Control Engineering
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Lecture – 36

We have been looking at the effect of feedback on the performance of the speed control system. So, let us take a look at the transfer functions involve once again. Here, is the transfer function from the armature voltage E a of s, I will call this transfer function G a of s going into this difference device, the load torque was the other input multiplied by the transfer function G T of s and the net the result is the output omega s and we are seen that G a of s and G T of s are 2 transfer functions whose denominators are of degree 2 or in other words, they have 2 poles and the numerator of G a is just a constant where as the numerator of G T is a first order polynomial.

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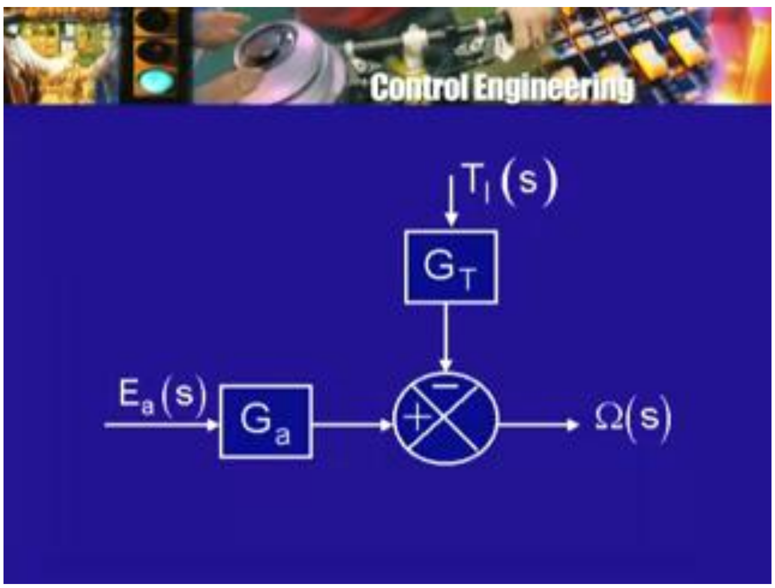

So G a has no zeros whereas G T has one zero, the values of the or the locations of the poles and zeros will depends on the parameter values of the motor and the drive. Now this was the system without feedback and as we have seen earlier, the transient response is going to be such that it will go to 0 as t tends to infinity and therefore there is no instability problem, as per as the open loop system is concerned. Of course, depending on the locations of the 2 poles in the complex plane, if they are both real and negative then the response will not have any oscillatory component but on the other hand if the poles are in the complex place like this.

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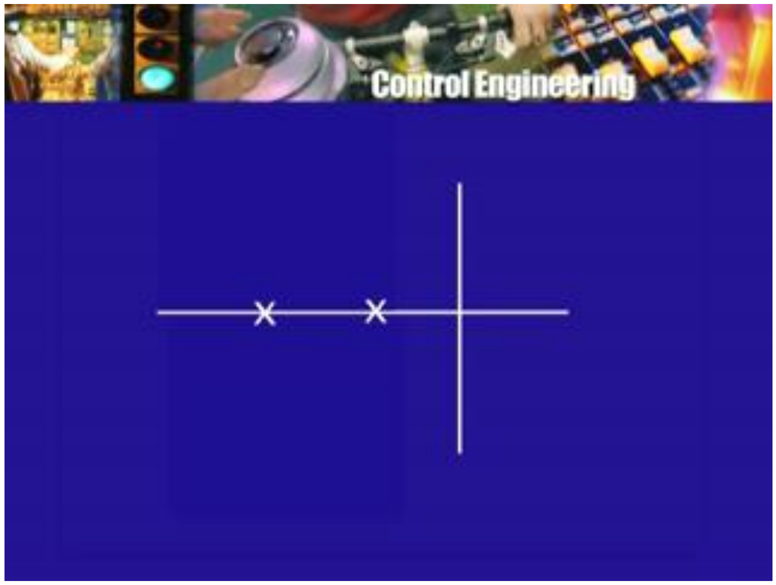

Then, the transient response or the response to change in E a or change in load torque will have an oscillatory component but the oscillations will be damped or we will go to 0 as t tends to infinity and by the time 5 or 10 time constants, I will have the transient response will be virtually zero and of course, the disadvantage of this open loop control system was that when that load torque changed from the rated value to some other value, the steady state speed was also change therefore there will be a steady state error in speed, when there is a disturbance input or there is a change in the load torque. Of course, the armature voltage is normally kept constant but if the armature voltage is also change, we expect the speed to change.

Now with feedback what is going to be the difference, now? As we have seen with feedback then we have to look at the closed loop characteristic polynomial which is given by 1 plus G h and therefore draw the pole 0 diagram corresponding to that. So, let us do that now first for the transfer function from E a of s to omega s that is considering only the effect of the armature voltage. So, the transfer function will be modified G a of s will go into this difference device I am suppressing the other input, I am only showing omega s and here is the k tacho generator and this output of the tacho generator goes into the other difference device for which the reference voltage is the input and the error signal, E of s is amplified by the amplifier k a to produce the armature voltage E a of s.

Now as we have seen the root locus method can be immediately applied and therefore when this k a is varied note that the tacho generator once it has been chosen one is not going to vary any thing associated with the tacho generator therefore the only variable thing in this will be the gain in the forward path from the difference device to the armature voltage, the reference voltage of course is kept constant for a desired speed and I repeat, we are only looking at the effect of E r on the drive, we are ignoring or not looking at the effect of the load torque, we will look at that very soon.

So, for this then what do have to do we have to draw the root locus and the root locus because there are only 2 poles of the transfer function G a of s, the h is only k tacho generator, this is just a constant. So there is neither a pole nor a zero, so the only poles and zeros if any come from this transfer function. Now this transfer function as I mentioned has 2 poles and no zero and therefore G a will have those 2 poles and no zeros and therefore, we can start with the root locus plot and we will consider 2 cases again.

Suppose, the 2 poles are like this which is usually the case the motor parameters the parameters of the armature and the mechanical parameters are usually such that the 2 poles of the open loop transfer function are both negative, real and in fact, they are reasonably separated, this is because the 2 time constants, the armature time constant tau a which is given by L a divided by R a and the motor time constant of the mechanical time constant tau m which is given by J divided k f, usually the mechanical circuit, so to speak is much slower than the electrical circuit or in other words, the motor of the mechanical time constant tau m is greater than the electrical or the armature time constant tau a because of this usually, the poles of the system are as I have showed here negative, real and somewhat separated.

