Control Engineering
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Lecture - 41

Let us take a look at a Bode's method of approximating the frequency response, what we are looking for is a method for finding out the modulus which corresponds to the gain and argument which corresponds to the phase shift of K G H j omega, where K G H is the loop transfer function.

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So we are interested in the modulars of this and in the argument of the same thing. Now of course K being positive the modulus of this is the same as the modulus of j h, j omega. So we need not worry about K, you can always take the effect of K into consideration later on and similarly K being positive the argument of K G H is the same as the argument of the G H. Now we can of course make use of the pole 0 diagram and get some idea about how these 2 things change with frequency, frequency response is the name given to the variation of this quantity with frequency omega that is the gain variation, the ratio of the amplitude of output to amplitude of input for a sinusoidal input signal and the phase difference or the phase shift from the input to the output as a function of frequency. It is called frequency response because we used it in our Nyquist criterion, it is called Nyquist plot or it is also called polar plot because the argument, the modulus and the argument are like the polar coordinates of complex number.

So from the pole 0 diagram, it is possible to get some idea about the variation of gain with frequency and the variation of phase with frequency. So for that let us look at the simple situation corresponding to our feedback control system with proportional feedback. So we have 2 poles here, one of them the 2 are a result of the 2 time constants, the electrical time constant and the mechanical time constants associated with the motor and the load.

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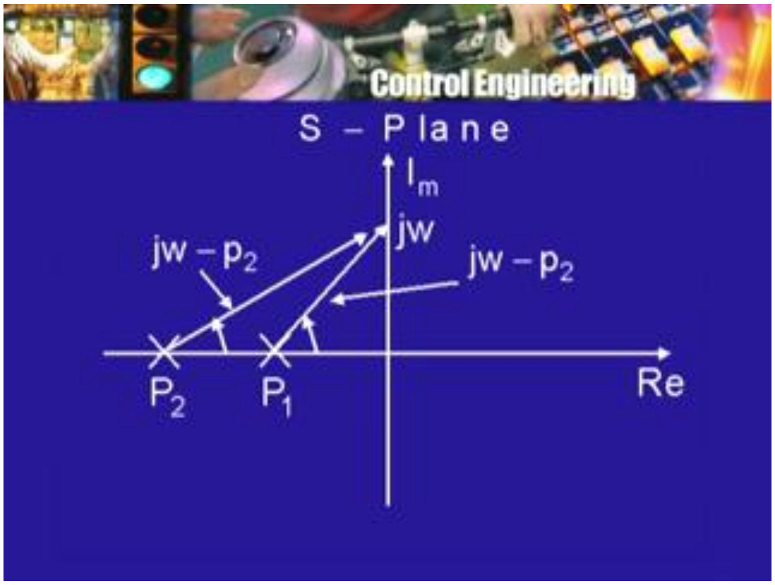

So we have 2 poles, no 0 and we want to look at g h j omega. Now, as we saw earlier we can think of writing G H as ratio of 2 polynomials in fact I am making the assumption that the system is described by a ordinary linear differential equation with constant coefficients of the kind that we saw a long time ago and because of the G H as a function of S can be written as ratio of 2 polynomials. So in this case then when there are only 2 poles, let us say one of the pole is called P 1, the other is called P 2. So what will GH as a function of S look like.

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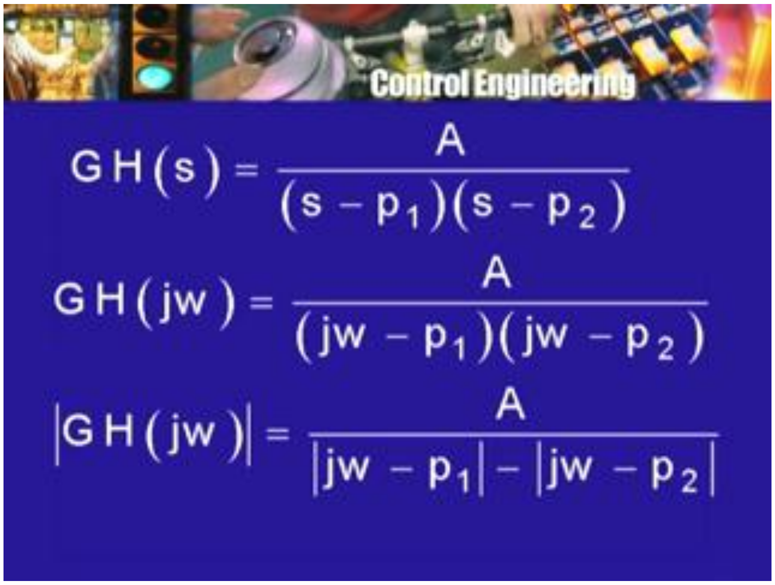

Well, there are no 0s, so in the numerator there will be just some constant or number A and in the denominator we will have S minus P 1 into S minus P 2. This will be the polynomial in the denominator because there are 2 poles P 1 and P 2, right. Now if you replace S by j omega what do we get, we will get G H of j omega equal to A divided by j omega minus P 1 into j omega minus P 2. Now as we saw earlier when we discussed the root locus method, if you plot j omega as a point in the complex plane then it will be a point on the imaginary axis. Here is the imaginary axis here is the real axes and all this is the S plane now, because you are raising S rather than z for the complex variable.

So here is the point corresponding to the number j omega, here is the point corresponding to the number P 1, then as we saw in connection with root locus method, then we derived the 2 basic principles. This vector that points from the pole to j omega represents the complex number j omega minus P 1. So I am going to write here by this side j omega minus P 1. This vector is not from the origin, so it is not the complex number j omega minus P 1 but it represents the complex number j omega minus P 1.

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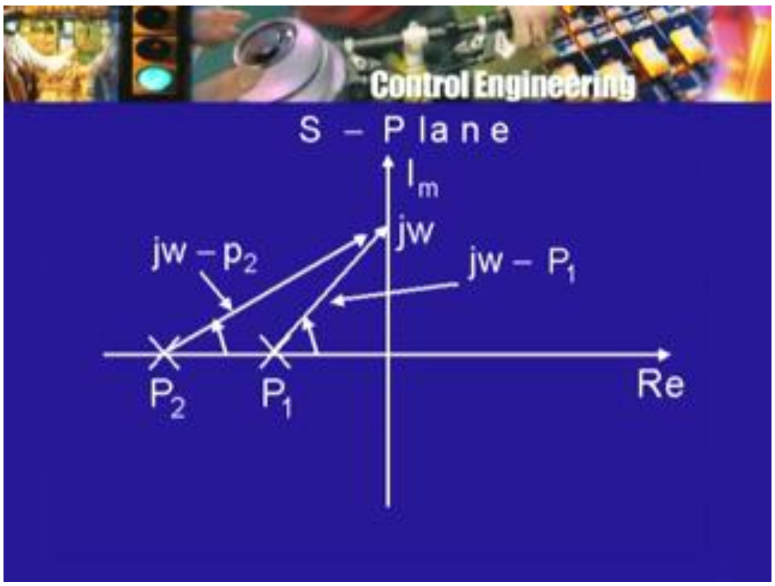

Similarly, this vector from here to here represents the complex number j omega minus P 2. So now, G H of j omega is A divided by the complex number corresponding to this vector multiplied by the complex number corresponding to this vector. Now you know that if we have a ratio of 2 complex numbers then, the modulus of the ratio is the ratio of their moduli. So because of this then, we will have mod of G H j omega equal to A, assuming A is positive modulus of A is A, A is real mod of j omega minus P 1 into mod of j omega minus P 2. But given a complex number j omega minus P 1, what is its modulus? Modulus is the length of the vector drawn from the origin to the complex number or because this vector represents that complex number the length of this vector.

Similarly, mod of j omega minus P 2 will be the length of this vector. So we have G H at j omega modulus that is the gain at any angular frequency omega equal to A divided by the length of this vector multiplied by the length of this vector that is the product of these 2 lengths dividing A that is the gain. You remember, in the root locus method we talked about distances to the poles and distances to the 0s, here of course there is no 0 there are only distances to the poles. So the gain equal to A divided by distance of this pole from j omega multiplied by distance of this pole from j omega.

Now what is going to happen as omega varies, so think of omega very small therefore this point is very close to the origin then, we have these 2 distances which are nearly equal to P 1 and P 2 absolute value respectively. So as omega tends to 0 the gain will tend to the value A divided by absolute value of P 1 into absolute value of P 2. Remember, P 1 and P 2 are negative numbers negative real numbers, so I have to write down absolute value of P 1 and absolute value of P 2.

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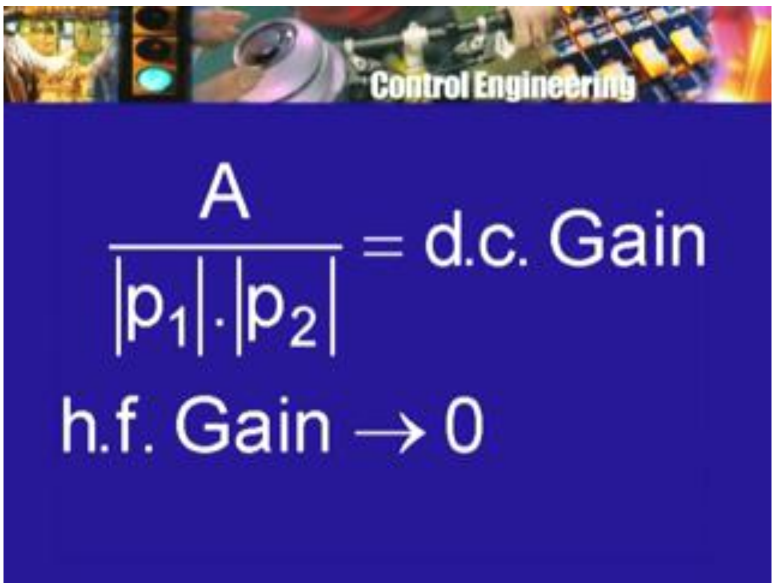

So this we can call as we have done it already as the DC gain that is the gain at very low frequencies. So the gain at very low frequencies will have some value like this. Now as omega increases what is the going to happen as omega increases of course A is the same but the denominator which corresponds to the lengths of these 2 vectors is going to increase and as omega increases more and more, the lengths will increase more and more as a result, the gain will become less and less and we can easily see that as omega tends to infinity or this point moves up indefinitely, the vectors become indefinitely long and therefore the gain will approach 0 and therefore I can write the high frequency gain tends to 0 and of course, this is something we had already seen in the polar plot, there was a DC gain which was non-zero and the high frequency gain was 0 or as the frequency increased the gain kept on reducing till in the limit it would be 0 or the limit of the gain would be 0.

