Control Engineering
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Lecture - 32

To determine whether a particular complex number s or a point in the complex plane is the root of the characteristic polynomial or characteristic equation or the point is on a root locus or a branch of the root locus. We saw that there is a condition or a criterion known as the argument or the angle condition or criterion which can be used and which, if you you remember states that a point s is on the root locus, if the sum of the angles subtended by this point at the poles minus sum of the angles subtended by the same point at all the 0s, this difference of the 2 sums is either 180 degrees or phi radians or plus minus 2 k phi radians that is plus minus a multiple of 360 degrees. This is called the angle condition.

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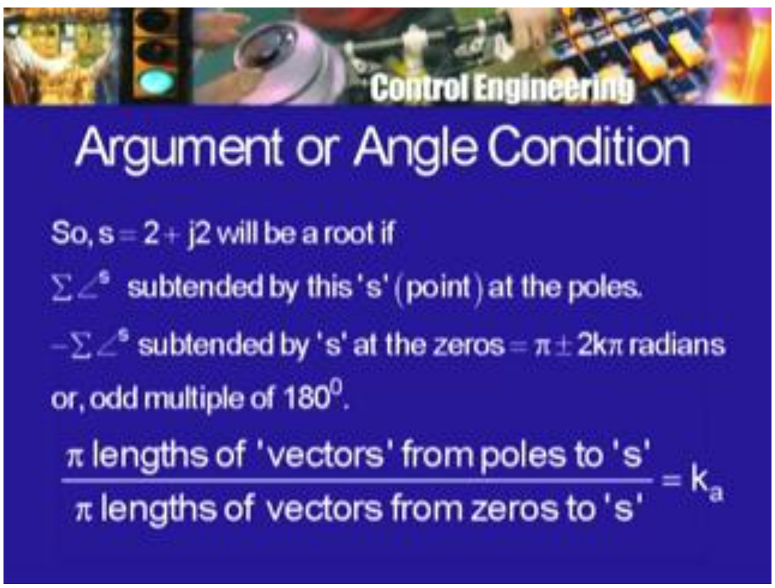

So whether, a given complex number s of the corresponding point is on the root locus or not depends on whether this condition is satisfied or not and I had asked you, for our particular problem to find out whether the point s equal to 2 plus j 2, can be on the root locus. To determine if the point s is on the root locus then to determine the value of the gain k or which it will be a root of the characteristic polynomial, we use the second fact namely the products of the lengths of vectors drawn from the poles to s divided by the product of lengths of vectors drawn from the 0s to s, this equals the gain k a.

Now the angle condition or criterion may appear to be a little complicated and at first glance it seems so, but let me remind you once again that the subject of control theory in particular and electrical engineering, in general does make use of quite a few mathematical ideas or concept. So if one wants to be in any branch of electrical engineering, even if it involves a lot of hardware for understanding electrical engineering, its principles, it is necessary to be familiar with some of the mathematical concepts or techniques more so in the case of control theory.

For example, by this time it should be absolutely clear to you that differential equations atleast the simpler ones, linear differential equations with constant coefficients, similarly Laplace transformation, complex plane, complex arithmetic, pole 0 diagrams, transfer function. These are all concepts which are going to be extremely useful in dealing with control theory problems. However, that particular rule that I have mention namely a point on the real axis belongs to the root locus under such and such condition that can be obtained more simply without using the angle condition and it goes like this, one I mentioning this because you should know various alternative ways of getting at results.

So, here was our example, we had some poles minus 1 minus 3 minus 4, two 0 s, 2 and minus 2 and we are trying to determine, whether a given point is on the root locus or not. Now, if one looks at it from the same view point as the angle condition. Remember, that we have written it as per our particular example s plus 1 into s plus 3 into s plus 4 divided by s plus 2 into s minus 2, this was going to be equal to minus k a and k a being positive, this was to be negative.

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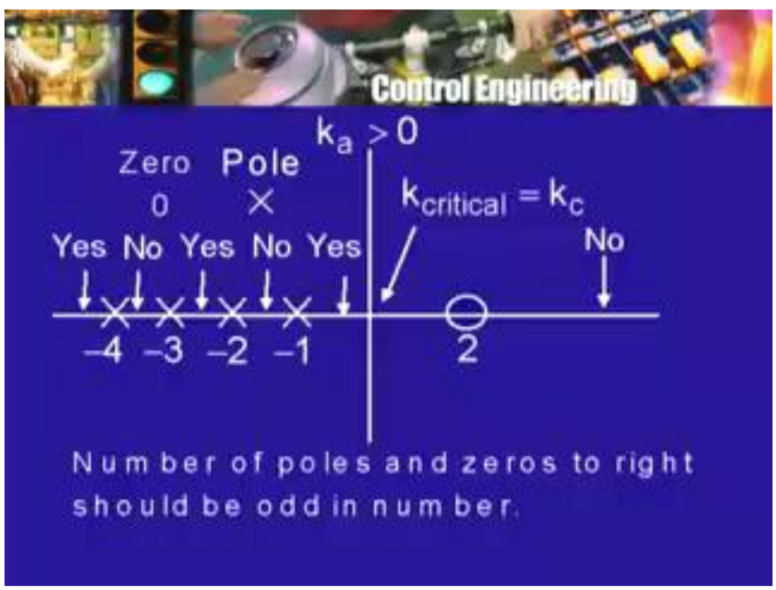

So the point s was on the root locus, if this was true now instead of thinking of s as a complex because we are going to look at portions of the real axis which may belong to the root locus. So, in other words we are only going to look at s which are real number points laying on the real axis and we want to determine whether this condition is satisfied. Now look at the right hand the right hand side is a negative number. So what this condition is says is that this product should be a negative number right.

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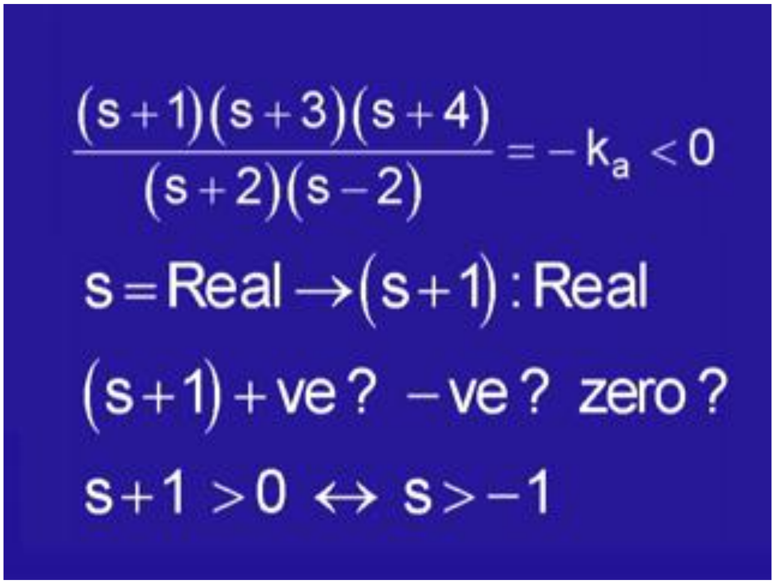

Now, when s is a real number each one of these s plus 1, s plus 3, s plus 4, s plus 2, s minus 2, each one of them is a real number only when s is complex like for our problem s equal to 2 plus j 2. Then, each one of these factors becomes a complex number but if s is real number then each one of these factors s plus 1 etcetera are all real numbers, if the real numbers they can be either positive or negative or 0. So, suppose we choose s such that number of factors is equal to 0, in other words we do not choose s as either a pole or a 0. So on the real axis there are some poles and 0s, we will exclude them, we will choose points other than the poles and 0s on the real axis. In that case each one of these factors is either positive real number or a negative real number, fine.

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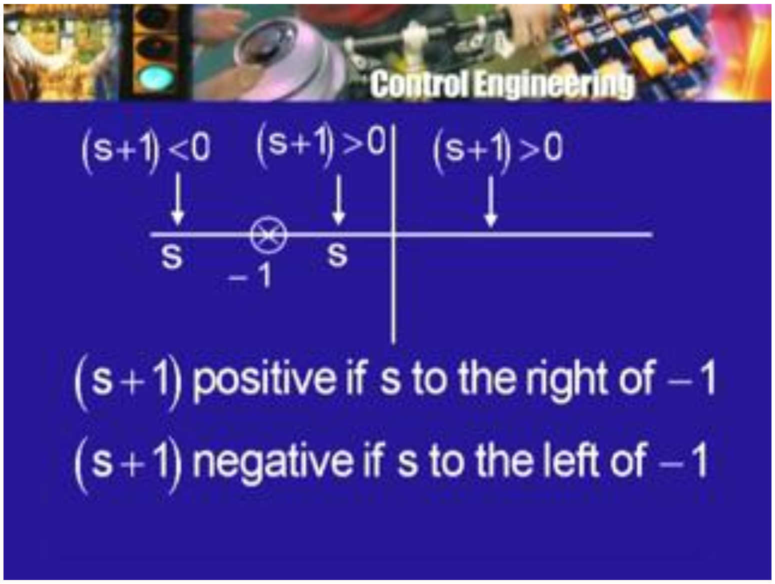

Now s plus 1, look at the factor s plus 1, when will s plus 1 be positive, s plus 1 will be and s plus1 greater than 0 or s is greater than minus 1. So, if I show on the real axis here is the point minus 1 which in our problem was a pole then for what s on the real axis will s plus 1 be positive, s which are to it is right here, here or anywhere, s plus 1 is positive, s greater than minus 1 on the real axis. So these are points to the right of minus 1. So s plus 1 is positive for s greater than minus 1, s plus 3 is positive for s greater than minus 3, s plus 4 is positive for s greater than minus 4. Similarly, for the 2 factors in the denominator.

