Control Engineering
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Lecture - 30

I hope you have done the homework that I had asked you to do namely, find out the relationship between the steady state speeds, the reference voltage. When we you use the integral feedback and as perhaps you should have expected the relationship is very simple, the reference voltage value E r, let us say, the constant value of the reference voltage is simply the tachogenerator coefficient K tg multiplied by the nominal or the rated angular speed of the motor omega 0. With this then, the feedback voltage is exactly equal to the reference voltage therefore the output of the difference device is 0.

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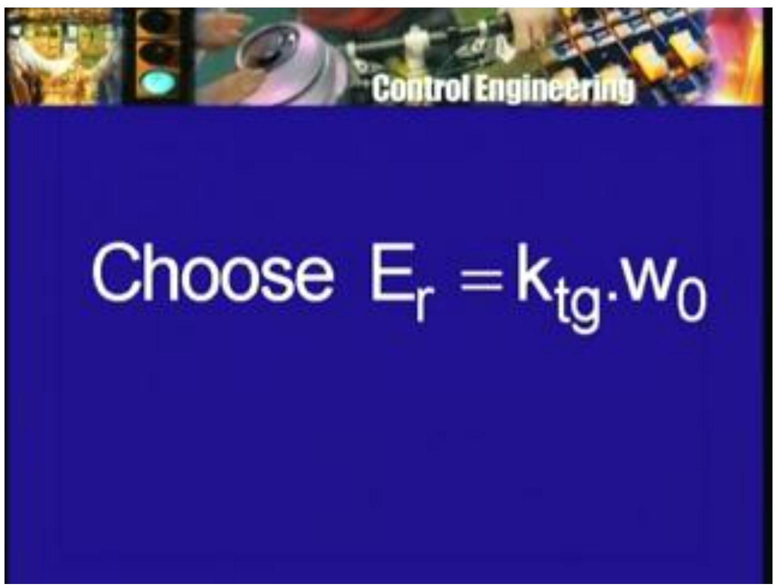

Notice that I am calling it a difference device and not an error detector. The output of the difference device is 0, this device feeds in to the integrator and as we saw earlier, the input to an integrator becoming 0 does not mean that the output of the integrator will be 0. In fact previous to the input becoming 0, the input was non-zero. As a result of which the output of integrator will reach a constant value, this will be the constant value which when amplified will become the applied voltage voltage applied to the armature which, under rated conditions maintains the speed at the desired speed omega 0 and even, when the torque changes even, when the load torque changes the speed returns to the rated value, after a transient period in which the speed will either drop or increase depending on whether the load torque is increased or decreased.

So the advantage of using integral feedback, in addition to the proportional thing K tachogenerator, is that the steady state error for changes in the load torque that is for disturbance change or a disturbance input becomes 0 and not non-zero unlike the earlier case, were we only had proportional feedback. You notice that the gain of the amplifier which amplifiers the output of the integrator K a does not occur in this relationship at all the reference voltage is equal to the tachogenerator coefficient multiplied by the angular speed. So it does not seem to depend on the gain of the amplifier.

So, even if the gain of the amplifier is simply one the integrator will produce enough output. So as to make the motor run at the desired speed then what role is the gain of the amplifier playing? Now, as I mentioned to you earlier what we have been talking about is only the steady state behavior that is after the load torque was changed to a new value and then kept constant at the new value, what happens in the long run that is, if we wait sufficiently. Let us say, beyond more than 5 or 10 time constants of the system. This is what we will find and the gain of the amplifier K plays no role here but as we will see soon today. The gain of the amplifier plays a very important role in the transient behavior that is what happens during this time, when the speed is either decreasing or increasing from the rated value only for a short time but still it is going to change. So in what way does it change? So, what is the transient behavior, this will be determined by the coefficient K a

Now, to see that I will redraw the block diagram of the system and I want you to go with me very quickly, so that we do not have to spend a lot of time repeating what we have done earlier. So in the block diagram now I am going to show starting with the reference voltage with the plus minus difference device, the output of course that is to be expected is omega S, input is the reference voltage E r of S and between the 2, you have the K tachogenerator coefficient because that feedback, we are keeping. The output of the difference device goes through an amplifier we combine with an integrator. In other words 1 by S is the integrator action K a is the scaling action and if K a is greater than 1, then we are amplifying the output of the integrator, this output of the integrator becomes the applied voltage E a of S.

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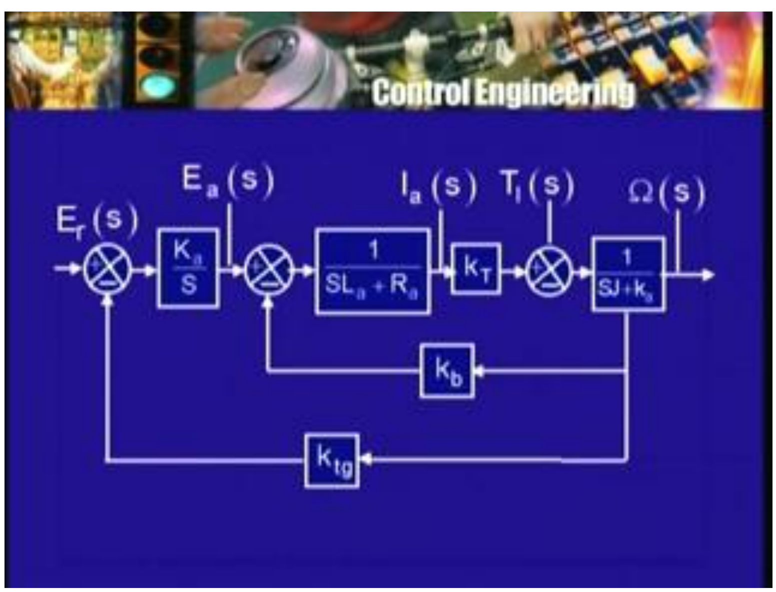

Now, from that gets subtracted the back EMF, so I am showing it here as K b into omega S. So that difference then determines the current, armature current and the transfer function here was 1 over SL a plus R a that produces the armature current I a that is multiplied by the torque constant K T, this produces the motor torque then, we have another difference device from the motor torque the load torque is subtracted. So there is T l of S there this difference is the torque which then overcomes friction or accelerates or decelerates the motor shaft. So in between we have a transfer function here given by 1 by S J plus K F.

So this is the block diagram of the speed control system now, let us go it over it very quickly E r is the reference voltage from that the tachogenerator output is subtracted, the difference is amplified and integrated, there is a combined action 1 by S integration, K a is the scaling and which will usually be amplification produces armature voltage E a from that the back EMF is subtracted gives rise to voltage that essentially drives the current through the armature inductance and resistance combination.

So there is this block 1 by SL a plus R a that gives rise to the armature current K T times that is the motor torque, from that subtract the load torque, the difference is the torque available for overcoming friction and acceleration. So that going through a transfer function1 plus S, J plus K, f produces the output quantity namely omega S. This is the feed completely system, of this note that this K b block and this difference device is only conceptual, it is not an external thing block here K b, an external difference device, the motor action is such that there is the applied armature voltage and there is the induced voltage generated by rotation of the armature in the magnetic field and that is where the difference action is taking place.

Of course, R a and L a are also not physically concentrated resistances and inductance respectively. But the motor armature has a resistance, the motor armature behaves also like an inductance and therefore, you have that combination then I a in to K T, K T is shown as a separate block but there is nothing physical like K T. The armature current in the magnetic field produces a torque and K T is only showing the relationship between the motor torque and the armature current. Likewise, the load torque although it exists separately the motor it may not be possible to measure it separately. However, again conceptually or in the model you have the difference between the 2 and that difference accounts for either the acceleration or deceleration thorough J and the overcoming of friction through K f. So this is the block diagram.

Now, what I am going to do is something we did earlier and for the moment. Let us only look at the effect of E r, the reference voltage that is when the reference voltage is present and the load torque is either 0 or we are not changing it. I told you earlier that the output omega S will depend on both E r, as well as T l. So it depends on 2 things and not only on one thing. For the moment, we will look at only the part that depends on E r and then, we will look at the part that depends on T l. In fact, we might even look at both of them although separately, one after the other.

So because of this I am going to reduce and this is something we did earlier using the Mason gain formula, I am going to reduce the effect of T l and the effect of E a, the armature voltage through 2 transfer functions. So in other words, I am going to replace this part of the block diagram which lies between this E a, T l and omega S boundary by an equivalent pair of transfer functions and this thing we had done earlier. If you remember now, if I am only looking at this part of the signal flow graph or block diagram I have E a as 1 input, T l is another input, there is a loop. So I have to look at the loop gain, there is only one loop, I will look at the loop gain, I will calculate the delta corresponding to the loop gain and then, I am ready to write down the expression for the 2 transfer functions from E a to omega S and from T l to omega S. We have done this already. So I will just ask you to quickly go over it.

