Control Engineering

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Lecture - 33

Let us take a look at the second method of determining the points, where the root locus branches may intersect the j omega axis or the imaginary axis. In the first method essentially, what we were doing is we were trying to find out if the characteristic polynomial had a root which was purely imaginary and of course, we know that if j omega is a root then minus j omega is also a root and so, to determine such an omega, you just replace s by j omega and equate the characteristic polynomial to 0 split it into its real and imaginary parts and equate them separately to 0 and you get 2 equations in 2 unknowns, one of the unknowns is of course, the gain k a for which there will be an intersection and the other is the value of omega corresponding to the point of intersection.

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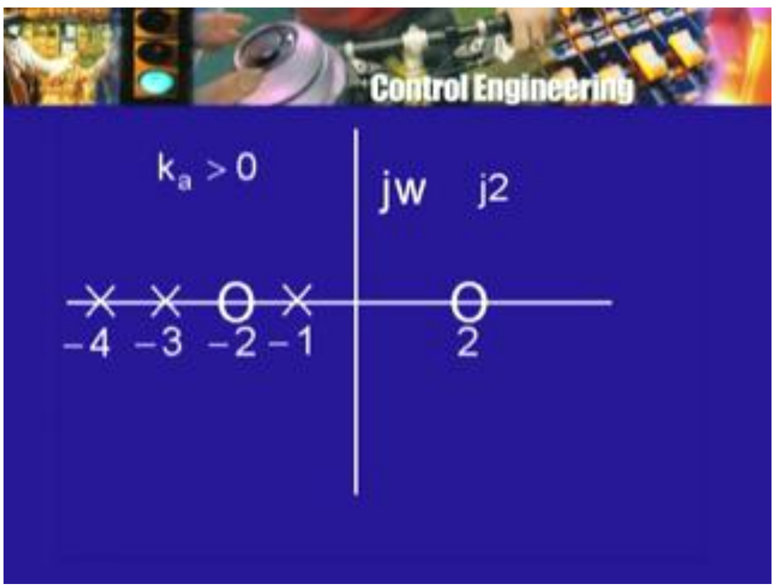

Now there are ways of solving these 2 equations for example, by eliminating k, the gain k a, you will get a polynomial in omega and then you have to find out if at as any real roots. The second method is a little different, it makes use of the angle or argument idea that is a angle condition or the argument condition and it is useful only if you are willing to use a calculator and make a number of trial calculations. As an analytical method, it is not a good one as we will see. So, what is the problem I want to determine whether there is point j omega which is on the root locus.

Now what does the angle condition tell you, the angle condition tells you that a point in the complex plane is on the root locus, if and only if, what happens, if and only if the sum of all the angles subtended at the 0s minus the sum of all the angles subtended at the poles. This difference is equal to phi radians or 180 degrees alternatively plus or minus a multiple of 2 phi radians or a multiple of 360 degrees that was the angled condition. So to find out whether a point on the imaginary axis which looks like this j omega whether it is on the root locus, we will have to find out the angle subtended by this point at the various 0s and at the various poles and then, write down that expression sigma angle subtended at 0s minus sigma angle subtended at the poles equal to phi plus or minus 2 k phi. Now, if I chose a particular valve for omega for example, I will chose let us say, j 2 instead of any arbitrary unknown omega, if I choose a point let us say j 2 then, it is not very difficult to find out if the angle condition holds. In fact, if you remember earlier I had asked you to find out whether the point 2 plus j 2 was on the root locus by using the angle condition.

So here for example we want to check whether the point j 2 is on the root locus or in other words, there is an intersection of the root locus branches with the j omega axis at least 1 point, namely j 2. Now of course, we will not be lucky that this j 2 will be on the root locus therefore I will have to repeat it for another point, let us say j 1 or j 3 and really there is no end to it because I may try 10 different points. I may find that none of them lies on the root locus still there may be point which does lie on the root locus. But by making this computations one can get some idea as to whether there is likely to be a point which lies on the root locus, how do the computation go, they go as follows, I first sum all the angles made at the 0s.

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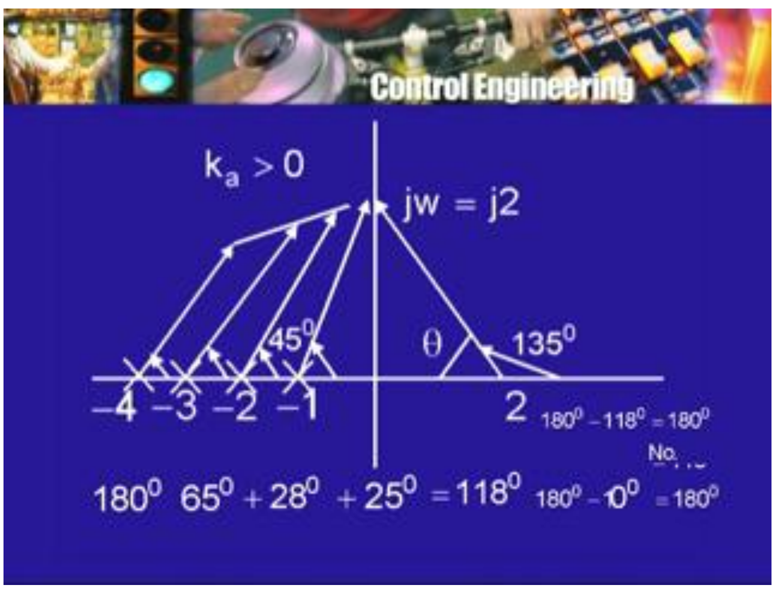

So here this point j 2 here is j 2 here is a 0 at 2. So I draw this vector going from 0 to this point and I have to calculate the angle here. Now incidentally when Evans introduced this method, he did not have the pocket calculator available, this was in the 1950, the pocket calculator had not been invented. Engineers of that time and even I use continue to use this tool which is know as the slide rule. Today, probably you are not even see one so multiplications and divisions were carried out and even trigonometric ratios were found out and logarithm and exponential with the help of what is called the slide rule.

So slide rule will enable you to add enable you to multiple divide but not to add or subtract. The slide rule was use for multiplication division rising to power even and exponential trigonometric calculations. But, here I need to note a angle and so, what Evans did was he combined slide rule with a device which we use for measuring an angle and what is that device, that is a protractor. With the help of a protractor, we can measure angles and so, Evans combined these 2 into a new instrument which he called a spirule and after this was invented, engineers found it quite useful and so it was being use for quite some time. I bought spirule for myself and learned how to use it. Today of course, you neither have this spirule nor do you have the slide rule. Fortunately, you still have the protractor, so you can measure the angle if you want it.

But today you have the pocket calculator, so you can calculate the angles more quickly. So for example, what will be this angle, I do not want to measure it, how do I calculate it? Now, you can see that to determine an angle, you need to know 2 sides of right angle triangle then, you can find out using the inverse trigonometric function, the angle. Now, what do I know in this case of this right angle triangle, I know the vertical side and I know the horizontal side. In other words, I know of the 2 side which enclose the right angle and therefore, if I call this angle theta then, I can immediately find out tan theta is tan theta will be this height which is 2 divided by this base which is 2. So tan theta is 1 and fortunately, I do not have to use a pocket calculator to find out what is the angle, whose tangent is 1and the angle is 45 degrees, if you want an acute angle. So this theta is 40 degrees.

So what is this angle, angle subtended at the point by the 0 measured in the anticlockwise or counter clockwise direction from the positive real axis. Here, is the positive real axis, here is this vector pointing towards j 2 point and so, I have to measure this angle in the counter clockwise direction. So what will be angle be, it will be 108 degrees minus 45 degrees or in other words, it will be 135 degrees. Now if this one is not true but this was something else then, I can do it on the pocket calculator by simply entering this number first dividing it by this number and then using the appropriate mode on the calculator to get tan inverse.

So just 2, 3 presses of the calculator, you know enter the height then, divide command into the base and then, tan inverse, for that you may have to use to inverse mode and then press the tangent key and you will get the angle but you will get this angle theta. The calculator does not give you all possible angles for example, the calculator will give you 45 degrees but will not give you 225 degrees which is also a correct answer because as we know, the tangent at the sin and cosine, repeat their values not only after 2 phi but even for earlier increments or differences. So the angle here is 135 and when using the calculator, we should have the figure in mind, so that you get the correct angle. Now what about the angle subtended at the other 0, which is this one.

