Control Engineering
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Lecture - 40

Let us go over an application of Nyquist criterion to the motor speed control problem once again.

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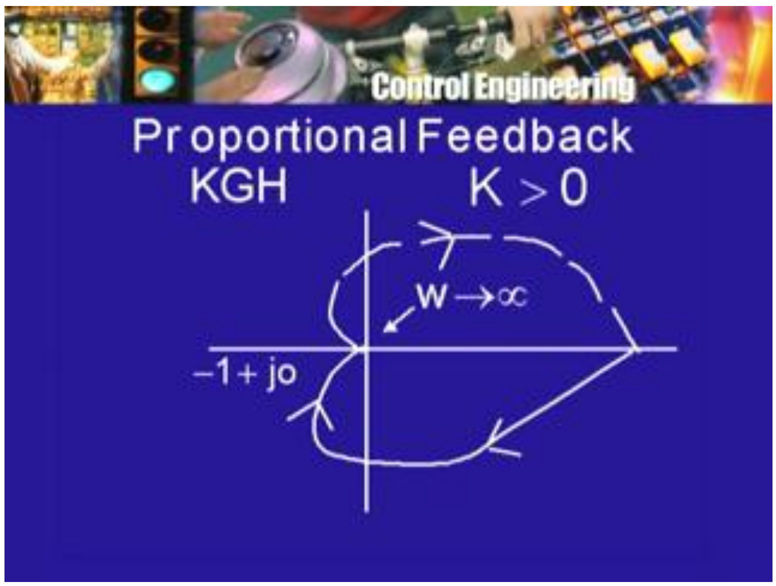

We first looked at the proportional feedback situation where the loop transfer function K G H had 2 poles and no 0, as a result of which the polar plot or the Nyquist plot had the following appearance, at DC that is for very low frequencies there was some definite gain but the phase shift was nearly 0 and as the frequency increase, the gain changed and eventually for large enough frequencies, the gain was very small and so this was the image of the top half of the j omega axis under this function K G H and then, we looked at its mirror image which is the image of the lower part and as I told you the image of the semicircle is virtually region very close to the origin.

So this is what it looked like and the whole contour was stressed in the same sense as the original contour, the number of encirclements of the point minus 1 plus j 0 which was out here. The number of encirclements of this point was 0, G H had also no pole in the right half plane. As a result the characteristic polynomial had no poles in the right half plane and therefore the system was stable and if you increase K, it is clear that this contour is going to blow up but in no case is it going to encircle the point minus 1 plus j 0 that is no matter by how much you increase the gain K, the system will remain stable. Of course, if you reduce it then it is definitely going to remain stable. So in other words for all positive values of the gain K, the close loop control system using proportional feedback is stable whereas the case of the integral feedback was different because now in addition to the G H that we are already there, we had put a 1 by S or a K by S in front and this was because of the integrator.

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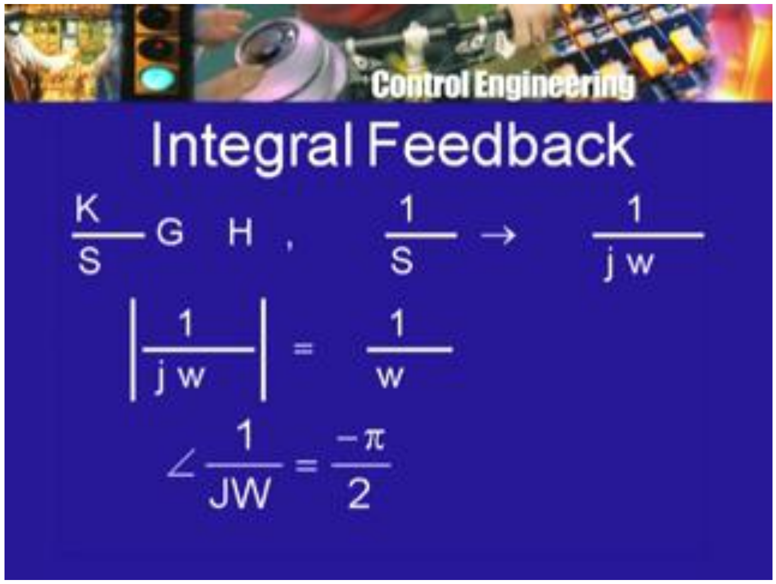

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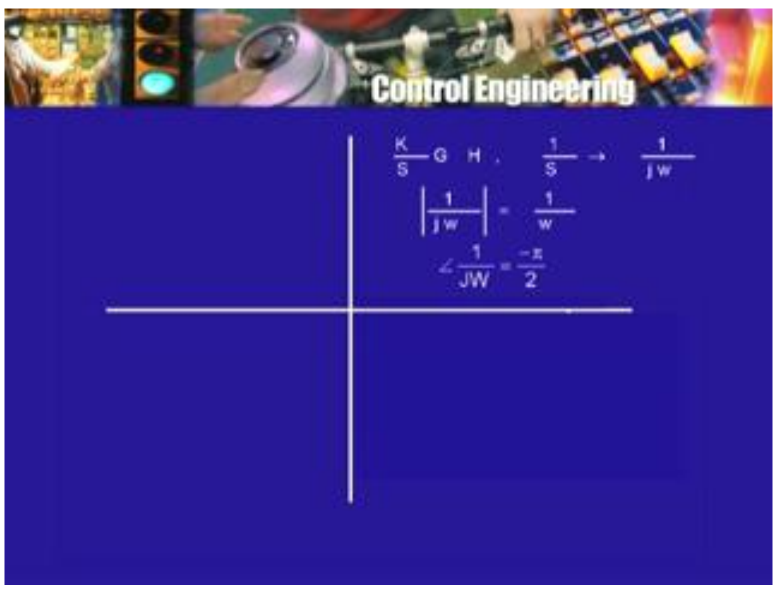

So the root transfer function was now K by S in to G H this S, 1 by S contribute 1 by j omega to the frequency response and I told you that the modulus of this 1 by j omega is 1 by omega that goes to 0 as omega tends to infinity and the phase angle or the argument of this however, remains constant at minus phi by 2. As a result there is a phase shift of minus ninety degrees which is introduced in the loop transfer function and there is a change in the gain of course, which goes to 0 as omega goes to infinity but if you have noticed carefully what I did last time, there was a small mistake that I committed there. This, in fact there were 2 mistakes which have to be noted, there were there was this factor 1 by j omega. Now let me repeat the modulus of 1 j omega is 1 by omega now, for a small omega this is very large.

So in other words, the gain, the loop transfer function gain at very low frequency its going to be very large, its not going to be some finite constant, in fact it is going to be increase as we approach 0 from above that is as omega tends to 0, you consider smaller and smaller frequencies, the gain will be greater and greater. So that was one mistake. Secondly, what is the argument of 1 over j omega as I told you just know because of this factor j in the denominator, the argument of this is minus phi by 2. So in this diagram 2 things are going to be added or acted upon it first of all there will be an additional phase shift of minus phi 2. Secondly, there is a gain which is going to be very large for small frequencies that is corresponding to this point and it is going to be approaching 0 for very large frequency.