So this is as far as the gain is concerned now, what about the phase shift, now that also is the very difficult to see from the diagram because once again, if you have ratio of 2 complex numbers, the argument of the ratio is the difference of the arguments, argument of the numerator minus the minus the argument of the denominator, if we have a product of complex numbers, the argument of the product is the sum of the arguments of the 2 terms of the product.

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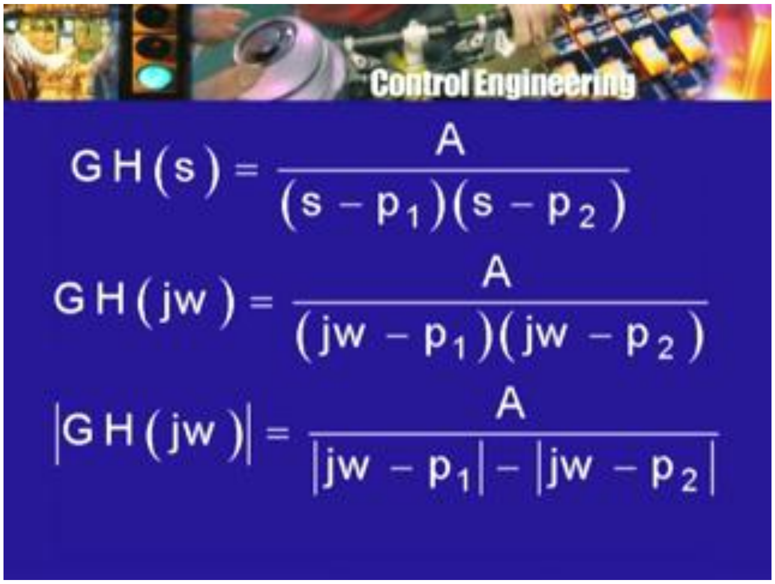

Now, what do we have here we have A divided by a complex number j omega minus P 1 and multiplied in the denominator by another complex number j omega minus P 2. So the angle G H j omega will be the angle of A is, A is positive then the angle of A is 0 minus the angle of j omega minus P 1 added to the angle of j omega minus P 2. Now j omega minus P 1 is precisely the complex number represented by this vector. So what is its angle or argument it is this angle and similarly, for the other vector, this is the angle. So the argument or the angle of G H j omega is 0 minus this angle minus this angle, when we have noticed this, we are now ready to look at the behavior as omega varies, when omega is very close to 0 what are the 2 angles going to be like, they are going to be very small, almost 0 degrees.

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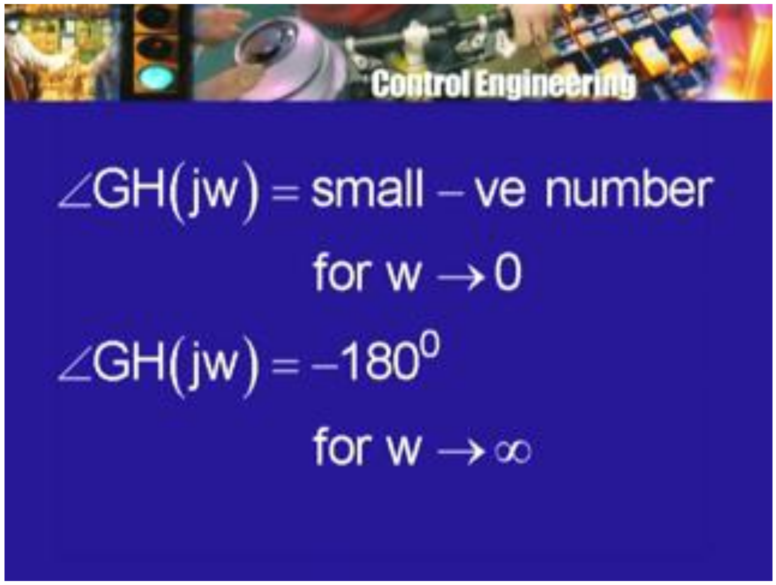

So their sum will also be very small very nearly equal to 0 degrees of course, not quite 0 there will be some small positive angles and therefore the argument of G H j omega will be a small negative angle. When omega is small the phase shift will be negative but small. Then, as omega increases these 2 angles keep on increasing but they do not increase indefinitely because as omega keeps on increasing, the angles always remain less than phi by 2 or 90 degrees each and they are getting added up because they are multiplying together in the denominator.

So as omega increases and becomes very large the 2 angles will be each of them the nearly 90 degrees but less than 90 degrees therefore, they will add up to angle which is almost 180 but less than 180 but because of the factors in the denominator, the phase shift will be close to minus 180 degrees for omega large and from this then we are able immediately to see that the kind of polar plot that I had sketched last time is correct for DC of course, there is no phase shift but for very small frequency there is a phase shift and therefore the angle is negative. The gain is of course not 0 not infinity but it some finite value at very high frequencies the gain is very nearly 0.

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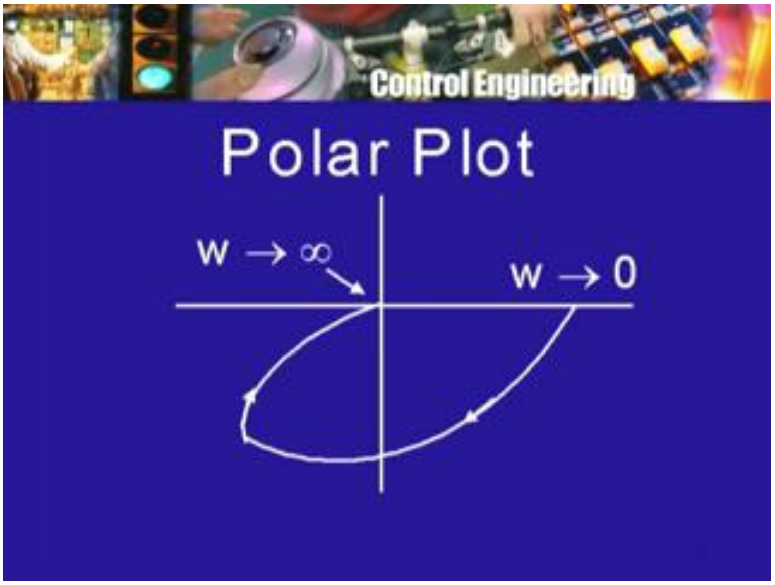

So I am near the origin but the phase shift is not quite minus 180 degrees, it is an angle little less than 180 degrees in absolute value and therefore this was the nature of the polar plot that I had drawn, when we looked at the Nyquist criterion. So merely by looking at the pole 0 diagram in this case there are only 2 poles by looking at the point j omega on the imaginary axes, by thinking of or visualizing these vectors looking at their lengths and angles and looking at what G H j omega involves terms in the numerator, terms in the denominator. It is possible to get some inkling of how the gain and phase shift are going to vary. Of course, this pole 0 plot had only 2 poles.

So the situation was really very simple if we put more poles we will see that the situation will be a little involved and therefore something like what bode device is useful. Note that I can, if I want calculate the lengths and angles exactly of course by exactly 1 means up to the precision provided by your pocket calculator or by the computer program, we never really get 100 percent exact results anyway.

So I am not talking about computing the gain or computing the phase shift over for one particular value of omega or another or 10 different values of omega. It is of course useful to calculate the values because then we can get a more correct idea of what is going to happen. But quantitatively, for qualitative understanding, I do not need to any calculations, I.. whatever calculations, I do are not really calculations but I look at the diagram and I look at some background knowledge, use some background knowledge I have and conclude that in this case, the gain will go to 0, the phase shift will go to 180 degrees but the phase shift will be with a negative sign and therefore the polar plot will be like this.

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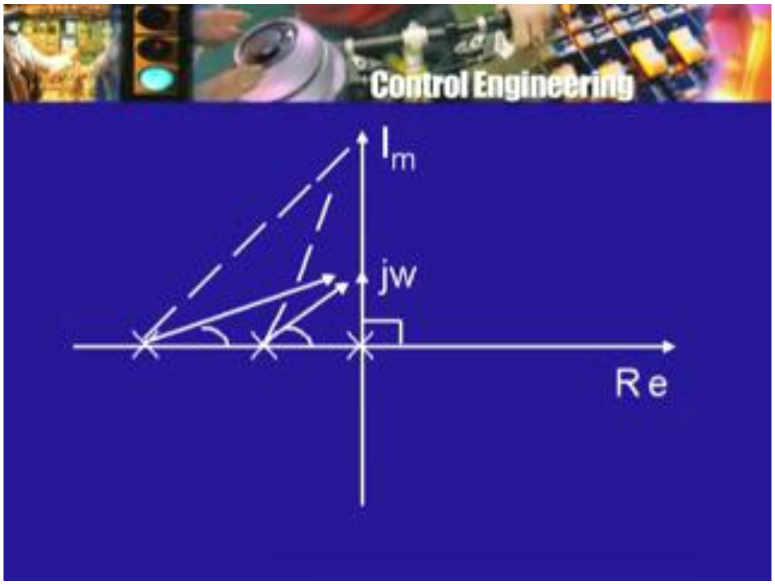

Let us take the next example which is the example of our speed control system once again with the integrator added. Now the pole 0 diagram has one more pole, so it is this pole here, another pole and a third pole. There are still no 0s and now, we want to figure out how the gain and phase shift vary. So once again we think of the point j omega which is on the imaginary axis, the upper half of the imaginary axes and then, we think of the 3 vectors drawn from the 3 poles to this point then, the gain is equal to whatever is that numerator constant divided by the lengths of these 3 vectors.

