Control Engineering
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Lecture - 34

Did you find out, what was happening when we looked at the Routh table for this polynomial S cube plus 6 s square plus 4 s plus 5.

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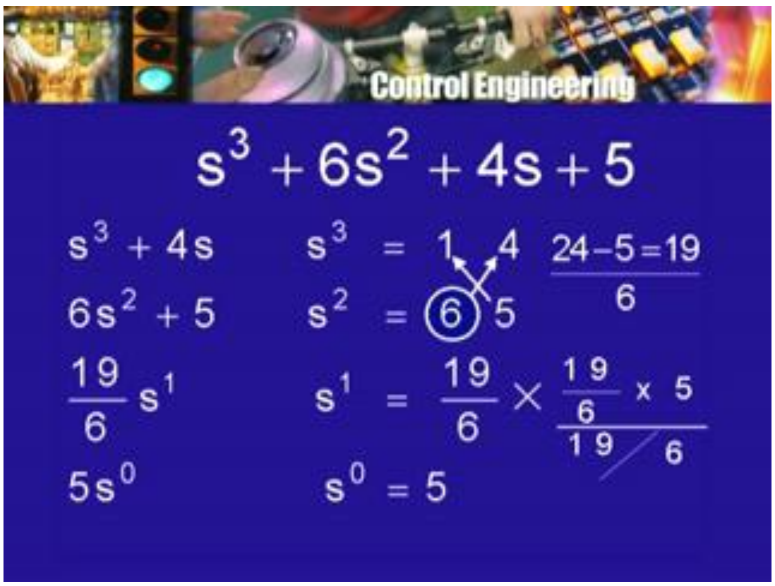

We wrote down the odd part of it, its coefficient first 1, 4 and then the even part 6, 5 and then by doing this peculiar operation, you know pivot element into this minus this into this divided by pivot. We got the entry 19 by 6 that means it corresponds to the linear polynomial 19 by 6 into s and lastly, we got just the constant polynomial 5 and my question was from the 2 polynomials S cube plus 4 s which corresponds to the first row and 6 s square plus 5 s which corresponds to the second row, we obtained the third row which corresponds to the polynomial 19 by 6 into s.

Now, what is the relationship between these polynomials, what you are doing to construct the array or table I hope it is clear but what is its significance in terms of polynomials.
In fact, we do these things on the entries of the table because they correspond to some polynomial. The whole algorithm or the whole idea is really based on looking at a set of polynomials. It is done in a compact fashion by not writing down the powers of s provided you do it in a systematic way and then, only computing the coefficients of the polynomials.

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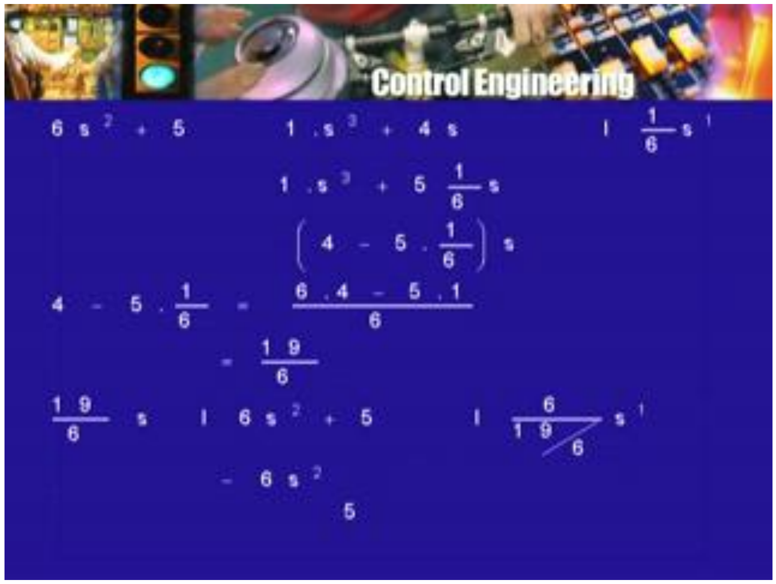

Now, I hope some of you have found out the answer and the answer is the following. Here is the first row polynomial S cube plus 4 s, here is the second row which also corresponds to another polynomial 6 s square plus 5 s. The original polynomial was of degree 3, so we wrote down the odd part first and then, we wrote down the even part. Now suppose, I were to divide this odd part by the even part then, what would be the quotient and what would be the remainder.

Now first of all because of the fact that this is an odd polynomial and this is an even polynomial and the degree of this polynomial is just 1 less than the degree of the other polynomial .You remember from school algebra or arithmetic this polynomial S cube plus 46 which is being divided is called what, it is called the dividend polynomial that is the polynomial which is being divided. The polynomial which divides it is called, what it is called the divisor polynomial. So the divisor polynomial has degree 2 the dividend polynomial has degree 3.

So the difference of degree is 1, as a result quotient will be s to the 1, this is called the quotient what I am going to work out is called the quotient. This quotient is going to have a power of s equal to exactly 1 and what will be the coefficient of this power. Well, I have S cube here I have 6 s square here, so to cancel of this S cube I have to write1 by 6 s here. So if I do that 1 by 6 s to the 1 and then, how does the division algorithm go well, I multiply this by the divisor and then, write down the product. So I will have S cube here plus I will write this now as 5 s into 1 by 6, yes I am deliberately going to write this as 5 into 1 by 6 into s.

Now, we subtract this, so the S cube power cancels out that is what is expected and what remains is what 4 minus 5 into 1 by 6 s, there is no constant term. So notice what has happened there is an odd polynomial which is the divided by even polynomial whose degree is just 1 less than the odd polynomial, the quotient is just s with some multiplier that is it is a polynomial of the kind k into s, in this case this k is 1 by 6 and the remainder is an odd polynomial. The remainder, when the odd polynomial is divided by any one polynomial is an odd polynomial. Now look at the table that we had and let us compare and see what we get. So here on the table I have 1, 4, 6, 5 and the quotient is the next entry is 19 by 6 and what did I do, I did 6 into 4 minus 1 into 5 divided by 6, 6 into 4 minus 5 into 1 divided by 6 and that was that 19 by 6.

Now do you see this 6 times 4 minus 5 times 1 here, almost what I have here is 4 minus 5 into 1 by 6. Now this can be rewritten as 6 into 4 minus 5 into 1 divided by 6. So the entry that we are writing here 19 by 6 obtained by following that rule is actually the coefficient of the remainder polynomial. This polynomial which remains after the division operation is called the remainder polynomial and so, what is really happening is that when we divide the polynomial which corresponds to the first row by the polynomial l corresponding to the second row, the remainder polynomial is the next row, what about the quotient. The quotient as we saw is s to the 1 or s multiplied by some coefficient and what is this coefficient is 1 by 6 and do you see 1 by 6 here somewhere, almost here is 1 here and here is 6.

So this 1 divided by the pivot element is the coefficient of the quotient polynomial and in this case, 1 and 6 have the same sign and therefore the coefficient of the quotient which is 1 by 6 is positive. So the Routh table construction involves not explicitly but implicitly starting with a pair of polynomials and then, carrying out this division, remainder operation repeatedly. Let us take one more example to illustrate this. The example will be a little longer and you should try to carry it out as I am doing it now. So that you would have understood it a little better by trying it out yourself.

In the previous example, we had only a cubic polynomial s to the 3. So there are 2 rows that we started of with s cube and s square and there are only 2 more rows to be filled up s to the 1 and s to the 0 or a linear polynomial and a constant polynomial. The first polynomial was odd, the second polynomial was even then, by division we obtained a remainder which was odd, the next step was this s square polynomial that is corresponding to the s square row 6 s square plus 5 s was divided by 19 by s, s to obtain a quotient 5, let me do that explicitly.