As a result, the polar plot is not going to look like what I had drawn earlier but it is going to look as follows. Remember, at very low frequencies the 1 by j omega contributes a constant phase shift of minus phi by 2 whereas the other 2 terms that is the other part of G H constitute a phase shift of nearly 0 degrees. So the total phase shift at low frequencies is not 0 but it is minus phi by 2 and the gain at low frequencies include the integrator is not a finite number but it is going to approach infinity. As a result at very low frequencies, the polar plot is going to start of something like this. This indicates a large value of gain and a phase shift which is very nearly 90 degrees. Of course, I cannot draw a very big diagram here. So you have to imagine that this is really going towards the negative imaginary axis side.

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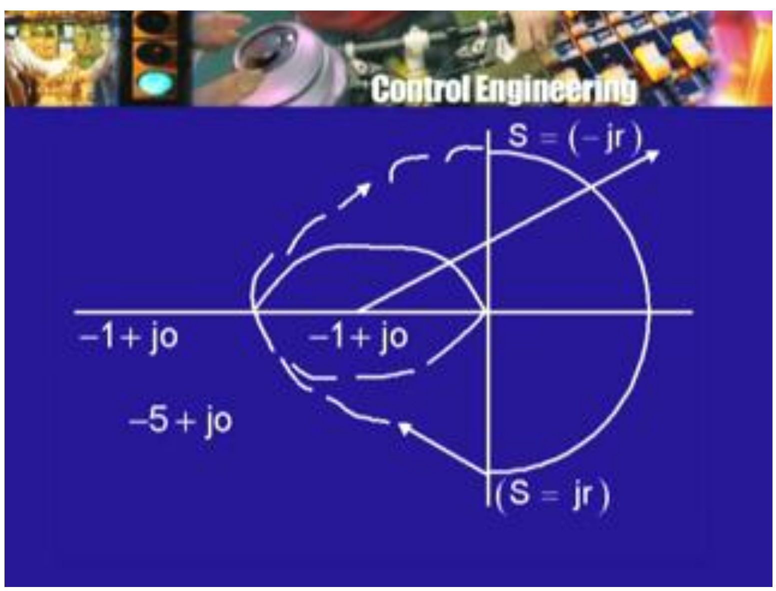

So it starts of from here at low frequencies, at high frequencies the phase shift is minus 3 phi by 2, the gain is virtually 0. So the diagram is going to look like what I had drawn earlier but in between it will have an appearance of perhaps this sought. So this will be the variation of the K by S, G H at j omega as omega varies from 0 to infinity that is the upper half part of the imaginary axis on our Nyquist contour gamma. Now, of course for the lower part we are going to have a mirror image of it. So I will draw the mirror image so it is going to be like this and then it will go this way and as before for very large frequencies, the plot is very close to the origin. So I can just show it by a big dot here in other words, this curve is very close to the origin this is what it is suppose to indicate, okay.

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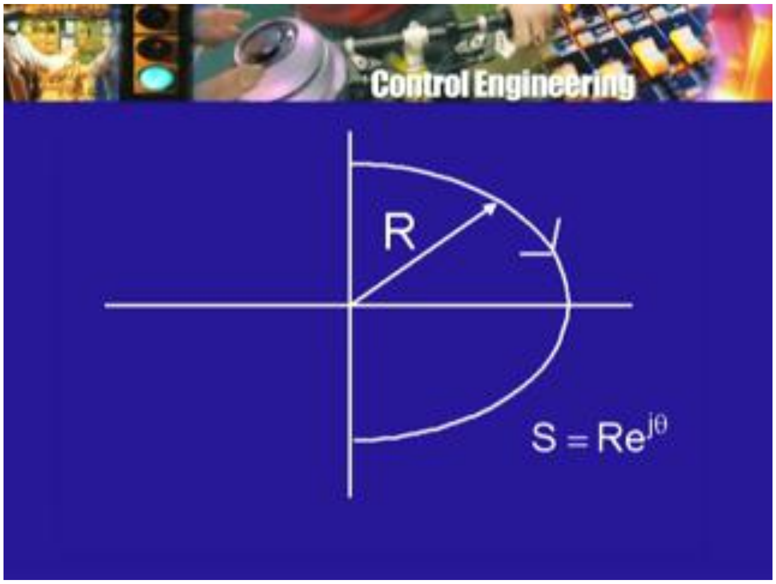

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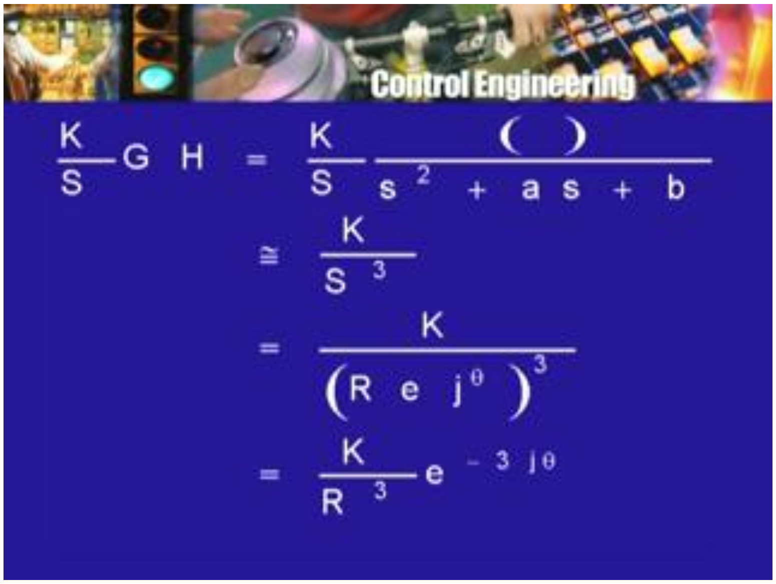

Now, I have to close the curve, so this is where one more thing that comes in to the picture. Along the semicircle, so let me draw the semicircle now once again, here is the semicircle, here is the radius R, so what was S? S was equal to R e raise j theta. Now I have K divided by S into G H, as we saw G H has no 0s. So there is a constant here and divided by a quadratic s square plus a s plus b is what I wrote earlier. Now as a result of this for S very large in magnitude which is what we are talking about on the semicircle, this can be approximated by K divided by S cube right because s squared over power S and constant b for very large s, large meaning large in absolute value. So there is this S and there is s square as a result we have K divided by S cube.

So K by S, G H is approximately K divided by S cube for large S that is points on the semicircle. Now what happens to S cube, S is R e raise to j theta as a result of that if I substitute then, I will have K divided by R e raise to j theta raise to 3 or this is therefore equal to K divided by R cube e raise to minus 3 j theta. So as theta changes when I move along the semicircle, theta changes from phi by 2 to minus phi by 2 therefore minus 3 theta will change from minus 3 phi by 2 to plus 3 phi by 2.