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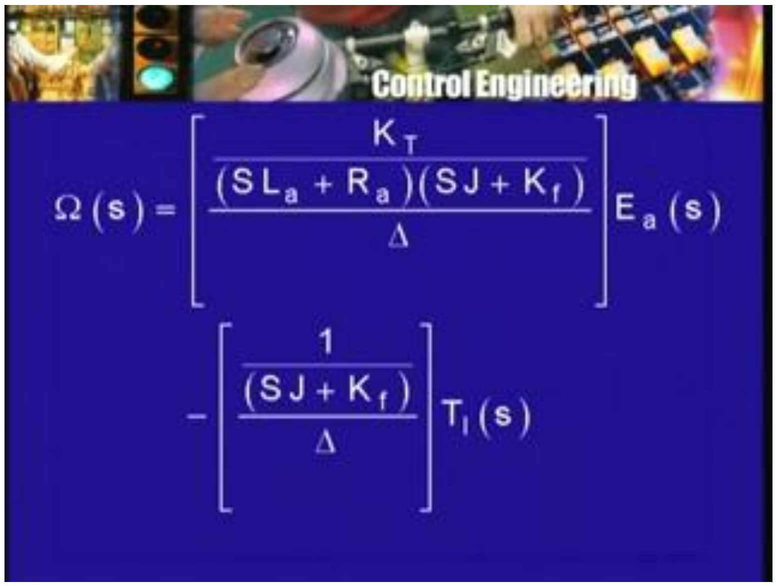

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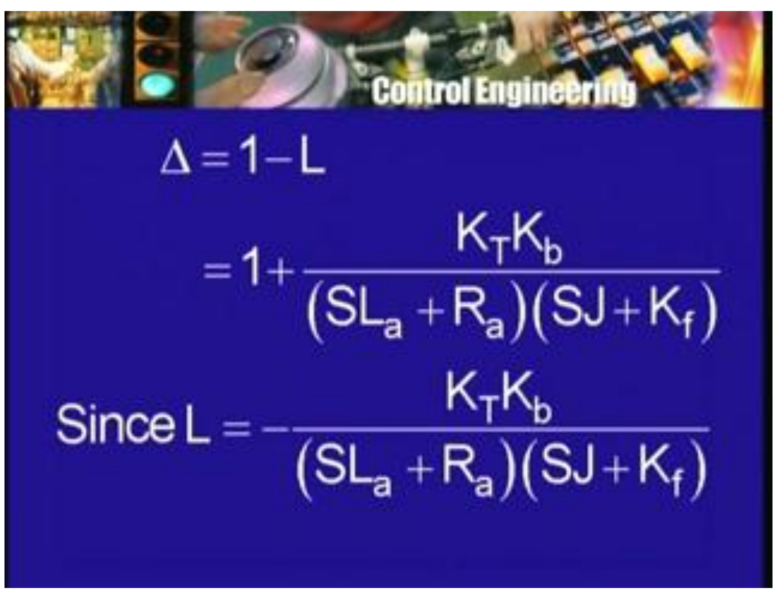

So the transfer function from omega S to E a of S or therefore, omega S will be equal to a transmittance involving E a of S minus another transmittance multiplying the load torque of S and what was delta well, L the loop gain was if you go back to this K T, K b and these 2 transfer functions which are first order factors. So L is equal to K T, K b divided by SL a plus R a in to S J plus K f and because it is going through a positive sign and a negative sign, Kb is the negative sign, therefore this has a minus sign. So L is minus of that there is only one loop. So delta is simply 1 minus L or it is equal to 1 plus K T, K b divided by SL a plus R a in to S J plus K f. So this is delta, so I will write it here as the common denominator delta and common denominator delta for both the transmittances and now, the 2 separate numerators in the case of E a, the forward path is from E a through this 1 over SL a plus R a, K T through the other transfer function to omega going with a positive sign.

Now that will be multiplied by what remains of delta but this forward path touches the loop therefore delta for the numerator is only one and therefore, what do I have here, I simply have K T divided by SL a plus R a into S J plus K f. For the load torque, the forward path only involves 1 over S J plus K f and with a minus sign and that minus sign, I have all ready put out here. So for the load torque then, I have to simply write here 1 over S J plus K f. So that is the expression we have for omega S in terms of E a of S and T l of S. Note, both these expressions have delta in the denominator, the denominator is the same, the numerators are different.

Now as we did earlier we will simplify the expressions by getting rid of the fractions in the delta term that is by multiplying by the 2 linear factors and therefore, we will get the following expression for omega S. Now this delta will change to something which will be common to both of them right. So, what will the delta change to I will have this 1 plus this now, when I multiply by this factor I will therefore have K, SL a plus R a in to S J plus K f plus K T, K b and I am getting rid of the denominator here. So I have to multiply the numerator by these factors.

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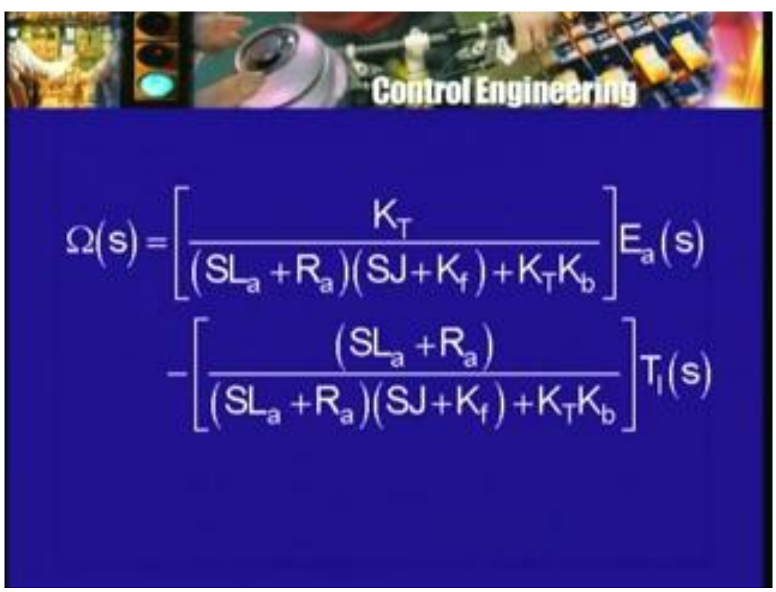

Now, when I do that for the armature voltage component then, I will get rid of the denominator all together I will have only K T left. So I will have in to that this whole thing multiplies E a of S, just check that, if it is correct minus the denominator of the second term will be exactly the same. The numerator, now the numerator was 1 by S, J plus K f, now I am multiplying that by SL a plus R a in to S J plus K f. So, what will I have there only SL a plus R a, the S J plus K f factor cancels out. So this is what the 2 transmittances from E L and T L of S to omega S look like, both of them have the same denominator.

Now, if you remember and of course if you do not you can still look at it right away that this denominator is going to be a quadratic, its going to be a quadratic and here of course the coefficient of S squared will be L a in to J. So if divide both the numerator and denominator by that product then the quadratic will look like S squared plus a s plus b. You remember, that quadratic we had looked at earlier the we have looked at the roots nature of its roots and so forth. So that is what we will get.

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$$\Omega(s)=\left[\frac{\left(K_T/L_aJ\right)}{s^2+as+b}\right]E_a(s)-\left[\frac{\frac{1}{J}\left(s+1/\tau_a\right)}{s^2+as+b}\right]T_l(s)$$

So I will write it again as omega S equal to now for the E a S part, what do I have I have this K T and now I am dividing by SL a in to J. So I will have K T divided by L a in to J divided by S squared plus a s plus b minus the second transmittance, this time I have SL a plus R a and I am going to divide it by L a in to J. So if I cancel off L a, L a, I can write this as when I divide by J, 1 by J and in the numerator I will have a factor left and that factor, if you remember was 1, S plus 1 by tau a, where tau a is the armature time constant, this comes from tau a being equal to L a by R a.

So the denominator is the same but the numerator for the load torque transmittance is a little different and if you remember, the denominator gives rise to 2 poles of the transmittance or the transfer function in each case, whereas for the transfer function from the applied voltage, armature voltage to the speed, there is no 0 but there is only a coefficient K T divided by L a in to J, whereas for the transfer function from load torque to the speed, there is not only a coefficient 1 by J but there is also a linear term which corresponds to a 0.

So, we have 2 poles which are common to both the transmittances but only the load torque transmittance has a 0 and so, as we did earlier one can draw the pole 0 diagram, in fact I am going to that very soon. But before we go to that now, I therefore have a system which can be represented as follows. Now from E a and T L to omega, I am not going to show all those different blocks, I am going to show only these 2 blocks and if you remember, I had called them earlier G A of S and G T of S meaning there are 2 transmittances from armature voltage to speed output and load torque to output speed.

