Control Engineering
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Lecture - 28

Before, I continue with a discussion of how to handle the complex roots case or complex poles case, let me a point out something which you can take a some kind of correction to a statement that I made earlier because I would like to keep the practice followed in most a control theory books. You, remember that we are looking at the quadratic factor S square plus A S plus B, for the case when the discriminant was negative, the discriminant delta being A square minus 4 B that was negative and because of that we had a pair of conjugate complex poles, a roots.

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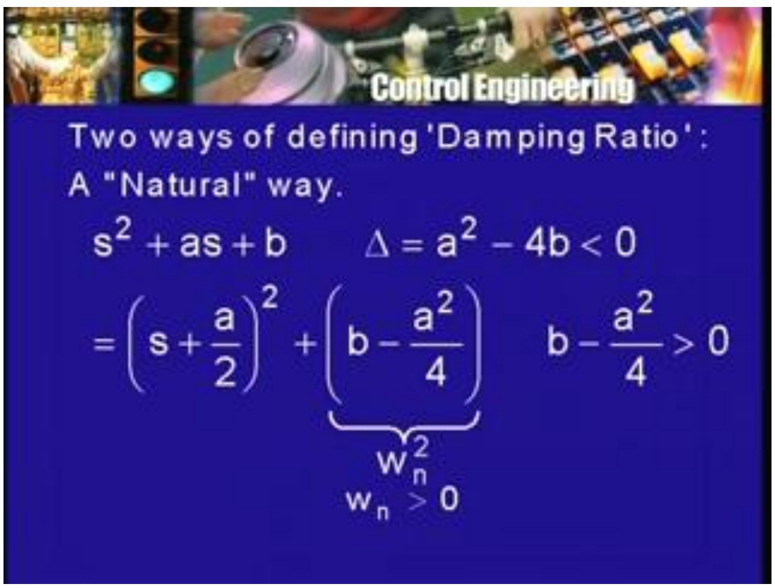

Now in this case, when trying to find out the Laplace inverse of the part of the response, one method was to factorize this quadratic S square plus A S plus B into its linear factors and then, use the regular partial fraction and expansion method for linear factors but as I mentioned earlier, this introduces complex numbers into the calculation and although the calculations are not really more difficult but if you make some mistakes, somewhere of a getting wrong sign for the real or imaginary part and when computing the coefficients of the partial fraction and expansion.

The final answer may turn out to be complex or even or imaginary rather than purely real. We expect that of course, the speed or the part of the speed that we are looking at will be a real valued function of time. Now such a mistake can be avoided or rather the use of complex numbers can be avoided in this case and I indicated to you, how that can be done by using the method of the completing the squares. So you write S square plus A S plus B as S plus A divided by 2 whole square and this is called completing the square, in other words you take the 2 terms S square plus A S and think of adding one more term. So that you get a complete square and so, I have to add A square by 4 and therefore I have to subtract A square by 4 and therefore I will get an expression which is S plus A by 2 square plus B minus A square by 4 and here because the discriminant is what it was, we saw B minus A square by 4 will be greater than 0.

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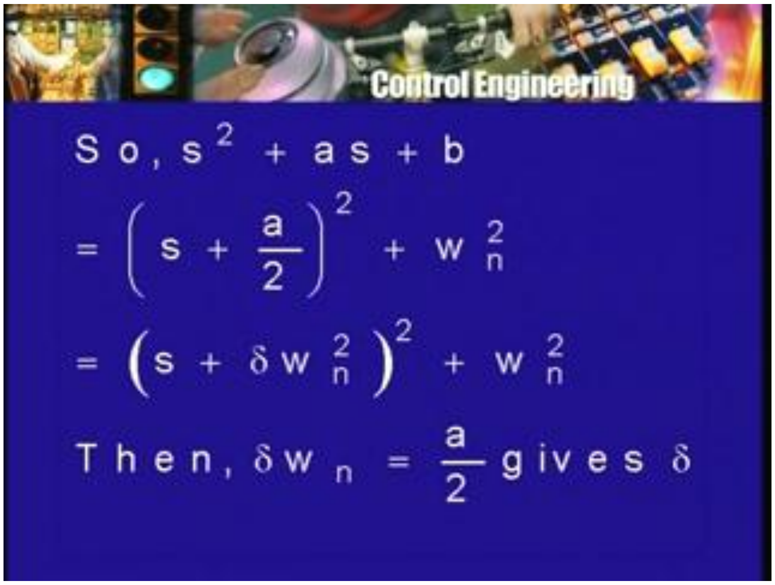

So this term will be greater than 0 or positive and therefore I said that you could write this as omega N square, where omega N is some real number and you can choose omega N also as positive. With this then the S plus A by 2 square term can be rewritten as S plus delta omega N square plus omega N square, where delta omega N then terms have to be equal A by 2. So with omega N square given by B minus A square by 4, from this formula delta omega N equal to A by 2.

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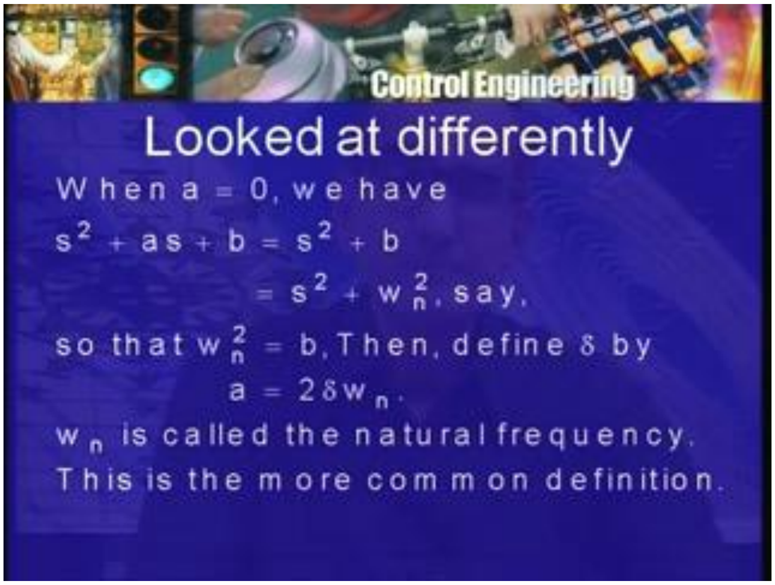

We can obtain the value of delta now, I told you that this omega N is called natural frequency and this delta is called the damping coefficient or the damping ratio. Now what I should have really said is that if would be more appropriate to call omega N, the natural frequency and this delta as associated with damping for a reason that you will see very soon but this is not normally done, normally what is done is, you argue like this I have A squared plus A, S plus B, what if I did not have the term A. So, if I did not have term A then, I will simply S square plus B and then you think of B as square of a positive quantity which is then called omega N.

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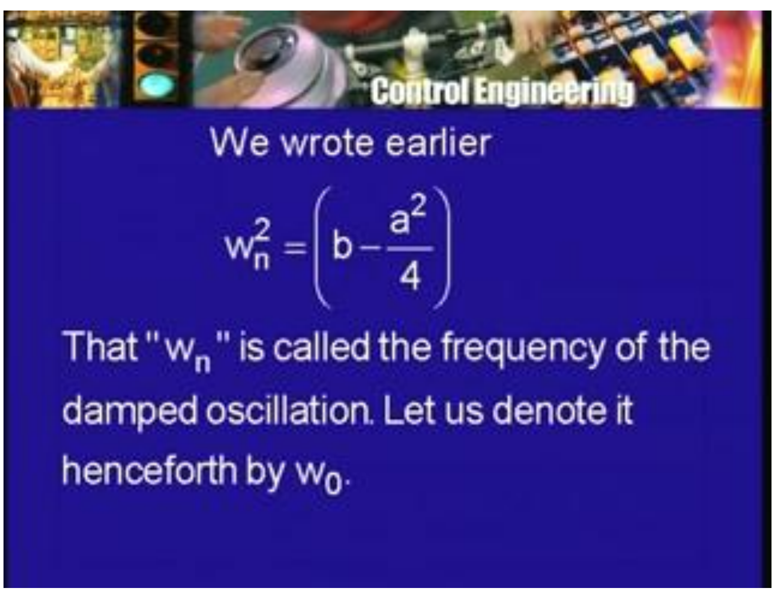

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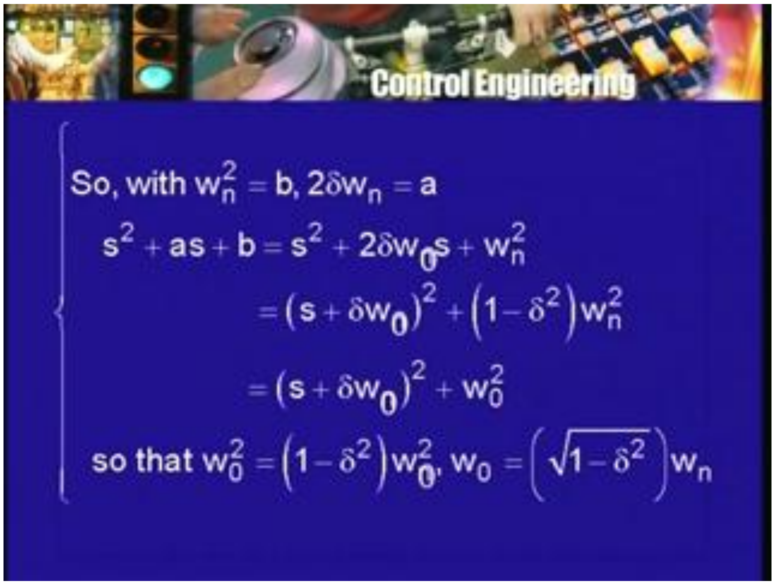

So I have write B as omega N square and then, we will write the term A as 2 delta omega N and if I do that then S square plus A S plus B becomes S square plus 2 delta omega N S plus omega N square and by taking these 2 terms, first 2 terms and completing the squares I will get S plus delta omega N once again, square plus, at this time I have added the term delta square omega N square. So I have to subtract it from here, so I will write is as1 minus delta square into omega N square and so finally, it will be written as S plus delta omega N square plus 1 minus delta square omega N square and this is replace by let us say omega 0 square. So I will have S plus delta omega N square plus omega 0 square.