Now that is not going to effect our earlier assumption very much because of this factor K by R cube in the denominator because R is very large, K by R cube is very small. So the conclusion is that what is happening at this point, there is a lot of phase shift that is taking place but the magnitude is very close to 0. So we do not really have to go in to any detail as far as the behavior here is concerned. So this is why, I will still continue to show a big blog here as the image of that semicircular part. So that is all right but there is one more thing which I did not mention and we need to be careful about it because this is a mistake which typically students commit and that is the following, what we are looking at is the function which looks like K divided by S in to G H.

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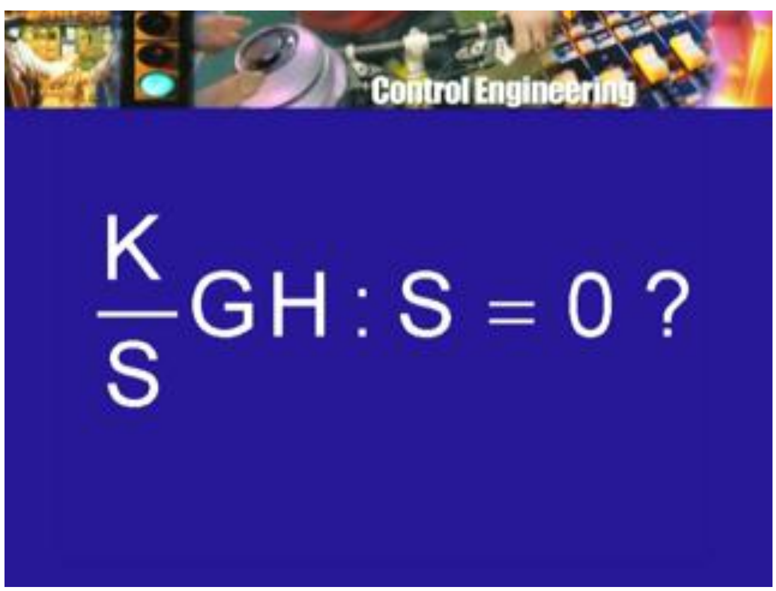

Now, what is this S this S indicates that this loop transfer function has a pole at the origin. Now remember ,one of our conditions for application of principle of the argument was that the contour should not pass through either a 0 or pole of the rational function but here is a pole of the rational function at S equal to 0, namely at the origin. So the contour that I choose earlier is no longer applicable and therefore I have to make a change in the contour, so as to avoid this pole at the origin, now this is sometimes called bypassing the pole and this we can do as follows.

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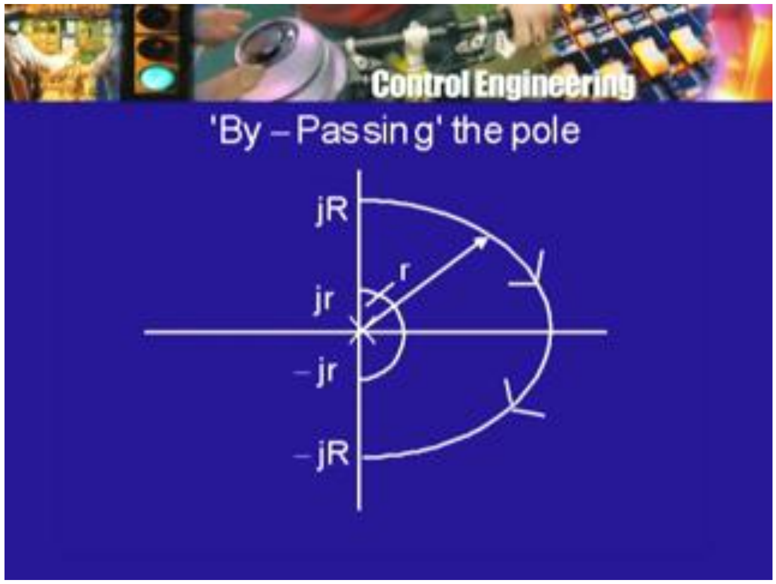

So here is the complex plane, I have to avoid this point, I cannot choose it 1 the contour. However, I will still have to make use of the negative and positive imaginary axis as before. Now this can be done as follows think of a small semicircular, now I am showing it big, so that you can see it clearly. But think of a small semicircle in other words, I am consider the limit as the radius of the semicircle goes to 0. So think of a semicircle here like this this after this we follow the positive j omega axis as before go up to some point here let us say j R, R is the radius of that semicircle. So I swing around that semicircle as before come to the point minus j R as I did earlier then, go up the j omega axis. Let us call the radius of this semicircle small r, so that this point is minus j r and this point is plus j r.

So the contour now consists not just of the straight line part of the j omega axis and the semicircle but it consists of 3 parts or if you wish 4 parts that is this small semicircle which is like a bypass or a detour in to the complex plane, the right half of the complex plane, the right half plane then, there is the big semicircle which we had as before and then, there are these 2 parts of the j omega axis which are symmetrically situated. Now, what is effect of this? The effect of this is the following suppose I start tracing the contour at some point and then find out the value of G H of j omega and do that plot that as a point on the polar plot, what is going to happen is the following this was the polar plot that I had drawn earlier and I said that you are starting of somewhere at a phase shift of minus phi by 2 and a gain which is very large.

Now this corresponds to the image of the original point j in to small r, in other words this corresponds to this location plus j r similarly, this other point here will correspond to the point minus j r on the Nyquist contour. I have changed the Nyquist contour, I have bypass the origin this way, on this bypass the variation is given by s equal to small r e raise to j theta. Now, when I am at j r my image is here, when I am at minus j r, the image is here but what happens when I go from here to here along this semicircle. Now that is something I have to figure out on that semicircle s is equal r e raise to j theta and once again, we have to look at K divided by S in to a constant divided by s square plus a s plus b for s given by r e raise to j theta, where r is small because of this now in the quadratic, I can ignore s squared, I can ignore a s and what remains is only the constant here and of course, there is a constant in the numerator.