So I will carry it out here, I have that 19 by 6 into s is the polynomial which divides 6 s square plus 5, what is the quotient and what is the remainder, lets try it out here, 6 s square plus 5 is the polynomial corresponding to the second row. Now that is divided by the polynomial corresponding to the third row which is 19 by 6 into s. So what is the quotient? The quotient will be of course here is s square and here is s. So the quotient will be s to the power 1 and what will it be multiplied by it will be multiplied by 6 divided by 19 by 6 and therefore, when I multiply this into this, I will get 6 s square.

So that is going to cancel of with the 6 square and then, what will I get nothing and therefore when I subtract the remainder is 5 and this remainder is indeed, what we had in our table 19 by 6 into 5 minus nothing divided by 19 by 6 plus 5 and that is indeed the quotient that we have here. So the Routh table construction involves doing compactly polynomial division and repeated in the sense start with the pair of polynomials get a remainder then, use the last 2 rows polynomials get a remainder and keep on doing this till you reach the constant polynomial, when you reach the constant polynomial it is not necessary to go any further because a constant polynomial will always divide any polynomial.

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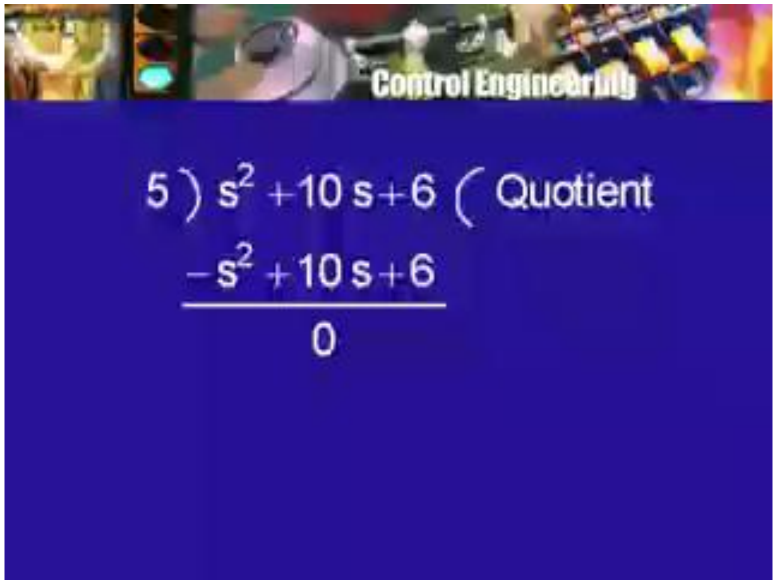

For example, here I have a constant polynomial 5 and I write down a polynomial, let us say s square plus 10 s plus 6. My claim is that I can find a quotient such that the remainder will be 0 and of course, this is very oblivious, I just take the quotient which is 1 fifth of this polynomial and then the remainder will be 0. So there is number point in doing this last step when the remainder is going to be 0 and therefore, it is not done we stop with the constant polynomial.

So, let us take now one more example, let the polynomial be s to the 6 plus 2 s to the 5 plus 3 s to the 4 plus 4 s to the 3 plus 5 s square plus 6 s plus 7. I have just chosen a simple example with coefficient 1, 2, 3, 4, 5, 6, 7 and it is a polynomial of degree 6. Now, you should try to construct the Routh table yourself, I will get you started with the first 2 steps and then, you should carry it on without looking at what I am doing. So here is the first row the highest degree term is s to the 6 notice that the polynomial is monic and I told you that you can always make the polynomial monic. So I have s to the 6 then, so coefficient is1 then the s power is 3, s square is 5 and s to the 0 is 7.

So I have the 4 numbers 1, 3, 5, 7, what I have done is I have written down compactly, the polynomial s to the 6, how do I know it is s to the 6 because there are 4 entries but better still there is this s to the 6 here. So s 6 plus 3 s 4 plus 5 s square plus 7, this is an even polynomial, next and this is the even part of the given polynomial. Next I write down the odd part starting with s to the 5, I have 2 then, I have 4 then, I have 6. So what is this polynomial, it is an odd polynomial it is given by s to the 5 multiplied by 2 plus 4 into s cube plus 6 into s.

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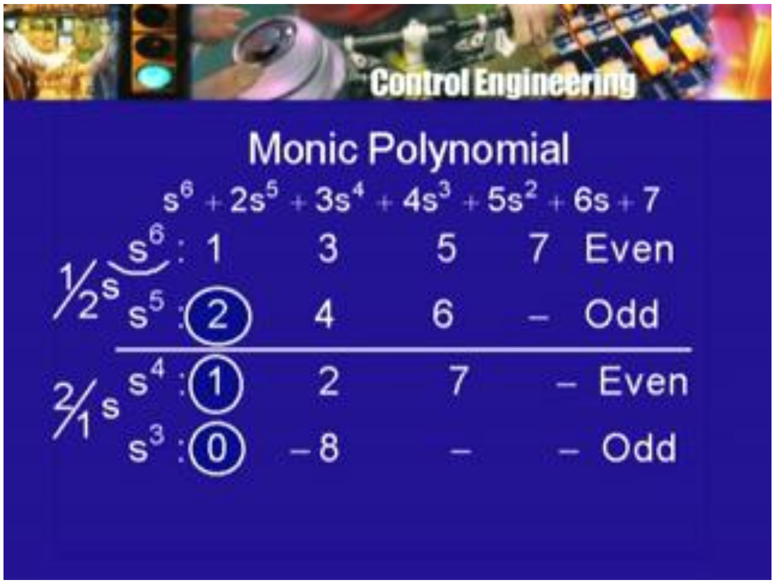

So I have broken the original polynomial into its even and odd part that step is easy. The next step is now involves this division of polynomial but instead of doing it by actually writing down the dividend, the divisor then working out the quotient step by step we do it compactly by doing that particular operation which involves starting with the pivot element and then, doing some things cross wise. So do not look at what I am doing, do it yourself, I am going to do it myself, have you finished the s to the 4 row the next row that is how did I get it 2 into 3, 6 minus 4 into 1.

So the difference is 2 divide by the pivot element that is 1, next 2 into 5 that is 10 minus 6 into 1. So the difference is 4, 4 divided by the pivot element is 2, 2 into 7 minus what minus nothing because there is number entry here. The polynomial is s to the 1 is the last entry so there is nothing here, do not put a 0, 2 into 7 minus nothing or there is nothing to subtract divided by the pivot element 2, so that gives me 7. So I have obtained now an even polynomial in the third row, the polynomial is s to the 4 plus 2 s square plus 7 and what is the significance of all of this. This polynomial which corresponds the first row when divided by the second row polynomial, first row is an even polynomial, second row is an odd polynomial leaves a remainder which is an even polynomial 1, 2, 7 are the coefficients and what is the quotient, the quotient is simply 1 divided by 2.

So if I write the quotient on this side, the quotient is 1 half of s or 1 divided by 2 into s where does that 1 by 2 come from, this is this 1 divided by these 2 that is the quotient. Now we will continue, so I will write down the s cube entries once again do not look at what I am doing do it yourself have you finished its an s cube polynomial now starting with s cube and the entries are 0 and minus 8, how did I obtain that. Now there is a new pivot element, now we forget about the first row we look at only the last 2 rows that we have obtained. So this is the pivot element now.