Now that is easy this is 2 divided by 2 is 1, tan inverse of that is 45 degrees and it is this angle. So this also, this is 45 degrees, so what is sum of the angles subtended at 0s that is 135 degrees here, there is 45 degrees here. So the total is 180 degrees, so sum of the angles subtended at the 0s is 180 degrees. Now we have to calculate the angle subtended at the poles and there are 3 of them. Now this is where you will have to use the calculator actually and you cannot use any ordinary knowledge of trigonometry. For example, now take this pole at minus 1, so what is the angle subtended it is tan inverse of height is 2 divided by 1 or tan inverse of 2.

Now you have to look it up on your pocket calculator, do it. Roughly speaking from the figure it looks like, it is an angle greater than 45 degrees, it is an angle less than 60 degrees. Let us say, it looks some what like an angle of 65 degrees but you check it on your calculator, what tan inverse of 2 is all right. Then another second pole which is at minus 3 here is the vector going from this pole to this point j 2. Now what is the corresponding angle here, the height is 2 and the base is 3. So it will tan inverse of 2 by 3, now what is tan inverse of 2 by 3. Well, once again you will have to use your calculator but let me say for the movement that it is something like 25 degrees, there is one more angle to be calculated from this pole to the same point, so 2 divided by 4.

So tan inverse 2 divided by 4, find it on your calculator, its tan inverse of one half. So it is actually the complement of the tan inverse of 2 and so it is going to be 25 degrees as a result of which this middle angel will not be 25 but it will something else let us say, 28 degrees all right. So we have 3 angles now, 3 angles obtained at the pole, you should be making exact calculation. Now what is there sum their sum is 65 plus 25 because tan inverse 2 plus tan inverse 1 by 2 is for 90 degrees, 90 degrees plus 28 degrees is above 180 degrees, this minus this is not close to 108 degrees at all. So this point is not going to be on the root locus, the point j 2 will not be on the root locus because it does not satisfy the angle condition.

Now this was just one sample calculation, I can do another calculation very quick in fact, we have done that already when looking at the real axis portion rule, what about the point 0, the origin, is it on the root locus, find out if the angle condition is satisfied. So what are angles the angle made here by 2 is 180 degrees and the angles made by all the other things on the left hand side is 0 degrees and therefore, what is conclusion? The conclusion is that sigma angle at 0s is 18 degrees minus sigma angle at the poles is 0 degrees and therefore, the difference is 108 degrees. So the point or the origin is on the root locus and of course, we had found that out earlier by using the real axis rule why, because if you consider this segment of the real axis then for any point on this segment, the number of poles and 0s to the right is odd, then in fact only one and therefore that point is on the root locus.

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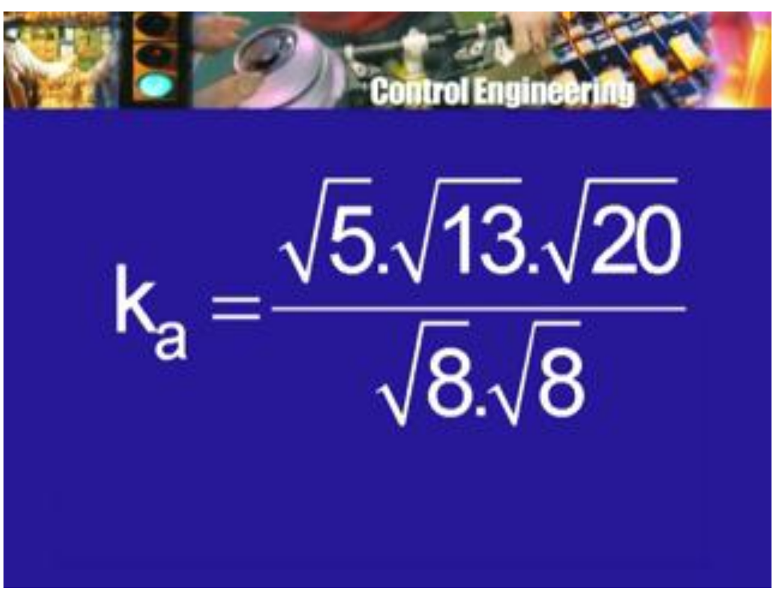

So we have found out one intersection with the j omega axis but there may be more, what is the value of k a, for this the value of k a can be calculated and what does the rule remember, the rule k a was the product of distances 2 poles divided by distances to 0. So for this point, the origin the valve of k, k a will be given by distances 2 poles minus distances to 0 and that can be found in this case again, it can be found out by calculation, if you have to use your calculator then, again there are simple calculations to be done, if you use a scale then you can measure the length also. For example, there are the 3 poles this one, this one and this one and what are the 3 distances.

So this is the hypotenuse of the right angle triangle, this is one side is 2, the base is 1 and so the hypotenuse squared is 2 square plus 1 square. To do that take the square root of that on your calculator, I will not do it using a calculator right. Now I will write down the exact expression for it, it will root 5, what about the distance to this pole 2, this way 3, this way. So 2 square plus 3 square that is 13 square root of that is root 13 that is the length. So root 5 plus root 13, third one is 2 here, 4 here. So 4 plus 4 square 16, 20 so plus square root of 20 distances to the poles divided by the distances to the 0s and there are only 20s here. This one, so this distance is 2 square plus 2 square that is 8 divided by that is the square root of that therefore, square root of 8 plus this is the second 0.

So 2 squared plus 2 square is again 8 square root of that plus square root of it. So I get this k a equal to this number. Remember, k a is the sum no not sum, I made a mistake, the sum occurs in the case of angle condition. In the case of the formula for the gain, what occurs is product and ratio, for the angle condition sum of angles minus sum of angles, for gain valve product of distances to poles divided by distances to 0s. So square root 5 into square root of 13 into square root of 20 divided by square root of 8 into square root of 8, whatever that number is that is the valve of gain k for which there will be root at the origin. But if you want to find out if there are any other intersections then I will have to repeat the calculations that I did for this point j 2 by looking at the point j 1 perhaps and j 3 and so on.

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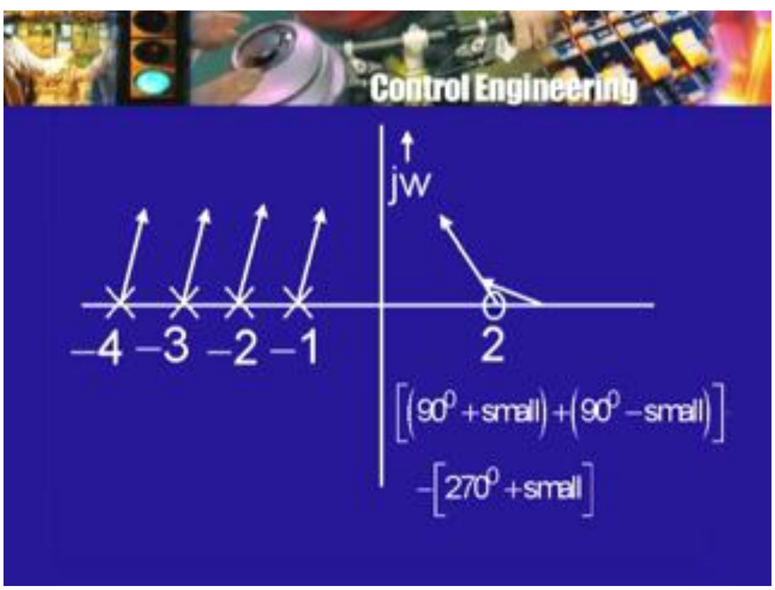

Now of course, what happens is that when I make the calculation for j 2, we found out that the angle or 180 degrees, at origin the angle was 180 degrees, 180, for j 2 the angle was about 118 degrees. We can take another point and we take the point which is sometimes said to be the point at infinity along the j omega axis. In other words, it is a point j omega, where omega is very large of course how large is very large, well that depends on the calculation you are making. So suppose you have a point which is say here, j omega j omega very large compared with the poles and 0 distances. So here is my pole which is a 0 which is 2 here are the poles minus 10 minus 2 poles minus 3, another pole minus 4.

Now, let us meet the angle estimation again. Remember, I am saying that j omega is such that omega I am going to consider very large. In fact, so large that I will not be able it on my paper. So I have to imagine that is way up here somewhere now, what about the angle subtended by that point at 2. Now it does not require much thinking to conclude that it will an angle, which will exceed 90 degrees but by a small amount. So it is going to be 90 degrees plus a small amount, how small will depend on where the point is, if I take omega equal to10,000, the angle will have some valve if I take omega equal to 1 million, the angle will be still smaller.