So, if I do this way then I have introduced now omega 0 and delta, sorry I should have written it as omega N, omega 0 here, rather than omega N, I will change to omega 0 and in textbooks usually, this omega 0 is call the natural frequency associated with the quadratic factor and delta is called the damping ratio. Now, let us see what is the a difference between the 2, if I write as I did earlier S square plus A, S plus B as S plus delta omega N square plus omega N square defining omega N square is B minus A square by 4 then the roots are given by minus delta omega N plus or minus J omega N and so I show them in the complex plane then here is what they look like, the imaginary part which is the length of this segment is given by omega N and the real part of for the length of this segment is given by delta omega N.

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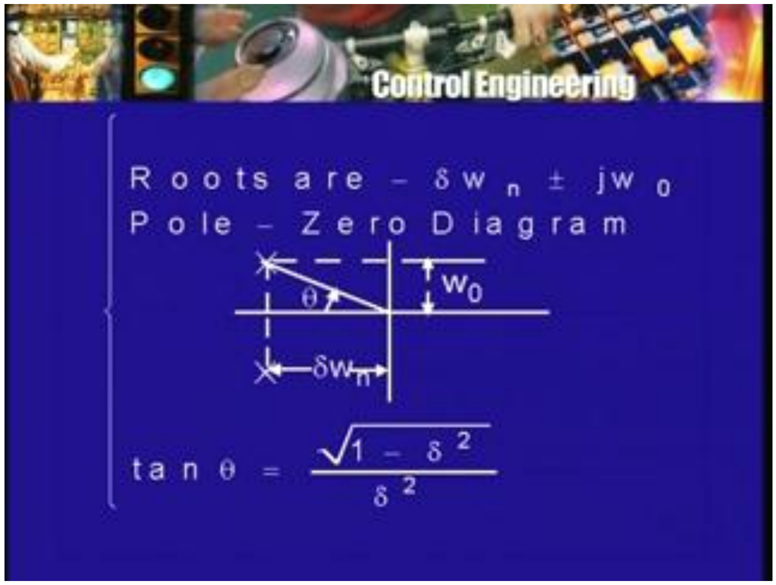

So, in other words delta omega N is the negative of the real part remember, the real part is negative and omega N is the imaginary part or the part that the number that multiplies J. Now I have complex number a like this depending on the value of delta, what will vary on this diagram. So as you can see if delta omega N is larger than omega N, the real part is greater than the imaginary part if delta omega N is less than omega N, the real part is smaller than the imaginary part, if delta omega N and omega N are equal then, the 2 parts real imaginary parts are equal.

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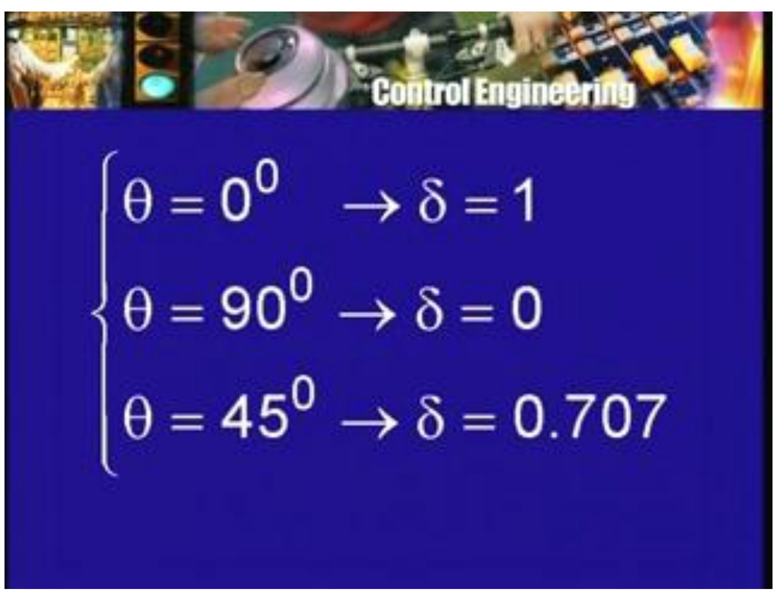

As a result now, if draw this radius vector to pointing to the pole and of course there is one and there is the other. So I can draw both of them if I look at let us say only one of them and call this angle theta then , I will have here tangent for theta is given by omega N divided by delta omega N or 1 by delta and now, I can consider the case where theta is close to 0 that is the imaginary part is virtually 0, the real part is of course not 0 or theta is very closer 90 degrees, in which case the imaginary part is large, the real part is much smaller. A third case will be theta equal to 45 degrees then the real and imaginary parts are equal. So accordingly then if delta which is one might an theta is much larger than 1, then the system will be such that the real part is greater than the imaginary part, if delta equal to 1 then the system will be such that the 2 parts are equal and if delta is less than 0, the 2 parts will be such that the real part is much less than the imaginary and we will see what difference it makes to the response.

Now the same expression can be rewritten as I showed earlier but it is rewritten using different symbols or different quantities instead of dealing with square root of B minus A square by 4 which are called the natural frequency, one deals with square root of B and calls it, the natural frequency omega N then, the whole expression written in terms of the natural frequency looks very different, it looks like S plus delta omega N square plus omega 0 square, where omega 0 is not omega N but it is given by omega N times square root of 1 minus delta square.

Now, if I write it this way then the corresponding poles will be as shown once again, where the imaginary part will be J omega 0 and the real part will be delta omega N. As a result, if I draw the diagram showing angle theta once again, the angle theta will be given by tangent of theta equal to omega 0 divided by delta omega N, but omega 0 is square root of 1 minus delta square multiplying omega N divided by delta omega N and therefore, this becomes square root of 1 minus delta square divided by delta which is different from the expression that I wrote earlier.

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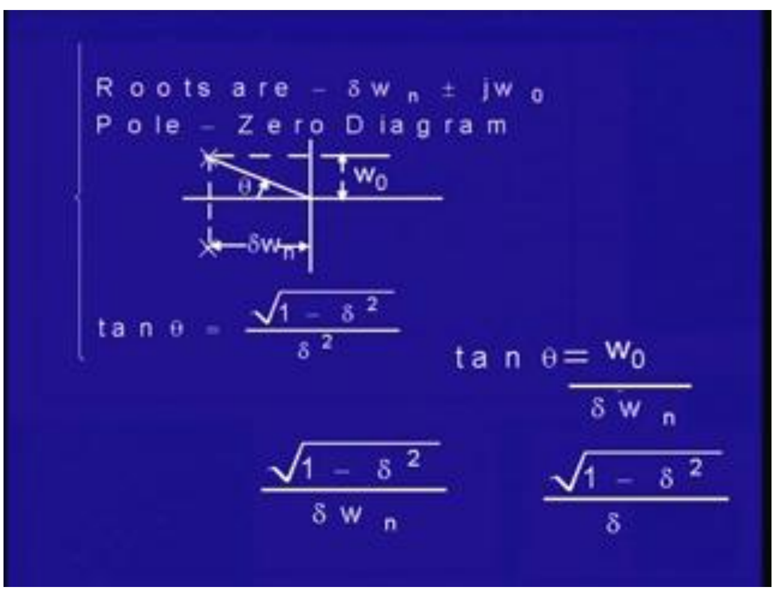

Now most textbooks follow this convention rather than the one that I mentioned earlier. So in other words, let me repeat it once again and emphasize, you take quadratic S square plus A S plus B, I am assuming that B is positive than the square root of B is called the natural frequency omega N and then, you define A to be equal to 2 delta omega N, this defines delta and then you write this quadratic as S plus delta omega N square that is okay but the second term is not written as omega N square but it is written as 1 delta square into omega N square or in other words, S delta omega N square plus omega 0 square.