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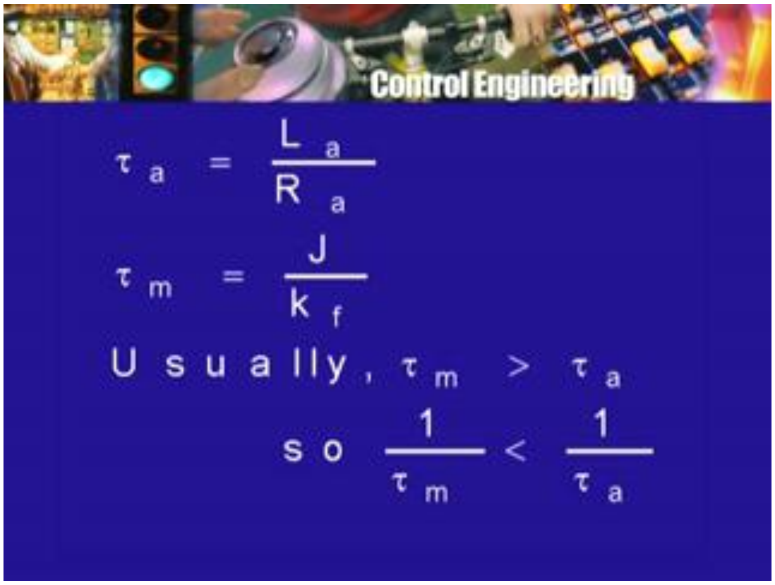

So what is going to be the root locus as k a is varied number of poles is 2 number of zeros is 0 p minus z is 2. So there will be 2 branches of the root locus, there will be 2 asymptotes, the point of intersection of the asymptotes as you can see will be just the midpoint of this interval, the angle made by the asymptotes will be plus minus 90 degrees, one asymptote going towards the upper half of the s plane and the other asymptote going towards the lower part of the s plane or the complex plane.

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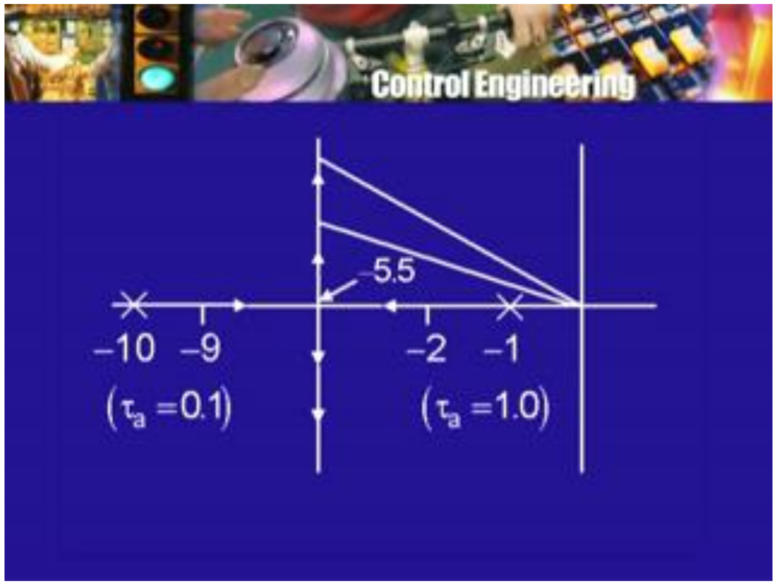

The root loci will start at the 2 poles, so like this and then we expect that they will come to better or need that means for some value of k a, there will be a repeated root by in this case because it is just a quadratic, it is very easy to work out the location of the roots in fact for different values of k a and one can see, from that that the 2 roots will really co inside at this midpoint of this interval from there to there, there after the root loci will breaks away from these real axis and in fact again one can show that the root loci will exactly lie along the asymptotes.

So first small gain k a, the 2 root of the characteristic polynomial which govern the response of the system or both negative and real therefore will correspond to exponentially decaying terms but the time constants will be a little different, the time constant will be different from these 2 time constants, one of the time constant will have increased where as the other time constant will have decrease. Till you reach a value of k a, when the 2 roots are equal and therefore the behavior is no longer purely exponential but also exponential multiplied by time t. However, as we have seen earlier t multiplied by e raise to alpha t, if alpha is negative still goes to zero as t tends to infinity. So, there is no problem of instability, neither of any oscillations continuing.

However, for k greater than this critical value corresponding to this point and this, the gain corresponding to this point as we have seen can be calculated using the magnitude rule. Again, here the polynomial is only a quadratic, so the condition for repeated roots is very simple, we do not really need to use any sophisticated rule. For greater values of k a the roots will be out in the complex plane and therefore, the response will be oscillatory but damped and as k a is increase, the imaginary part of the root will go on increasing and therefore the amount of oscillation or the percentage overshoot that takes place during the oscillations will go on increasing also the frequency of the oscillation will go on increasing. The frequency of the oscillation corresponds to the this distance of a point on the root locus from the origin and this vertical distance or the imaginary part of the point on the root locus and therefore the frequency will go on increases.

So the oscillations will be more rapid the oscillations will have a larger amplitude but still as time increases, they will go to zero, so the system is going to reaming stable. Now, how large a value k a should one choose then, now these are considerations which are very much dependent on the user of the drive and on the particular application. For example, if k a is very small then the roots are very close to the 2 poles of the open loop transfer function therefore the 2 time constants will be very nearly what these are, one of them corresponding to a root it has the smaller absolute is the larger time constant, the other one is the small is the smaller time constant. For example, suppose this was minus 1 and this was minus 10 then the time constant corresponding to this is one second, whereas the time constant corresponding to this is .1 second.

So one of the time constant is very small, the other one is what we should consider. Now when the root moves away the time constant here is going to decrease, for example if it goes from 1 to minus 2 then this time constant would have become point 5 whereas this time constant would have moved away, let say it has move to minus 9 then, it could have become 1 by 9 that is nearly .11 second. So this time constant would have decreased where as this time constant would not have increase very much. Suppose, they 2 meet somewhere in the middle and let say for the sake of simplicity, this is minus 5 then, the time constant corresponding to this is .2.

So, as we increase an value of k, one of the time constants decreases but that time constant is already very small, the other time constant which was large, decreases and therefore there is a net speeding up of the response. So which is why you may prefer to have some value of k a, so that you are may be close to this break in and break away point. The main reason for doing this will be the speed of the response. As you move away from the imaginary axis, the time constant decreases of the 2 roots, one is moving away, so that time constant is decreasing, the other is moving towards that time constant is increasing but this time constant is already smaller than this time constant.

So, it is this time constant which matters and therefore one may prefer to let it move all the way and choose the value of k such that the roots coincide and you have a much smaller time constant. Both the time constant are in nearly equal to point 2 as against 1. So the response will be speeded up what about going further now going further the real part of the root as you can see is not going to change they asymptotes and the actual root locus lies along this vertical line, so the real part of the root is not go into change and therefore the time constant is no longer going to change, what is going to be introduce by increasing k a is a oscillation and as you increase k a further the frequency of the oscillations will increase and the amplitude of the oscillations will also increase.

Now, because a mechanical systems sometimes have what is called static friction that is the behavior which we are describe by k f friction being proportional to speed is not really 100 percent accurate, therefore sometimes one prefers a little bit of oscillatory nature of the response, you just make sure that the may motor is not struck in some position because it is forced to move away from the equilibrium position.

So, what is known as static friction if any can be overcome, of course I am assuming that the motor is already running then, it makes no difference but if the motor is to start then at start then, may be an amount of friction which is more than what we have modeled and therefore very often, one who prefer to choose a value of k such that the 2 roots are not both real but have a imaginary part and therefore, they are complex but again not very for away from the real axis that is not a large amplitude of oscillation nor a high frequency of oscillation.

Now, what exactly should be the value of the location of this point on the root locus is up to the designer, very difficult to say anything, where it definite. Sometimes, the delta or the damping coefficient is specified but if the damping coefficient is not specified then the designer has to think in the manner I have told you about and so, one we will choose the value of k a, a corresponding. Now this is the case which is normal going to take place because as I told you, the 2 time constant the armature constant and the mechanical time constant are set that the mechanical time constant may be 10 times as large as the armature time constant and therefore because of the way equation in the parameters are the 2 open loop poles will be as I have shown and therefore, what I have told is as now will be valid.

