequency gain as omega tends to 0 which will be given by minus 20 log to the base 10 of absolute value of the pole value minus 20 minus sign because it is in the denominator and when I take log, if something is in the denominator logarithm gives you minus sign. So minus sign here similarly, the asymptote as omega tends to infinity will be what it will be given by minus 20 log to the base 10 of omega. So if you think of it as a function of log 10 of omega then, again it is given by a straight line but this slope of a straight line is now minus 20 or it has a negative slope and a result therefore, if I show the low frequency or d c asymptote and the high frequency asymptote then the dc asymptote once again will be horizontal line.

The high frequency asymptote will be now sloping downwards and the downward slope therefore is shown as minus 20 db per decade or minus 6 db per octave. For every multiplication or increase in the frequency by a factor of 10, there will be reduction in gain by 20 db. The point of intersection of these 2 asymptotes is as you can expect absolute value of p, where p is the location of the pole. So if there is a pole at minus 2 then it will be absolute value of minus of 2 that is 2 radians per second, if the pole is in the right half plane say p is equal to 2 than an absolute value of 2 is just the same as 2. So it is again 2 radians per second.

So the point of intersection is not a negative frequency or a negative value of omega, it is the absolute value of the location of the pole. So this is the point of intersection, so this is the corner frequency corresponding to a pole. Now what about the actual curve in the case of the 0 the actual curve as I told you was above the asymptotic approximation. So in the case, this is the case of the 0 the asymptotic approximation is below the actual in the case of a pole, we expect just the opposite to happen.

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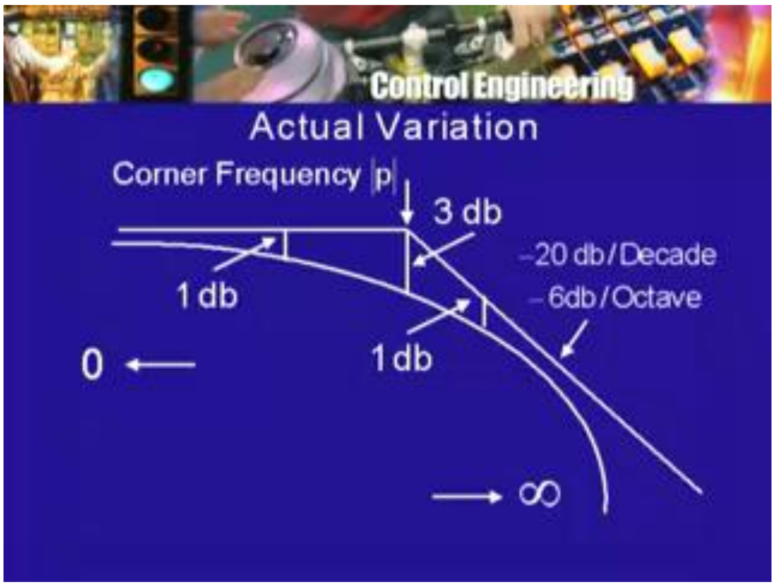

So the asymptotic curve is going to be above the actual and this time at the corner frequency therefore, once again the error will be 3 db but the actual will be 3 db below one octave above the corner frequency, the error will be just 1 db below and 1 octave above the error will also be 1 db. The actual curve is below now, I have 3 points and I know that these are the asymptotes. So I can sketch a curve passing smoothly through these 3 points and meeting these asymptotic.

So this will be a very good approximation to the variation of gain corresponding to a pole. Now as I said, we just do not have 1 pole or 1 0 but we have may be several poles and several 0s and so we have to combine the effects of all the poles and all the 0s and as we saw, the log to the taking the logarithms simplifies our life by requiring us only to add and subtract but we can go 1 step further, am I going to draw all these separate curves and then am I going to add them just as you could in coordinate geometry and earlier work on sketching of curves you perhaps did it by drawing 2 separate curves and then adding their ordinates to get their sum.

Now that is of course one way of doing it but in this case the approximations are so simple and what are what is the nature of the approximation horizontal line towards the low frequency end and the sloping line towards the high frequency end. The slope is 20 degree per decade positive or 20 degree per decade negative and the corner is at a corresponding corner frequency which corresponds to the location of the pole or 0. So with this much information one can see that it is not necessary to plot these separate curves but we can add them up sort of visually and mentally and obtain the resultant graph.

Now this will require practice and of obviously, I am not going to do it here. So I will ask you to look up any problem in your text book in fact I will put down one problem and what I will do is I am not going to work out the whole thing in detail, I am not going to calculate the dc gain etcetera, etcetera. I am just going to show you the highlights of what is the result. So let us take a simple example, so let us take a transfer function which say looks like and let me put an I simple factor in front, let us say 10 and let me have a say a 0 and let us say located at minus 1.

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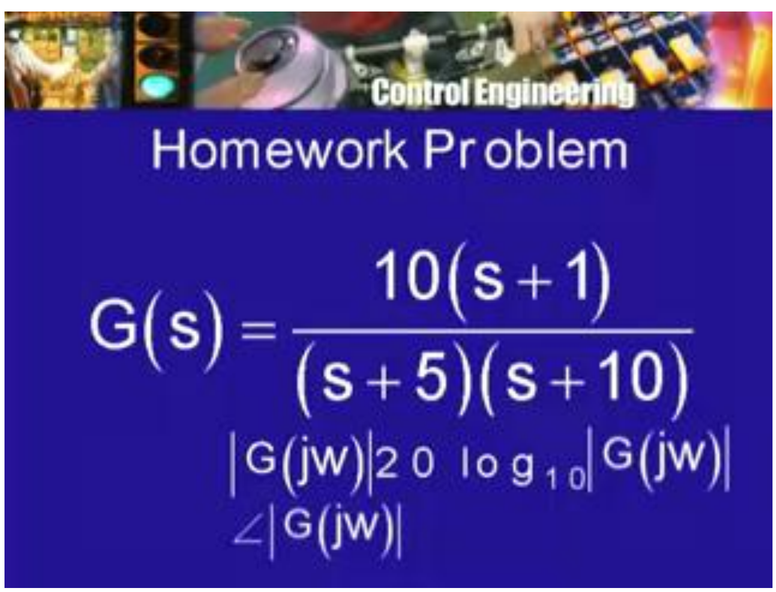