So this multiplied by 4 and that is 4 minus this product which is also 4, so the difference is 0 divided by the pivot element 1 therefore I write a 0 here and next this one cross wise is 6 minus this 1 cross wise is 14. So 6 minus 14 is minus 8 and that I am dividing by 1 and I get minus 8 and what is it that I have here now, I have an odd polynomial and what is the polynomial, it is 0 into s cube minus 8 into s. Now there is something which has happened here, which will not allow us to go further. In fact the usual approach is to say that when this happens, we know immediately that our polynomial is not good, it is a bad polynomial, what has happened is now I have got a 0 here, what about the quotient, when this polynomial is divided by this polynomial, what is the quotient, the quotient is 2 divide by 1 into s.

So the quotient is again a power of s first power of s. Now what is going to happen, I have an even polynomial which is s to the 4 plus 2 s square plus 7 and the next polynomial is minus 8 into s, the degree of the even polynomial is 4, the degree of the odd polynomial however is not 3 but it is 1. Therefore, the quotient is not going to be s to the 1 but it is going to be something different and therefore, when this happens you have to use some other rule or some other device to proceeded further.

So this, 0 here in the first entry of a row gives you a warning not that your calculations are wrong but that the polynomial that you are working with is a bad polynomial and what did I mean by a bad polynomial or what is a good polynomial. From the point of view of stability meaning that, if I look at the 0 input response, when the initial state is not 0 that 0 input response as time increases will go to 0, if that is the case then, we call the system a good system and for that the characteristic polynomial should be such that all of its roots have negative real parts.

So that is what was mean by stability and that is what I mean by a good polynomial. If a polynomial has even one root which is not in the strict left half plane that is one root which is not negative real or it does not have a negative real part, it could be 0, it could be purely imaginary, it could be positive real number or it could be in the right half of the complex plane. All those polynomials are bad polynomials, in the sense that the 0 input response will not go to 0 as t tends to infinity and we do not want, the 0 input response either to remain constant at some non-zero value or to keep on oscillating or to keep on growing because that is the case, when the system response will keep on increasing and therefore, some where or the other something will happen, a fuse may blow or the system will be no longer in the linear region of operation. So, all our analysis is just no longer applicable. We are looking essentially for a test to find out weather a polynomial is good and of course, for what values of the gain K a, the polynomial is good that is what we are looking at.

So the moment you get a 0 as a pivot element for the next operation that is a warning that you have got a bad polynomial. Now in such a case one can do something and proceed further in somewhat different way and get some information out of all this but, if you just want to check that the polynomial is bad then, you can stop here and say that the polynomial has at least 1 root which is not in the strict left half plane. So that is the question that you are interested in when it is answered right away. Now there are techniques and I will not go into details and the Routh table construction or the Routh theorem or the algorithm, as I told you is about 120 years old, naturally a lot of people have looked at it and the situations like this have also been looked at and this has been discussed in many journals not all of it appears in text books.

So your text book may tell you some way out or something else to be done, when you get a situation like this. But we will not have enough time to discuss all those details but such a case in general is called a singular case and a case, when this does not happens is then called a regular case. So unfortunately, with this case we are ending up with the singular case and what is bad is from the point of your polynomials, the degree difference is not 1 but its 3 and from the point of view of the algorithm, there is an immediate problem because now my pivot element is 0 and if I am going to divide by the pivot element then, I will be committing the sin or dividing by dividing by 0 and dividing by 0 is meaningless I can write infinity but that does not help me at all, as you can see from here for example 0 into 2 plus 8, so that is plus 8 divided by 0.

So should I write plus infinity or minus infinity and similarly, 0 into 7 minus nothing divided by 0 should I write 7 or it is 0 by 0 or should I write it as some other number. There is no point in just proceeding arbitrarily and then, there in that does not guarantee that you will get the correct result. So let us leave this singular case here it is, it is an indication that the polynomial is bad and if that is all you are interested in we stop. There are techniques of proceeding further by doing some things but I am not going to discuss them right now.

So we have encountered a singular case but the earlier example was a regular case because with this polynomial, we could go all the way till the end without any problem, there was no 0 encountered in the first column, pivot element was not 0. So there was no problem one could go ahead and get the answer. Now let me take one more example which we had looked at earlier but we are not carried out the Routh table construction. Now that is s to the 4 plus 3 s cube plus 6 s square plus 12 s plus 8, this is the polynomial, same algorithm to be used. So start with s to the 4, the coefficients are 1, 6, 8 next row s cube 3 twelve there is no entry here. So I am putting a dash now, the next step the division process this is the pivot 3 times 6 is 18 minus 12.

So difference is 6 divided by 3 is 2, 3 times 8 is 24 minus nothing divided by 3 is 8, no problem so far I have not encountered a 0 here. So I can continue one more step, I write down s to the 1, now this is the new pivot element mind you. Now and we look at the last2 rows that we have obtained. So 2 into 12 is 24 minus 8 into 3 is 24, the difference is 0 divided by 2 is 0. Now we have encountered the same trouble pivot element has become 0, so the polynomial is bad, the polynomial is bad, the original polynomial is bad. It will have at least one root which is not in the left half planes, strict left half plane.

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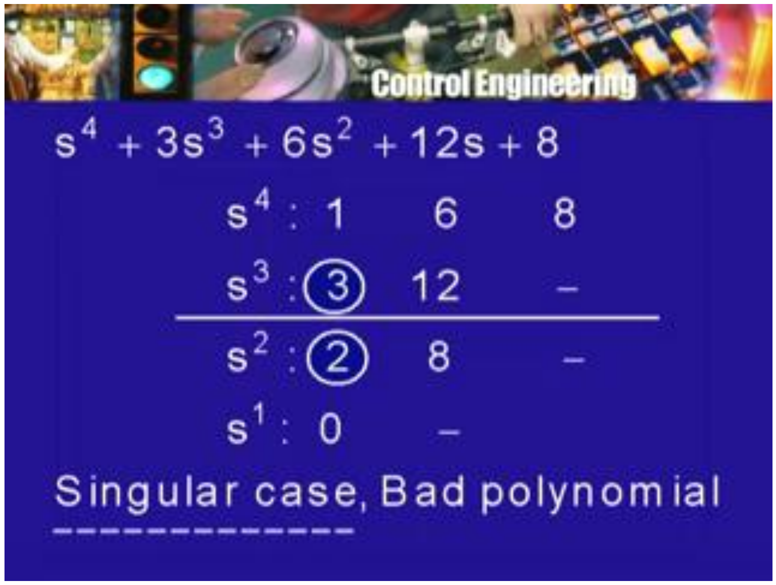

Now this therefore looks like a singular case, now is it any different from the previous singular case, where we had a 0 as a pivot element. On the face of it, no but there is a difference because this is a polynomial which is odd degree 1 and therefore, you just have only one term. So this entire polynomial is 0, so although this is a 0, 1, 0 here, the conclusion is that this whole polynomial is 0, where as in the previous case I had a polynomial which was not identically 0, not all the coefficients were 0, one coefficient was 0 but the other coefficient was not 0.

So this polynomial is not 0 while it is 0 into s cube minus 8 into s. So the whole polynomial is not 0 but only a part of it namely the leading coefficient which should have been s cube is missing the coefficient is 0. So this is a different singular case, this is also a singular case because it is not regular, but it is a different kind of singular case and we can take one more example to see what is happening, where we can really see that the remainder polynomial is an identically 0 polynomial and it is not difficult to construct examples of this kind and therefore, I will tell you, how you construct an example of this kind. So that you will realize how text books or authors who write papers think of these examples. If I ask you to write the polynomial, may be you will write like a wrote, s to the 6 plus 5 a 1 s to the 5 plus etcetera, etcetera some such and what is known happen to it you just do not know before I and now, I will construct a polynomial and I will show you, what is going to happen and why it is going to happen.