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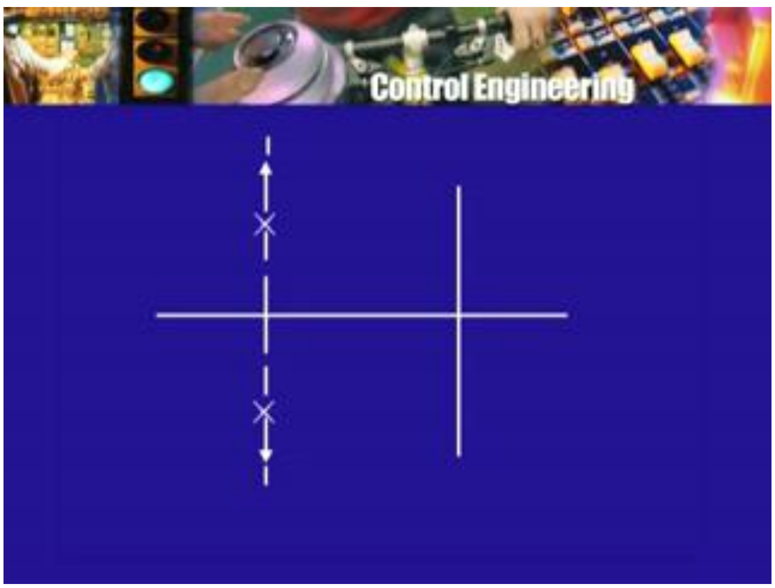

Now in case, the 2 roots are not on the real axis but are already in the complex plane then what is going to happen though suppose the 2 close loop, the open loop poles are located like this then, what will the root locus look like. By applying the rules, we can see that there will be no point of the root locus on the real axis or putting it the other way around, no point on the real axis will lie on the root locus, why because choose any point on the real axis look at the number of poles and zeros if any on the real axis which are to its right there are intending.

So this entire real axis is prohibited from being on the root locus, what about asymptotes there will be of course 2 asymptotes, what about their point of intersection as one can immediately see it will be this point and the angels are as before and so, you can expect that the root locus branches will actually look like this that is they will start at these poles and simply go away towards infinity. In this case now, as we increase k the time constant is not going to change, the time constant is going to remain what it is and therefore you may not want a very large value of k, in such a case because already there are oscillations and you do not want to increase the amplitude of oscillations, nor the frequency of oscillations because nothing is gained by doing that, the time constant remaining unchanged and therefore if the situation is like this which is very unlikely the case then, a small value of k a will do.

Now, what is the effect of the choice of this value of k a on the design of the feedback control system. Now going back to the steady state considerations, if this k a is large then you require a certain armature voltage in the steady state therefore the difference voltage available here will have to be small, larger the k a, the smaller this difference voltage. Now of course, it depends on what tacho generator you have chosen corresponding to the given motor speed, so that will determined roughly the reference voltage.

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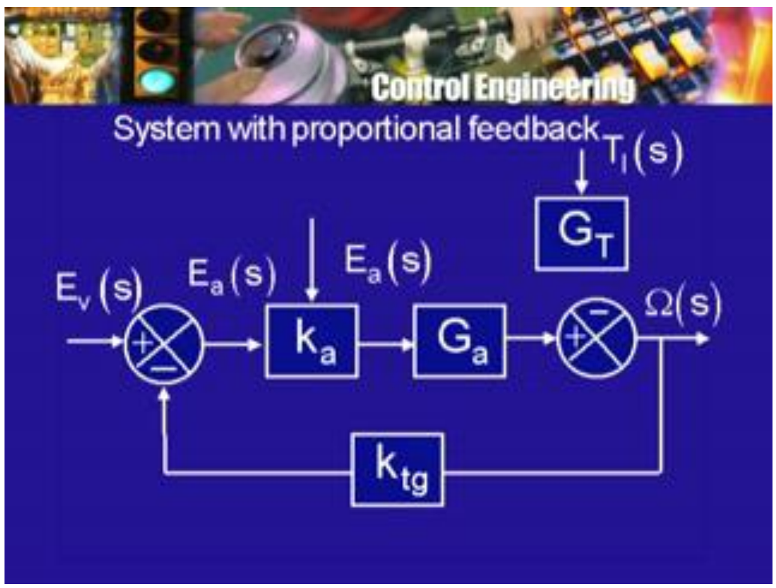

So if k a is made very large then this error voltage is very small, so the reference voltage will be very close to the tacho generator output, whereas if k a is not so large, so that the difference voltage E s may be comparable to the tacho generator voltage. Then the reference voltage may be larger than the tacho generator voltage by an amount which is not very small but that is usually no problem because the reference voltage may be coming from a battery or some stable DC source and this reference voltage actually does not supply much power to the drive, it is only function is to produce this difference voltage E s which then, goes into the amplifier. Now in our case if you remember, this amplifier actually consists of perhaps an amplifier which speed the DC speed winding of a DC generator which drives the of DC generator which is driven by a separate motor and the DC generator is speeding into our motor which is being controlled.

So, this E r is not really a source of power and therefore, this voltage being small or large is not really of much great significance. Of course you do not want to use a very large tacho generator to produce a larger amount of voltage because the main purpose of the tacho generator again is not to generate power but to produce of voltage proportional to the speed. So the effect of K a, on the choice E r will be there but it will not be significant. Now let us look at the other part of the response namely the effect of the load torque. Now, here is the load torque input and here is that transfer function G T going in there.

Now, what about the root locus in this case? Now in this case, I can assume that E r is equal to zero and therefore the diagram looks as if that has only one input T l and as you can see is the same close loop and therefore the closed loop transfer function or the close loop characteristic polynomial is going to be exactly what it was before, you have this G a and this k t tacho generator and k a as before. Only thing is you have this transfer function G T of s which multiplies this transfer function from this point to omega s and therefore the response is more or less going to be like, what we have discussed as for as changes in E r or E a are concerned.

So, one need not worry about this part of the diagram, this part of the diagram is what is really crucial because that governs the characteristic polynomial and that governs the transient response of the system. Of course, as for as the steady state is concerned k a has an effect on the amount of error introduce by changing T l, as we saw earlier and therefore larger the k a, the better it is from the point of view of steady state error introduced by change in load torque. Now, this is where one has to again look at the problem from the practical point of view, how much steady state error are going to tolerate for that you require a larger value of k a or you may require. A larger value of k a, the larger of k a, on the other hand may result in a system performance which is very oscillatory therefore there has to be some kind of a compromise that is how much oscillations can you tolerate and how much error can you tolerate.

It is the designer or really it is not the designer it is the user of the drive who has to tell the designer what he can tolerate, how much allowance can he make for the designer. It is for the user who has to decide between these 2 conflicting things, one is the steady state error because of change in load torque, the other is a possible oscillatory nature of the transient response. Of course up to some extent increasing k is useful, so up to this point there is no difficulty in going, it is only beyond this point that one is discussing the situation.

So the user of the drive is the person who has the give the specifications and then, once the specifications are given the designer then tries to choose the configuration. In this case we are only looking at proportional feedback and choose the appropriate parameters of the configuration. In the our case, the only parameter to be chosen is the gain of the amplifier k a. Now, what if this person the user has given specifications which cannot be met for example, he may want you to reduce the steady state error to a certain level which requires a large value of k a but then he does not want oscillations beyond the certain level which requires a smaller value of k a.