So 10 into s plus 1 divided by and suppose I have 2 poles, let us say there is a pole at 5. So I have s plus 5 and let us say I have another pole at minus 10, this is pole at minus 5 not plus 5 so there is a pole in the left half plane at minus 5, there is a pole in the left half plane at minus 10 there is a 0 in the left half plane at plus at minus 1 and there is a over all multiplying factor of 10, let us say this is my transfer function. So this is my g s and now I want to find out the frequency response that is I replace g s by g j omega corresponding to a sinusoidal input of an angular velocity or a corresponding to a frequency omega equal to 2 phi f, where f is the hertz frequency I want to look at the modulus and angle or argument and once again, I am going to look at the d b counter part of this that is 20 log to the base 10 of this.

So 20 log to the base 10 of absolute value of g j omega, we will look at that first and then of course we will look at the angle of g j omega and we want to find out and draw and the or sketch a good approximation to this as a function of omega and we are going to use a logarithmic scale for omega that is we are going to use a semi logarithmic paper equal increments along the horizontal axis will not correspond to equal increments of omega but will correspond to equal multiples of omega and as I showed you earlier, usually you have decade sections of the semi logarithmic graph paper. So it could be labeled omega equal to point 1 omega equal 1 omega 10 etcetera, etcetera.

So let us do that here, now if I look at this I see that there is a 0 at minus 1 and if I were to have this only then there will be a corner frequency which will be given by the 0 is at minus 1. So the absolute value of minus 1 or 1 radians per second, so there will be a corner frequency at 1 radians per second there is a pole at minus 5. So there will be if I just look at the plot of corresponding to this factor I would have a plot which would have a corner at 5 radians per second and the third 1 corresponds to minus 10 and so it will have a corner at 10 radians per second.

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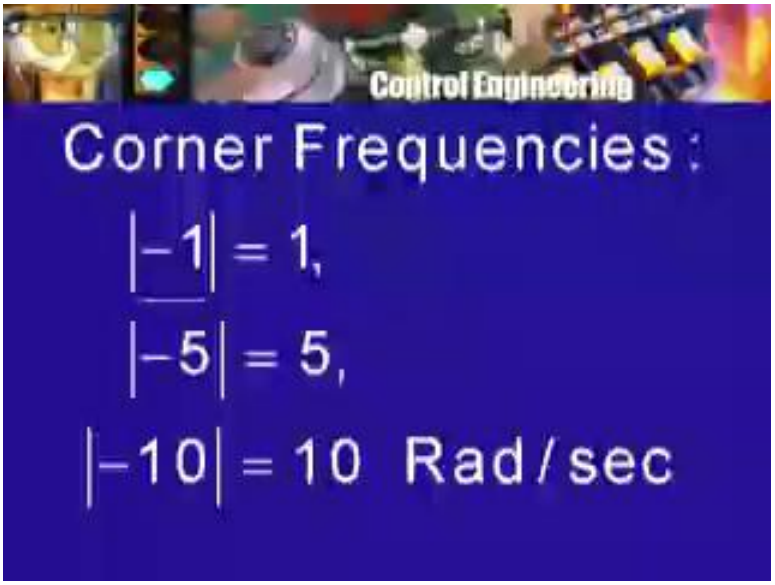

So if I have plotted this 3 approximations separately each one of them would have a low frequency of asymptote, high frequency of asymptote, a corner frequency, the corner frequency would be different for these 3, the corner frequency for this is 1, for this 5, for this 10. The high frequency slopes of all of them are 20 db per decade for the 0 and minus 20 db per decade for the pole what about omega tends to 0, what happens as omega tends to 0. Now that in fact is very easy to evaluate you do not really require lot of effort because s is going to be replaced by j omega and then, omega is going to be replaced by 0. So you might as well replace s by 0, so if I do that what do I get 10 into 1 divided by 5 into 10.

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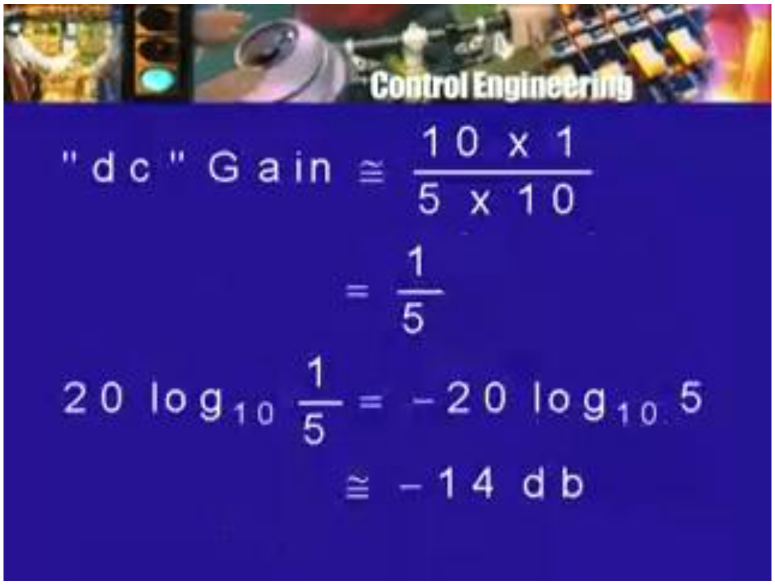