So the signs of each one of these factors will depend on whether the point s is to the right of the particular pole or 0. This is the first observation now secondly if you have complex poles then as I mentioned several times they occur in conjugate pairs. For example, if 2 plus j 2 is a pole not in our problem but suppose in some situation then, we have a factor s minus 2 plus j 2 and we have another factor s minus conjugate of this which is 2 minus j 2. Now this can be rewritten as s plus 2 plus j 2 into s plus 2 minus j 2. Now it is easy to see that this thing is s plus 2 square plus 4. Now for s real, s plus 2 is real, s plus 2 square is real positive or 0, s plus 2 square plus 4 is greater than 0.

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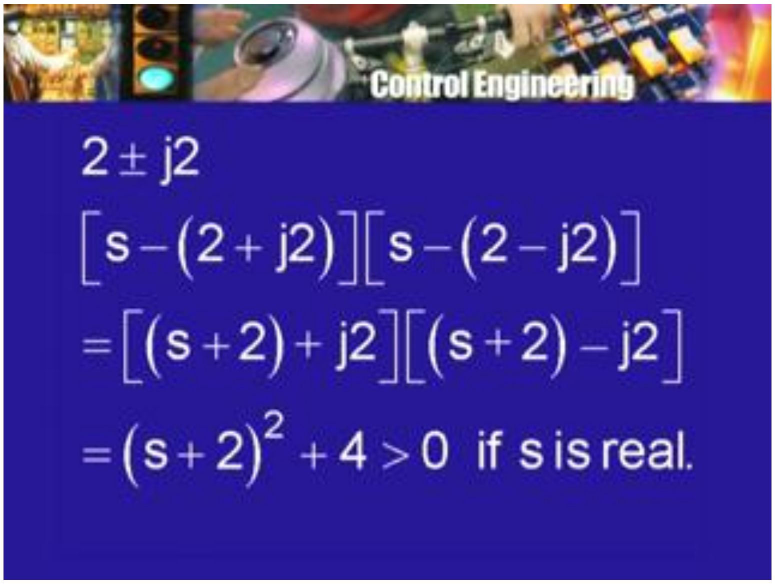

So, when you have a pair of complex conjugate roots like this, the corresponding 2 factors for s real will always be positive and you can verify that this holds no matter, where the poles are or where the 2 roots are the conjugate roots are, whether they are in the left half plane or right half plane or on the j omega axis. So long as they are not on the real axis and therefore, they are truly complex the product of the corresponding 2 factors is always positive, all right. Now let us go back to our example, so here we have a number of factors in the numerator, a number of factors in the denominator. The factors which correspond to the complex poles and complex 0s, we need not worry about because their product will always be positive no matter where the point s is.

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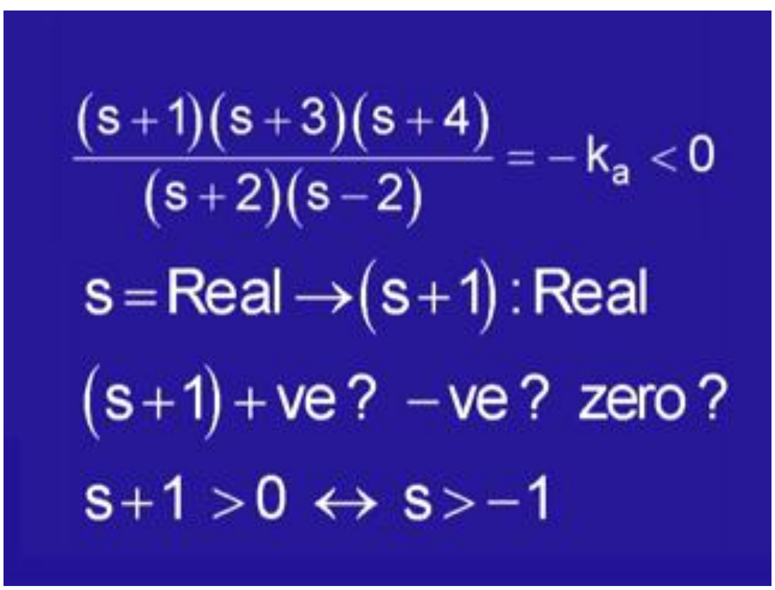

So now, what will be sign of the numerator s plus 1, s plus 3 into s plus 4, each one of the factors is either plus or minus and it is plus for points s which are to the right of the corresponding root s plus 1 is positive for s to the right of minus 1, s plus 3 is positive for s to the right of minus 3. Similarly for s plus 4, s plus 2 is in the denominator that is positive for s greater than minus 2, s minus 2 is positive for s greater than 2. So from this it is easy to see that if we have a point which is to the right of all of them, here was the point we looked at yesterday, then each one of the factors s minus 2, s plus 1, s plus 2, s plus 3, s plus 4, each one of them is positive.

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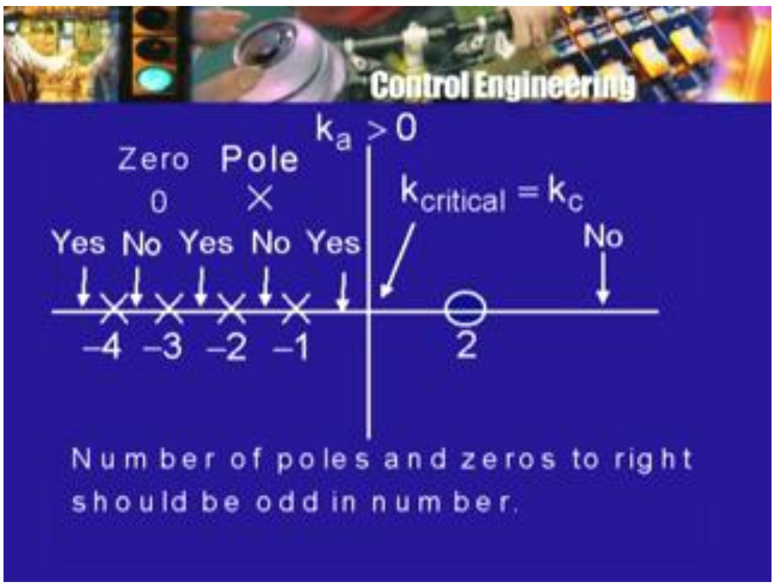

So, when we take the products and divide it by the other product, this whole thing on the left hand side will be positive but for the point to be on root locus, it has to be equal to minus k a or negative, therefore this number or that point cannot be on the root locus.
So this point cannot be on the root locus, now it is so happens that the location of the point is such that the number of poles and 0s to its right is 0 and we treat 0 as an even number so it is not an odd number. Now, let us move to this region between these 2 points. Now, what is this situation there is a factor now, corresponding to this s plus 2, s minus 2 which is positive, all the other factors are negative.

Now, this is where you have to look at the rule and get the correct conclusion that if the number of poles and 0s to the right is odd number then, the point will belong to the root locus. I will leave it to you to work it from this point of view and make sure that the rule as we have stated is correct. Evans, when he stated the rule looked at the angle condition in fact, he gave a number of additional rules many of which make use of the angle condition and so the angle condition is very important but you do not always have to use that angle condition. You can use for some simple facts like position of points on the real axis, you can use or some simpler arguments and I have given you an example of one such simpler argument.

I stated that the root locus starts at the poles, it end the 0s but how does it proceed from the pole to the 0s. But I had concluded yesterday and I showed it to you that this particular branch of the root locus starts at minus 1, ends at 2, but goes along the real axis as I have shown. Now how do I know that so for that there is a rule which comes about by the use of the angle condition and that rule concerns, what is called angle of departure.

