Control Engineering
Prof. S.D. Agashe
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 39

Let us look at an application of the Nyquist criterion for checking the stability of a system. As I said earlier the Nyquist criterion is useful mainly because in cases where the model for the system is not known but it is believed that the system has a linear time invariant model of the kind that we have been looking at described by a, nth order linear deferential equation with constant coefficients etcetera. An experimental measurement of the gain corresponding to the loop transfer function can be used to determined stability.
If we recall this loop transfer function was G H and instead of s, we put j omega then this is a complex number G H of j omega and its modulus gives you the gain that is the ratio of the output amplitude to the input amplitude in the steady state when the input is a sinusoidal signal of frequency omega.

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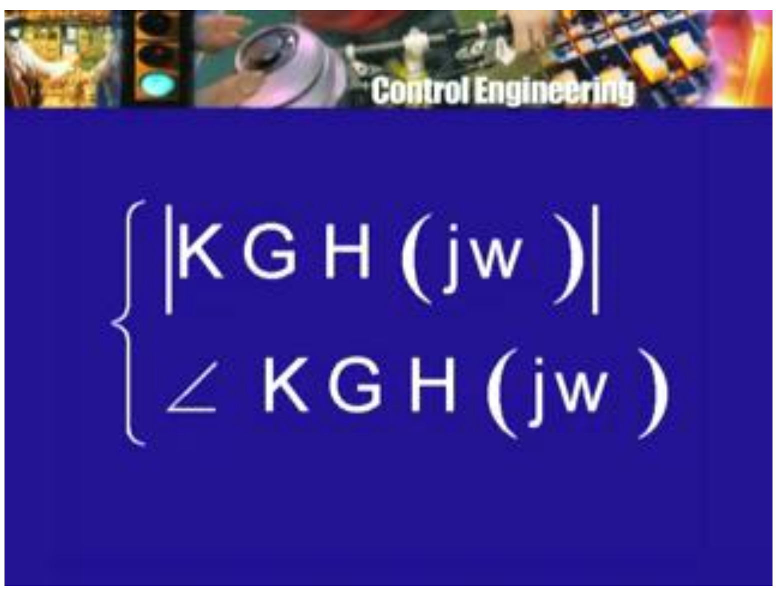

Now this is something which can be experimentally determined, you apply an input and then measure the output. Now where do we apply the input and where do you measure the output, let me repeat here is this gain K and here is this feedback transfer function H, the forward transfer function G, the difference device etcetera. We open the system here and then apply the input at this point to this input of the amplifier K. So the signal then is amplified passes through the transfer function of the system G and then passes also on to the transfer function H and then, you measure signal at this point.

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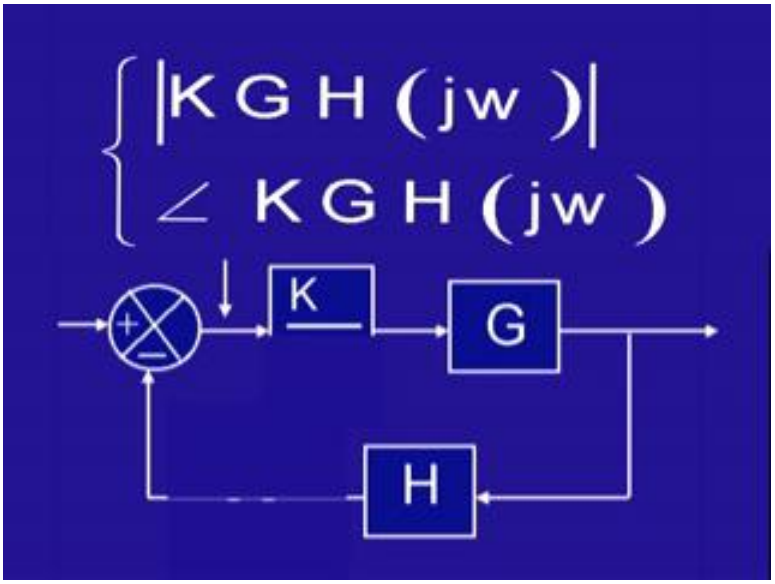

So the ratio of the transform or the Laplace transform the signal here to the Laplace transform of the signal here is exactly G H, K G H of s and if the signal is sinusoidal then, what we are discussing right now will hold namely I may put the K here also, if I look at the modulus of K G H at j omega then that will be the ratio of the amplitude of the sinusoidal signal here divided by the amplitude of the sinusoidal signal here. In practical terms also this means some thing for example, think of our motor control example, what you going to do is you are going to apply a sinusoidal input voltage to the amplifier which then results in a amplified voltage which is applied to the motor, the motor will run and the motor shaft has the tachogenerator on it.

So the tacho generator also will produce an output. The only thing is in normal operation the input here is a dc voltage, it may be changing but it is unidirectional, the output of the tacho generator voltage is also going to be a dc voltage but under test conditions you are applying an ac voltage may be of a small amplitude here and measuring an ac voltage of another amplitude here. So this test can be actually experimentally perform, of course in all of this we are assuming that load torque is either 0 or when doing the theoretical calculations, we are only looking at the effect of the input, the reference input on the performance.

So here in this case for example the motor will not be loaded it will be run under no load conditions, of course it will not run at the rated speed and all that. Similarly, the argument of this complex number G H j omega or K G H j omega is equal to the phase difference of the phase shift from the input at this point to the output at this point and this is some thing which also can be measured experimentally. So in other words, you know the modulus and you know the argument or the angle corresponding to this complex number K G H j omega. So all you have to do is from these measurements, you can plot this complex number in the complex plane.

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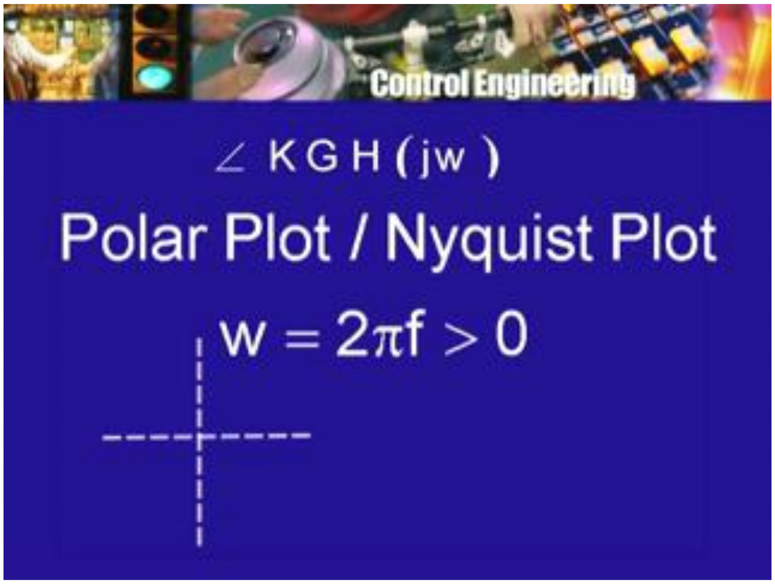

So this plot is known as the polar plot and why polar plot because the modulus of a complex number and the argument of a complex number, these correspond to the polar coordinates of a point in the complex plane but it is also known as a Nyquist plot because Nyquist showed us, how to make use of this experimental information. Now a polar plot may look like the following at the moment, you may not be able to guess what kind of a system will it be for which the polar plot will be as shown but let me draw a typical polar plot. So let say its starts form a point here and then, it goes like this towards the origin.

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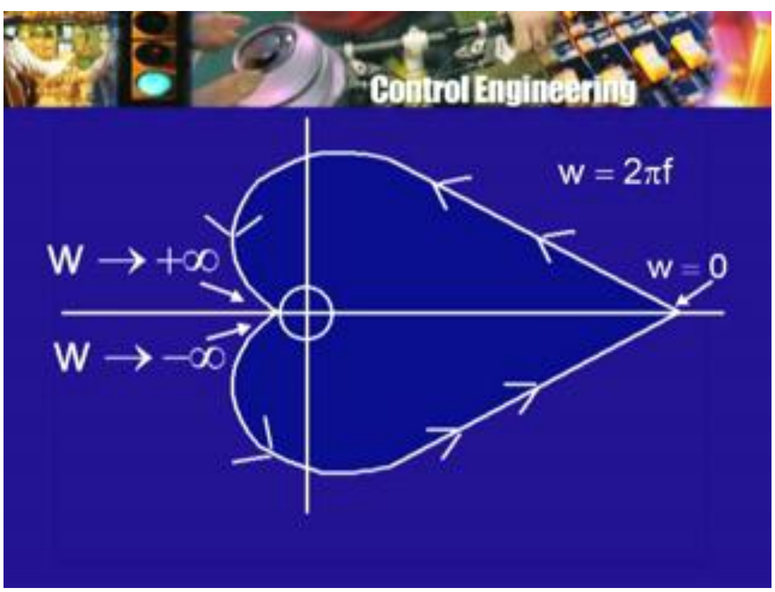

Now, let us remember one thing when you do an experiment, you apply a sinusoidal signal of angular frequency omega of course, omega is 2 phi f, where f is the frequency hertz or cycles per second. Now, obviously when you do an experiment, you can only chose a number of different values of the input frequency, say starting from omega equal to as low as point1 hertz, may be 1 hertz, 10 hertz, 100, 1000, 10,000 or may be a few more points in between. So in other words the data that you are going to get will only consists of a number of points like this and the angular frequency omega of course being 2 phi f is necessarily only a positive number.