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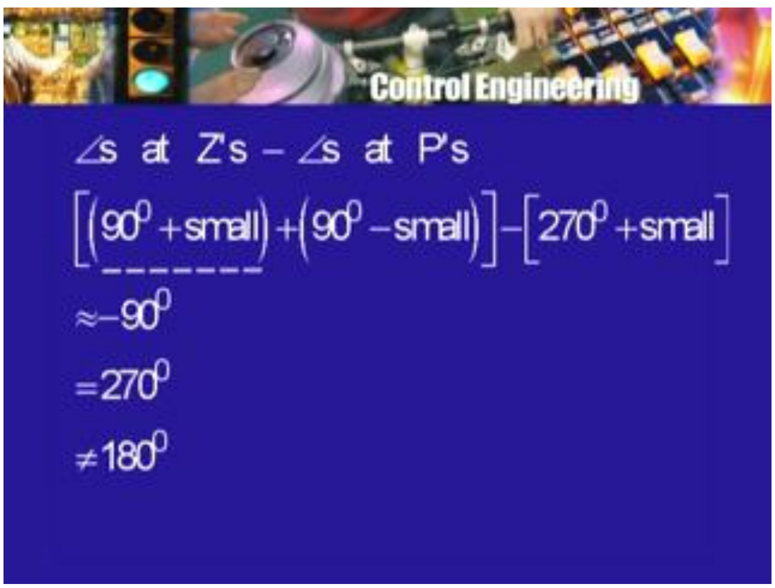

So it will get closer and closer to 90 degrees as I increase omega but it will always little greater than 90 degrees, what about the other angle at the other 0, it is this one by a similar argument, it is also going to be very close to 90 degrees but it is going to be less than 90 degrees by a small amount and so the total contribution of the two 0s will be almost 180 degrees plus or minus a small amount. Larger the value of omega smaller the amount all right. Now what about the contributions of the 3 poles by a similar argument each of the poles will contribute 90 degrees plus a small amount.

So the 3 pole together will contribute 270 degrees plus or minus a small amount. So some what angles at the poles will approach 0s will approach 180 degrees some of angles at the poles will approach 270 degrees. So this difference will approach therefore, I was writing will be very nearly equal to minus 90 degrees or alternately, I can write it as 270 degrees close to it nowhere near 180 degrees. So in other words there will not be any intersection with the j omega axis for a sufficiently large value of omega. This is a conclusion I can draw by looking at the angle condition and considering the limit as omega tends to infinity or as omega becomes very large. For the origin the calculation was very simple and we found that the origin was a point on the root locus. The problem is there any other point of the root locus on the j omega axis unfortunately, in this method I cannot do anything else but to keep trying and then, may be interpolate between 2 points. For example, going back here if the angle this one is 118 degrees, the angle here is 180 degrees most probably the angle for any point omega here may be between 180 degrees and 180 degrees, 118 and 180 degree, it may be, there is no assure.

Now there is a way of using this method which is harder namely to write down the angle condition for a general point j omega and I will write it and you will how difficult it is going to be. So if I have general point j omega, now I am going to write down expressions for the angles and you will check that there are correct. So the angles made at the two, 0s one of the angles will be tan inverse of omega divided by 2 but not quite that because of the location of that angle to its phi radius minus tan inverse of omega by 2 plus the other angle is tan inverse of omega by 2. This is the other angle subtended at the other 0, as we notice that in this case these 2 angles add up to phi that a coincidence because the two 0s, one of them was at plus 2, the other was at minus 2.

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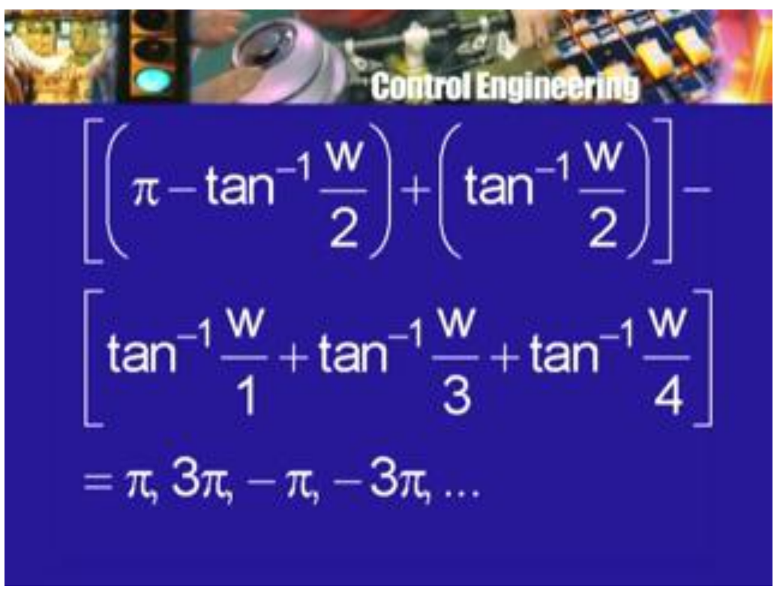

So the sum of 2 angles just adds up to 2 right angle for phi radians. So omega drops out here but that is by luck, in general its not going to be like that. Now minus there are 3 and what are the there are tan inverse of omega divided by 1 minus tan inverse of omega divided by 3 minus tan inverse of omega divided by 4,this whole things should be equal to phi radians or 3 phi radians or minus phi radians and so on. So I have an equation for omega unfortunately, it is not a simple equation it involves the tan inverse function. So, how do I solve it, one way as I told you its just to substitute various value of omega and see whether you get close to a root or a solution. The alternative is to take the tangent of both the sides of this equation, when you do that and then you have to use certain trigonometric formulae. For example, you will have to use to formula for tan of theta 1 plus theta 2, tangent of theta 1 plus theta 2, this tangent of the sum of 2 angles, what is the formula, it is given by tan theta 1 plus tan theta 2 divided by 1 minus tan theta 1, tan theta 2.

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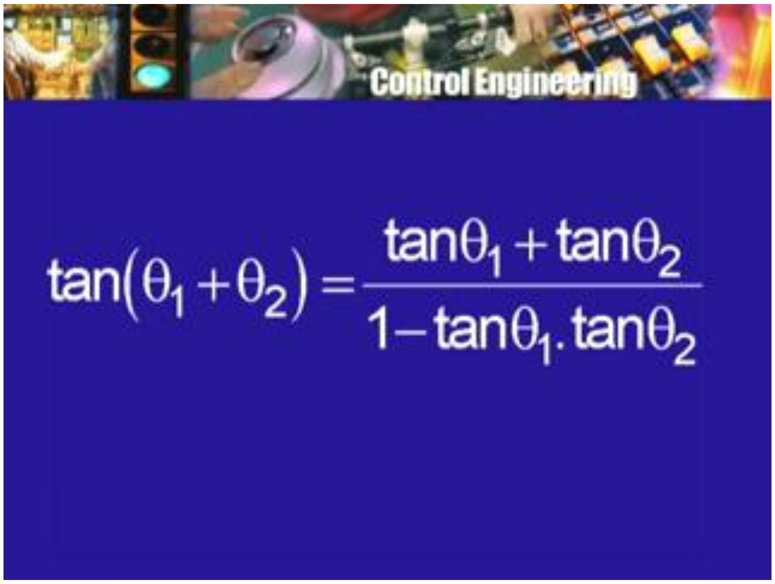

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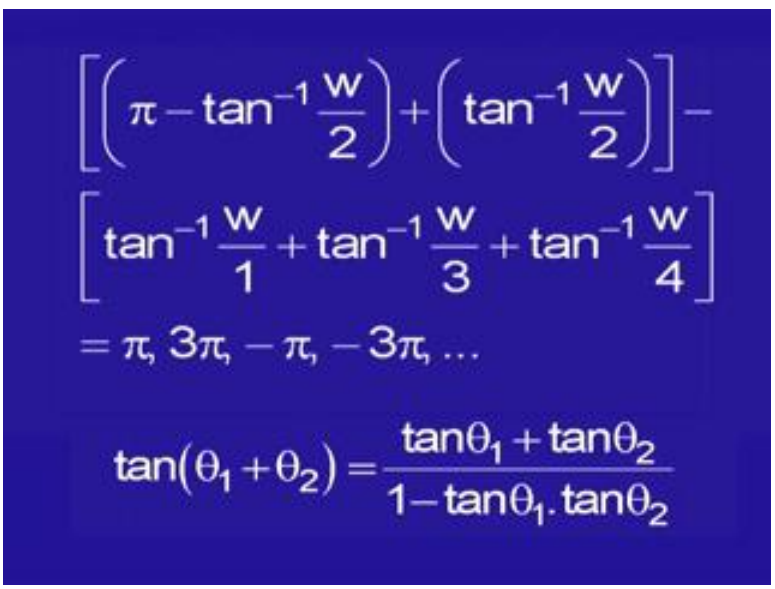

So apply this formula repeatedly to this equation by taking tangent, apply this formula repeatedly, you will get a polynomial equation in omega after simplification and then, you can find the root of that polynomial equation either by trial and errors or by using some computer program. In general, to find out the root of a polynomial there is no alternative but to use a computer program or to use a method of approximation like Newton-Raphson method or Horner's Method and there are so many of them to get an approximation for the root.