So, what is going to happen when omega is very small, when omega is very small this vector that points from the origin to j omega has a very small length. The other 2 vectors of course they do not have lengths which are small or going to 0 but the length of this vector as omega approaches 0 is going to become smaller and smaller and in fact, approach 0 and that is in the denominator therefore the gain will tend to infinity or if I want to talk about the DC gain then, I can qualitatively write the DC gain is infinity. Actually, there is no DC gain because with an integrator if I apply an input voltage to the integrator which is constant, the integrator output will go on increasing indefinitely therefore in principle, the motor speed will be go on increasing indefinitely. Of course nothing of that sort will happen because the motor is linear device only within certain limits.

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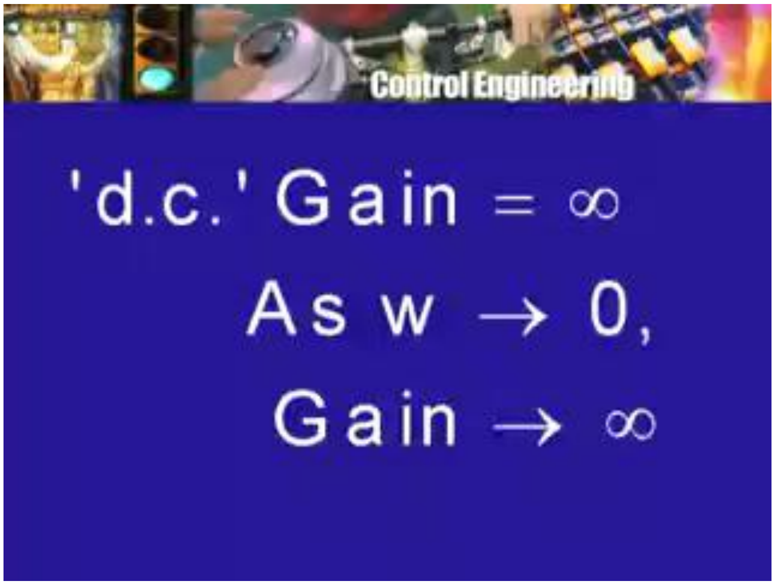

So what we mean is really as omega tends to 0, the gain tends to infinity, the gain will be very large for frequencies which are very small. So that information we have obtained from the pole 0 diagram, the next piece of information is about the argument or the phase shift and that again it is not difficult to figure out, here is j omega. Now what do I have to do the argument for G H j omega is argument of a which is 0 degrees minus the sum of the 3 angles made by the 3 poles at the point j omega.

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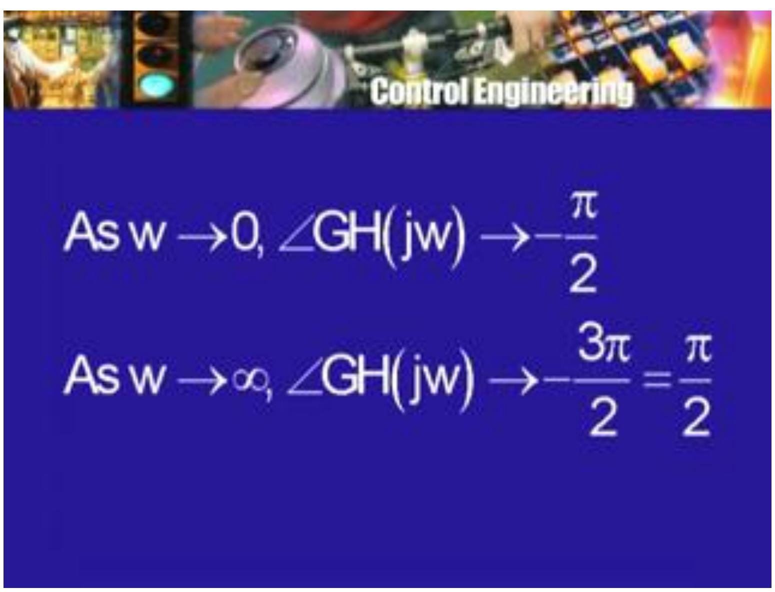

Now this time this pole at 0 makes an angle of 90 degrees at this point j omega and this angle remains constant as omega varies as this point moves along the imaginary axis, this particular angle stays put at 90 degrees whereas these 2 angles they will start of with small values and then go on increasing to large values. So this makes a constant contribution of 90 degrees in the denominator. These 2 make contribution which varies from 0 degrees to 90 degrees in the denominator and therefore, what can we say about the angle of G H j omega as omega tends to 0, the angle will be phi by 2 with a minus sign moreover these 2 other angles made by these poles are not quite 0 and therefore the angle is going to be slightly greater than minus phi by 2 or in the third quadrant whereas as omega tends to infinity, this angle is phi by 2, the other 2 angles also are phi by 2 each. So the net phase shift will tend to minus 3 phi by 2 which is also the same as plus phi by 2, all right. So with these 2 items of information about the gain and phase shift then, we are ready to sketch the polar plot at low frequencies the gain is very high but the phase shift is minus 90 degrees minus a small amount.

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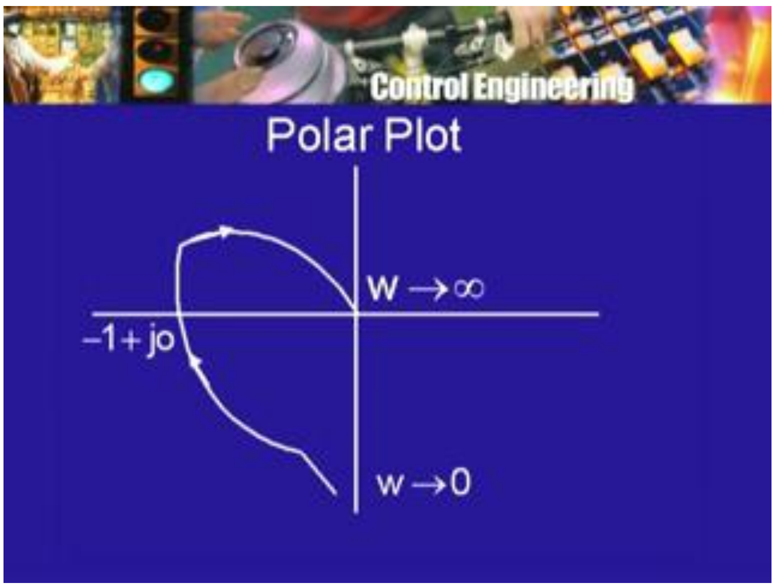

So it will start of let us say somewhere here, for very high frequencies the gain is very low, so I am very close to the origin but the phase shift is almost minus 270 degrees and therefore the nature of the polar plot or the shape of the polar plot is like what I had drawn earlier. Now of course this is only a qualitative diagram, I have not calculated anything, I have only found out or thought about the limits, limit as omega tends to 0, limit as omega as tends to infinity. So for example, unless I do some calculations I cannot figure out what is this point of intersection of the polar plot with the negative real axis. Remember, that we have to worry about the point minus 1 plus j 0 when applying the Nyquist criterion.

So you would like to know where the point minus 1 plus j 0, is it here or is this point itself minus 1 plus j 0 or is it in here. So the intersection of the polar plot with the negative real axis is something we have to find out. Now for this point the phase is 180 degrees or the argument is 180 degrees. So in other words, you have to find out of the angular frequency omega for which the phase shift or the angle of G H omega is 180 degrees. Now that is a numerical computation which one can do and of course in practice one does choose different values of omega and gets a value of omega close enough to the value where the phase shift will be exactly 180 degrees, knowing that value of omega then we can calculate the gain by may be calculating the lengths of the various vectors or by evaluating G H j omega as a complex number by doing complex arithmetic on the calculator or with a program, it does not matter what you do. So qualitatively, this is okay but we may need this information and for that we will have to do some calculation.

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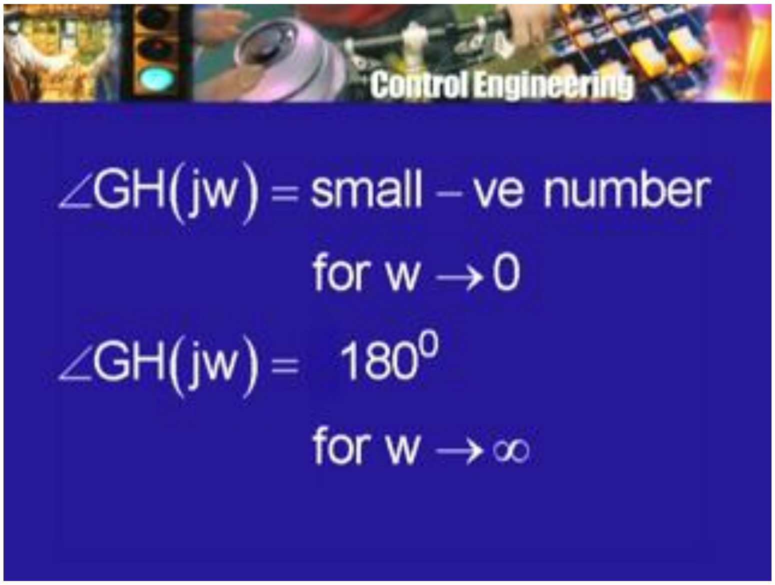

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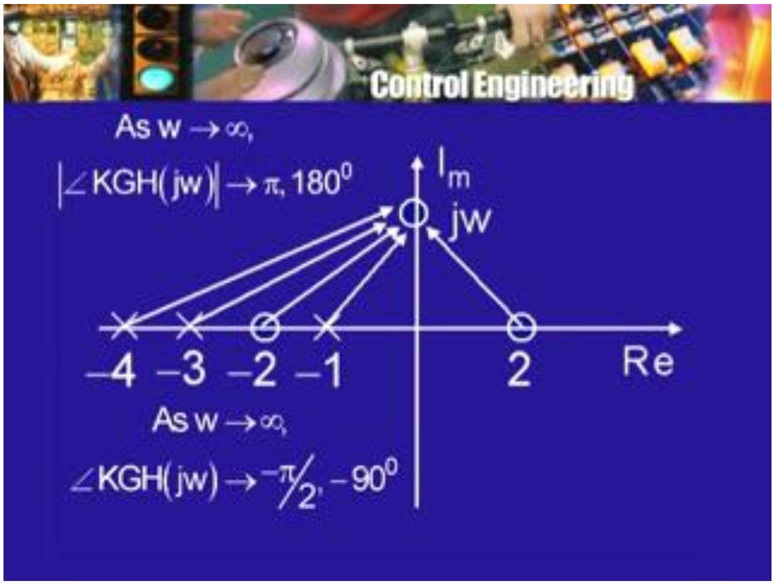

So it is not that no calculations are required at all but qualitatively we know what is going to happen. Now this was with 3 poles but what is the transfer function as poles as well as 0s and not just to 2 or 3 but more. So let us go back to an example which I had taken in connection with the root locus plot, so there was a 0 at 2, there is a pole at minus 1 is 0 at minus 2 and then, 2 more poles and minus 3 and minus 4. So we had 3 poles and two 0s. Suppose, the loop transfer function was like this now what will be the variation of G H j omega as omega varies once again, qualitatively as far as the 2 limits are concerned, omega tending to 0 and omega tending to infinity. It is possible to work out, what the limits will be why because look at this point j omega here, all have to do is think of omega very small, so I am somewhere here, so what is the gain.