So here omega is only positive omega equal to 0 corresponds to dc but dc is not ac. So in fact, all of this does not hold for dc, although you can do an experiment for the dc case also that is apply a voltage, dc voltage at the input and the measure tacho generator output voltage. So one can do dc test also in this case and we will get this number corresponding to say omega equal to 0 and then, you go on increasing omega all you get is a number of points like this and then, as we do for any experimental data, we join this points by as smooth a curve as possible.

So this will be the kind of curve that will result and as is obvious, you can apply frequencies up to a certain upper limit, if a frequency is to high for the case of system like a motor, it will not just make any sense, the motor will hardly turn if the frequency is very high because of its inertia, inertia of 2 kinds. So there will be an upper limit on omega may be omega will be up to 10 kilohertz. So in other words this response will stop somewhere the measurement points will end of somewhere here and one is now extrapolating to go beyond that and trying to sort of guess, what will happen when the frequency is increased indefinitely.

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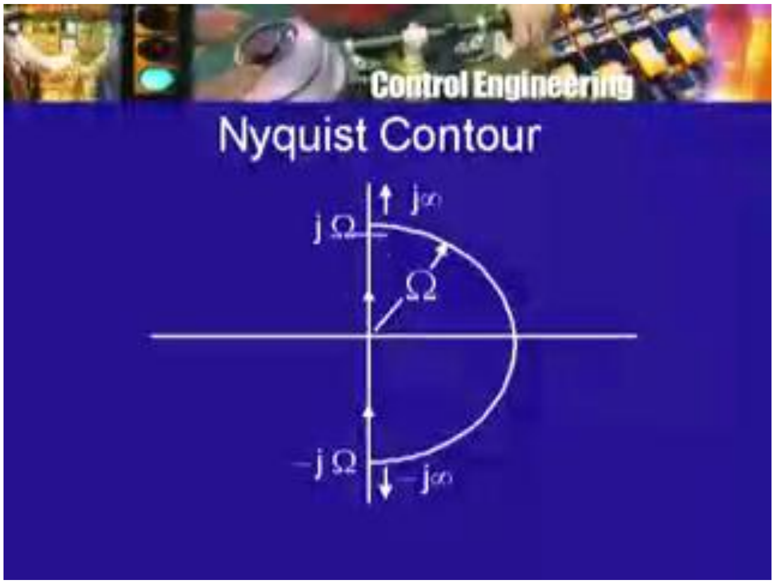

So remember that the polar plot may actually start under dc conditions but it never practically is going to correspond to the situation which can be indicated as a limit as omega tends to infinity, this limit we have to guess how far we can guess it depends on the system is 10 kilo hertz sufficiently high or do you have to go further up. From the trend of the data one can see what is likely to happen. So let us say here we have chosen some number of values of values of omega and this is the kind of plot that we are going to get now this is the plot of K G H j omega and if you remember, if you go back to the Nyquist contour. The Nyquist contour consists of a part of j omega axis closed by what I called or what I do like a semi circle in the right half of the complex plane and in other words, this part corresponds to minus j omega to plus j omega and then, there is a circle of radius say omega and the argument changes from phi by 2 to minus phi by 2 in the clockwise sense.

So this was the whole Nyquist contour what we are doing now is we are only looking at this part of it corresponding to the upper half of the j omega axis. The data that we get for the polar plot, from the experiment corresponds essentially to a number of frequency values or omega values or j omega values on the j omega axis, so this is the polar plot.
Now as I told you the transfer function usually has only real coefficients and therefore, G H j omega will be the polar plot, the image of the Nyquist contour will be symmetric, the Nyquist contour is symmetric around the real axis and therefore the image contour will be symmetric around the real axis and because of that if I know that the polar plots for positive omega that is for values of j omega which lie on the upper half of the j omega axis looks like this.

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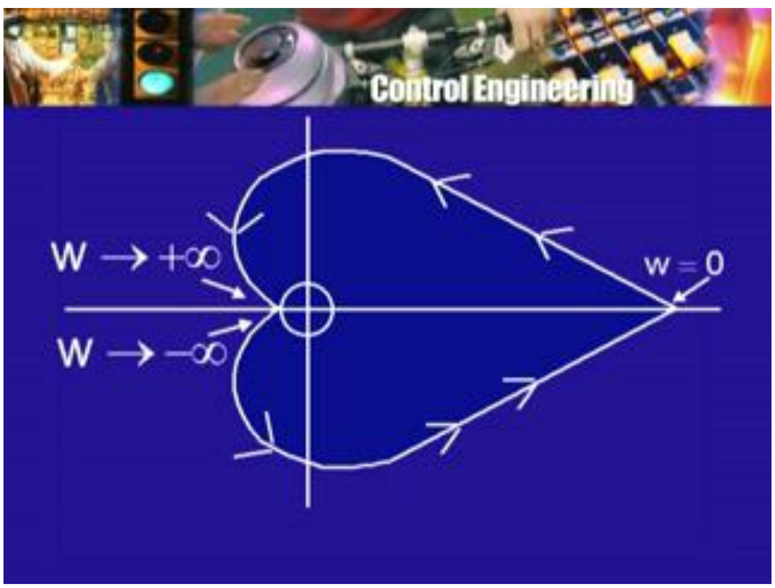

Then, the values for omega negative or in words for points lying on the lower half of the j omega axis will be the mirror image of this in the real axis and so, it will look like this. Experimentally, we only determine this part but we immediately infer that the part corresponding to the lower half of the j omega axis will look like this and of course, it

goes through this point omega equal to 0 omega or j omega equal to j 0 but what about omega tending to infinity, will this upper part corresponds to omega tending infinity.
So the lower part will then correspond to omega tending to minus infinity or in other words one end of it corresponds to this point on the Nyquist contour, the other end of it corresponds to this point on the Nyquist contour. But these to 2 points are not the same points they are separated although one of them is j omega with omega very large, say 10 raised to 5, the other is minus j omega with omega very large, same value minus j 10 raised to 5 but this not quite the same thing as j infinity and minus j infinity and as I told you, the contour has to be closed and therefore it is this part of the contour which also we have to consider, experimentally so what happens on this part by symmetry, we know what is happening on this part but what is happening on this part.

Now this is something, we have to figure out from some knowledge about the system as we will see. So the Nyquist or the polar plot experimentally will consist of this half of we almost the full figure expect for this transition from j omega omega large positive to minus j omega through the right half plane of the z plane, along that semicircle. Now, let me remind you once again about the principle behind the Nyquist criterion and the rule that the point minus 1 plus j 0 played in the whole thing.

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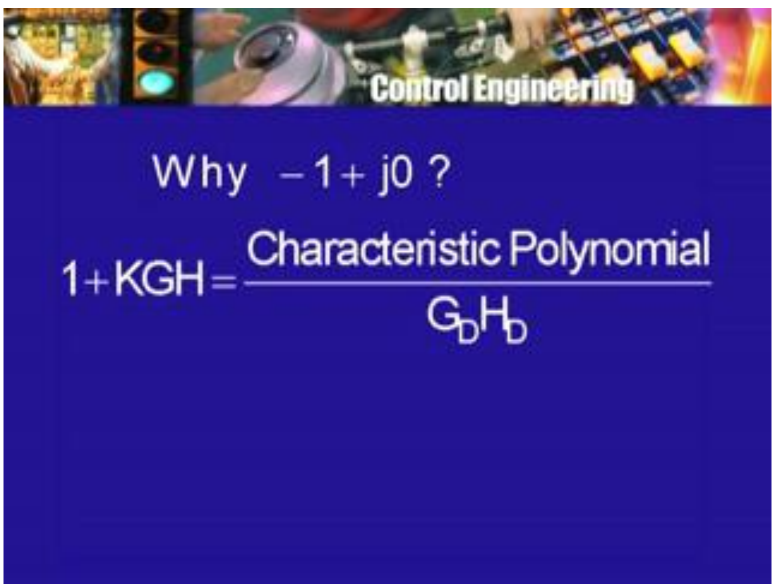

If you remember, we were looking at the rational function 1 plus G H or 1 plus K G H whose numerator was the characteristic polynomial and our main interest is in the roots of the characteristic polynomial whether there are any roots in the right half plane. The denominator of it is G D, H D, the denominator polynomial of G and H respectively. In other words, the poles of the forward transfer function G and the feedback transfer function H. So it is 1 plus K G H that we are looking at and we wanted to we applied the principle Cauchy's principle of argument to1 plus K G H but we observed that the plot of 1 plus K G H is going to differ from the plot K G H by only the addition of this one.