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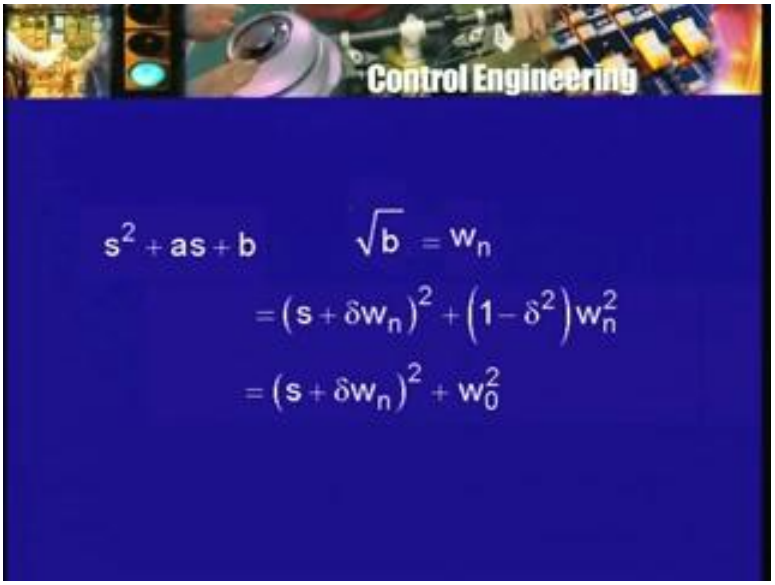

Now, in this case when delta is define this way, the limits of delta will not be between 0 and infinity as they were in the earlier case but will be between 0 and 1. Going back to the figure once again, from this figure you will see that the real part is delta omega N, the imaginary part which is omega 0 is square root of 1 minus delta square into omega N and therefore, when delta is very close to 1, the real part will be much greater than the imaginary part and when delta is close to 0, the real part will be much less than the imaginary part. So in one case, delta will approach one in the other case, the delta will approach 0 and we will see now, why one calls that delta whichever it is define as the damping coefficient of the damping ratio.

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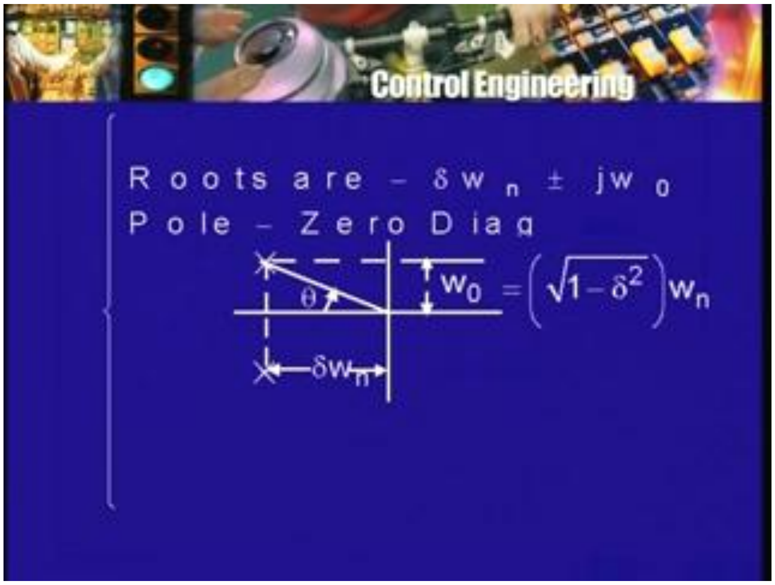

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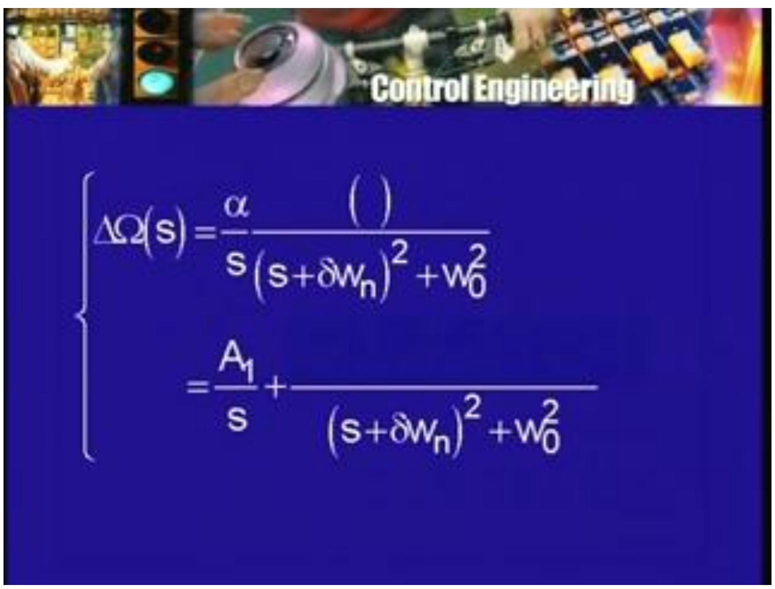

So, let me use the more familiar or the more commonly used abbreviation or symbolism as I have indicated here S plus delta omega N square plus omega 0 square. So, with this then I will rewrite quadratic of the whole expression, for the part for the response that we are looking at as follows. So let us look at the expression for the change in the speed, the Laplace transform of that that we were looking at earlier which is alpha divided by S multiplied some expression in the numerator divided by this quadratic and this quadratic now, I am writing as S plus delta omega N square plus omega 0 square then, when we do the partial traction, we will have a term which will be given by A 1 divided by S as before that is there will be a partial traction that goes with the factor S and there will be a partial traction which I will not write, in term of the linear factors and that will be as follows, S plus delta omega N square plus omega 0 square into a linear term in the numerator. This linear term in the numerator, we will write as A 2 into S plus delta omega N plus A 3 into omega 0 square.

Now, why am I writing it this way because as I told you earlier, I am going to make use of a theorem formula laplace transformation which tells you that if you have function of time and if you consider it laplace transform, what happens if the function of time is multiplied by an exponential factor, the theorem says that in that case all you have to do is, in the laplace transform you replace S by S plus an appropriate number. So, anticipating that I am writing this expression in in this form.

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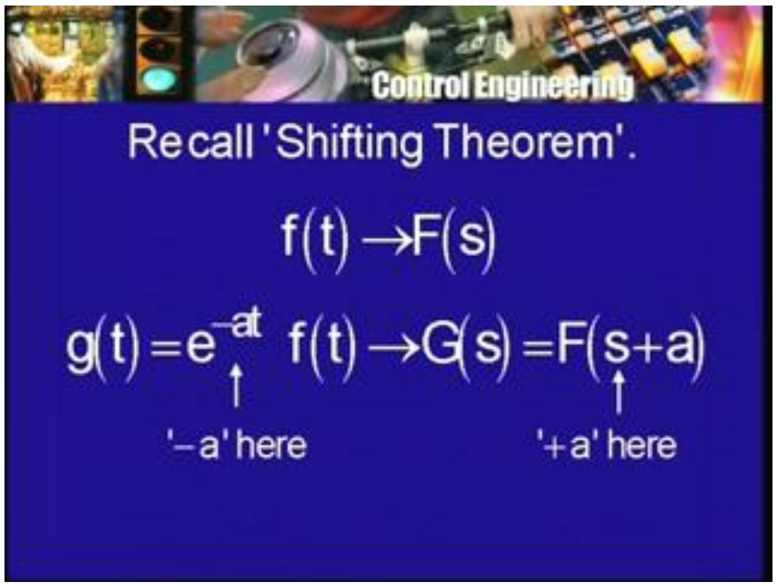

Let me mention that result once again, if I have function F of T whose Laplace transform .We are denoting by capital F of S then, if the function of T is multiplied by an exponential factor. So I write F of T into E raise to minus 80, notice the minus A here and I call this function G and then I ask the question, what is G of S, what is the Laplace transform of the function G of T at a complex number S, what is its value at a complex number S then, the answer is G of S is simply F, not evaluated at S but evaluated at S plus A.

So what is the point of this theorem now, to use it here? If I have G of S which looks like F of S plus A that is, if I can rewrite a function G of S as F of S plus A then the Laplace inverse G of T can be recognized as T raise to minus a t into the Laplace inverse of F of S. So that is what I am going to use now. So what I have now is the problem of inverting let us say A 2 into S plus delta omega N divided by S plus delta omega N square plus omega 0 square. I want to find out the Laplace inverse of this, so let me write this specifically, Laplace inverse of this function of S, what is it as the function of time that is what I am trying to find out.

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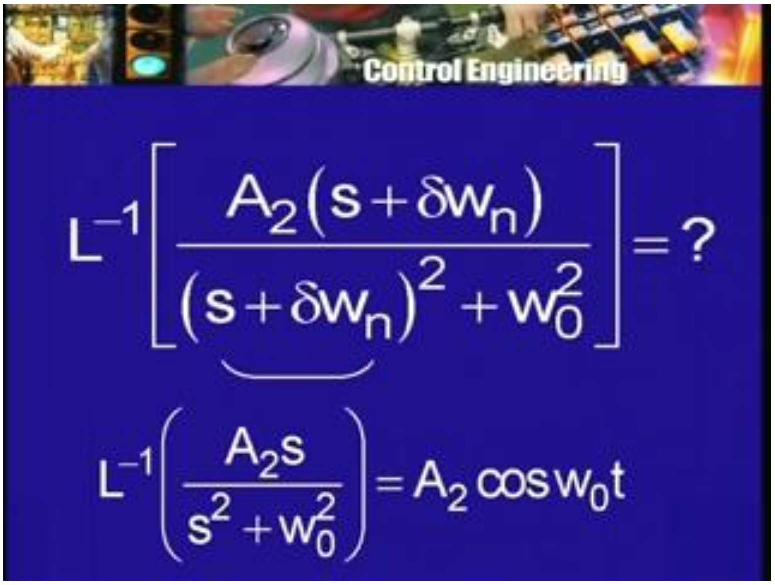

Now I see S plus delta omega N, so in place of S plus delta omega N. Suppose, I have S then what would I have? I would have A 2 S divided by in place of S plus delta omega N, if I had S I would have S square plus omega 0 square. Now, what is the Laplace inverse of this? Well, look at your table of Laplace transforms and Laplace inverse transforms or if you remember it and this is something you should remember, the Laplace transform of cosine of omega T is given by S divided by S square plus omega S square therefore S divided by S square plus omega 0 square will be the Laplace transform of cosine omega 0 T and this is multiplied by A 2 therefore, the Laplace inverse that we are looking for will be A 2 cosine of omega 0 T.

