Control Engineering
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Lecture - 38

Let us go over Cauchy's principle of the argument once again because I want to be sure that you have understood what exactly is involved. We are talking about a mapping of the Z plane into the W plane through a function f of z, w equal to f of z for each complex number Z, a unique value of it is determined by the function f and that is the point in the W plane. Then, we look at a contour in the Z plane and look at its image in the W plane. So here is a contour in the Z plane and let me choose the contour as the unit circle but centered around the point 1 plus j 0 with radius 1 and I am going to choose for the function f simply Z minus 1 as we did earlier.

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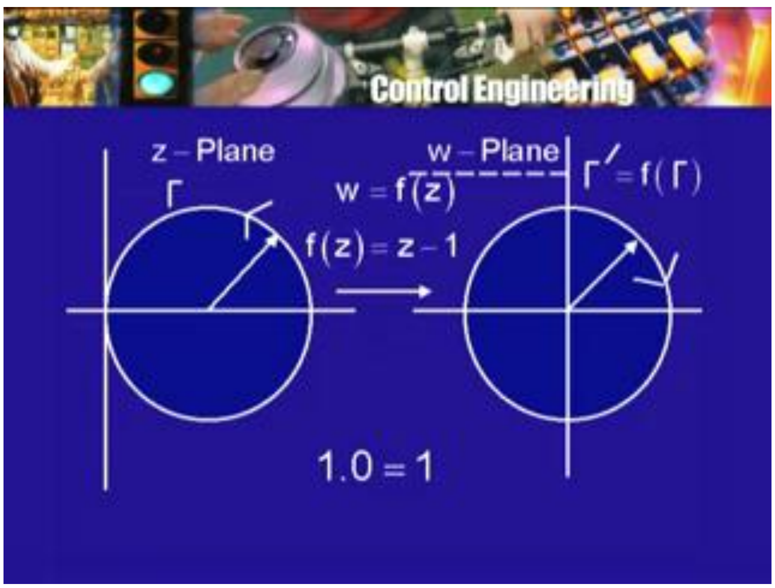

So, what will be the map or image of this contour in the W plane? As we saw, we are simply subtracting 1 from Z, so this whole circle will shift by 1 unit to the left and therefore the image will be the unit circle in the W plane centered at the origin. Now I told you that one talks about a direction of traverse around the contour gamma and that has to be indicated by putting an arrow on the contour.

So for example, if I put the arrow in this way this means that I travels the contour in this direction or sense, instead of direction it may be better to use the word sense clockwise sense and counter clockwise sense rather than direction because the direction is usually applied to a vector or a straight line segment where as what we are talking about is a rotation of a radius vector. So this contour gamma is traversed in the counter clockwise sense, notice that I have chosen the contour so that it does not pass through the 0 of the function, this function has only one 0 namely, at 1 it has no poles. So that condition is fulfilled that the contour should not passed through either a 0 of the function or a pole of the function.

So that condition being satisfied the function f z will never become 0 nor will ever become infinite on the contour gamma. So, we have the image contour gamma prime or f of gamma.

Now depending on what is the function is as we trace the contour gamma in the chosen sense, I will choosen the counter clockwise sense the contour gamma prime may get traversed in the counter clockwise sense or in the clockwise sense that depends on the function f. For this simple function f equal to z minus 1 it is easy to see that the contour gamma prime will also get traversed in the same sense, this is traversed counter clockwise and this is also traversed counter clockwise. Now, so the senses are the same.

Now, we look at the number of encirclements of the origin by this image contour gamma prime.

The number is 1 and because the 2 senses are the same we do not change the sign of this number that is we count it as plus 1 only. Then, Cauchy's principle of the argument tells you that the number of encirclements by the image contour of the origin in the same sense as the original contour equal to the number of 0s of the function minus the number of poles of the function which are enclosed by the original contour gamma, the original contour gamma has enclosed 10.

So 1 but there are no poles, so it has enclosed no poles, so 1 minus 0 and that is indeed equal to 1 the number of encirclements by gamma prime of the origin in the same sense as the contour gamma, note that if I had changed the direction of traversed gamma here, if instead of this I had this then, the contour gamma prime would be traversed in the clockwise sense and so the sense of gamma and gamma prime would still be the same both clockwise therefore the number of encirclements which is 1, I will not attach minus sign to it and still keep it as 1 and therefore 1 is still equal to 1 minus 0.

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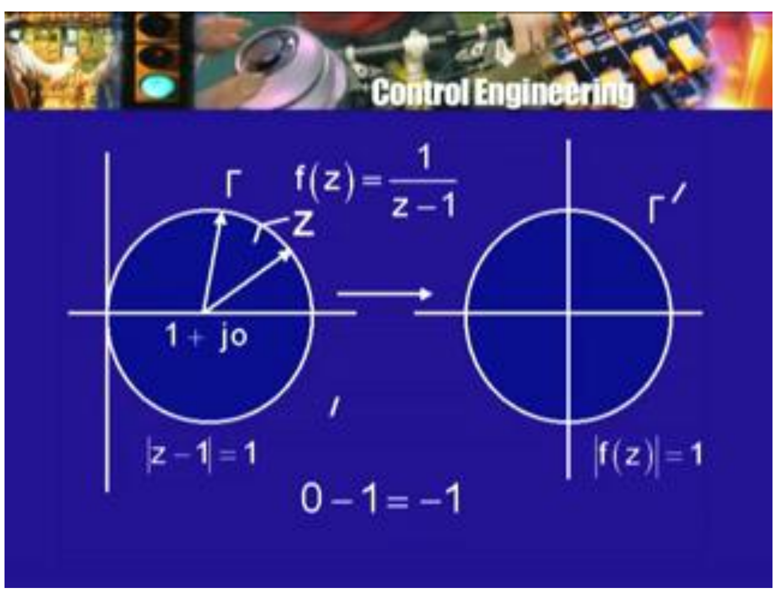

So that is how the principle of argument is being applied. The image contour gamma prime, the original contour gamma the direction of traverse along the original of the contour of the sense, the direction of traverse along the image contour, we have to look at the number of encirclements in the same sense as the original contour. So that number may be positive or negative then that number is the difference of number of 0s of f inside the contour gamma minus the number of poles of f inside the contour gamma. So let us change the example only slightly, we will take the function of f z as1 over z minus 1.

So, what is the difference now, there is no 0, now this function is not going to become 0 at any point in the z plane. However, it will blow up at the point z equal to 1, so it has 1 pole and the pole is at the point 1 plus j 0. So for the z plane, I will choose the same contour as before so here is that gamma centered around 1 plus j 0, radius 1 and let us say that we traverse gamma in the same direction as we are same sense, we have chosen earlier namely in the counter clockwise sense. Now what will be the image contour now the image contour is no longer simply displaced version of the original contour because what we are doing is 1 divided by Z minus 1.

Now from complex function theory you may recall what are called bilinear transformations and both Z minus 1 and 1 over z minus 1 are examples of bilinear transformation and bilinear transformation, if you remember do certain things to certain very specific curves. For example, if you take straight lines or circles, they go into either straight lines or circles. Now which is which depends on the bilinear transformation in this case, it is not too difficult to figure out what the image control is going to look like because look at the denominator z minus 1, now z is any point on the contour gamma. So what is z minus1 geometrically, it is this vector, this is z, this is 1, so this is z minus 1.