Similarly, the plot of K G H will differ from the plot of G H by the multiplication or the scaling by the factor K and so it is enough to consider the plot of G H or under the action of G H rather than under the action of 1 plus K G H. Now in the principal of the argument this whole 1 plus K G H, we want to find out whether it encircles the origin and what number of times and in the same sense or in the opposite sense as the original contour. Now, what is the relationship between the plots of 1 plus K G H and G H. For the sake or simplicity, let me show let us say the plot of G H turns out to be a small circle like this. This is the plot of G H that is whatever is the contour in the Z plane, the image contour let us suppose looks just like this, a small circle here. This is the plot of G H, what will be the plot of K G H, K G H is a scaled version of this by the factor K.

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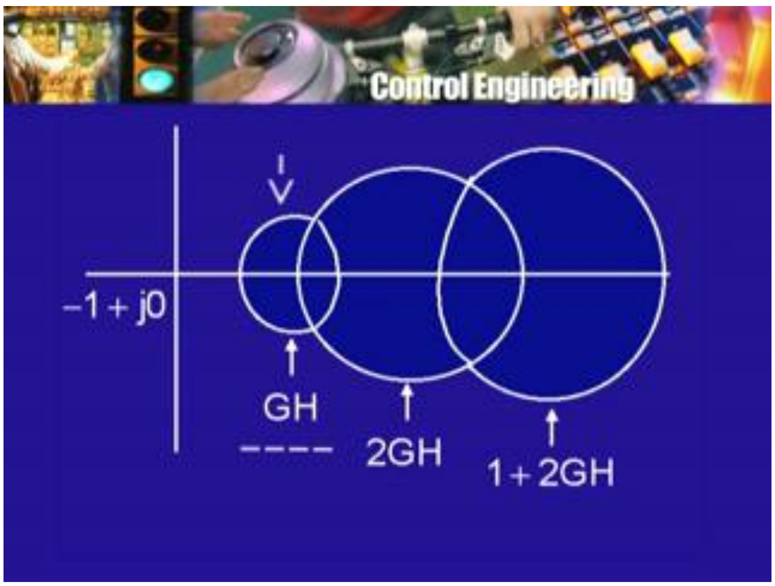

So let us say if K equal to 2 then, what is going to happen each one of these points is going to be multiplied by 2. So in particular this circle, it will be such that its center now be at this point and the circle will have double the radius. So this will be the plot of 2 G H, if a plot G H is like this, the plot 2 G H will be like this. It will be scaled all the radius vectors from the origin will be doubled in length. So it will be a circle of the double the size in the sense, the double the radius and the center has shifted to a place which is 2 times as far from the origin, so this is the plot of 2 G H.

Now, what about the plot 1 plus 2 G H, so I to add 1 to it, so I am going to shift this circle. Now by 1 unit, let us say 1 unit on this scale is such that this plot looks like this. So this will be the plot of 1 plus 2 G H, so this was the plot of G H, this is the plot of 2 G H and this is the plot 1 plus 2 G H. Now you are looking at the relationship between this plot and the origin. Cauchy's principle of the argument tells you that the image contour, the number of times it encircles the origin is related to something else.

So we are interested in the encirclements by this of this point but now we can see the relationship between this point and this, is the same as the relationship of this 2 G H contour with the point minus 1 plus a 0 because this is 1 plus 2 G H therefore the relationship between this plot and this origin is the same as the relationship between this plot and point minus 1 plus j 0 that is the origin instead of the origin we move 1 unit to the left.

So the number of encirclements by 1 plus 2 G H of the origin will be exactly the number of encirclements by 2 G H of the point minus 1 plus j 0 and so instead of talking about encirclements by of the origin which is what is involved in the principle of the argument. In the Nyquist method, we talk about encirclements by the plot of KGH of the point minus 1 plus j 0 and this is the reason for the special rule that this point minus 1 plus j 0 plays and of course, as I told you that we really a do not vary the gain in practice.

So we have to find out the effect of the gain by simply considering scaling and we will see, how this will affect our calculations. So remember this this is why we have to look at encirclements of the point minus 1 plus j 0 by the polar plot. Now going back to the example that I have drawn earlier, we are now to think of the rest of the original contour that is here is this contour once again, what about the image of the this part of the contour. Now above this this cannot be experimentally determined because the frequency response method applies only to this part of the contour, this part of the contour cannot be experimentally determined but if you have some knowledge about the system and this of course we do, we are not just looking at a system about which we know nothing. For example, we know that in the case of the motor, there is a first order transfer function for the armature circuit, another first order transfer of function for the mechanical part etcetera, if we use an integrator we know that there is one more first order transfer function 1 by s and all of this we know.

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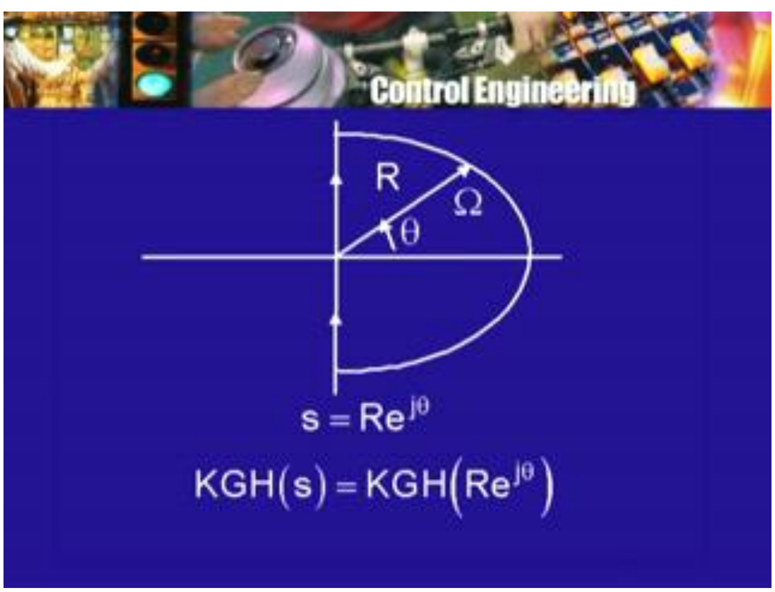

So because of this we can find out something that happens along this part of the contour. Now on this part of the contour I can say that the complex number s is given by what in polar form R e raised to j theta, where R is the radius of the circle semi circle and theta is the angle or the argument of a point on it. Here is R of course, I have call this omega so instead of R, you can think of omega. So the complex number s along the semi circle is simply omega e raised to j theta and now what I am in doing, I am just evaluating G (s) H (s) that is G H at s or K times G H at s, for any complex number s on the contour. For the point on the contour corresponding to that semicircle s is R e raised to j theta therefore this is going to be K G H, R e raised to j theta.

Now this is where we make use of the fact, this R is going to be a large number and we will come to understand, what is meant by large and why. Now if we recall our root locus method there we were one of the rules that Evans had given was the following to find out whether a point in the complex plane lies on the root locus, you have to apply the angle criterion that is you have to measure or draw lines and measure the angles made by that point at poles and 0s of the transfer function G H, H (s) and then if you the that point was on the polar part to determine the gain corresponding to it, you have to determine the distances and if you remember, at that time we drew vectors from the poles and 0s to a point in the complex plane.

Now we can imagine that exactly the same thing is done here, for trying to find out what is going to happen to this K G H of s along the semicircle, when the radius of the semicircle R is quite large. So to do that let us do what we did earlier we look at the poles 0 plot of the transfer function and let us say, the pole 0 plot looks like this, I am just showing some poles and some 0s I do not have any specific system in mind, there could be poles in the complex plane, there could be 0s which are not real but on the complex, all that is okay.