So the 2 specifications are conflicting. In that case what is to be done, of course one way out is to say well sorry I cannot meet both the specs or specifications, you tell me which is your more preferred part of the specifications, steady state error or oscillatory performance. But, fortunately we can do a little better because we can think of other control schemes other control configurations and in fact we already been talking about one namely integral feedback that is with integral feedback perhaps one can meet both the specifications reduction in the error in speed which change in torque that is one part of the specification, the other is the oscillatory or non oscillatory nature of the response. Now it is possible that even with that modified feedback scheme, the 2 performance specifications are conflicting, what to do in that case.

Well, in that case you can think of something more and I have already mention to you that there is what is called derivative feedback which is an idea, which can be tried out and in beyond that also there are many other things that one can do. Because, control system designers have been working on such problems from the 1930's onward and as the result they have invented various configuration and so, it is extremely unlikely that the given specifications are such that there is no known configuration using which both the specifications can be met. Of course, as a student I do not expect you to know all the configuration but a few configurations or a few methods of using feedback must be known to all of you and therefore we started with proportional feedback.

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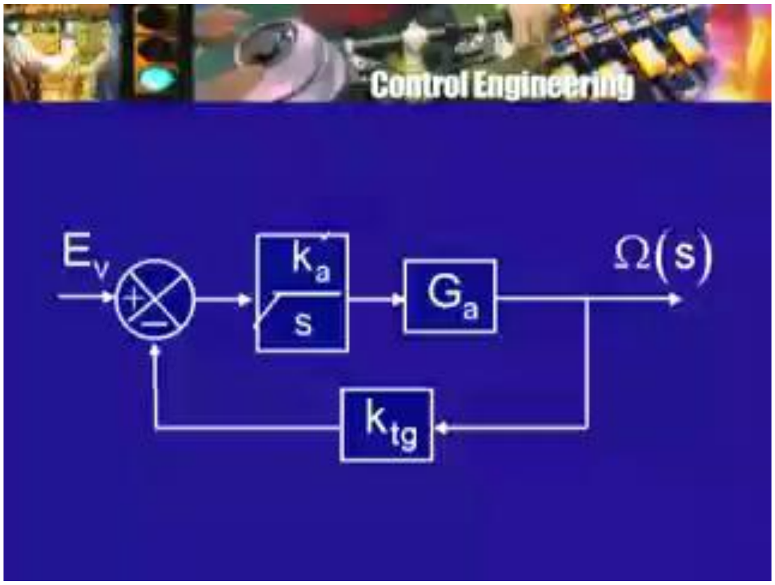

We now take a lit quick look at integral feedback and derivative feedback and then, after some time will return to this and we will talk about various other kinds of controllers. For example, lag controller, lead controller, lag lead controller and lead lag controller and there are many others which of course, we will not have time in the course to discuss. So, with this then less move on to the integral feedback situation. Now in the integral feedback situation the main difference is that in the forward path, we are going to use an integrator with a gain k. So, I am going to put the transfer function as ka divided by s this drives the transfer function G a as before and again I am not looking at the torque dependent part of it, here is E r, here is the tacho generator negative feedback.

So this is the modified block diagram, we have this integrator the difference device as before the output of it instead of just being multiplied and fed to the armature goes through an integrator. The transfer function of course this is the ideal integrator, the transfer function of an ideal integrator is 1 by s and we are introducing the gain k a. So I have written down k a by s and of course, we are going to consider the effect of varying k a on the transient behavior of the system.

So this is the configuration now, what about the root locus, where G a as before has the open loop poles but now we have added one more pole namely, the pole at the origin because of this integrator. The tacho generator part remains unchanged therefore the pole 0 diagram is now going to change. In addition to the 2 poles that already were there now, let me take the same values that I had shown earlier, I am not write now drawing things to scale. So here is that pole at minus 1, here is that further or a pole at say minus 10 and now we have added this pole at s equal to zero that is the integrator.

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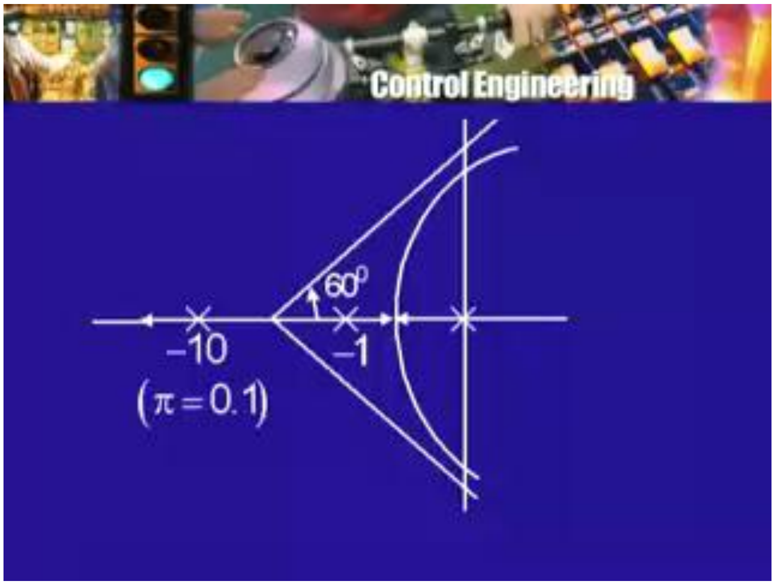

So, now what do we have we have 3 poles no zeros. So how many branches of the root locus, 3, how many asymptotes, 3, what will be the intersection of the 3 asymptotes minus 10, minus 1 that is minus 11 divided by 3 that is roughly somewhere around say minus 4. The angles made by the asymptotes plus 60 degrees minus 60 degrees and minus 180 degrees. So these are the 3 asymptotes.

Now the characteristic polynomial is of degree 3 or it is a cubic as a result it is not very easy to factorize it or think about its factors. But we can still use the root locus method rules to be able to say some thing about the root locus, real axis portions of the root locus,
what about this part, does it belong to the root locus, no because the number of poles and zeros on the real axis to the right is zero, what about this part of the root locus, does it of the real axis, does it belong to the root locus. If I consider point lying between these 2 points the number of poles and zeros to the right is one which is odd and therefore this part does belong to the root locus, what about this part lying between minus 10 and minus 1 of the real axis, considering the point here, the number of poles and zeros to the right is 2, 2 is not odd and therefore this part does not belong to the root locus considering points beyond this minus 10. A point here the number of poles to the right is 3 and therefore 3 being odd, this entire potion of the real axis to the left of this point minus 10 will belong to the root locus.

So, now this part belongs to the root locus and of course, there is an asymptote in that direction to one can almost guess that one root will be in fact negative real and on this side of the minus 10 point or in other words, the absolute value of the root will be greater than 10. The time constant corresponding to this is.1 and therefore the time constant in fact is even going to decrease. Now, what is happening here? The root loci will start at the poles, so we expect that the roots will move towards the each other and they will meet for some value of k a and therefore there will be a break in and break away phenomenon.