So as the point said moves along the contour gamma in the counter clockwise sense, what is happening here this vector is going to rotate but its length is going to remain fixed. So the length is just going to be 1, so the modulus of z minus1 on this contour is just 1, so 1 divided by 1, so will the modulus of 1 divided by z minus 1, it will be 1. So f z will have a constant modulus 1, so here mod z minus 1 was equal to 1, this vector, so here mod f z is 1 it is a constant, what about the argument? The argument of this of course goes on changing but then f z is 1 divided by z minus 1, so the argument of f Z is the negative of the argument of z minus 1 and so, we will see now that the image will be a contour which will be the unit circle in the w plane. As it was earlier except that the traversed direction will be reversed that is gamma prime will be traversed in the clock wise sense as point z traverses gamma in the counter clockwise sense.

So this traverse is opposite in sense to this traverse. Now, what about the number of encirclement the number of encirclements is still one. Of course now, with this one I have to put a sign minus 1 minus sign because the directions of traverse are not the same, this is clockwise, this is counter clockwise. The two senses are not agreeing, they are opposite. So the number of encirclements is 1, I have to put a minus sign before it so that is minus 1 now that by Cauchy’s principle of the argument should be equal to what the number of 0s of f enclosed by gamma but f is no 0s. So the number of enclosed 0s is simply 0 minus the number of poles of f enclosed by the contour gamma. The contour gamma does enclose the point 1 plus j 0, 1 plus j 0 is inside the curve gamma and therefore this number is 1 and we do check that 0 minus 1 is equal to minus 1.

So we have verified Cauchy’s principle of the argument, so this other simple function 1 over z minus 1. Now, the going is not going to be as simple as this when the function f Z which for us will be a transfer function or related to a transfer function has 1 or more 0s and 1 or more poles. Then, the mapping becomes complicated and just to take an example, let us say I write down a mapping f Z equal to z minus 2 into z plus 2 divided by Z plus 1 into z plus 3 into z plus 4.

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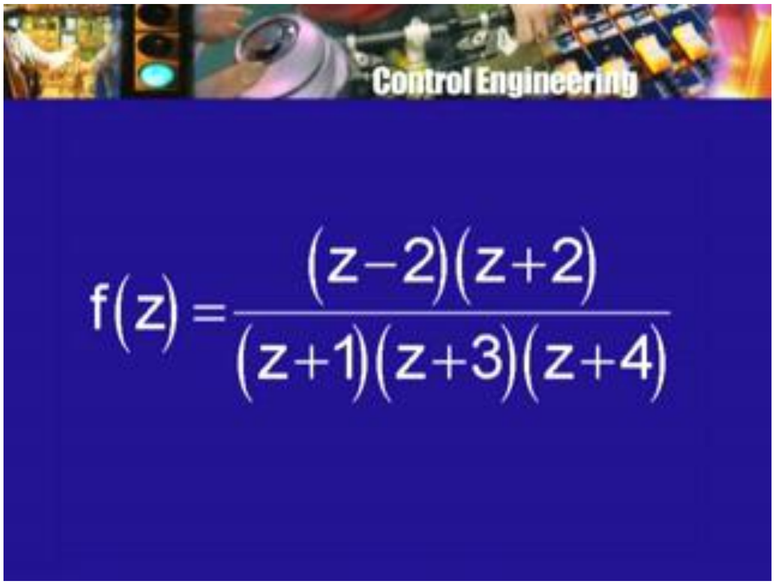

Now there are 2, 0s and there are 3 poles, what kind of a contour can I choose or to apply the Cauchy principle of the argument. The contour must avoid all the poles and all the 0s. So it must avoid the points minus 2 plus 2 minus 2 which are the 2, 0s minus 1 minus 3 and minus 4 which are the 3 poles of this fractional function f z. You are free to choose any contour which avoids these points that will be your contour gamma and of course to start with one need not choose a contour gamma which is not a simple closed curve. So we will choose it as a simple closed curve

So we will choose a simple closed curve, you can choose a simple closed curve traverse it in either clockwise or counter clockwise sense for each point on that gamma, we will have to find out what is the corresponding w, plot all those points and you will get a simple or a non simple closed curve in the w plane that is the image of the original contour gamma. Then as you traverse gamma, the contour gamma prime will be traversed, in the same sense or perhaps in the opposite sense. So if it in the same sense then plus line, if it in the opposite sense then minus line this sign is to be attached to the number of encirclements of the origin by the image contour then, that number by Cauchy's principle of the argument is the difference of number of 0s enclosed by gamma minus the number of poles enclosed by gamma.

Now how does this principle of the argument help us in discussing questions of stability of our closed loop feedback control system. In fact, the function that I have chosen f z is such that it corresponds to the transfer function that we had looked at earlier, if you remember I had taken an example in which we had this loop transfer function G (s), H (s) which was given by exactly the same kind of function except in place of s I have z here or in place of z, I had s there.

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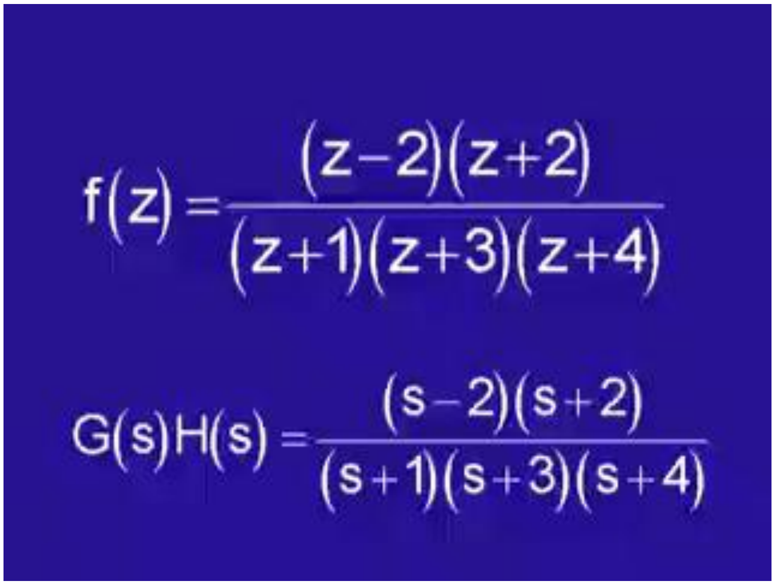

Now this is the just matter of standard practice somehow the letter z was used by mathematicians for work with complex variable for a long, long time. The letter s somehow was the preferred letter, when discussing the Laplace transformation and so we talked about G (s), H (s) rather than G (z), H (z). Unfortunately, the letter Z has been used by control theory and system theory people when talking about the Z transform or the discrete Laplace transform.

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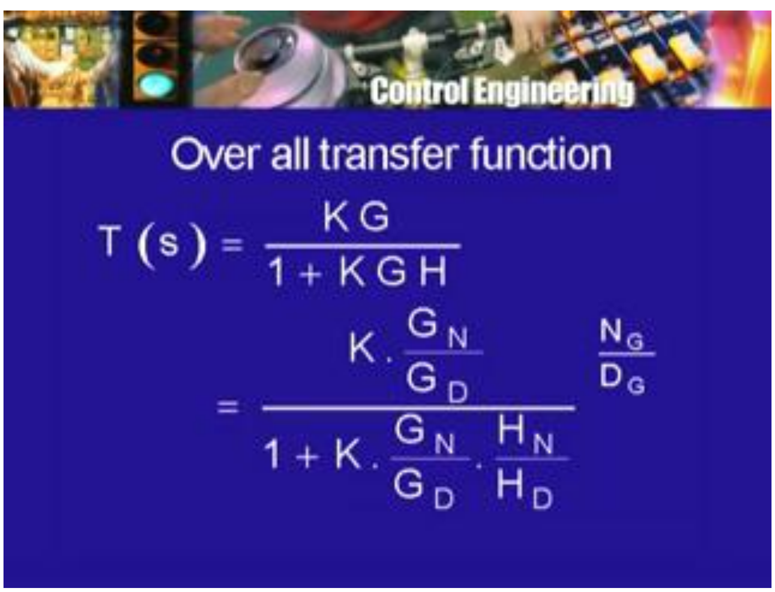

So the mathematicians said and the system theory or the control theories, Z transform again can cause some confusion but just keep this in mind that this s or that z is just a complex number and therefore in place of the Z plane one talks about the s plane that is the only difference, it is the only difference of a symbol. So here is the transfer function that we are looking at the loop transfer function that we are looked at earlier and hence forth now, we will use s in place of z. So this was the loop transfer function now, if you remember what we were looking at was the overall transfer function t of s which was given by G multiplied by K of course, the gain plus K times G H. This was the overall transfer function of a closed loop system with gain K G in the forward path that is a transfer function G in the forward path, the transfer function H in the feedback path and a gain K preceding the block G and this K and this K in general corresponds to some kind of an amplifier gain.