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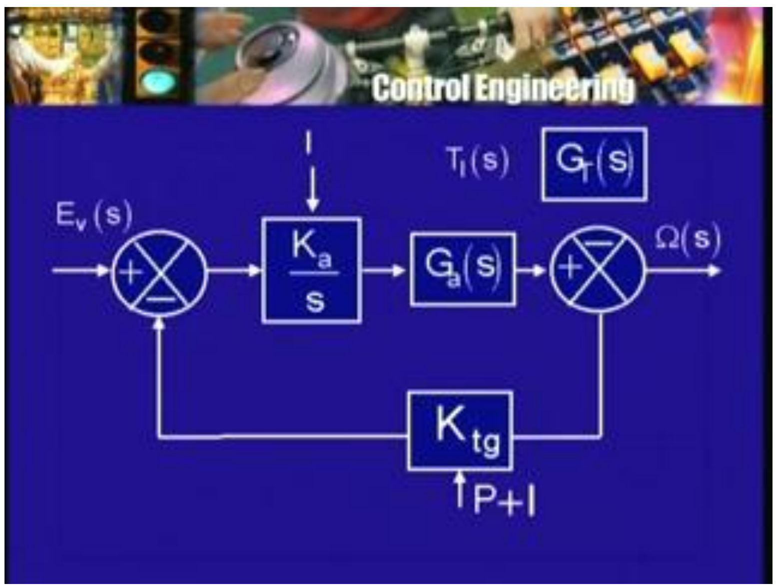

So, if I show that I will put here again that plus minus thing for convenience, here is omega S, here is the load torque and this is going through a transmit, I am sorry, a transfer function which we have called G T of S and this G T of S is what it is going to be exactly this, 1 by J in to S plus 1 by tau a divided by S square plus a s plus b and the other transmittance is from the applied voltage. So here is G a the input to that is E a of S. So, we have got rid of several blocks which were there earlier and we have represented in a different way but the rest of it I am going to keep the same because I have here K a by S, the amplification and integration operation, I have tachogenerator which provides the proportional negative feedback and here I have the difference device as before.

So here is E r it is the tachogenerator output and it goes in to this. So this is what the somewhat simplified block diagram looks like and now, we want to study the effect of K a, this gain of the amplifier integrator on the transient behavior of the system. We saw that it plays no role in the steady state behavior of the system, the steady state speed is simply such that E r equals K tachogenerator in to the steady state speed irrespective of what the load torque is. So the motor will run at the desired speed, irrespective of the load torque changes.

So in the steady state there is no problem but what about transient, when the load torque changes suddenly from, one level to another, the speed is not going to remain constant, the speed is going to change, change for a small interval of time before it goes back to its old value and it is the transient that we are interested in and that transient, we will see will be governed by this K a. Now the analysis that I am going to do for this particular case can be generalized and that gives rise to a method or an approach which is called the root locus method and this was introduced by an American engineer by name Evans, in the 1950, the method is called root locus method.

So what we are going to discuss applies to a more general class of systems however, which look like the system that we are looking at namely there is an output, there is an input and there may be 2 inputs, one is the reference input or the true input, the other is the disturbance then you have transfer functions from them but in between you have inserted a negative feedback which is proportional and another feedback term are using, are applied to that feedback signal is this integrator action.

So what we have is called a proportional plus integral control system or a P I control system. Of course, we can apply the technique to a system where there is no integrator that is there is straight or there is only an amplifier but no integrator. We will do that also that is only the P control but we got in to this whole thing because P controller was not satisfactory. So, we let us first look at the P plus I or P I controller. This is the block diagram that we have and as I said earlier we will look at the effects of the 2 inputs, the reference voltage input and the load torque separately instead of looking at them both at the same time.

Now, remember that we have a signal flow graph in which there are 2 inputs, there is 1 output, the relationship between the various signal is what is called linear. So the whole block diagram corresponds to a linear arrangement and in fact, we had shown that omega S is equal to E r multiplied by something plus T L multiplied by something else and therefore we can consider, the effect of E r and T L separately. This is sometimes referred to as the principle of super position but it is also known as the property of linearity of the system. The system is additive with respect to the inputs and its multiplicative or homogeneous with respect to scaling of the input. So because this is true we can look at the effect of E r alone or the effect of T L alone.

So, let us look at the effect of reference voltage E r alone, so I can put T L equal to 0 as it where and only look at what remains. So what do I have I have a simple forward path and there is a feedback path and that is all I have. So I can think of a system and this is what the root locus method thinks about, where I have a transfer function say G a of S which produces an output omega of S and there is the speed back arrangement E r, I can combine this K a of S along with G a because ultimately one is followed by the other.

So I can put that K a by S here itself. So I have a single block K a by S in to G a of S and here is the feedback element K tachogenerator. We are assuming that the tachogenerator has been already chosen. So, K tachogenerator is not going to be changed and we are looking at or we are going to look at the effect of the amplifier gain K a on the transient performance of the system or on the behavior of the system, when E r may change and later, we will see the effect T L changes right.

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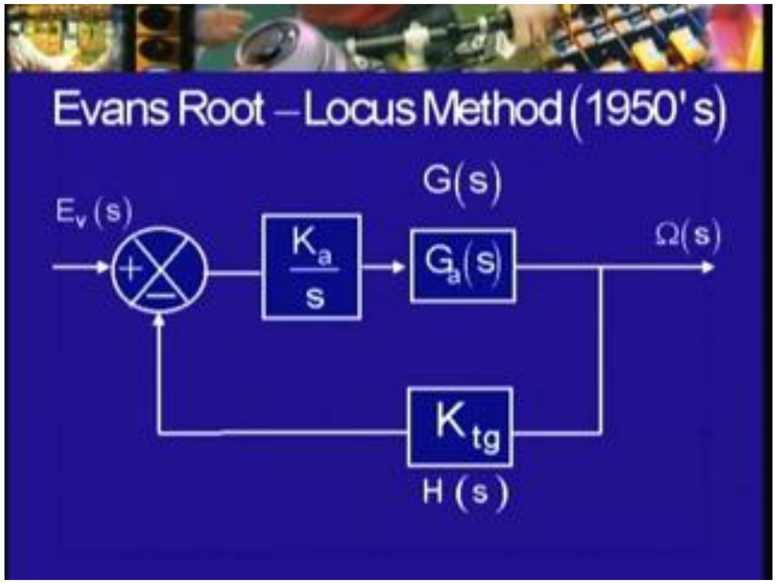

So this whole thing can be thought of as a single transfer function G of S and the transfer function in the feedback path as I have told you is called H of S. So we have the traditional standard block diagram for a feedback control system which is a forward path. Now I am going to show that K a separately, so I am going to write it as K a in to G of S. In the feedback path, there is H of S there is not variable gain there and there is the simple difference device, E r is the input and omega is the output of the whole thing.

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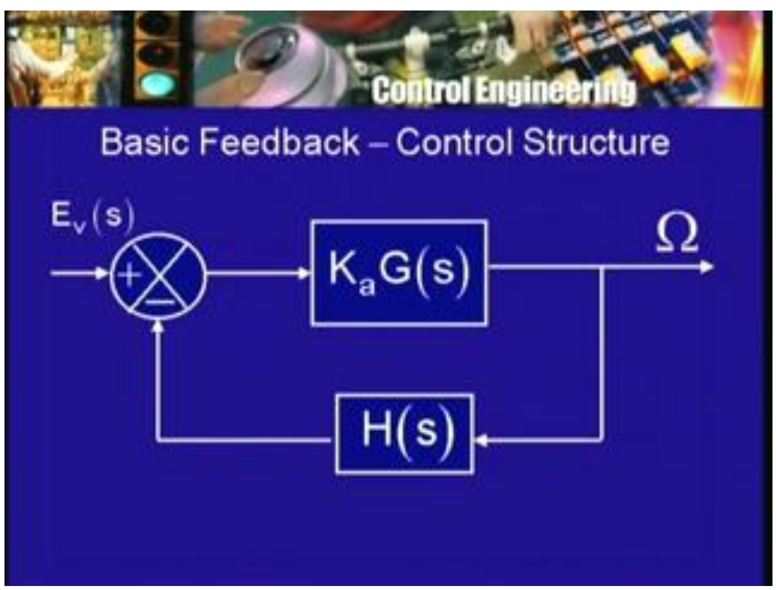