So I am going to construct the polynomial like this. I am going to write down a polynomial s square plus 1. Now, what do we know about this polynomial s square plus 1. Well, we have looked at such things earlier, what are the roots of this polynomial plus minus j. So they are on the j omega axis imaginary axis, they are not in the left half plane. Let me multiply it by s square plus 2, what are the roots of this plus minus j root 2. So, now I have a product of 2 polynomials and all the roots of it are on the imaginary axis. There is no root in the left half plane, now let me change it further by multiplying by s plus 1, now what is happening there is 1 root which is in the left half plane.

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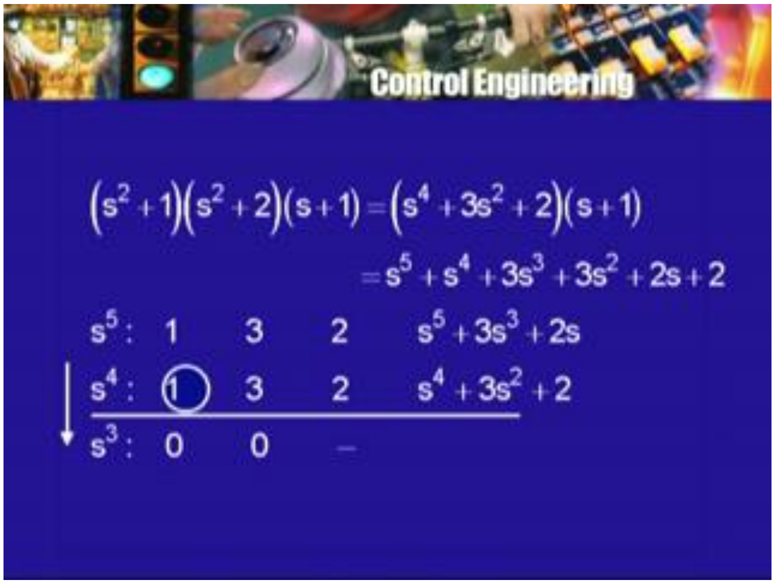

So there are 4 roots which are on the j omega axis and there is 1 root which is in the left half plane but now, let me work out this polynomial, what are all the coefficients and that is not very difficult to do here. I have s to the 4 plus 3 s square plus 2 multiplied by s plus 1 and therefore, what do I get, I will get s to the 5 plus what is the s 4 term s to the 4 plus, what is the s cube term, s to the 4 is coming from here, s cube term is 3 s cube, what is the s square term, s square term is 3 s square, what is the s term, s term is 2 s and the constant term she 2.

So this is the polynomial that I will get. Let me check this is s 4 plus 3 square plus 2 into s plus 1 so s to the 5 plus s to the 4 plus 3 s cube plus 3 s square plus 2 s plus 2. So that is the polynomial. Now let us start our Routh table, so s to the 5, I will get 1, 3, 2, s to the 4 I will get 1, 3, 2, this is the polynomial s to the 5 plus 3 s cube plus 2 s, this is the polynomial s 4 th plus 3 s square plus 2. This is odd polynomial this is even polynomial, I carry out the division operation. So this is s to the 3, this is the pivot element what do I get this times this is 0, I am not finished because there is this operation also to be done this time this is 2 minus that is 0. So I get 0, so the next polynomial not only is this entry is 0 but the other entry is also is 0, so the entire polynomial is 0.

In fact, what is happening is if your check for yourself this odd polynomial which is s to the 5 plus 3 s cube plus 2 s square which is being divided by the even polynomial s to the 4 plus 3 s square plus 2. This is such that when I divide this by this, there is a quotient and the quotient is of course 1 divided by 1 into s namely s but the remainder is identically 0 or I can say the remainder should have been a polynomial of degree 3, for me to proceed further but it is a polynomial whose coefficients are both 0 and therefore, I cannot any further. But this was different from the earlier singular case, where I encountered only a pivot element which was 0, the other element was not 0. The pivot element is 0, the other element was not 0 that is it is the difference between this case and the case that I am looking at where the entire polynomial is 0.

So this is a singular case which is different from the earlier one. Now such a singular case indicates what we see here then, there are roots on the imaginary axis or on the j omega axis. So a singular case where one says the entire row is 0s or vanishes then, this implies or this indicates that the original polynomial has roots on the j omega axis, it has roots on the j omega axis and in fact, you can find out what those roots are or at least make some progress towards finding out what those roots are what you do is when you encounter this row of 0s look at the previous row. The previous row is this polynomial s to the 4 plus 3 s square plus 2 then, this is a polynomial which is a factor of the original polynomial and then you factorise this polynomial and then, we look at its roots.

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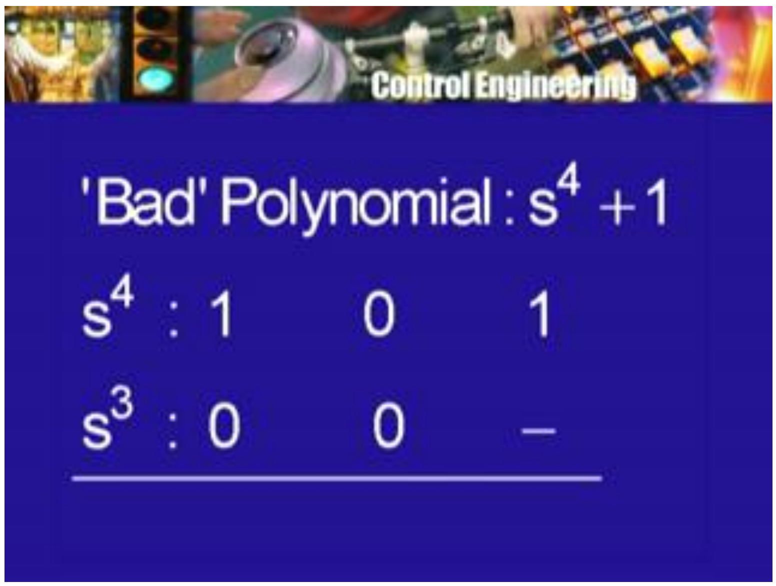

In this case, it is not difficult to factorise, this polynomial. So the roots will be simply s square plus 1 and s square plus 2 factors therefore the roots will be plus minus j 1 and plus minus j root 2. Now, unfortunately it is not as simple as that so it is not really correct to say that when the entire row vanishes, there are roots on the j omega axis, it has happened in this case but it is possible to construct an example where this will not happen in fact, it is possible to construct and example, where the Routh array construction cannot get started at all and I will take a very simple example which is important because it indicates a particular very special situation. Let us look at the polynomial s to the 4 plus 1 of course, I told you earlier that if a coefficient is 0 or if there is a change of sign in the setup coefficients then the polynomial is already bad.

So we know that this polynomial is bad and so, if you really wanted only that much information, we will stop but suppose I try to apply the Routh algorithm to it, what will I get, start off with s to the 4, no problem leading coefficient is 1, s square term is 0, constant term is 1, what about the s cube polynomial. Well, there is an any, this polynomial does not have any odd part or the odd part is really the 0 polynomial, 0 into s cube plus 0 into s. So I am stuck pivot element is 0 but it is the second case, singular case of the singular case where the entire row is 0.

