Control Engineering
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Lecture - 29

Let us now take a look at, a different scheme of feedback which I mentioned earlier but we have not gone into it, namely integral feedback, what we had been looking at was proportional feedback. Now the first time one looks at integral feedback, it looks rather mysterious and difficult to figure out exactly how it works. But, we will look at it first from the point of view of the transfer functions involved and then try to understand it from a more dynamic point of view.

So, we have our old block diagram in which I had shown omega s as related to 2 inputs, E a of s, the applied voltage input and the load torque input through 2 transfer functions. So, let me redraw the diagram here is T l of s and that goes through a block whose gain I had called GT of s, T for torque. Then, there is E a of s the applied voltage and that goes through a block which I had called G a of s and these 2 are added together, one with a plus sign, the other with the minus sign to produce omega s, the Laplace transform of the angular speed or velocity of the motor.

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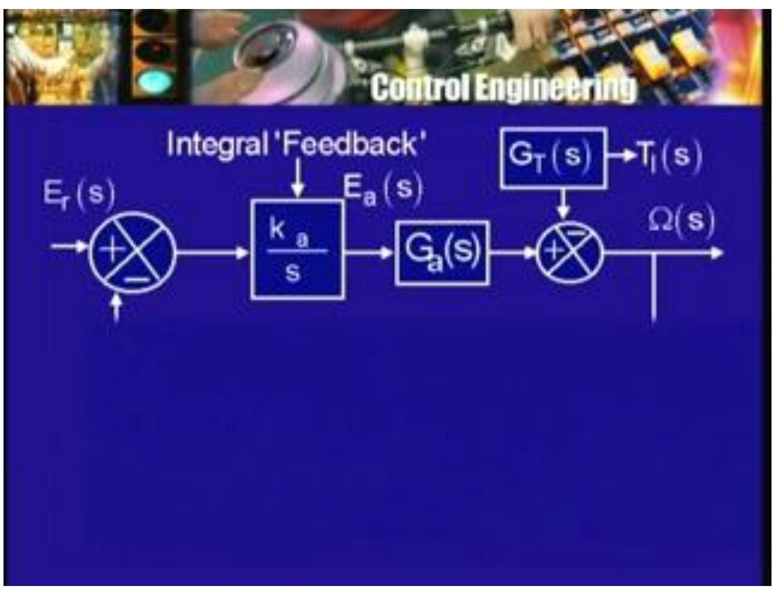

So, in other words this block diagram represents only the equation omega s equal to E a into G a that is the transfer function G a multiplies the applied voltage E a minus the transfer function G T multiplies the load torque T l. We have of course had a look at the expressions for these 2 transfer functions G a and G T earlier. We will need them a little later as we go on. Now, in the open loop control situation or that is in the situation of control, where no feedback is used. You remember that what we do is we just apply a constant voltage to the input of the system that is at the armature, whether on or the load torque changes, whether or not the speed changes. We just apply a constant input voltage, this means we are not going to look at, what the output is doing whether the output speed is at the correct value or is it any different.

We just apply a pre calculated voltage at the input and under normal conditions rated torque parameter values being exact etcetera. We expect that the output will be at the expected value at the rated value and that is what, we have referred to as the open loop of course, there is no loop yet, there is no question of opening a loop I had also called it preprogrammed or pre calculated input or pre calculated control action. I want a certain output speed under certain conditions, what should be the input voltage, I calculate that and try to maintain it at that value. As I told you of course, no control system is operated exactly like this, there is always at least the human operator or supervisor of some kind who will look at what is happening and who will usually be have some means at his dispose of or making some adjustment.

In this case, the adjustment would be that of the armature voltage that is the probably if you are using it in this fashion then, there will be a provision for adjusting the armature voltage and there will be an operator, who will be looking at the speed and seeing, whether it is the same as the desired value or not. When we saw that we can change the scheme of things by providing for an automatic adjustment of the applied voltage and that was the Lenard system of control, in which the speed is measured or a voltage proportional to the speed is obtained through a tacho generator.

There is a reference voltage, the difference between the two is then amplified and fed to the armature of the motor. The reference voltage of course is kept fixed, it is kept fixed at a pre calculated value depending on what speed you want under rated conditions and I told you that contrary to what many text book say, such a system will work with 0 steady state error that is, if the disturbance is or the load torque is at the rated value. The parameter values are exactly what they were assumed to be then, one can calculate a reference voltage such that the motor will run at the desired speed although, if when I compare the reference voltage with tacho generator output voltage, the 2 will not be equal. There will be a small difference between the two and this difference amplified by the amplifier produces the armature voltage but this difference is not the error, it is a difference between the reference voltage and the feedback voltage. But this, the difference does not represent any error. It provides you with the required voltage or required output which we have amplified will generate the required applied voltage, voltage to be applied to the armature.

So there is no steady state error however, with that system which we had just studied, if the load torque changes then, the speed is not going to remain constant because we are keeping the reference voltage constant, the speed is not going to remain constant. However, the scheme is such that the change in speed because of change in torque that is because of disturbance signal will be less then, what it was in the preprogrammed case and we saw, how we can figure that out with the help of the Mason gain formula applied to the block diagram. We will do the same thing now to our system, what we will now put is what is called an integrator in the forward path. Now, what is an integrator, now an integrator some of you have already perhaps in your electronic circuits, analog circuits, encountered integrator circuits. For example, an operation, an amplifier can be configured suitably to produce this action of integration. So, if I show it as a block, what will be the transfer function of any integrator. The transfer function of an integrator is 1 divided by s, now this follows because of a another theorem of the Laplace transformation that is if I have a function f of t and its Laplace transform is F of s, capital F of s.

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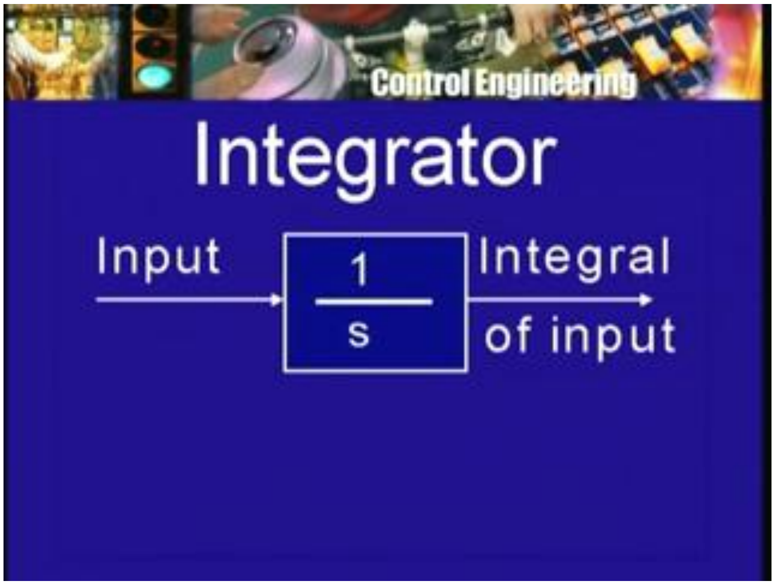