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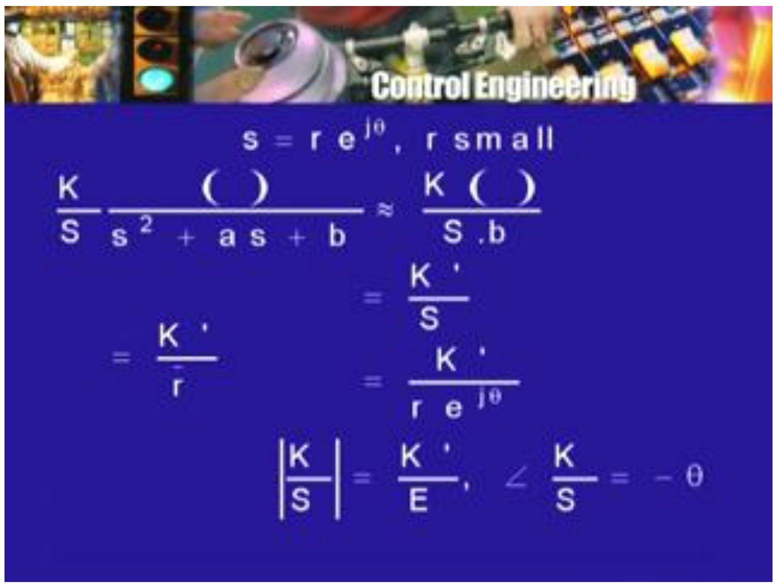

So, this whole thing looks approximately like say another constant K dash divided by S. Now I substitute for S K dash divided by r e raise to j theta, now if I look at the modulus of this the modulus of this is given by K dash divided by R. So when R is small, the modulus is going to be large and what is the argument of this? The angle of this, the angle of this is just minus theta because of this e raise to j theta in the denominator. So the angle is going to be minus theta where theta is the angle of the corresponding point here. Now when I move along this bypass what is the variation of theta, theta is varying from minus phi by 2 to plus phi by 2 in the counter clockwise sense. In other words, minus phi by 2 minus phi by 4 0 phi by 4 phi by 2 but what is the angle of K by S, G H doing it is given by minus theta.

So it is going to change in the opposite sense. So corresponding to this point where the phase is minus phi by 2, the phase of K by S G H will be plus phi by 2 and that is indeed correct I am here, corresponding to the other point j R here, the phase is plus phi by 2 K by S G H we have a phase of minus phi by 2 and that is correct I am here but in between now when I move along this semicircle, the modulus is remaining nearly constant because the modulus is K dash by r but the phase is now minus theta therefore, you will see that the phase will change from phi by 2 to minus phi by 2 in the opposite sense and therefore I have to complete contour in this way. So this is now the entire image gamma prime of this contour gamma. For this small semicircle the image turns out to be this large semicircle and the sense of traverse is just opposite to the sense of traverse here because of this minus theta in the argument, whereas the image of the big semicircle is a small thing very close to the origin. In fact, it is not a semicircle it is a circle it is in fact one and half times a circle because of that S cube in the denominator but the magnitude is small, so it does not matter.

The image of this upper half of the j omega axis is this part omega starting from small value and going to infinity. The image of the lower part of the j omega axis is this dashed curve and now, we have a close contour notice that this is not a simple close curve. I had told you earlier that when we apply Cauchy's principal of the argument, the image curve may not be a simple close curve but the contour that we choose in the Z plane, we do choose to be a simple close curve. This is a simple close curve whereas the image is not a simple close curve.

Now we have to look at the encirclements of by this on of the point minus 1 plus j 0. Now let us suppose the particular value of gain K that we have chosen and the transfer function G H is such that the point minus 1 plus j 0 lies here and this point of intersection of the image contour with the negative real axis. Let us say for simplicity it is minus point 5 plus j 0. So let us assume that the gain K that we have chosen and the parameters of the system are such that this is the image of the Nyquist contour, this is our gamma prime and the numerical values are such that the point minus 1 plus j 0 is out here. In this case now, what is the number of encirclements by gamma prime of this point minus 1 j 0, it is 0. So the number of encirclements is zero.

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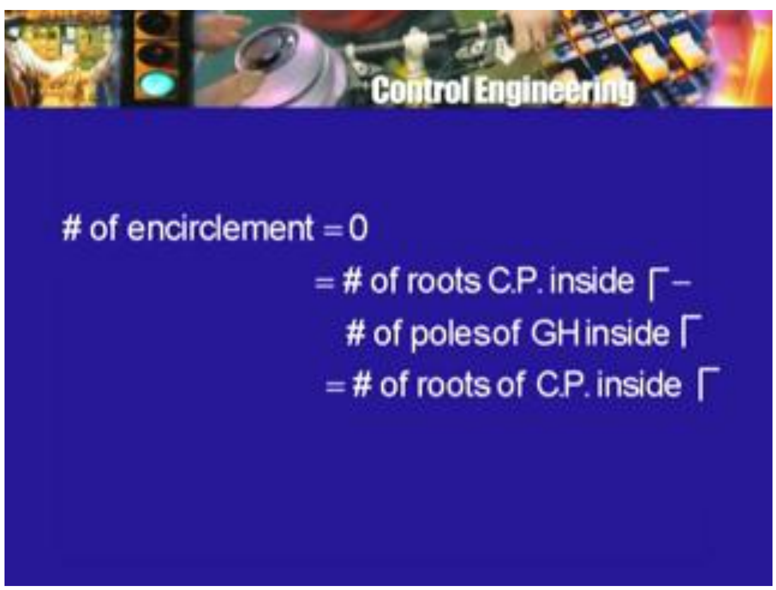

So the number of encirclements is 0, this is equal to what, this is equal to the difference between the number of roots of characteristic polynomial inside the contour gamma minus the number of poles of G H inside gamma. Now, let us look at our contour gamma once again. Gamma is it the entire right half of the Z plane not quite because it is this region but if this radius r is very small and if this radius r is very large then, whatever roots are there in the right half plane of whatever function or polynomial will be included in this contour. So this is the whole idea of arguing that r the radius of this semicircle is to be made very large or to be thought to be very large, the radius of this semicircle is to be considered to be very small. This is because whatever roots that we are thinking of the rational function will all be captured inside this contour and that they will be, if R and capital R is chosen sufficiently large, little r is chosen sufficiently small.

However, in our case there are no poles of G H in the right half plane and therefore this number is 0 and therefore this is simply equal to the number of roots of the characteristic polynomial inside gamma. So the number of roots of the characteristic polynomial inside gamma is simply 0 which means that for this particular value of K, for which the polar plot or the image of the Nyquist contour looks like this, the system is the close loop system using proportional and integral feedback is stable. Now as I said earlier we may have to think of values of K which are not decided upon beforehand, in fact we have to try and see whether we can increase the gain K or decrease the gain K or whatever, what was the reason for introducing integral feedback in the first place.

If you remember, the reason for introducing integral feedback was to reduce the steady state error due to disturbances in the torque to 0, there is no matter what change in the torque takes place in the steady state, there will be no steady state error in the speed if we have integral feedback. But as we saw earlier with the root locus method with the use of integral feedback, there was this possibility of instability if K becomes too large. Now you can see that using the Nyquist approach also for a particular value of K that you have chosen this point was .5 plus J 0.

Now suppose I double the value of K then, what is going to happen this whole figure is going to be scaled by a factor of 2, as a result this point of intersection will move out to minus 1 plus j 0 which indicates that characteristic polynomial has a root which lies on the j omega axis. So this is a conclusion that we can draw, if the image of the Nyquist contour goes throw this point minus 1 plus j 0. The conclusion is that there is an intersection in the root locus language of the root locus with the j omega axis or in terms of characteristic polynomial root, there will be roots of characteristic polynomial on the j omega axis and what will be the value for K. Well, what was the original value, the double of that will be the value of K. So you can think of it as some kind of a critical value of K.