Now the gain is the coefficient that is the numerator multiplied by the lengths of the vectors from the 0s or the distances of this point to the 0s divided by the distances of the point from the poles. So the whole thing will be sum number, what it is of course we can even calculate for example, here when this point is very close to the origin distance to the two 0s is 2 and 2 and distances to the 3 poles are 1 and 3 and 4 and whatever is the multiplying coefficient k then, this is going to be the gain.

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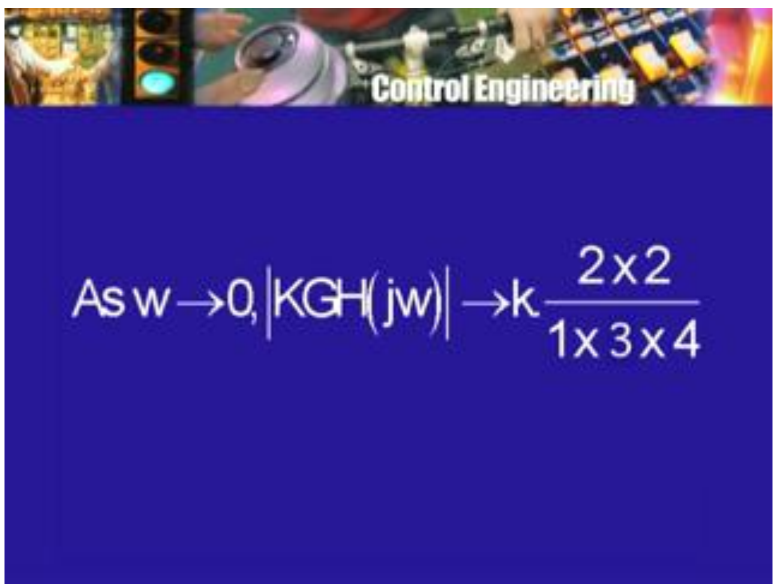

So the DC gain can be calculated quite easily omega tending to 0, this will be the limit of the gain, what about the limit as omega tends to infinity, as omega tends to infinity there are 2 distances to the 0s which are becoming large but there are 3 distances to the poles which are also becoming large and all the distances will very nearly be equal to omega itself, j omega is this point. So the length of this line is omega, so as I go moving up that length will essentially be the length of the vectors therefore I will have k multiplied by omega into omega divided by omega into omega into omega or this is nearly k divided by the omega and as omega tends to infinity, this goes to 0.

So the high frequency gain will approach 0, so qualitatively then we know that the low frequency gain will have some finite value which can in fact be calculated and the high frequency gain will be 0. This much for the gain, what about the phase shift well look at the vectors drawn from the 0s and the poles to this point j omega therefore there are these various vectors and look at the angles made by them with the positive real axis, add the angles made by the 0s from that subtract the sum of the angles made by the poles and see, what the result is going to be.

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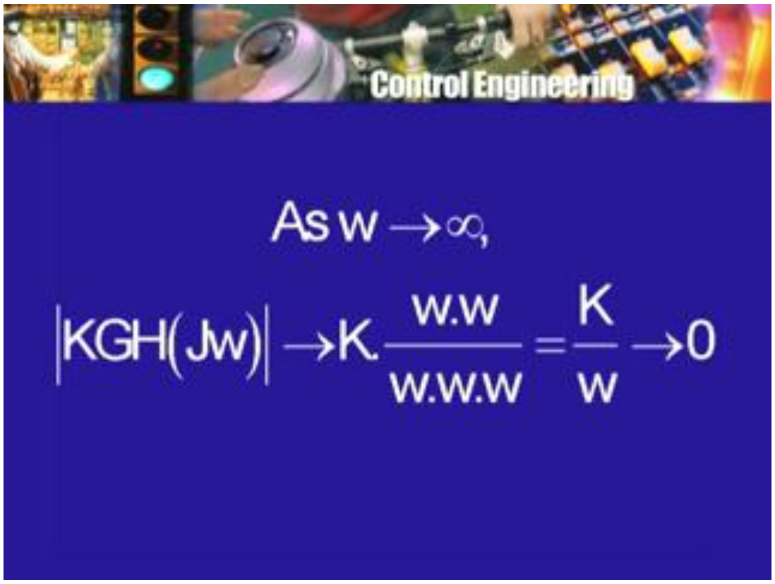

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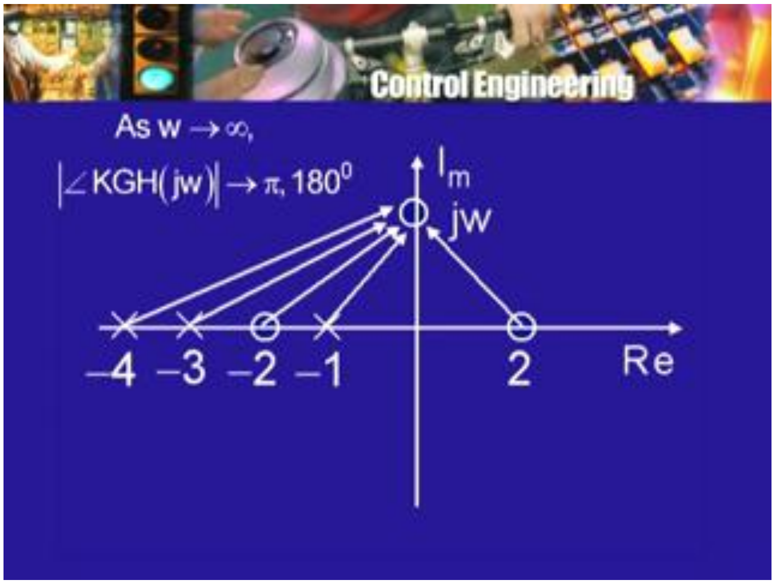

Now here, again the limiting values are not difficult to calculate suppose omega is very close to the origin then, what are these angles what is the angle made by this 0 at placed at 2, at this point near the origin on the j omega axis, it is almost 180 degrees or phi, actually a little less than phi but almost, what are the other angles, the other angle made by the 0 is almost 0. The angles made by the 3 poles are also 0, so as a result the phase shift or the angle of G H j omega as omega tends to 0 will be very close to phi or 180 degrees.

So as the frequency goes down or as we approach the DC end, the phase shift will be almost 180 degrees or phi radiance. Of course, will it be a little more than 180 degrees or little less than 180 degrees, we cannot immediately say, one will have to a little bit of calculation or estimation to find out what is going to happen. But the limit is going to be this. At that end there gain was not infinite the gain was not 0, the gain was some finite number. So the gain is finite and the phase shift was 180 degrees at the DC end, what about the high frequency end at the high frequency end all these vectors will make angles which are nearly equal to phi by 2 each of course, the angle made by this 0 is not phi by 2, it is actually more than phi by 2, the angles made by the others are all less than phi by 2 but in the limit there are all very close to phi by 2.

So what is the net angle of the phase shift, from this 0 and from this 0 phi by 2 each, so 2 times phi by 2 from these 3 poles phi by 2 each, so 3 times phi by 2. So subtract 3 times phi by 2 from 2 times phi by 2. So that gives us minus phi by 2 or minus 90 degrees that will be the phase shift at high frequencies of course, whether it will be slightly more or slightly less than this this, we cannot right away say without making some more estimates or some more calculations but we know qualitatively that in the limit, this is what is going to happen. As a result, now I can sketched the polar plot what information do I have low frequency gain is non-zero, low frequency phase shift is nearly 180 degrees, high frequency gain is 0, high frequency phase shift is minus 90 degrees.

So what will be the polar plot look like at low frequency, the gain is not 0 but the phase shift is 180 degrees. So I am somewhere here or near the negative real axis, I do not know whether I will be here or here, somewhere there. For high frequencies the gain is 0, so I am very close to the origin but the phase shift is minus 90 degrees again I do not know whether I will be on this side of the j omega axis or this side of the j omega axis.

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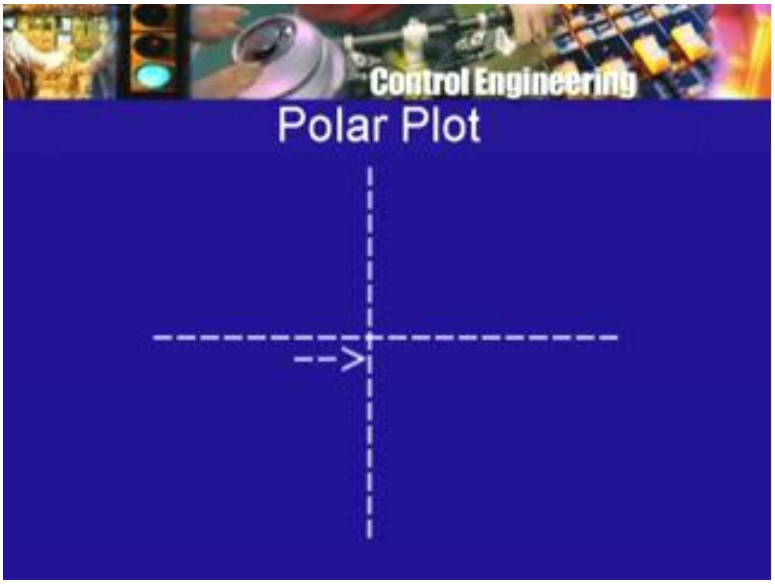

Now I have to go from here to here. Now this is where it is not immediately clear which way the curve will be if I want to go from here to here of course, I can be very optimistic and just draw a straight line going like that, it is very unlikely that the phase shift is just going to you know remain almost constant and then, do something miraculously and then, go like this, it is very unlike, may be it will go like this, may be it will go like this, may be it will go like this, all know at the limiting value. So, which one of these is likely to be the correct or close to the exact variation of gain and phase with frequency. This is something which we cannot immediately figure out by only using this approach that is pole 0 diagram, vectors drawn to the j omega point angles made by those vectors, the lengths of those vectors etcetera, etcetera.