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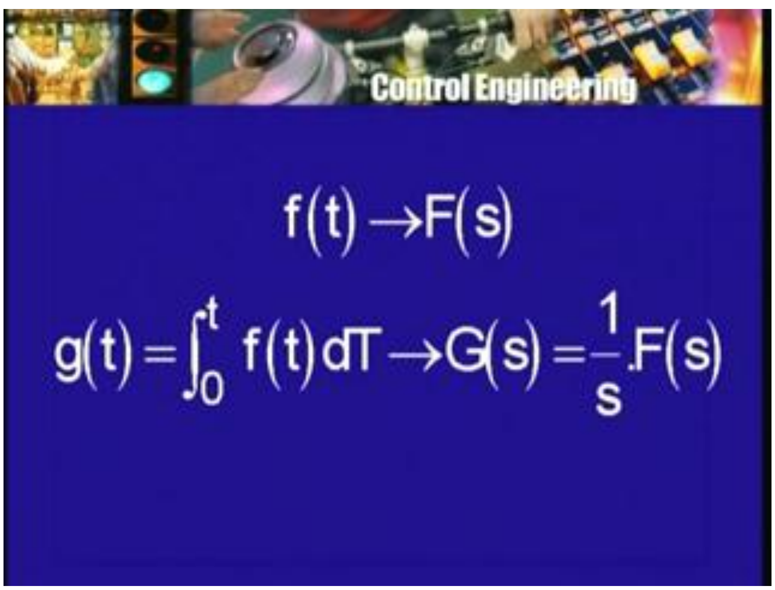

Now I defined another function g of t, which is an integral of this function f. Let us say starting from 0 up to a running time t and therefore I have write this as f tau, d tau. So from the signal f through this action of integration with lower limit 0, upper limit is the running time t, I obtained another signal g of t then, the Laplace transform of this g of t which is G of s is related to F of s by the simple relation G of s is 1 by s times F of s.

So, if I look at the integrator, its output is g t, its input is f t, output transform is G of s, input transform is F of s and therefore, the transfer function of the integrator, what was the definition of the transfer function of the integrator, a transfer function of a system. It is the ratio of the Laplace transform the output to the Laplace transform of the input. If the initial conditions are 0 or alternately, it is the Laplace transform of an appropriate part of the output divided by the Laplace transforms of the input and the concept transfer function is useful only when this ratio of the 2 transforms does not depend on the input. In this case of the integrator, the ratio of the 2 transforms G s over F s is simply 1 by s and so, it does not depend on what F t is.

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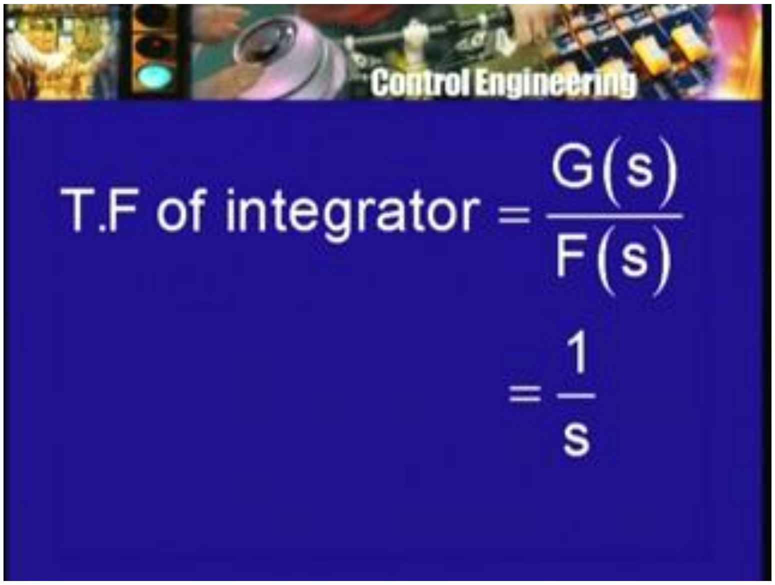

Now of course, graphically we can imagine, what the output of the integrator will be, if we know the input to the integrator. So let me draw the block diagram once again and I will put the integral symbol here 0 to t, the transfer function is 1 by s, suppose to that I give a constant input starting from time t equal to 0. Let us say, its value is 1, so this is the constant function 1 which is the input to the integrator then, what will be output of the integrator. The output of the integrator will be integral 0 to p of 1 d tau and therefore, it will be a unit ramp function. It will be a function whose graph will be a straight line passing through the origin and it is the formula for it will be simply t at time t, if instead of a constant function 1, I apply a constant function of some other value. Let us apply constant function then the output of the integrator will be 2 t, remember that the integration from 0 to t. If we do not integrate from 0 to t then, we may get an additional constant, an additive constant as a part of the output.

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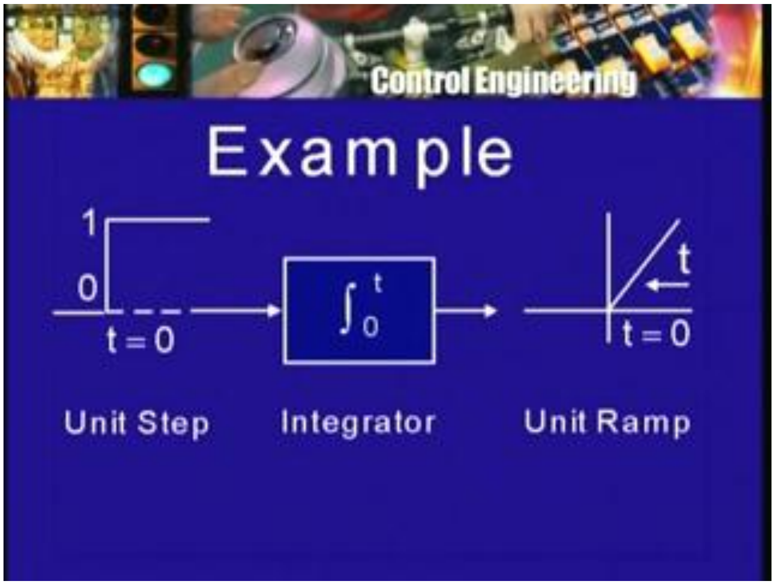

So that is why I said that we are going to look at only a part of the output and its transform rather than the entire output, in the general situation. So this is the integrator, its transfer function is1 by s, the point is that if I have a constant input to the integrator then the output of the integrator goes on increasing. It will increase without limit, of course in practice, you cannot have a physical system whose output some variable can simply go on increasing and you know that when you use an operational amplifier in an integrator circuit, you never apply a constant voltage to the input of the integrator because if you did, what happens to the operational amplifier circuit, the output gets saturated.

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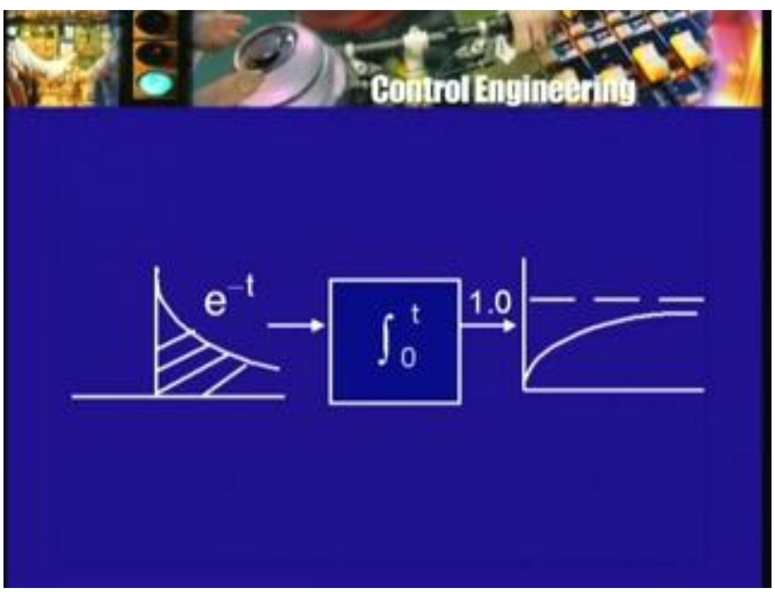

So the output never goes on increasing beyond a certain limit which is set by the supply voltage. So a practical integrator, if you give it a constant input, it will never produce an ideal ramp which simply goes on increasing but suppose, now to the integrator you do not apply a constant voltage but you apply some other waveform of voltage. For example, you may apply an exponential input. So, let us say to the integrator I apply the input e to the minus t, it is an exponential function. So, here is the integrator, integral 0 to t and to it I apply e raised to minus t, the input signal will go on decreasing as time increases, the time constant in this case will be 1.