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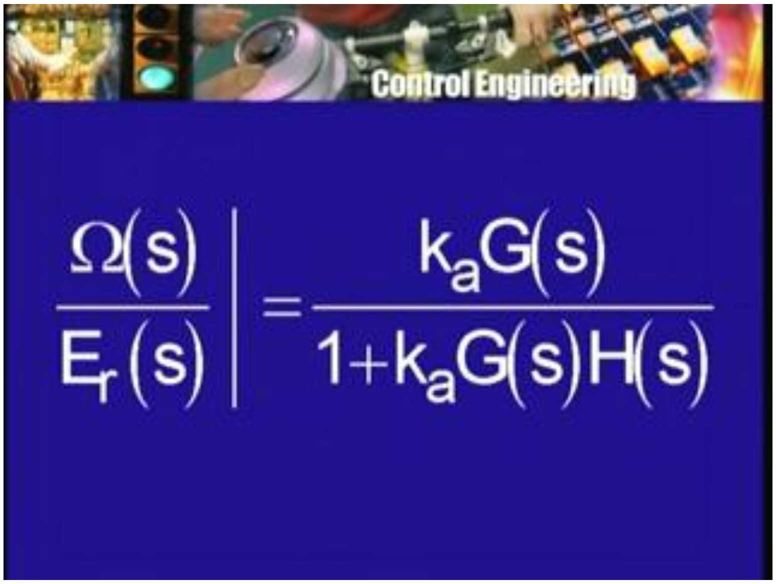

Now, I told you and in fact now, we can still apply the signal flow graph idea and the work is so simple that we can derive the relationship between omega and E r almost immediately and what is that relationship, omega S divided by E r of S and actually, what we mean is omega S is equal to E r S in to something plus another term which involves the load torque. We are only going to look at one term. So this, I will put that vertical line to remember us that we are only looking at a part of omega S. This is given by what by Mason's gain formula delta what is delta here 1 minus loop gain, what is loop gain K a in to G in to H. So the denominator will be 1 plus K a, G of S, H of S and the numerator is what forward path gain K a in to G a multiplied by delta for that but delta for that is 1.

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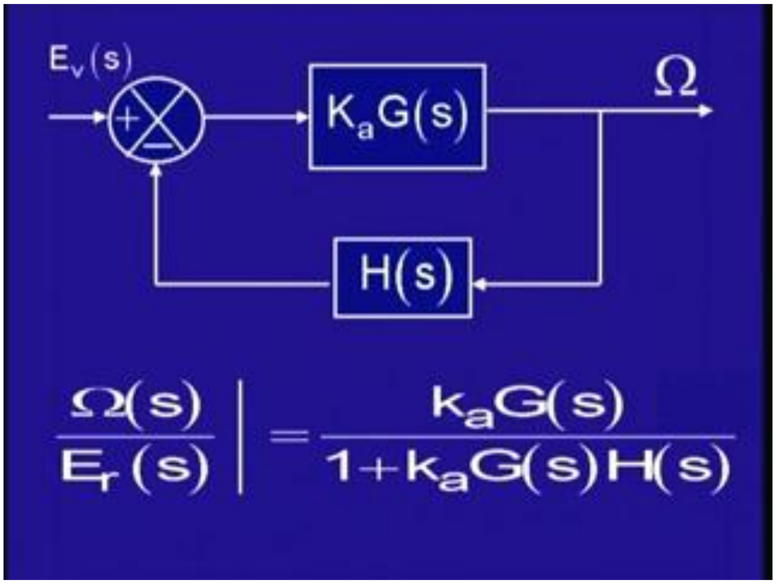

So it is simply K a in to G of S. So that is what we have the transfer function of the close loop system or the overall transfer function as it is called equal to K a G the forward path transfer function divided by 1 plus the forward path transfer function in to the feedback path transfer function or alternately 1 plus the loop gain or the loop transfer function. The transfer function going around the whole loop start with a point go through K a, G a go further through H. So when you are back it is as if you are gone through transfer function K a in to G in to H all right. So this is what the overall transfer function looks like.

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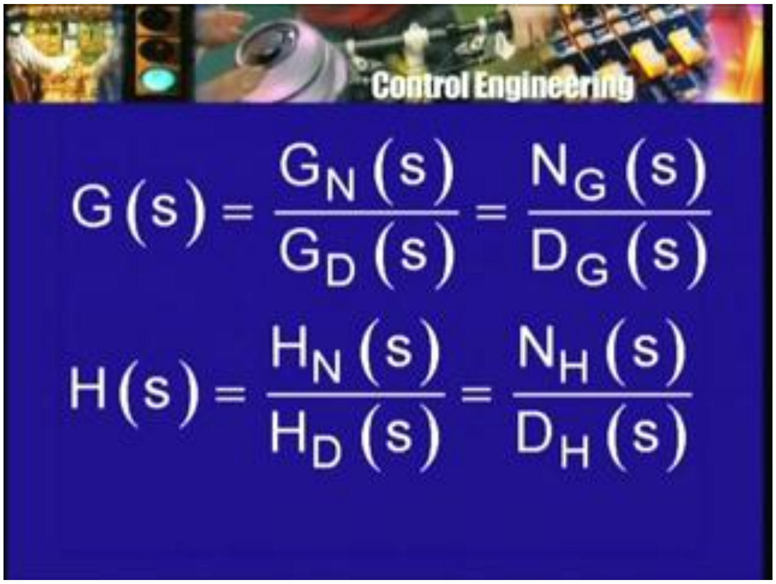

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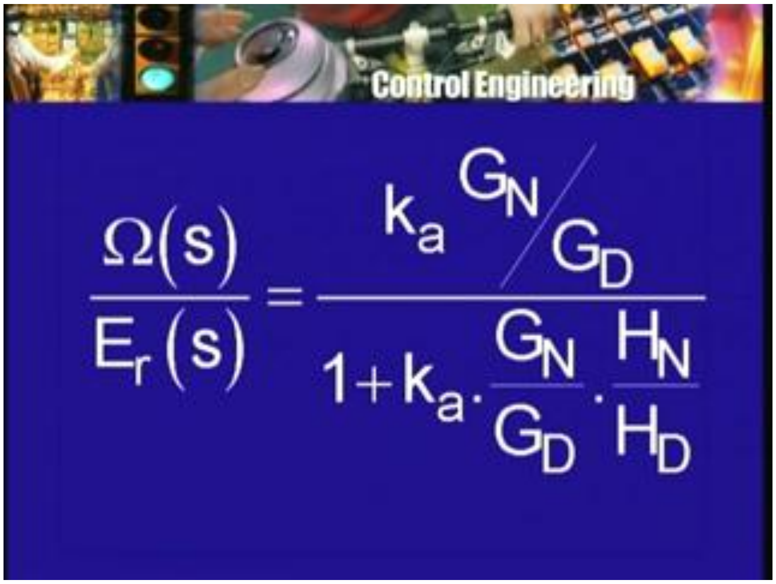

Now, as we saw earlier each one of these transfer functions G S and H S, in general will be a ratio of 2 polynomials and remember, the terminology that we had introduced for the polynomials, n for numerator, d for denominator. So, G of S, I can write as G of S equal to G N of S, the numerator polynomial divided by G D of S. Alternatively, some books may write N for numerator G of S and D for denominator G of S depends on which you like to think about it. The G transfer function its numerator and the G transfer function its denominator that is what G of S is likewise, H of S will be equal to H N of S divided H D of S, the numerator part of H, the denominator part of H. Alternately, the numerator of H divided by the denominator of H so we think of the 2 transfer functions in terms of their numerators and denominator and as you know by known with the numerator part one associates what are called the 0s and with the denominator part one associates, what are called the poles.

So my omega S by E r of S looks like K a in to G consists of G N by G D divided by 1 plus K a in to G N by G D multiplied by H N by H D. Here, G N, G D, H N, H D are all polynomials which can be thought of as being factorized for our convenience, if necessary. So this is what we have. Now we can simplify this further by getting rid of this G D, H D term in the denominator, in the denominator part of the denominator and therefore, we with get omega S divided by E r of S equals K a in to G N in to H D divided by G D, H D plus K in to G N, H N, K a in to the numerator of the forward path transfer function multiplied by the denominator part of the feedback transfer function divided by the product of the denominators of the G and H transfer functions plus K a times, the numerator parts of the 2 transfer functions.