You should go back to this very quickly and find out for yourself that this was indeed the transfer function of the standard feedback system configuration, K and G in the forward path, H in the feedback path and a comparative device with a negative sign at the input. So t of s was K G plus divided by 1 plus K G H then, what did we do we looked at each G and H in most of our applications is a rational function that is it is a ratio of 2 polynomials. So, what we did was we simply wrote it as a ratio of 2 polynomials and we wrote it as G N divided by G D, G is the numerator of G, G D is the denominator of G, I could have written of course N G, the numerator of G and D G the denominator of G. I have written this some books use the opposite symbolism but just keep in mind that the G is the transfer function, N for numerator.

So K times that plus K into G N divided by G D but the G transfer function and H N divided by H D for the H transfer function. So simplifying what did we get T of s gave us G D, H D plus K times G N, H N. In the denominator all of these are polynomials G N, G D, H N, H D, so this whole thing is a polynomial and what was this polynomial called this was called the characteristic polynomial of the closed loop system. Remember, we encountered this when we looked at the root locus method also.

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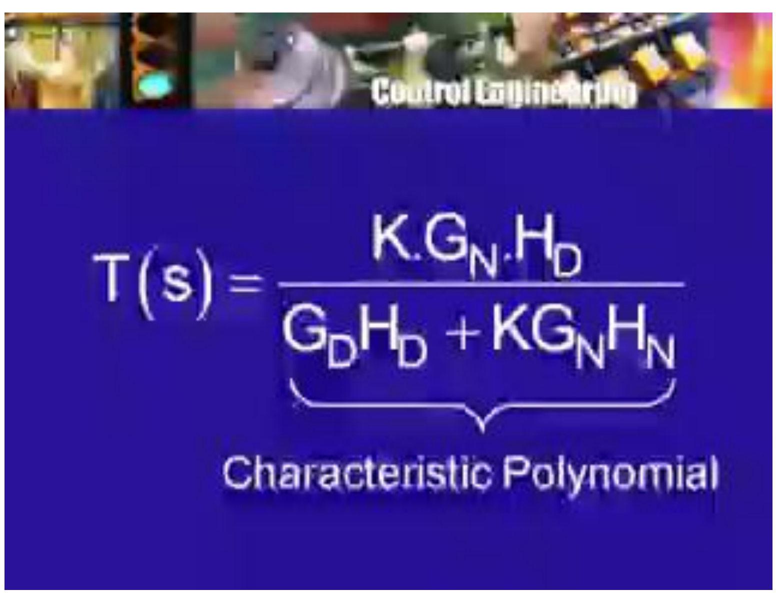

In fact, the root locus method enables you to show or to study the location of the roots of the characteristic polynomial as K varies. In the numerator, what do I have I have K, G N remains there and multiplied by because I am there multiplying numerator and denominator G D, H D, G N into H D. So the overall transfer function T (s) is a rational function of s or of a complex variable. Of course we are not interested in T s as such in the sense, we were more interested in the response of the system to a particular input or a general input or the dependence or the part of the response that depended on the initial condition and then, we found out when we looked at the response and if you remember, when we did that we looked at a general, fairly general differential equation of the form P (D) operating on the response variable and for us the response variable is usually y equal to Q (D) operating on an input variable and in control, the input variable is usually u.

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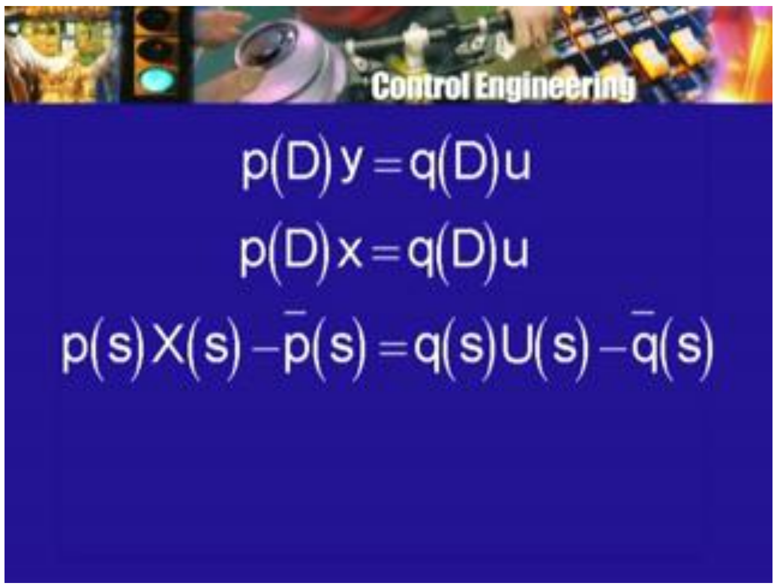

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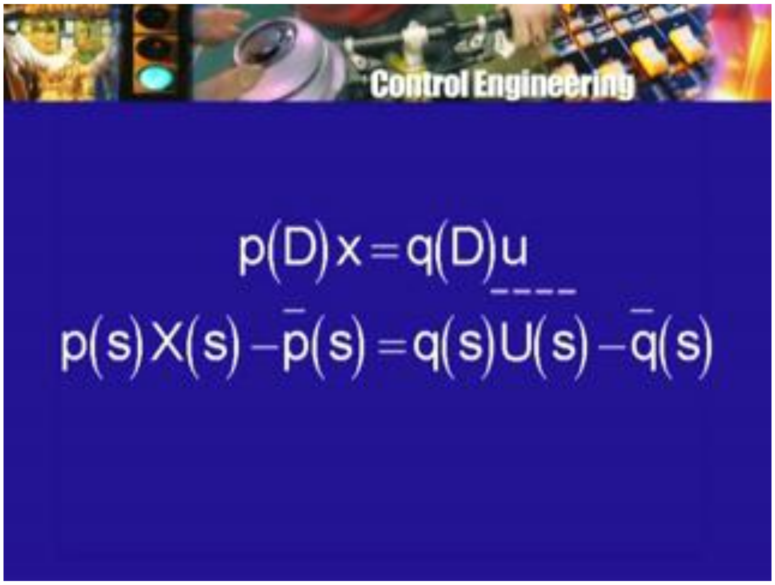

So P (D) y equal to Q (D) u was the differential equation that described the control system or a system in general. Then from this applying the Laplace transform, we got p s of course in place of y, I may have put X also because X itself may be the desired of the output of interest. So if I use X I will get p (s), X of s minus p bar of s equal to q (s), U of s minus q bar of s, where p bar of s and q bar of s are 2 polynomials that depend on the initial values of X and the initial values of U that is x 0, D x 0 etcetera U 0, D U 0 etcetera.