So after t equal to 10, the signal is virtually 0, the input is virtually 0 but what about the output, what will be the output of the integrator, when the input to it is e to the minus t.
Now one way of finding out will be to use the Laplace transformation, let us do that what is the Laplace transform of e to the minus t, it is 1 divided by s plus 1. This is the Laplace transform of the input, what is transform function of the integrator, it is 1 divided by s.

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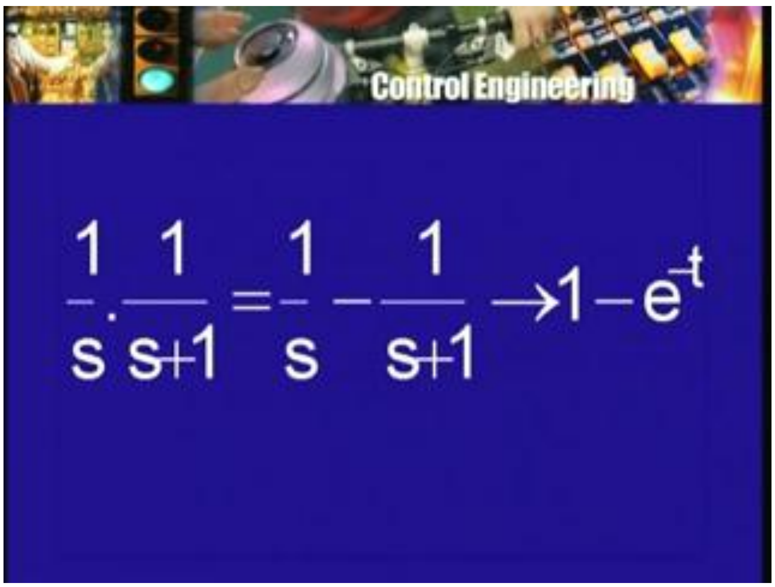

$$\frac{1}{s} \cdot \frac{1}{s+1} = \frac{1}{s} - \frac{1}{s+1} \rightarrow 1 - e^{-t}$$

So what is the Laplace transform of the output 1 upon s multiplied by 1 upon s plus 1. Now in this case, the partial fraction expansion is very simple. So it is simply 1 by s minus 1 by s plus 1, as you can verify, this s plus 1 goes here, s goes here difference is 1 divided by s into s plus 1. So this is the partial fraction expansion of the output transform and now it is very easy to invert this. If I invert this what do I get 1 by s gives me 1 and 1 by s plus 1 gives me e raised to minus t.

So the output will be 1 minus e raised to minus t. So, what will be the output waveform now this is the waveform which should be familiar to all control people and we have already had look at such a waveform. It is what is sometimes called a signal which rises exponentially to a steady value, it is not the signal e raised to t which keeps on growing without any limit. It is a signal which rises to a steady value and the waveform of it or the graph of it will be something like this.

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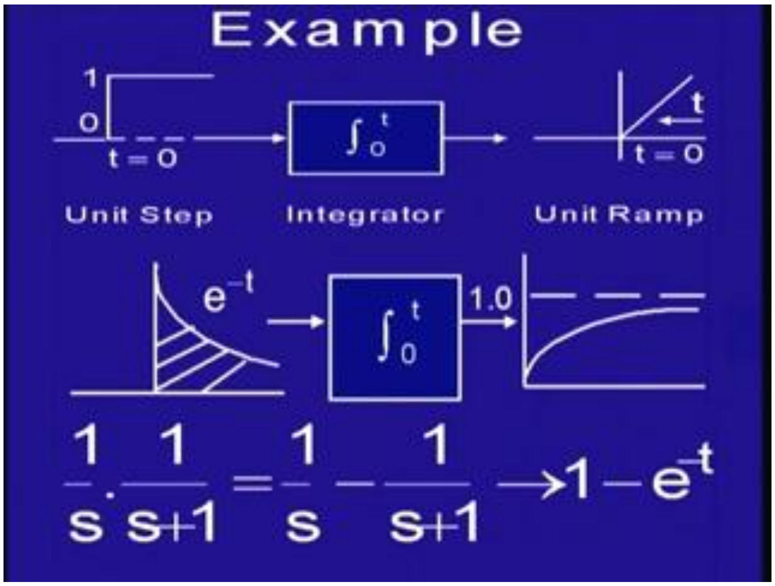

So to an integrator, if I given an input which is the exponential decaying function. So that the input eventually becomes 0, after some time the output however, has not become 0 the output approaches the limiting value 1 and so after the same amount of time that is elapsed which makes the input virtually is 0. The output of the integrator is not 0 but it remains constant. Now this is the special property of the integrator and it is not really puzzling, if you are looking at the process of integration because you have in the calculus courses looked at the operation of integration, integration is area under the curve.

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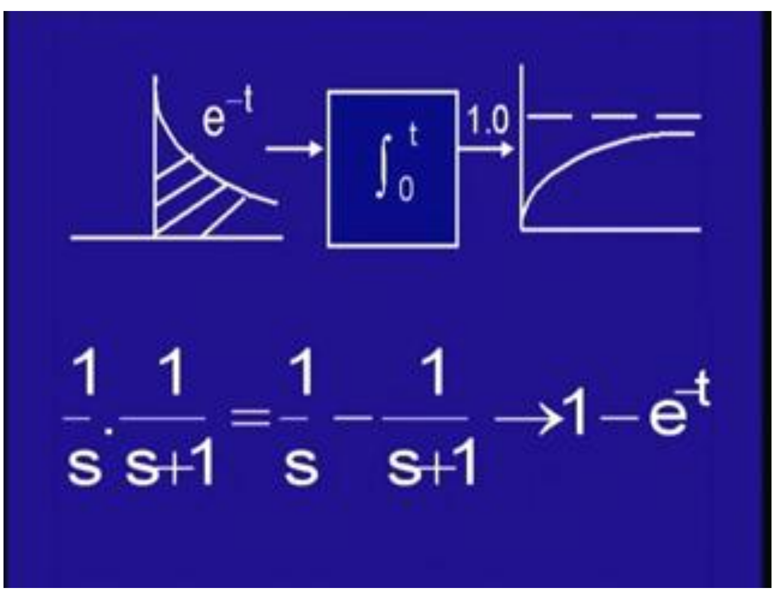

So, if you look at it that way then, one can see why, when the input goes to 0, the output does not go to 0, but the output approaches a constant value in the limit. This is because the integral is the area under the curve and as time increases I get more and more area under the curve. So the output will go on increasing however, and that is not clear from the graph. The area under the exponential curve is finite, it does not keep on growing, it keeps on approaching the value 1. So looking at integration as the operation of area under a graph or under the curve that represents the given function, we can see that even when the input approaches 0, the output may not and will not very often approach 0 but even it may approach a constant value which means that eventually after say time 10 time constants in this case, the input is 0 but the output is non-zero that is from a non from an input which appears to be 0. When you look at it after certain time, the output has a reason which is not 0 and this is something which may appear to be little mysterious that even when the input has become 0, the output of the system has not become 0 but in fact, it has approached a constant value. But remember, the input was not 0 all the time, if to an integrator and so, this is of course very trivial but let us look at it once again.

