Control Engineering
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Lecture - 27

The calculation of the partial transfer function from the applied voltage to the motor speed must have looked rather complicated. Of course, one of the reasons for taking it up was to remind you about the signal flow graphs and use of the gain formula, for making some calculations of transfer functions or transmittances using the signal flow graph. We will do it in a different way now, using the earlier concept of a block diagram and in this case the block diagram is also going to be fairly simple. In fact, the block diagram is more meaningful than the signal flow graph but remember that if, we look at the block diagram, we can apply a rule similar to Mason's gain formula to the block diagram as well.

Let us see how we can do that, what we will do is now again, our starting point is the 2 equations that is all we know about the motor and the drive but we will draw the block diagram. So as to represent those 2 equations in a particular sequence, so first look at the voltage equation we have the applied voltage E a on one side and we have the back emf and the armature current multiplied by the operator sL a plus R a. Remember now, we are looking at the transient of the non-steady state behavior therefore, we have the derivative term and therefore, we have the armature inductance term and we are taking Laplace transforms of all the variables involved both the current, the speed and the voltage, as well as the load torque, all of them.

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We are looking at only Laplace transforms or sometimes one says that we are looking at the problem in the transform domain or in this so called S domain, whereas in contrast the differential equation is set to be in the time domain or T domain and the Laplace transformation method allows us to go from t domain to S domain and then, back to T domain. So we are now in the S domain or in the Laplace transform domain. So from that equation, I can write E a as a single which is coming in to a difference device, this is not a physical difference device, this is only a conceptual difference device.

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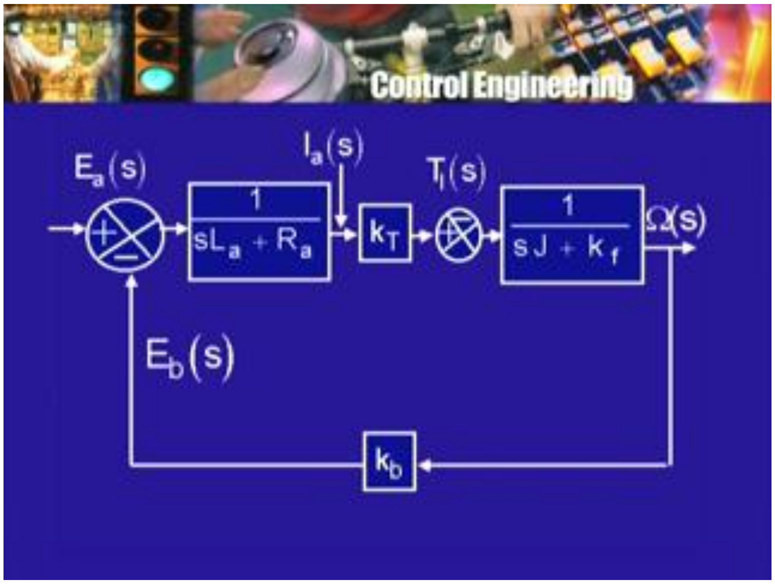

So, it takes the difference of E a and the back emf term which is k b omega. So I write that omega out here as the output signal transform and I will show a gain factor or a block here k b. So we have E a minus k b into omega, this is the output of this difference device but that equals R a, I a plus L A d I a Dt in the time domain and therefore, sL a plus R a in the S domain and therefore, in order to get the transform of the current I a, capital I a, I will have to put into this block 1 divided by sL a plus R a. These also looks like a transfer function but remember that this E a minus k b omega is not a actual physical signal and I have told you earlier it is not that back emf appears as separate voltage which gets subtracted from the applied voltage. The motor action is described through a back emf.

So this 1 by sL a plus R a sitting there is not really transfer function of system or device as such but it represents relationship between the armature current and this difference E a minus k b into omega so I have written there1 over sL a plus R a. So there is a block there. Now this I a produces torque and therefore I have that proportionality constant k t, the torque constant. So this produces the motor torque, from this motor torque I will use put another difference device the load torque gets subtracted. So I am showing T l as an input to this difference device with a minus sign.

So, k t I a is the motor torque subtract from that the load torque and that is the torque that is available, for accelerating the motor as well as for overcoming the viscous friction and therefore, this will be multiplied by what, if you look at the torque equation, we have k t, I a equal 2 T l plus 2 terms, one corresponding to friction, the other corresponding to moment of inertia and therefore in this block, I will have 1 over s J plus k f. So this will be the block diagram for the system this looks simpler than the signal flow graph does not it. The signal flow graph had 2 input nodes, 2 output nodes and there where transmittance says to one of them to other and it looked a little messier also instead of writing transmittances along the lines in the signal flow graph, if I show blocks like this, it is stands out little better. Remember that these are not actual physical devices, this k b is not a physical block, the motor action is such that there is back emf and the motor action such that the difference between the applied voltage and the back emf is what determines the current.

Similarly, the motor torque is not produced in a separate place the motor torque is produced at the same time, there is the load torque opposing the motor torque plus the friction and the difference is available for accelerating or decelerating the motor. So none of these blocks really stand for any separate physical device as such except perhaps the moment of inertia and coefficient to friction k f part, this could stand for the drive part of the whole thing, whole system that is the load and the friction and the moment of inertia of the, all the masses on this shaft.