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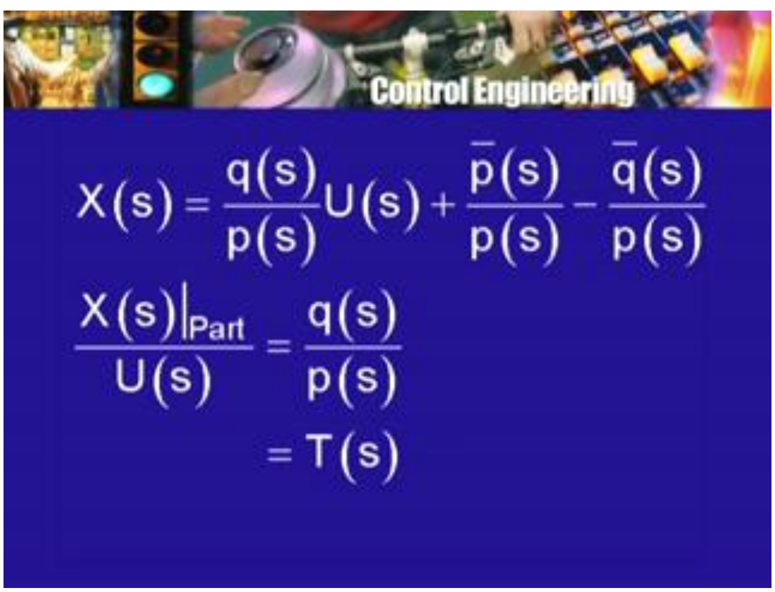

So from this we obtained xs as a sum of 3 terms, one of the terms was q (s) by p (s) into u (s) the other term was p bar of s divided by p s and the third term was minus q bar of s divided by p (s) and if you recall this term p bar of s divided by p (s) depends on the initial values of X, if the initial values of X are 0 this is 0 so you can say that either we are looking at the special case when the initial values are 0 or we are not looking at this part of the response transform. Similarly, this part involves only the initial values of the input, if the initial values of the input are 0, this part will be 0. So, once again we can say that we are looking at the special case when the initial values of the input are 0 or we are not looking at this part of the response. The third term depends on the Laplace transform of the input u of s and it involves 2 polynomials q and p and when we look at only this part, then we write x (s) divided by u (s).

So in other words, we are not looking at entire x of s but we are looking at part of it. This is q (s) by p (s) and this was our definition of transfer function. Transfer function was the ratio of the Laplace transform of the output or any particular part of it to the Laplace transform of the input. Now this polynomial p (s) occurs in all 3 places and if you remember, when doing the inverse Laplace transformation, we have to do the factorization of p (s) and therefore the roots of p (s), the roots of p (s) are the simply the poles of the transfer function p (s). The roots of p (s) determine the of the transient response or the terms that depend on initial values of x, initial values of U and this part also may give rise 2 terms that involved the poles of p (s).

So for stability understood in the sense that all these terms will go to 0 as t tends to infinity and this is one concept of stability that I have been talking about for that concept or for that stability to hold the roots of p (s) must all lie in the strict left half plane that is the roots of p (s,) if they are real must be negative if they are complex, their real part must be negative, they must lie strictly in the left half of the s plane, this was the condition of stability. Now going back to our problem of the closed loop control system, we have the transfer function t (s) for which this is the denominator and we are now interested therefore in the roots of the characteristic polynomial, their nature will determine the stability of the system.

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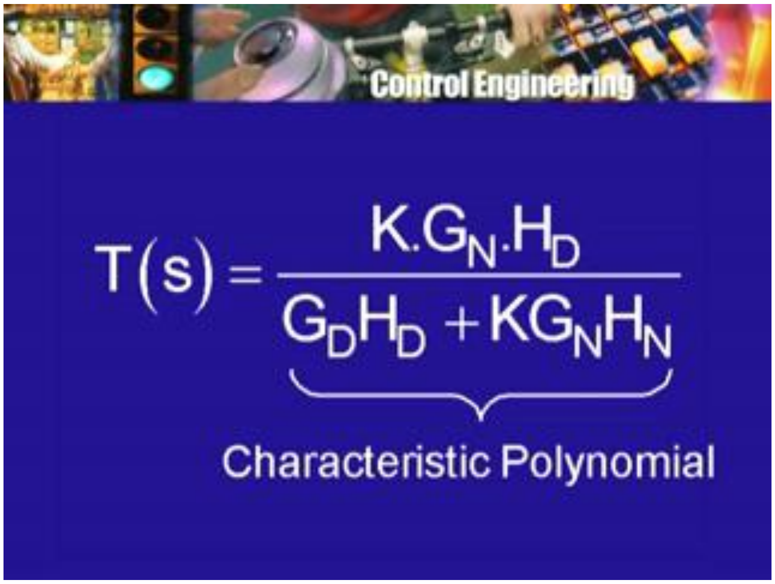

So we want to find out what are the values of K for which this polynomial will have all the roots which are strictly in the left half plane and therefore the closed loop control system or the feedback control system will be stable in this sense of the term. So we have to look at this characteristic polynomial but in the Cauchy principle of the argument approach, what you have is not a polynomial but a ratio of the polynomial and then, you have number of encirclements and so on. So, how to take care of that now that can be taken care of in the following way. Now this is where some confusion is possible, so you should pay attention there is a numerator of this T of s G N into H D which is a mixture which has the numerator part of G but it has the denominator part of H.

Now this numerator also can have influence on the response but I am not going to look at it more involved a kind of thing and these contribute the 0s or T (s), I am not looking at the 0s of T (s), we are only going to look at the poles of T (s) for stability the poles of T (s) should all we strictly in the left half plane. So this is what was are going to look at G D, H D plus K G N, H N. I want to find out the location of the root of this polynomial whether they are all in the left half plane for K or for what values of K. Now of course, we saw that the Routh algorithms or criterion was a method which enable us to determine that but now, we want to try a different method and this is what Nyquist did and there is another person whose name is sometimes also associated with this approach, he was the German engineer, so his name was Barkhausen.
So sometimes it is referred to as the Nyquist Barkhausenhe criterian or technique okay. So we are interested in the roots of G D, H D plus K times G N, H N. Now for a reason which you will not see right now but I will give it a little later. Instead of this polynomial, we will create a rational function related to it and we will do that in a very simple way. We will simply divide this by G D, H D. So instead of looking at this polynomial of course, we are interested in the roots of this polynomial. Let us divide it by G D, H D and let us look at this rational function think of that our F (z) in the earlier notation or let us say F (s).

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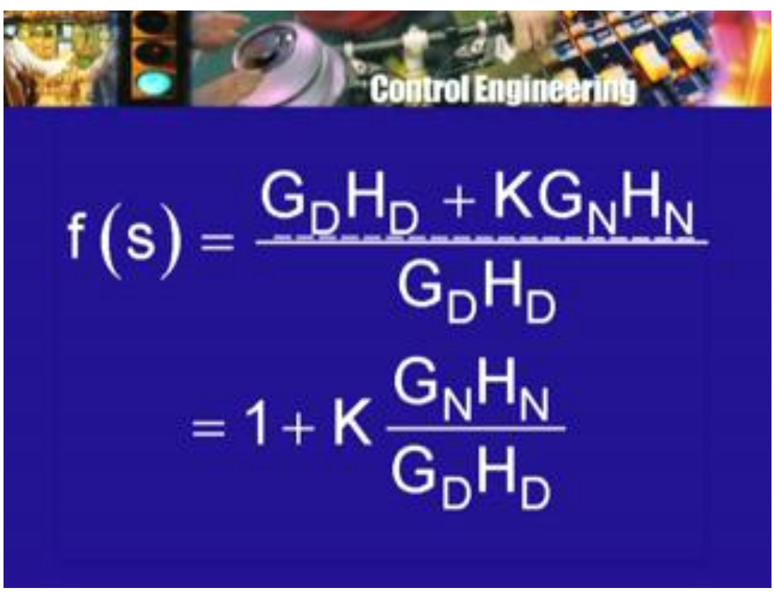

So this F (s) can be thought of this transforming the s plane into another plane of course, I can still call it the W plane if you wish. The function is given by this but of course because I have divided by G D, H D it is very simple I can write this as 1 plus K times G N, H N divided by G D H D. So this function F s is given by 1 plus K times G N, H N divided by G D, H D. Now what is the effect of this first of all the function F is simply 1 plus something. So the image of F can be thought of as add 1 to the image of this part, so instead of looking at F, I could look at this function K times G N, H N divided by G D, H D and look at the image of any contour in the S plane for the, under this function under the action of this function.