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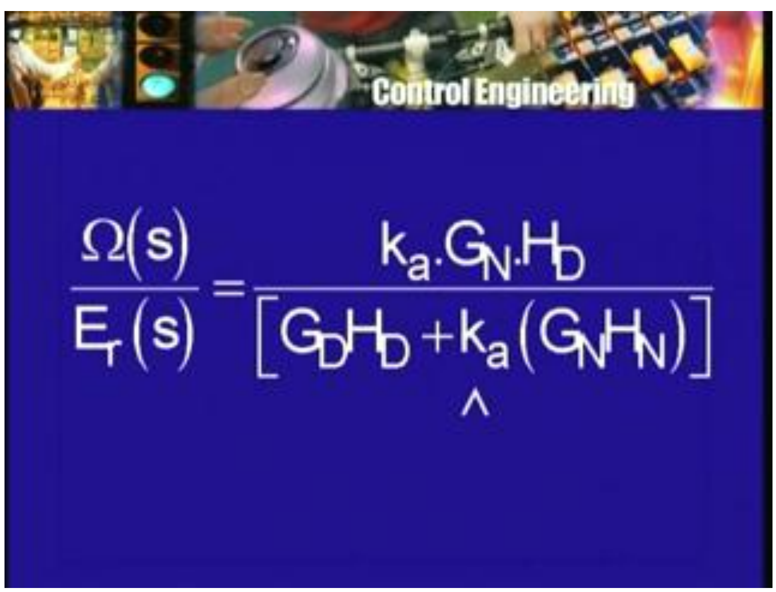

Now, the information about the G and H transfer functions as I mentioned to you earlier is normally shown in the form of what is called the pole 0 diagram and so, if here is the complex plane, on this complex plane I can show, G N by showing the linear factors indirectly by showing the linear factors by showing the 0s corresponding to G N. Similarly, there will be 0s corresponding to H N, there will be poles corresponding to H D and there will be poles corresponding to G D. Once, I do that then on the pole 0 diagram then, I can show the various poles and 0s. For example, I am just doing it completely arbitrarily this has no relevance to the example that we are looking at although we will take that up immediately, there will some 0s corresponding to the 0s of G and H.

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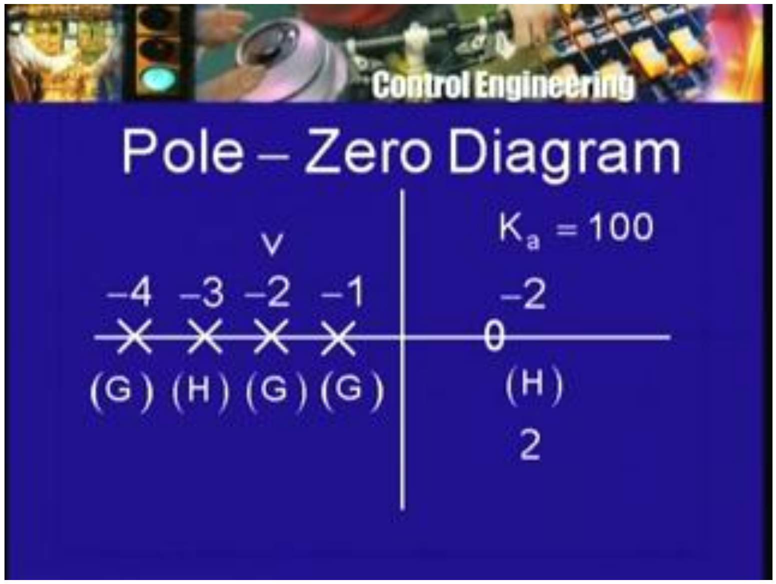

So, they can be shown, for example there may be a 0 of G. So I will put here G N in bracket that is this corresponds to the numerator of G. So I can even put G here for this 0, the H transfer function may have a 0 or it may not have. I will put say a 0 here and I will say it is a 0 of H that is all right. So I have shown a 0 of G and there is a 0 of H, I have to show then the denominators of G and H. So I have to show the poles of G and I have to show the poles of H. Let us say here are the poles of G, so here is a pole of G, here is another pole of G and may be H has only 1 pole. Let us say here is a pole of H. So here is a pole of H shown like that and so in this way, I can represent my block diagram with K a in to G in the forward path, H in the feedback path by a pole 0 diagram like this and I will put here K a as the overall multiplying scale factor and I will assume that the numerator will consist of terms like S minus something in to S minus something else etcetera, there are all the 0s similarly, the denominator will consists of S minus something in to S minus something else they are all the pole term.

So we can assume that they are all being factorize with each linear factor having the coefficient of S equal to 1. With this then this pole 0 diagram completely specifies all the information that we need to know about the block diagram that we started with, all right.
Now what is our problem, we have this transfer function omega S by E r of S and do you remember now, I am going to look at not only the steady state behavior. In fact, I want to look at the transient behavior now. So suppose I assume that E r is a constant input from T equal to 0 onward. So the E r of S is say capital E r divided by S then from this, I can

calculate omega S and then from that I can find out omega T, how do I do that by using partial fraction expansion.

So partial fraction expansion how does that proceed this denominator that we have, I have to factorize it and once I have factorize that denominator then this ratio or fraction can be split in to the partial fraction. Now, what does the denominator look like, the denominator will perhaps look like this. Now look at the pole 0 diagram that I have all ready drawn, let me put some numbers here. For example, this pole of G may be at minus 1 the 0 of G may be at minus 2, the pole of H may be at minus 3, the pole of G, the other pole may be at minus 4 and the 0 of H may be at say plus 2 and the value of the gain K a may be let us say 100. If that is the case, what is my denominator going to look like denominator is G D, H D plus K times G N, H N all right, what is G D, G has 2 poles, one is at minus 2, the other is at minus 4. So G,D is going to be S plus 2 in to S plus 4, G D pole is at minus 1, so S plus 1 in to S plus 4 that is a contribution of G D, what about H D, H had a pole at minus 3. So this will be multiplied by S plus 3 and that will correspond to H D.

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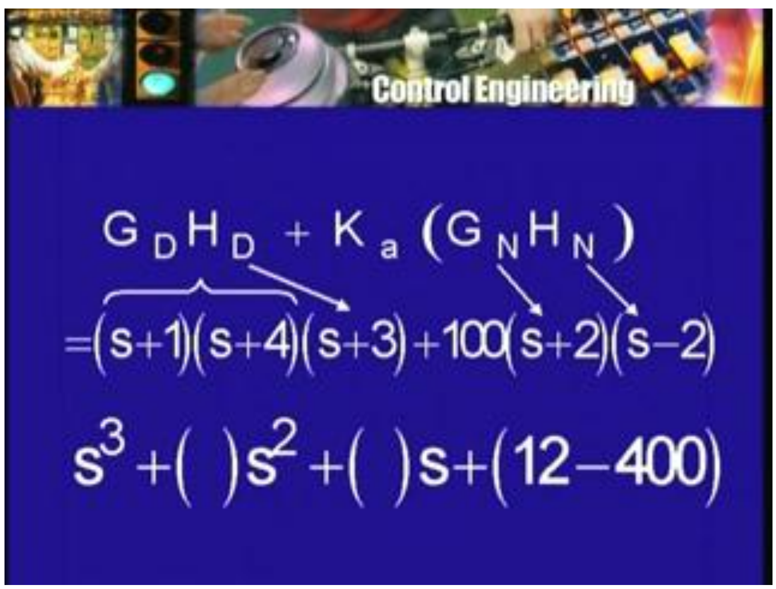

So I will have this product of 3 factors that is G D, H D plus K a, K a is 100, so 100 times the numerator of G, the numerator of G, the 0 of G was minus 2. So it will S plus 2 multiplied by the contribution of the numerator of H this is at plus 2. So this multiplied by S minus 2. So the denominator is a polynomial which is G D, H D plus K a times G N, H N, in this case it is a product of 3 times linear terms plus 100 times a product of 2 linear term. So what can you say about it as a polynomial, what is going to be its degree, there is this S, S, S 3 time. So I will get an S cube term from there whereas from this I will only get an S square term.

So the S cube term will remain therefore I will have S cube as one of the terms in the polynomial plus what there will be an S square but where will that S square term come from. It will come from this but with also come from this. So it will be some term in to S squared what that is can be worked out plus what about the coefficient of S. Again, this products will contribute this product with contribute, so something in to S plus what about the constant from this I will get 1 in to 4 in to 3 that is some term 12 plus from this, I will get 100 in to plus 2 in to minus 2 or minus 400. So it is going to be a cubic, if it is a cubic then, I can conceptually imagine factorized in to 3 linear factors. So I can talk about its 3 roots.

Now it is not going to be easy to factorize a cubic as you know. In fact, I have asked you earlier and mentioned that factorization of the quadratic is very easy, we have learnt it back in school, factorization is a cubic, you may have learnt in your college algebra course but you have most probably already forgotten, it its difficult, if it is a quadratic, biquadratic that is 4th degree polynomial, it is still more difficult but there is a formula and beyond the 4th degree, there is unfortunately no formula, we have to use numerical techniques to find out the roots. But we can conceive that this cubic is factorized in to 3 linear factors and therefore we have 3 different roots. Once we factorize the denominator then, we can go back to the ratio the numerator divided by the denominator.