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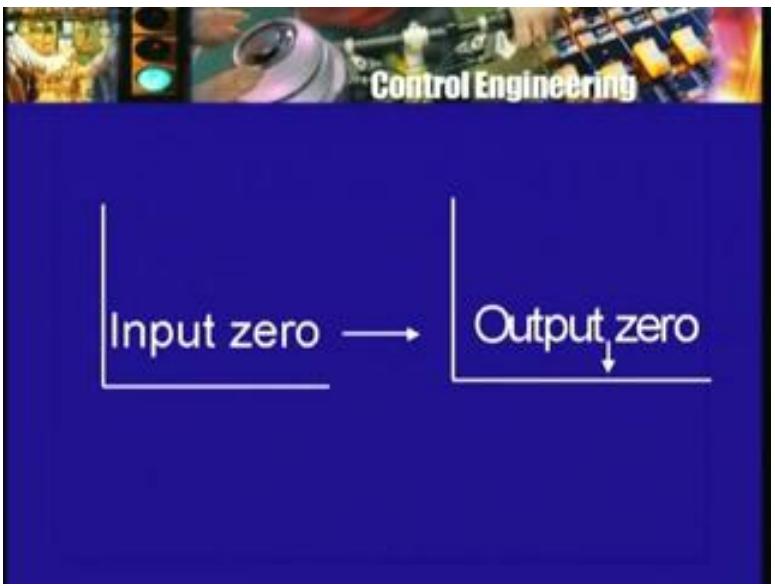

If to the integrator, I give the 0 function as the input then what will be the output. The area under the 0, curve is 0, so the output will be 0. So if you give an input which is all the time 0. Then, the integrator output will not be non-zero but in fact will be 0 but if to the integrator, you gave an input which has been non-zero for some time and then, becomes 0 or very close to 0, then its output need not be close to 0 at all. Of course, depending on the input waveform the output may not build up to steady value like one, for example if I apply a sinusoidal input to an integrator, what will be the nature of the output.

Now remember, your calculus course, if the derivative of sine is cosine then, the integral of cosine is sine. So if I have an oscillating input given to an integrator, the output of the integrator will also be oscillating although it in addition, it may have a DC component depending on when you start the integration process or in the operation if I had integrator, if the capacitor which is put across the feedback link, if it is initially charged then there will be a DC shift and so, the output will not be the sinusoidal but will be sinusoidal with a DC shift. If the input is not sinusoidal but goes through may be positive and negative variation then, the output will not approach a constant value, it may go through variations also depending on, how much of positive and negative variation was there in the original signal, the output signal may finally be positive or negative or may even go to 0. To take just a very simple example, suppose I take an input which has this form that is it is constant for some time and it is negative constant for an equal amount of time and then it stays 0.

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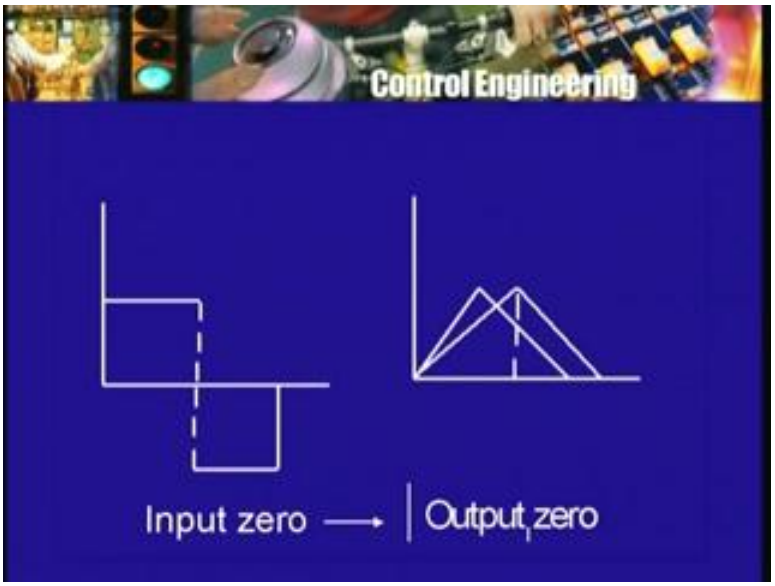

So it is a square pulse, symmetrical square array but only one pulse applied, what will be the output of this integrator. Now you perhaps have done experiments of this kind applying a single pulse or applying a pulse repeatedly after sufficient time delay, what does the output waveform look like and so on. So in this case, if I apply a square, single square pulse, symmetrical square pulse to the the integrator input ,what will be the output. Well, the input is constant for certain part of the time, so the output goes on increasing but then the input certainly becomes negative and remains constant for another interval of time. So the output goes on decreasing there after the input remains 0, so the output now being 0 already, remains 0.

So, if the input is a square pulse the output is a triangular pulse. So the operation of integration can do interesting things of this kind and therefore it is used in electronic circuits as you perhaps aware. We can use it in our control application as well. Going back to our problem what I want is, a constant voltage here at the armature terminal. I want a constant voltage at the armature terminal, can I get it from a voltage which does not remain constant but which can go to 0 also, yes, the answer is I can do that, if that input to the integrator has been non-zero for at least some time, if it has been non-zero for at least some time then the integral will build up to a constant value and therefore even when that input become 0, the output of the integrator and therefore the input to the armature can remain constant at a certain value. So this is the idea.

Now that voltage which is to be integrated, we can think about it as obtained from the actual speed and the desired speed that is if there is an error between the actual speed and the desired speed then, this difference will be non-zero and if that difference is integrated, it can result in something which is non-zero and in fact may remain constant, if the difference becomes 0 after some time. One immediate conclusion from this is that, if you are going to use an integrator somewhere in the system then, there will have to be a non-zero error for some time, to produce an output which is constant after that time. In other words the system will have to have a non-zero error for some time, if you want no error after that time.

So, if you want the steady state error to become 0, you will have to tolerate a non-zero error but that is all right because in the case of the proportional feedback, in any case when the load torque changes, we saw that the speed will change. So the transient state also the speed changes and there is a steady state error. There is the possibility here however of having non-zero steady state error, when load torque changes. The steady state error is produced earlier was produced earlier by changes in load torque. Now, there is a possibility that with changes in load torque, there will not be any steady state error because of the action of the integrator.

So, we will have to put this integrator block, to where shall we put it as I said the constant armature voltage which you have to generate, I will put it here. So that it is the output of the integrator block and what should be the input to the integrator block. I need some thing which is non-zero for some interval of time thereafter, it can very well become 0. The output will be a constant voltage at the integrator terminal. So this signal which goes into the input of the integrator, what should it be. It can now be a difference between a reference voltage which represents the speed and the output on the tacho generator voltage. Remember, we still need to look at what the speed actually is because we will take any corrective action or any adjustment only if the speed is not at the correct value. If the motor is running at the correct speed there is no need to make any adjustment at all.

So it is necessary to look at the speed of the motor therefore I will have to take feedback from the speed of the motor and of course, as before we do not measure the speed, we simply put a tacho generator whose output is a voltage proportional to the speed of the motor. We will have a reference input E r and then, we will have as before the difference device or the comparator E r with one input and the tacho generator output as the other input. The only thing, the difference between this scheme and the previous scheme is the following. Previously, when we did not have the integrator block but you simply had an amplifier here, the reference voltage was such that the output of the tacho generator corresponding to the rated speed of the motor was not the reference voltage equal to the reference voltage. The reference voltage had to be slightly greater than the tacho generator output voltage corresponding to the rated speed, if you remember we said that okay the tacho generator output was 20 volts then, the reference voltage may have to be 21 volts.