Now, look at the previous row, it is s to the 4 plus 1. Now, what are the roots of this polynomial s to the 4 plus 1, little bit of thinking will enable you to find out the roots put s squared equal to z squared equal to z, we are making a change of variables. Then, what does s to the 4 plus 1 become it becomes z square plus 1. So, when is z square plus 1 equal to 0 when z equal to plus minus j. So, here is the complex plane, here is plus j and here is minus j. So the roots of the polynomial z square plus 1 are plus j and minus j but our polynomial is polynomial in s, s 4th plus 1 and not z square plus 1 and we have this relationship between s and z, s square equal to z.

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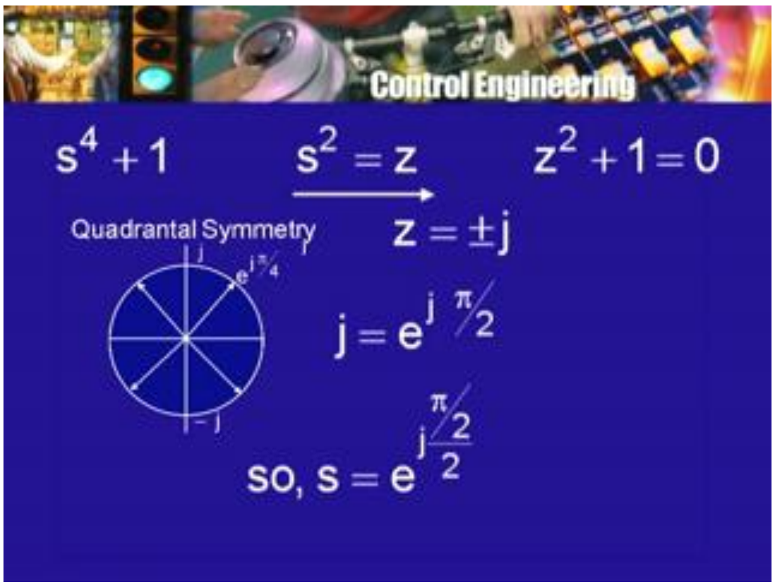

So, s is a square root of z, so I have to take the square root of j and in fact, when I take a square root there is not 1 but they are always 2 square roots in general or always really. Similarly, for minus j I have take this square root of minus j, how do you take the square root of j converted into the polar form in the polar form j is the same e raise to j phi by 2 and therefore, one of the square roots of j will be e raise to j phi by 2 divided by 2 or e raise to j phi by 4 and where is that complex number e raise to j phi by 4. Well, it is located here, if I draw units circle around the origin then, this is the root which is square root of j.

So this is one of the roots of the polynomial therefore, it is negative will also be a root and then from minus j we get 2 more roots like this and so, this polynomial s to the 4 plus 1 will have 4 roots which are located like this in the 4 quadrants and they are located in a symmetric way and such a symmetry is called quadrantal symmetry. Because, there is a symmetry that involves roots lying symmetrically in the 4 quadrants, it is called quadrantal symmetric. So here is an example of a polynomial where the Routh algorithm results in row which is identically 0 and not just the leading coefficient of a row, the pivot element or few elements 0, the entire row is 0, still there are no roots on the g omega axis whereas in the earlier case that I took took here the entire row was 0 and the roots of the polynomial, where on the j omega axis and indeed they were the roots of the polynomial s to the 4 plus 3 s square plus 2.

So this result that I wrote is not really correct an entire row vanishes does not imply that there are root necessarily on the j omega axis, but the what is true is that if I entire row vanishes then, the roots which are may be on the j omega axis will be factors of this polynomial which is last non polynomial. So this result is not true. However, the result as I sated in the modified formed is true and this rule can be use to find out the intersection with the j omega axis. In this second singular case, where you have a row, entire row which is 0, again there are techniques which have been devised to proceed further to get some other information out of the Routh table. In principle with the simple rule that cross product divide by diplomat element I cannot proceed any further, I have to give up I get of course the information that the polynomial is bad, I get that information because I cannot continue. I encounter the singular case, the moment you have a singular case, the conclusion is that the polynomial is bad but can I get some information other than the polynomial being bad the answer is, yes provided, I do something further.

Now, what further is to be done when you have a situation like this. Again will require quite a bit of looking into your text book probably has some indication but as I told you this is a subject which has been studied for long time. So there are quite a few techniques and some of the results which I have mentioned in the text books.

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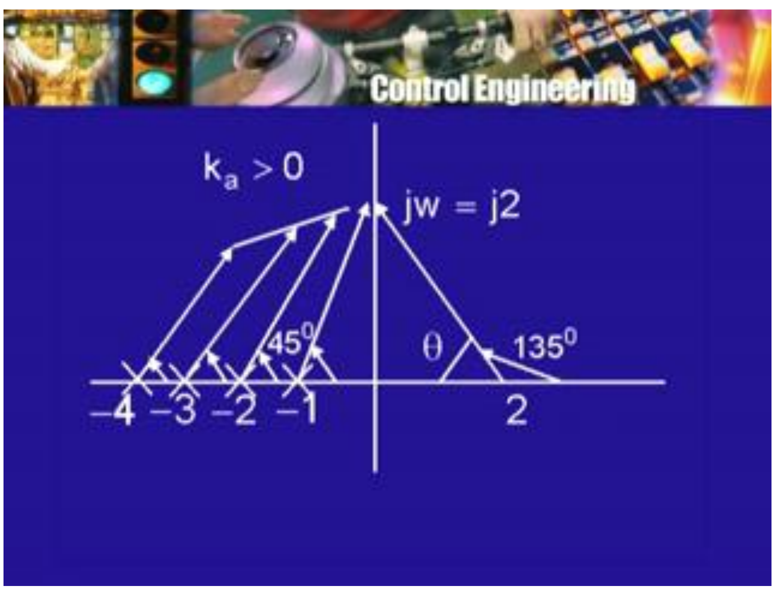

So, some of the techniques which are given in the text book may also fail. So this problem is as simple as it looks but we really are not interested in full problem of given a polynomial, say something about it is roots. Our main problem was the root locus method where we wanted to find out intersections with j omega axis and how does the Routh algorithm give us a way of finding out the intersection that is what we are interested in.
So, let us get back to our original problem and try to apply this Routh are a technique to that problem. So here was the pole 0 diagram which have been looking at for quite some time now. There are 3 poles minus 1, minus 3, minus 4, there are two 0s plus 2 and minus 2. So what is the characteristic polynomial? The characteristic polynomial if you remember was G D, H D that is

the denominator of G H plus the gain k a in to G N, H N. Let me call it p of s, this is the characteristic polynomial.