So this is the second method for determining imaginary axis intersection or find out whether the root locus branches intersect the j omega axis at all for a non negative valve of k or for a positive valve of k a and then, once in all at point where it intersects finding the gain k a is very simple. On the calculator simply involves calculating distances taking the product of distances to the poles and dividing it by the product of distances to be 0. So this is the second method but as I said I would not recommend it to you because it involves lots of calculations and mostly trial and error kind of thing.

Fortunately for us, we have this third method which is based on a result which was discovered and proved by Routh more than 100 years ago and this involves the construction of a table or sometimes it is called an array of numbers and the array is a called Routh array or a Routh table and then, it can be used also to provide or to enable us to deduced a condition or stability which is sometimes know as the Routh criterion or the Routh test. The method of constructing the array or the table may be called the Routh algorithm, I hope you have come across the word algorithm already in your computer science courses, any regular procedure, step by step procedure for solving a problem is called a algorithm for a problem.

Now the Routh table or array procedure, is a procedure which enables you to find out, whether a given polynomial has any root in the strict right half plane that is whether, it has any roots with a real part which is strictly positive. Now I will illustrate the Routh array by taking a few example, but first a few simple thing is not too difficult to show that if you have a polynomial of some degree in which, one of the coefficients of degrees of course, less than the degree of the polynomial is 0 or as we may say, the corresponding power of s is missing then, this polynomial will be such that it will not have all its roots in the left half plane, it may have a root on the imaginary axis or it may have a root in the right half plane.

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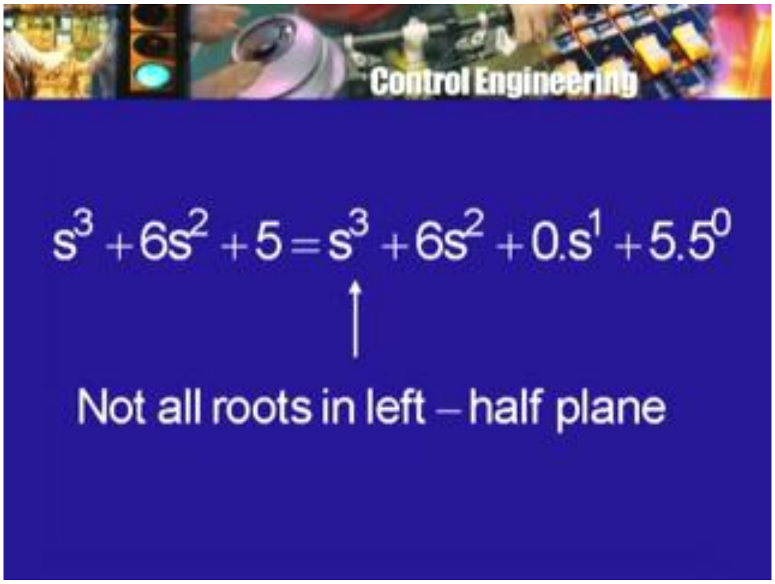

So it is going to be a bad polynomial, from the point of view stability criteria, condition, it is going to be a bad polynomial and that valve of k a, the gain k a is going to be a bad choice of the gain. For example, here is a simple polynomial s cube plus 6 s square plus 5, what is the degree of this polynomial 3 because the highest power of s is of degree 3 s cube. Now this is a polynomial in which of the coefficients is missing or it is the coefficient of that power of s is 0 which power, the first power. I can of course, rewrite this as a s cube plus6 s square plus 0 into s or s s to 1, if you wish plus 5 into I will write even s, s to 0.

So here are power especially, third power, second power, first power and 0 th power, one of coefficient is 0. So this polynomial is bad that is its roots will be such that not all of them will lie strictly in the left half plane. Now for this you do not have to try to factorize it at all. This is surprising but this can be proved that if a coefficient is missing then that polynomial cannot have all of its root in the left half plane. At least one root will be either on the j omega axis or in the right half plane. Another simple result, here I have polynomial s cube plus 6 s square minus 4, s plus 5.

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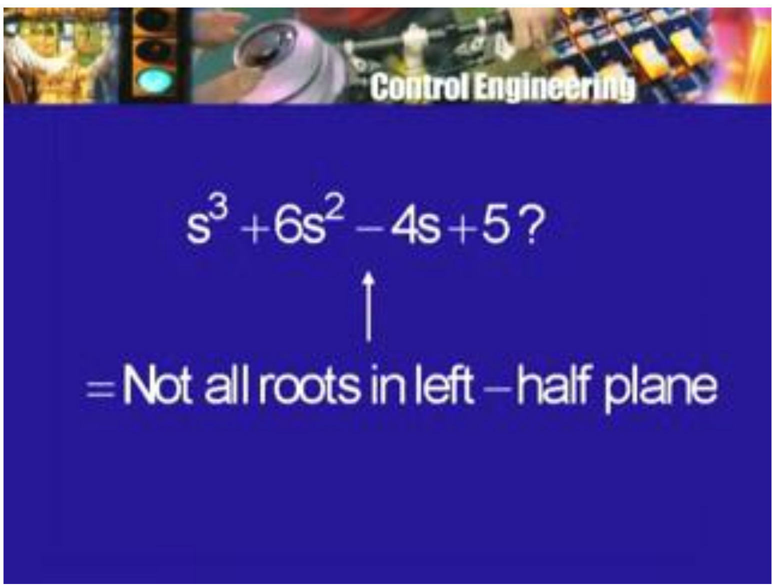

Now, here the power of s is not missing but one of the coefficient is negative while all the coefficient are positive. Well then, such a polynomial is also bad, what about the polynomial minus s cube minus 6 s square minus 4 s minus 5, all of its coefficients are negative. Well, I cannot concluded it is bad. So the rule is if in the coefficient or the polynomial there is change of sign anywhere then that polynomial is bad. If a coefficient is missing then that polynomial is bad. So sometimes you can tell a bad polynomial by what may be called inspection, inspection meaning just check whether a power of s is missing or whether there is a change of sign as you go down the coefficient of the polynomial all right.

So I have the polynomial s cube plus 6 s square plus 4 s plus 5 no power of s is missing all the coefficient are positive. So what can say I right now, nothing I will have to invoke or use some further rules and the Routh array or table will enable us to answer this question but what about the polynomial minus s cube minus 6 s square minus 4 s minus 5. Well, it is just nothing but the negative of and therefore minus 1 into s cube plus 6 s square plus 4 s plus 5. So the roots of this polynomial and the roots of this polynomial are going to be exactly the same. So all the coefficients negative is not necessary bad, we do not have the all the coefficient positive. Of course one can always change the sign of all the coefficient throughout by saying that I am multiplying the polynomial by minus 1 that does not change its roots.