Now, this is the Laplace inverse of this thing. So going back what will be the function whose Laplace transform is this, I go from this expression A 2 S divided by S square plus omega 0 square to this expression by replacing S by S plus delta omega N and therefore, the function which I am looking for will be this function which is what, the Laplace inverse would have been if it was the Laplace inverse of this part multiplied by an appropriate exponential and what is that exponential, that exponential is E to the minus delta omega N T and therefore, the function is A 2, E 2 the minus delta omega N T cosine of omega 0 T or if I write it with omega N in place of omega 0 that is omega 0 expressed in terms of omega N. I will have it as A 2, E raise to minus delta omega N T cosine of omega N square root of 1 minus delta square into T. Now let us not worry too much about, what exactly appear as the argument of the cosine function and the exponential term here but let us concentrate on their behavior.

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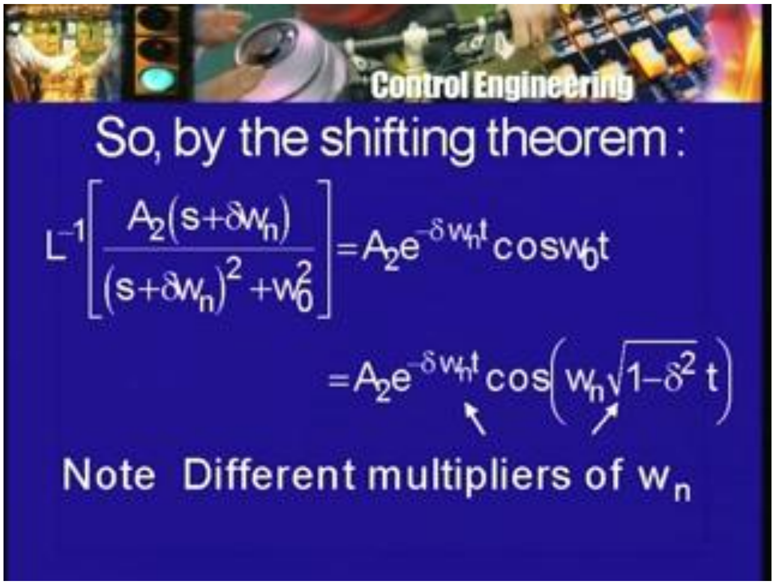

Of course, it does not require me to tell you that this function is simply an oscillating sinusoidal function which will continue oscillating forever. This part since delta is positive, omega N is positive is an exponentially decaying function and now, that we have product of the 2, the exponential decaying function goes to 0 as T tends to infinity, the cosine function keeps on oscillating between plus 1 and minus 1 and therefore, this whole thing will be oscillating all right. But the amplitude of the oscillation will go on decreasing as time passes and therefore, it will be what is called a damped sinusoidal function, which I had mentioned earlier.

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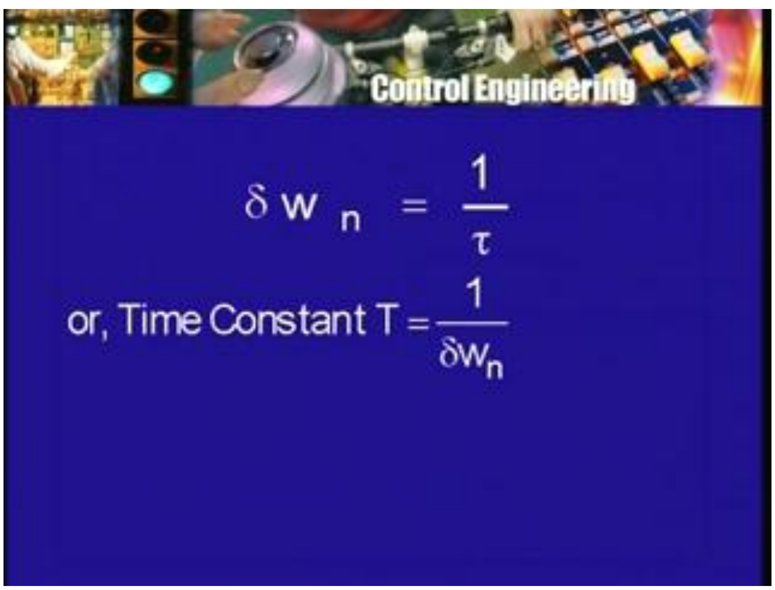

So in other words, there will be oscillations all right but the oscillations will go on decreasing in the their amplitude as T increases and at what rate that rate that rate is given by the exponent associated with the exponential part. The exponent in this case is delta omega N and that is given by 1 by tau because tau is time constant or alternately, if there time constant tau is given by 1 by delta omega N and as before, if you wait for 5 time constants or for 10 time constants perhaps, the exponentially a decaying part is virtually 0 and therefore the oscillations will be almost 0 by the end of 5 time constants or may be 10 time constant.

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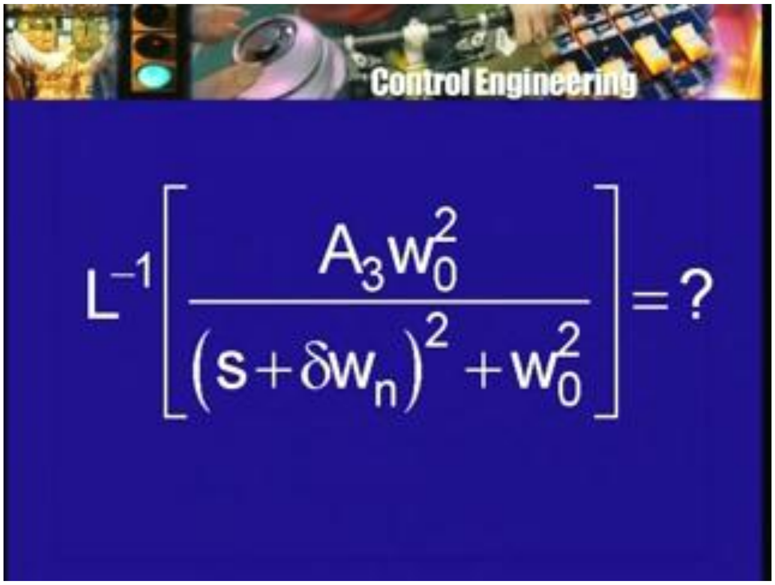

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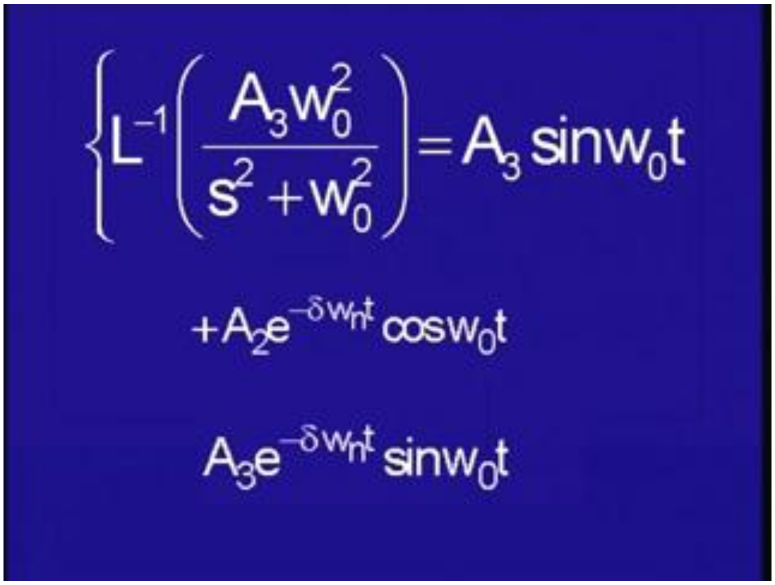

So this takes care of one of the 2 terms the other term can be handled also equally easily. The other term is given by A 3 omega 0 square divided by S plus delta omega N square plus omega 0 square and I am asking, what is the Laplace inverse of this. Now here, I have S plus delta omega N in the denominator but there is not S or anything involving S in the numerator. So, if I know the Laplace inverse of not this but of A 3 omega 0 square divided by S square plus omega 0 square. Then, from that I can obtain the Laplace inverse of the function of S that I have already. Now what is the Laplace inverse of this once again, this is something which all of us are to remember the Laplace transform of sign omega T is omega square divided by S square plus omega square. Remember, omega square not omega omega square divided by S square plus omega square.

So this thing is the Laplace inverse of the Laplace transform of sign omega 0 T and therefore, the Laplace inverse of this whole expression is A 3 sign omega 0 T. But I do not have S, I have S plus delta omega N and therefore the Laplace inverse that I am looking for will be given by A 3 once again, the exponential minus delta omega N T multiplying sign of omega 0 T. So this is also an oscillating but exponentially decaying function. Now of course, I have this and I have the other term which is A 2 E raise to minus delta omega N T into cosine of omega 0 T because A 2 may not be 0, A 3 may not be 0. So I have 2 terms now, these 2 terms can be combined to write as simply equal to say some number A 4 in to the exponential factor is pulled out common multiplied by cosine of omega 0 T plus what I can write as some angle theta 0 and so, it can be seen as oscillating function which is modulated by an exponentially decaying function or it is a exponentially decaying, sinusoidally oscillating function.