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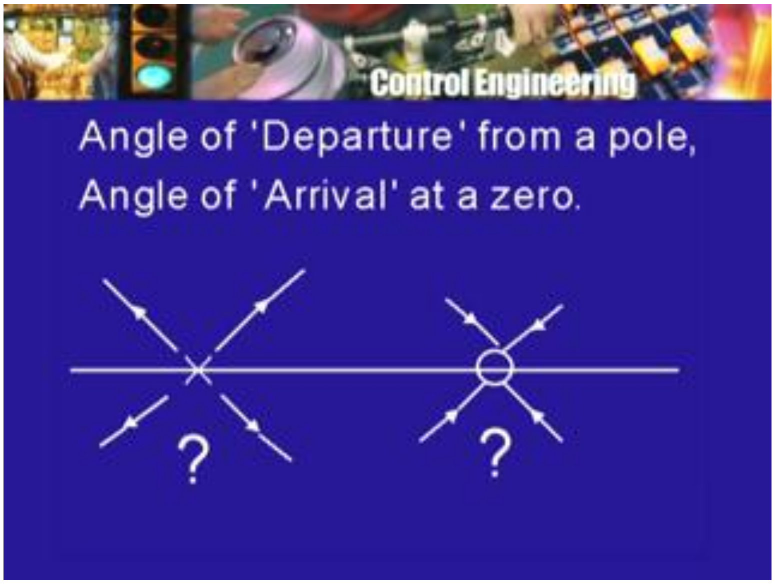

Let us say here is pole, so I know that the root locus branch will start from there that is for k very small, there will a root very close to this point. So there will be a root locus branch which starts at that point but how does it proceed, will it go like this, will it go like this, like this, like this, like this. For very small k, it can be approximated by the tangent to the root locus. So it can be thought of as a small straight line but which direction, in which direction does the root locus branch take off, that direction is determined by the corresponding angle which is called the angle of departure, angle of departure from the poles of the transfer function or the poles on the pole 0 diagram. Likewise, there will be angle of arrival at the 0s for example here is the 0 that we added 2.

Now, we know that for very large k, there will be points on the root locus which are closed to this location of the 0 and therefore, as k increases beyond any limit that is as we say k tends to infinity, the root locus branch will end up being at the 0. But in which direction will, it approach or arrive at the 0. Now that direction or the corresponding angle is called the angle of arrival. Now using the angle condition namely, sum of angles at the pole minus sum of angle at the 0s is an odd multiple of phi or 180 degrees using that condition for points very close to the pole or to the 0 apply that condition you can find out the angle of the departure and the angle of arrival at the poles and 0s respectively.

It will take some time to go through the derivation of the rule but you can look it up and it is a fairly simple rule about the angle of departure and the angel of arrival at the poles and 0s. It does involve some calculation of angles made by various vectors that one has to draw but that calculation is not a very complicated calculation, it is essentially you draw vectors and find out the angles made by them with the positive real axis, one can either measure them, if one plots the poles 0 diagram or one can calculate them with the help of the calculator, without any difficultly.

So, the angle of departure and the angle of arrival can be calculated using that rule and that is the rule that could enable you to check that in our example, this the root locus branch which leaves at minus 1 goes in the direction of the positive real axis and therefore, the assumption that it goes along the real axis is a good one. Similarly, the branch which arrives at minus 2, arrives at it from the negative side of the real axis. So that part is okay further we know that the all the points between these 2 are on the root locus for some value of k or the other and therefore, it is a reasonably good conclusion that this whole segment of the real axis is on the root locus, not only that it is the branch of the root locus. As I told you, the root locus method is a what may be called a qualitative method is more a qualitative method, for getting some knowledge or making some good guess about what is going to happen, it is not a substitute for calculation, when calculations are required, calculations will have to be made even when plotting the root locus, one will have to make some calculations.

So it is not that calculations can be entirely avoided but qualitatively doing very few calculations or very simple one by looking at a figure, by looking at a pole 0 diagram, by imagining various situation, one can work out or figure out quite a bit about the root locus and that is the beauty of it. It is mainly a qualitative tool and therefore, it is useful or used by human beings not by computer program and so, I repeat that for us to solve simple small problem or even problems which are not so small. To get some qualitative idea, the root locus method is a good method and therefore one should learn it.

It is not a substitute for quantitative investigation but quantitative investigation or using a program package is not a substitute for understanding, it is not a substitute for thinking about what can happen, what will happen or what cannot happen. The Laplace transformation, the root locus method, Nyquist criterion which we are going to look at, all these are helpful tools for getting an understanding of what is going on rather than for finding out exactly what is happening. All right then, so we have how many rules, so for we have rules that for small k the root locus is near the poles.

So they start from the poles for large k they are near the 0s, so they end at the 0s then, I have talked about asymptotes and will state a rule about the number of a asymptotes and something more about them. Real axis portion of the root locus, I have discussed in detail, angles of departure from the pole, angle of arrival at the 0. I have just mentioned that there is a rule which uses or based on the angle condition and you should look up that rule from your textbook. Now let us look at the asymptotes, the rule about asymptotes. Now there will be asymptotes only in the case, when number of poles N P is not equal to the number of 0s N Z, if a transfer function or if a pole the 0 diagram has the same number of pole and 0s then, there are no asymptotes that is there are no branches of the root locus which either go away, move away towards infinity as we say or start from positions which are very far of and then, come towards the finite part of the complex plane.

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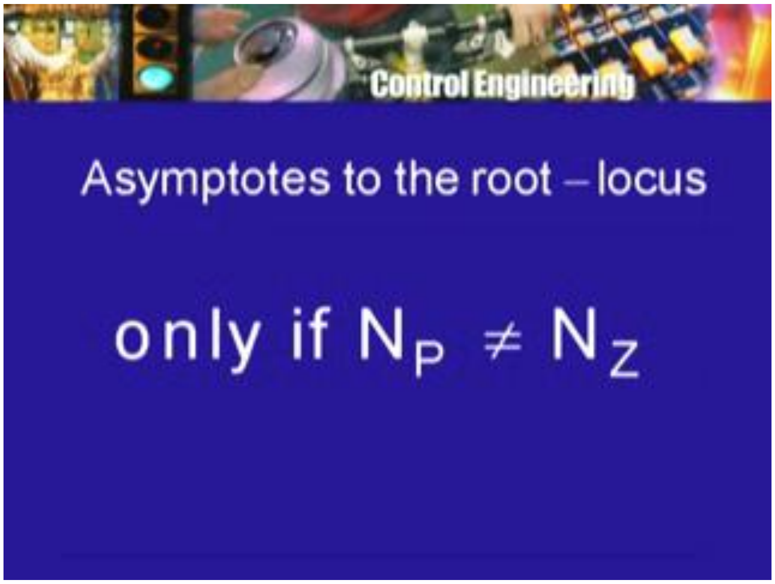

So there are asymptotes that is something of that sort happens going away to infinity or starting from infinity only if N P is not equal to N Z. The number of poles is different from the number of 0s. So if the 2 are equal there is no need to worry and think about asymptotes there are none. Now N P not equal to N Z there can of course be 2 possibilities, one is the number of poles is greater than the number of 0s, if this is so then the number of root loci is the larger of the 2, it is N P.

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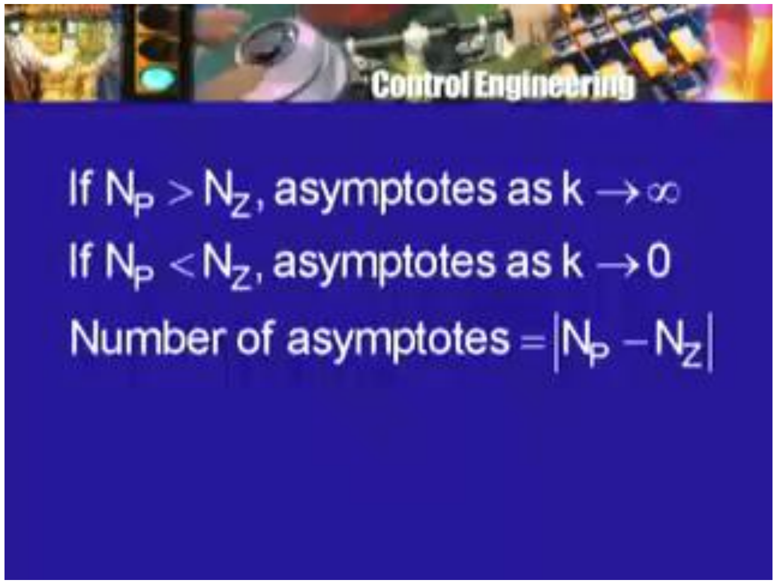

So all the root locus branches will start at the poles N Z of them will end at the 0s and the remaining will go towards or approach asymptotes, that is go towards infinity, that is increase in absolute value, may be with a change of angle as k increases. So there will asymptotes for the case when N P is greater than N Z, there will be N Z branches which end at the 0s and the remaining branches will go of asymptotically. In the other case, when N P is less than N Z, it is going to be just the opposite the number of branches of the root locus is the larger of the 2 that is the degrees of the characteristic polynomial therefore, there will be N Z branches, there are N Z 0s. So N Z branches will end at the N Z 0 but there are only N P poles.

So N P branches will start at poles, the remaining a difference between the 2 will start from a very region which we referred to as the region at infinity that means for small value of k, there will be roots which are very large in absolute value that is what we mean by saying that, the roots or the branches start from infinity. Now one can show that in such a case, when N P is not equal to N Z and therefore there are asymptotes, the asymptotes can be actually located that is one can actually find out the asymptotes or the branches of the root locus which either approach them as k increases that is go away as k increases or start or k small that is come towards the finite part of the plane as k increases from 0.