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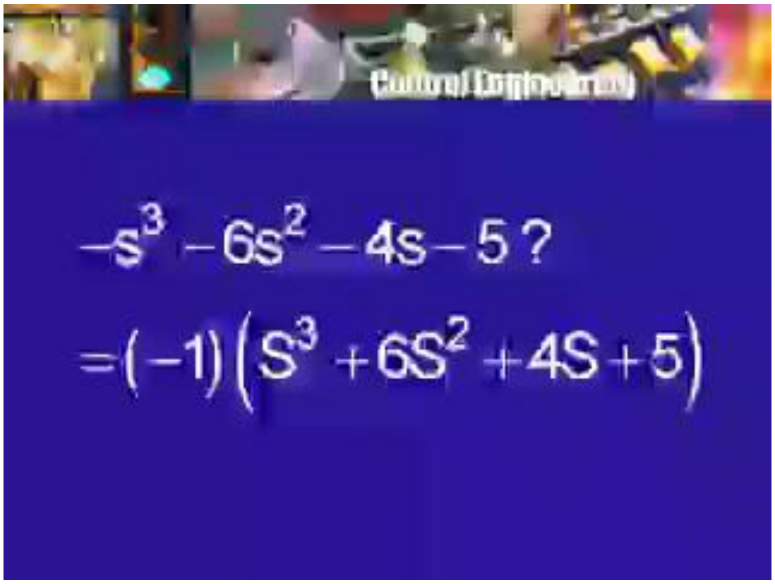

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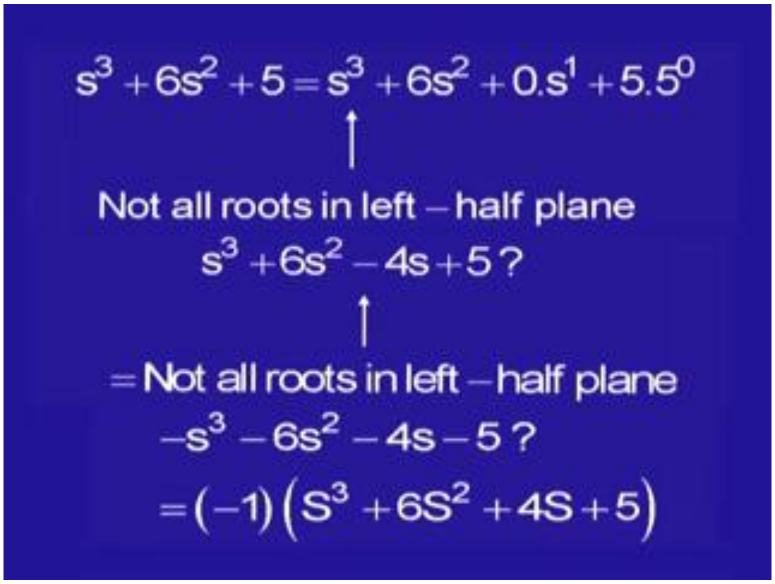

So, if you do not want to see negative numbers, if there all negative you can make them all positive, no problem but there must not any change of sign right. There are some coefficient which was positive and coefficient which was negative if I multiply by minus 1 then, obviously then plus will become minus and the minus will become plus. So there will be some coefficient which is plus and some coefficient which is minus. So the polynomial is bad. Let me look at another polynomial which is 2 s cube plus 12 s square plus 8 s plus 10, this polynomial is different from the first polynomial but are they really terribly different, no because this second polynomial can be written as 2 times the first polynomial.

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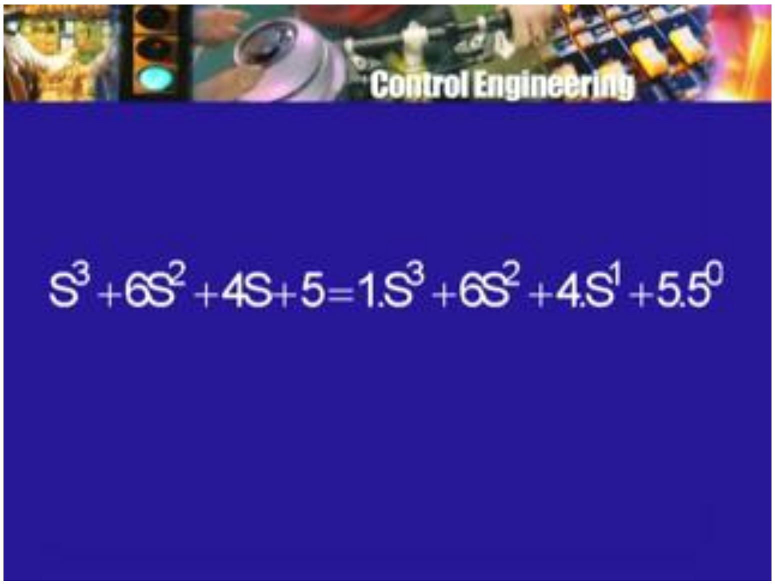

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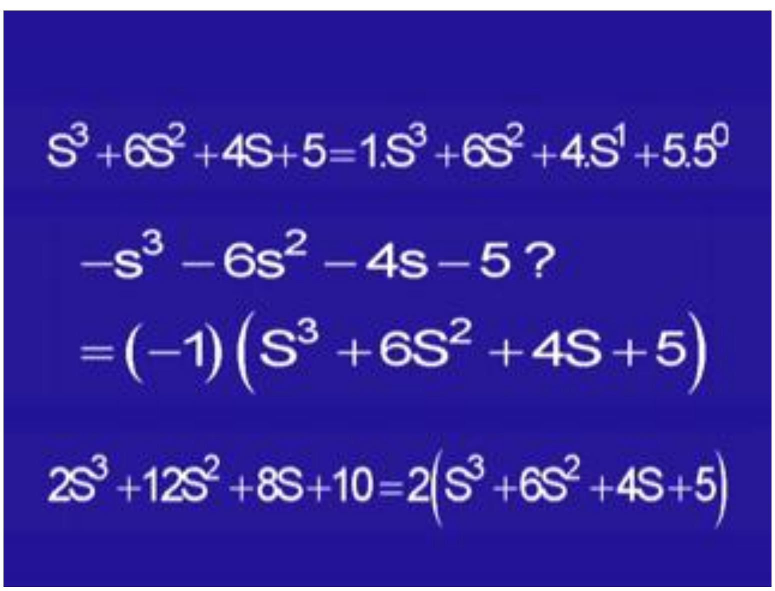

Now, again it does not require too much thinking to see that the roots of first the polynomial and this polynomial are exactly the same because wherever this is 0, the other one is 2 times 0 and therefore 0, wherever this one is 0 so 2 times something is 0. So that something must be 0 and therefore this polynomial is 0. So, what is the upshot of this the upshot of this is that whenever you have a polynomial and you are going to look at or you want to study where the roots of this polynomial may be using some of these tests. You can look at the highest degree coefficient and change it to 1 that is multiply the polynomial by a number either positive number or a negative number to make the coefficient of the highest power 1. The coefficient of the highest power is called the leading coefficient and so, I can always change the polynomial into a polynomial whose leading coefficient is equal to1 and therefore, I do not even write it, I do not write 1 into s cube, I simply write s cube. It is convenient to do this because there at least one of the coefficient is a standard coefficient namely 1 such a polynomial is also called a monic polynomial. It is not necessary that your polynomial should be a monic polynomial but its useful to make it monic because of a reason that you will see and it is very easy to make it monic, all you have to do is multiply it by suitable number, all right.

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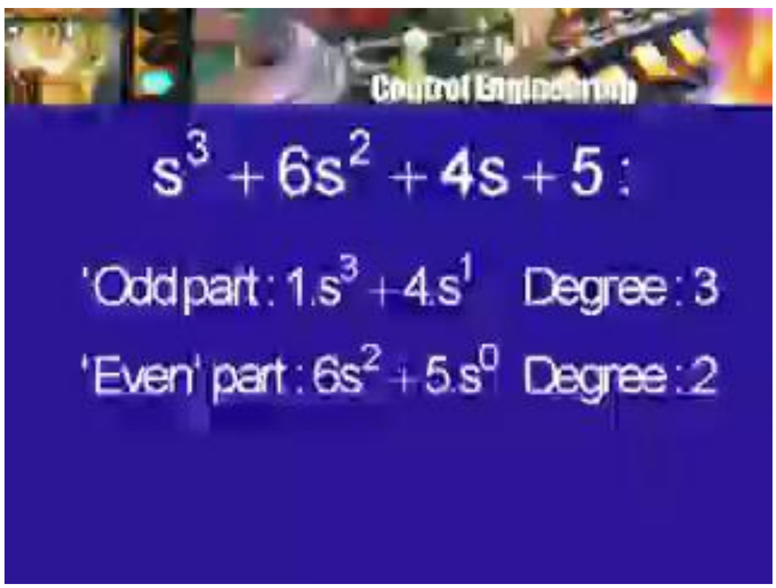

So, let me look at the polynomial once again, s cube plus 6 s square plus 4 s plus 5. Now some of you may try to guess the factors of this polynomial but they are not going to be easy to guess. Here are some simple rules for example there is what is called the remainder theorem, if you use the remainder theorem you can find out whether a particular number is or is not a root of the polynomial. But in fact, you do not have to use the remainder theorem, all you have to do is replace s by any number and check whether this thing comes out to be 0, then you can see if I replace s by plus 1 that is not going to be 0.