Now suppose, I increase the value of K beyond that critical value. Suppose, I make the value of K 3 times then what is going to happen this figure is going to blow up further. As a result this point minus 1 plus j 0 which clearly lies outside of this contour is no longer going to be so and now instead of drawing a figure on the same scale because then, my image contour will just run out of, go out of the paper, I am going to ask you to think of a change of scale. So that the contour remains the same but the point minus 1 plus j 0 changes its location.

So suppose I increase K by a factor of 3 then, I can show it effectively by showing the point somewhere here. So my point minus 1 plus j 0 will now lie here for that value of K. In other words I have increased K by a factor of 3 from the original value and now the point minus 1 plus j 0 lies here. Now what is number of encirclements, as you can see if you trace the curve or if you trace the image contour, it is in the same sense as the original contour but the number of encirclements is 2. I can also do it by drawing a ray from this point in any direction finding out the number of intersections of this ray with the image contour, the number of intersections in the same sense is also 2.

So, now for this new value of K the situation is different the number of encirclements is not 0 but it is 2, of course that is still the number of roots of the characteristic polynomial inside the contour gamma minus the number of poles, the number of poles of course are 0. So that is still equal to the number of roots of the characteristic polynomial but that is no longer 0 but it is 2 and what does it mean? If the characteristic polynomial has even 1 root in the right half plane it means that the system is unstable. So for this value of K that I am thinking of which was 3 times the original value. The system has become unstable, this point originally was point 5 plus j 0, so if I double the value of K then, this contour will pass through the point minus 1 plus j 0 that is the critical value but if I increase it further then, it is going to be enclosed by the contour, the system is going to become unstable.

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Now this possibility of increasing the gain the system being unstable but if you increase the gain not too much, the system still remaining stable is reflected in an expression that one uses in this context and that is known as the gain margin of the system and that is for an obvious reason because if this was the plot for a particular value of K then, I could increase the gain by a factor of 2, not add but multiply by a factor of 2. When I do that this point will move out here and the system will be on the verge of instability. I would have reached the critical value of the gain K and so the gain margin in this case will be by a factor of 2 or you can covert that 2, a factor of 2 in decibel language you can think of it as 20 log to the base 10 of 2 and that is therefore nearly equal to 6 decibels.

So one speaks about in this case that if for a particular value of K this was the polar plot, this was the location with a point minus 1 plus j 0, the point of intersection here was .5 plus j 0 then in this particular situation the gain margin is of a factor of 2 or 6 db. So one says that the system is stable for that value of K that we had chosen and there is a gain margin of 6 decibels that is we can increase the gain up to a factor of 2 and the system will still remain stable but we increase it beyond that factor of 2 or you increase it beyond 6 decibels, the system is going to become unstable.

Now as a result of the work by Nyquist on this problem and in particular, they were not be basically interested more in feedback amplifiers rather than feedback control system, gain margin has become 1 of a possible specification of the control system that is when you specify a control system requirements, we talked about them earlier for example, what were the control system specifications like, rise time, settling time, maximum percentage, overshoot percentage, steady state error that you can tolerate and so on.

This is one more performance specification that which does sometimes get added that is you can say that the system design should provide with a gain margin of 6 db, why because it may happen that during an actual operation, the gain K maintained at the value that you had chosen, it may accidentally go up or it may go down. Usually, of course going up causes problem for example in this case if the gain K is reduced then, this is going to even shrink the point minus 1 plus j 0 is going to remain well outside this contour, the system is going to remain stable and of course, we knew that before hand from the root locus approach. So this is the significance of the term gain margin and this may be one of the specifications in the control system design.

Now in the case of an amplifier one is not only worried about the gain but one is also worried about the phase shift, in fact phase shift usually it depends on frequency, it varies from one frequency to another similarly, the gain varies from 1 frequency to another. An ideal amplifier is one whose gain and phase shift remain constant over a large band of frequencies but practical amplifiers are not ideal and therefore the phase shift as well as the gain can change. Now just as there is a possibility of the gain of the amplifier because of some reason changing there is a possibility that the phase shift that the amplifier may introduce.

I have assumed that the gain K was just a pure positive constant but any actual electronic amplifier at a particular frequency may have a gain K which is not just a change in amplitude but also a change in phase and therefore, just as one talks about gain margin because of possibility of a change of gain, Similarly, amplifier design people at that time in the 1930's and 40'sintroduced the concept of phase margin because of the possibility of a phase shift. Now that is a little difficult to consider here, let us say this was the polar plot, the gain has not change but the amplifier has introduced a phase shift. For example, the integrator the integrator, ideal integrator has a transfer function 1 by S or K by S, the practical integrator however does not have this ideal transfer function. As a result of which the practical integrator will produce a phase shift not just of minus 90 degrees but of a slightly different amount which will change with frequency.

Now if that happens then imagine that this polar plot part of it which corresponds to the change in frequency from 0 to high frequency is going to suffer not only a change of scale but it is going to suffer a change of phase that is the whole diagram may sought of rotate around this figure that I have drawn and therefore, this point of intersection which I have shown as .5 plus j 0 may change for you all you know, it may become a larger amount and in fact you can see how that is going to happen, if this figure were to turn in the counter clockwise direction from my diagram it looks like, the point of intersection may move further to the left of the point .5 plus j 0.

So in other words, if there is a phase shift which is positive the whole diagram will turn as a result the point of intersection may go further towards this point minus 1 plus j 0, this minus 1 plus j 0 is the danger point. So a change in phase shift may take this point of intersection closer to minus 1 plus j 0 and of course, if the phase shift is very large and if the particular system is such that with that large amount of phase shift, this diagram turning may even may even pass through the point minus 1 plus j 0. So the system will become unstable because of this unforeseen phase shift introduced by the amplifier and therefore one may specify a certain amount of phase margin for the design that is you should stay sufficiently away from the point minus 1 plus j 0 both with respect to a change in gain and with respect to a change in phase shift.