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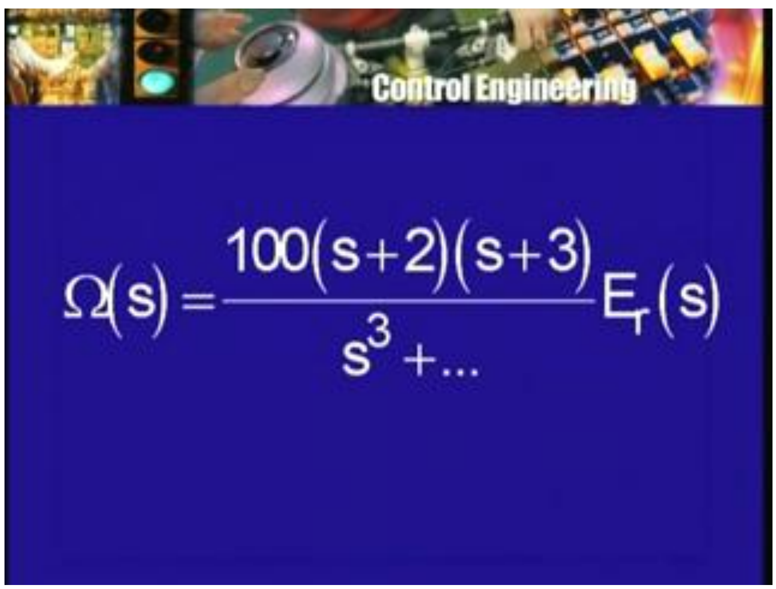

In this case because K a is 100, it will be 100 G N, G N is the numerator part of G. So that will be in to S plus 2 once again in to H D, H D is the pole of D therefore there will be S plus 3 divided by the same cubic in the denominator. So this is what the whole transfer function is going to look like and now, we can expand it in a partial fraction expansion. I have all ready mention that and I have asked you to read up from you algebra book as to how to carry out the partial fraction expansion. You do that of course this is going to be multiplied by E r of S and for many inputs E r of S will also be a ratio of 2 polynomials. So actually therefore I will have a polynomial divided by another polynomial, I have to factorize the denominator polynomial, do the partial fraction expansion and then do the inverse Laplace transfer from that I will get omega T, that is the approach.

The hurdle is that I get a denominator polynomial which is such that it depends of K a and there seems to be no nice way of determining, what its roots are, that is the problem. Now Evans, in the 1950's thought about this problem and gave a qualitative approach which can be made partially quantitative to get some idea of what the roots of this complicated thing, may be like. So this is the root locus method of Evans like the signal flow graph, gain formula of Mason, the root locus technique is useful for really understanding by you and me, for small order systems. When you have larger order systems, the root locus method does not become so useful if you and I have to do it by writing expressions by manipulating them and so on. Fortunately, today we have program packages which can do all of this and in fact which can plot what is called the root locus for you, all you have to do is enter the information for the G and H transfer functions, specify the gain K a or the gain of the transfer function of the system, the program will plot for you the root locus that is what happens for that value of K a or as the value of changed.

So, if you have a higher order system not a polynomial of degree 3 or 4 or may be 10. Similarly, the numerator also is more complicated then factorizing, doing partial fraction expansion, finding the coefficients of the partial fraction expansion and all that. We can avoid it by using the programs which give you the location of the roots or therefore, we say that they plot the program, plots the root locus. But to repeat what Hamming said the purpose of all of this is not computation but inside, we want to understand, what may happen, what can happen therefore, we with follow the root locus method for a lower order system and see what one can get out of it.

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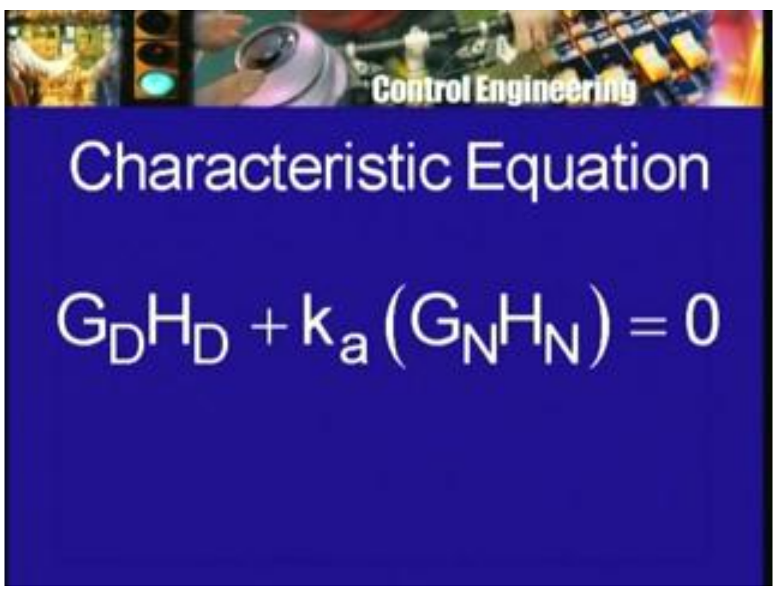

So the problem then is when I have a polynomial which looks like G D, H D plus K a in to G N, H N, what are it is going to what are going to be its roots, not for just one value of K a because we want to see the effect of the gain but when the gain is changed, what is going to happen to the roots. So, we are looking at the equation this equal to 0 or in other words, we are looking at the roots of the polynomial on the left hand side. This kind of an equation is called a characteristic equation of the system. So one says that we are finding the roots of the characteristic equation of the close loop feedback control system and that involves finding the roots of a polynomial which looks like this. There is a part G D, H D which corresponds to the denominator of G H, there is a part G N, H N which corresponds to the numerator of G H and there is this multiplying coefficient K a that multiplies the numerator part and the sum of these 2 is the polynomial that we are looking at.

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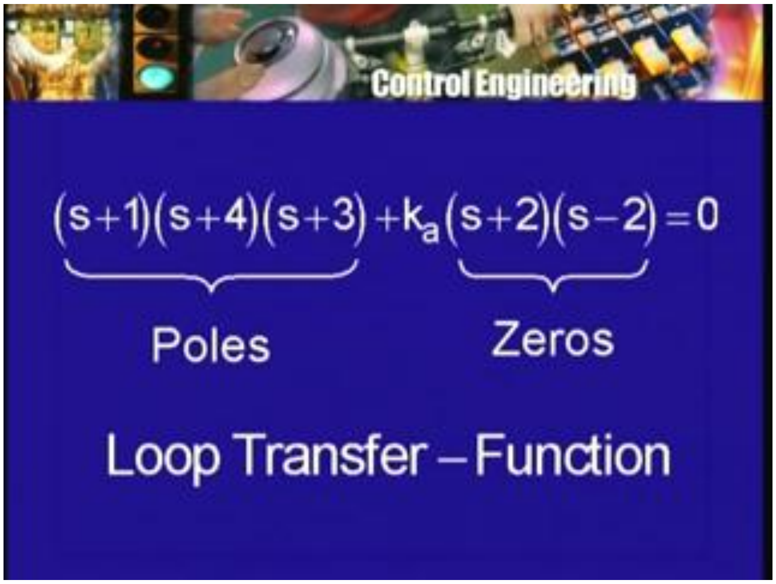

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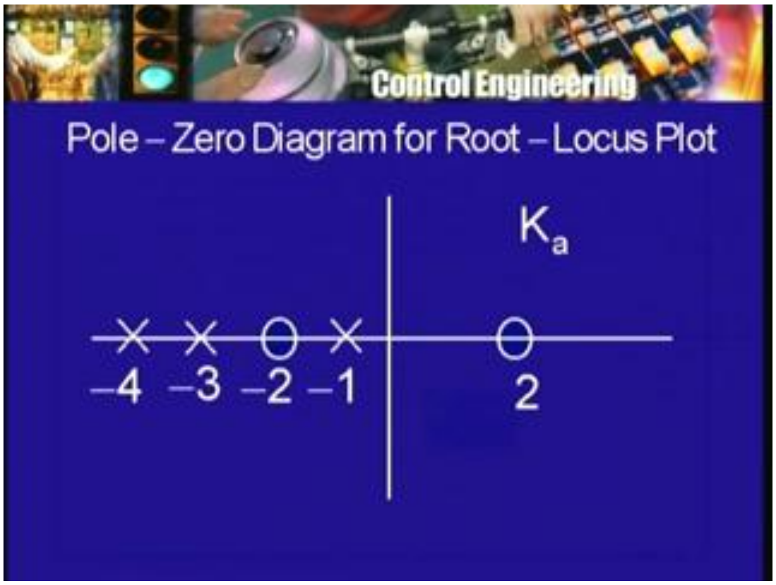

In our case, this polynomial there the characteristic equation was S plus 2 in to S plus 4 in to S plus 3 plus keeping K a and not writing 100 in to S plus 2 in to S minus 2 which results in a third degree polynomial and therefore, we are trying to find out its 3 roots as a function of K a, what about this part of it, that part of it comes from the denominator G H. So that part comes from the poles of both G and H, what about this part, this part comes from the numerator or G H therefore, it comes from the or it represents the 0s of the transfer function or the product G H which is called the loop transfer function without the multiplying coefficient K a.