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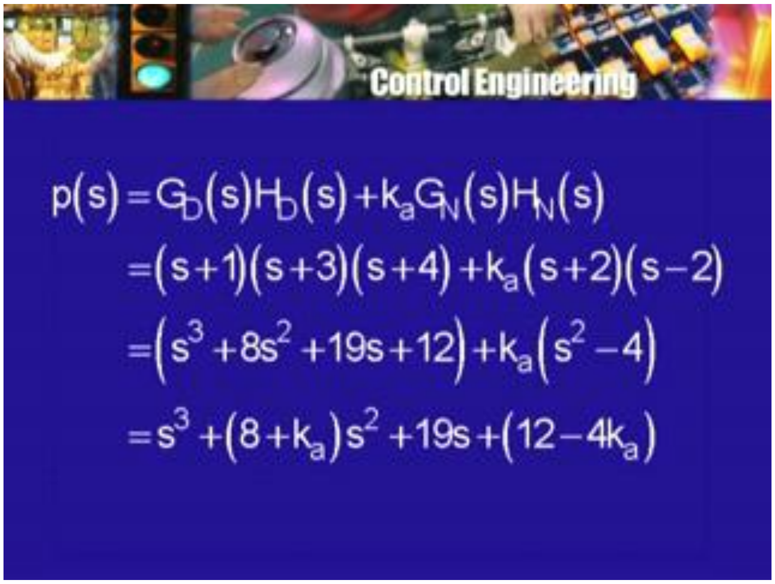

In this case, the denominator there are only 3 poles. So, I have s plus 1 into s plus 3 into s plus 4 plus k a. In the numerator I have s plus 2 and s minus 2. So, s plus 2 s minus 2 because there are two 0s, these correspond to the 0s, these correspond to the poles. So the polynomial corresponding to the pole part plus k a, this gain of the amplifier whose effect we are trying to find out multiplied by the polynomial corresponding to the 0s. Let me expand this out then, this is s cube plus as I told you earlier, 4 plus 3 plus 1. So 8 square plus 3 times 1 is plus 4 times is 3, 12 plus 3, 15 plus 4, so 19 s plus 4 into 3 into 1 plus 12 plus k a into s square minus 4.

So, collecting coefficient then, I have s cube plus and I have here 8 s squared and I have k a s squared, so 8 plus k a in to s square plus 19 s is not affected plus however, 12 minus 4 k a. So this is the characteristic polynomial and as you can see depends on k a. We would like to find out, where are the roots of this polynomial or really, we would like to find out for what values of k a will p s be bad and if possible, for what values of k a will p s have roots on the j omega axis. This the problem we are trying to investigate for what values of k a will the polynomial become bad.

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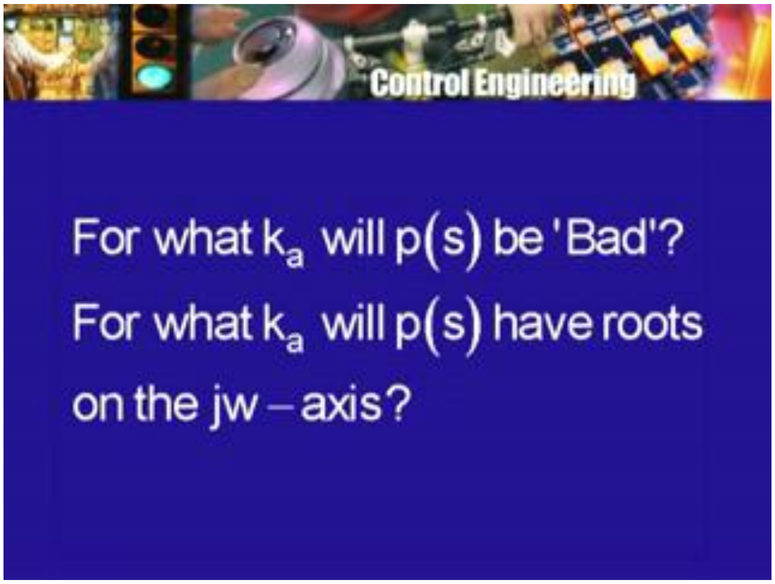

So, we know the that the roots no longer be all of them in left half plane at least 1 root will not be in the left half plane and secondly, for what values of k a will the polynomial have roots on the j omega axis and if possible, we can find out the location of the roots. So, let us proceed further so with the table construction then, I start with my s cube row because the degree of the polynomial is 3. So s cube the coefficients are 1 and 19 s square, I have 8 plus k a and the second entry is 12 minus 4 times k a. This is an odd polynomial, so s cube plus 19 s this is an even polynomial 8 plus k a into s square plus 12 minus 4 k a. Next step this is pivot element s to the 1 power. So I will get only entry and that entry will be what this into this minus this into this divided by the pivot.
Now, when I do this multiplied by this I going to divide by the pivot. So, I need not carry out the multiplication and then divide by the pivot.

So I can write down here directly 19, this is the term here minus this product 12 minus 4 k a divided by 8 plus k a. This is the whole coefficient okay. Now can I proceed further well I can if this coefficient is not 0, so the question that we are to ask, is this coefficient 0? Can this coefficient become equal to 0? So that is the question I am going to the ask, I going to give you little bit of time to work out, whether this coefficient can become equal to 0. I am going to do the calculation, you should also do the calculation side by side.

Okay, here are the calculations 19 minus 12 minus 4 k a divided by 8 plus k a, is it equal 0 that is the question that we are asking. Now carrying out the product operation then, 19 into 8 plus k a should be equal to 12 minus 4 k a and therefore, collecting the coefficients will get 23 k a should be equal to this 12 minus 152 or minus 140 and therefore, the answer is this will be equal to 0 for a particular value of k a, namely k a equal to minus 140 divided by 23 minus 140 divided by 23, this is a negative value of k and we are only looking at positive values of the gain k. We are not looking at any negative values of the gain k.

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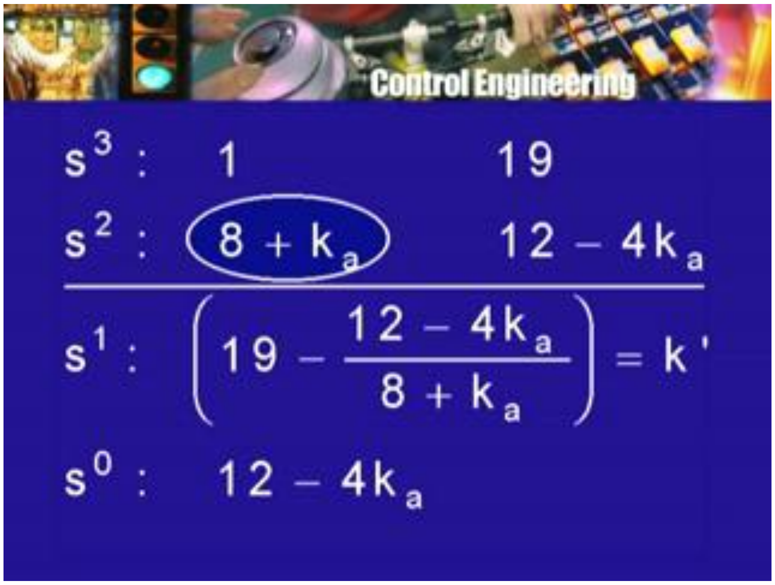

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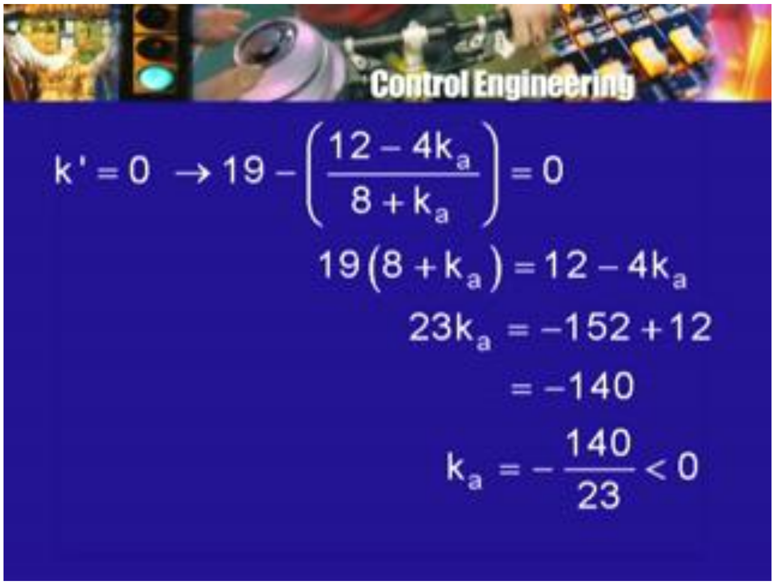

So this pivot element will not become 0. It will not become 0 for a positive value of k. Of course, what this is be dependent on what k a you are chosen but for positive values of k a this coefficient will not be 0. Now the question is then, can I proceed further yes, I can, if this pivot element is not 0 then, I can proceed further. Now, what is the next row now, are the 2 rows that I have obtain from that I have to obtain the next row. This is pivot element this multiplied by this entry minus what minus nothing because in this there is only s to the 1 power.