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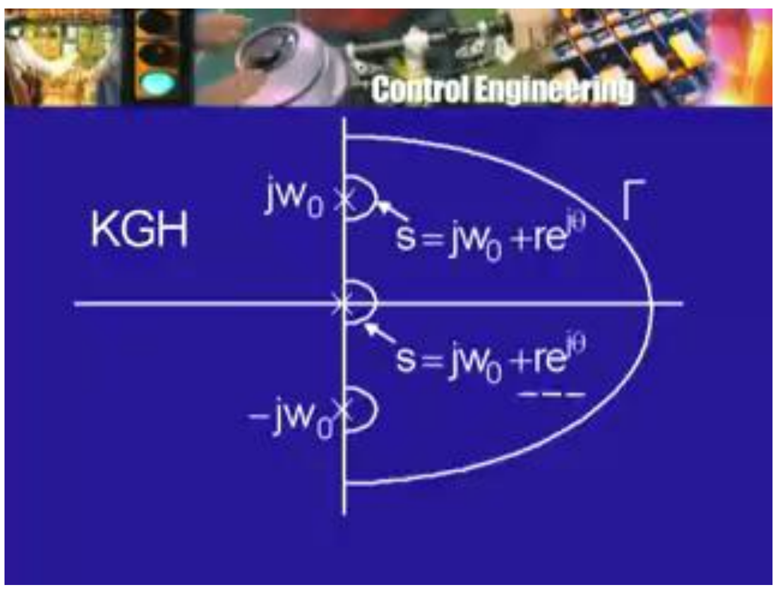

So before, we go further let me remind you about a few pitfalls, when applying the Nyquist criterion, the most important one which I brought to your attention just now is the proper choice of the Nyquist contour. We are using the principle of the argument and because of that the contour must be chosen to avoid any poles and 0s of the rational function that we are talking about. As a result whenever you have a transfer function which may have a pole at the origin or very rarely a 0 at the origin, one has to choose a contour which will avoid that pole or the 0. In very exceptional cases, there might be poles of the loop transfer function which are not at the origin but which are on the j omega axis or there could be a pole at the origin in addition there may be a pole at the j on the j omega axis and therefore not one pole but 2 poles because there are always symmetric.

So the situation may be like this the G H transfer function has a pole at the origin and it may also have a pair of poles like this. In this case, how am I to choose the Nyquist contour well it should be not very difficult to see what is to be done just as we avoided this pole by going around it, by using a bypass neutral speed, one of my professors used to call it digging a canal that is you have to bypass this by digging a canal around it.

Similarly, you have to bypass this pole by digging a canal around it and symmetrically, this pole by digging a canal here and then, of course we have this infinite semicircle or what we have to really imagine as an infinite semicircle. So the contour now has this infinite semicircle has one part and these 3 circles of small radii as other parts and the j omega axis has these 4 portions now. So from a frequency which is very close to 0, to a frequency which is very close to the location of this pole but less than it then, a frequency which is just greater than this frequency increasing all the way to as high value as possible and then symmetrically on the other side. So this will be the Nyquist contour for such a transfer function.

So, if my K G H is such that it has poles like this then the contour that I must choose must look like this. Then, of course when I look at the image contour I have to be careful because what happens to these 3 by passes or canals. Now in the case of this bypass it was very simple S was simply give as R e raise to j theta and therefore we could make an approximation for K G H along this contour, what about this bypass and this bypass. Well, you can see that if I call this pole j omega 0, so that it corresponds to the angular frequency omega 0 then, what is going to be a point S on this bypass. Well it is going to be j omega 0 plus R e raise to j theta, j omega 0 is not a small complex number small n modulus because omega 0 is not necessary small but to that we are adding this part whose modulus is small.

So substitute this in the expression for K G H and then, see what is an approximation to it considering the fact that r is small. Now this require some careful work and we do not have time to go in to all the details and in any case such examples are very rare but one should not get to be at loss as to what to do, when the loop transfer function does have a pair of poles on the j omega axis. You have to choose the contour gamma carefully and for these bypasses, we have to look at the behavior of K G H as to what kind of an approximation we can make because of the fact that the radii of these semicircle are small just as the radius of this semicircle was large and as a result of which we could make some approximation to the behavior of K G H. So this is something which needs to be remembered.

The second is the orientation or the sense of traverse around the contour, there is the contour gamma for which as I told you usually the preferred sense of traverse is the clockwise traverse and why was that, that was because on the j omega axis you go form low frequencies to high frequencies and typically, when you plot the frequency response of an amplifier you start at the low frequency end and then, you think of increasing the frequency to the high frequency end right. So as the only reason why the contour gamma is chosen to have the clockwise sense. Now the contour gamma prime what is going to be its sense of traverse.

Now that will depend on the transfer function K G H, we took an example where it was the same, the sense of traverse of gamma prime as well as gamma was both clockwise, as a result the number of encirclements ended up to be a positive number. But if the sense is opposite the number of encirclements has to be attached to a sine. So that is another thing that you have to remember, the sense of traverse must be considered and taken in to account and therefore the number of encirclements may have to be taken down as a negative number, negative integer rather than a positive integer. So this is the second thing that you have to watch again.

The third is in the principle of the argument application we have to look at number of poles of G H in the right half plane. Now usually K G H will not have 1 any pole in the right half plane but there can be an exceptional case, where K G H does have a pole in the right half plane. So in that case I should not forget that number and therefore when I apply the principle of the argument number of encirclements equal to number of roots of the characteristic polynomial minus the number of poles of G H in the right half plane that number may not be 0. So I have to keep that in mind. So these are 3 important considerations you should not forget, first is the proper choice of the contour gamma, the second is the proper orientation or sense of gamma and gamma prime and properly compare the two as to whether gamma prime is in the same sense as gamma or not third is any right half plane poles of G H must be considered.

The fourth will be of course the point of intersection of the image contour gamma prime with the negative real axis because as we saw in our example, it is that point which turns out to be very critical as shown here, what is happening here is usually of no concern to the encirclement because the encirclement is at the point minus 1 plus j 0, what is happening on this side one has to be very careful. So one will have to obtain the value of this point the location of this point, now this requires some calculation, now how does one do it.

Now if my K G H j omega corresponds to this point then, what can I say, I can say that the phase shift is 180 degrees. So I have to look for that frequency omega at which the phase shift of K G H will be 180 degrees. This will require some computation usually but suppose I am able to find out that frequency omega then, the modulus of K G H at the j omega will give me the location of this point. Of course it will come with a minus sign, once I know that then provided it is like this, I know that there is a gain margin whereas if already the point minus 1 plus j 0 inside, I know that the system is unstable for the chosen value of K and so I will have to reduce the value of K sufficiently.

So this is another thing that one has to remember and because of this minus 1 plus j 0 and 180 degrees phase shift and the corresponding gain whether it is less than 1, these becomes some of the key words in the application of the Nyquist criterion. But one should not use them without thinking or blindly so to speak, phase shift 180 is important because it gives you this point the real axis or the real part or the real number that corresponds to this point is important because it determines very often, the encirclement of the point minus 1 plus j 0.

Now there is one more thing that I would like you to remember and I have see many books do not emphasize it sufficiently that whenever, they talk about the polar plot, they do not show the whole plot, they do not show the whole plot. Of course, usually only this part is referred to as the polar plot because this is a plot obtained from the frequency response but this part is not enough if I just draw that part here, so frequency low start here frequency high end up here, I cannot apply the Nyquist criterion or I cannot apply the principle of the argument because what has happened here, I do not have a closed curve. In fact, this is only the image of the upper half of the j omega axis, this is not the image of the entire contour, this is an image only of the part of the contour.