So 1 is not a root of this polynomial, in fact any positive real number will not be a root of this polynomial because all of this will added up to a positive number but there could be roots which are negative real numbers or there could be roots which are complex. It is not easy to figure out at all from this polynomial, what its roots may be. Of course, this is a very special polynomial it is what is called a cubic and there is even a formula for the roots of a cubic but I would advice you not to use that formula for one thing, the formula is fairly difficult to remember and secondly the formula may require the use of complex arithmetic, it is better to avoid that as far as possible. Fortunately, the Routh table method gives us a method of answering this question. It does not tell you, what is the root of polynomial will be but it can tell you, where the roots of the polynomial may or may not be. I will explain as I go along, now the first step in the application of the Routh algorithm. Take the polynomial then, the books will of course tell you do this this this but let me tell you what is that you are doing and then, may be if you spend a little time you can understand why one is doing that.

Now this polynomial highest degrees term is s cube, s cube is an odd power of s is the third power of s, it is an odd power is there any other odd power of s in the polynomial, yes 4 s, 4 s is a term which is an odd power of s. So think of these 2 together s cube plus 4 s, s cube is an odd power s or s to 1 is an odd power. Of course, they are multiplied by some coefficients 1 here, 4 here. So this part that we have taken out from the whole polynomial is called the odd part of the given polynomial. Now what remains, what remains is the term 6 s squared, now what is the power of s here, 2, 2 is an even number, the other term that remains is 5 or I can write is as 5 into s, s tends to 0 by the law of indices s, s to 0 is 1 and we treat as an even number then, of course you should be able to answer, why 0 is treated as an even number and not as an odd number. Now if you take this 2 terms together then, this is said to be the even part of the polynomial.

So given the polynomial think of splitting it into its 2 parts, the odd and even part. The odd part is here s cube 4 s, the even part is here 6 s square plus 5. The odd part has the highest power 3 s to the 3. So the degree of the odd part in this case is 3, what about the even part, even part highest power is s to the 2. So the degree of the even part is 2. So in this case, the degree of the odd part is greater than the degree of the even part. Now from the polynomial, how do you pick out the odd part because in the procedure, once you have understood it is not necessary to write down the 2 parts separately.

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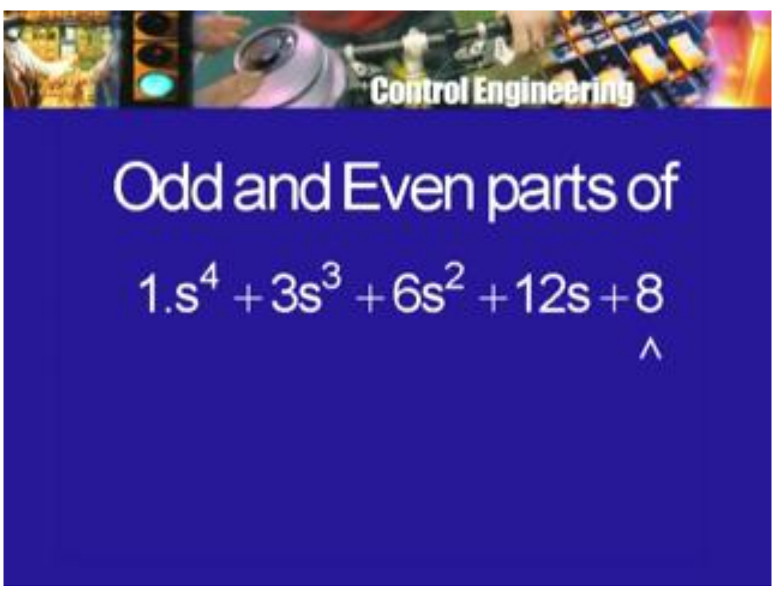

So, I start with the highest power that is odd, that is cube then I leave one term which is 1 degrees less go to next term or the next odd power of s, this is s to the 1. So I will get 1 s cube plus 4 s that will be the odd part, what will be the even part start with the highest even power that is this 6 s square. So 6 s square skip 1 term and go to the next lower even power of s, in this case s to the 0. So I will get 6 s square plus 5 into s to the 0 and in this case because the degree of the polynomial is an odd number 3, the odd part is of degrees 3, the even part is of degrees 2 and therefore the degree of the odd part is greater than the degree of the even part. Now, I can take an example in which the situation may be reversed and so, let us take a look at that example.

So here is a polynomial which we will make use of later on, I have s to the 4 plus s cube plus 6 s square plus 12 s plus 8, what is the degree of this polynomial? The degree of this polynomial is 4 because the highest power of s is s to the 4, is it monic, yes because its coefficient is 1. Now what is its even part and what is its odd part. Well, start with the highest power which is even s to the 4, the next lower even power is 6 s square, the next lower even power is 8. So the even part will be s 4 plus 6 s square plus 8, what about the odd part, start with the highest odd power, it is 3 s cube, in the next lower odd power that is 12 s. So the odd part is 3 s cube plus 12 s. So in the Routh test or Routh array or table construction, what we are going to do is we are going to split the polynomial into its odd and even part that is the first step but we are going to do it in such a way that we will start constructing the table with this odd part and the even part.

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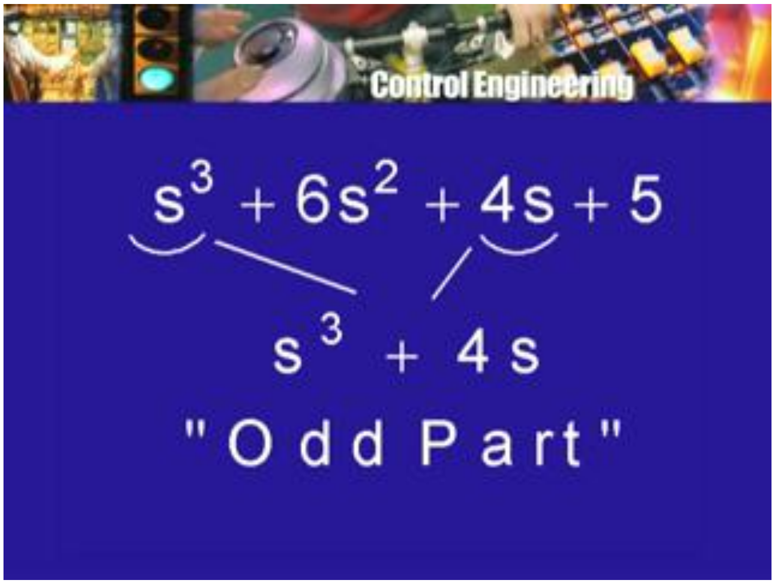

So for the polynomial that I started with namely s cube plus 6 s square plus 4 s plus 5 starting with the highest power, I will get s cube plus 4 s plus and so, I will starting writing my table or array in this fashion. Since, the highest power is s cube, I am going to write here s cube on the left hand side then, I am going to put 2 dots here. This is the the English punctuation symbol, whereas colon instead of that of course, you can draw a vertical line you finish. So I am going to write here s cube colon and then, what I am going to write are not the 2 terms s cube and 4 s.

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So the polynomial is actually s cube for 4 s but what I am going to write is simply s cube, the coefficient of the s cube power which is one and the coefficient of the next lower odd power which is 4. So this is the first row of my table that I am going to write s cube here 1 and 4. Now the text book may tell you do it this way but if you do it this way without understanding what you are doing then, it is not really worth learning the whole thing, what you are doing is from the polynomial you have separated out its odd part and this is just a compact way of writing the odd part. The s 3 tells you that this one is multiplied by a s 3 and the next number 4 tells you that 4 multiplies the next lower odd power of s namely s to the 1.