So for some value of k a, the 2 roots will be equal and I have told you that there are some rules that enable you to determine the location of this break away point and the value of k corresponding to it. But, we know qualitatively there will be a break away point like this for some value of k a, what is going to happen to the time constant? The time constant corresponding to this minus 1 was one second the other time constant corresponding to this integrator 1 by s is infinity but of course, we are not making k equal to 0, we are putting some positive k. So it would have moved away from the origin as a result the time constant will have decrease but how for can it decrease.

Now, you can only go up to this point, so let say for the movement for simplicity that this is minus .5, then the time constant corresponding to this will be 2 second. So although the time constant corresponding to the open loop pole was 1 second and .1 second respectively, the close loop system will have a time constant which is greater, which is 2 which means that the response will be slowed down not much perhaps instead of 1 second you have 2 seconds, so instead of waiting for 5 time constant which is 5 seconds, you will have to weight for 10 seconds

Now, once again is it acceptable to the user if it is that is fine. For a larger value of k what is going to happen now, the roots are going to move out into the complex plane. So there will be an oscillatory nature of the response but worse than that because of the asymptotes which are going this way, which are moving out into the left half of the complex plane therefore the root locus branches will go like this. Therefore, we expect the there will be intersections with the j omega axis, for some value of k or the system will actually becomes unstable for a larger value of k and even for a value of k which is not as larger corresponding to this point, the real part of this roots will have again become very small, the time constant would have become very large and so, the system response would not be acceptable.

So with the integrator that is with the integral feedback now, the situations seems to be little worse than what it was with the proportional feedback. The time constant has increased the system response of course can become a can be made oscillatory, slightly oscillatory if you want. But then, why do this now if you remember the reason for doing that was essentially that introducing this integrator, reduce the steady state error cause by change in the load torque to zero and that did not require k a to be large. In other words, near presence of this 1 by s with k a which need not be large was enough to reduce this steady state error to zero.

So we do not really need a large value of k a, unlike in the proportional feedback is where you may want the roots to become a little oscillatory or a little complex so to speak. So here probably the best value of k a will be where the 2 root locus branches meet because that is where the time constant would have become perhaps not too large and this may be adequate. So in this case the steady state error can be made completely zero, so that specification will be met the oscillatory response of the system can also be almost avoided although you can have a little bit oscillations if you wish and that time constant however would have increase a little bit.

So if this is acceptable to the user then, this design is going to be quite satisfactory but if not, the steady state error is 0, we may have even done better than what the specification was but the transient response may have been a little slower than what the user at specified then, what, then we have to look for some other scheme of feedback. Let us spend little bit of time on one more feedback idea, I have talked about proportional feedback, the tacho generator was providing the proportional feedback then integral feedback in fact, what we are done was we had combined proportional and integral feedback because we had both the tacho generator as well as the integrator being used.

So, we had p and I proportional and integral feedback and now, let us take look at the third kind of feedback which is known as derivative feedback. Now, you know that using operational amplifiers one can design a circuit which is very close to being an ideal integrator. Of course, no physically device circuit using op amps is an ideal absolutely ideal integrator and those of you who have looked at analog electronic circuits and have studied operational amplifier, characteristics and so on, know that the operational amplifier gives an integrator which is not 100 percent ideal therefore the transfer function is not really exactly 1 divided by s.

It is worse when it comes to differentiator that is again using an operational amplifier, if the operational amplifier is ideal that is infinite gain, infinite input impedance, 0 output impedance etcetera, etcetera. Then using operational amplifier and an RC configuration one can device a differentiator that is the circuit which is like the ideal differentiator what is the transfer function of a differentiator while what is the differentiator, if f is the input then, the output of the differentiator will be Df.

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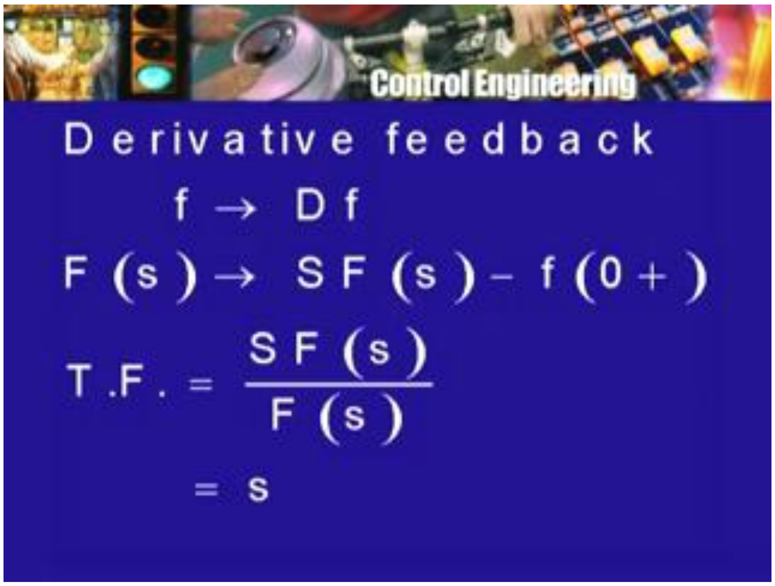

So, if F s is the Laplace transform of the input, the output Laplace transform will be SFs minus f 0 plus. So if I ignore this part and that of course contributes there is a transient error response as we have seen then, this part is only SFs. So the transfer function is s, so the differentiator, ideal differentiator multiplies the transfer function of its input by s, practical differentiators are much less ideal than practical integrators. So that is one thing that is to be kept in mind. Secondly, it is not immediately clear what the use of a differentiator is going to do. Let us go back and think about, why did we introduce proportional feedback and why did we introduce integral feedback.

We introduce proportional feedback because with the help of proportional feedback, there is by looking at the output quantity during an actual operation, when the load torque may have changed, it may be possible to reduce the effect of load torque change, on the change in the steady state speed and in fact, we saw that it was possible by using proportional feedback, it was possible to reduce the steady state error caused by torque changes but not make it absolutely zero then can be idea of the integrator.

An integrator is a device for which an input may eventually become 0, still the device may have an output which is non-zero because the input was non-zero earlier and the effect to the integration is to produce an output which is non-zero and remains non-zero thereafter and therefore, there was a possibility that with 0 voltage given to the integrator from a certain time onwards the output of the integrator can remain constant and therefore can drive the motor and on the other hand, when the load torque changes during the transient period, the input to the integrator remains non-zero but the integrator output is built to a new value. So that it can compensate the change in the torque value for example, if the load torque decreases then you want a smaller motor to the applied to the armature where as if the load torque increases, you want the voltage applied to the armature to be increased.