Now the location of these asymptotes can be calculated by a rather simple rule of course, the proof of it is a little complicated , so we will not look at that and the rule is simply this. First do the following just calculate the number given by I am going to write here sigma P by that I mean the sum of all the pole whether real or complex or imaginary, sum of all the poles from that subtract the sum of all the 0s. So compactly sigma P minus sigma Z sum of all the poles minus sum of all the 0s divide this by the difference between the number of poles and the number of 0s.

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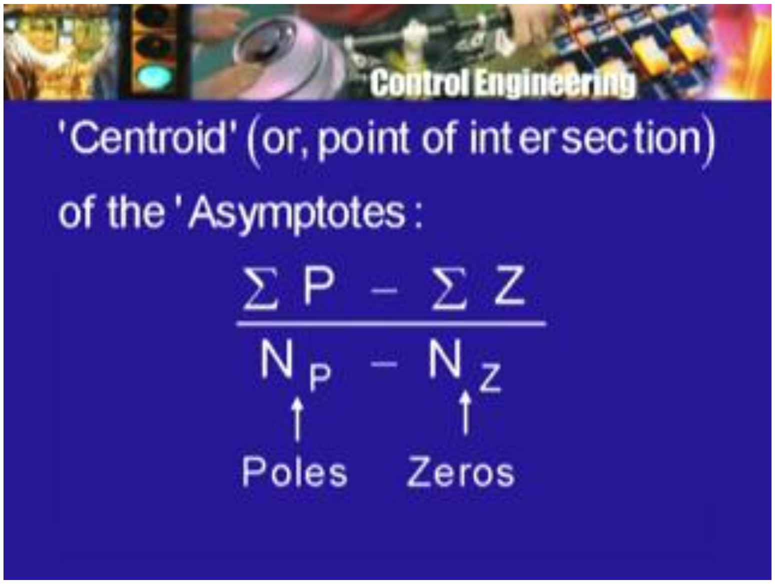

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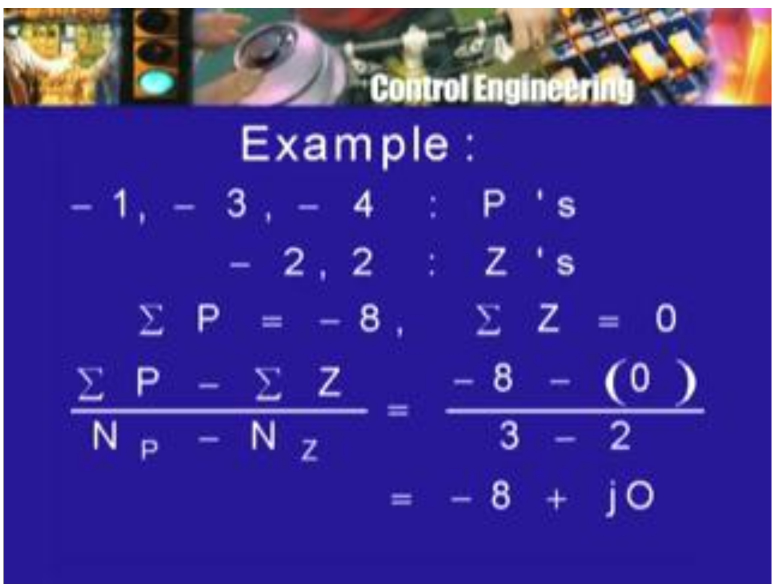

So N P minus N Z in some books N P itself is denoted by P, N Z is denoted by Z and therefore it will be P minute Z in the denominator, do not worry whether N P is greater than N Z or N P is less than N Z, do not worry about the sign of the denominator, take it if you have sigma P minus sigma Z into the denominator you will have N P minus N Z. Now this gives you a number and therefore a point on the complex plane which is what is called the point of intersection of the asymptote, sometimes it is also referred to as the centroid of the pole 0 diagram this gives you the point of intersection of the asymptotes that is, if there are any asymptotes then they all pass through this point. Let us apply that to our particular example, there we had poles at 3 points, the poles were minus 1, minus 3 and minus 4, these were the poles, there were two 0s, one was at minus 2 and the other was at plus 2.

So there are two 0s, so what is sigma P sigma, P in this case is just minus 1 minus 3 minus 4 sum of them, so that minus 8, what is sigma of the 0s minus 2 and plus 2 therefore 0. So sigma P sigma Z divided by N P minus N Z, in this case will be equal to minus 8 minus 0 divided by N P was 3 poles m N Z was 2, 0sm so 3 minus 2. So this is equal to minus 8 or as electrical engineers will prefer to write it as minus 8 plus j 0. So, this is the point of intersection of the asymptotes, the point is on the real axis, on the negative real axis is the point minus 8 plus j 0. So that is one information about the asymptotes the asymptotes are all straight lines passing through this point that is another point of information, what about their number, how may asymptotes N P minus N Z, if N P is greater than N Z and N Z minus N P, when N Z is greater than N P that is whatever number either go to infinity or start from infinity. It is the difference between the number of poles and the number of 0s

In our problem, N P minus N Z is 1, so that means there will be one asymptote and because N P is greater than N Z, this asymptote is what the root locus branch will approach as k increases. So in other words as k increases one branch of the root locus will go close to this asymptote, so other will approach the straight line asymptotically. I hope you remember in coordinate geometry, the concept of asymptote for example, the hyperbola. The hyperbola given by xy equal to 1 has the x axis and the y axis as if asymptote that is the branches of the hyperbola or one part of it and the other part of it approach these 2 axis as one moves along the hyperbola in one direction or the other, all right

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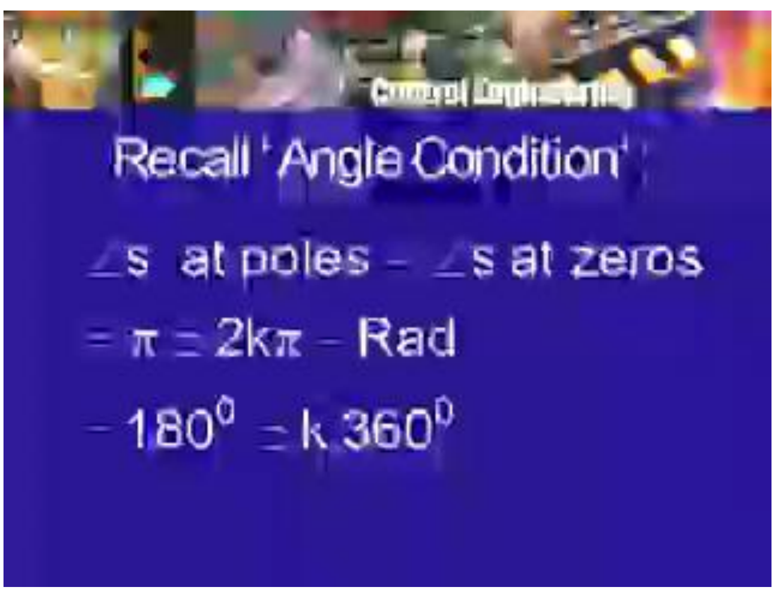

So there is going to be only one asymptote, it is going to pass through this point. Now in order to specify a line in the complex plane, if I have a point that is not enough, there are

100s of 1000s of lines that pass through this point. So, what do I need to know I need to know the angle made by a line or a ray with the positive real axis. Now that is precisely what we can obtain, so this will be then the angle made by an asymptote with the positive real axis. Now what is the formula for that, the formula for that is as follows. Take the number phi, radians as radians that is the angle or alternately 180 degrees as an angle to that if necessary add multiples of 2 phi or alternately multiples of 360 degrees.

Now whatever, this number is divide it by the difference between the number of poles and the number of 0s, all right. In our problem, so we start with phi equal to180 degrees and I divide it by the difference between poles and 0s. So divide it by 1, so what do I get 180 degrees so that is an angle made by one of the asymptote. Now I said take180 degrees and add to it a multiple of 360. So let me add 360 degrees divide by 1, what do I get 540 degrees but a line which makes an angle of 540 degrees is the same as the line which makes an angle of 180 degrees. So this is the same in that sense as 180 degrees and indeed therefore because there is only one asymptote.

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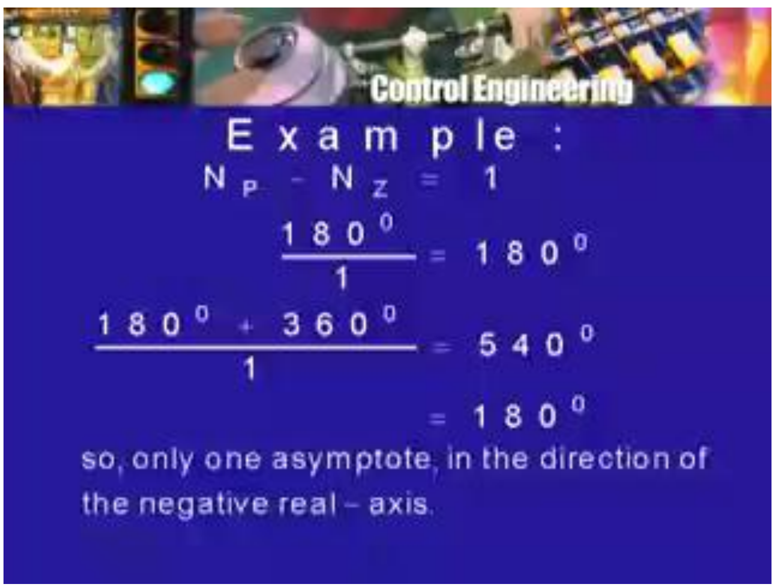

So the angle made by the asymptotes is 180 degrees with the positive real axis, with this then we can sketch the asymptote as to where it must be. So here is the point minus 8 plus j 0 say from here draw a ray which makes an angle of 180 degrees with the positive real axis. So what can it be here is the angle of 180 degree therefore it must be this and so this is the direction of that asymptote and indeed, it will go back to our problem. You will see that from this pole at minus 4, I had drawn a branch you are simply going away towards the negative part of the real axis. So I was thinking of the rule about the direction of asymptote, when I drew it this way.