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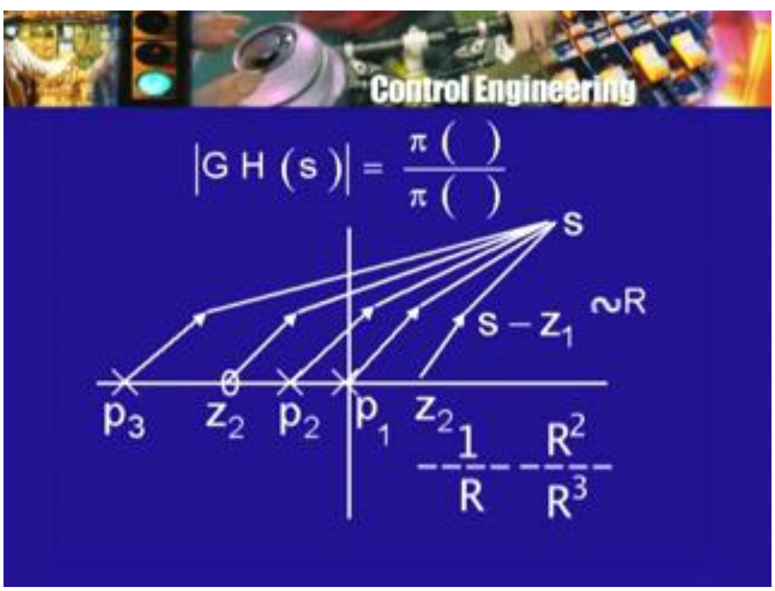

Now here is this point s and this point S is at a distance R from the origin and this R is going to be considered very large then, G H of s as we saw earlier, looks like a product of a linear factors in the numerator divided by a product of linear factors in the denominator and what are those factors essentially they correspond to the poles and 0s. So, for example here is a0 here let us call this point Z1 then, there is this factor s minus Z 1 corresponding to it but what is the vector S minus Z 1, the vector s minus Z 1 is very well represented by this line drawn from Z 1 to S, there is this other 0, Z 2.

So I can draw the vector of the line from Z 2 to S. So this represents S minus Z 1, this represents S minus Z 2. Similarly, there are poles let us say P 1, P 2, P 3, in the denominator I have factors S minus P 1 corresponding to this line, S minus P 2 corresponding to this line and S minus P 3 corresponding to this line. So what do I then, I have product of the 2 vectors corresponding to the 0s divided by the product of the 3 vectors corresponding to the poles and remember now, we are considering this radius to be very large.

Now because of that if you look at the length of the vectors, the length of the vectors is also going to be very large. In fact, it is going to be very nearly equal to the radius R. On this diagram it does not appear to be so because I have not chosen R sufficiently large in comparison with the magnitudes of the poles and 0s. The poles and 0s are all here I have shown a point s which is not really very far away from the origin. But imagine that the point S was at a distance which was 10000 times anyone these distances then, all these distances would be nearly equal, equal to R and therefore what will I have, if I look at magnitude or the amplitude of G H of s, it is going to be what roughly corresponding to this factor R corresponding to this other 0 another R.

So R square in the numerator divided by RRR for each one of the poles and therefore R cube in the denominator and therefore, it is going to be roughly like1 by R and if R is very large then, it is going to be nearly 0. Essentially, what I am saying is that the modulus of GHs for s equal to R e raised to j theta, when R is sufficiently large and now what do I mean by sufficiently large, R is much larger compared with the distances of the poles and 0s from the origin. Now for the given motor for example or some other system we may have some ideas about the poles and 0s. In fact, we can make measurements of the various parameters of the system and in fact, this determine the various poles and 0s but even, if we do not know them exactly we may still have some idea as to how large they might be and we are only considering, we are not doing any actual experimentation we are only considering theoretically points S, which are sufficiently away.

So in that sense we can always say that by choosing R sufficiently large, this is what is going to happen. The whole modulus is going to be nearly equal to 0, now why is it nearly equal to 0 because in the numerator I have R square, in the denominator I have R cube. Now where does this R square come from this comes from the 2, 0s and the R cube it comes from the 3 poles. So what is happening here the number of 0s here is less than the number of poles or the system has more poles than 0s and because of this along that semicircle, when R can be considered very large, modulus of G H of s will be nearly 0 and because the modulus is nearly 0, the argument is not really going to matter that much and therefore, when I look at the image of that part of the contour the semicircle under the action of G H is going to be some curve here very close to origin, it is not going to be a closed curve as I have shown here, it is not a circle or anything, I do not know what its shape is but it is going to be so small that it can be considered to be virtually at the origin and therefore in our experimental figure, if this was the polar plot coming down to high frequencies, this was the mirror image of it.

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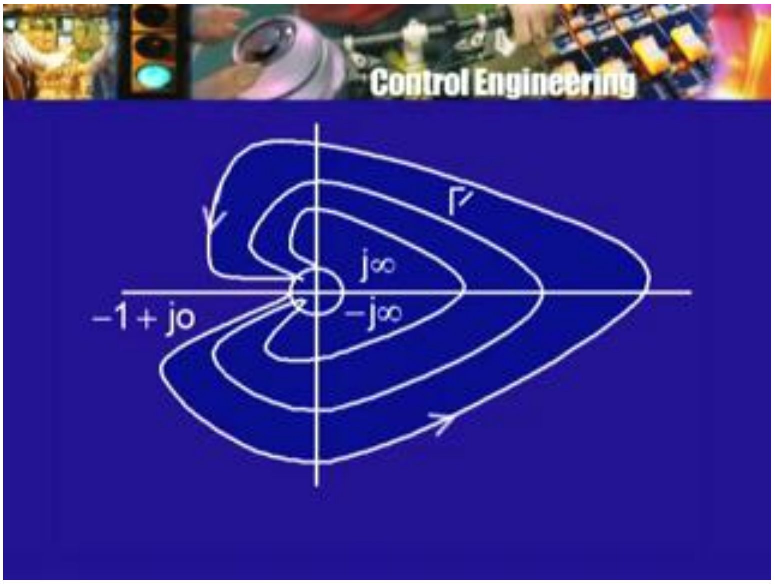

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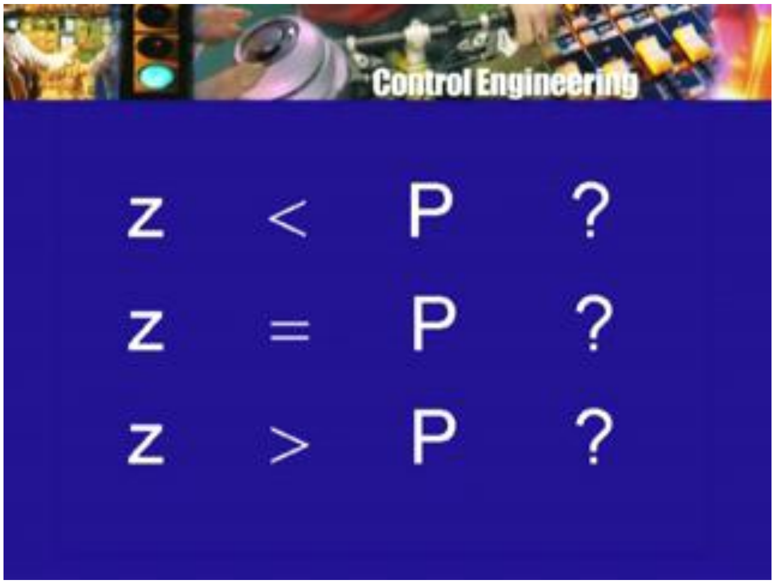

So this corresponded to the j omega axis part the semicircle image will be something which is very close to the origin. So some curve joining the 2 ends of it here. So this is what we can conclude of course this is what is happen if the number of 0s is less than the number of poles and this is case in practice. In most practical systems, the system has more poles than 0s if the number of 0s is equal to the number of poles then, the situation is going to be different because then in the numerator and denominator I have equal powers of R. As a result the ratio will not go to 0 as R goes to infinity and should it happen that the number of Z, 0s is greater than the number of poles then, it is going to be a of course drastically different because now, the numerator has higher power of R than the denominator.

So the modulus is actually going to go to infinity instead of going to 0. Let us not looks at these 2 cases, once you have understood what it happening, it is not very difficult if one comes across. A practical example or even to study it in theory these 2 cases, Z equal to P and Z greater than P. So we will not look at those cases, we will only look at the case Z less than P. Now with that then from experiment I can determine this upper parts, what looks like the upper part here I take the mirror image in the real axis.

So I get this figure and because the image of the semicircle is virtually something occurred which is very close to the origin, I can show it like this actually by joining these 2 and going to the origin. On doing practice, you know if I write here j infinity I do not really mean that I apply a signal of infinite frequency and I make any measurement that is all non-sense, there is no signal of infinity frequency that I can ever apply. But I can apply a signal of whatever high frequency is under my choice whatever signal generator which will say provide some power to drive the motor in the case of the motor, it may not really be of the order of megahertz or even kilohertz.