To get the image of this function F (s), I simply have to add one arithmetically, so geometrically what do have to a do have to displace or shift the contour or the image by 1 to right okay. Now second thing is this factor K, now as I said earlier in most application this gain K is a positive real number. So what is the effect of this K, it multiplies this, so it acts only like a positive scale factor that it will just blow up the image by a scale factor. A complex number W is simply going to multiplied by the real number K, now of course it will blow up if K is greater than 1, if K is less than 1 then it will reduce it but the action of it is very simple, it is simply going to either blow up or shrink the contour.

So we need not worry about K right now, we need only worry about G N, H N divided by G D, H D but the G N, H N divided by G D, H D is nothing but our loop transfer function G H. So essentially, we will be looking at the values of this loop transfer function G H at various points in the S plane okay. So this is 1 factor of course, this is not the image of the function F to get the image under the function F, I have to multiply this by K that is I have to imagine it as scaled by the positive number K and then, I have to shift it by 1 to the right multiplication by K and shifting by 1 to the right.

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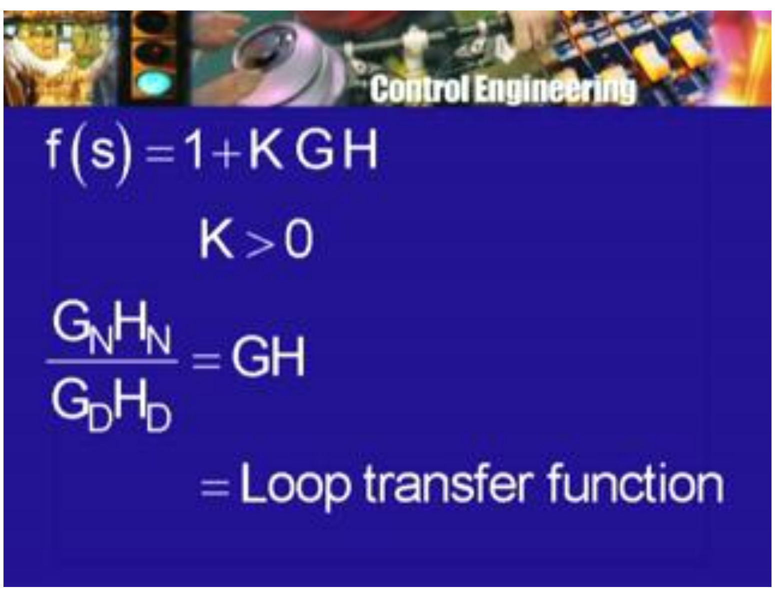

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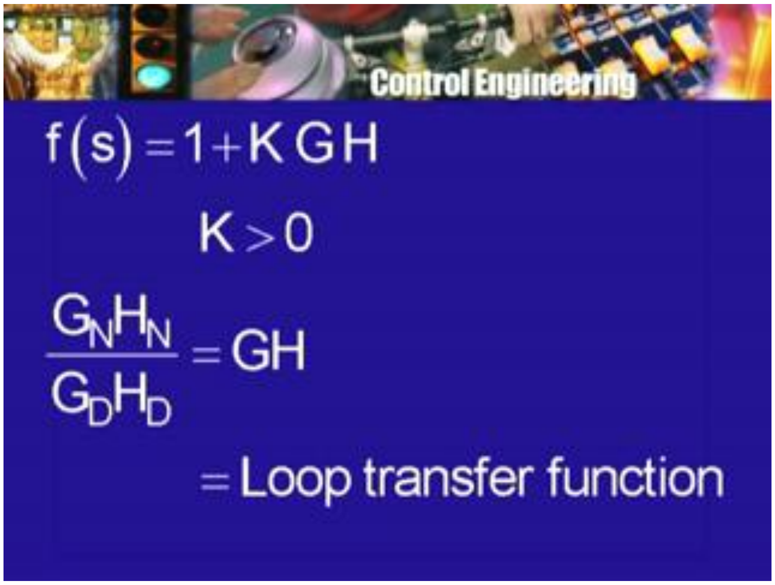

Now, we are going to apply the Cauchy principle of the argument to this function F and why because the numerator of it is the characteristic polynomial. Unfortunately, we do not know where the roots of the characteristic polynomial lie for different values of K. We can calculate them using Routh, we can even calculate some special intersection points and so on. But in general, if you do not use the root locus approach we just do not know where the roots of this characteristic polynomial are going to be what about the denominator G D, H D. Well the transform using G S is known to us, so if is denominator can be factorized and I will assume that it can be factorized because these are the poles of G and these are the poles of H.

So the denominator polynomial, we know the location of its root it is the numerator of which we do not know the roots. So suppose I will choose a suitable contour in the S plane apply Cauchy principle of the argument for this function F then, will I get the information that I want what is the location of the root of this characteristic polynomial or in particular, I want to make sure the system is stable, so there should not be any roots in the right half of the complex plane nor on the j omega axis. Now, let us first be a little optimistic and let us assume that there are no roots of this characteristic polynomial on the j omega axis.

So all the roots are either in the left half plane or in the right half plane all are sum, we want to find out whether it for some value of K, positive K, there could be a roots in the right half plane, strict right half plane. With this in mind now, we can choose a contour and this was of course the contribution of Nyquist and Barkhausen that is they choose a contour which will do the job and the contour is this one. Remember, it has to be a simple closed curve but it has to be such that it will be useful to us.

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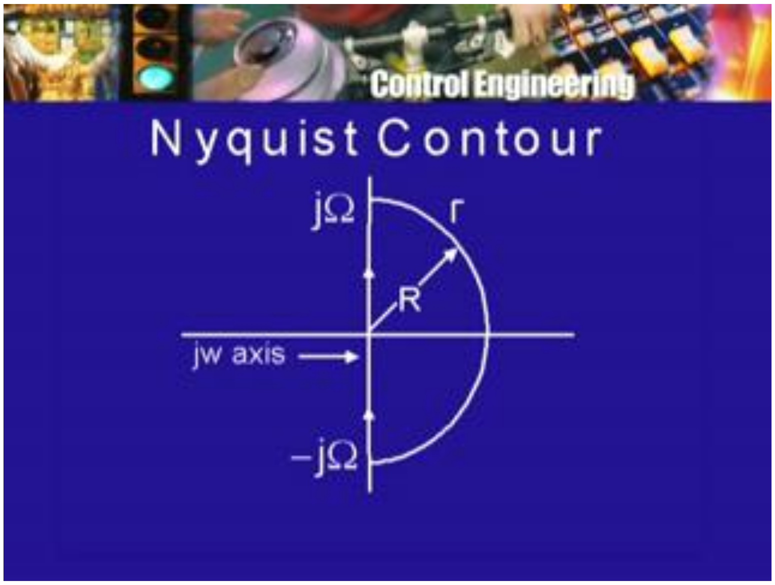

So this will be the contour choose a part of the j omega axis and we can choose it in a symmetric passion. So let us say an upper limit here is j capital omega, this omega is not to be confused with speed it is just some real number of course, we can interpret it as frequency as you will see the lower end point is minus j omega and to close the contour, we will take a semicircle and you will see that if it really does not have to be a semicircle, it can be a curve for which the only condition that you will see is that this part of the curve should be far away from all the poles and 0s of the transfer function G H, this is the only condition that needs to be satisfied.
So this will be our contour there is a part of it that lies on the imaginary axis and the rest of it lies in the right half plane and we will think of it as a semicircle whose radius r, we will consider to be sufficiently large. In other words, we are almost considering the limiting argument as R tends to infinity except that we do not really make it go to infinity, we only consider R large enough, we have to choose a direction of traverse of the contour. Now I can choose it to traverse in the counter clockwise sense or in the clockwise sense.