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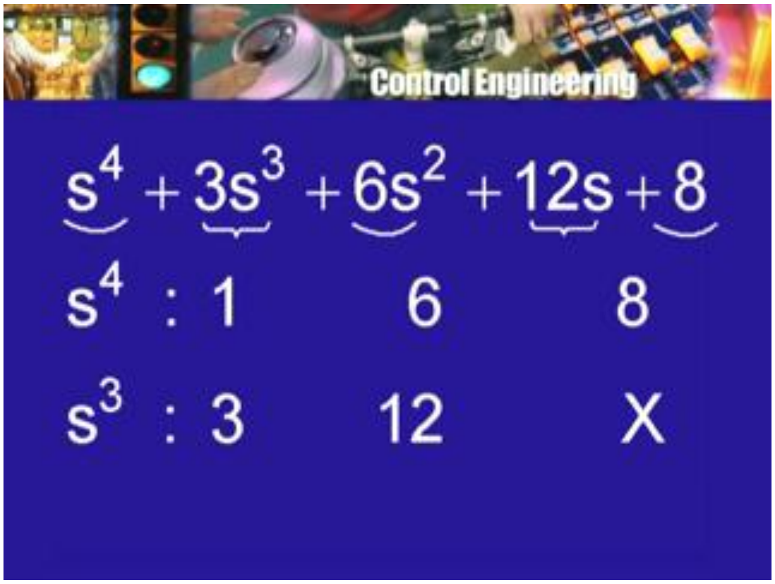

So s cube 1 and 4 from that I can conclude that the odd part is 1 into s cube plus 4 into s. Now the next row will be the even part, the even part therefore I will write down s squared here and then, write down the coefficient. The coefficient of s squared is 6, so I will write down that coefficient here and the next lower even power of s the coefficient is 5. So I will write down 5 here. So this is the way to start of the table and the first 2 rows of the table are written like this. In other words, the first 2 rows of the table give you in a compact way, the odd part and the even of the original polynomial because here the odd part was at higher degree, so I wrote it first, the even part was of lower degree, so I wrote it next. For the other example, that we are looking at it will be just the reverse.

So if the polynomial is s to the 4 plus 3 s cube plus 6 s square plus 12 s plus 8 then, how do I begin the highest power is s to the 4. So I will write down s to the 4 here, colon the coefficient of the even power s to the 4, one it is also monic, next 1 is 6 and next 1 is 8. So these are 3 coefficient in the even part of the polynomial the even part is 1 into s to the 4 plus 6 s square plus 8. Below that, I will write down the odd part of the polynomial the odd parts start with 3 s cube.
So I have written s cube here, I will write the coefficient of s cube 3, the next lower power or power of s is s to the1, the coefficient is 12 and what should I write here well, nothing because there is no further power of s.

So it is better to put a cross or a dash. There is no entry to be filled here, do not write a 0, there is no power of s here, there is no s to the minus 1 because we have a polynomial, we do not talk about s to the minus 1. So do not write a 0 either put a cross or put or dash but do not forget to put a cross or a dash because then, you know that we are not forgotten to write something. There is no entry here the polynomial therefore I can read now is looking at this s to the 4, 1 into s to the 4 plus 3 s cube plus 6 s square plus 12 s plus 8 or if I go back to the earlier example, s cube is the power here. So 1 into s cube plus 6 s square plus 4 s plus 5.

So in this case, the odd part appears first followed by the even part because the whole polynomial is of odd degree. In this case the even part appears first and then, the odd part because the polynomial is of even degree. You do not have to remember this, if you understand what going on, you do not have to really remember a rule that if it is odd do this, if it is even do this. Look at the polynomial it requires very little practice to see the odd part that is the odd powers of s and the even part that is the even power of s and start writing them down. The only thing is the higher degrees polynomial should appear first because that is the way, I am going to tell you the rule okay.

So the first step in the constructional as the Routh array of the table is to write down these 2 arrays correctly. Let us take example that I mentioned earlier in which one of the coefficients was equal to 0, what happens in that case. Of course, I told that it is a bad polynomial and Routh’s rule tells you also something about it in that case, the polynomial was s cube plus 6 s square pus 5. So let me split it into its odd part and even part, degrees is odd. So I will start with s cube what are power of s cube, s cube is 1, what are the powers of s, odd power s cube as coefficient 1 s to the 1. Now, I cannot put a cross or a dash here I have put 0 the coefficient of s to the 1 is 0. So s cube is 1 s 2 the 1 is 0, what about the even part s square is 6 and the constant term or s to the 0 power is 5.

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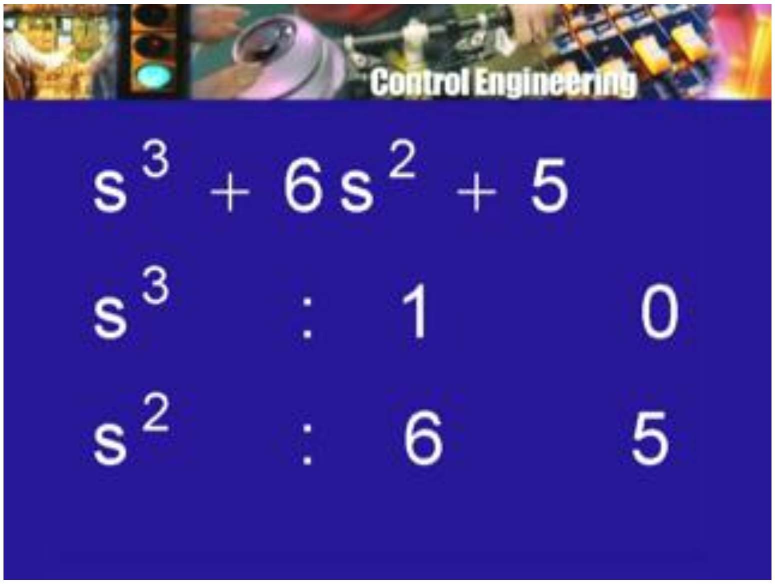

So these will be first 2 rows for this polynomial and if the polynomial as a negative coefficient, you can still write down the Routh table but as I told you, the polynomial can be immediately concluded to the bad but how bad, if you want to find that out using the Routh algorithm then, you should work out the Routh table.

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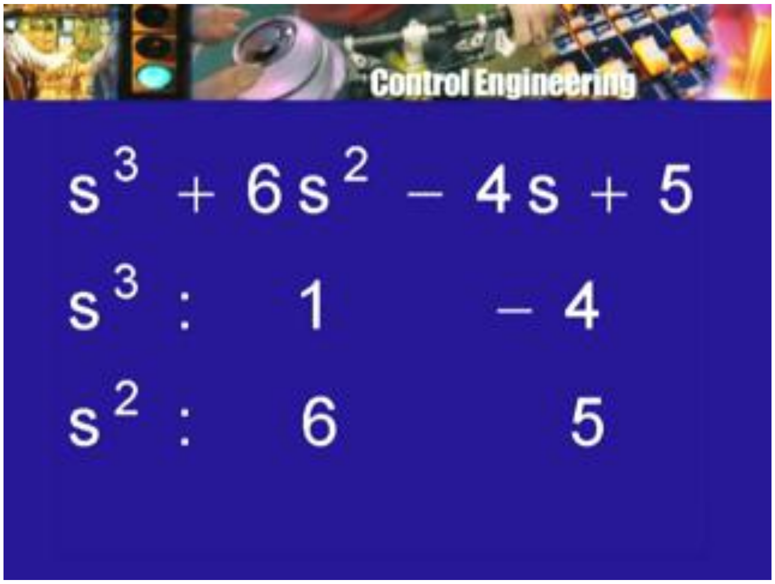

So I have s cube plus 6 s square minus 4 s plus 5. So where do I start, start with s cube, so s cube coefficient is 1, s to 1 coefficient is minus 4, next the even powers starting with the s square, s square coefficient is 6 constant term is 5. So this is how I start my table. The first 2 rows of the table therefore correspond to the odd and even part of the given polynomial. So, we are splitting the polynomial into its odd part and its even part and we start the table by putting down these 2 polynomials in a compact form like this. Now, the next step is a little involved but with practice one can get used to it without any difficultly. Now, the next steps involves something like the following and I have illustrated it not by taking actual numbers but by showing 2 rows let say, I have some power of s.