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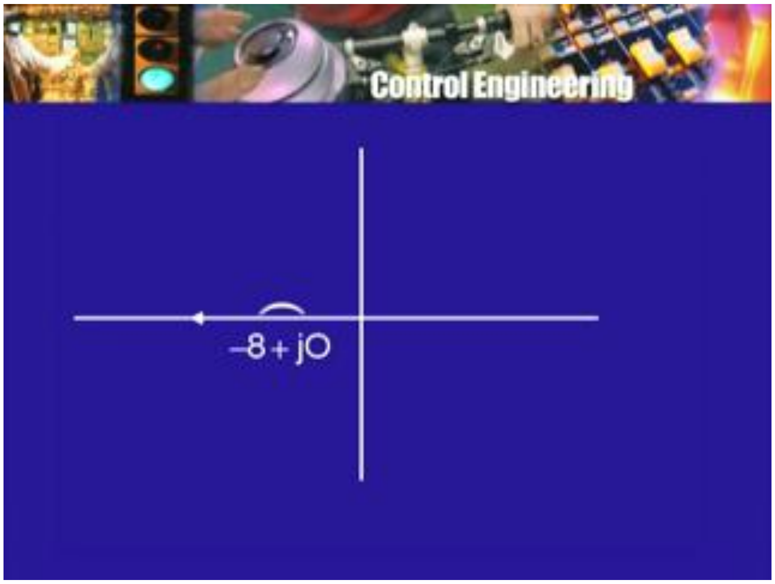

Now, that I have told you we can use this rule any time you want, 180 degrees plus minus multiple of 360 degrees divided by the number of asymptote that gives the angles made by the asymptote, if there is only one asymptotes, there will1 angle, if there are 2 asymptote, there will be 2 angle and so on. Let us see suppose, there are 2 asymptote then what is my rule now, I said 108 degrees divided by 2. So I divide by 2, what do I get 90 degree, so I get one angle but there are 2 asymptote, why 2 because the difference between the number of poles and 0 is 2. So what is the angel made by the other asymptote.

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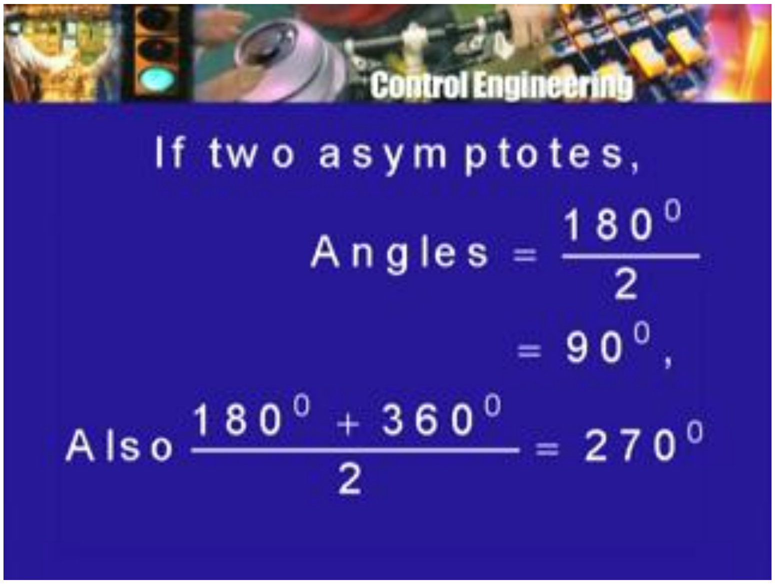

Well in addition to 180, I can now consider 180 plus 360 and divide it by 2. Now again an angle which is 270 decrease which is not the same as 90 degree. So I get 2 different angles, so if there are 2 asymptotes the angles made by them will be 90 degrees and 270 degree, if there are 3 asymptotes, what will be the 3 angle, think about it, 180 degree by 3 is 1, so 60 degree, 180 plus 360 divided by 3, so 180 degrees and the third one,180 plus 720 degrees divided by 3, so that is 300 degrees.

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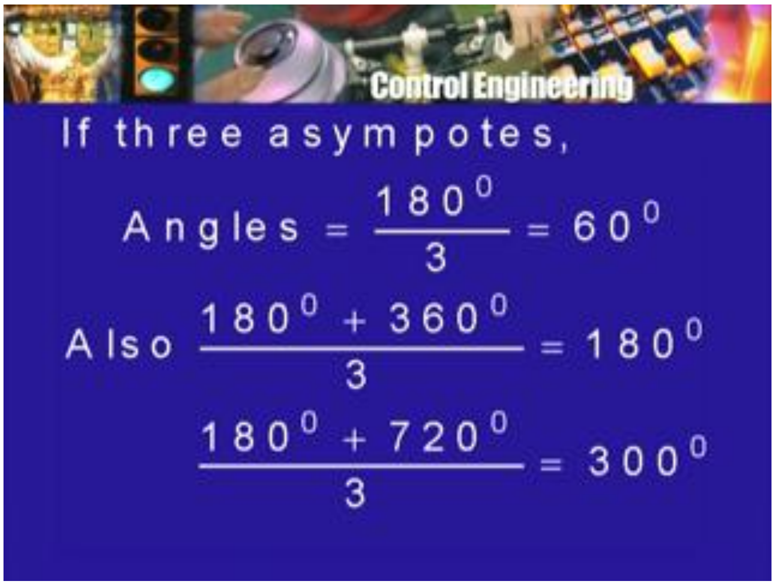

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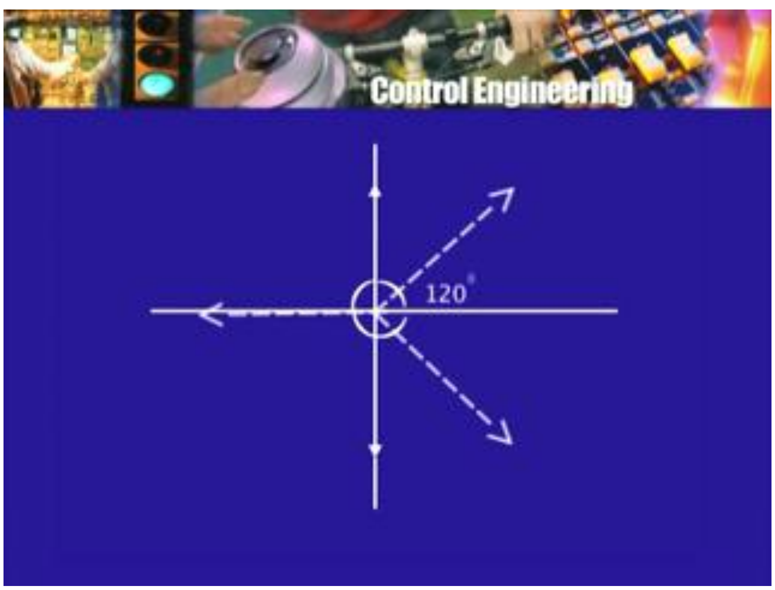

So the 3 asymptotes will make angles of 60 degrees, 180 degrees and 300 degrees or if I show them on this diagram then, one asymptote will go away like this the other asymptote, if the center of a point of intersection is the origin another asymptote will go like that, the third asymptote will go like this and between each other, they will make angles of 100 of 20 degrees. So the number of asymptotes is the difference between the number of poles and 0s and the angles made by them with the positive real axis can be found out by this rule. In our problem there is only one asymptote and that asymptotes goes towards the negative real axis by this rule.

So point of intersection of the asymptotes can be obtained by the rule sigma poles minus sigma 0 divided by number of poles minus number of 0s. All the pole whether real or complex or imaginary, all the 0s have to be included in the those summations sigma P minus sigma Z divided by P minus Z is the rule for the location of the common point of intersection of the asymptotes of the root locus. There are some further properties, some of which are so simple that as soon as I state them, you will agree that they are true the characteristic polynomial let me go back it looks like, what we had G D of s, H D of s plus k a into G N of s, H N of s equal to 0, right. Now G H, whether numerator or denominator have all real number as coefficients.