Normally I would have chosen the counter clockwise sense because that is the preferred direction of rotation, when you talk about phases and so on. Again, historically and again for a reason that you will see very soon one chooses this direction going upward from the minus j omega point to the plus j omega point and therefore, in the clockwise sense to complete the contour we will travel it this way. So this is sometimes called the Nyquist contour, named after the Nyquist who used it for the first time. So this will be our contour gamma all right. There is a part of it that lies on the j omega axis and of course, if I choosen this radius R then this j omega can be replaced by j R and this minus j omega can be replaced by minus j R.

So this is the contour that I am going to choose. So let us now apply Cauchy’s principle of the argument choosing this Nyquist contour as the contour gamma in the S plane and we are going to look at the image of this contour under the function f of s of which the numerator of is the characteristic polynomial and the denominator is simply G D, H D, as we saw earlier. Now, what is going to happen Cauchy’s principle of the argument will tell us something about what, the number of encirclements by the image contour of the origin in the image plane is related to the number of 0s and poles of the function of f or w of f of z or in this case f of s enclosed by the contour.

Now we have chosen this has our contour. So what was the reason and what is the effect of this, if the function f has any poles or 0s in the right half plane then, if R is large enough they will be enclosed by this contour gamma and we do not want to miss out any poles or 0s of a f which may be in the right half plane and this is the reason, why we have to consider this contour as sufficiently large. Of course, if you look at the poles and 0s of f, the 0s of f are unknown to us but we know that they are all finite or they are all complex numbers, none of which is infinite because this polynomial, a polynomial of any degree has finite roots, the denominator is G D, H D. These are the poles of G and the poles of H, if there are any then they are also known.

So these poles are known these poles are unknown but they are going to be finite therefore, we have to consider the situation of a contour such that the radius of it is sufficiently large, how large exactly we do really do not need to know essentially what you have to consider is, we have to consider the situation that R keeps on increasing and then see what happens, all right. So this is the reason for choosing the semicircle, the whole right half plane cannot be enclosed by a closed curve, it is not possible to do it. But any finite part of it can be enclosed by a closed curve like this and therefore Nyquist choose the contour gamma in this passion. Therefore, the result now will be when I apply Cauchy principle of the argument it will be like this the number of encirclements of origin and instead of origin, I am going to write 0 plus j 0 by f, the number of encirclements of the origin by the image contour f gamma.

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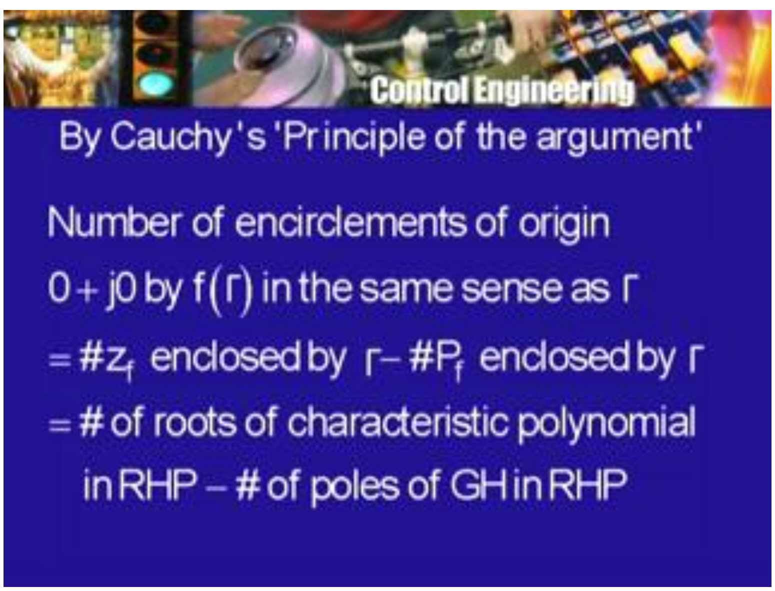

In the same sense, in other words, if the image contour is traversing in opposite sense we prefix a minus sign, if it is traverses in the same sense as gamma. We do not prefixed a minus sign that we take it as plus in the same sense as gamma. This number by Cauchy principle of the argument is the difference of these 2 numbers, the number of 0s of f enclosed by gamma minus the number of poles of f enclosed by gamma, the number of 0s of f enclosed by gamma. Now, where are the 0s of f coming from the 0s of f are precisely the roots of the characteristic polynomial.

So the number of 0s of f enclosed by gamma will be precisely equal to the number of roots of the characteristic polynomial in the right half plane and for a stable system, we want this number to be 0 but let us be prepared to deal with unstable systems as well because we do not know for what value of K, it will happen or it will not happen. So the number of 0s of f enclosed by gamma is simply the number of roots of the characteristic polynomial in the right half plane minus the number of poles enclosed by gamma, now what where are the poles of f, they are the roots of G D, H D. So they are simply the poles of G H in the right half plane. So this number is simply the number of poles of the loop transfer function G H in the right half plane.

Now in many practical situations, the loop transfer function G H that is the character, the transfer function G, as well as the transfer function H, do not have any pole in the right plane that is they are stable usually as they are. In very exceptional cases and such exceptional cases occur in aerospace engineering G H may have poles in the right half plane, if so then we have to count them and include them here. So the number of encirclements is equal to this difference number of roots in the characteristic polynomial in the right half plane minus the number of poles of G H in the right half plane.

Now let us consider a special case, the special case is when there are no poles of G H in the right half plane that is all the poles of G and H are in the left half plane and we want the system to be stable. So the number of roots of the characteristic polynomial is required to be 0, in that case what do we have here, this number is 0, this number is 0, so 0 minus 0 is 0 therefore, the number of encirclements of the origin by f gamma in the same sense as gamma should be 0 that is it should not encircle the origin at all, okay. Now comes this idea of Nyquist, the function f was simply 1 plus K G H, I will rewrite it here 1 plus K G H.

So the image contour f gamma is simply 1 plus the image of this part but this is K times G H. So it is simply a scaled version of the image of G H. So in other words instead of looking at f, I can look at the image of the contour under G H, under the loop transfer function. So image of gamma under the loop transfer function G H, the only thing is that I have to keep in mind that there is this one and there is this scaling K, we will see how this scale factor K comes in but what about this one that is easy to see that f s the f gamma is obtained by shifting this by 1 to the right. So the following then will hold look at G H and the image of gamma under G H and look at image of gamma under f what is going to be the difference, the difference is going to be the scale factor K and this shift of one. Let us the think of it little bit and if you draw a figure for yourself, now this is something which one has to do it oneself in order to realize that it is slow.