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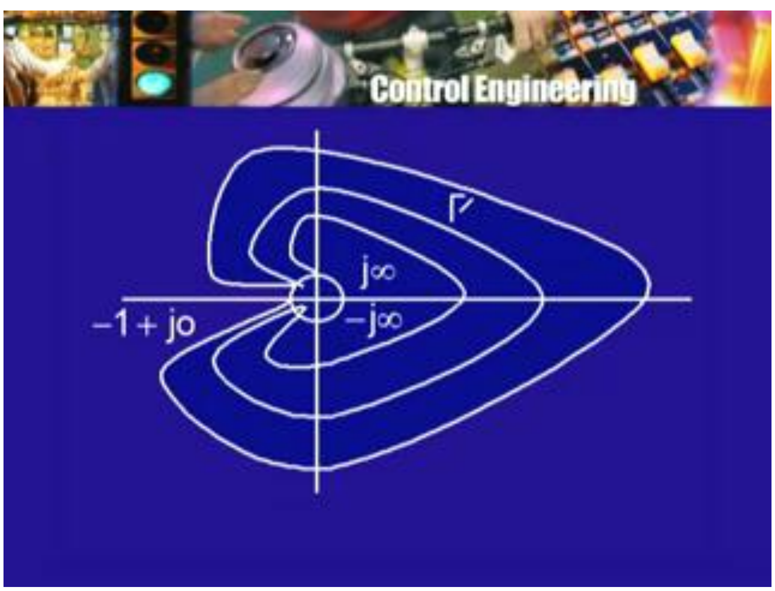

It may be of the order of a few hundred hertz still that may be adequate because the image or the frequency response will be such that it is very virtually very small. So this is what we mean here now, therefore this is the contour gamma prime which is the image of our original contour gamma and now, we are ready to apply the Nyquist criterion to this plot. Now remember, we have to consider the encirclements by this plot of the point of which point not the origin but of the point minus 1 plus j 0, now for high frequencies I am very close to the origin.

So the point minus1 plus j 0 might be somewhere here. Let us say, now it is clear from this figure that this plot that is gamma prime does not encircle this point minus 1 plus j 0. Now this is for some particular value of k because experimentally, I have chosen some value of k now, what if I increase k, what if I decrease k. Now again as a engineers, we want to be able to visualize, what is going to happen if I increase k then, what is going to happen is this plot is going to blow up so to speak that is it going to be up scaled by that factor k. As a result, you know everywhere the values will increase they will be multiplied by some positive number.

So this point instead of here will start here and the new curve may look like this. Similarly, its image mirror image will be in the enhanced in size and it will look like this but will it encircle the point minus 1 plus j 0, no it does not look like that. Even if I increase k by 100 times one can now see that the new plot, the new gamma prime corresponding to that new value of k will still not encircle the point minus 1 plus j 0. On the other hand, if I consider values of k which are smaller then, this scale is getting reduced then it is even easier to see that the new plot or the plot corresponding to that value of k will not encircle the point minus 1 plus j 0.

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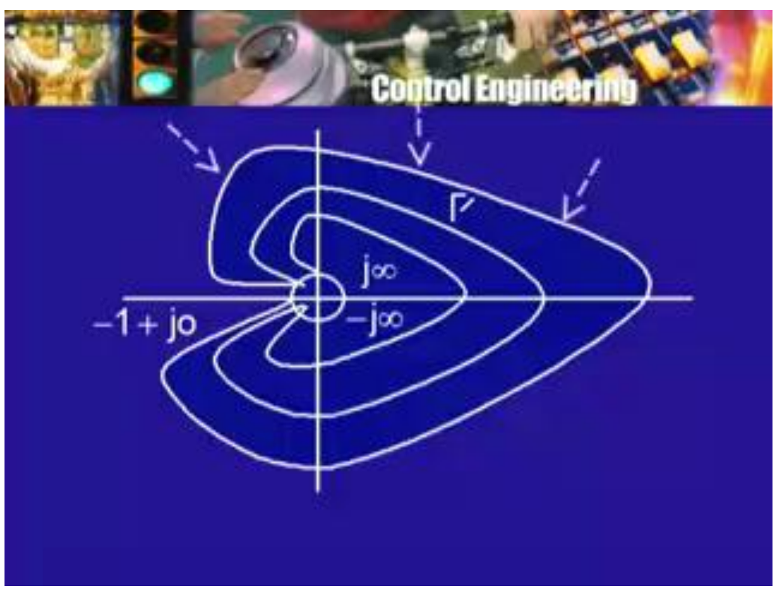

For example, if I reduce it by factor a factor of 2 then, this plot will become 1 half in size and will look like this and this certainly is not encircling the point minusb1 plus j 0. So this is how with the experimental Nyquist plot completing it with this way by using this fact that their this mirror image business also is the number of poles is greater than a number of 0s then, this is what is going to be the image of the infinite semicircle. We can look at the relationship of the plot with respect to the point minus1 plus j0 and determine the number of encirclements of this point by this plot and that corresponds to what that by Cauchy's theorem equal to the number of 0s of1 plus G H divided by the number of poles of1 plus G H and that enables us to determine the number of 0s of 1 plus G H in the right half plane because according to that the number of encirclements therefore is equal to the number of 0s of 1 plus K G H minus the number of poles of 1 plus K G H but as we saw earlier, 1 plus K G H essentially is the characteristic polynomial divided by G D, H D, the pole part of G and H.

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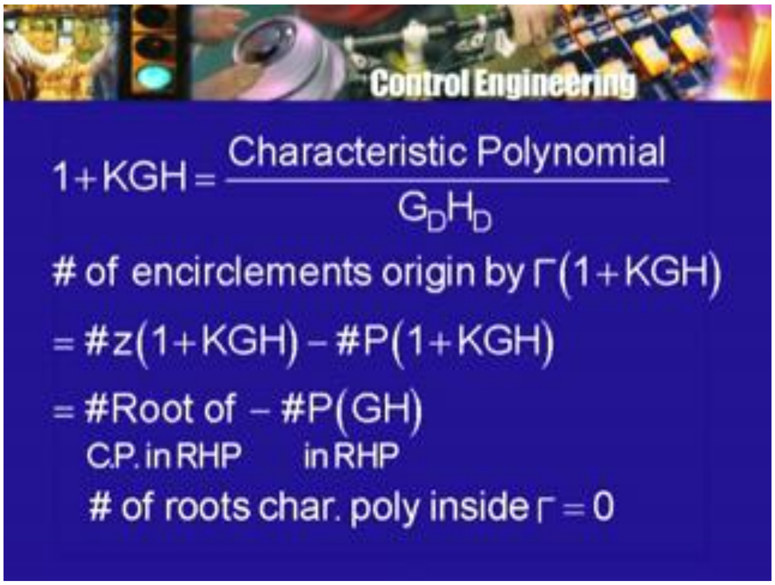

So this number, number of 0s of 1 plus K G H is exactly equal to the number of roots of the characteristic polynomial in that contour gamma, the Nyquist contour gamma and minus the number of poles of 1 plus K G H is exactly the number of poles of G D, H D in the inside the Nyquist contour. So the number of encirclements is equal to the number of roots of the characteristic polynomial in the right half plane, actually not in the right half plane in that contour, inside the contour gamma minus the number of poles of G H inside the contour gamma.

Now suppose, we know and as I said we have to know something about the system otherwise we really cannot do much. We know that the G H transfer function has no poles in the right half plane, in that case this number is 0. Now if the number of encirclements is also 0 then, you have 0 equal to the number of roots of the characteristic polynomial minus 0 therefore the number of roots of the characteristic polynomial inside the Nyquist contour gamma is going to be equal to 0. In other words, the characteristic polynomial has no roots inside the right half plane and therefore the closed loop feedback control system is stable not only that it is stable for all values of k, positive values of k because no matter what positive value of k we choose, the number of encirclements is just going to be 0 for all those values therefore for all those values of k, the characteristic polynomial will not have any root in the right half plane and therefore for all those values of k, the closed loop system will be stable.