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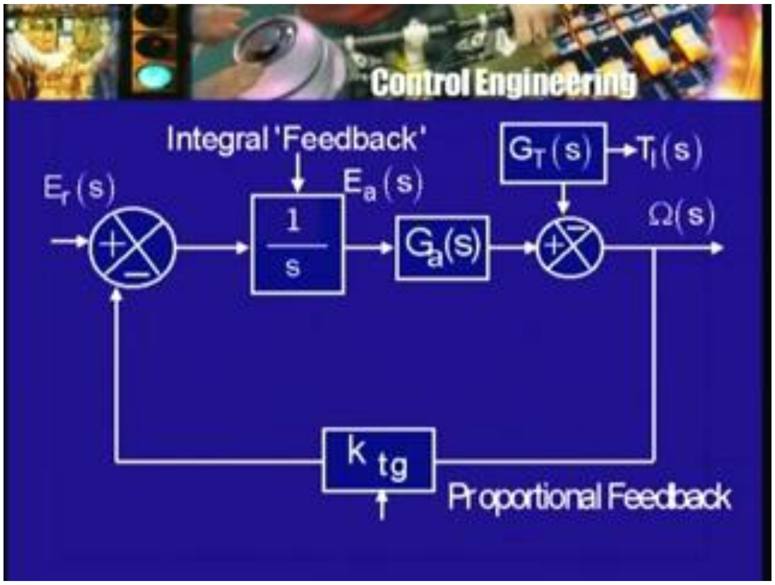

So that there is a difference of 21 minus 20 equal to1 that when amplified produces, say 230 volts at the armature terminal. If the reference voltage was made exactly equal to the tacho generator output at rated speed then, the difference device will have voltage 0 under rated conditions and then, therefore there will be no armature voltage. So you cannot have the motor running on 0 volts and of course this unfortunately was misunderstood, has been misunderstood as steady state error, but that is not so.

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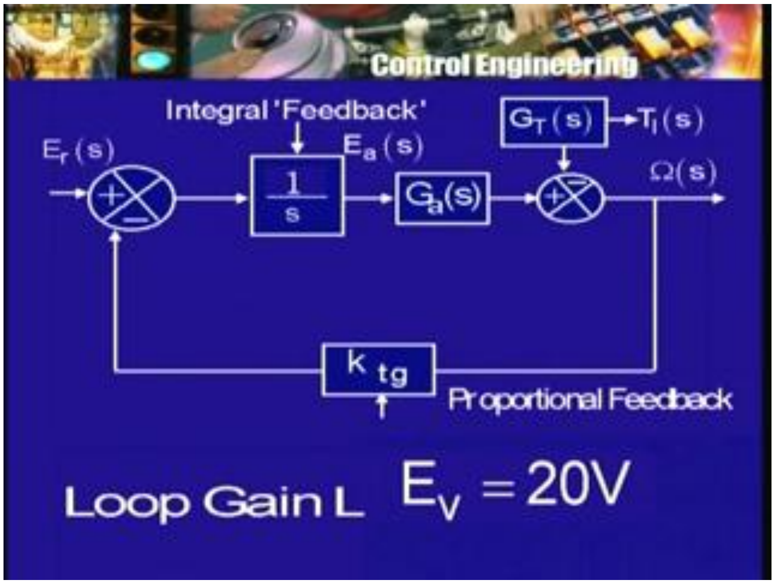

The reference voltage with proportional feedback had to be slightly greater by an appropriate amount than the feedback voltage produced under rated conditions. For rated speed, whatever is the tacho generator voltage output, the reference voltage has to be more larger than it by an appropriate by a suitable amount. It is the difference which is amplified and we saw last time that because of this action, the effect of the change in the load torque on the speed is reduced. Remember, what was the factor that was involved it was the loop gain L and the loop gain L in that case depended on of course, the transfer function G a, the tacho generator coefficient k tg and in place of the integrator I had only a simple gain k a and L can be made large by having the gain large and therefore, the effect of the disturbance could be reduced to any extend you wanted by increasing the gain of the amplifier in the forward path or in effect increasing the loop gain.

But I said that that is not desirable, as we will find out very soon. We cannot increase the loop gain without to any level that we a like because of transient performance, it is effect on transient performance. Now that again is something we have to be careful about but at the moment we are only worried a steady state operation, all right then. So continuing with the old calculations if the tacho generator output voltage under rated speed condition is 20 volts then, the reference voltage will be exactly 20 volts now. So E r is 20 volts, the tacho generator output under rated conditions is 20 volts. So if the motor is running at the desired speed, the reference voltage is 20, tacho generator output is 20, the difference is 0 therefore is the armature voltage 0, no the armature voltage is not 0 because you had the integrator in between. So if this difference had been non-zero then because of the integration action, it could build up to a level such that the armature voltage is at the correct value.

Now, can the difference be non-zero, well the answer is yes, because if I start this motor from 0 speed condition then at T equal to 0, the output quantity of the speed is 0, tacho generator output voltage is 0, the reference voltage of course is not 0. So the output of the difference device is not 0 but it is actually a positive quantity and the integrator starts integrating this function therefore, the integrator output will start building up. Now it is interesting and fortunate that the output of the integrator will build up, keep on building up to exactly the required value. So that the input to the integrator eventually becomes 0.

Now this as I said, sounds a little mysterious but that is the way the integrator works,
if you assume that the integrator faithfully integrates the input then, this is something that will happen as we will see. So although if the motor is started from rest therefore of course, there is speed error right away, the motor should be running at full speed but it is at 0 speed. Well, you have to give it some time to speed up as the motor speeds up, the output of the comparator goes on decreasing but the output is not 0, the output is positive at t equal to 0, it goes on decreasing but unless the speed has increased to a value larger than the rated speed, the output here will be positive, the integrator goes on integrating it and therefore, the integrator output goes on building up. As the integrator output builds up, the armature voltage is increased, so the motor speeds up and as we will see, all this will come to an end so to speak, after enough number time constants have elapsed.

When the motor will run at the desired speed, the tacho generator output will be equal to the reference signal, the output of the difference device will be 0 but the output of the integrator will not be 0 but will be exactly the required armature voltage which produces the desired speed. Now, what about change in torque, what is going to happen when the load torque changes. We will find out exact expressions for the steady state value under changed torque as we did earlier but before we do that let us looks at it from again the qualitative point of view. So, suppose the motor is running at the desired speed now because load torque was at the rated value and we had done pre calculations in such a way that the motor did not run at the desired speed.

Now, we will see there fore that we may have to make some adjustments somewhere but lets comes to that only later. Now suppose the load torque suddenly increases may be the mechanical job that is working encounters more resistance, the metal there is some hard part of it or whatever, so the load torque certainly increases, then what is go into happen.
Now, the way I am going to talk about it, it will be as if there is a sequence that this happens first then this happens, then this is followed by that and so on. When we modeled the system, when we write it in the form of a differential equation, the differential equation does not show you anything like that the differential equation simply says the derivatives are related to the function in such and such a way.