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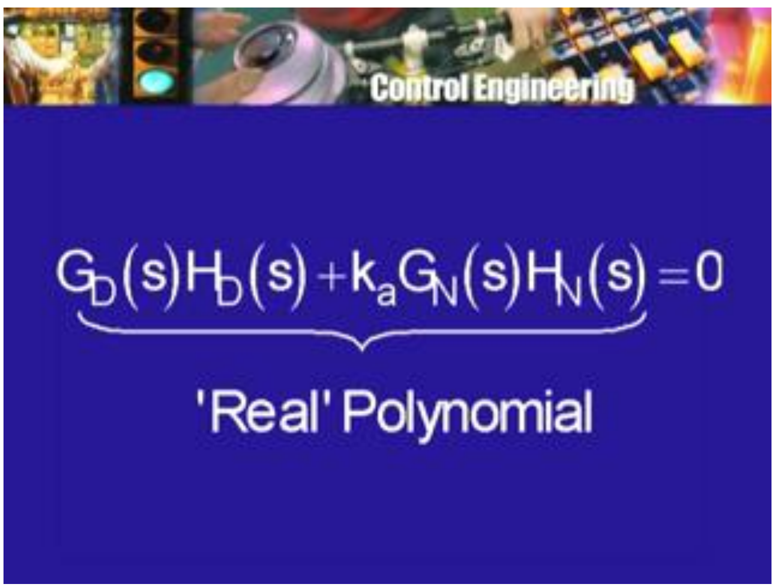

Our system parameter like resistance inductance back emf co-efficient, co-efficient of friction, all of them are real numbers. In fact, most of them are chosen the concept is such that they are all positive if I said that a resistance has value minus 20 volts then, you will suspect that there is something wrong either I am talking about the resistance in a different way or there is some mistake but they are real numbers. I do not and I cannot find the resistance which has a value of 2 plus j 3 ohm. Notice I am talking about parameters, I am not talking about frequency response, gain, phase or complex gain and things like that. We do come across complex numbers in our work but the system parameters are all real number.

So G s, H s whether D or N have all their coefficients real. So this whole polynomial is what is called a real polynomial that is it is a polynomial all of whose coefficients are real numbers because of this, the roots of this occur in conjugate pair. So not only the poles and 0s, which are roots of either G D or H D or G N of H N they occur in conjugate pairs. So in the poles 0 diagram are symmetry, symmetry of what kind, symmetry about the real axis. The pole 0 diagram is symmetric about the real axis but not only the pole 0 diagram, the root locus plot that is the root locus branches, the whole figure consisting of all of them must also be symmetric about the real axis and why, because of this fact that points on the root locus are roots of a real polynomial.

So they must occur in conjugate pair. So if there is a phi is a point on the root locus above the real axis, there must be a point on the root locus below the real axis which is like its mirror image in the real axis. So, this is called the symmetry of the root locus about the real axis. This is one more rule which follows almost immediately from our knowledge of polynomial. If you think of the example that we have taken I had said that the root locus will have a branch which will cross the imaginary axis of the j omega axis and therefore one of the roots of characteristic polynomial will become 0 for some value of k.

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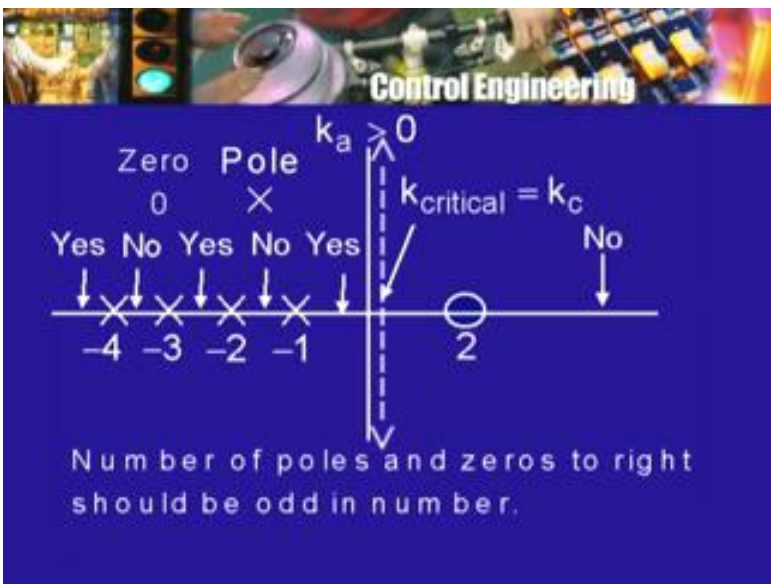

So this is called intersection with the j omega x axis or with the imaginary axis. Now there is a method for determining the value of k for such intersection and the corresponding location of the intersection. But I mentioned it earlier that there is an algorithm or a procedure which was given by Routh more than 100 years ago using that one can find out the intersections of the root locus with the j omega axis or the imaginary axis and the value of k corresponding to those intersections. Now, before I discuss this method based on the Routh algorithms because the Routh algorithms is not a very difficult one to understand and to use once you have found out, what exactly is to be done and it is a very important tool not only in control theory but also in network theory or system theory.

It occurs in investigations of stability and so every control engineer should know about the Routh algorithm but there is another way of finding the intersection with the negative with the j omega axis, that rule is not really very simple to apply, as you will see in a moment. But sometimes it does give you some answer for example, let us go back to our problem.

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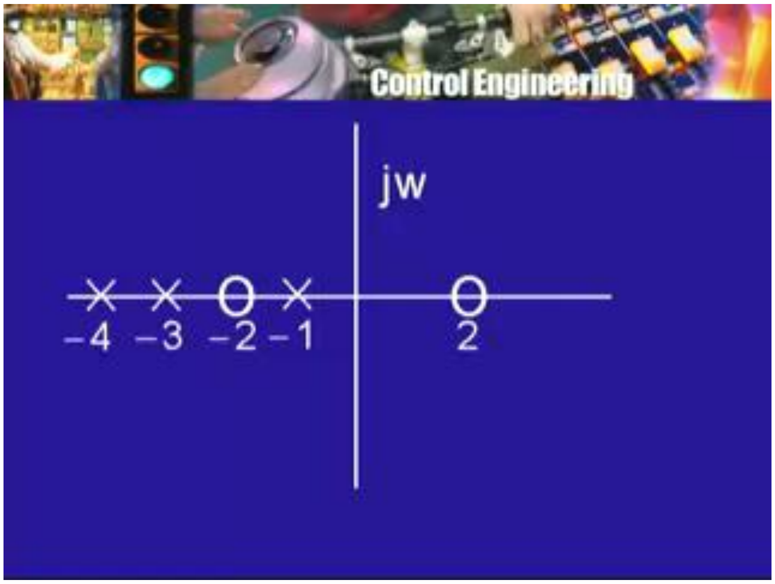

So, let me redraw the figure now and here are the poles minus 1, there is a 0 at minus 2, there is a pole at minus 3, another one at minus 4 and there is a 0 at plus 2. Now, I want to find out if any point on the j omega axis or on the imaginary axis will belong to the root locus. So what is it that I am checking whether for any point on the imaginary axis that complex number and this case, the purely imaginary number will be a root of the characteristic polynomial, all right. Now, we can proceed in 2 different ways this characteristic polynomial, I can write down explicitly, what it is.

So, I will do it for example and you will see that the situation becomes more complicated, if you have a larger number of poles and 0s. I had s plus 1 into s plus 3 into s plus 4 plus k a into s plus 2 into s minus 2, that is the characteristic polynomial. Let us call it P of s and we are trying to solve the equation or find the roots of the equation P s equals to 0, they are the points on the root locus, the complex number stays on the root locus right. Now, let us look at this, now you expand this out and you expand this out, all right.

So if I do that I will get, what I will get s cube plus, from this you can see that I will get 8 s square plus, there is a way of finding out 3 into 1 plus 4 into 3, 15 plus 4, 19, so 19 s plus 4 into 3 into 1. So 12, if you were good at algebra and if you remember it, there is a nice way of multiplying this out mentally, instead of writing it down in 2 steps, s cube plus 4 plus 3 plus 1. So 8 s square plus 3 into 1 plus 3 into 4 plus 4 into 1, so that is 3 plus 12, 15 plus 4, 19, the last term is 1 into 3 into 4, so 12 plus k a into this is easy of course s plus 2 into s minus 2 s square minus 4.