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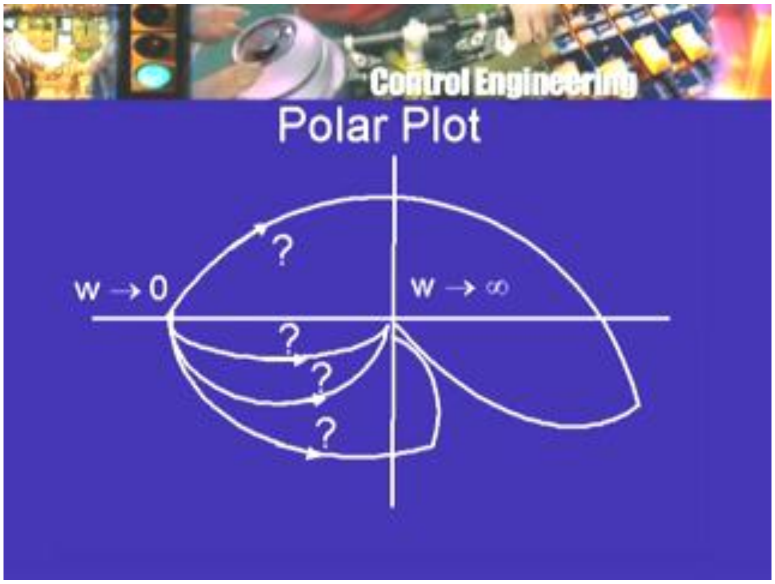

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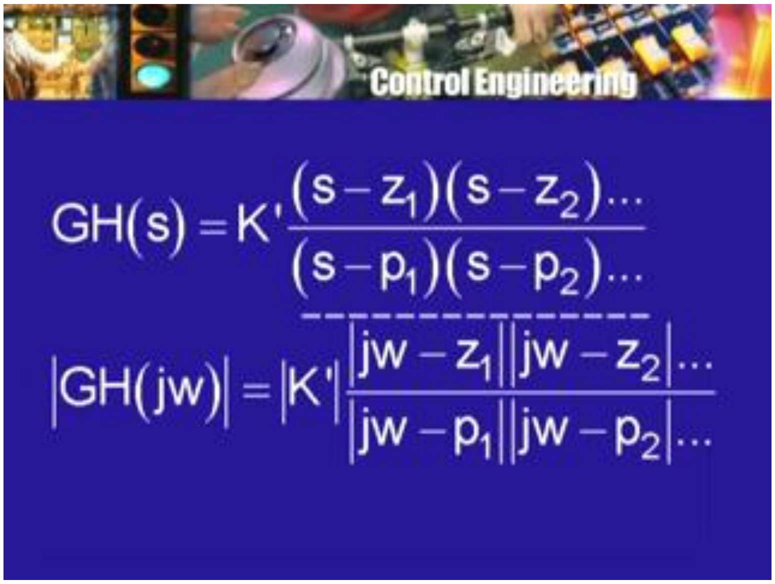

This is something which you have already done in connection with the root locus method, there it was use to determine whether a particular point s was on the root locus or not and if it was on the root locus, what was the value of the gain k corresponding to that is that point that is where we use this idea, all right. So it will be useful to have some other and a little better way of finding out the frequency response and this is exactly what Bode did. So for that let us write down the transfer function G H of s as some coefficient, I will call it as a K prime which is positive multiplied by s minus z 1 into s minus z 2 and so on divided by s minus p 1, s minus z 2 and so on, where there are 0, z 1, z 2 etcetera there are poles p 1, p 2 etcetera, this coefficient K is 0 is just some real number.

Now I want to find out the gain and phase shift. In other words, it want to find out the modulus of G H j omega and I want to find out the argument of the G H and j omega as before the modulus of this is K prime, modulus of K prime, if K prime is positive, it is just K prime. In the numerator modulus of j omega minus z 1 modulus of j omega minus z 2 and so on. In the denominator modulus of j omega minus p 1 modulus of j omega minus p 2 and so on. In fact this is what we did earlier, these are then lengths of vectors from 0s to j omega, lengths of vectors from poles to j omega take the product of these divide the product of these etcetera, etcetera.

Now this is where our engineering way of thinking or approach comes into play. Of course, if I want to calculate I just multiply out these numbers and divide by these numbers if I am doing it on the calculator it is not really difficult at all, remember these are complex numbers. So I will have to use the complex mode of arithmetic if I have a computer program then I can write a program which will do the product that is the program which will do complex arithmetic, there is no problem what so ever. But as I told you this was done in the 1930’s and 40’s then, there were no pocket or desk calculators we had only the slide rule of course, we had the commercial calculators which essentially did addition and multiplication of large numbers involved in financial computations but there were mechanical calculators very clumsy.

So the engineer’s tool at that time were essentially the slide rule and the logarithmic and the trigonometric tables. So given that background and this is where the engineering comes in. Instead of a product, I would like to use a sum now that is possible how, how do you convert a product into a sum or a quotient into a difference. The simplest thing is to take logarithm. So we take the logarithms or take the logarithm of this number rather than work with this number and therefore instead of mod G H j omega which is the ratio of the output amplitude to the input amplitude at the angle of frequency omega, we will look at the logarithm of this.

Now once again it turns out that although in mathematics in calculus and so on, the base e is more convenient for practical purposes the base 10 is a good enough base. So we look at logarithm of this modulus to the base 10 and then it turns out that a that time people who had already been working in acoustics, in fact they had thought of this because the volume of sound that the human being can hear the faintest sound to the loudest sound that you can hear without going mad that is a very large range, it is called the dynamic range. Therefore, if you look at the pressure variation or if you think of it as an amplitude or oscillation then, the range is very large and therefore acoustic people were already using something like the logarithmic scale that they were doing log to the base 10.

Now it turns out that this number may turn out to be less than 1 for a range of omega values therefore, it is good to multiply it by say 10. So that it becomes not a fraction but can have a integer part and again it turns out that for historical reasons, it was multiplied by 20. This 20 has something to do with power as against voltage. So instead of modulus of G H j omega which is the ratio of amplitudes and which therefore it is the gain properly speaking through the loop transfer function that is the output voltage amplitude or output quantity amplitude divided by the input quantity amplitude. Instead of that we take logarithm of that to the base 10. So remember, the base used here is 10, so I am not writing ln I am writing log to the base 10 of modulus and that also is scaled by a multiplying it by the number 20.

So we have 20 log to the base 10 of mod G H omega and this is referred to as the db gain. This is taken as a unit although there is nothing like a unit because the gain is simply a ratio for this is referred to as db gain that is a number calculated like this is said to be the gain in decibels. In fact the number log to the base 10 is referred to as a Bell named in honor of the physicist, scientist Bell, Alexander Graham Bell and then, of course if you scale it 10 times it is a decibel but because of this power and voltage quantities, we multiply by 20 rather than by 10 and therefore 20 log to the base 10 of the modulus or the gain ordinary gain will be called the dc gain. We can very quickly look at some numbers for example if the modulus is 1 then, what is db gain? Log of this is 0, so the db gain is 0.

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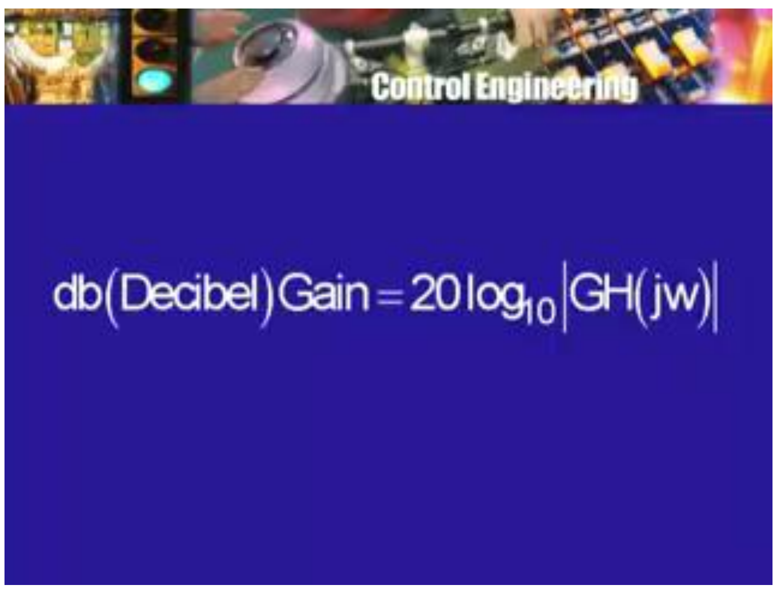

Now do not think of this as there is no gain in the sense output amplitude is 0, in fact contrary to it the output amplitude is equal to the input amplitude therefore, there is no gain in amplitude, the amplitude remains unchanged decibel gain is 0, what if the output amplitude is double the input amplitude. So the modulus is 2 then, what is 20 log to the base 10 of 2. Well, approximately and as engineers we do not really care all the time about so many places of decibels, this will be equal to 6 db's. So the doubling of the amplitude corresponds to the db gain of 6 or a 6 db gain, what if they amplitude becomes 1 half then, they will not be a gain. So it will really be should be called attenuations but the gain will be minus 6 db or the attenuation will be 6 db or having making something 1 half is attenuating by 6 db or there is a gain loss, there is a loss of 6 db but the gain is minus 6 db.