So I have to draw the full contour image which means complete this symmetrically as I have done it and which also means that what is happening here, how do I join this to this and therefore that consideration about the pole at the origin etcetera was extremely important because if I have this as the image then, this image is still not a closed curve and this is because I forget about that pole at the origin and therefore then, I had to bypass it and then I had to take the image of that bypass portion which then turns out to be a very large semicircle of this. Now when I do that I do get a close curve, not a simple close curve but I do get a close curve.

So do not just plot this plot and rush to any conclusion about stability, complete the full image and then only proceed to apply the Nyquist criterion and of course at the base of it or behind it is really the principle of the argument. So remember this, these possible errors that one can commit by not paying attention to some of the important aspects. Now, if we look at the 2 methods that we have studied so far, one was the root locus method and the other was the method based on the Nyquist criterion frequency response, Nyquist plot and so on. You will see that there are good points and bad points or advantages and disadvantages or both the methods.

The Nyquist criterion or the method which is based on the frequency response, one of the major advantages is that it can be based on experimental data, the polar plot can be obtained experimentally without knowing what the transfer function is, just apply input signal vary the frequency over a large enough range note the gain and phase shift and then, you are ready to do the polar plot. Once you have the polar plot then, you can complete it symmetrically as I have done here and then, with some knowledge of what kind of a pole at the origin.

We have we can complete the semicircle and therefore get the entire image contour, apply the Nyquist criterion and we are through, if you want to draw the root locus, we need to know the transfer function, I need to know the locations of the poles and 0s as precisely as possible then only I can start drawing the root locus. But on the other hand, if one studies the root locus method sufficiently then, there are some conclusions which one can draw from the root locus method almost immediately. Of course, with some practice the same conclusions can be drawn using the Nyquist approach.

For example, we had let us say to start with only 2 poles, no 0, so what is going to be the nature of the root locus well, these 2 coming together breaking away and going in to the complex plane, knowing instability but may be oscillations beyond a certain value of the gain K but now introduce a pole at the origin this is the integrator then, now we have 3 poles like this, no 0 because of this there are 3 asymptote at angles 60, 180 and minus 60 therefore 2 of the asymptotes will go out in to the right half of the complex plane and therefore there is going to be instability for a sufficiently large value of the gain K, I do not even have to draw the root locus by simply looking at this 3 poles, no 0s I know that for a large value of K, the system will be unstable.

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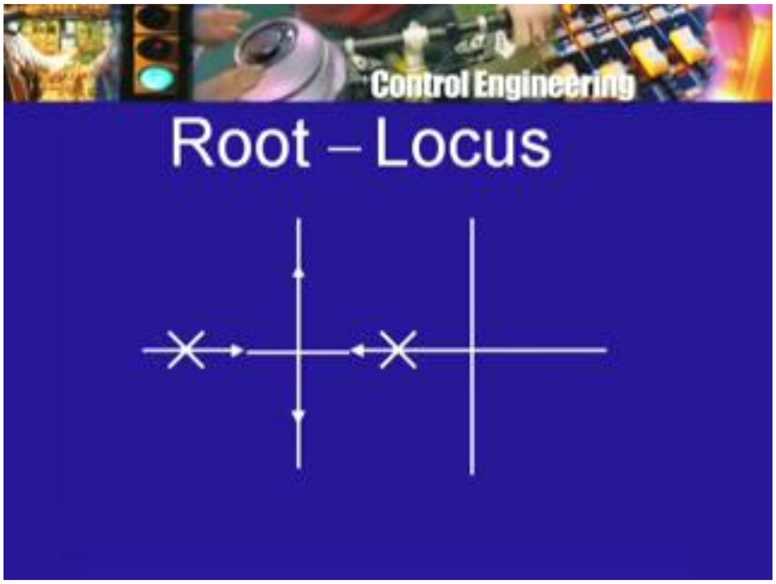

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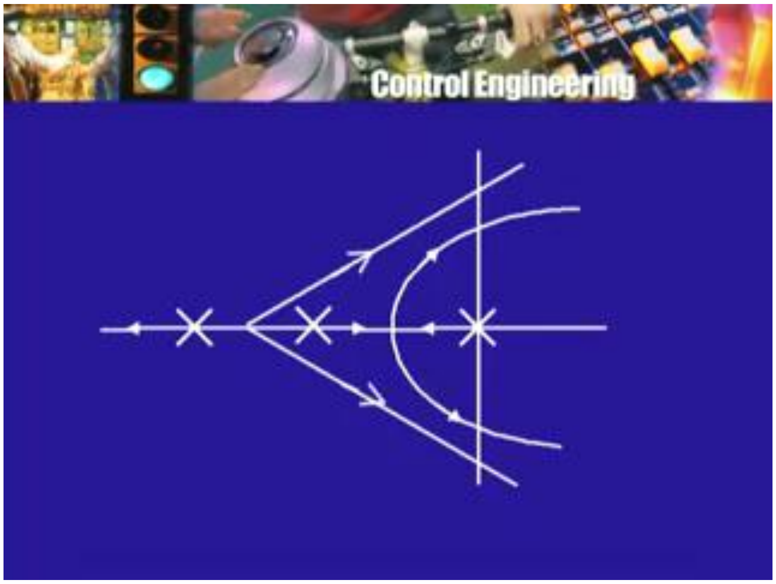

Now such a conclusion it is really a marvelous conclusion that we can draw provided of course one has studied the root locus method by merely looking at the pole 0 diagram, we are able to say something of course one can proceed a little further. There is one asymptote toward minus 180 degrees therefore this is one branch of the root locus, these 2 will come together then, they will break away and they will go away out like this approaching the 2 asymptotes and of course to find out the particular value of K corresponding to this intersection one will have to use the Routh array technique and therefore the transfer function will have to be known precisely.

So this is what will be, can be done with the help of the root locus qualitatively, sometimes one is able to get quite a lot information about what is going to happen if the gain K is increased, without integrator with only proportional feedback, with only 2 poles like this, no difficultly at all, no instability. Of course, oscillations will result the oscillations amplitude will increase as the gain K is increase but no instability. Here with 2 poles as before but 1 pole added because the integrator, we know immediately that for a sufficiently large value of K there will be instability and beyond that value of K, the system will remain unstable.