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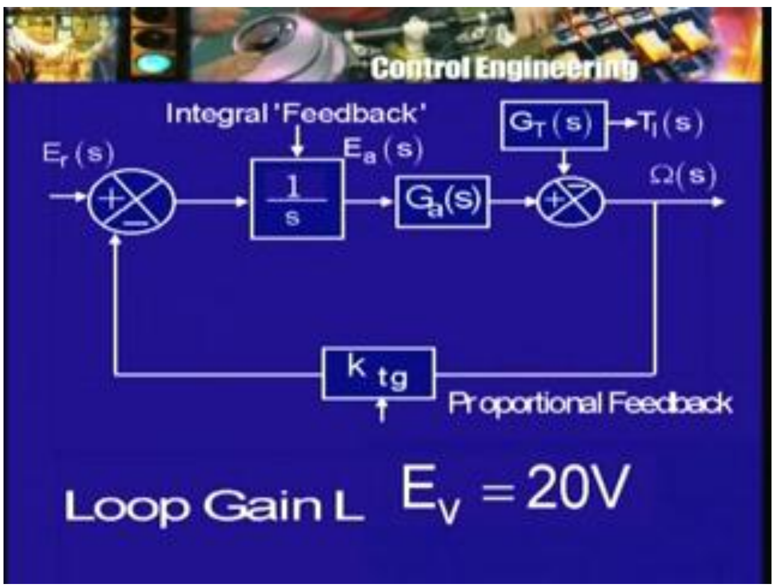

So in the, when I look at the differential equation I will not be able to say that as this happens then, this will happen then, this will happened and so on. So, remember this that this is only an approximate way of understanding, what is going to happen, what is going to happen, if our models are good will then be determined by whatever transfer functions or differential equations that we write. So first qualitative what will happen, I increase the load torque suddenly, the effect of increase in the load torque is, if the armature current in the motor had remain the same.

So the motor torque remained the same, load torque has increased therefore there will be a deceleration of the motor. Of course, the motor does not decelerate instantaneously because it has a moment of inertia. So the motor speed will start falling below the rated value therefore the tacho generator output voltage will be less than E r, as a result the output of the difference device which was 0 till then, will become positive. Now the integrators starts integrating it, the integrator output is not 0, the integrator output is already at the exact level, say 230 volts.

Now, we have a non-zero input applied to the integrator, when its initial value was not 0 but was 230 volts. The integrator integrates the input from that moment onward therefore the output of the integrator will increase to a value greater than 230 volts. In other words, the integrator output will be greater than what the voltage was earlier. The load torque had increased but now the armature voltage has increased therefore the armature current will increase, increase in armature current will produce a greater amount of torque and we will say that finally, things will becomes steady, when the new voltage is such that the new torque exactly balances the load torque and the frictional torque and the motor is backed at its rated value. The motor speed is back at the rated value. On the other hand suppose, the lower torque decreases assuming that the armature current does not decrease immediately and or does not change immediately and why is that so because the armature circuit has inductance, it has also its electrical inertia.

So the armature current will not change instantaneously even if the torque is suddenly reduced. So the armature current has not changed instantaneously, the motor torque has not changed, the load torque has decrease therefore the motor will start accelerating, the speed will increase start increasing beyond the rated value. The tacho generator voltage will become greater than the reference voltage, the output of the integrator now will become negative, the integrator already was at 230 volts. Now the input is no longer 0 but it is some negative quantity. So to this 230 will be added the integral of the input. So the output of the integrator will start decreasing from 230 volts.

When the output of the integrator decreases or the applied voltage decreases, the back emf has reduced a little bit but not much, the difference which produces the armature current is decreasing. So the armature current will start decreasing. Finally, we will say that the whole thing will come to an end, when the speed is at the same value as before the error, output of the difference device has become 0 therefore but the output of the integrator is no longer 230 volts but it is now perhaps 220 volts which is enough to make the motor run at the rated speed because the load torque has reduced.

So qualitatively, it seems that there is a possibility that with this kind of a feedback arrangement and remember, that we have not only the integrator here but we also have some kind of a proportional device here. Because, I need to measure the speed of the motor anyway, I can take the of course the action will be automatic provided the speed is being measured and so, this is necessary. Somehow, the arrangement is that when the load torque changes the speed will change for a short period of time but it will eventually return to the desired value and therefore, in this sense there will be no steady state error, in speed or the output quantity even when there is a permanent change in the disturbance signal, in this case the load torque.

So this is true 0 steady state error situation. Now to understand this in terms of transfer functions. Let us look at it once again, from the point of view of the transfer function idea, we have a block diagram, already we have 2 inputs one of the input now, instead of E a of s. I am showing it still here but we are not interested in it, the reference signal is the input and the reference signal once again is going to be kept constant. So, I have omega s which is affected by E r of s and T l of s. So I will have an expression for omega s which as before, we will have one transmittance multiplying E r of s minus another transmittance multiplying T l of s and to the find the transmittances, one has to once again apply the Mason gain formula.

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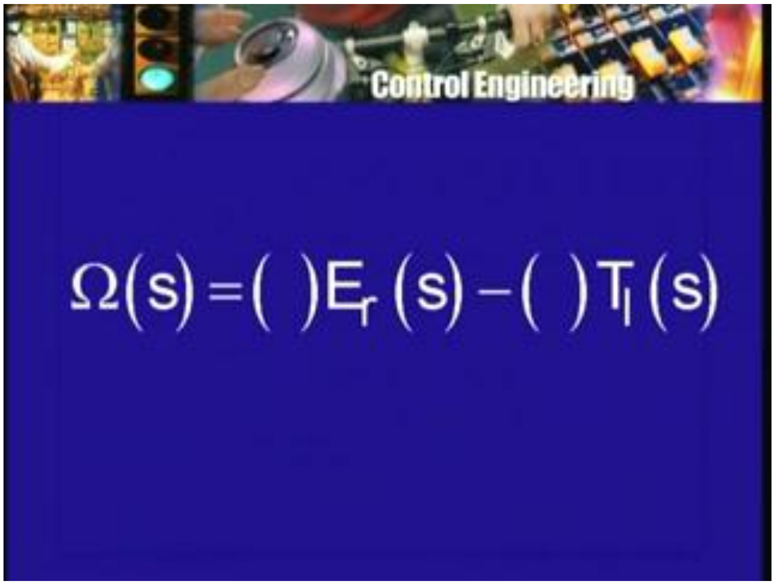

So, we will have to look at the block diagram or you can think of the corresponding signal flow graph and we have to find out the delta of the signal flow graph. Now what is the delta here, there is only one loop. So I can write down and expression for the loop gain L, the loop gain L will be what is the loop starting here, say 1 by s then G a, through the plus to omega then through k tg, through the minus back to1 by s. So the loop gain L will be 1 by s multiplied by G a, multiplied by k tacho generator, multiplied by this minus 1 through the difference device and therefore it will be minus G a, k tg divided by s.

Earlier, we did not have s but in its place we had k a. Now I am going to put that k a here, for above reason that you will see right away little later. So the integrator out transfer function instead of being 1 by s, I will put the scale factor k a by s. So then, I will have k a here and L is this. So because this is L what is delta, delta is 1 minus L therefore it is 1 plus G a, k tacho generator, k a divided by s, this is my delta. Now I have to look at the transmittances. Let us first look at the effect of the load torque because that is the disturbance signal which is really the cause of all this modification. So I have to look at the forward path from the load torque to omega s. Well, there is no change in it, the forward path simply goes through GT with a minus sign because there is a minus sign that we have here.