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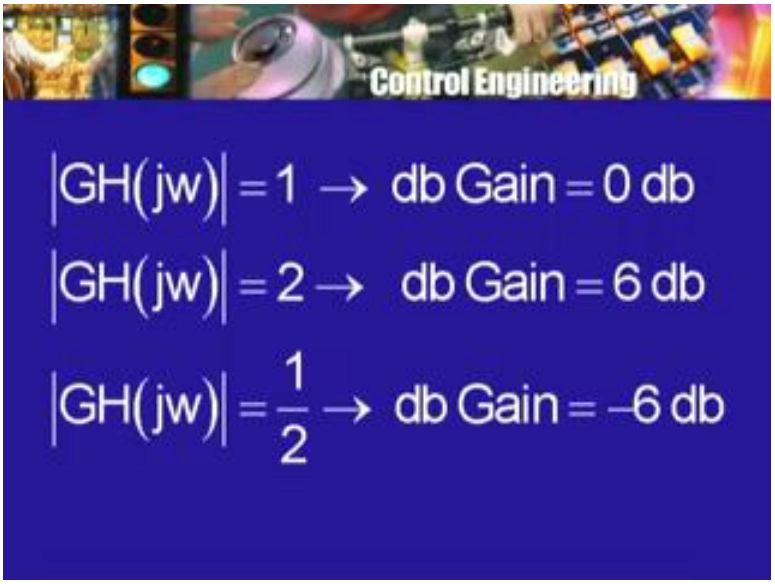

Now as for as the gain is concerned, we do use this engineering approach of converting the product into a sum or a difference by taking logarithms. As far as the phase is concerned or the argument is concerned fortunately, we do not have to be engineers because the angle of a product is already the sum of angles or the angle of a quotient of 2 complex numbers is the difference of 2 angles.

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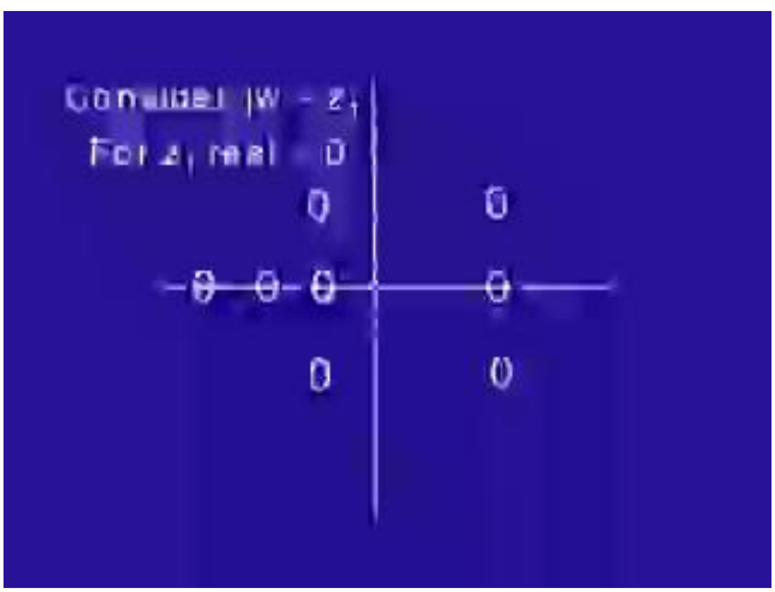

So there is nothing like log required there and therefore, we do not have to really do anything more than what anybody else would have done. But remember, as far as the gain is concerned the gain that one talks about is therefore not the ratio of output amplitude to input amplitude but the decibel version of it that is 20 log to the base 10 of the ratio of the output amplitude to the input amplitude or 20 log to the base 10 of one would, what one would normally called gain all right then. So what does this mean this means that the gain, the db gain that is corresponding to G H j omega that is the db gain in which we are interested is given by a sum of db gains of K prime or their modulus of that plus the db gains corresponding to these factors in the numerator namely the 0 factors minus the db gains corresponding to the denominator factors okay.

So now let us look at one of the numerator factors let us say j omega minus z 1. Now this is where we will have to consider several cases. So the first case will be that zone is real that is the real then, the 0 is a real number not only that I will assume that the 0 is in the left half plane that is if I am going to draw the pole 0 diagram again, the 0 will be somewhere here or here or here, if the 0 was here then the z 1 will be real but positive and of course, if the 0 was out in the complex plane somewhere then z 1 is no longer real.

So, we first consider the case when z 1 is real and less than 0. In that case, what about modulus of this. Remember now, z 1 is simply a real number although it, I am writing z 1, it is a real number, so what is the modulus of this. Well, this is real, so this is the real part with the minus sign this is imaginary, so omega is the imaginary part, so the modulus is going to be square root of omega square plus z 1 square, square root of the sum of the squares of the real part and the imaginary part. So it is going to be square root of omega square plus z 1 square. Now this is the actual gain or the contribution to the gain of this 0 but remember, I have to take log to the base 10 of this and multiply it by 20.

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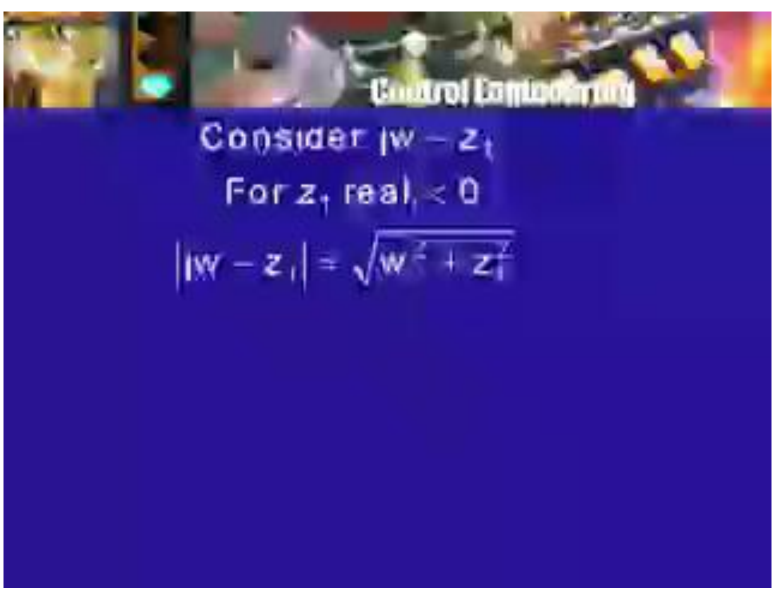

Now, if I take log to the base 10 of this what I am I going to get log to the base 10 of this because there is a square root it is going to be 1 half of log to the base 10 of omega square plus z 1 square

Right. Now this is going to be multiplied by 20. So the net result is that the db gain corresponding to a factor like j omega minus z 1, with z 1 the real and I have said less than 0 but way it does not really make a difference if z 1 is greater than 0, why because I am squaring it anyway. So the db gain is given by 10, log to the base 10 of omega square plus z 1 square.

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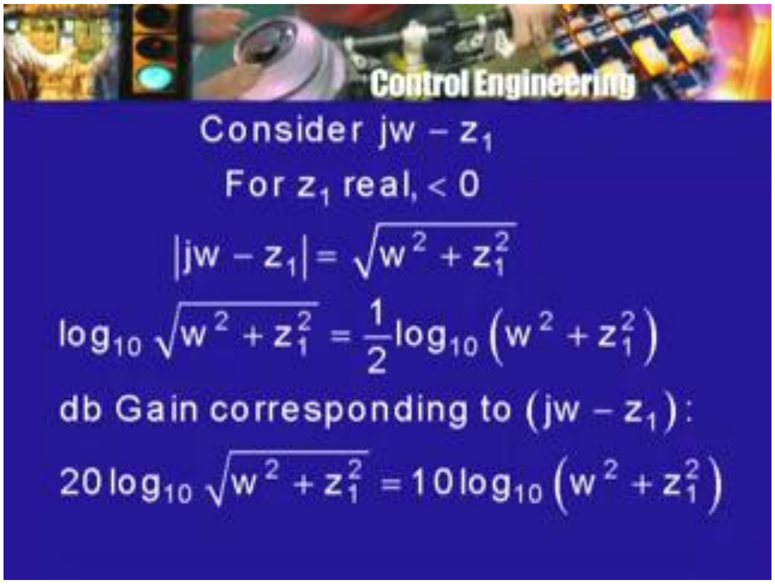

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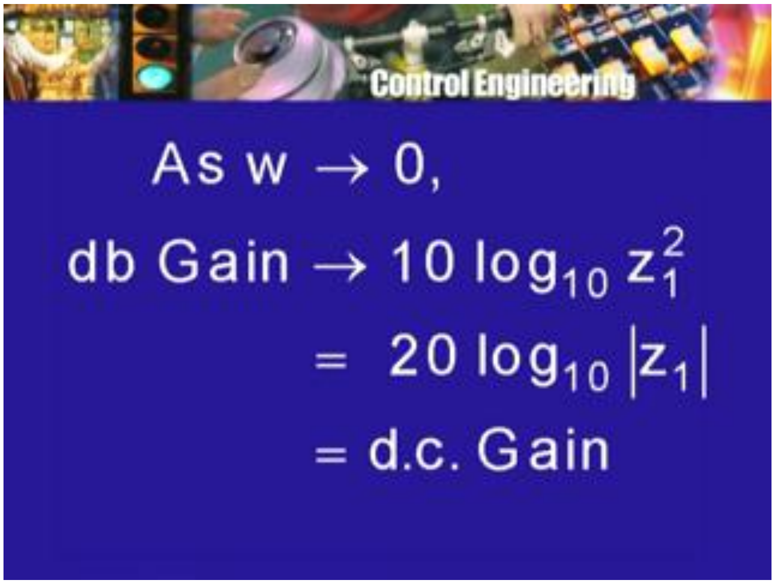

So this is the exact expression for the db gain corresponding to that factor j omega minus z 1. Now of course, if you are in a calculating mode, you cause change or take different values of omega and calculate these numbers. But it as said Bode's intention was not to do any detail calculations but to get some qualitative information. So as before we look at the 2 extreme conditions but we look at something else. So as omega tends to 0 the low frequency end on the dc end, what is this going to look like omega square is very small, z 1 square is what will remain I am assuming that z 1 is not 0, so z 1 square is a positive number omega square however, is going to 0.