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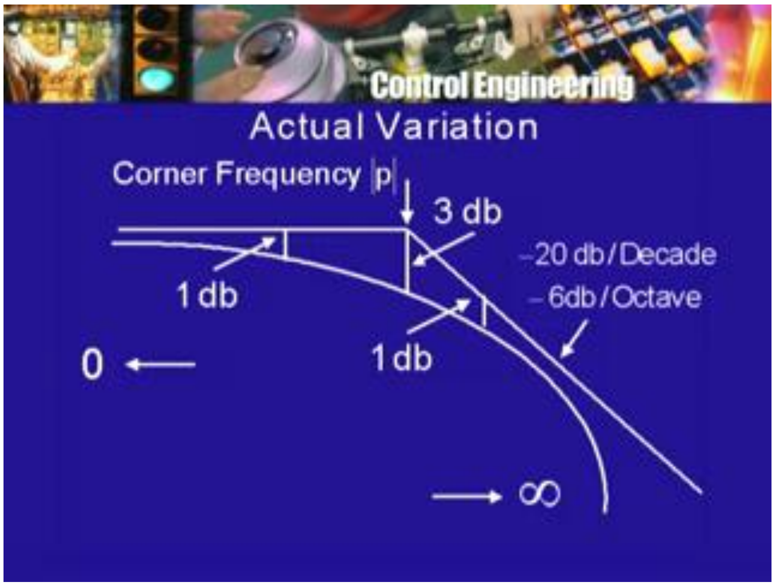

So that gives me a dc gain of 1 by 5, so there is a d c gain of 1 by 5 now remember I have to convert this into decibels I do not work with gain but I work with decibels gain when doing this Bode plots. So what is the db gain corresponding to this that is 20 log to the base 10 of 1 by 5 well, that is minus 20 log to the base 10 of 5 and what is the logarithm of 5 to the base 10? It is approximately point 7 because the logarithm of 2 to the base 10 is point 3, 3010, the logarithm of 5 to the base 10 will be 1 minus that and therefore it is nearly point 7 and remember, we are doing approximations. There is no point in writing down the third and 4th place of decimals, point 7 is good enough, so how much is this minus 20 into log of 5 to the base 10 which is point 7 nearly. So this is approximately would minus 14 db so the dc gain is going to be approximately minus 14db.

Now, whether it is a pole or it is a 0 the asymptotic approximation for low frequency as we have seen earlier, for a 0 it is a horizontal line of some value for a poles it is again a horizontal line of some value depending on what, depending on this pole or 0 location, right. So when I add them up at the low frequency end, I am simply going to add these horizontal lines which are a different levels, add or subtract. So I do not really have to do all that the net effect is I draw a horizontal line at a level of minus 14 db.

So the db gain will approximate minus 14 db as omega tends to 0 an horizontal line at that level will be a good approximation. So I can think of so that starting my dc at the low frequency part of the total approximation at a level of minus 14 db. Now what is going to be happen after this, so think of sort of moving along the omega line. Now there are 3 corner frequencies, there is 1 corner frequency at 1, there is another corner frequency at 5 and there is third corner frequency of 10. So as you start increasing omega and remember we do not increase omega from 0, we increase it from some low value like say point 1 hertz or 1 hertz and so on, where should we start, well the 3 corner frequencies are 1, 5 and 10.

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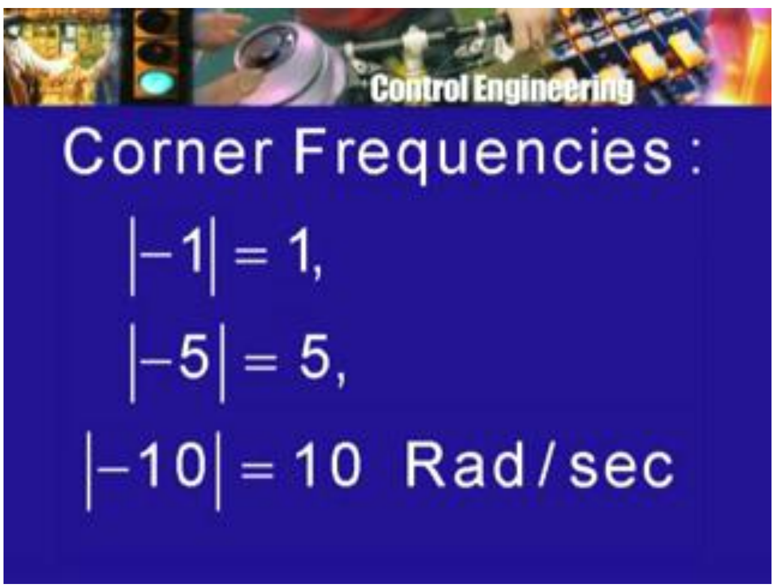

So, we should start at a value lower than 1. So maybe we can start at point 1, where should we end and how far should we go the highest is 10. So perhaps we should we go up to 100 and therefore when we are going to do this plot, we will chose this semi logarithmic paper and the frequency range or the range of omega will be from point 1 radians per second to 10 radians per second right or no 1 decade above the 10 radians. So point 1 to 100 which is how many decades point 1 to 1 is 1 decade, 1 to 10 is the second decade and 10 to 100 is the third decade, so there are 3 decade. There is change in frequency by a factor of 10 raised to 3, 10 times, 10 times, 10 and therefore on the graph paper I will have 3 cycles. So if I just show that for the moment without showing all the details.

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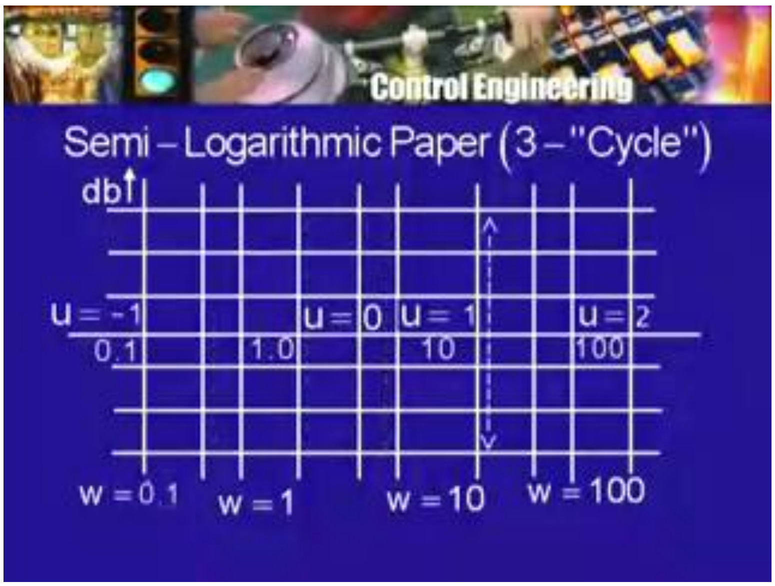