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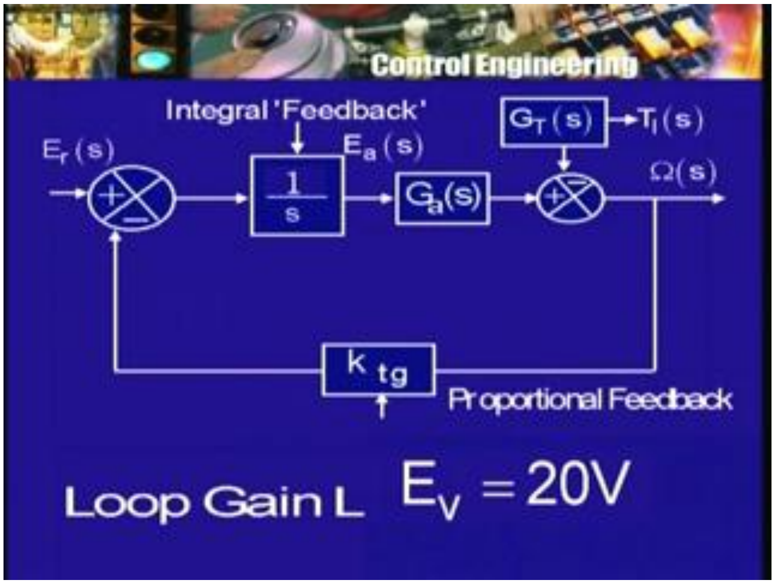

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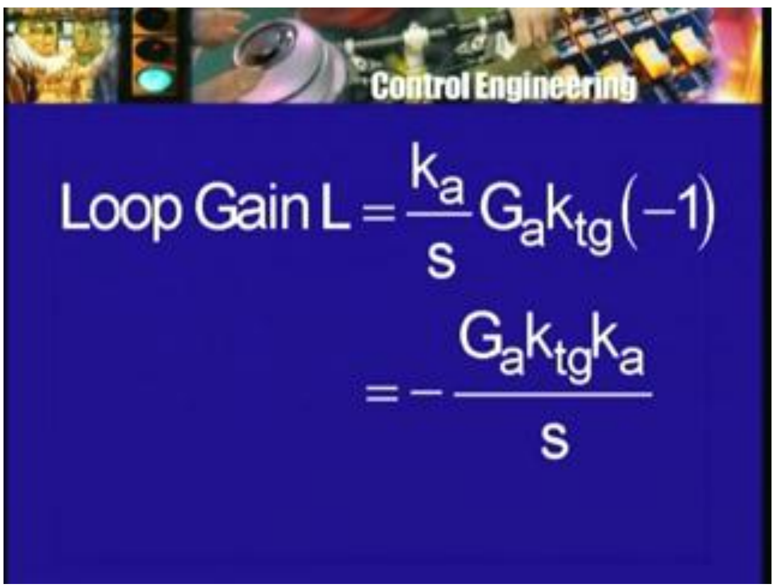

So the gain of the forward path is minus GT that is multiplied by delta or what remains of delta, when I consider only loops which do not touch the forward path. However, as you can see, this is the loop and this is the forward path and therefore the 2 touching each other and therefore, the numerator delta will be simply 1 and therefore I will have minus GT multiplied by 1. Let me write that explicitly divided by 1 plus G a, k tacho generator, k a divided by s. This is the transmittance and therefore, the part of omega that depends on T l and so I am putting this vertical line here to remind us that is not really the entire omega but only a part of it that is effected by T l of s then, it is given by this formula. The difference now is I have this term s in the denominator here. Earlier, I did not have the s term, the s factor here and that is going to make a big difference as we will see. Now, I want to find out the steady state part of this or the steady state value of this. So by the final value theorem, I am multiply this by s and take the limit as s tends to 0 there whether, I have multiply this whole thing by s and take the limit as s tends to 0.

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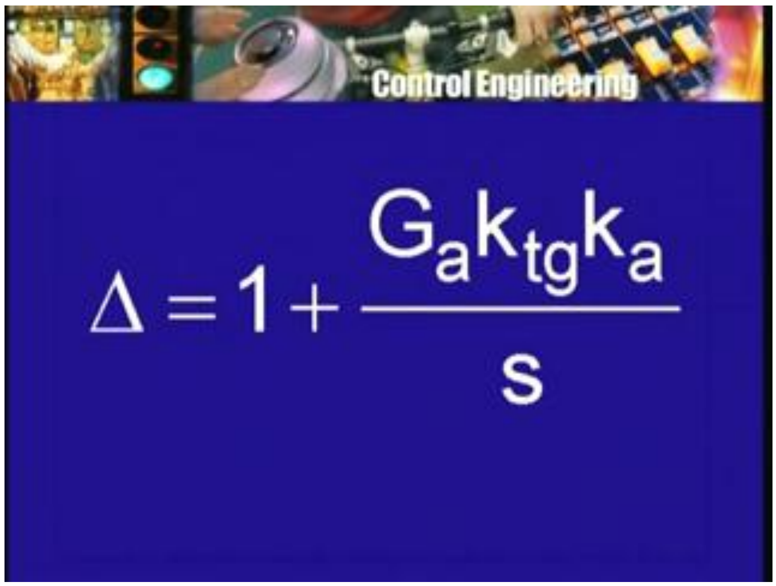

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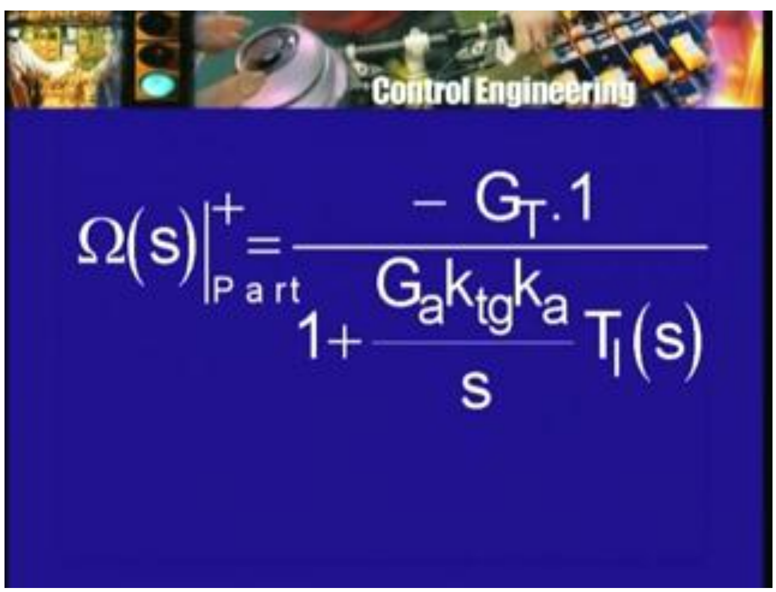

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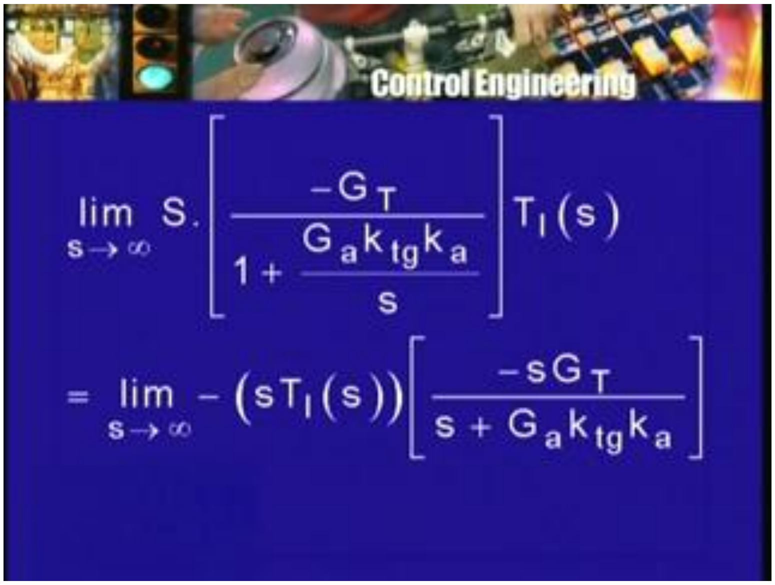