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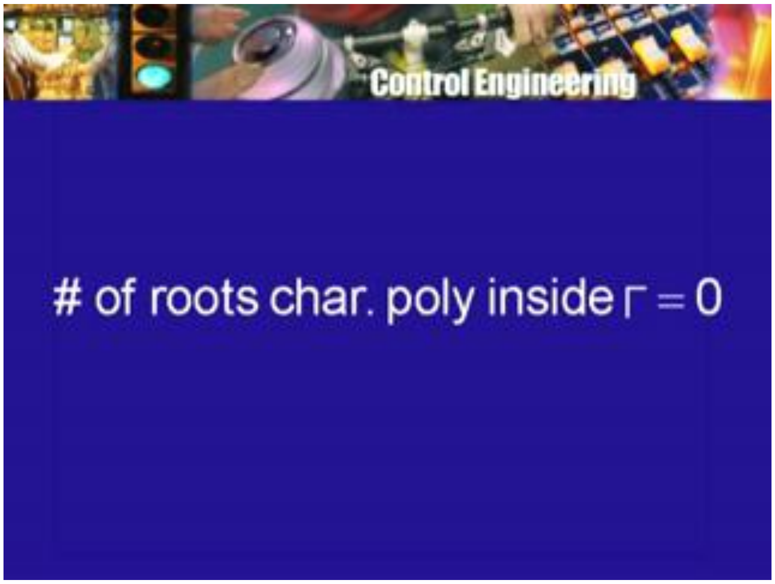

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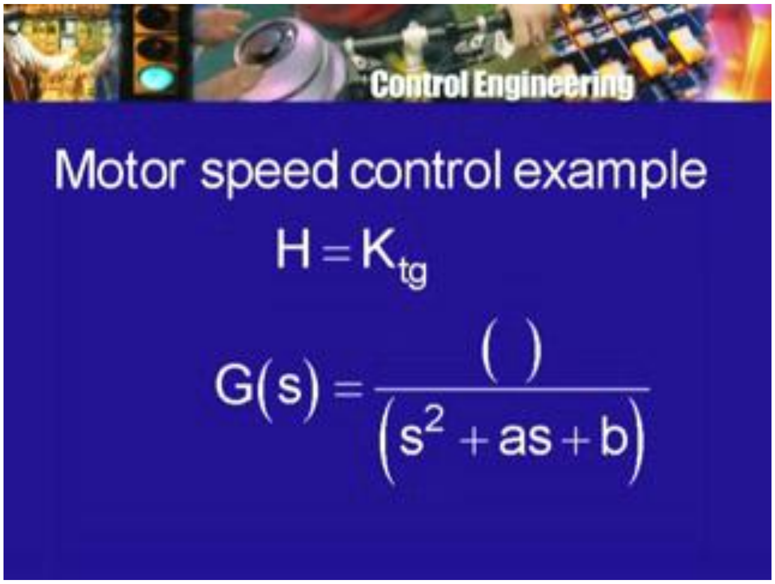

Now the plot I drew to illustrate the application of the Nyquist criterion is almost the one that corresponds to our motor control problem with the proportional feedback, may be some of you have been able to see the relationship. Of course there is going to be a slight difference between the2 as you will see. Now, if you recall the analysis that we did for the motor control system, the feedback transfer function H, what was it, it was essentially nothing but the tacho generator gain, K t g because the feedback element for simply the tacho generator and this was the gain from the input speed to the output voltage produce by the tacho generator.

So H is not really a function of s or s does not appear that is just a constant function of s, what about the function G of s, if you remember there were 2 parts to this G of s, one was the relationship between the armature voltage and the motor current or it dependent on that and the second was the relationship between the motor current which determined the motor torque and the speed. So there were these 2 parts of that transfer function and as a result that transfer function G of s had a denominator which was a quadratic and I have written it earlier as something like s square plus a s plus b and then, it had some term in the numerator.

So G s was a transfer function like this or in other words, it was a transfer function of second order and therefore if we look at G H then, there are 2 poles and as far as the armature voltage concerned or the relationship between the reference voltage of the armature voltage and the speed is concerned, the numerator is just a constant and so, this G H has only 2 poles and no 0. Now because of this what is the polar plot going to look like.

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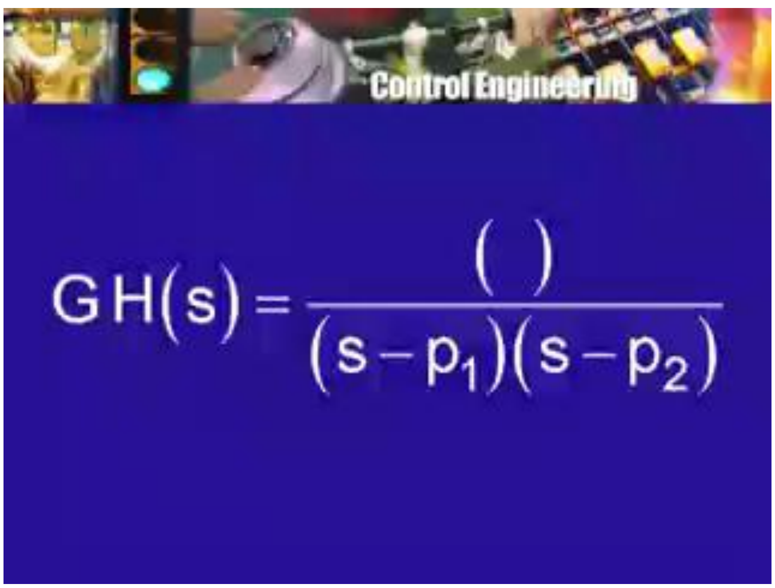

So let us factorize this s square plus a s plus b into the 2 poles therefore G H of s is going to be essentially some constant number in the numerator divided by s minus p 1 into s minus p 2, where p 1 and p 2 are the 2 poles. Now these 2 poles as we are seen earlier are in the left half of the plane and they may both be real or they may be complex. So let us consider the situation when the poles are real. So here is p 1 and here is p 2 and I have to look G H is corresponding this p 1 and p 2 for what points s. Now for the part of the contour which is on the j omega axis I have to consider point s here. Now this is what corresponds to the experimental data, so here is a typical point j omega, now what about G H of j omega.

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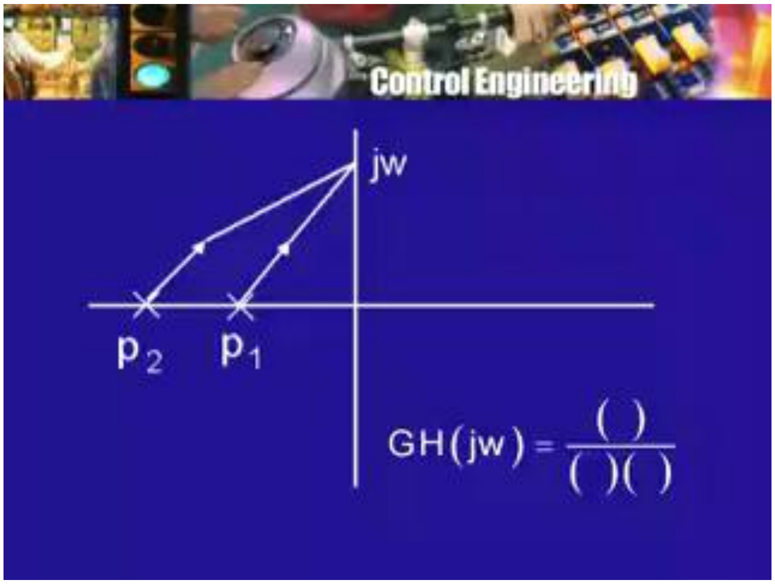

Now once it is not necessary to substitute any numbers and compute it to find out what is going to be the nature of G H of j omega. Once again, we draw the vector from the pole to the point j omega from the other pole to the point j omega and what we have is then, G H of j omega will be the complex number which will be some number in the numerator divided by the product of 2 complex numbers in the denominator, what are these 2 complex numbers, they are the complex numbers, they are the complex numbers corresponding to these 2 vectors drawn from the poles to the j omega point.

Now it is easy to see that the argument of this complex number G j omega is going to be what, what is the argument corresponding to this factor that is j omega minus p 1, it is the angle made by this vector with the positive real axis. So it is an angle that lies between 0 and 90 degrees perhaps it looks here something like say 60 degrees, what about this angle? It is also an angle made with the positive real axis, it is a positive angle may be of the order of 30 degrees however, these2 factors are in the denominator and therefore as far as the argument of G H is concerned j omega it is not going to be a positive angle but it is going to be a negative angle and it is going to be a negative angle lying between 0 and 180 degrees. As a result when I do the experimental plotting, what I am going to get is not going to look like what I drew earlier but it is more likely going to look like this.

Now this corresponds to the dc case and therefore, this is referred to as the dc gain of the motor and then, as you apply ac and increase the frequency, we will get points along this part. So the plot corresponding to the positive or the upper half of the j omega axis is really going to be this and this is what will be obtained experimentally and the other part will be inferred because of symmetry and of course, I will show them both as actually passing through the origin although in practice they will not come to the origin because of the argument earlier that in this case, the number of poles is 2, the number of 0s is 1 therefore the image of the semicircular part of the Nyquist contour will be essentially very close to the origin. So will be point like this.