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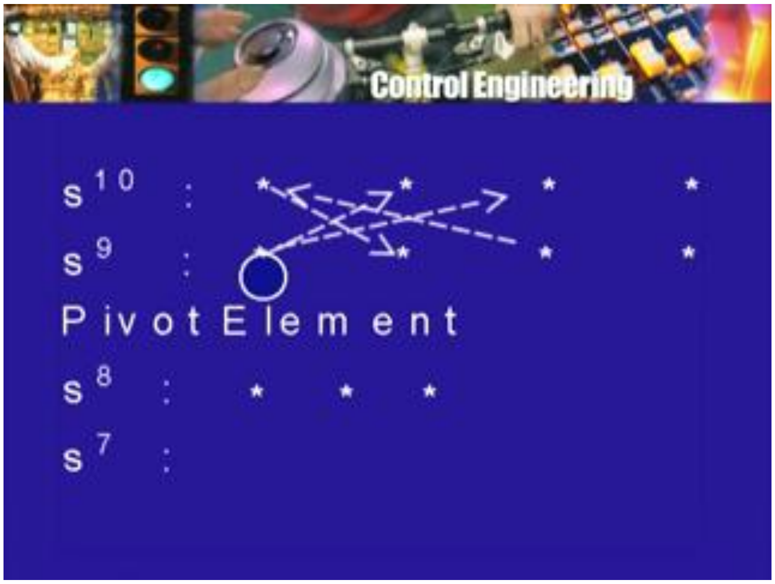

Let say it is 10, so I have some coefficients here. So I am going to put down x, x, x, x meaning that there are some numbers not all equal necessarily, s to the10, s to the 8, 6, 4, 2 and 0. So whatever there are the next power of lower power is the odd power s to 9 and there will be some numbers once again, I am going to put x, x, x that does not mean, there are all equal, there are some unknown entries at the movement. Now from these 2 rows I am going to construct a new row, I am going to give it a row heading which is the next lower power of s.

So 10 here, 9 here, so I am going to level it, s to the 8 and I am going to write down the entries of this row by following a particular rule but before I tell you, what the rule is. If these 2 are really polynomials, this is an even polynomial and this is an odd polynomial and if I write a new row and this new row the heading is s to the 8, then what would this row mean or what would it stand for. It would stand for an even polynomial of degree at most 8 an even polynomial of degrees at most 8. So in other words from the even polynomial of degree 10 and even polynomial of degree 9, by following a certain rule I am going to obtain an even polynomial of degree 8 and of course the procedure does not stop here. From these 2 rows, I obtained the next one then, what I do is from the 2 rows that I have the last 2 rows that I have, I calculate the next one.

So from that I will calculate s to the 7 row and I will keep on doing it. We will take a numerical example to illustrate this but first from this 2 rows, how do I obtain the next row. So the basic idea of the Routh array is from 2 appropriate rows generate the next row, from 2 rows whose coefficients are given calculate the next row. Now what is the rule for calculating coefficients. As I told you the first time you see it it may appear to be very difficult but with practice, you can get use to it and you can apply it without difficulty. Now there are these 2 rows and I want to find out the next row. First of all in the second row, the first entry that occurs here is going to play a different role in what we are going to do.

So, this element we will call the pivot element. So think of this as a very crucial element, think of it as a pivot element, a pivotal person is somebody who is very important in an organizations. So this element plays a very important role. So think of it as the pivot element, now what you do you do the following and you can see that when you write the table properly, the rule can be understood without any difficultly. From the pivot element, go like this easier shown than said that is go to upper row but go in the towards the right. So it is this element across it this way, take the product of this 2 entries.

So that way I like to remember it is, the pivot element into or multiplied by this element, take that product, from that product subtract this product. The product of this 2 elements which are located crosswise like this. So, this product minus this product calculate that difference and divided by the pivot element that will reach the next element, first element in the next row. So this element in the next row will be take the pivot multiply it this way take that product from that subtract this product and divide by this pivot element itself that is the first entry in the next row, what about the second entry in the next row. For the second entry do a similar thing, start with the pivot element but now go up but go one step further to the right.

So this number into this number minus what minus again one step further to the right into this number. So this minus into this minus, this into this divided by that gives me the next entry of the third row or the next row. Now, keep on doing this till you run out of such products to form. So, let us take an example and illustrate. So, let me take the example, s cube plus 6 s square plus 4 s plus 5. So here is the first row s cube 1, 4, s square, 6, 5 there are no further entries. So I am not even writing dash or cross.

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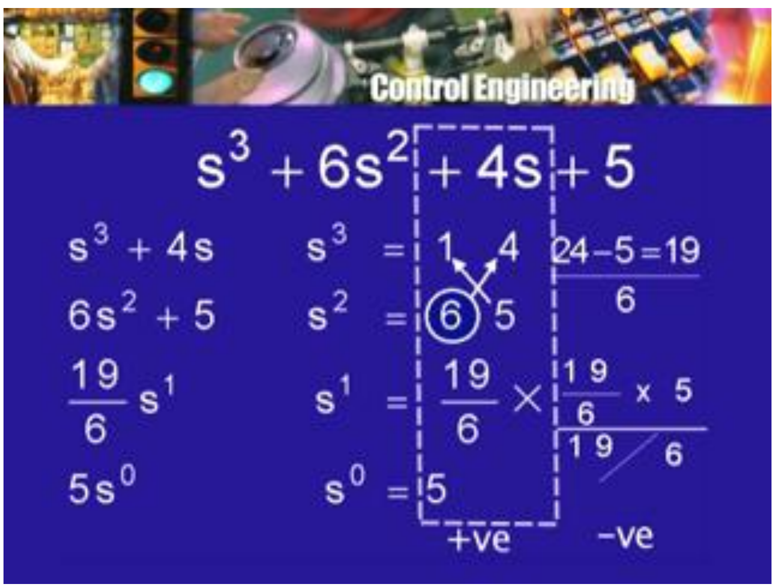

Now, the next row the heading is going to be s to 1 and I want to find out the entries. So this is my pivot element, so 6 times 4, how much is that 24 minus 5 times 1 that is minus 5. So the difference is 19 divided that by the pivot element, what is the pivot element 6. So the number here is 19 by 6. So that is the first entry here. Now, what will the second entry well, none because there is no entry here to go crosswise there is nothing, do not think of it as 0, there is nothing. So even if there is nothing here, so there is no more calculation to be done and am finished and that is right because s to the 1 row, what is polynomial, it is a polynomial whose highest power is s to the 1 and is there go our polynomial, is there going to be any other term, no because s is the smallest odd power of s that you can think of. So that is it, if you want I can put a dash here that there is no number to be filled in here all right.

So my next row calculations are over but my table calculations are not over because I have the power s to the 1. So I will one step further, s to the 0 and how many coefficient will it have only one coefficient because the polynomial of degrees 0 is just 1 constant. So, I have to find out this number here, same rule now. But now instead of the first 2 rows, I am now to look at the last 2 rows that I have obtained this one and this one. Now what is my pivot element it is not 6 anymore, it is this, this is my pivot element right, so the same rule. Now this pivot element into this number minus this number into this number but there is no number here.

So minus nothing, so in other words I have 19 by 6 into 5 that is the product from that nothing is to be subtracted divided by 19 by 6. So, what is the number here that number is just 5. From the even polynomial corresponding to this row and the odd polynomial corresponding to this row, I have obtained an even polynomial corresponding to this row and now, I have reached the end of table because the power is s to the 0 constant, there is nothing less than that the lowest degree in a polynomial is the 0th degree term, or the constant term.

So, now table ends here, so it is as simple as this of course, the degree of polynomial was 3 therefore the work was not really very much involved but what if the degree is higher then, I will have to carry out my calculations for a larger number of steps at this point. Let me just tell you very quickly what the Routh criterion is that is after this calculations have been made Routh's this theorem tells you a very powerful result about this polynomial. Have I factorized the polynomial, no I have not even guessed the factors is s plus 1 a factor, I do not know, s plus 2, I do not know. Have I found out any root for the polynomial, no I have not factorized the polynomial, I have not found out any root of the polynomial, I have not use my pocket calculator for that purpose at all.

I have not used any computer program, all I have done is written coefficient of the polynomial and found out some new numbers while using this cross multiply subtract and divide, kind of arrangement and I kept on getting numbers. Then I filled up the table like this, now Routh tell you that what you should do is look at the first column of this table. The first column of this table has entries starting from the top 1, 6 ,19 by 6 and 5. There are all positive, more generally they could be all negative but if they all have the same sign and that means they are all positive, no number is 0 or all there are all negative, no number is 0 in the first column. Then, this polynomial will have all the roots strictly in the left half plane that is if the root is real, it will be negative ,if the root is complex, it will have a negative real part and there cannot be a purely imaginary root of this polynomial, s cube plus 6 s square plus 5.

So really how wonderful is it, without factorizing the polynomial, without trying to find out any root by trial an error, by computing the table in a particular way and by looking at entries in a column of the table and merely by looking at their signs, are they all positive or are they all negative. We are able to say something about the location of the roots of that polynomial namely all the roots of the polynomial are strictly in the left half plane and therefore, this polynomial is a good polynomial. Now this is really a result which the first time you look at it, looks unbelievable that without findings the roots of a polynomial, you are able to say something about the roots of the polynomial, thank you.