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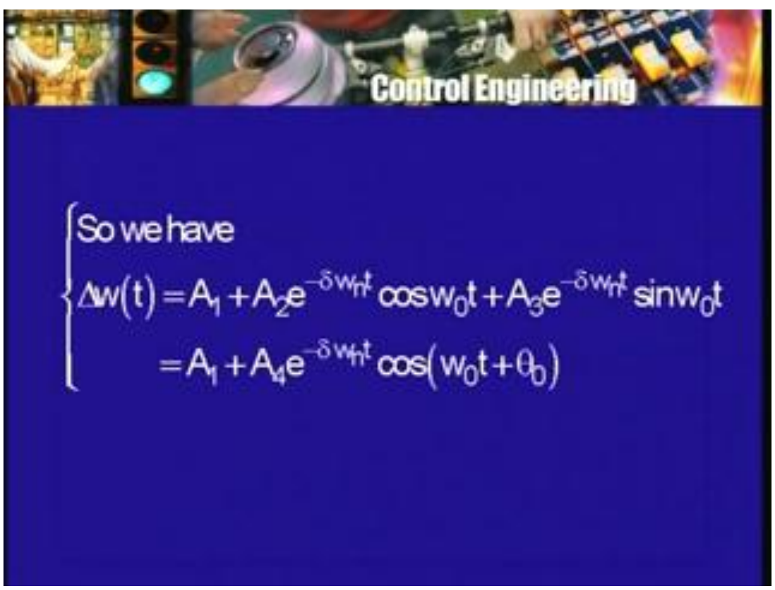

Now, this is something one comes across as I have told you a network theory, when you look at the response of an RLC series circuit which is connected to a battery through a switch, the capacitor is initially uncharged let us say and then, you find out the build up of voltage on the capacitor. Then, depending on the values of R L and C, if the resistance value is not too large then, the circuit does involve oscillation, the capacitor voltage does not build up steadily instead the capacitor voltage keeps on rising but has overshoots and undershoots before, after enough time has elapsed it becomes constant equal to the battery voltage.

So, you have perhaps experienced or encountered this function and its graphs earlier. Once again, I am not into an exact sketch but this is what it will roughly look like there is the final value towards which the function is tending because I have that additional term, A 1 divided by S whose Laplace transform is the constant A 1. So here is that A 1 and here is this additional part which is oscillating but decaying in amplitude and the whole thing therefore, we look somewhat like this. I am drawing this figure only to give you some idea of what it looks like. So, I am not been very careful in locating the various points on the graph.

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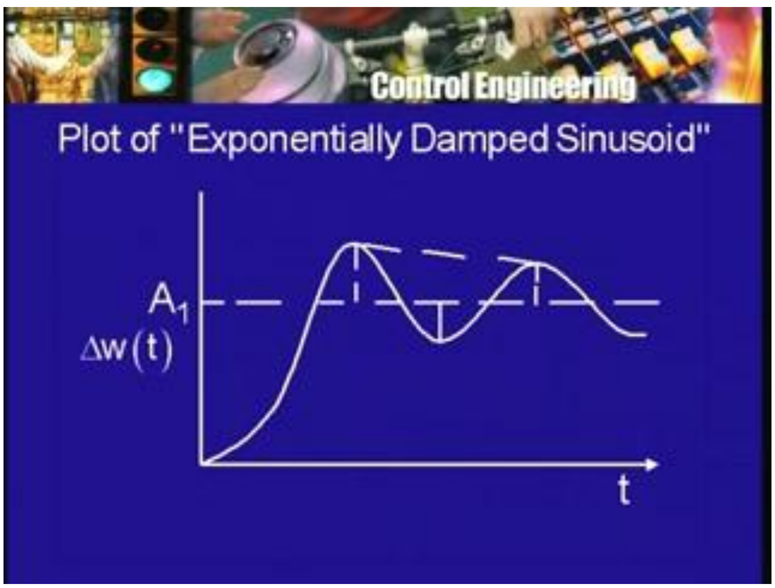

So, here is the final value of the steady state value starts up with 0 but crosses the steady state value goes beyond it therefore there is an overshoot, this is the maximum overshoot then, start decreasing crosses the final value once again but does not stop there goes on decreasing further. So this is the undershoot once again, it starts building up but goes beyond the mark, there is the second overshoot which is not as large as the first overshoot then, keeps on decreasing but overshoots the mark because once again in the opposite direction or undershoots the second undershoot is less than the first undershoot and keeps on doing this but after 5 or 10 time constant, the function is virtually A 1.

So after 5 or 10 time constants have elapsed, the change in speed will be constant or the speed will now, the at a new value. Now this was what this was the effect of change in load torque or the change in speed, we are looking at the factor or the amount by which the change in load torque affects the speed and we see now, that in the steady state, when the steady state is reached the speed will be different from what it was originally but the during the transient state, when we have a pair of complex poles like this, there will be oscillations.

So in other words the motor speed, when the load torque is increase will not simply go down to the new value but it will go down more than it will finally remain at, then keep going up and down up and down like this and go on oscillating like this, till enough time has elapsed. Now this will depend on the nature of the poles of the transfer function or the nature the poles are the roots of the factor A square plus A S plus B and we are looking at various possibilities, one possibility was real distinct roots, second possibility was real coincident roots, third possibility was a pair of conjugate complex roots and the 4 th possibility which does not arise in this case is that, we have a pair of imaginary roots.

We are looking at possibilities for a particular set of parameter values, for the motor drive, one and only one of these possibilities will hold but it is good to know, what can happen the what are the various situations which are possible and this is what we have done. I would leave it as homework to you, to lookup the section in your network theory course or network theory textbook, where this RLC series circuit with a battery and a switch is studied in detail and the response is plotted in full or look at, if the corresponding chapter in the control theory book where you look at the response of second order system to a step input. We will proceed now with feedback, why it is that we want to introduce feedback and what kind of feedback and what will it do.

Now, what we have talked so far of course we have gone beyond the steady state by using the Laplace transformation, by using the idea of transfer function and by using Laplace transformation and inverse Laplace transformation, partial traction expansion and all that. We are able to find out how the speed varies during the transient period, we are just looking at this steady state values now. So we have gone beyond saying that okay, if the load torque was at the nominal value and the motor speed was at the rated value say 1500 RPM, when the load torque increases to this amount, the motor speed will decrease after some time lag to this amount.

So this old speed 1500, what will be the new speed in the new steady state that was the very preliminary calculation that we did but now, we are able to go beyond it, we are able to say that during the transient period, the motor speed may oscillate or it may not oscillate and how long will it take for the final value of the speed to be taken, as usual it is given by or it is expressed in terms of a certain time constant and one will say that 5 to 10 time constants is enough, when that much time is elapse, the you can say that the steady state has been reached.

So the Laplace transformation technique has enabled us to do. The Laplace transformation technique will enabled us to go further now and that is we want to look at the effect of feedback.
In order to do that I am going to write down the expression for omega S, now in a somewhat different way. Remember, what we are done earlier using the signal flow graph, idea or by looking at the equations and transforming them. We were able to write down omega S as some coefficient multiplying E A of S and some other coefficient multiplying T L of S, where E A of S Laplace transform of the applied voltage, T L of S is the applied a Laplace transform of the load torque. We are able to obtain these 2 coefficients and of course, the second coefficient of T L had a minus sign and so, the increase in T L cause decrease in speed and so on.

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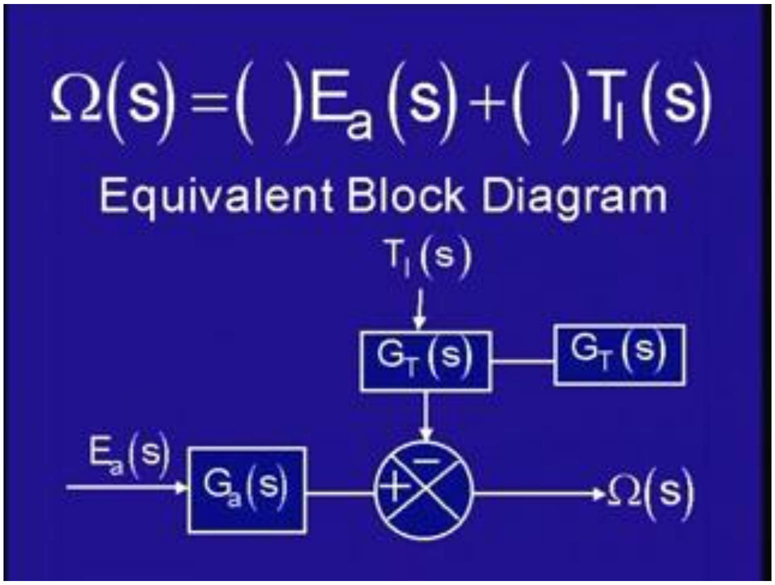

Now, this we can represent for the purpose of further work by a simple block diagram which does not correspond to any actual part or component of the system and I can represent it like this. I will show here a summation block plus sign and I will show 2 incoming arrows. So this is partly the signal flow graph idea but I am drawing a block diagram. I will have E A of S as an input to this block and this block, I am going to represent or a denote by G A of S, why A because it is something that multiplies E A of S and why, G because it is a transfer function I told you that in control literature, G is the preferred symbol for the transfer function, except for feedback path we use the letter H.