So, for this all we need to know the locations of all the poles and the locations of all the 0s in the loop transfer function. So for our particular example, then the locations are once again I will draw the diagram, there are 3 poles, one is at minus 1, another is at minus 3 and the third one is at minus 4, of these of course, 2 of them are coming from G, the other one is coming from H but as far as our treatment is concern, it does not matter which is coming from which there are 3 poles. The other part are the 0s, one of them is coming from G, the other is coming of H, once again it does not matter which is which. So I am just going to show here 1, 0 at minus 2 and 1, 0 at plus 2 and I want to look at it as K a varies and not with K a equal to 100 only.

So, I will just write K a there, so this is the problem or represented on a pole 0 diagram, the 0s are given, the poles are given, the gain K a that sought of seats between the 2 is to varied what happens to the roots of the characteristic equation or the characteristic polynomial that is the problem and that is the problem which Evans solved and he gave a graphically technique, therefore an approximate technique or a technique really to get some idea of what is going to happen rather than a technique for exact computation, that is the root locus method. Now, it can come as a surprise that just looking at this and of course applying once thinking, following the steps of Evans, there is quite a bit that we can say about the roots. For example, if K a, the gain is very small by that we mean I am not going to put K a equal to 0 because if the gain is 0 then, there is nothing in the forward path. So I will get 0 output but K a is very small.

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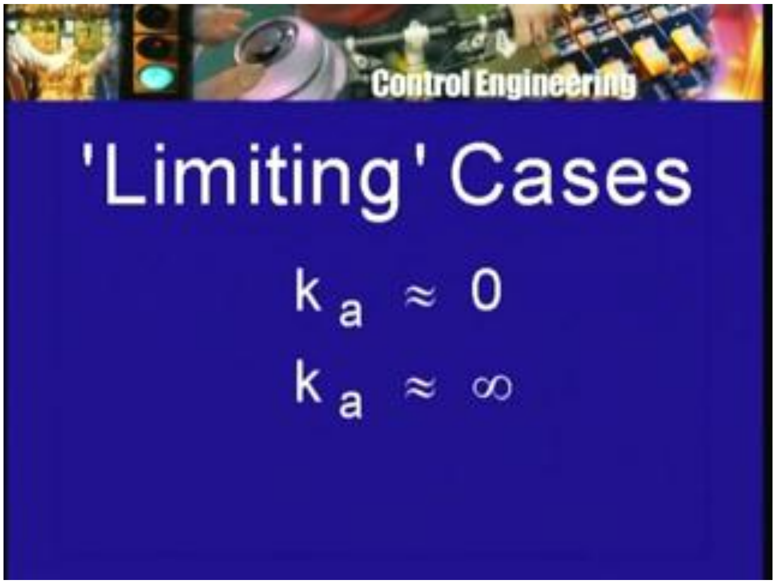

So K a is very close to 0, now what do I mean by very small, I cannot really say but think of the limit therefore as K a is decreased, where will be the roots of the characteristic equation that is one case, second is you make K a very large, may be 10,000 not in practice of course but in theory, in our study. So I will put it as K a equal or nearly equal to infinity meaning that if K a is very large, where will the roots of the characteristic equation be Evans gave rules, 2 rules from which you can get this information for values of K a which are not very small nor very large, in other words for normal values of K, therefore which lie between 0 and infinity.

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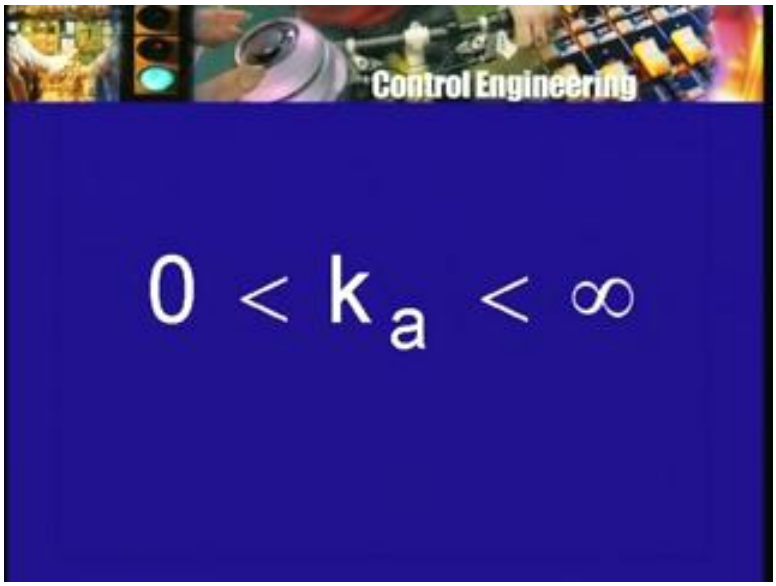

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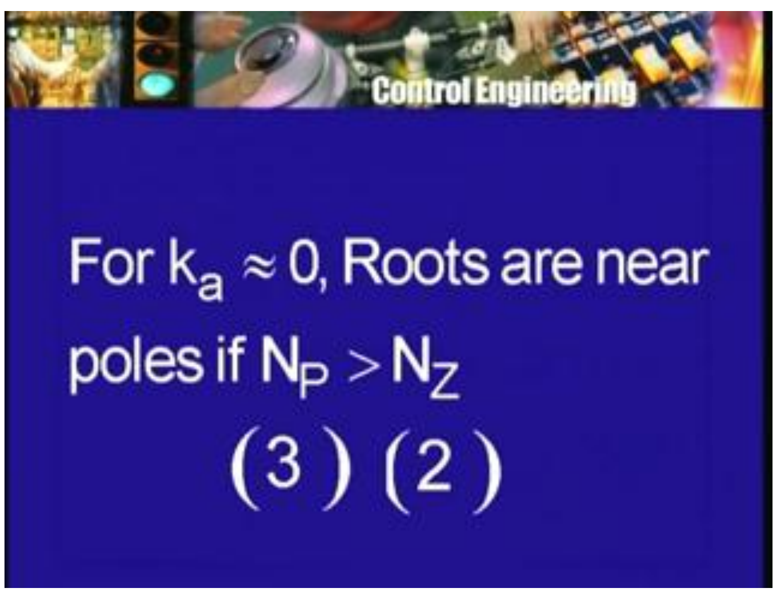

You have to do some more work to in order to find out, where the roots will be but the 2 extremes, approaching 0 and approaching very large gain that is infinity, there are 2 very simple rules that tell you what happens to the root. Now of course that depends on whether the number of poles is equal to the number of 0s or is greater than the number of 0s or is less than the number of 0. So, if I write N p for the total number of poles of G and H, N z for the total number of 0s of G and H, in our case there are 3 poles of G and H together. So N p is 3 there are 2, 0s of G and H considered together so N z is 2, N p is greater than N z, the number of poles is 3, the number of 0s is 2.

There can be a situation, where the number of poles is equal to the number of 0s and in principle, you could consider a situation, when the number of poles is even less than the number of 0s. So one has to consider these 3 cases, usual case is this the number of poles is greater than the number of 0s. For, our motor problem that is exactly what is happening the number of poles is greater than the number of 0s because you remember, the forward path transfer function G had a quadratic in the denominator therefore only 2 poles, the feedback transfer function was only k tg. So, no pole nor 0, it was just a coefficient.

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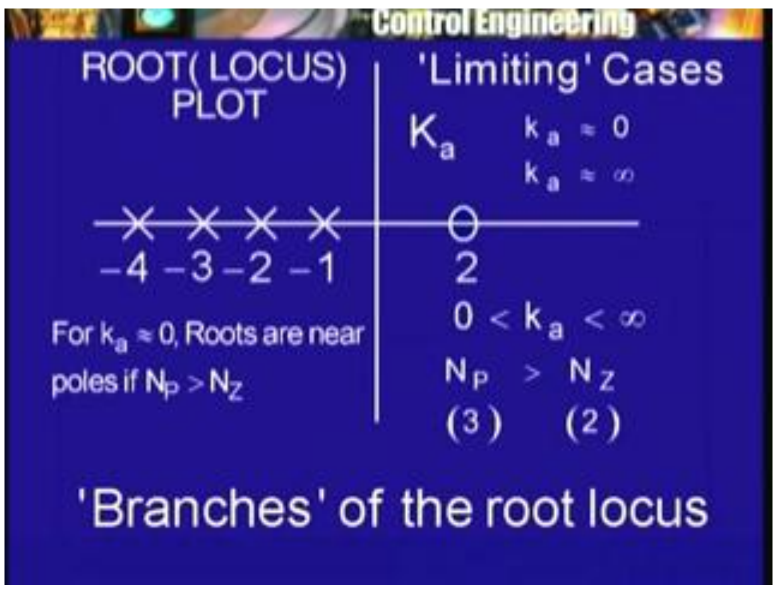

So the loop transfer function G H or the loop gain G H had only 2 poles and no 0. So N p was2 and N z is actually 0. So, certainly N p is greater than N z, if this is the case if the number of poles is greater than the number of 0s then, one of the rule of Evans says that when the gain is very small, the roots will be very close to the poles. So for K a very nearly equal to 0, the roots are near the poles and the smaller the K a is, the closer the roots will be to the poles of the loop transfer function and therefore, one can say that if you think of starting at 0, K a equal to 0 and then gradually as if you have a knob by which you are changing the gain, you change increase K a starting from 0, slowly and you keep on increasing and in theory, you increase it indefinitely make it very large.