So this multiplied by this divided by the pivot will give me simply 12 minus 4 k a, all right. So I will write down the Routh table, I will call this k prime because it is a complicated expression. So my Routh array is going to look like s cube is 19, s square is 8 plus k a, 12 minus 4 k a, s to the 1, is this k prime and this k prime is not 0 and in fact, this k prime is positive as you can verify s to the 0 is however, given by 12 minus 4 k a, right. You can check that this k prime is actually positive for k a which is positive because if you cross multiply you have this 19 k a plus 152, on the other side you have 12 minus 4 k a and therefore the left hand side will be greater than the right hand side and therefore, k prime will be positive, all right.

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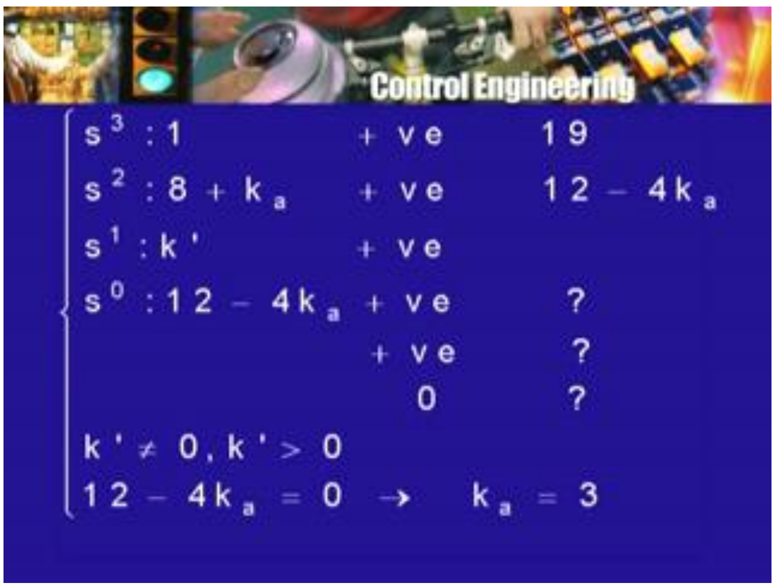

Now of course, I have gone all the way up to s to the 0. So I have not encountered any singular case however, somethings can happen. This coefficient can become 0, yes when will it become 0, 12 minus 4 k equal to 0 will become 0, when k a is equal to 3. Of course, there is no need to go further but the moment I have a 0 here, this means that for k a equal to 3, my polynomial will be bad, will my characteristic polynomial be actually bad. We will look at the original polynomial and substitute k a equal to 3 and see what you get. The polynomial was s cube plus 8 plus k a s square k a is 3, so plus 11 s square plus 19 s plus 12 minus 4 k a.

Now, what is 12 minus 4 k a in this case, it is 0 and therefore the polynomial is actually cube plus 11 s square plus 19 s or it is s multiplied by s square plus 11 s plus 19 and so, there is a root which is at the origin or the origin is the root for this value of k a equal to 3. Now of course, this is exactly what we had guessed earlier, when we are looking at the root locus method. When we use the root locus method we argued that there is a portion of the root locus lying on this part of the real axis and therefore, there will be a root which will be at the origin for some value of k a.

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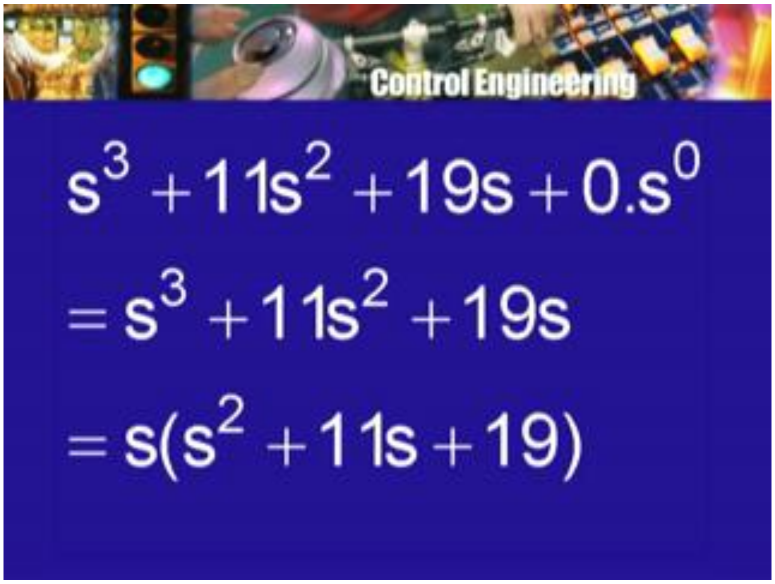

Now by using the Routh table, we have obtained it in a different manner all together and that has involved a pivot element becoming equal to 0 or equally well, the previous row becoming equal to 0. Now is there anything more than that we can conclude from this, the answer is, yes. Remember, what I told you earlier that if the table is completed, so that is, if you do not involve there are no hurdles involved if you do not get stuck because of a pivot element becoming 0 or an entire row becoming 0. Then, once the table is completed you can look at the first column of the table and this is the first column of the table and then, what is it that you have to look for, you have to look for change in size of the entries as you go from top to bottom.

So, I start with the top this is 1, it is positive second entry is 8 plus k a. Remember, we are looking only at positive k a. So this is positive the third entry is k prime I told you and we agree it that for k a greater than 0, k prime is also positive, so this is positive, what about the last entry, is the last entry positive, no because in fact, it becomes 0 for k a equal to 3. So we see that if k a is less than 3 then the last entry is positive, if k a becomes greater than 3 then, the last entry will be negative and if k a equals 3 then, the last entry will be 0. So excluding that 0 case which is the bad polynomial case, what about the other cases.

So when k a is less than 3, let us look at that first what are the sings of the one entries plus, plus, plus, plus, plus is there any change of sign, no. So the number of sign changes is 0, there are number sign changes, where as in the other case, when k a is greater than 3, I have plus, plus, plus, plus and then minus. So how many sign changes, no sign change in going from here to here, from here to here but there is one sign change is going from here to here. So for k a greater than 3, the number of sign changes is 1. Now, this is what the original result of Routh tells you

that if you are able to complete the Routh table without any difficulty that is there is no singular case, you have what is called the regular case, none other pivot elements is equal to 0,
So, you can go till the very end then look at the first column and look at the number of sign changes in the first column. The number of sign changes in the first column, in this case, in the regular case, when there are no 0s encountered in the first column, in this regular case the number of sign changes in the first column is the number of roots in the strict right half plane that is the number of roots with positive real parts is exactly this number. The number of sign changes in the first column and if you go back to our pole 0 diagram once again and we had started sketching the root locus.