Well, here is this say point 1, here is 1, here is 10 and here is 100, if was than ordinary graph on an ordinary paper, this would not be correct because the distance between 1 and 10 seems to be the same as the distance between the point corresponding to 10 and 100 but this one stands for omega but what we are plotting horizontally is log to the base 10 of omega which of course, we are not showing here if I did want to do it then I could write here u equal to 0 and this is omega equal to 1 omega equal to 10 and therefore u equal to 1 omega equal to 100, so u equal to 2 and omega equal to point 1, so u equal to minus 1.

Now if I do not look at w but if I only look at u, then it is correct 0 to 1, 1 to 2, minus 1 to 1 equal increments. So, uniform scale for u but it is a non-uniform logarithmic scale for omega. So I will chose 3 cycles on the semi logarithmic paper and so if I draw the vertical lines then this is what it is going to be like. So omega equal to point 1, 1, 10 and 100 and as I told you earlier and I hope you have procured for yourself a semi logarithmic paper and are looking at it side by side, if this is point if this is 1 let us say then, where will the point 2 be it will not be midway because omega equal to 1 and omega equal to 10 is what these 2 points are it will be at a distance of point 3 because log to the base 10 of 2 is point 3.

So here will be the frequency corresponding to 2 roughly. So here will be the vertical line corresponding to 2, the next frequency then there will be 3 than 4 and so on and it is gets crowded as you go towards the right. Then you are 10, so 20 is again one-third almost one-third the distance away, so that is the line for omega equal to 20 then, 30, 40 and so on or if it is point 1 here, here is point 2, point 3 and so on, okay. So we have to take the semi log paper and then my make markings on it like this corresponding to omega equal to point 1, 1, 10 and 100, one does not usually even write omega it is understood that it is omega which is being shown logarithmically along the horizontal axis and then, we have to choose the vertical location of our graph because all we have on the graph paper are simply equally space lines, lines which are equally spaced vertically. They may be 1 centimeter, the major lines may be 1 centimeter of part or depending what kind of paper you have at a closer or greater distance.

So that does not matter, now where do I put my 0 db line where do I put my 10 db line, 20 db line minus 10 minus 20 that is again something one has to decide and usually therefore what one does is one goes through this exercise sort of quickly almost mentally or just putting down some numbers before actually starting to draw the asymptotic Bode plot. So this is what we will do now our transfer function is 10 into s plus 1 that is there is a 0 at minus 1 divided by s plus 5, a 0 at pole at minus 5 rather s plus 10, there is another pole at minus 10. Remember, the approximation for 0 is horizontal towards the 0 end, omega tending to 0 end and it is aligned with positive slope at the high frequency end. The approximation for a pole because it is a factor in the denominator is again horizontal at the omega equal to 0 or towards omega equal to 0 end and a line with negative slope for the high frequency end.

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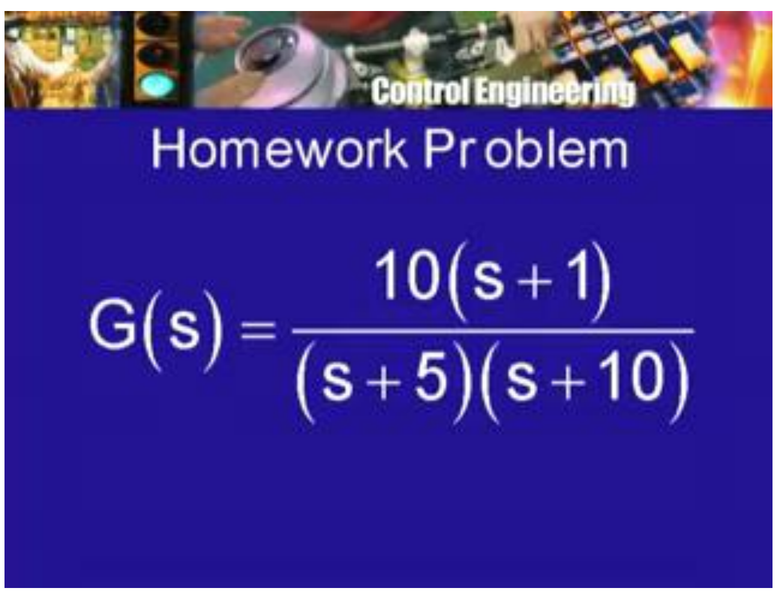

So now you start thinking of it as follows suppose, I start at look at a frequency which is small, how small well the corner frequencies are 1, 5 and 10. So if I take a frequency of say point 1 radians per second then that is quite small compared with 1 is 1 decade below the smallest corner frequency, all right. Now for that frequency what are the 3 approximations, what is the approximation corresponding to the 0 and corresponding to the 2 poles, you can immediately see that s is going to be put equal to 0 literally and there for the dc gain that we are had calculated is a good approximation which takes care of the effect of all the 3, the poles 2 poles as well as the 0, so the dc gain end will be minus 14 db.