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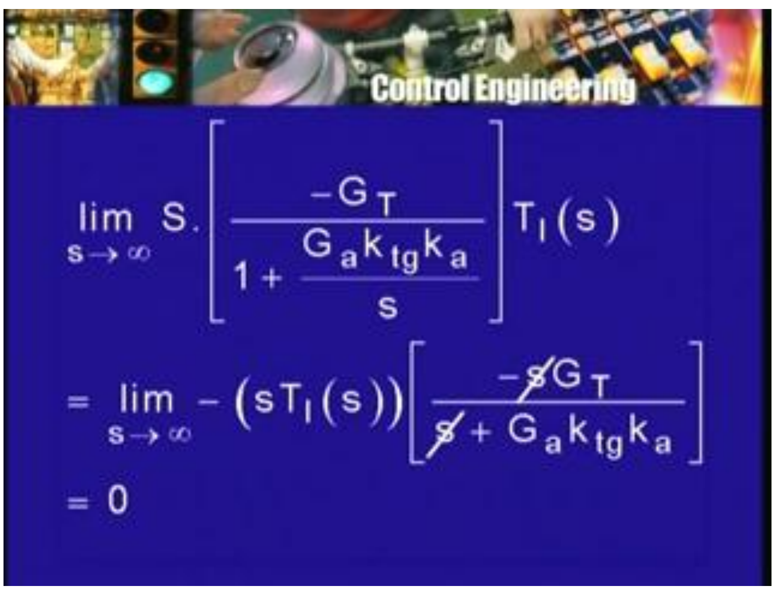

So let me do that, so I have s into minus GT divided by1 plus G a, k tacho generator, k a divided by s into T l of s. Now as we did earlier, I will combine this factor s with T l of s and I will therefore rewrite this as this is equal to I will put the minus sign out there, s into T l of s multiplied by this bracketed expression and I will now simplify the bracketed expression, by getting rate of this s in the denominator. So I multiply both the numerator and denominator by s therefore I will have minus s GT divided by s plus G a, k tacho generator, k a and now, I have to take the limit of this as s tends to 0 of this product. The limit of this as s tends to 0, is what, it is the new value of the torque. The torque has changed it has therefore assumed a new value, the limit of this as s tends to 0, by the final value theorem is the steady value of the torque. So that is the new value of the torque, what is the limit of this as s tends to 0. In the denominator, I have s plus something, so this s goes to 0, so this remains but in the numerator, I have s and as s goes to 0, this goes to 0 and therefore the limit of this whole thing is 0.

So the steady value of the load torque makes no contribution to this part of the speed and this is why, the speed in the steady state, in the long run, after you wait for enough number of time constants, will not change or the change in speed because of change in load torque will be 0. This s factor which we have introduced, its appearance here in the denominator, when you multiply through, its appearance in the numerator and therefore the numerator becoming 0, whereas denominator does not become 0 and so the ratio become 0. This is the crucial thing, this is the think that is making the system work in a such a way that with load torque change to a new constant value, the speed as t tends to infinity will go back to the old value that is to the rated value and therefore, there will be no steady state error.

We can look at the part of the speed that depends on the input reference voltage. So that part of the speed is given by omega s and so, I am putting line, vertical line here to remind us of that part of the speed and let us look at the block diagram once again, what is the forward path now, I want the forward path from E r to the omega. So through the plus sign k a by s followed by G a though plus sign again. So the gain for the forward path is k a by s into G a.

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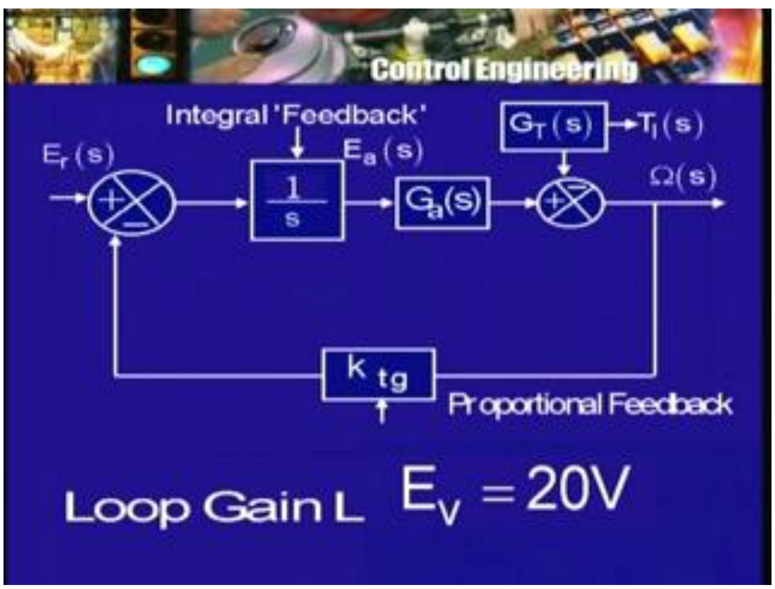

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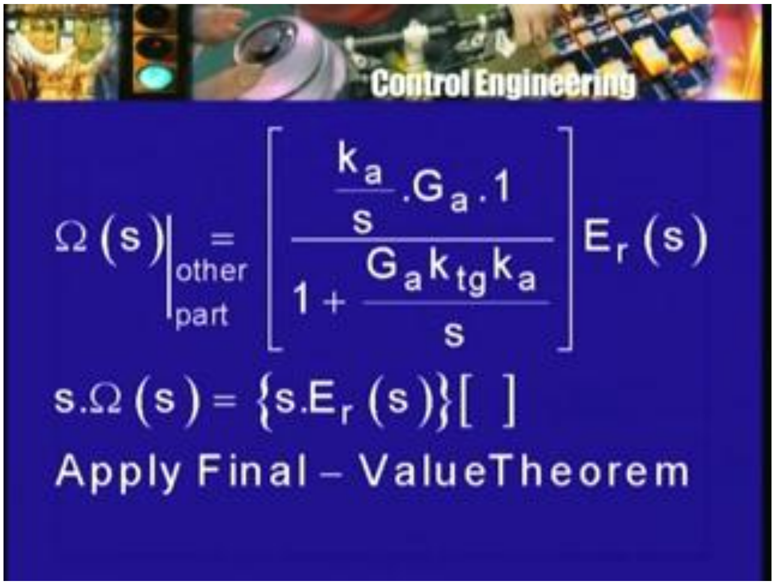

Now, what about delta for then the, for this particular path ,well it touches the loop. So the delta for that is 1 divided by the same delta that we had earlier for the other transmittance namely 1 plus G a, k tacho generator, k a divided by s. This whole thing multiplies E r of s, right. So this is the part of the speed that depends on the reference voltage. We already looked at the part of the speed that depends on the torque. Now, I want to find out the steady value of this. So I will multiply both sides by s, so a into omega s therefore on the right hand side I will have s into that once again, I will rewrite this as s into E r of s multiplied by this bracketed expression.

Now take the limit of both sides as s tends to 0, on the left hand side I will get the steady state value of the speed, on the right hand side I will get the steady state value of the applied or the reference voltage and that of course, I have not changed. So it is as before, what about this bracketed expression, if this bracketed expression went to 0, as say s tends to 0 then, the whole thing will be hopeless because the speed will be 0. Fortunately for us, as you will see this bracketed expression will tend to a certain non-zero value and you can find out what that non-zero value is and from that you can figure out that under the new steady state conditions, even when the load torque has changed, the steady state speed will be equal to the reference voltage multiplied by that constant, do this homework and we will get back to this.