Likewise I have the other term, so I will write this. So I have to change this plus symbol now, instead of that if I use the usual block diagram symbol I will have the cross type thing with a plus sign here and anticipating that there is the minus sign here I will put a minus sign here and I will put a small block here which I will call G of T for torque S and the input to that will be T L of S. The load torque Laplace transform and the output of this device which has a plus and a minus although I can have and a plus there will be a minus sign associated with G T will be omega S. So this simply represents diagrammatically in the form of block diagram this equation, the Laplace transform of the speed is given by E A of S, the Laplace transform of the applied voltage multiplied by one transfer function G A of S minus the Laplace transform of the load torque T L of S multiplied by another transfer function which I am calling G T of S and this G A and G T, one can calculate they will involve the system parameters.

Now, we can see from this that if I increase T L of S or if I increase or decrease E A of S, it is going to change omega S, in one way or the other and that is have we figured out that a change in the load torque and increase in the load torque will result in the decrease in the steady state value of the speed. Similarly, an increase in the applied voltage will result in an increase in the steady state value of the speed. Now, when we did that of course we made use of what is call the final value theorem of the Laplace transformation.

So, let us apply that final value theorem once again. If I am looking at the final value of the speed which is the limit as T tends to infinity of omega T, then what did the final value theorem tell us that this limit that is the limit of a function as T tends to infinity which is called the final value of the function is given by limit as S tends to 0 of S times omega S, the Laplace transform multiplied by S, the limit of that as S tends to 0, do not forget this multiplying factor S, it is not the limit as S tends to 0 of omega S, it is the limit as S tends to 0 of S omega S, very often students forget this factor S, do not forget that of course one can verify it very easily, if take omega T equal to constant 1, then the limiting value is 1 omega S, the Laplace transform is 1 by S.

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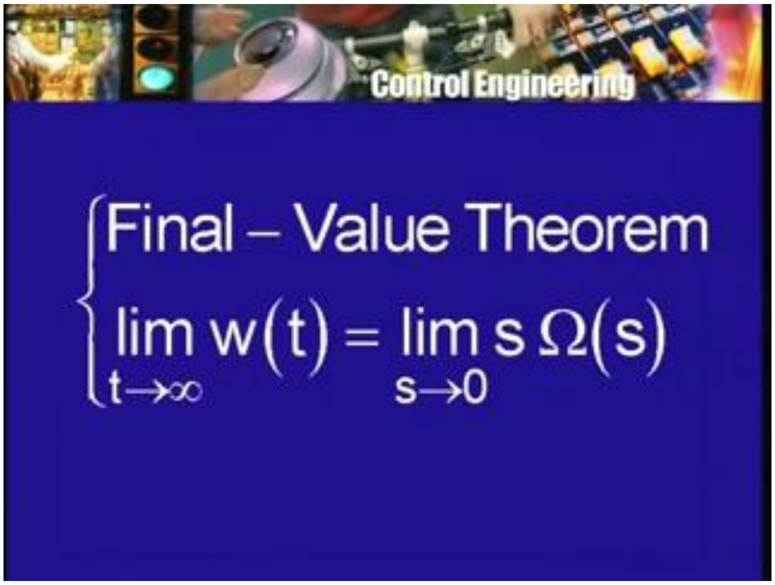

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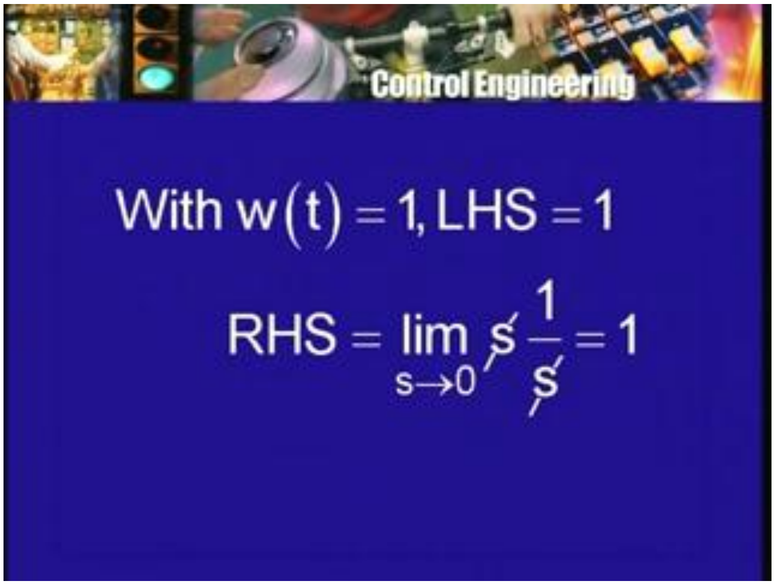

If I forget the S factor then I have the limit as S tends to 0 of 1 by S that is not 1 that is infinity but if I have multiplying factor S, then course canceling S and S, I will get 1. So this is just a way of sort of rechecking that what we are written down is correct or not but you should remember, the correct result. The final value theorem says limit T tends to infinity of omega T is equal to limit S tends to 0 of S omega S, now here I have omega S consisting of 2 different terms. So I will have E a (s), G a (s) T l (s) minus G T of S as the 2 parts making omega S, now if I would look at the final value of the speed then, you can see that it will involve the final value of the applied voltage and the final value of the load torque multiplied by G A of 0 and G T of 0.

Let us go through this, a little carefully although it is really very simple omega S equal to G a of S, E a of S minus remember, now I am writing this minus sign here earlier, I had a plus here but there was a minus sign with the multiplying factor minus G T of S, T l of S multiply both sides by S. So, S omega S equal to S, G a (s) into E a of S minus S G T (S) into T l of S and now, I am going to use the final value theorems. So I am going to take the limit of this as S tends to 0, with that mind then I will rewrite this as G A of S into S into E A of S minus G T of S into S into T L of S and now, let me take the limit as S tends to 0.

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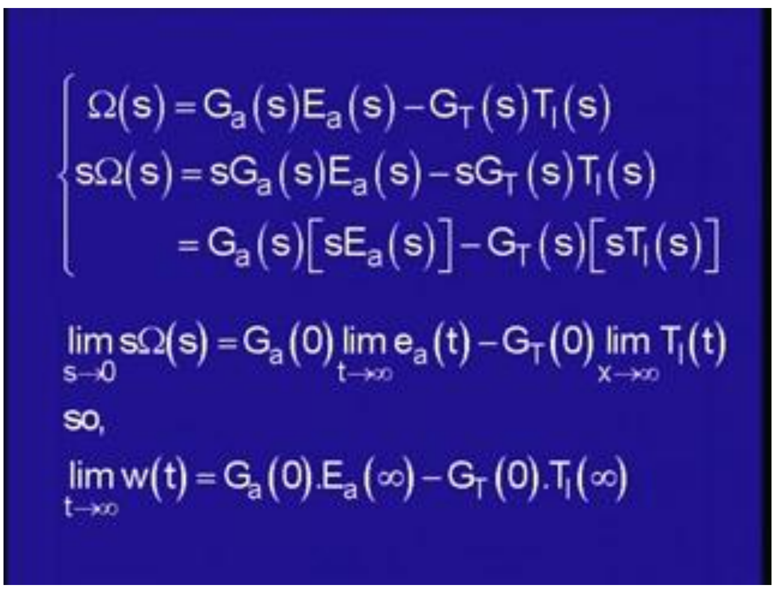

So limit as S tends to 0 of S omega S which is nothing but omega limit as T tends to infinity of omega T, the final value of the speed that is given by the limit of this as S tends to 0. Now, if G a of 0 is not infinity that is I can put S equal to 0 in G a of S, without getting into any difficulty that is there is no infinity occurring then, I can write this as G a (0) multiplying the limit as S tends to 0 of S E S but what is that, that is the final value of the applied voltage or in other words it is limit as T tends to infinity of E a of T minus G T (0) limit as T tends to infinity of T L of T.

So it will be given by, the final value of the speed will be given by G a (0) into the final value of the voltage, applied voltage, let me rewrite it as E a infinity. This is just a symbol indicating that I am taking a limit as T tends to infinity of E a of T minus G T (0) into T L infinity. So these coefficient G A (0) and G T (0), tell you how the final value of the speed is related to the final value of the applied voltage and the final value of the load torque and from this, then we can calculate the effect of change in load torque or change in voltage. If there is change in load torque find out what is the new value of the load torque that multiplied by G T (0) will give you this part, if the applied voltage has changed, find out what is the final value for the applied voltage that multiplied by G A (0) gives you this part and so, the difference between the two will give you the new final value or if we are making no change here. Let us say the applied voltage is just remaining constant throughout then, there is no change in the final value of the input from the earlier situation and the new situation whereas, the load torque had change from one value to another that change in load torque is what we will substitute here and that will give us the change in the speed of the motor.

So, G a (0) and G T (0) are in some sense 2 numbers which determine the effect of change of applied voltage and change of load torque on the motor speed or change in speed equal to G a (0) and to change in voltage minus G T (0) into change in load torque. Now because G T (0) is not 0, when there is a change in the load torque, there will be change in the speed of the motor. This is the unfortunate part of the drive as we have it without feedback, when the load torque changes, there is a change in the motor speed, this is not error because of anything other than disturbance, this is an error cause by the presence of the disturbance, the drive can be made to run at the correct speed by a choosing correctly the value of the applied voltage, when the load torque is at its nominal value, this thing is we have something, we have done earlier okay.