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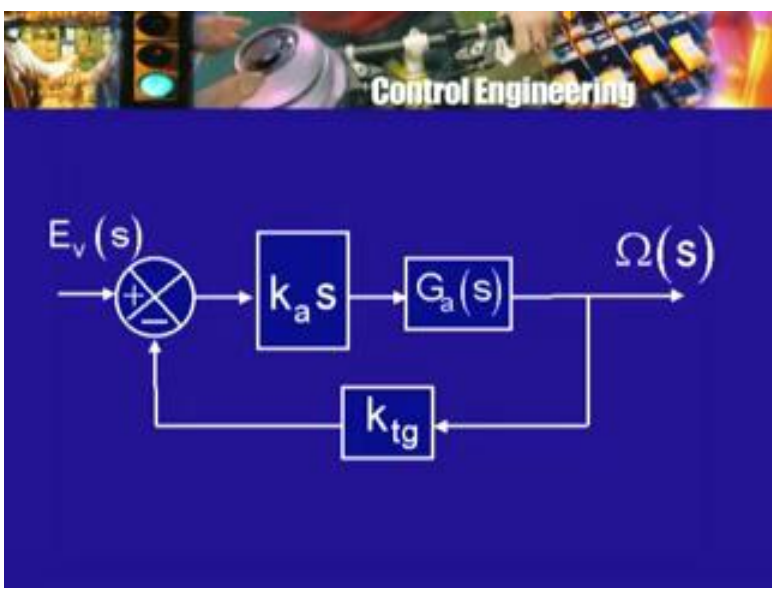

Now, this increase or decrease of this adjustment can be done automatically with the help of the integrator device. The input to the integrator can eventually become 0 but the output to the integrator can have 1 value or another depending on what has happened in the past and so using integral feedback in addition to the proportional feedback, we have the possibility of the steady state error becoming absolutely zero and therefore, the idea was used but of course, we saw some other side effects of the integral feedback.

Now with derivative feedback, what is going to happen, where do we apply the derivate feedback. Now, we integrated the output of the difference device, so perhaps it is reasonable to apply the derivate at the output of the difference device but what is it going to do. The derivative network of the differentiator is going to differentiate the difference voltage. So what effect is it going to have, we really do not know. Now this is where of course one can write down the differential equations describing in the system which in this case is not very difficult but the root locus approach can come to our rescue because now, all I have to do is in that forward path instead of k a divided by s, I put the transfer function k a into s and that is the ideal differentiator, the rest of it is unchanged and so that is the block diagram. You are by now very familiar with it now, what has changed therefore.

Now, if we look at the closed loop here the loop transfer function as change. Now, we have G a as before with its 2 poles but now, we have this term k a into s, this k a into s corresponds to a zero. So now we have introduced a 0, what is the effect of introduction of this 0. The root locus method will enable us to find out without too much trouble. So here once again is our original pole 0 diagram minus 1 and minus 10 and now 0 has been introduced, this 0 is at the origin because it corresponds to the term k a s right. Now, what is going to be the root locus like 2 poles, one 0, number of branches of the root locus 2 the 2 branches will began at the pole, number of asymptotes 1. So one asymptote will go towards the negative real axis, what about real axis portion of the root locus?

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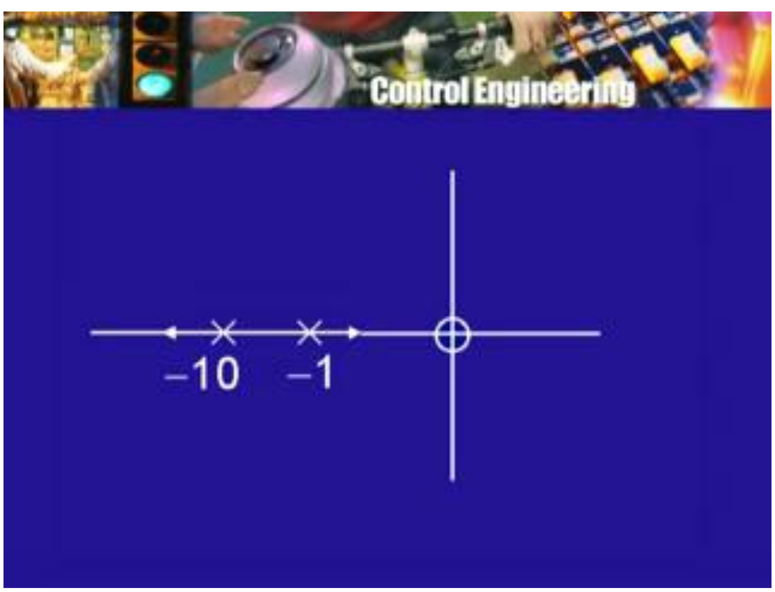

Now you can immediately see that this part of the real axis will belong to the root locus because for any point here, the number of poles and zeros on the real axis to its right is odd, namely one.

So this part will belong to the root locus and again, we can see immediately that this part will belong to the root locus, all right. So what is going to happen as k is increased as k is increased from 0, this time constant which was 1 is going to in fact increase this time constant which was point 1 is going to decrease but already it is small but this time constant which is going to increase is a dangerous thing because if, I increase k further, the root is coming closer and closer to the j omega axis and therefore the time constant side increasing further and further. Therefore, if I introduce a differentiator the response is going to become slower and slower, if I introduce the differentiator as I have shown here in this way then, it is going to make the response slower and slower because of the particular nature of the root locus.

Now this is happening because this transfer function has only 2 poles and no 0. If I have the different transfer function with poles may be more than 2 with zeros then, what happens will be different. So no general conclusion can be drawn from this example but for this motor drive derivative feedback in the forward path there is not seen to be of any use it will make the situation worse what about its effect on this steady state error, will it make the steady state error, zero, steady state error cause by load torque changes, will it make it 0 or will it make it smaller or will it increase it. I will leave it to you work it our as a homework problem and we will look at it later.

So look at the effect of change in k a, the differentiator gain on the steady state error caused by a fix change in the load torque, is it going to be less, when the differentiator is there than it was with only proportional feedback, all right. So perhaps this configuration has no promise because it is increasing the time constant, it is making the system response slower. It may not reduce the steady state error significantly and therefore, it may not be a good think to look at, what other possibilities are there. Well, we can think of transfer functions which are different from the 2 transfer functions that I have talked about namely, the integrator transfer function k a divided by s or the differentiator transfer function k a into s.

Now, I can think of an operational amplifier network which has a transfer function which is not k a into s or k a divided by s. In fact, using operational amplifiers, it is possible to design network using resistors and capacitors only, no inductors are required whose transfer functions can be of a variety of sort for example, the transfer function can be ratio of 2 polynomials both the polynomials may be of the same degree or their degrees may be different from one another. There is quite a lot of freedom that is possible with the help of the operational amplifier as a circuit component to be used in connection with control system.

Now there was a time in the 30s and 40s, when the operational amplifiers use vacuum tubes as a result they were very expensive, they were very heavy, bulky, power consuming, so nobody would have really thought of implementation of integrators or differentiators. But in subsequent times with the advent of the transistor and the integrated circuit, as you know the operational amplifier has shrunk in size and the power consumption of the operational amplifier is also significantly less and therefore complicated transfer functions can be implemented today, using the op amp and RC devices.