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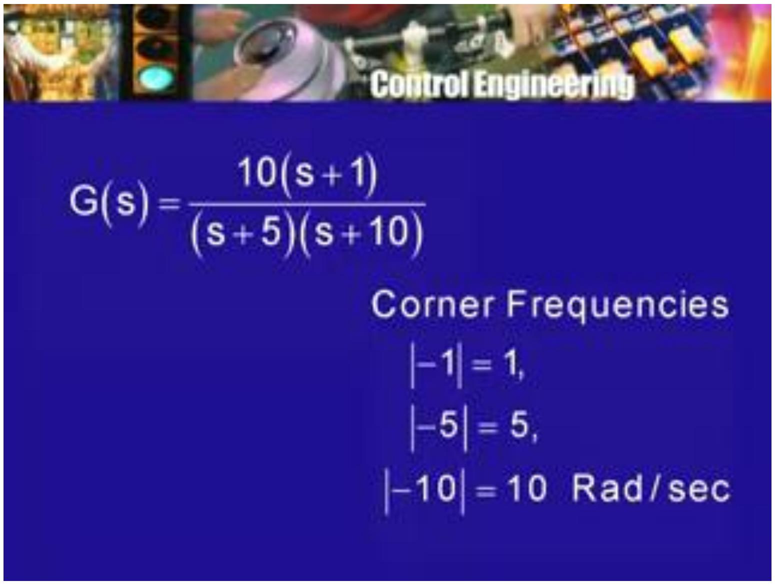

Now I think of increasing omega as it were slowly so I am starting to the move to the right from omega equal to point 1. Now all the 3 curves are horizontal till their corner frequency, so this curve corresponding to s plus 1 is horizontal till the corner frequency of 1, the other curve s plus 5 corresponding to s plus 5 is also horizontal till that corner frequency third one is also horizontal. So up to the lowest corner of frequency 1, all the 3 curves are horizontal.

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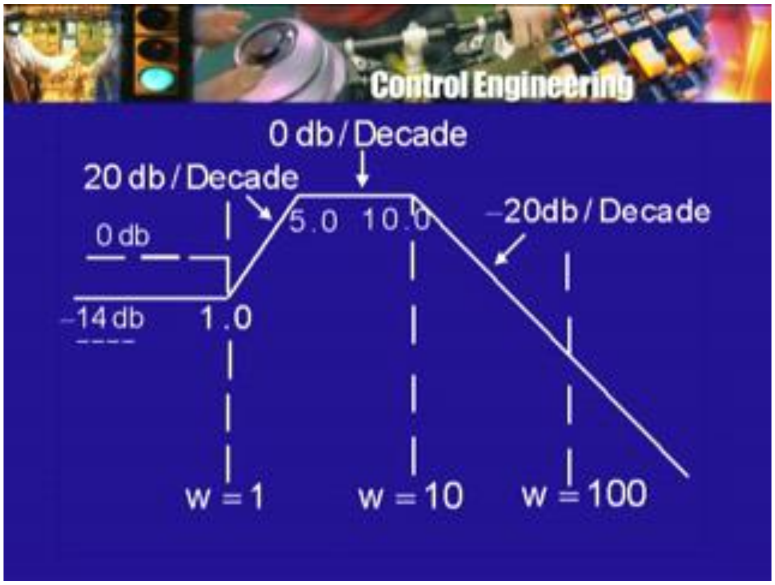

So if I add them up I will get a horizontal line right therefore the net effect will be somewhat like this. Now I am not putting down the axis right now, I will do that later. So it will be like I had a constant value of gain of minus 14 db's, up to the time that I reached the first corner frequency. So I am now considering increasing omega or increasing log to the base 10 of omega, if you wish. Now when I reach the first corner frequency which is corresponding to this 0 then, this has an approximation which is no longer constant but which increases with frequency, the other 2 are however are below the corner frequencies and therefore there are as good as being constants. So what is the net effect, the 0 contributes an increase in gain the 2 poles are yet to make any significant difference therefore the net result is an increase in gain, an increase in gain at what rate so to speak 20 db per decade and so, if this was the curve so far up to the corner frequency of 1, then from here it will start growing at the rate of 20 db per decade.

So one shows it one thinks of it like this. Now will it keep on going indefinitely it would if this was the only factor but then there are the 2 poles. So what is going to be happen as we increase the frequency further you reach the second corner frequency 5. So when you reach the second corner frequency 5, the effect of this pole will start coming into the picture. In other words the gain will now start decreasing or this will contribute a decrease in gain and at what rate minus 20 db per decade. So let us say here is the point corresponding to the 5 radians per second up to this frequency, there was this increase at the rate of 20 db per decade beyond that frequency of course the effect of the 0 will cause the increase at 20 db per decade but we have the pole also whose effect is now to be seen and that effect is a at the rate of minus 20 db per decade.

So what is the net result and this is where our engineering judgment comes into play, if something is increasing in 1 part is increasing at 20 db per decade another part is decreasing at 20 degree per decade than what will happen to the sum. The sum will be very nearly constant. So the net result is a horizontal line from 5 rad per second onwards but will it continue indefinitely no why because the second pole will play its role, starting with what frequency, starting with its corner frequency which is 10 hertz.

Now if I show it on this scale then because it is logarithmic 1 and 5, so 10 may be some here. So up to this point this is a good approximation. Now beyond this point 10, the 0 of course has its own contribution the first pole makes its own contributions. So these 2 will be cancel as they did earlier but the pole had 10 will now make one more contribution and therefore there will be a decrease in the gain at the rate of minus 20 db per decade and here also it is horizontal but one can emphasize it by showing 0 db per decade and so this is going to be a fairly good approximation to the variation of db gain with frequency. Now this is an approximation one can improve it little bit and in practice one really does not bother too much to increase it because here for example, this is the first corner frequency I will have to go 3 db above but then here this is the corner frequency of the pole.

So I have to go 3 db below but the contribution of the 0 may not be quite negligible. So I have to add and subtract and it is not really worth doing it this horizontal at the level of minus 14 db's vertical or not vertical are thus sloping at 20 db per decade positive slope up to corner frequency of 5, then horizontal again up to the corner frequency of 10 and then, finally sloping downwards at minus 20 db per decade is a good approximation. Now I said that we have to do it on the graph paper, so where do we choose the 0 db line and so on where should we choose it.