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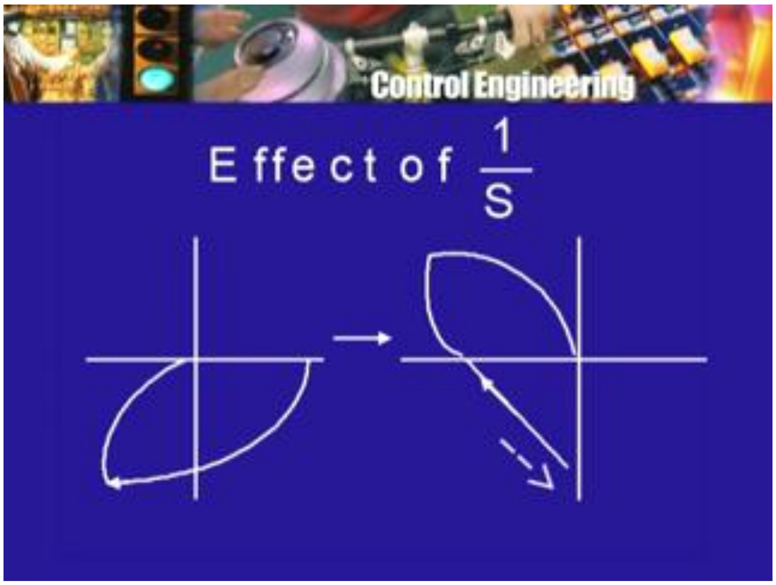

On the other hand using the Nyquist criterion, we started with the same situation, the polar plot turned out to be like this corresponding to the 2 poles, the proportional feedback case. The effect of introducing that 1 by S, the integrator was 2 fold 1 was at low frequencies, the gain increased and the other was a constant phase shift of 90 degrees. As a result, when I use the integrator this diagram is going to be changed in 2 ways, you can imagine that its going to be turned 90 degree in the clockwise direction but not only that at low frequency end, the gain is going to be increased and because of this, from this we went to this.

Now as a result now, this plot is crossing the negative real axis that is at some frequency the phase shift is becoming 180 degrees and therefore we can be in the danger zone because if the gain corresponding to this is 1 or more then, the point minus 1 plus j 0 may turn out to be inside the overall contour gamma prime, of course it is too soon to say anything because I have to complete the whole contour, then only I can draw any conclusions. There may be poles of the transfer function G H in the right half plane, so I have to take care of all those factors but you can see that the effect of this factor 1 by S, the integrator was to turn this figure by introducing a phase shift of 90 degrees minus 90 degrees. So the whole diagram was getting turned as a result of which whereas there was no intersection with the negative real axis.

The phase shift was never 180 degrees, here the phase shift could become 180 degrees for some value of frequency moreover there was a stretching of this point to infinity because of the factor 1 by S. The gain at low frequency is becoming very high, so there was this stretching. Of course, this stretching should have been the warning because then I have to think of that closure of that contour by that semicircle in the right half plane but that indeed also gives you the possibility of instability because of this intersection.

So there are few things which can be inferred from the Nyquist approach, there are something is which can be inferred from the root locus and therefore, control engineers would like to make use of both of them. The more the number of tools you have the better your design will be because design is always a matter of trial and error or approximation to what exactly is wanted. As we have seen we can only achieve so much, we may not be able to achieve everything that we are asked to achieve and so these alternative approaches do help.

Now I am going to spend a little more time along the Nyquist approach in the following way. I said that K G H j omega is what is used in the Nyquist criterion, this set of points is referred to as the polar plot and of course one can obtain it experimentally, by opening the loop, by injecting an input at 1 point, measuring the output at another point, simply measuring the ratio of the amplitudes and the change in the phase shift, we can obtain K G H j omega for various value of omega experimentally. What if you did not want to do the experiment because you knew what the transfer function was fairly, accurately. For example, our motor control problem one can assume that we know the parameters of the motor fairly accurately not precisely 100 percent and that is why we introduce feedback but still it is not too bad, it is not as if we just do not know anything about it.

So if you know the transfer function then do you have to go to the laboratory to find out this K G H j omega of course not, we can calculate it. So one calculate the frequency response if you know the transfer function. Now just as Nyquist was working on this amplifier stability problem, there was another engineer cum mathematician who looked at this and his name is Bode and he saw that there was a quick way of calculating this frequency response approximately. Once again not exact calculations exact calculations will take time. Today of course, it is not time it is the programming effort or using a particular programming package which of course because the programming has been done by somebody already, will give you the result almost instantaneously and as I told you there are many program packages which with give you this.

You can specify the value of omega by entering the data omega of course you have to enter the transfer function appropriately and then, the computer program works out the gain and phase shift and even plot and you can even see the polar plot display. Things have improved to such an extent. At the time when Bode was working, this was in the 1930's, you still had to use your calculator, the electronic calculator had not been invented essentially one had to use either the slide rule or logarithmic tables and trigonometric tables. At that time then, Bode gave a very good approximation technique for finding out K G H of j omega and the resulting approximate plot is referred to as a Bode plot.

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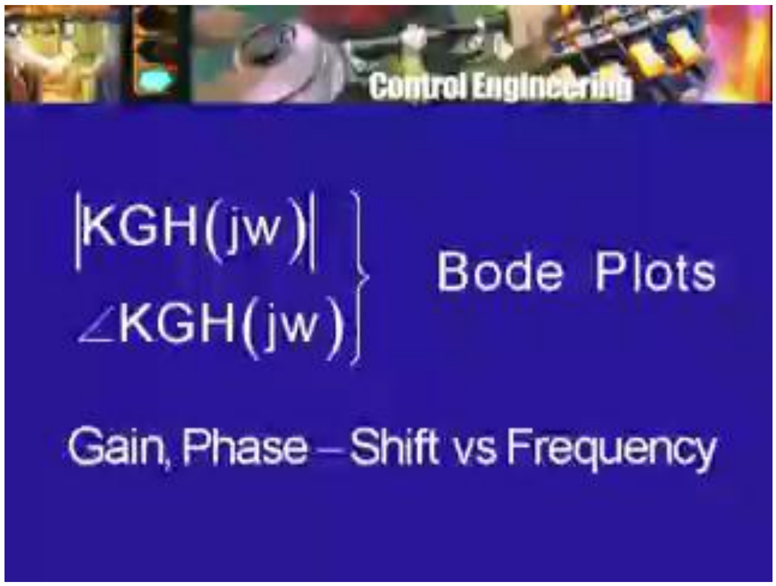

It is not the exact polar plot, it does not give you the exact values of K G H j omega, this is a complex number so modulus and argument of it, if you wanted it separately for various values of omega. It gives you an approximation to this exact polar plot and it is convenient to take the modulus of this number that is the gain separately and the argument of the number that is the phase separately. As a result, we do not have one Bode plot but we have 2 Bode plots, 1 Bode plot corresponding to the gain and the other corresponding to the phase shift.

So we will spend a little time on Bode's technique of obtaining a good approximation to the variation of gain with frequency and to the variation of phase shift with frequency. The method is a graphical one and therefore, it is convenient for a quick understanding it is not necessarily a very good method for precise calculations. I have been emphasizing that over and over again because as engineers we not only want to compute but we also want to get an insight in to what is likely to happen and therefore, these techniques like Bode plots then using that of course the Nyquist criterion or the Evans's root locus method, the Routh array which of course does involve some little bit of calculation but the root locus plot method, these are qualitative techniques or they are really useful more to give insight in to what is likely to happen rather than to be used for making precise calculations. But remember, that in design even today this insight is as important as making precise calculations, you may have the best of computer programs but if you have not understood what was likely to happen, you would not be able to solve the problem. So we will took take a look at Bode plots.