We were seen there that there was a part of the root locus here. there was a part of the root locus here and the third part of the root locus was asymptotic and the conclusion that we obtained is indeed correct that there is at most one root which is in the right half plane, other 2 roots are in the left half plane and that root which will be in the right half plane. Using, Routh algorithm we have found out will occur when k a is greater than 3 and so, combining the root locus rules, real axis portion, asymptotes, number of branches of the root locus, those where the simplest rules with the Routh table kind of investigation tells us that for k a less than 3, the characteristic polynomial will have all the roots in the left half plane and so, the system will be stable there will be no problem what so ever. The 0 input response will go to 0 as t tends to infinity and the 0 input responds the part of it which depends on the initial conditions of the input also will go to 0 as t tends to infinity.

However, for k a greater than 3, there will be one root which will be strictly in the right half plane and this case being simple the root locus approach tells us that the root will actually be positive and real. Therefore, it will correspond to a growing exponential and so the 0 input response could, if the initial conditions are not properly chosen, could have a term which goes to infinity as t tends to infinity. The system will be unstable therefore the system will be bad and this is something we do not want to happen. The case, when k a equal to 3 is the singular case of the Routh technique.

In this case of course, the answer or a simple when k a is equal to 3, there is a root exactly at the origin. In general, when you get a singular case, the conclusion is not so simple and as I have told you, there are 2 singular cases with pivot element 0 but not the entire row 0 and the second case, when the entire row is 0 then, there are some additional rules which enable you to proceed further and complete the table and get some conclusions about the location of the roots but those rules are not very simple and some of the rules which are given in some of the textbooks are not even correct, there has been discussion in the journals about the correctness of some of these rules.

So, let me just warn you that the methods which are given in the textbooks sometimes may not work or they may not give you the correct answer but essentially the problem that people have been looking at subsequently to Routh and in fact, if we remember Routh was looking at a problem which was posed by Maxwell, which came out of his investigations Maxwell's investigations of the governor which was designed by James watt and others earlier. The problem is what is called qualitative theory of polynomial root location.

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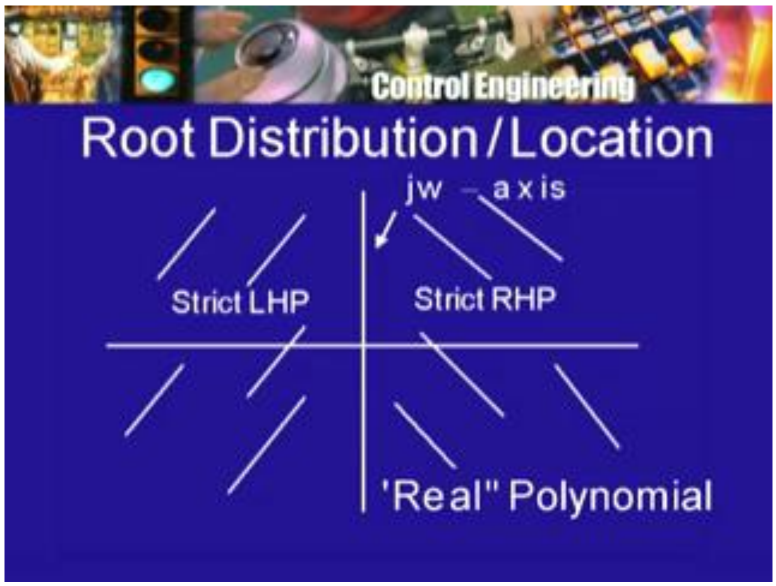

The complex plane can be divided into 3 parts, the j omega axis, the strict left half plane and the strict right half plane, what one is looking for is a method where by starting with a given polynomial without factorizing it, you would like to get some information about the root distribution. For example, from the point of view of system stability, all the polynomial roots should be in the strict left half plane or the polynomial should be a good polynomial.

So we would like to find out knowing the polynomial if all the roots are in the strict left half plane without factorizing the polynomial, without finding out the roots of the polynomial individually. The Routh algorithm enables you to get that information. The Routh algorithm also enables to get you the information, when not all the root lie in the left half plane some of them may lie on the j omega axis, some of them may lie in the right half plane.

Now just as I can find out the number of roots which are in the right half plane. They are precisely the number of sign changes, when there is no singular case, I can find out the number of roots which are precisely in the left half plane and therefore, I can find out how many roots are there on the j omega axis and so, the root distribution problem can be completely solved for a polynomial whose coefficients are all real numbers or as one calls them, what is called a real polynomial, a polynomial with all coefficients real.

It is really very surprising and a significant achievement that at least in the case of all polynomials which have real coefficients, information about the root distribution can be obtained without factorizing the polynomial, without finding out even one root by simply working with the coefficients of the polynomial and then, using some kind of an algorithm which involves multiplication, subtraction, division of numbers and if, there is no singular case the whole thing goes till the end, there is no problem what so ever. There are other problems for example, this is a root distribution into left half plane right half plane and the j omega axis.

Now, when you discuss discrete time systems and in our course, we just do not have any time to discuss discreet time system. Although, I have mentioned sample data on discreet time systems in the beginning but there is a different problem which is to be solved, there is this unit circle in the complex plane and this unit circle of the circumference of it divides the complex plane into 3 parts. The part which is inside the unit circle circumference itself and the part which is outside.

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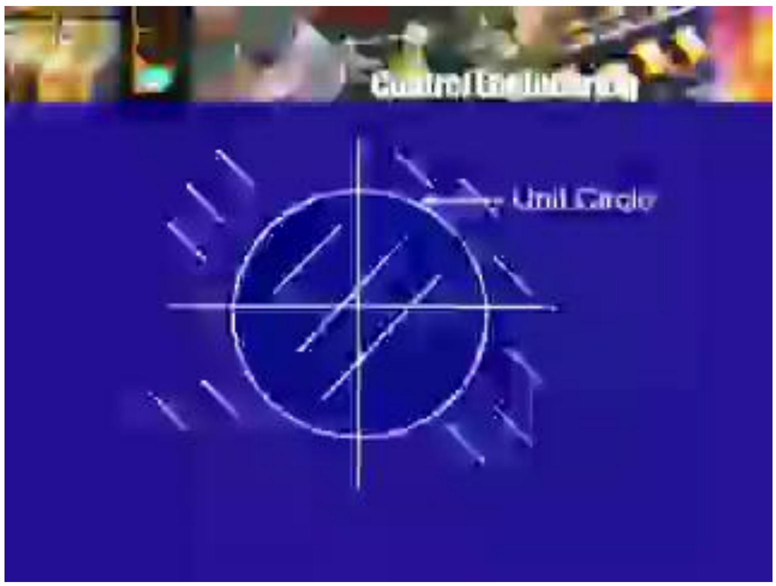

Now, you can ask a similar question given a polynomial by just working with the coefficients without solving for the roots by doing some simple, what are called rational operations on the polynomial coefficients, can I determine the root distribution, how many roots are inside the unit circle, how many on the unit circle, how many outside the unit circle. This problem can also be solved and there are algorithms which are similar to the Routh algorithm. Techniques that are used vary from very simple once like just the complex plane and the complex function theory some of which you have studied to what I have called quadratic forms which involves looking at matrices and what not but that is a different and a deep subject.

So, we are not going to get into it, as far as we are concerned. When working with the root locus method, the Routh algorithm can provide us with a way of finding out if there are going to be situations where, there will be intersection with the j omega axis or at least it gives you a way of finding out, whether the characteristic polynomial can become bad and if so, for what values of coefficient k a, the polynomial will become bad. Now we will go back and apply this to our speed control problem with proportional control and with integral control and check whether that system can become bad for some values of the gain, the proportional gain and the integral term gain, this we will do next time.