Well, again some quick calculations and approximations which one can make quickly are enough here it is minus 14 db's. So on my graph paper I must be able to show this line minus 14 db somewhere what about this point I can quickly calculate the approximate gain corresponding to this frequency, why it is minus 14 here, the slope is 20 db per decade what is this change here from 1 to 5, 1 to 5 is logarithm to the base 10 of 5 that is point 7 multiplied by 20. So that is a change of 14, this is minus 14, so what will be the gain corresponding to this level it will be 0 db so this point will be at 0 db level then of course, this point is at 0 db level.

So at frequency of 10 radiance per second the total gain will be roughly 0 db's that means the gain will be just 1, absolute gain will be 1 or there is no gain, there is no loss, there is no gain, there is no attenuation. Subsequently, what is going to happen there is going to be decrease at the rate of point 20 per db per decade. So if this is 10, then I will have the point 100 for that frequency 100 rad per second what will be the net decrease 0 here decrease of 20. So this level will be minus 20. Now if my range of interest goes beyond 100 then I will have to make provision for it.

Similarly, if I interested in the frequencies much less than the first frequency that we started of namely point 1 then, I will have to provide for it in the graph paper but what is the change in gain for this frequency range of interest. Let us say 1 decade below the lowest corner frequency, 1 decade above the highest corner frequency and this is what one usually does for this point at the low frequency end. It is minus 14 db's then it is increasing going to 0 and then it is decreasing and by the time I have reach hundred radians per second it is nearly minus 20.

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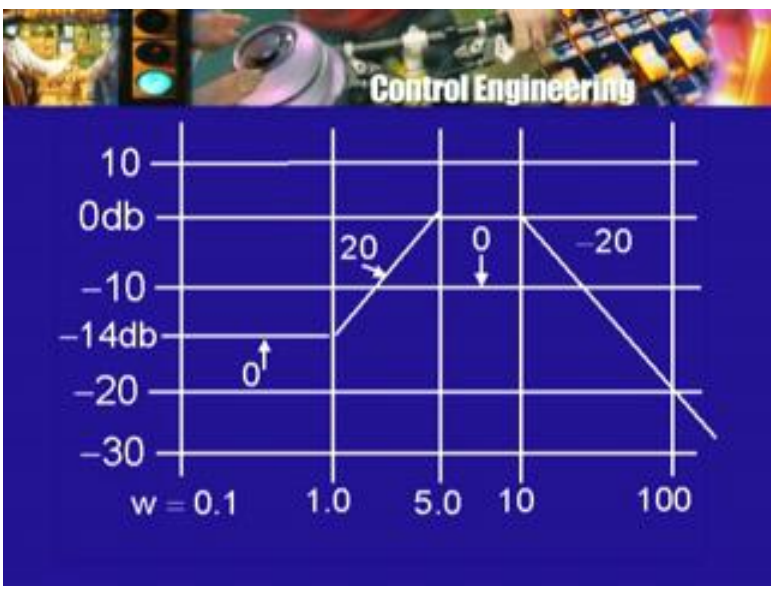

So along the vertical direction I must a provision of variation of minus 14 db's to 0 db's to minus 20 db along the horizontal I must a provision of point 1 radians per second 1, 10 and 100. Now with this kind of a quality to work done, one is then ready to use the semi logarithmic paper in fact, it is not even necessary to use the semi logarithmic paper all though as an engineer one should make use of such a paper as far as possible. So if I show it now it will be something like this, I am have vertical level which ranges between minus 14, 0 and minus 20.

So I may allow may be 10 db's in the positive direction although I am not likely to use it. In the negative direction I have minus 20 but I may go up to say minus 30, so the vertical variation the graph papers scale that I will chose will be from say 10 db to minus 30 db. So, 10 db, 0 db minus 10 minus 20 minus 30. So let us say here is the 10 db line than I will have what 0 db minus 10 minus 20 minus 30. So here is the 0 db line, here is the minus 10 minus 20 and minus 30 line. I have to choose the vertical scale appropriately depending on the width of the graph paper that I have the height and the width of the graph paper I am not going above 10, I am not going below minus 30.

So this is adequate, so appropriate scale has to be chosen for the horizontal axis I have already chosen because of the frequencies 3 decades. So here is let us say 1 decade, one end starting at point 1 and here is the next one, the second one and here is the third one. So this is point 1 this is 1 point 0 rad per second, this is 10 point 0 rad per second, this is 100 point 0 rad per second of course, I do not really have to write point 0 but this is just to make sure that I have written the number correctly. Now where do I start at the low frequency end at point 1 and even up to 1, the approximation was minus 14 db's.

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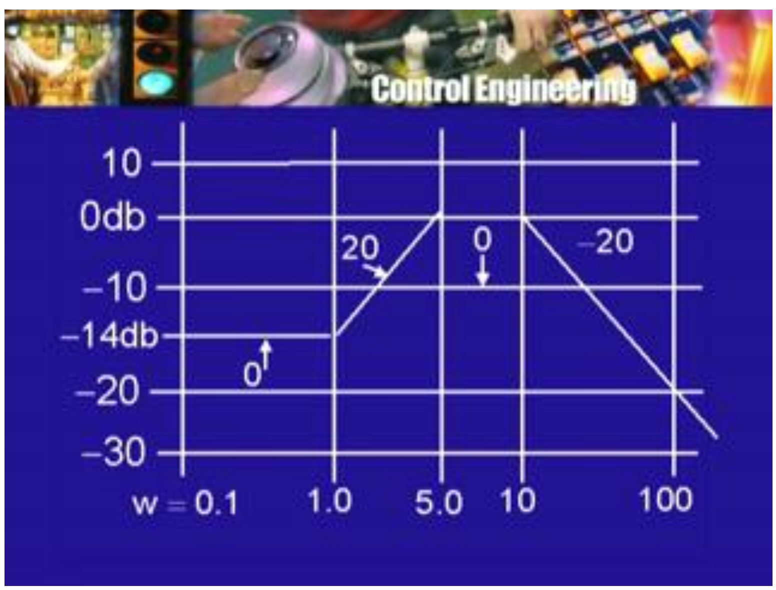

So here is just minus 14 db line then from here and of course here is the vertical line corresponding to 5 rad per second and we found out that the net gain at that point is going to be 0, so this is the upward sloping line with 20 db per decade minus 14 has become 0 here because in terms of decade, they have gone point 7 of a decade, so 20 times point 7 is 14, then we have this horizontal portion from over this range from 5 to 10 because this 0 contribution has cancelled the pole contribution and then beyond that from 10 to 100, there is a drop of 20, so here is the curve and then of course it goes beyond.