Now, how can we reduce the effect of the change in load torque on the speed, can we make it completely independent of the speed. Now as a first step towards that we thought of introducing feedback and what was the kind of feedback that we talked about, we called it proportional feedback. So, let us now see what will be the effect of proportional feedback whether it will reduce the change in speed or the error in operation of the system, when there is a disturbance or when there is change in the load torque, whether we can reduce it. Let us see, if we can do that now I will continue with the block diagram that I have drawn just now.

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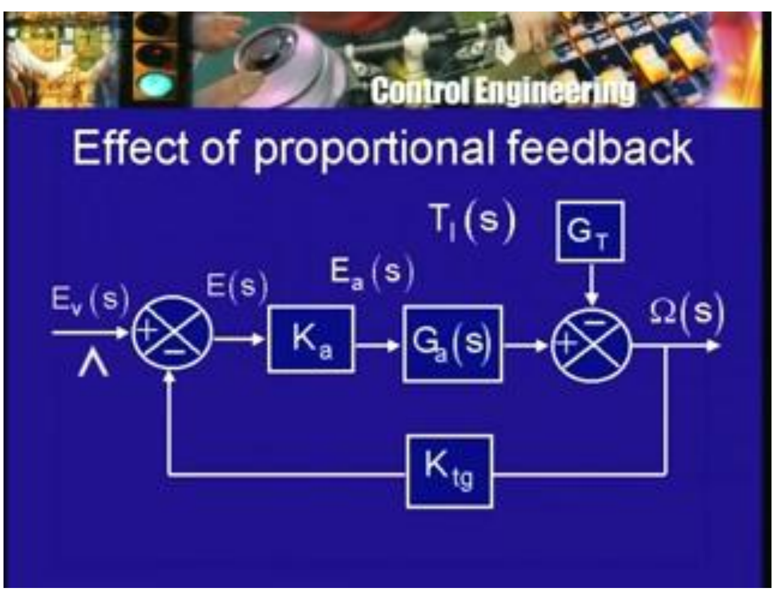

So, here is T L of S that goes into the block which I have called G T, here is another block G a and into it goes E a of S and out comes omega S and now, I am ready to use proportional feedback. So, I take up from here, here is something that represents my tachogenerator, K tachogenerator, the output of that is the tachogenerator voltage, the applied voltage is not obtain now directly but I obtain it by having, what I have called a reference voltage then, there is this difference device which takes the difference between the reference voltage and the feedback voltage. The feedback voltage in this case is simply the tachogenerator coefficient, EMF coefficient multiplied by the speed, this difference is then sent through an what is effectively and amplifier and we have call the gain of that amplifier K a. In the Leonard type of speed control, this K a is represented by something which is not that simple, it is a prime mover which drives the generator produces the output E A and the field current of the generator is what is driven by this difference between the reference voltage and the tachogenerator voltage.

So, this is the new block diagram now, for the system with feedback and now, we want to find out how does it affect the relationship between omega S and T l of S that is the disturbance, how is the speed effected by the disturbance, will the effect of disturbance be reduced. Of course, how is this speed related to the reference voltage that is also something which we have to know. Now we have all the tools that are required or that can be used the solve this problem very quickly and this is why, I had to spend some time on the signal flow graph the Mason gain rule or gain formula, Laplace transform, transfer function block diagram all this is really to enable us to see, what will happen.

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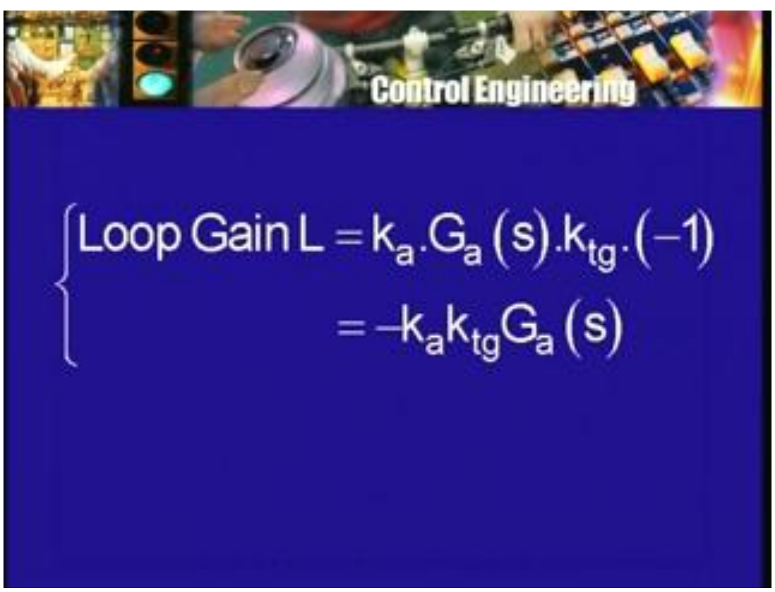

Of course, this is only a simple example we are going to take somewhat different kind of feedback and we are going to study the effect of that feedback and then, we will go to a very general situation but first this very specific case. I want to write the expression for omega S in terms of T l of S and E r of S. I want to use signal flow graph idea and in this case, you will see that it is extremely simple, here is the signal flow graph which has input E r, T l, omega S, the non-input quantity the intermediate variably E A of S, we are not interested in, I have not even bothered to show to the error variable although, here I could put E of S as the error variable. I want to find out this relationship. First of all, we clearly see a loop here say I start here, go through K a, go through G a, go through this difference device, come out here, go through K TG, go through this difference device and you come back. So there is the loop, yes there is a loop and what is therefore the loop gain, there is only one loop and what is the loop gain remember, the symbols for the loop gain L, here is the loop, what is the loop gain, starting here I have K a multiplied by G a, then I go with a plus sign.

So that then, I come here K tachogenerator but then here I go through a minus sign. So into minus 1 or the loop gain is minus K a, K tachogenerator into G a that is the loop gain L, what is the next quantity which is to be determined using the signal flow graph and Mason formula, delta. What is delta? Delta equal to remember 1 minus sum of all the loop gains, we have only one loop. So 1 minus L plus the sum of products of 2 loop gains for non-touching loops but there is only one loop.

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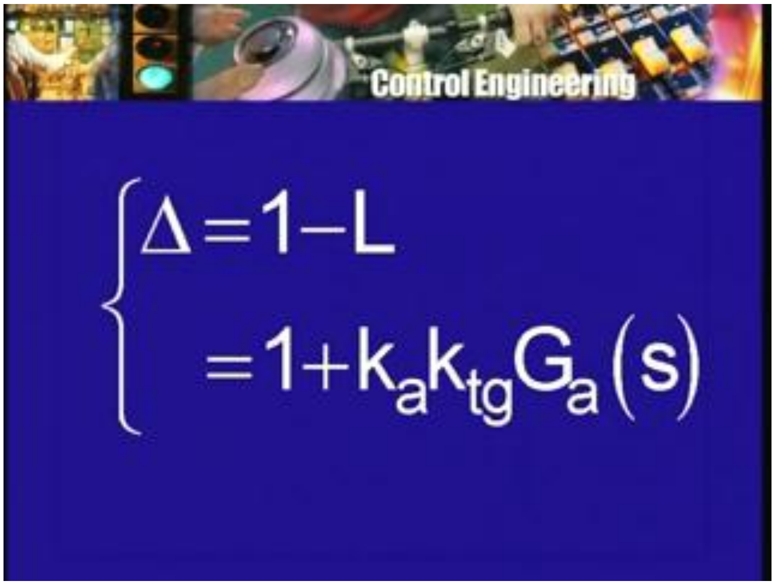

So there is nothing more, so as simply delta equal to 1 minus L or delta equal to 1 plus K a, K tachogenerator into G a. This is what I have for delta. Now, I am ready to write down the expression for omega S equal to there will be transmittance from E a of S and there will be transmittance from T l of S. Remember, all these transmittances have the same denominator, so in the denominator I will have 1 plus K a, K tachogenerator G a, I am not writing the S here explicitly here also I will have the same 1 plus K a, K tachogenerator, G a, same denominator for both transmittances. What about numerator? For the numerator, I have look at the forward path. Now, what is the forward path from E r, this should be E r of F not E a (S), I have to look at the forward path from E r to omega. There is only one forward path and what is the gain of the forward path? I go through this with a plus sign K a, then G a with the plus sign out to omega.

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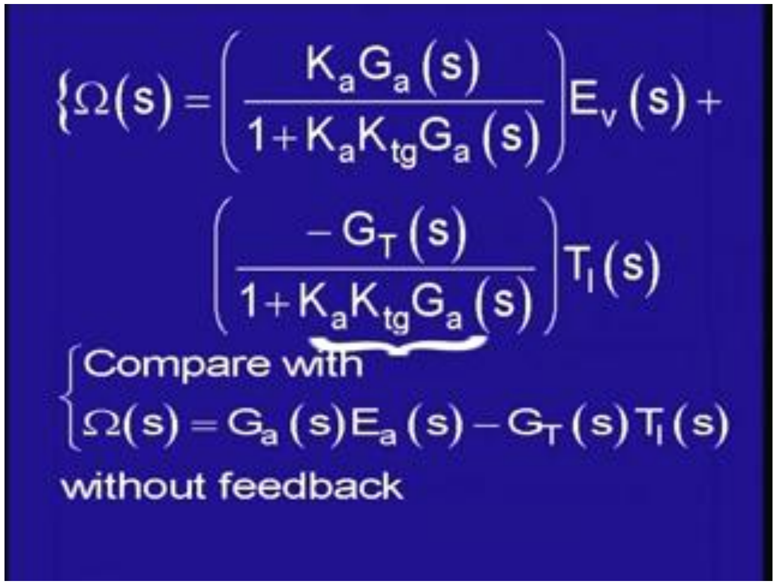

So, the gain of the forward path is simply K a times G a. So let me write this down as K a times G a that is the gain of the forward path from E r to omega S that multiplied by what remains of delta but what remains of delta, only one because that forward path touches the only loop that is there. So delta for the numerator is simply one, the other path, from the load torque to omega S that goes through G T with a minus sign here. So it will simply minus G T, so I will write here that multiplied by what remains of the delta but what remains of delta once again is only one. So this is all that there is.