So this whole this will approach 10, log to the base 10 of z 1 square or therefore it will be the squares of 20 log to the base to the 10 of z 1. So if I know the location of the 0 then 20 log to the base 10 of z 1 will give me the dc gain. So corresponding to this factor there will be a dc gain of this amount 20 log to the base 10 of z 1. So that is the behavior as omega tends to 0, now the behavior as omega increases without limit or as you say omega tends to infinity in that case, what about 10 log to the base 10 of omega square plus z 1 square.

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Now, z 1 is of course fixed it is omega which is now increasing. So this becomes negligible compared with omega square for a large omega therefore this whole thing comes down to 20 log to the base 10 of omega. So as omega changes the gain changes and of course, it changes logarithmically because the gain is db gain and not the original gain, no wonder, do not get worried by this log 10, we already taken log that is why there is log 10. Otherwise, the gain will be simply proportional to omega as we saw earlier, so it is as a function of omega it looks like log to the base 10 of omega.

Now here is where one more engineering example of engineering aptitude or approach comes in. If something is a function of log 10 of omega rather than omega directly of course I can say, I always calculate log 10 of omega, I need a calculator or a log table but why do not I consider log 10 of omega as the independent variable that is instead of frequency omega as say, so many radians per second corresponding to some practical frequency in hertz, why do not I think of log to the base 10 of omega as my independent variable that is instead of omega varying I am varying log to the base 10 of omega, this was the idea of that Bode used.

So for a moment suppose instead of log to the base 10 of omega, I think of this as a variable let us call it is a u, then if u is my independent variable the db gain is nearly equal to 20 u. As u tends to infinity geometrically, this is very easy to interpret because if a function is given by 20 u then, what its graph is going to look like. This as a function of u it is going to be a straight line passing through the origin with the slope of 20 right, this is exactly what Bode did. Now, when you plot log to the base 10 of omega rather than omega, this is said to be plotting omega on a logarithmic scale this is something which you may have studied but it does not matter, if you have not studied it, we will look at what it means, what it means is that I am going to show log to the base 10 of omega on a ordinary therefore, linear or uniform scale.

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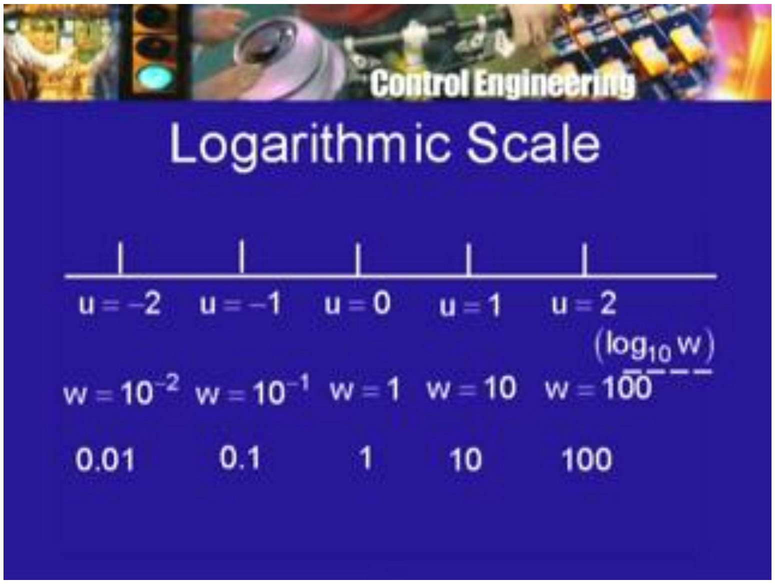

So suppose I have this scale here I have a straight line here which is my u axis now and I am going to call it log to the base 10 of omega axis then, let us say what are the various values of u that are possible. Now the first thing that you realize is that u cannot be 0, why because u is the logarithm so what are the values that are possible. So with omega equal to 1, what is the value of u, u equal to 0. So let us say here is the point u equal to 0, with some other value of omega I will get some other values of u, let us take omega equal to 10 and we see why, because log 10 of omega is 1, so u equal to 1, so here is the point u equal to 1.

Let us take omega equal to 100 that corresponds to log to the base 10 of 100 which is u equal to 2. Now I am going to draw our u is uniform scale on the u axis, so this will be u equal to 2, of course u equal to 0, corresponds to a frequency of 1 rad per second, u equal to 1 corresponds to a frequency of 10 and u equal to 2 corresponds to a frequency of 100 or 10 raise to 2 rad per second. Now, you will get imagine that instead of thinking of frequency in rad per second omega, we can think of f which is in hertz or cycles per second. However, this is a uniform scale so what about the point u equal to minus 1, what will be the angular frequency to which it will correspond, what will be omega such that log to the base 10 of omega is minus 1.

Well, it will be point 1 or 10 raise to minus 1 rad per second and similarly, u equal to minus 2 will correspond to a still lower frequency of point 01 Rad per second or 10 raised to minus 2 rad per second. So we have these various values of omega their corresponding values of u but omega equal to 0 cannot be shown here because strictly speaking, if I do calculate or take the limit of log to the base 10 omega at omega tends to 0, I will get what minus infinity.

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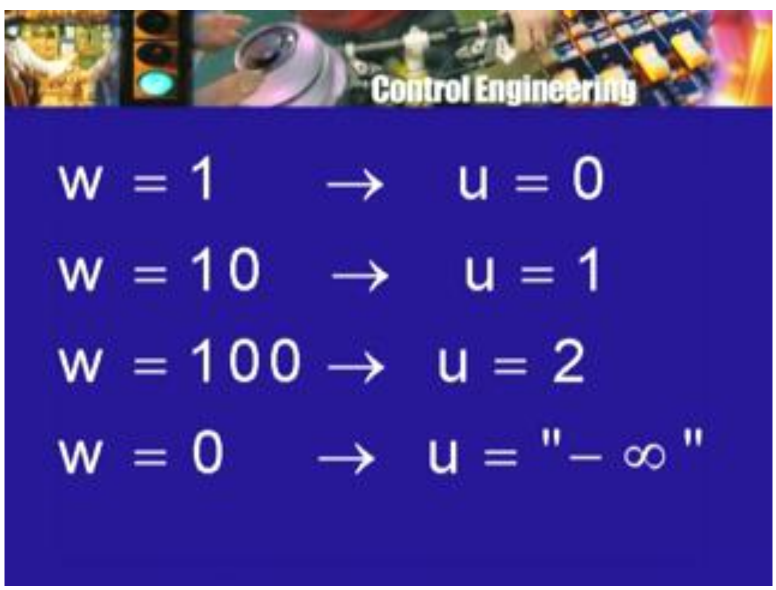

So that point will be ray out here towards the left. So I am not going to be able to show it. Now this interval say from u equal to 0 to 1, there is a change in the value of u by 1, what about the change in the value of omega, it is from 1 to 10 that is, it is by of factor of 10. Now instead of you saying factor of 10, one uses a term from music or acoustics which is called a decade. In fact the acoustic term is not really decade, the acoustic term is an octave, what we have here is a decade, u equal to 0, u equal to 1, omega equal to 1, omega 10 times that u equal to 1 to u equal to 2, the interval here as far as u is concerned is 1 but the frequency range is from 10 to 100.

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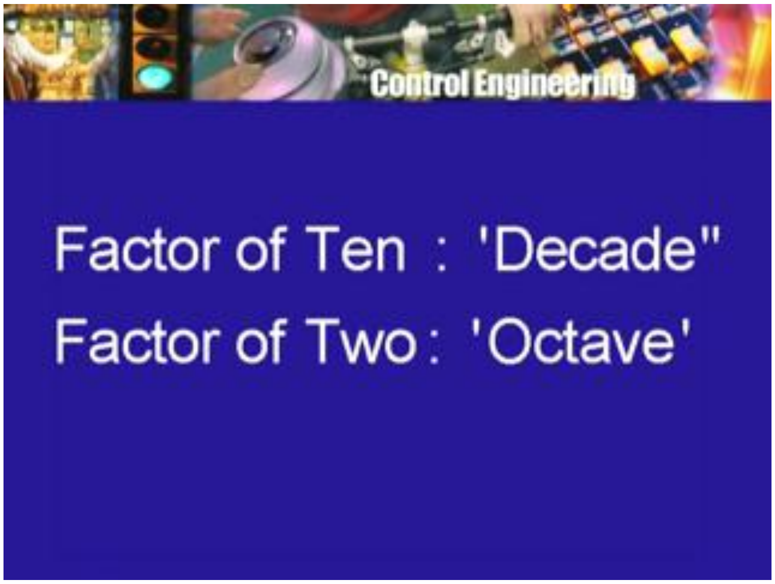

So here the frequency ranges was from 1 to 100 but the difference of only 9 radians per second where as here, it is from 10 to 100, a difference of 90 radians per second. So that is why one refers to what we have drawn as a logarithmic scale for omega although it is a linear scale for log to the base 10 of omega and this, all this is being done because in the magnitude expression or in the gain expression, we have log to the base 10 of omega and we sort of disguise it by calling it u, so that we get a linear function like 20 u.