So this is the approximation to the variation of gain with frequency and this is what Bode's contribution was that without going through actual detail numerical calculations, you can get a good idea of what the variation of gain with frequency is going to be like. This was his contributions and this is the importance of Bode plots not actually evaluating the gains at various frequencies and joining them by a nice and smooth curve that is not worth it because as we saw in the Nyquist criterion, you do not really need the full polar plot at each and every point experimentally also you do not really get a full plot, what you get is gain at a number of frequencies and phase shift at the same set of frequencies, what you want is the qualitative kind of information which is therefore available here and this is the main idea behind the Bode plot.

So I hope you will be able to work out problems where the poles and 0s have different locations and there may be several 0s and there will be several poles and the multiplying factor in front may not be 10 but it may be something else. Now there are 1 or 2 things which one can remember and in some books, it is done one way in some books, it may be done the other way. I have written a factor like s plus 1 either in the numerator or in the denominator and I have talked about a 0 or a pole at minus 1 but as we saw earlier, this has something to do with the transient response and the time constant of the transient response.

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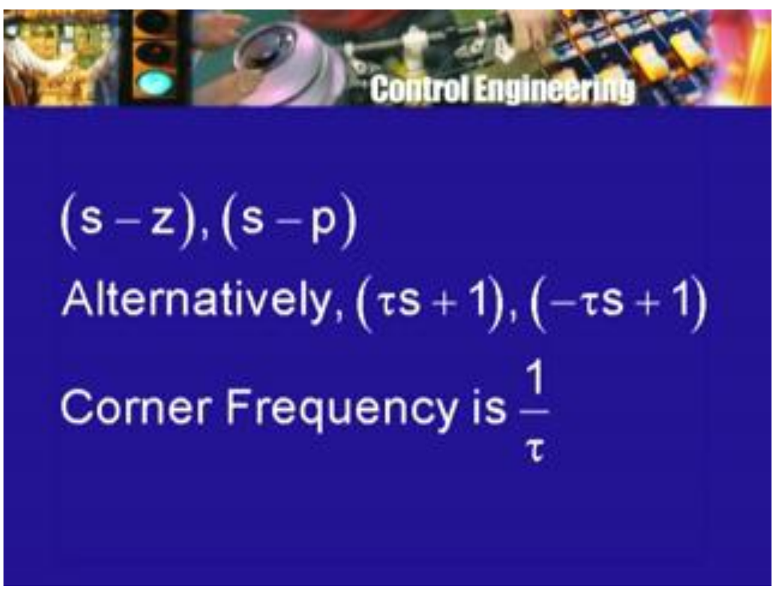

So instead of writing s minus z or s minus p, one can write it in the form of tau s plus 1 or this tau although it is a time consistent there will be minus sign here it will be of the form minus tau s plus 1, if something is in the left half plane it will be like this to tau positive, if something is in right half plane it will be like this with tau positive. Now when you do this tau is not the corner frequency, what is the corner frequency then you have to go to this form s minus z or s minus p form. So the corner frequency corresponding to this will be 1 by tau 1 divided by the time constant that is the reciprocal of the time constant is the corner frequency. So if you write the transfer function in this form then the coefficient of s is not the corner frequency but the reciprocal of it is the corner frequency, there is one advantage in writing it in this form because when I write it in the form tau s plus 1, when I put s equal to 0 what remains only one.

So that tau just disappears and I have 1, 1, 1 for each of the poles and 0s, only the overall multiplying factor is what remains so the d c gain is very easy to figure out. But I do not think we have you will consider yourself unable to figure out the d c gain in the case like this that that is you cannot put s equal to 0 mentally and see that what you have is 10 into 1 divided by 5 into 10 or 1 by 5 but it is some time advantages to show the time constant explicitly because ultimately they tell you what the transient response is going to be like.

So if your expression has factors of this form, remember that this tau is not the corner frequency 1 divided by tau or if it has a negative sign here then of course take the positive part and the 1 divided by that is the corner frequency and when you are looking at the dc gain, then of course ignore the tau altogether and ignore this 1s because the answer is multiplying and divided and adding to a amount into 1. So that is one thing, the second thing is that we have only looked at real poles and real 0s what if the poles are not real but then as we know they occur in complex conjugate pairs or they occur in conjugate pairs and for the example on pole 0 diagram, there may be a pole here there may be a pole here, this may be say minus say minus 1 plus j 1 and this is pole minus 1 minus j 1, what about the frequency response corresponding to a pair of poles like this.

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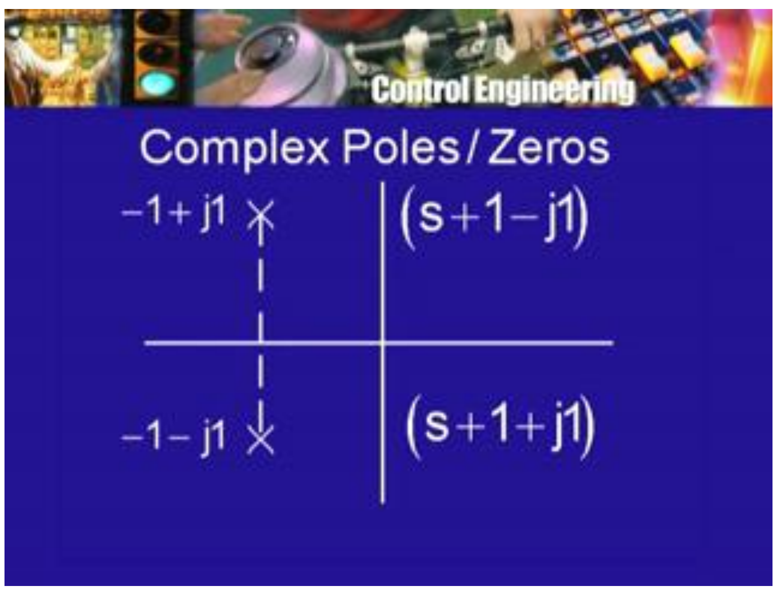