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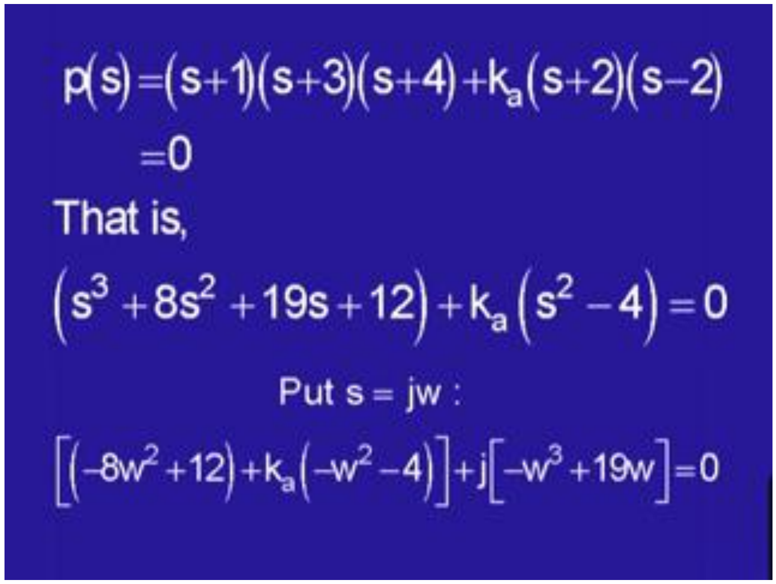

Now, what is it that I am looking at, I am trying to find out, if a purely imaginary number is on the root locus, a purely imaginary number is j omega for omega real. So, I am trying to find out whether j omega is a root of this polynomial or j omega will make this equal to 0. Now you can see what is going to happen, there are terms like this s squared and there is this constant, when I put s equal to j omega, they will become purely. real On the other hand there are terms like this s and this s cube.

So when I put s equal to j omega, they will become purely imaginary and so the left hand side can be written as one purely real expression plus another purely imaginary expression equal to 0, let me do that the purely real thing ,where I am going to get it from 8 s square s equal to j omega. So what will be 8 s square with little practice. You will be able see immediately that it will be minus 8 omega square then I have plus 12 which is real, from here k a is real s square is minus omega square. So this is the real part of the left hand side at s equal to j omega and what will be the imaginary part, it will be given by j times what s cube, s cube, s equal to j omega.

So j omega into j omega into j omega, so how much is that that is minus j omega cube. So I pulling that j outside I will have minus omega cube s is j omega. So from here I will get plus 19 omega and that is all from the real part I missed out this term not only minus omega square but also that minus 4. So this whole thing is equal to 0 right this is by equation minus 8 omega square plus 12 plus k a into minus omega square minus 4 plus j times, this is equal to 0. Therefore, each of the 2 parts must be equal to 0, the real part must be 0 and the imaginary part must be 0. So from this I will get 2 equations and these equations will be Minus 8 omega square plus 12 plus k a into minus omega square minus 4 equal to 0, that is one equation the other equation is minus omega cube plus 19 omega equal to 0, all right.

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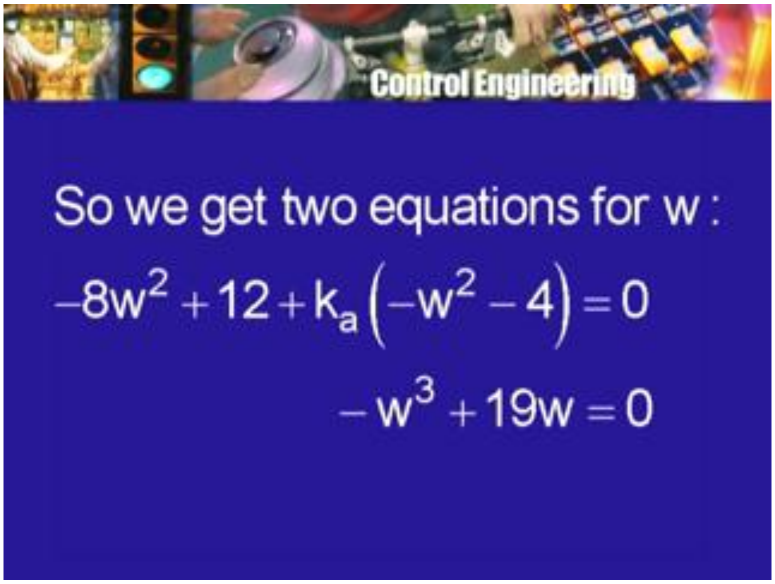

I get 2 equations and what are the unknowns omega is unknown I am trying to find out what point j omega is on the root locus, I do not know what omega is and I do not know the value of k a for which, it is a point on the root locus. So there are 2 unknowns, there are 2 equations, this is what is going to happen in general I take the characteristic polynomial expand it out replace s by j omega, separate the left hand side into 2 parts, the real part plus j times another part and equate the real part and the imaginary part to 0, I will get 2 equations for the 2 unknowns. So gain k a and the point omega corresponding to that point I will not call it frequency although it can be given an interpretation of frequency as we have seen earlier.

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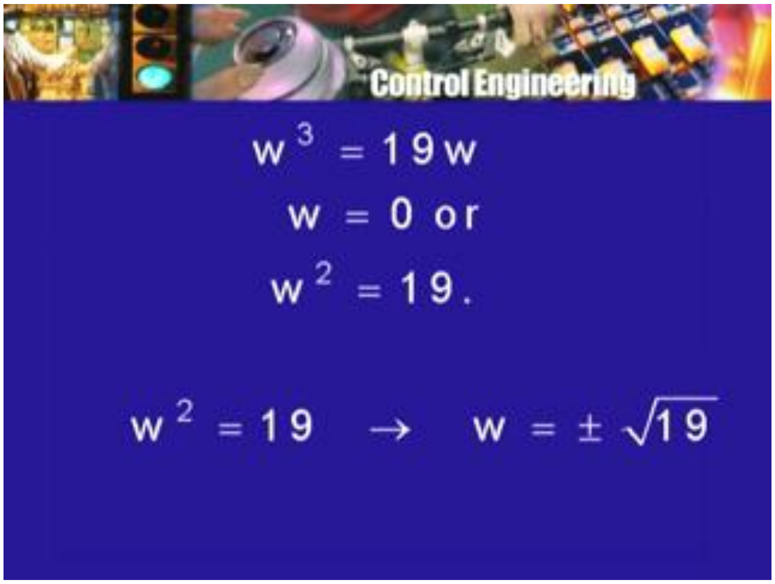

Now, in general it may not be immediately clear how to solve these 2 equations. In this case for example it is easy because in the second equation k a is not appearing at all but that may not happen, there will be k a in the second equation also. So both the equations may contain both the unknowns k a and omega and therefore, you have to use some algebraic trick or manipulation to solve for omega and k a. But for our particular problem it is not that bad because this second equation does not contain k a at all, what are the solutions of this omega cube minus omega cube plus 19 omega equal to 0 or alternately omega cube is equal to 19 omega and what are the solution, there are 2 solutions, one is omega equal to 0, do not forget that, do not cancel omega because when omega is equal to 0, this is true, 0 is equal to 0.

So, there is a solution omega equal to 0 and the second solution gives you omega square equal to 19 and therefore, omega equal to plus or minus square root of 19. So corresponding to omega equal to 0, there could be a point on the root locus and corresponding to omega equal to plus minus root 19 that is j plus minus root 19 that is the point on the imaginary axis that could be also on the root locus. Now we have to check the other equation because the other equation has to be satisfied. The other equation now omega is known and I can solve it for k a and you check that if I put omega equal to 0, we will get some particular value of k a.

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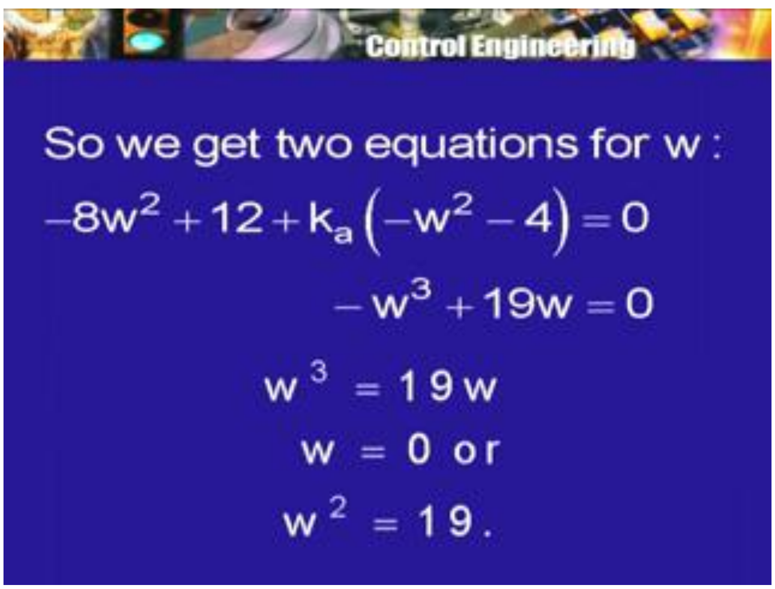

Let me substitute if I put omega equal to 0, I will get twelve plus k a into minus 4 equal to 0 or I get k a equal to 3. So that is a value which I can accept because it is positive real number. So I have found out that for k a equal to 3, there will be 1 root or there will be 1 point on the root locus, the location is j omega where omega is 0 or j 0 but j 0 is nothing but the origin of the complex plane and so, for k equal to 3 origin, the point 0 plus j 0 or j 0 or 0 will be on the root locus and I have shown earlier, the root locus branch like this implying that there will be intersection with j omega axis and this is the point where it will intersect. Now what about the other values of omega.