The roots are not going to stay unchanged, when K a is small the roots will be having some values, when K a is very large they will have some other values, when K a is say 50 they will have still other values, when K a is 1000, there will be still some other value. So one can think of the roots changing there location as K a is changed and Evans's method enables you to sketch or get some idea of what this would look like, if you were actually going to plot the root locations for various values of K a and therefore it is called a root locus plot or we could simply call it the root plot but as in coordinate geometry one uses the word root locus. So the root locus plot is just a plot of the location of root of the denominator of the close loop transfer function. As the gain K a in the forward transfer function is varied from 0 to infinity or between 0 and infinity that is the gain, may be considered to be very small, it may be considered to be very large and then, we want to look at the values in between.

So one of Evans's rule the first one for example, we will say that if the number of poles is greater than the number of 0s then, for small value of K a the roots are very close to the poles and therefore, one can say that the root plot or the root locus plot or the plot of the roots starts at the poles. There are 3 poles, so there will be 3 branches of the root locus, this is the terminology that is used. There will be 3 branches of the root locus which will start at the poles, it is a very simple rule not very difficult to remember, the root locus branches or the branches of the root locus plot and there are branches why because there are 3 root in this case, in general there will be more than 3 or less than 3 roots not just one root, the characteristic equation is a polynomial not of degree 1 necessarily therefore, there will be more than one root therefore, there will be branches of the root locus.

So one says in this case that if N p is greater than N z then, looking at our earlier formula here for example, we can see that always there will be 3 root. So the root locus will have 3 branches so the number of branches of the root locus will be equal to the number of poles of the root transfer function and one says that these branches will start at the poles, meaning that when K a is very small, the roots will be very close to the pole location and in theory when K a is 0 then, what does the polynomial become this part is already gone, it is simply the part that corresponds to the poles. So the roots are the poles but of course this is not of interest because K a equal to 0 will simply give you a 0 output but no harm in saying that the root locus branches start at the poles for K a equal to 0.

Now, what happens when K a becomes very large the answer is there will be branches or there will be roots which will be very close to the 0s, for large value of K. Now I have 2,0s, so there will be 2 roots which will be close to these 2, 0s but the number of roots is 3. So what about the third root. So now for the third root, one says that it has an asymptotic behavior and something will happen to it as K tends to infinity, it will be not be near the 0s of the transfer function but the third root of the remaining root will be in fact going far away in the complex plane, with an asymptotic behavior and therefore one can talk about what are called asymptotes to the branches and how many asymptotes will there be.

Well, here we have N p equal to 3, so there are always 3 branches of the root locus, all 3 of them start at the poles N z equal to 2. So there will be only 2 of them which will be close to the 0s and therefore, one says they will end at the 0s and the third branch will approach an asymptote and Evans gave a rule for determining the number of such asymptotes and the location. So that one can draw the asymptotes and expect that for large values of gain K, the roots will be close to the asymptotes. In our particular example, the number of asymptotes is only one and I will give you the formula very soon. In this case, the asymptote which is only one will really consist of the negative real axis going towards the negative real part becoming infinite end. So this is the asymptote that means what, the third root will lie close to this asymptote and therefore, the third root will become large and negative and very close to being real, whereas the other 2 roots will be close to the 20s of the loop transfer function.

So the 3 roots will start of at the 3 poles, so think of some curve starting from here, the branches of the root locus then, we do not know what happens to them right now, 2 of them end up at the 2, 0s. So they will end up here, the third one will go towards the asymptote and the asymptote is going towards the negative real axis or the negative real axis is the asymptote. So, the third root will approach the negative real axis going towards infinity. So the third root will become very large, negative real or very close to being a negative real number, whereas the other 2 roots, one of them will be very close to minus 2, the other root will be very close to plus 2.

So think of it just 2 very simple rules which depending on N p and N z, tell you that the number of roots will be so many therefore the root locus will have so many branches and where will the branches start that is for K a very close to 0, where will the roots be, where will the branches end that is for K a very large or approaching infinity, where will the roots be, if N p is greater than N z the number of branches is N p all of them start at the poles among them N z will end at the 0s and the difference N p minus N z will approach asymptote. So the number of asymptotes will be N p minus N z.

In our case, there are 3 poles there is only one 0, where therefore, there are 2, 0s therefore there will be only one asymptote, if N p is less N z the situation will be reversed, the number of the degree of the polynomial will be the degree N z that is of the 0 part, the branches will be the number of 0s then branches will all end at the 0s but N p of them will start at the poles and the remaining will start from a region of the z plane which is far away from the origin and therefore, there will sought of come in from infinity, starting being very close to again asymptotes. So, one can find out the position of the asymptotes in both the cases. In the special case, when N p equals N z now that is not a very common case but it is useful to think of it, when N p equal to N z then what then the number of branches is simply each one of them the same number. So they will start at the poles they will end at the 0s and therefore there are no asymptote.

So there will be no roots which will be very large out there in the complex plane or towards infinity as one calls it, whether for small value of K or for large value of K. For all values of K, the roots will not become very large although they may become large but they will not increase to infinity or they will not for very low gain K, as you reduce the K also increase the infinity. They will start at the poles, they will end at the 0s, equal in number. So these are the 2 simple rules of the root locus or 3 if you wish, the number of root locus branches is the large of N p and N z, secondly the root loci start at the poles some of them, if not all of them, the root loci end at the 0s some of them, if not all of them and the remaining branches either go asymptotically out in the complex plane, if N p is greater than N z or they start from infinity as it where that is being very large amplitude in the complex plane and as K increases from 0, they will come closer to the location of the 0s.

So these are the 2 simple rules of the root locus method. We will looking at some rules of the root locus method but in order to emphasize the power of the root locus method. Let me give you the solution that the Evans root locus approach is finally able to give you. I have this pole 0 diagram, I am going to start at these 3 poles, I am going to end at these 2, 0s and one branch is going to end towards the negative real axis, all right. Now what is going to happen in between I am going to say something which we will look at in detail. So there will be some more rules which have to be looked in to. There is a rule that determines the real axis portions on the root axis, there is what part of the root locus can be expected to lie on the real axis or the other way rather, what part of the real axis will correspond to some root location, for some value of K.

Now, there is a rule which is a little difficult but not that difficult which can be used to determined the parts of the root locus which are on the real axis or the parts of the real axis which belong to the root locus. So this rule is called rule for determining the real axis portion of the root locus, the portion of the real axis which belongs to the root locus. Now there is a simple rule for that that requires looking at the pole 0 diagram, looking at the real axis and then observing something and I will not tell you what it is right now, but I will simply give you the rule as it is.

In this case, using that rule this portion of the real axis will belong to the root locus I am going to put an arrow on it to indicate that as K increases there will be a root which will be here then, it move that is for a larger value of K, the root will be here for still another value it will be here and so on. For very small of course the root will be very close to minus 1, for very large value of K it will be close to 2. So one branch of the root locus will simply look like this and one says, it starts at minus 1 and goes towards 2 along the real axis in this direction, what about the other 2 branches of the root locus.

Well, again some further rules we will tell you that there will be a branch or there will be a portion of the root locus here between these 2 poles and nowhere else now, on the real axis and therefore there will be a breakup of the 2 branches, the 2 branches will go towards each other and then, they will breakup or things of that sought. Now this is something which we will have to look in to but it is interesting that simply from the pole 0 diagram with the help of some very simple rules, one can get some idea of the branches of the root locus, what they are going to look like and then, from that get some idea of what the response of the close loop system for certain inputs is going to be like.

We will able to conclude for example that, if the gain is K a is too large with integral feedback then the system will oscillate that is, it will involve oscillation. The speed when it changes momentarily before going back to the same value will oscillate. Now, if the behavior is tolerable well and good, if it is not then the value of K a that you have chosen is too large you will have to reduce the value of K a, information like that can be obtained by looking at the root locus plot as given by Evans and his rules. So that is the thing that we are going to look at next, we will be looking at Evans's root locus method. Thank you.