Now it is not very difficult to get an idea of what it is going to be and again we are interested not in the exact calculations but good approximations. So once again one can look at omega tends to 0 and omega tends to infinity situation and then one will see that there again, we will get a exactly 2 asymptotes a low frequency asymptote and high frequency asymptote and to see what it is, let us take this example and complete the job. So there is a pole here so corresponding to these there is a factor s plus 1 minus j 1, corresponding to this there is a factor s plus 1 plus j 1and so what really I have if I want to get rid of the complex numbers is the factor s plus 1 minus j 1 into s plus 1 plus j 1 and one can see immediately that this is s plus 1 square plus 1 square and this will remind you the of the form in which we wrote the quadratic expression, when the factors were not real or alternatively, we wrote it as s square plus a s plus b.

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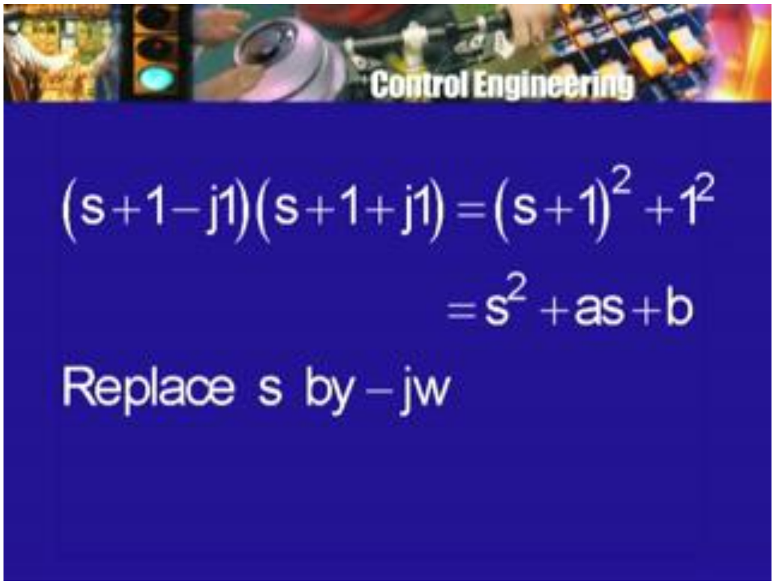

So instead of looking at the poles or 0s in their complex forms look at the quadratic expression like this, s square plus a s plus b now replace s by j omega okay. We do that what do we get and than consider the case where omega tends to 0 and omega tends to infinity, what about tends to 0 it is very easy to see this simply amounts to be. So at the d c end we will again have a constant gain given by this number b or 20 log to the base 10 of b or minus 20 log to the base 10 of b depending on whether we have the numerator that is 0 or denominator.

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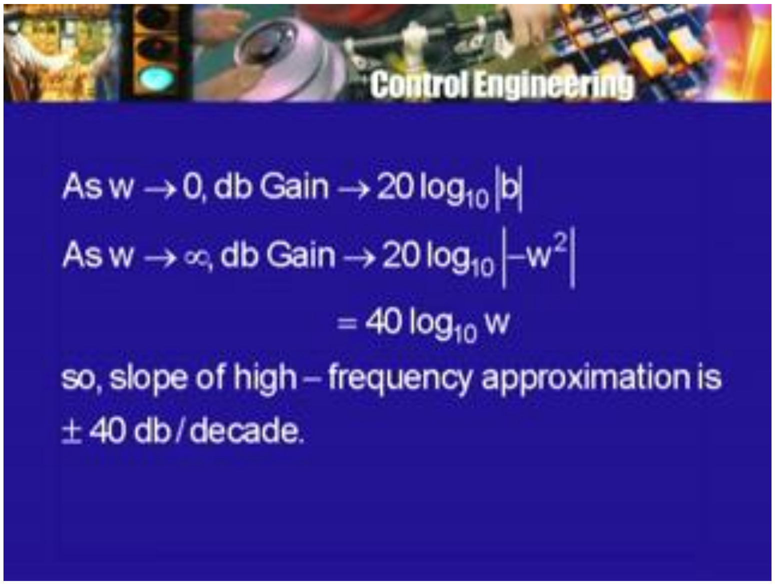

So that is taken care of so the dc asymptote is going to be horizontal line what about the high frequency asymptote. Now if you replace s by j omega you will see that you will get a omega square from here you will get a omega from here and this is of course a constant, so for large omega you can forget about the omega and you can forget about the constant and what remains is only omega square and therefore, we have 20 log to the base 10 if you have s square absolute value of omega square or minus omega square if you wish.

So this is what this is 40 log to the base 10 of omega, so instead of 20 log to the base 10 of omega I have 40 log to the base 10 of omega. So the high frequency asymptote has a slope of 40 db per decade rather than 20 db per decade or minus forty d b per decade rather than 20 db per decade and that is not very difficult to remember because we are looking at a pair of complex poles, we are looking at the effects of not 1 pole but of 2 poles together or of not one 0 but 2 0s together.

So earlier, each pole or 0 contributed minus 20 or 20 db per decade now, we are considering them in complex conjugate pairs so the contribution will have a slope of 40y db per decade, for a 0, for a pair of 0s and minus 40 db per decade for a pair of poles. So that is one thing there is to remember, the second thing is what about the actual curve and the asymptotic curve, what is the difference between the 2, now there is something that one has to take a little bit of a care of and some little computation may be necessary. So we will take a look at that next.