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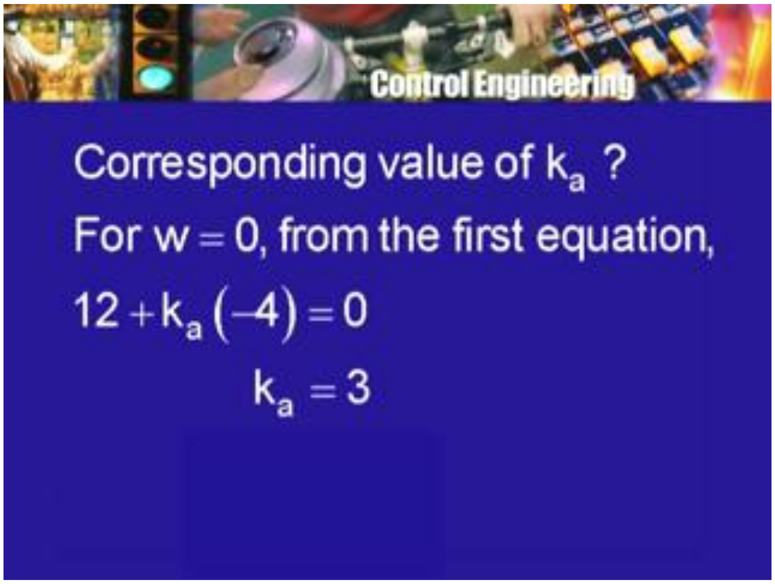

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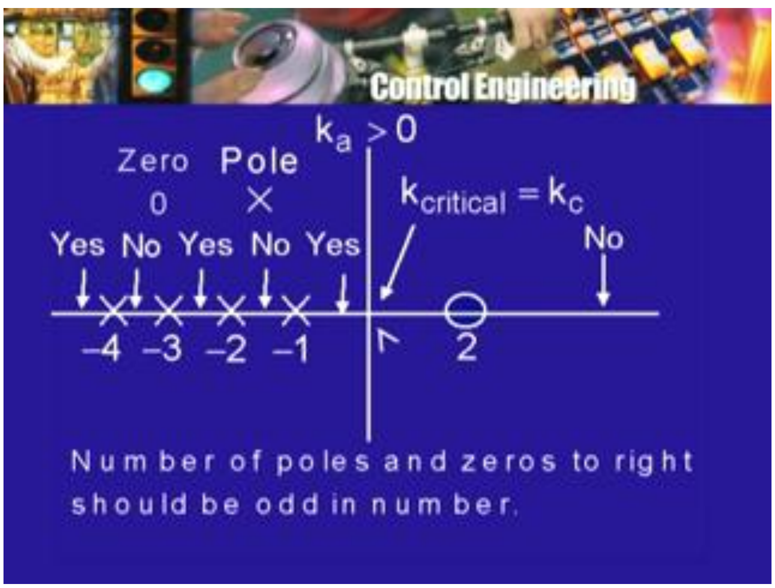

I will leave it to you to work out put omega square equal to 19 or omega equal to plus minus square root of 19, substitute that in this equation and try to find out, where the values of k. You will find out that the value of k a that you get will be less than 0 but we are not looking at gains which are less than 0. Therefore, this thing we have to rule out or exclude so the only intersection that we can have with the j omega axis is at the origin. This will be an intersection, if a negative value of k a was allowed in some investigations one does look at negative values of k a and the corresponding root locus that is obtained is therefore called the inverse root locus. Normally, when you have k a positive one does not call it the direct root locus. So simply call it the root locus but if you allow or consider the values of k a less than 0 then, we are looking at the inverse root locus.

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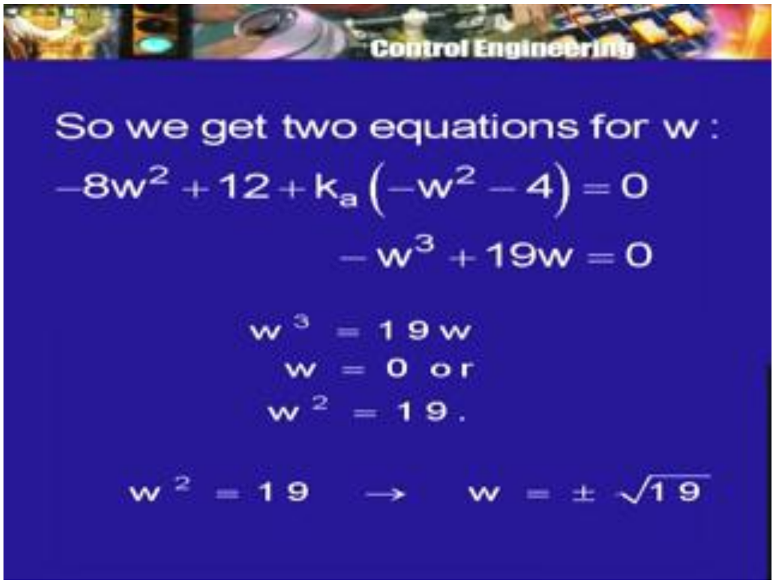

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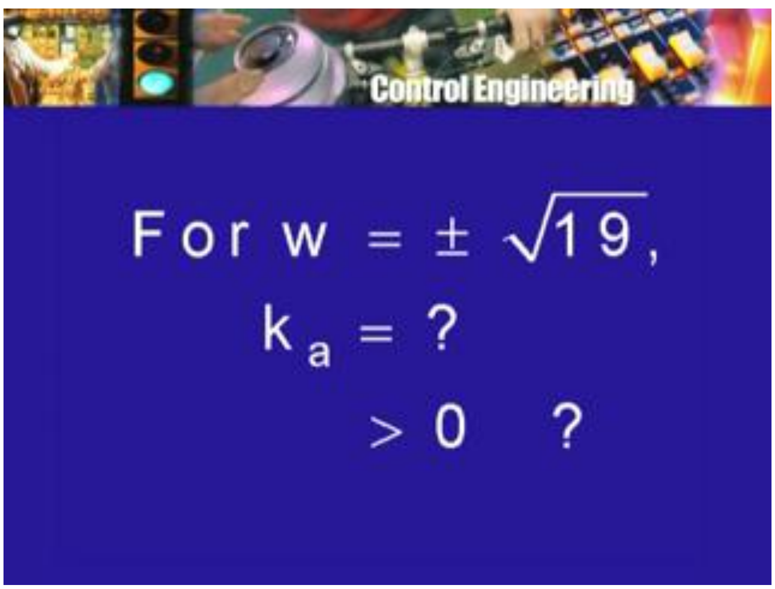

So then, it will show that for the inverse root locus there will be a point on the root locus corresponding to omega equal to plus minus root 19, in other words the point plus minus j root 19 will be on the inverse root locus. Now this is one way of proceeding but this requires as you can see a lot of work, I have to expand the polynomial replace s by j omega equate real and imaginary parts equal to 0, solve the resulting 2 equations for omega and k a and choose the values of k a greater than 0, if they do result that will give you intersections with the j omega axis, this is one way of doing it.

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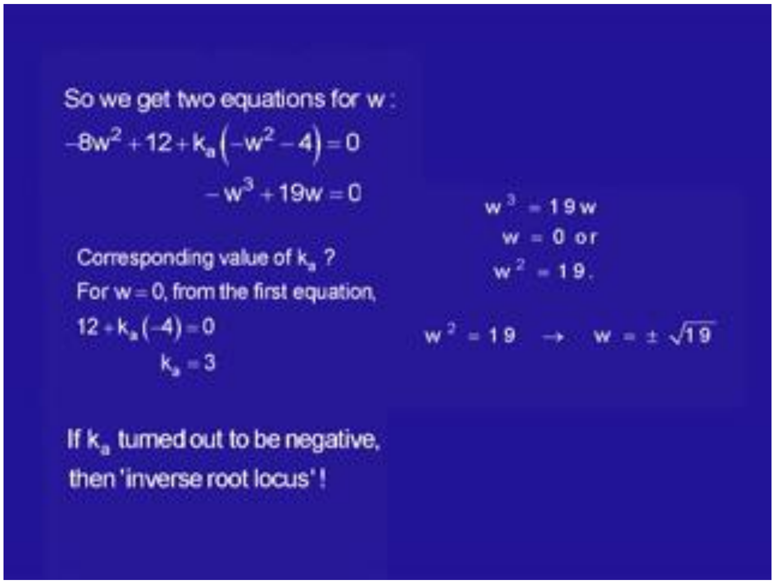

Another way of doing it ,we will look at the angle condition that will involve of course calculation of or writing down expressions for some angle and from that you can get an equation which only involves omega but unfortunately, it will not be a simple polynomial equation. Using, a calculator however one can without too much difficulty find out atleast an approximate solution, for the omega values that is a second method but the preferred method is the one that uses the Routh algorithms or the Routh criterion. So we will take a look at the second method for a very quick exposure, I will not talk about it in great detail, I will leave it to you to look it up from your text books, how what kind of calculations are to be made but then we will spend more time on the Routh algorithm of the Routh table method, for determining intersections with the j omega axis or with the imaginary axis.