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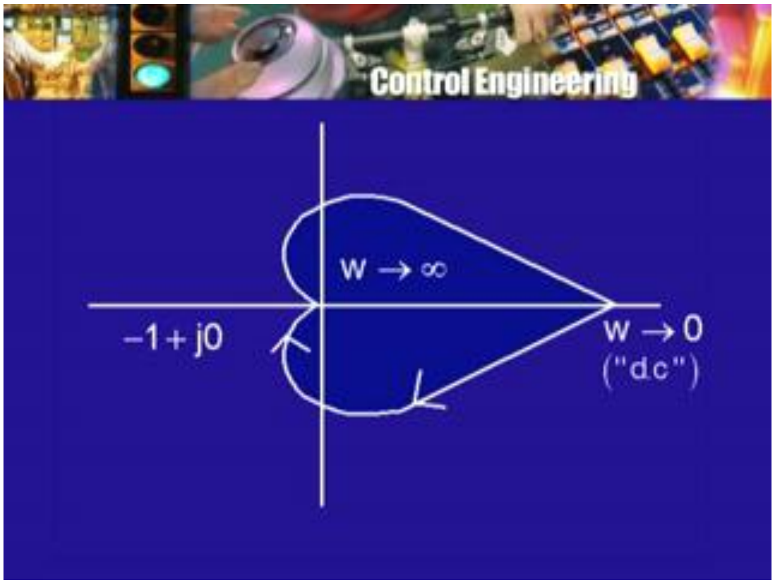

So except for this interchange, when I drew the figure first, I do the upper and then I said that the lower half is the mirror image, where as experimentally for the motor, you will get this part and the upper half will be the mirror image. So the total shape remains the same and the total conclusion therefore remains the same that the closed loop system consisting of tacho generator feed back that is proportional feedback, no matter what the gain k which is applied at the input to the armature, no matter how large it is, the system will remain stable.

Now when we use the root locus method, we were able to get more information than this because we were able to find out the location of the closed loop poles and from that we could find out information about time constants and things like that. Now that information we cannot obtain here atleast not immediately, although it is possible to from this polar plot to conclude something about even time constants but that is not very easy that requires a lot of argument and we may not go through it right.

Now I will refer to it later on but you will have to read up your textbook carefully and in detail to get some information about that aspect. But as per as stability is concerned there is no problem here. So the feedback control system, proportional feedback control system that we had studied for the motor control problem, motor speed control problem is going to be stable.

Now that conclusion we had drawn earlier, by actually finding out the transfer functions by using the root locus plot. Here, now the same conclusion we can reach by looking at the experimental data namely, the Nyquist plot or the polar plot obtained from the experimental data then, completing it this way by using the symmetry of the Nyquist part and by using the fact that the number of poles of system is greater than the number of 0s therefore, the encirclement relationship around the point minus 1 plus j 0 is going to be 0 and therefore from that we concluded that the number of roots of the characteristic polynomial inside the or in the right half plane or inside the Nyquist contour is going to be 0 and therefore the system is going to be stable.

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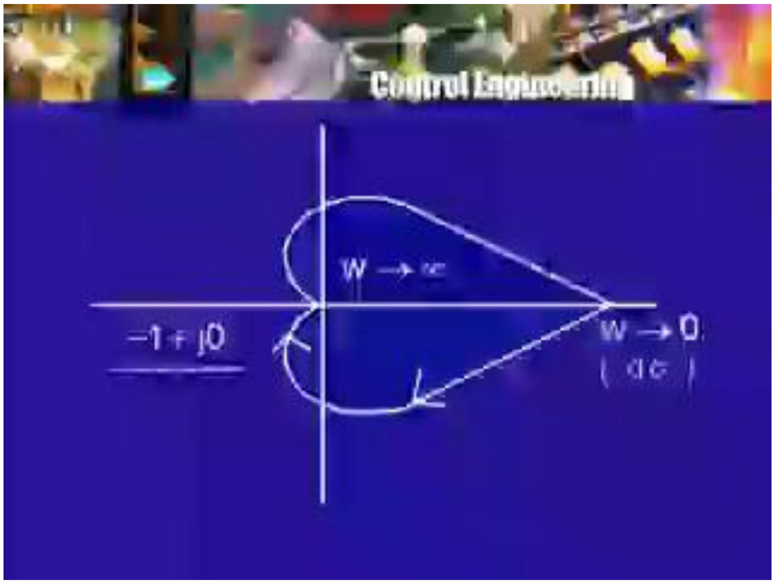

Now let us apply this to the integral feedback case, now what it is going to be the difference with the integral feedback case. Now the main difference is that in the forward path, we have introduced the integrator. So in place of k now, we have K by S as the block that precedes the motor armature. So after that I have G as before and then I have H the tacho generator as before there is no change, the rest of the arrangement is exactly the same. Now what about the stability of the system, when I use an integrator that is when I use proportional and integral feedback, when the gain of the integrator or that is the proportionality of the integral part of the feedback is increased.

Of course, we knew we had found out earlier using the root locus method that the system was going to become unstable and how did we conclude that because then, G has 2 poles, H has no pole, G has no 0, H has no 0 but there is this additional pole at the origin corresponding to the integrator S therefore, the total system now has 3 poles, one 0 because of that there were 3 asymptotes and therefore 2 asymptotes went off in the right half plane and therefore for a large enough value of K, the system would become unstable.

This was the conclusion that we have drawn, using the root locus method. Of course for what value of K to determine that one has use the Routh for its criterion of Routh criterion of the Routh table from that we have been able to calculate the value of K. Now let us look at it from the point of view of the Nyquist criterion or the polar plot. Now what is going to be the polar plot of the new loop transfer function G is there as before H is there as before, I have this factor K by S.

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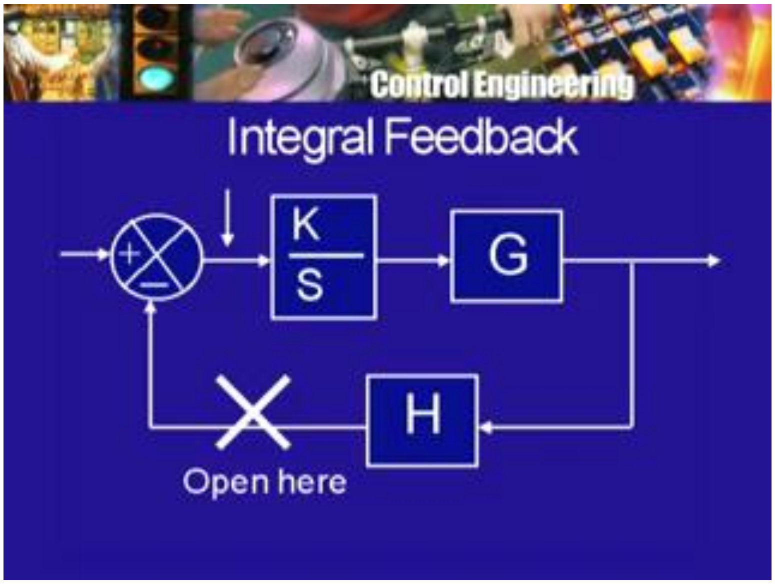

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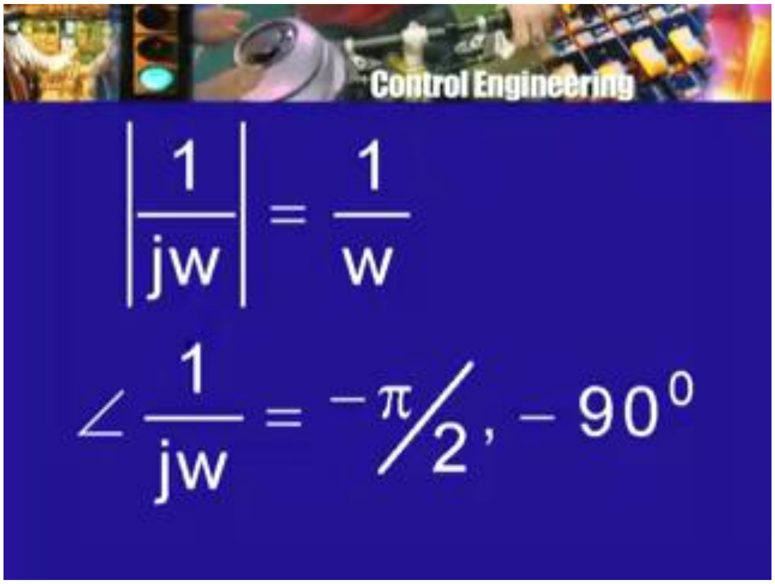

Now it is not necessary for me to repeat an experiment breaking open the loop here, applying an input here and then seeing the output here. I have already done the experiment for the G H part, it is not difficult to figure out the effect of the S part and that is because we have to replace S by j omega. So if I do that I will have a1 j omega multiplying the original K G H. Now what is the modulus of this and what is the argument of this. What is the modulus of this? What is the modulus of1 by j omega, it is 1 by omega. So when omega is increased the modulus is going to become small, it is going to become number approaching 0 but what about the argument of 1 by j omega.