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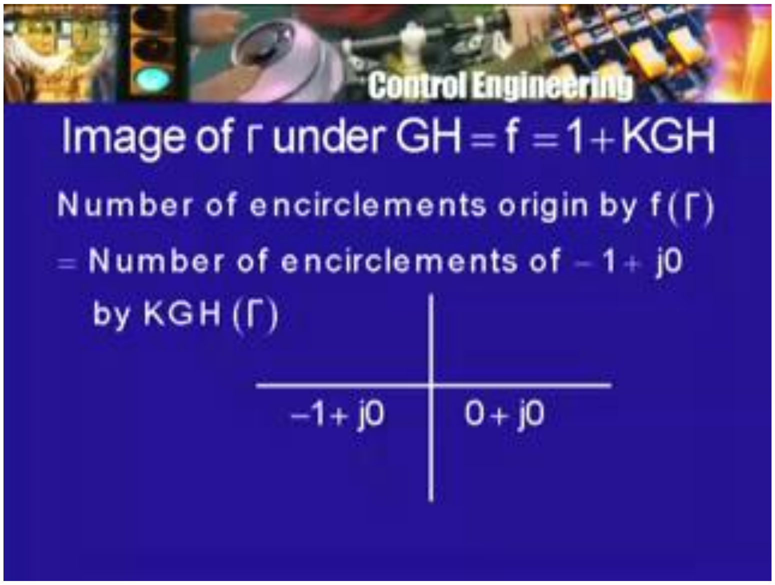

So but you will realize that the following is true. The number of encirclements of the origin, if gamma is equal to the number of encirclements of not the origin but by of the point minus 1 plus j 0 by not f gamma but G H gamma that is the image contour of gamma under G H, instead of looking at encirclements of the origin by it, you should look at the encirclements of the point minus 1 plus j 0 and the reason for this is that f is obtained from G H by multiplying by K and adding 1. So encirclements of origin by f will correspond to the encirclements of minus 1 plus j 0 by G H because minus 1 plus j 0, if I add 1 to that will become 0 which becomes the origin.

So this is the reason for this change in our point of view, instead of looking at the number of encirclements of the origin by f gamma. We look at the number of encirclements of this point minus1 plus j 0 by G H gamma, so this point minus 1 plus j 0 plays a very special role in the use or the application of the Nyquist criterion minus 1 plus j 0 but the reason, why it comes about is because the origin plays a special role in the Cauchy’s principle of the argument and because of our choice of f as 1 plus something G H of s, where s is j omega, where omega is a real number.

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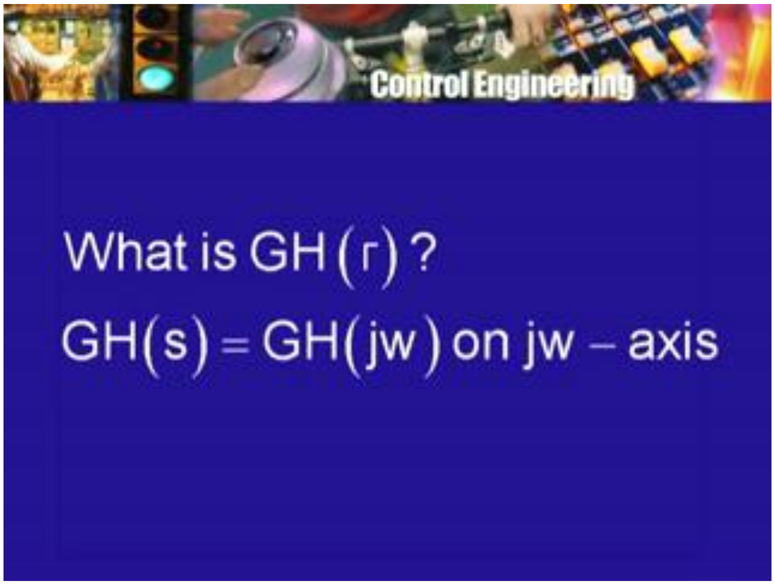

Now we will see in a moment that this complex number G H of j omega has a particular interpretation. But before we do that let us note one more thing about the contour I have chosen the contour so that it is symmetric about the real axis. The transfer function G H has only real coefficient and therefore the image contour will also the symmetric about the j omega axis. Unfortunately, in some books you will find the statement where instead of a contour one only talks about this part of the j omega axis corresponding to positive omega but principle of the argument requires a closed curve and therefore, it requires a closed contour like this one and therefore you have to pay attention to the negative half of the j omega axis.

So it is always better to think of this full Nyquist contour rather than only a part of it that corresponds to the upper half of the j omega axis, keep this in mind. Now, what about the interpretation of G H of j omega. Now this is something which goes back to Laplace transform theory and also the concept of transfer function and I am not going to go into details but the following is true, if you look at our expression for the response that we had worked out which was given by this, X of s equal to sum of 3 terms and if you look at only the first term here that involves only U of s then, one can show that if the input is a sinusoidal function, if the input is a sinusoidal function and we are going to look at only a part of the response that remains when all the transients have vanished. So I am also assuming that the system is stable, so all the transients really are transient that is they go to 0.

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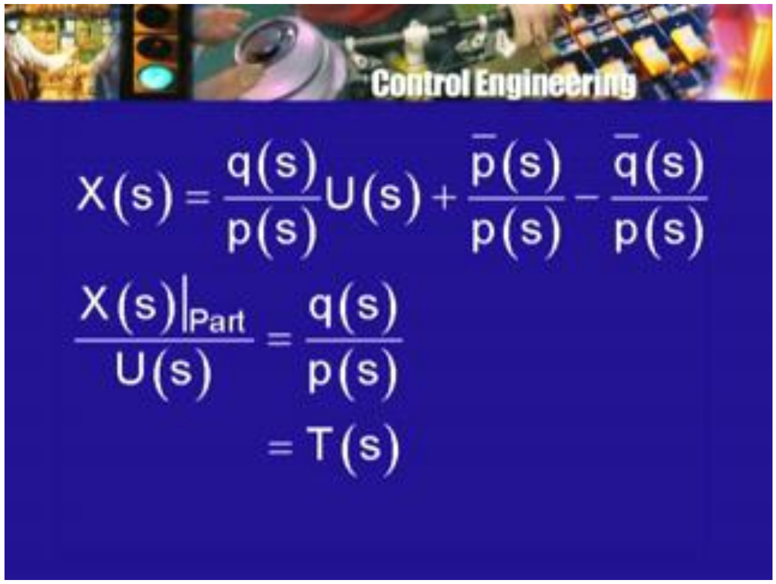

So under these 2 conditions and let me write them down, the input may have a certain amplitude, the output will have a other amplitude and second is the phase and as you aware are when you studied the amplifiers in your electronics circuit one talks about the ratio of the output amplitude to the input amplitude for a sinusoidal input signal and this is called the gain of the amplifier. So the ratio of the amplitudes of the output to the input or sinusoidal input applied to us, say stable system is called the gain at that frequency, at the frequency corresponding to that sinusoidal signal. It is called the gain, gain of the transfer function. The difference between the phase, output phase minus the input phase is called the phase shift corresponding to the transfer function.