So this is the expression for the speed in terms of Laplace transform of the reference voltage and the Laplace transform of the load torque. Let us compare that with the earlier expression that we had which was simply omega S was given by G a of S, let me write that S explicitly into E a of S minus G T of S into T l of S. So, what has now happened is the following and let us look at the torque term first because that is the disturbance variable and we are interested in reducing the effect of the disturbance on the system performance. So let us look at that part first earlier, it was simply G T or with a minus sign minus G T.

Now, it is what now, it is minus G T divided by 1 plus K a, K tachogenerator, G a and this K a, K tachogenerator, G a is simply the loop gain, if there was no loop, if remove the feedback path go back here. If I remove this feedback and there is no gain and if take this as my E a, then there is only G a because of the feedback, there is a loop created and because of that loop now in place of 1 in the denominator, I have 1 plus something and so, the denominator is going to be greater than 1 therefore, I have G T divided by a number greater than 1, earlier I had just G T. So the effect of the load torque will be reduced and will reduce by what factor by this factor 1 plus K a, K tachogenerator, G a or if you want to look at this steady state value then, I have to put S equal to 0.

So the use of this feedback will be that the steady state error cause a change in the load torque that is cause by the disturbance variable will be reduced, will be reduced by the factor 1 plus K

A, K tachogenerator, G a (0), let me write this down in place of 1 I have now 1 plus K a, K tachogenerator into G a of 0. Now this can be not small it can be even comparable with 1 or even larger than 1 and if it is so, then the effect of the disturbance will be reduced very much. For example, if I can make this equal to 10, let us say then earlier I had G T now, I have G T divided by 11.

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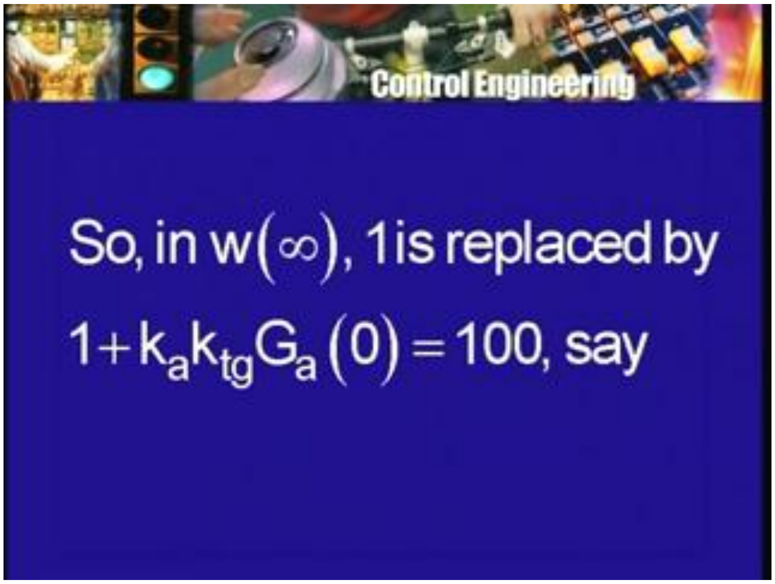

So effect of the load torque or the disturbance will be reduced by a factor of 11. So if earlier the change in speed was 11 percent. Now with this change in speed only will be 1 percent. So how can we reduce the effect of the load torque changes, can we make it absolutely 0 no, we cannot, we can make this term large, I can make this instead of 10, 100 then the effect will be reduced by 100 but I cannot make this infinite because this is just gain of that error signal. So called error signal to the armature voltage, input, tachogenerator coefficient and G a (0). The value of the transfer function involve that multiplies the motor voltage at 0.

So, this cannot be made infinite but it can be made large. Unfortunately, as I have been telling you earlier we are only looking at steady state, the effect of making this large will be to reduce this steady state error cause by the disturbance, but what about the transient behavior and we will see very soon that the transient behavior can become bad in the sense, originally without feedback, the speed may be just changing from one value to another without any oscillations, but with feedback the speed may now perform oscillations before going to the new value and the oscillations need not be small although the error may be small, the oscillations could be large and this is something we are going to look at as our next topic. But before we get into that let us look at the other term, what has happen now is E r is multiplied by this expression earlier, E a was multiplied by G a of S.

So now this 1 plus K a, K tachogenerator, G a is appear occurring in the denominator here also. So that means the value of the applied voltage which was used earlier will not be now the value of the reference voltage and this is of course something we have seen already. I was talking about in the numerical example armature voltage of the order of 232 volts and whereas I was saying that the reference voltage may be of the order of 20 volts but that is all right because this E a is multiplied by G a, whereas E r is multiplied by G a into K a divided by this denominator factor 1 plus K A, K tachogenerator, G a and therefore we can find out the appropriate value of the reference voltage. So that under rated conditions, the speed will be at the nominal value and we can complete this by using the final value theorem therefore, I will be able to tell, I will be able to write omega infinity, the value of the motor speed of the enough time has elapsed will be given by K a into G a (0) divided 1 plus K a into K tachogenerator into G a (0), multiplying the reference voltage, the steady value of it minus the second term will be G T (0) divided by 1 plus K a, K tachogenerator, G a (0) multiplying the steady value of the torque.

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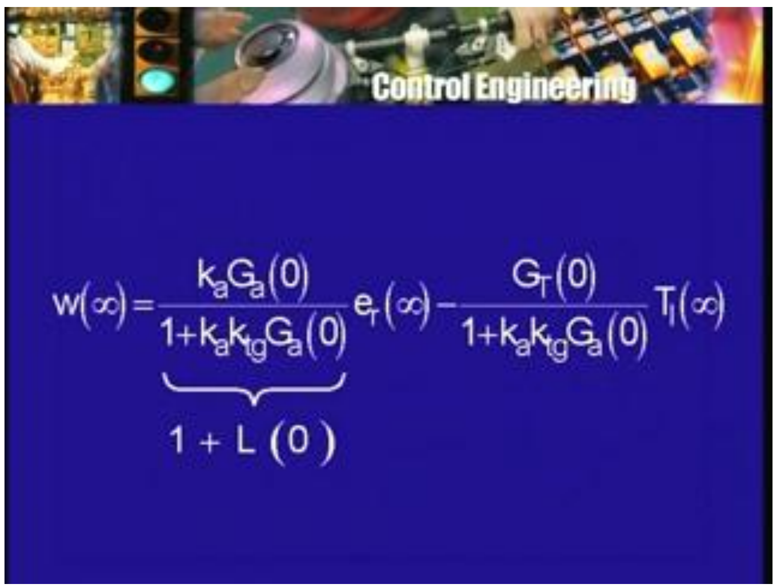

So, if I know what is the required steady value of the speed, if know under what torque condition I want that value, I know Tl infinity then, if know the tachogenerator back EMF or voltage coefficient K TG, if I know G a and G T involve the parameters of the system. I can calculate though and I choose the value of K a, the gain of the amplifier, so to speak. I can find out the reference voltage that needs to be applied to produce the correct speed. So this drive will then run at the correct speed with an appropriate value for reference voltage, when the load torque is at the nominal value. The signal e that I talked about earlier, the output of the comparator will not be 0, but that 0 is not a steady state error at all, there is no steady state error at rated torque conditions. However, when the reference voltage is kept unchanged of course and there load torque changes then, there will be a change in the speed or there will be a steady state error in the speed.

However, this steady state error is in the speed, when the load torque changes will be much less with feedback then what it would have been without feedback and the factor by which you have been able to reduce this error cause by load torque changes is given by this 1 plus L 0 and very often, L 0 is much greater than or is greater than 1 therefore, it is this loop gain its value at S equal to 0 because there is a transfer function involved this loop gain is what determines the reduction in the effect of the load torque changes.

So, what appears as a gain is really a gain in the sense of the term, the higher this loop gain is the lower will or the lesser will be the steady state error due to load torque change. But to remember, remind you once again, this L 0 increase will go with a worse performance in the transient state that is as the load, as the speed changes from an original value to a new final value and this is what, we will take a look at after, we are looked at another feedback scheme which I had mentioned long time ago, what we have looked at right now is only proportional feedback. We will look at another feedback scheme, which is, what it was called integral feedback and we will see that with integral feedback, the effect of the load torque changes can be made 0 that is no matter, how the load torque changes, if it changes from one steady value to another steady value, the motor speed although it will change during the transient period will go back to the desired value.

So the system will operate without steady state error cause by disturbance input or by load torque changes. We will see how the integral feedback achieves this thing once again, there will be a negative factor associated with this namely transient performance. We will have look at the transient performance when we have integral feedback and we will see that with integral feedback, there is a danger that the transient performance will become even worse than it is with proportional feedback and for that we need additional methods of investigation and one such method I have mention to you earlier is called the root locus method. So we will first look at the integral feedback and then, we will go on to the root locus method.