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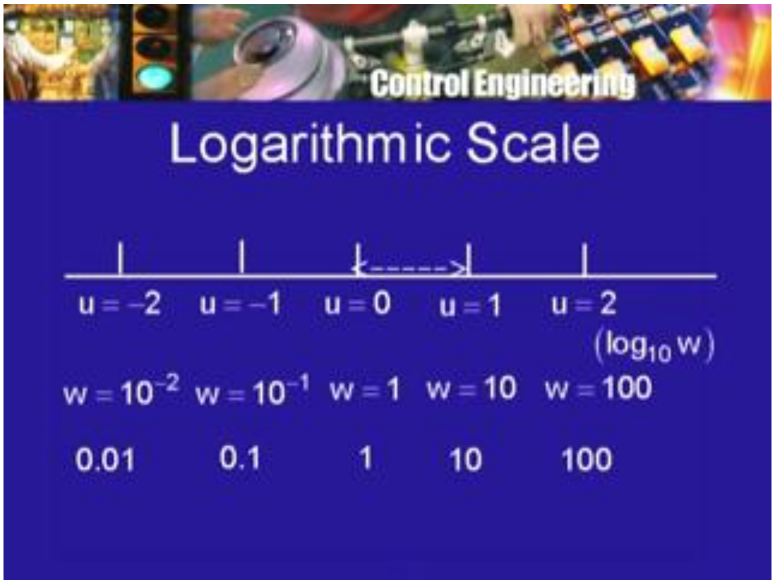

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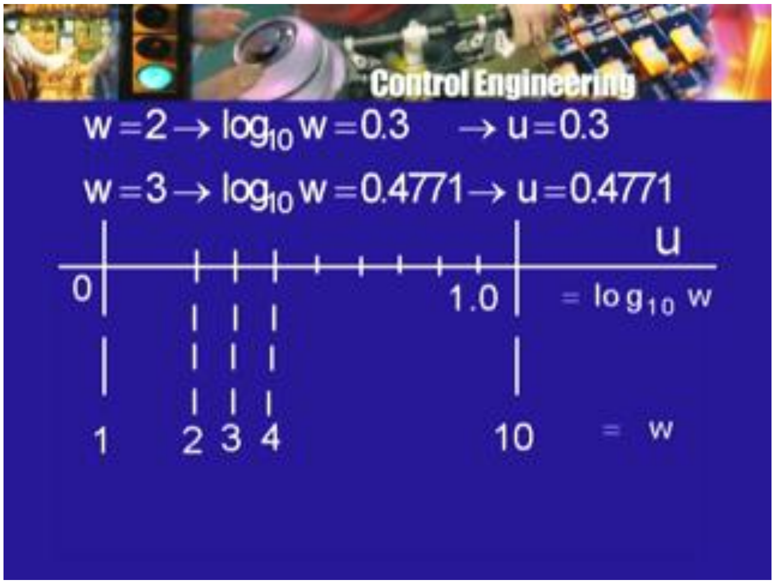

Fortunately, we have one can go to the market and buy a paper in which the horizontal axis is not only calibrated with the uniform scale, so to speak from one point to another which covers a range of a decade but with the points marking between in a very special way and such a paper is known as a what we need is a paper known as a semi logarithmic paper. So semi logarithmic graph paper is what, one can use in this connection. Now what is this nature of this graph paper let us look at this interval from u equal to 0 to u equal to 1 and let us look at some intermediate values of u. So here is u equal to 0, here is u equal to 1, so these are the values of u the values of omega are of course 1 and 10 remember, u is equal to log to the base 10 of omega right.

Now let us take some intermediate values of omega, let us take omega equal to 2 rad per second, what will be the value of u, log to the base 10 of omega is approximately, point 3. I hope you have not forgotten the logarithms of some of these simple small integers. So log to the base 10 of omega is point 3, therefore the corresponding value of omega will be point 3. Now u equal to 0 here, u equal to 1 here, so where will be the point u equal to point 3, it will be something here about 1 third the distance. So this corresponds to a frequency of 2 rad per second, so here is 1 rad per second, here is 2 rad per second, what about another point, let us say omega equal to 3, what is log to the base 10 of 3, it is .4771.

So the point corresponding to it will be almost half way. So this will be 3, what about 4? 4 of course is square of 2, so the log to the base 10 of 4 is nearly point 6 and so the point will be here, so this will corresponds to 4, what about 8? 8 is the cube of 2, so it will correspond to u equal to .9, so I will be very close to the 1 point here, so here will be 9. See if I now, draw the ordinates through these points. So here is the ordinate corresponding to 1, here is the ordinate corresponding 2, 3, 4, 8 and this is the ordinate corresponding to 10. So these ordinates get closer and closer to each other as we proceed from the lower end of the decade, say 1 to the higher end of the decade 10.

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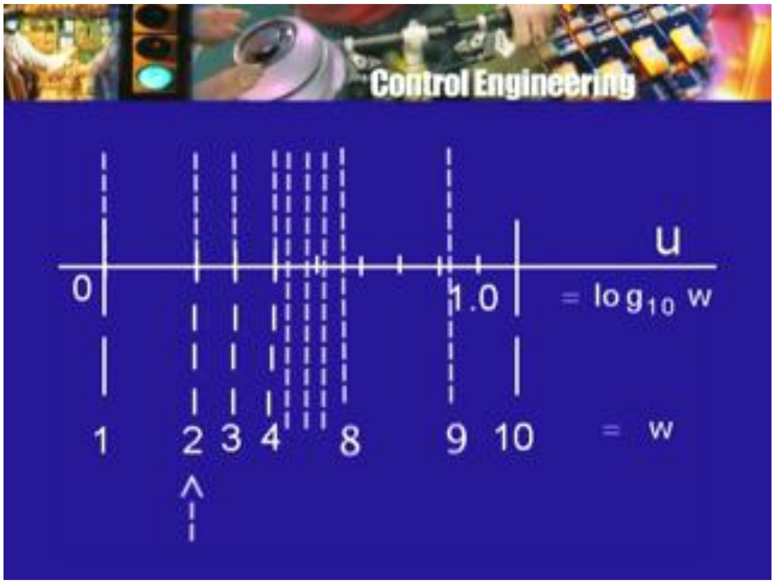

So if you look at the semi logarithmic paper the vertical lines, if you hold the paper properly will not be equally spaced unlike an ordinary gridded graph paper but it will have non-uniform markings of this kind of course, no numbers will written there. So you have to look at it and then interpret it appropriately that starting at 1 point you have a decade then the successive markings are 2, 3, 4 of course 5 is .6991, so its somewhere here and 6 and 7 and 8 and 9 and so on. So the ordinates get sort of crowded together as you proceed from one end of the decade to the other end of the decade. In fact this is something you have to learn the first time you have used the semi log paper, you can use it upside down.

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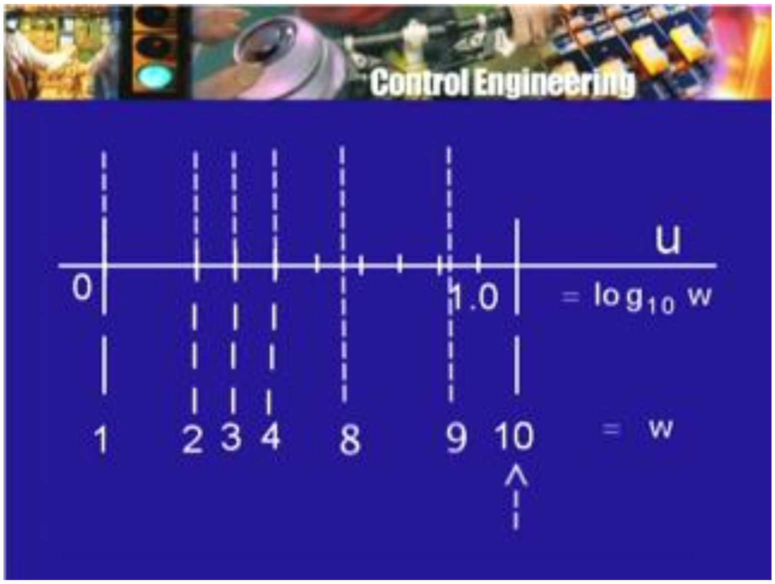

So you can do it this way and then this will correspond to 1 and this will correspond to 10 but then these markings are not correct, this is the mistake which you students very often do. So first of all you must hold the semi logarithmic paper properly and then use this semi logarithmic axis for writing down the values of omega and the typical way of doing it is to start with the frequency, which is a frequency like 1 or 10 or point 1 and then, go to the next decade, mark it down as say 10 then, the next decade mark it down as 100, the previous decade mark it point 1 and so on. So in other words what you may end up doing then depending on the frequency range of interest will be something like this.

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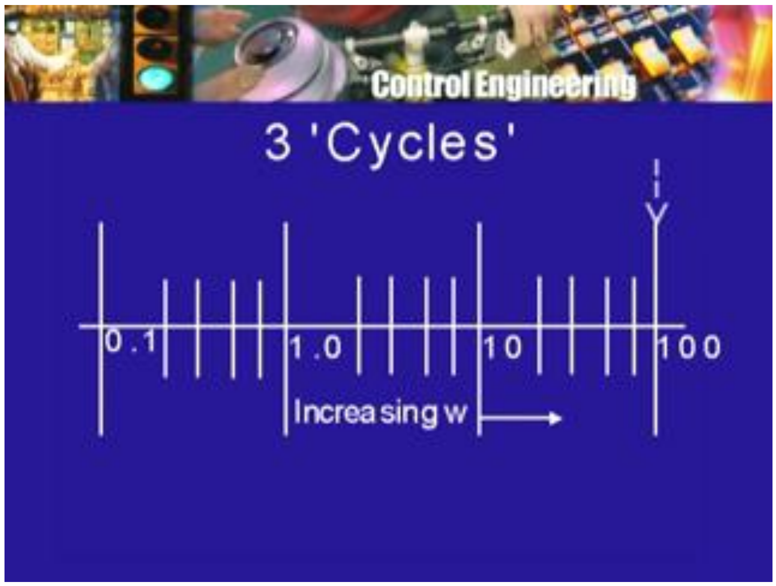

So, let us say here is my 1 here is 10 here is hundred and here is point 1, so I have used 3 decades or 3 cycles then, there will be this ordinate here, there will be ordinate here, there will be the ordinate here and the ordinates here will be in what way, 2 will be nearly in point 3, 20 will be here, point 2 will be here, then 3 will be closer, 4 will be closer, 5, 6, 7, 8, 9 like that. Similarly, 2, 3, 4, 5 like that, here also 2, 3, 4, 5 like that. So they get closer and closer as we proceed from one frequency to a frequency which is 1 decade above it to another which is still 1 decade and the third one which is a 1 more decade above it. The total range covered is a frequency range from say 1 radians per second to 100 radians per second. This is typically the frequency range that at 1 uses in control system practice, you gone really ran into kilo hertz or mega hertz frequencies or corresponding frequencies in radians per second. So this is the semi-logarithmic paper, so get for your self a semi logarithmic paper and we will do some plotting on the semilogarithmic paper when we look at a Bode plots.