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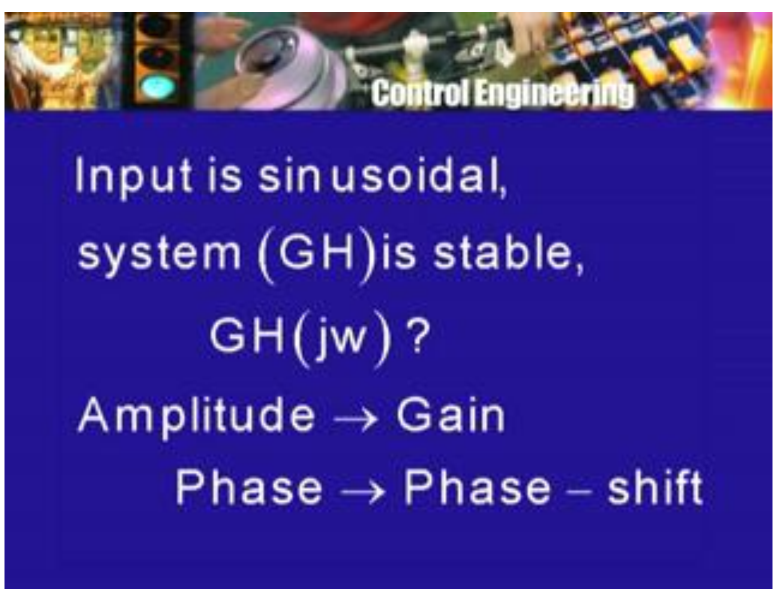

So when you apply a sinusoidal input to a system of the kind that we have been talking about which is stable then, in the steady state there will be these 2 things, there will a gain, there will be a change of amplitude and there will be a phase shift the output phase will not necessarily be the same as the input phase. Now this is the result which can be proved that the gain and the phase shift can be obtained from the transfer function t of s by simply doing the following. Replace the s by j omega, where omega is the angular frequency of the sinusoidal signal that one is looking at then this also will be a complex number you look at its absolute value or modulus. This absolute value or modulus will be the gain produced by the transfer function that is it will be the ratio of the output amplitude to the input amplitude and the argument or the angle of this complex number will be precisely the phase shift that occurs from the input to the output.

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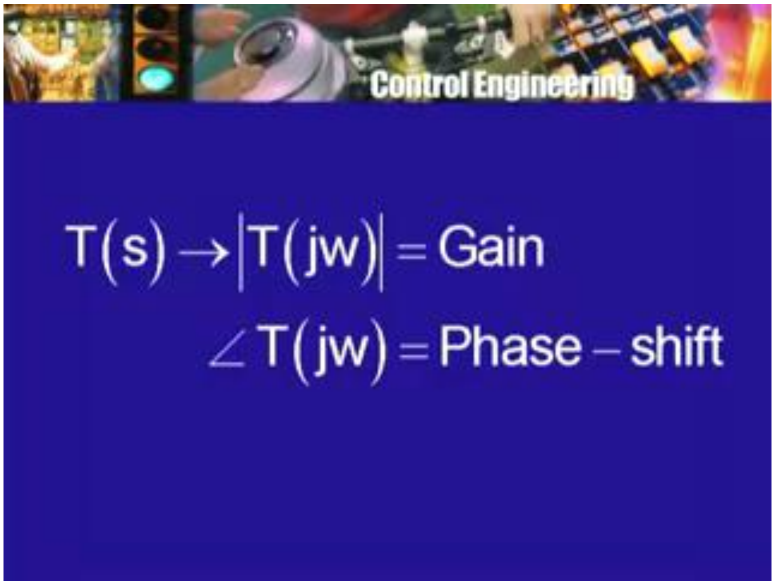

So if you know the transfer function T(s) of the system then the gain of the system that is output amplitude divided by input amplitude for a given sinusoidal of frequency omega is just the modulus of this T j omega and the phase difference between the output and the input is simply the argument of this complex number T j Omega. Now in practice one can do a laboratory experimentation where, you really it to the system apply a sinusoidal signal, we have signal generators which will generate sinusoidal signals of a good waveform of a range of frequencies going may be from as low as point1 hertz to may be 100 megahertz, if not 1 signal generator, you can have several of them.

So in the laboratory, I can take the system and I can apply to its input, a sinusoidal signal of a frequency which I can vary over a large range, I can of course measure the amplitude of the input signal then, I can observe the amplitude of the output signal and I can observe the phase of the output signal in relation to the input signal and as you know, this is very conveniently done by using a cathode ray oscilloscope or a CRO. So on a CRO, you can observe the input, you can observe the output, you can measure the amplitudes therefore from the measurements, you can calculate the gain of the amplifier or the system under study and the phase shift of the system under study at a particular frequency omega and you can vary omega, see you can obtain this for various values of omega and this is what is known as the frequency response of the system. This frequency response is determined purely experimentally, I take the system, I apply to it a sinusoidal input of so an amplitude which I will measure and I look at the output in the steady state that is if I wait long enough, the output will be sinusoidal of a different amplitude perhaps and a different phase, I can measure the output amplitude, I can measure the therefore the gain of the system and I can measure the phase shift of the system.

So this can be done experimentally, I do not have to know what the transfer function looks like as a ratio of 2 polynomial, what is its numerator, what is its denominator, I do not need to know that, I can experimentally determine the frequency response and that is T j omega the modulus and argument of it for various values of omega. This is one main advantage of using the Nyquest approach that is, it makes use of experimental data experimental data which is called the frequency response of the system. Of course, we are assuming that the system is described by a set of linear differential equations or in our case just one linear differential equation. In other words, we are assuming that the system is linear time invariant of the kind that I have talked about namely, it is described by differential equation like this p D x equal to q D u, single input u, single output x. The system is described by differential equation and I am also assuming that the system is stable that there is something called steady state that there will be a transient part which will indeed go to 0.

So under this assumption then, G H of j omega can be obtained experimentally, how I am to look at the transfer function G H. Now, where does the transfer function G H come about now, if you remember we had this G in the forward path then, the gain K in the forward path then, we had H in the feedback loop and this was the configuration. With this difference device R of s here, C of s or for else now I am using U of s and X of s and here is the feedback 1. Now if I want to measure G H, j omega for various frequencies, how do I get it from this.

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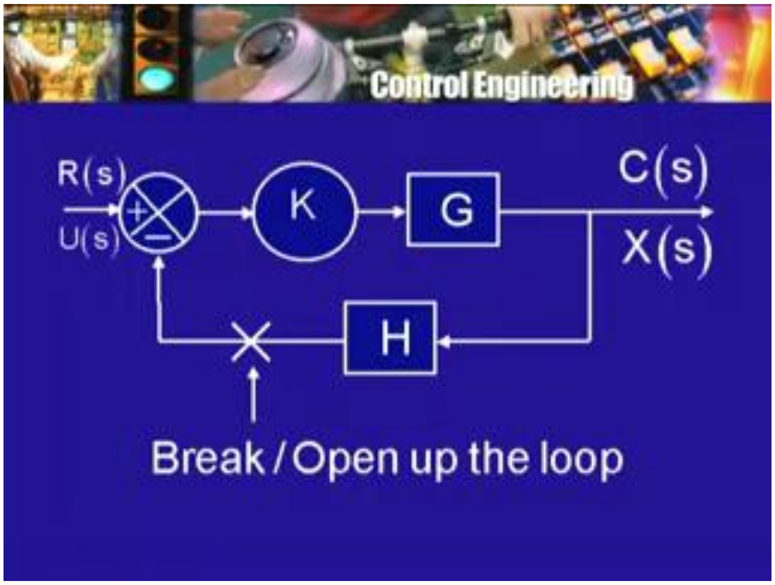

Well, I can get it if this is the closed loop control system which has been set up then, I can break the feedback, break the feedback loop here, apply an input here and measure not the output of the system but measure the feedback signal here. So if a system has been set up already a closed loop system has been set up all that I need to do is I need to break the feedback signal going to this difference device and I need to inject the signal here at the input to the G block or I can even input inject it at the input to the K block. So that will then take care of K and I will be looking at K G H, usually one leaves out this K because this K is the amplifier gain which would like to determine in your design consideration, you just applied to G and H, may be with some particular value of K that you chosen otherwise, your output may be too small.

So why, so the closed loop system is deliberately opened up and we make measurements of this transfer function G H, we make measurements of its frequency response by that we will know G H j Omega and therefore, we will know the image of that part of the contour namely that part of the contour which lies on the imaginary axis under the action of G H. This is one part of it, the other part of it now involves of course, the part of the contour which lies out in the complex plane namely it is the semicircle of a very large radius, we will look at what happens to it then we will put the 2 things together.