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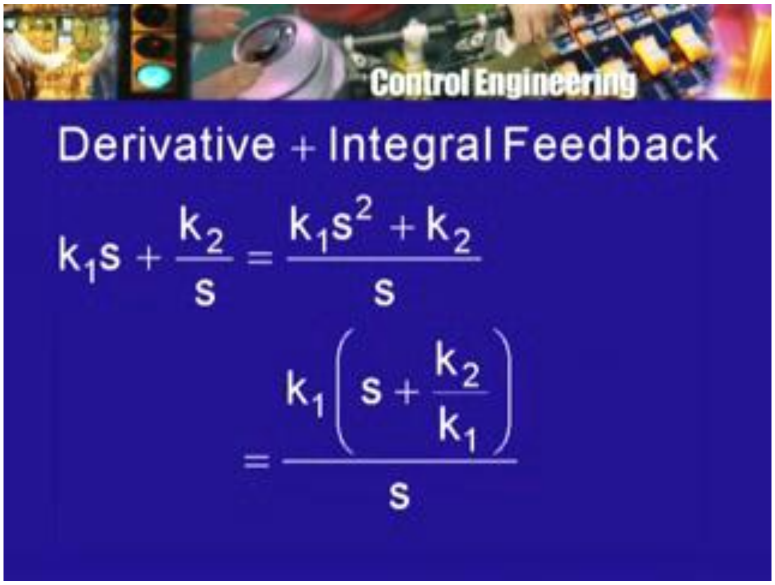

As a result one can consider many options, other than just the integrator and the differentiator and of course, the proportional feedback which one might still keep using all the time. Now, one can look at for example, a combination of derivative and integral feedback for example, a simple way of combining will be think of a transfer function which is k 1 (s) plus k 2 divided by s that is all I do is I take the output of the differentiator, I take the output of the integrator and I just add they to and in fact, one single circuit may be able to do the job that is involved here. What is going to be the transfer function of this kind of a combination, as you can see the transfer function is going to be like if I take s as the common factor, I will have k 1(s) square plus k 2.

So with this in the forward path, what is going to happen to the pole 0 diagram, this s now introduces the pole whereas this k 1 s squared plus k 2 that is the derivative and the integrator combination produces what, a pair of purely imaginary zero. So if I draw the pole0diagram for the system using this kind of a feedback. Here, are the original poles of the open loop transfer which are unchanged, here is an additional pole which has been introduced namely this one corresponding to the integrator and because of the combined effect of the integrator and the differentiator, we have 2 zeros which are on the j omega axis.

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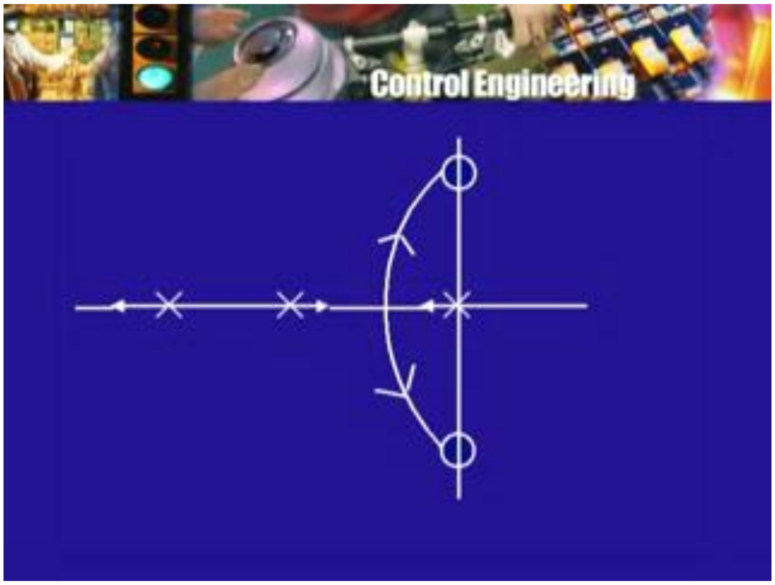

So this is the pole 0 diagram because the pole 0 diagram has change the root locus will of course be different what will it look like once going through the rules 3 poles, 2 zeros 3 branches of the root locus. So there will be 2 asymptotes which go towards infinity which means for large enough value of the gains k of 1 of the appropriate gains then, the system will become unstable. So that is a conclusion which one can draw immediately from the very simple rules of the root locus. What about real axis portions of the root locus, once again you will see that this part belongs to the root locus because the number of poles and zeros to the right lying on the real axis is odd, this part belongs to the root locus because the number of poles and zeros on the real axis lying to the right is odd 3.

So, there is going to be this break in and break away phenomenon and one root locus branch goes towards infinity in the negative real axis direction, the other 2 branches will approach the 2 zeros and therefore, we can expect that the other 2 branches of the root loci will move out like this and will approach these 2 zeros. So this is a little better than what we had with the pure differentiator in the forward path because now, the time constant there is the possibility that the time constant will increase but not very large, it will not become very large as we have seen in the case earlier. However, this may not be preferred to the configuration which uses only the integrator. In the case of the integrator also of course as we increase the gain the system became unstable. In this case, what is going to happen is as you increase the gain then, the system will be or the roots will be approaching these 2 roots on the j omega axis.

So the system will be approaching instability which is only sustained sinusoidal oscillations rather than oscillations which will go on increasing in amplitude. With the integrator alone, the situation was different there was this possibility of going out into the right half plane. As a result oscillations which keep growing exponentially here that possibility may be avoided. Of course, when was talk about the root locus remember we had only 1 parameter gain k to talk about, now here I have 2 coefficients k 1 and k 2.

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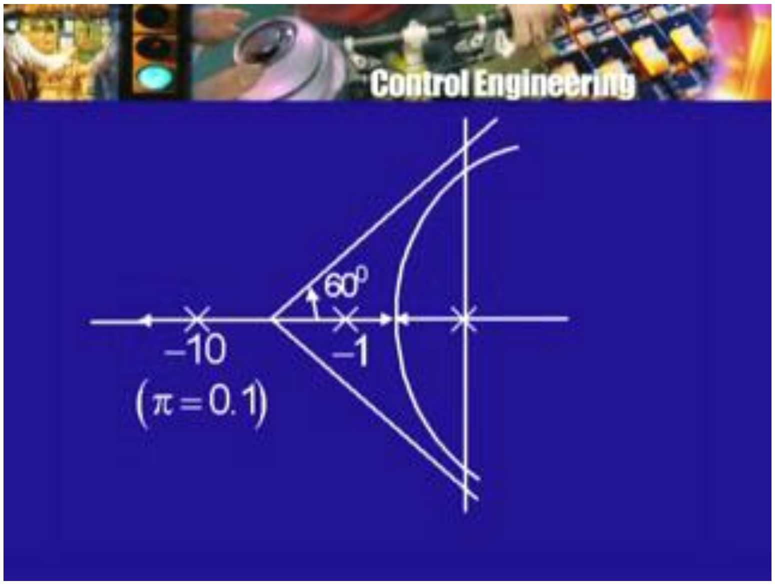

So how do I think of it as only one parameter where one way is simply to pull out, let say k 1 as a factor and write this as s square plus k 2 by k 1 divided by s and this k 2 by k 1 will determine the location of these purely imaginary roots and this k 1 will be the gain which can be treated as the gain of the root locus method. So this way although there are 2 parameters to be chosen, you can reduce that 2 parameter choice in 2 steps, 1 is the ratio k 2 by k 1 which determines the location of these 2 zeros. So, whether you want them like this or whether, you want them for away like this then, in that case the response will become oscillatory, the frequency of oscillation can increase etcetera, etcetera. For a given choice of k 2 by k 1 ratio then, k 1 is varied to draw the root locus and this is what I mean by the root locus diagram that I have drawn here.

Once again, you can look at the effect of this on the steady state error due to load torque changes, find out whether the steady state error will go to 0 or it will not become zero. The s factor in the denominator makes you suspect that the steady state errors due to load torque changes will become zero just as it was when we had only s in the denominator and constant in the numerator. Now I have some things in the numerator all right but this s in the denominator may have the effect of making the steady state error zero.