Now I hope you have not forgotten you j manipulations the argument of 1 by j omega when omega is positive is minus phi by 2 radians or minus 90 degrees or this integrator produces a 90 degree lagging phase shift therefore, the phase shift of the original G H loop transfer function is increased by an amount 90 degrees and the phase shift as I have pointed out just now was a lagging phase shift. As a result of which the lagging phase shift will be increased further by 90 degrees and therefore the polar the polar plot or the Nyquist plot or the frequency response data will give you plot which is going to look like what?

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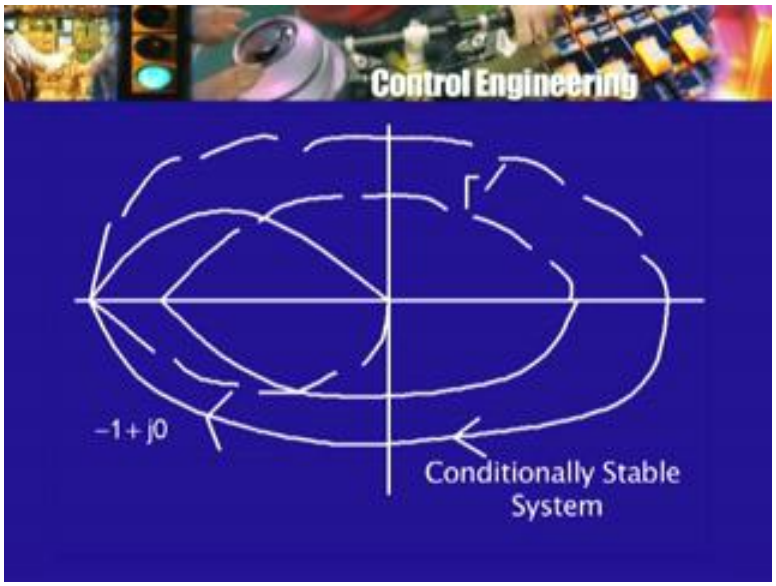

At very low frequencies there is no phase shift difference , so it will start here as it did and then, it will go around because of the lagging phase shift. However, as the frequency increases to a large value the phase shift is not going to be180 degrees or minus 180 degrees but it is going to be minus 270 degrees or plus 90 degrees which you are where you want to look at it and therefore if I actually did an experiment the polar plot that I will obtain will look more or less like this. Remember, that this is not what you actually get what you get are a number of points which we then, are going to join by a smooth curve and we really do not get may not get very close to the origin although we can try to apply as higher frequency as possible. However, we know from the fact that the number of poles is greater than then number of 0s that is it is going to become very small anyway.

So this is what you will obtain experimentally, now this looks different from what we had looked at earlier because of this presence of this factor 1 by S. So these were the curves that I have drawn earlier. Of course, this was for the case when the thing was in the upper half but actually it is going to be in the lower half as we have seen. So this is what is going to look like now this is not the whole of the image contour gamma prime. So I have to complete it by taking its mirror image. So let me draw this dash line to indicate the mirror image.

So this is the one part of it this is the other part of it and this is corresponding to that semicircle business, a point which is very close to the origin or the part of that curve is very close to the origin. So I can essentially show it as going through the origin. So this is going to be the polar plot or the entire Nyquist image of the Nyquist contour, this is going to be gamma prime. Now remember, I have to put arrows and did not do it earlier. So let me now put the arrow on the dash part also, so this corresponds to the upper half of the j omega axis, frequency increasing from dc to a very high frequency then, this blob here corresponds to the images of the semicircle which is virtually the origin itself then, I am coming to the lower half of the j omega axis as the frequency is very high, I am here and as the frequency is decreased to dc, I go this way.

So this is what is going to be the entire contour gamma prime. Now this is for some particular value of K, remember. Now what about the relationship of this contour with the point minus 1 plus j 0. Now I do not really know what is the actual experimental data so for whatever K, I have chosen I do not know where the point minus 1 plus j 0 will be on this diagram. But let us assume for the purpose of illustration that the point was here minus 1 plus j 0. So for that particular experimental value of K the point minus 1 equal to j 0 happen to lie here then, what is our conclusion. Now first of all let us look at the sense of the travels the sense of the travels of contour gamma prime is clockwise, the gamma contour was also travels clockwise, so the 2 senses agree.

So when I look at the number encirclements I should take out number for which I do not have to change the sign that is put negative sign but what is the number of encirclements by this contour gamma prime of the point minus 1 plus j 0, it is 0. So the number of encirclements is 0 that is equal to the number of characteristic polynomial roots minus the number of poles of G H in the right half plane which is 0. So the number of characteristic plane characteristic polynomial roots is equal to 0 or the system is stable for this value of K but now suppose, I change the value of K, what is going to happen.

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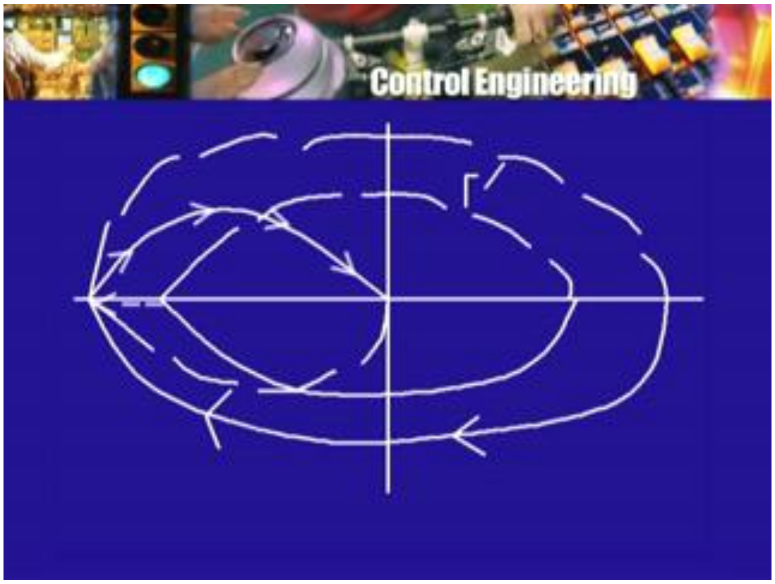

Now this is where one has to be very careful suppose I change the value of K. So that it is say double the original. Now this whole contour is going to be scaled, it is going to go out into the complex plane by a factor of 2, as a result this point here is going to move out, it is going to be scaled by a factor of 2. So this point may move out all the way here and therefore this curve will now look more or less like this and so I am going to show it as a heavy figure here like this and this will be the part here like this and similarly, for the mirror image.

The mirror image, the dash curve will now be like this and the remaining dash curve will be like this. The S along the semicircle part still remains very close to the origin because multiplication by 2 is not going to change it very much. So now the image contour looks like this and let me trace it, starting at low frequencies I go like this, high frequencies along the semicircle then, low frequencies around the lower part of the j omega axis and back to here.

Now what is the number of encirclements of the point minus 1 plus j 0 by the image contour gamma prime. It is no longer 0, it is 2, by drawing a radius vector from this and by looking at the intersections also you can see that the number of encirclements is going to be 2. In the same sense therefore it is going to be plus 2, let me trace it again going here like this then, going like this and then, coming back. The total of number of encirclements of this point is 2 therefore 2 is equal to the number of characteristic plane roots in the right half plane minus the number of poles of G H in the right half plane that of course, it is not change which is 0 and therefore the number of characteristic plane roots in the right half plane is now 2 and not 0. In other words, the system will now have become unstable.

So for this new value of the gain K the system is going to become unstable and we can see that if I increase the gain the further, the system is going to remain unstable. When as if I reduce the value of gain K from the original value, the system is going to remain stable for all smaller values of K. Now this was exactly the conclusion that we had obtained by using the root locus method that is for values of K lying between 0 and some number, the system was going to be stable but for values of K greater than that number the system was going to be unstable.

Therefore, the system was going to be what is called, a conditionally stable system. This was the conclusion that we had drawn using the root locus method and we can draw the same conclusion using the Nyquist criterion or the Nyquist method, what about that critical value of K corresponding to the j omega axis intersection that we can obtain from this also if we know the location of this point then, what should be the value of the gain K, so that this point become minus 1 plus j 0 that will be the critical value of the gain and that can be calculated.