So here is a configuration which is another possible configuration, it is more complicated it involves both an integrator and a differentiator, it has the advantages of the integrator of the integral feedback namely, the steady state error as become zero. It does not really seem to have much more of an advantage over the pure integral plus proportional feedback scheme. But again there could be systems with a different kind of transfer function, open loop transfer function such that using the derivative feedback may have certain specific advantages.

So this is where the matter stands as for as proportional, integral and derivative feedback is concern, I have not shown the proportional term but the proportional term has very much been there all along because of the tacho generator in the feedback path. So what we have really looked at is proportional, proportional plus integral, proportional plus derivative and proportional plus integral plus derivative feedback.

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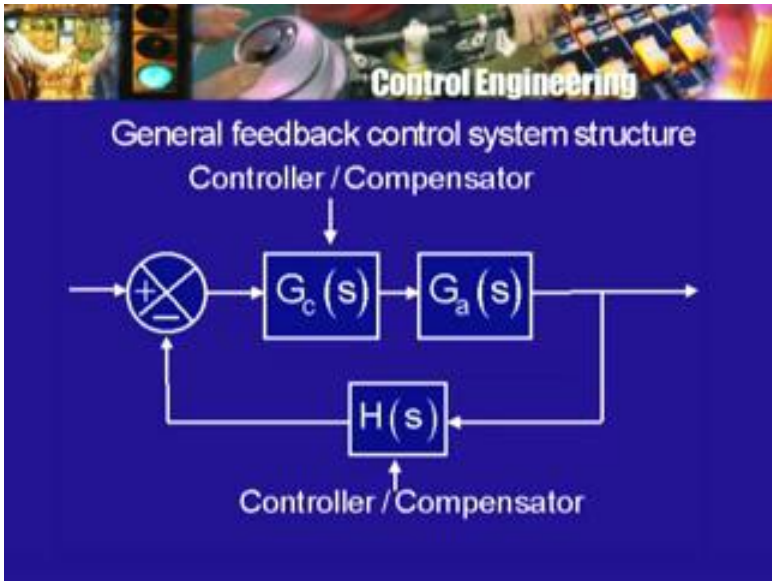

Now, as I told you there are many other transfer functions that you can think of and corresponding physical devices put in the forward path such devices are called controllers and so one can think of various controller transfer function, not only that at the movement it is not clear that I could put something in the feedback path, why should I have only keep the tacho generator here, why can I not act on the output of the tacho generator, may be do some processing of that tacho generator voltage output, maybe I can differentiate it, maybe I can integrate it or operate on it by some other block or a circuit.

So in other words, in place of single multiplying coefficient k tacho generator, we can have a transfer function H s in the feedback path then, in addition to this G a of s which is the armature voltage to speed transfer function. We can put a transfer function in the forward path itself and it is called G c for controller and it need not be just 1 by s or k a by s or k a into s, it could be something more complicated than that and therefore, our feedback system configuration will now become more flexible or become more general, you have to choose the controller in the forward path G c (s).

You have to choose the transfer function G c (s) and you have to choose the transfer function in the feedback path and you have to see whether, these 2 can be chosen in such a way that the system performance will need the specifications, will become better than what it was, in the absence of these 2 complicated transfer functions in the forward path and the feedback path. These are referred to as the forward path controller and feedback controllers and various kinds of transfer functions can be a talked about the integral, derivative, integral plus derivative or proportional plus integral plus derivative are the simplest one, one can think of various kinds of transfer functions for example, just to mention 1 example, here is the transfer function, tau1 s plus 1 divided by tau 2 s plus 1.

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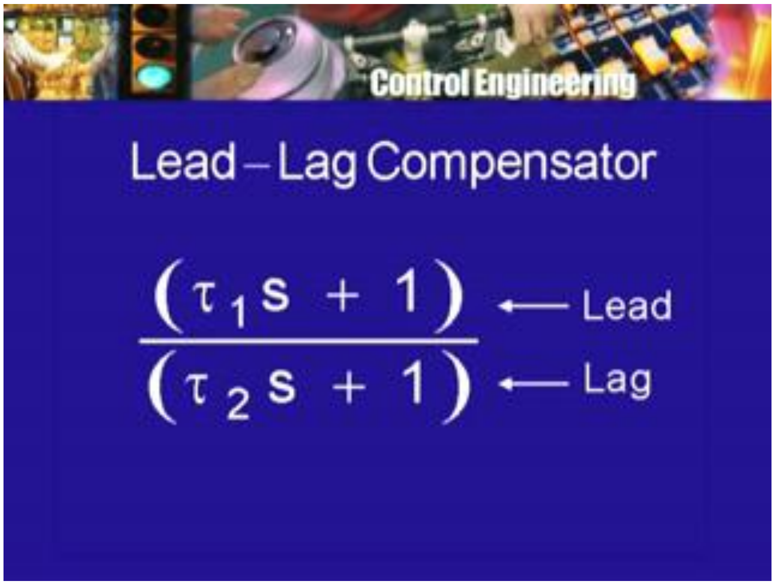

So there is a 0 here corresponding to this numerator factor, there is a pole here corresponding to this denominator factors and there are 2 time constants that are associated with these 2 factor. Now as an example, you can try to find out how an operational amplifier can be combined with the resistive, capacitive network to produce this transfer function, probably you can do it only using 1 op amp but if I go to higher order transfer functions, you may require 2 op amps.

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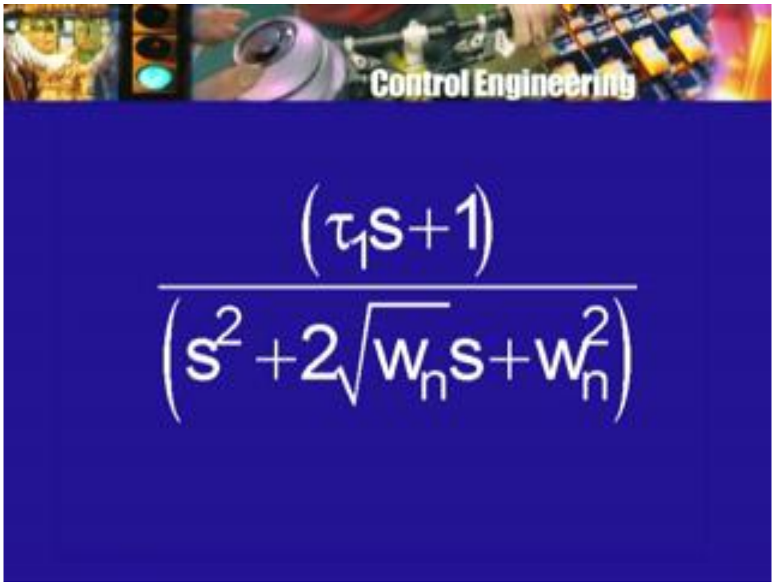

For example, can I device a circuit which has a transfer function which is given by let us say, tau 1 is plus 1 divided by let say s square plus 2 delta omega and s plus omega n square. As we had seen earlier that is a secondary transfer function in the denominator that is 2 poles and 1 zero, a first degree transfer function or a first degree polynomial in the numerator, is it possible to design an op amp circuit may be not using 1 op amp but 2 op amp, which has such a transfer function with specified values of tau 1 delta and omega n, using op amps and RC component. Think about this, we are not going pursue this too much because there are so many possibilities here but will come back to them a little later, after we have taken a loop at what is know as frequency response of the control system.