Now from this block diagram, we can write down the transfer function and remember, what I told you right at the beginning today that we can think of this in way similar to signal flow graph. There are arrows here one can look at and see loop here, one can talk about forward paths and so on and so, one can deal with this block diagram, very much like a signal flow graph and we can therefore, use the Mason gain formula, for the block diagram as well and so, if I do that then what will I get. I will get the output signal omega of the Laplace transform equal to a transmittance form E a plus a transmittance from the load torque. So there are 2 incoming arrows E a is 1 T l is another.

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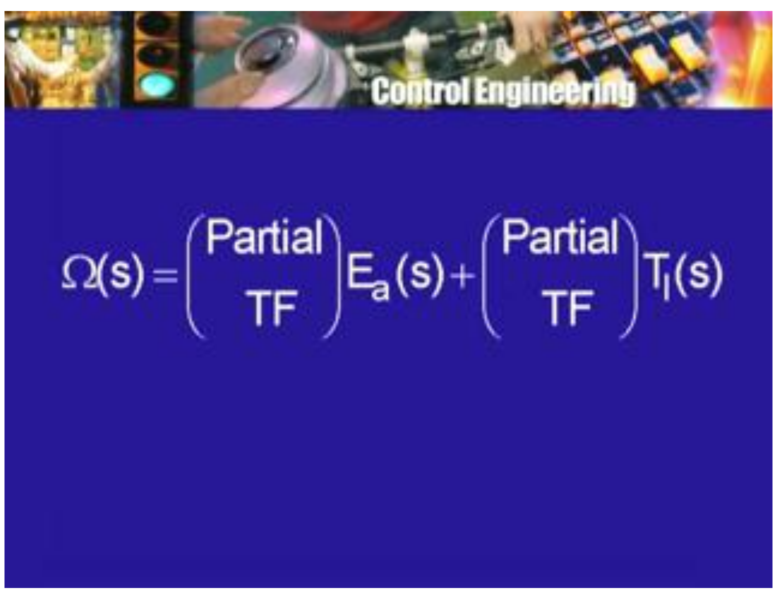

So I will have omega as a sum of 2 terms, one of them is transmittance-multiplying E a and this, I have called the partial transfer function from E a to omega that is gives you the effect of E a on omega and the other is a partial transfer function from load torque to the speed. Remember, all of this is only a part of the response, we have ignored or we have kept outside the part corresponding to the initial conditions. The armature current at time 0, t equal to 0 and the motor speed at time equal to 0, we are left have out that part we are only looking at part of the response. Moreover, the initial conditions of the input of course, in this case the inputs are E a and T l, E a also could have a initial condition that also is being kept outside of this calculations or this block diagram.

Now, what will be the partial transfer function once again, we can look at the loop and one can look at loop gain and the loop gain will turn out to be exactly, what it was earlier. So looking at the loop then we have, one transfer function 1 over sL a plus Ra that gets multiplied by k t then, there is also 1 over s J plus k f and as we go through the difference devices, we have to take care of the sign. So far it is okay 1 over sL a plus R a into k t into 1 over s J plus k f because the signal from k t output signal from k t goes through it plus sign. So no change of sign then, this is multiplied by k b, we come back to the our starting point but then, we go through minus sign.

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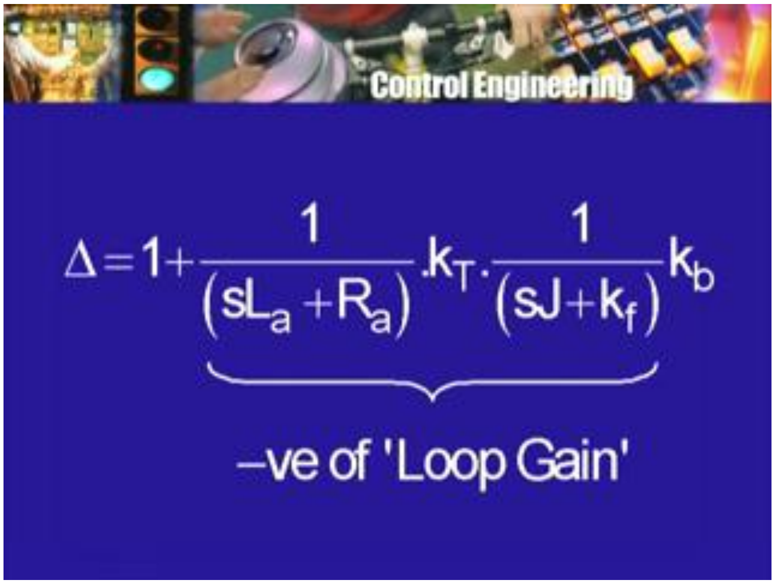

So this multiplied by minus 1 or with a negative sign is the loop gain and this is precisely our delta in the signal flow graph, gain formula and you can check that we have got the same thing as earlier. So, we have k t, k b divided by these 2 linear factors in the denominator, sL a plus R a and s J plus k f and we can write them using the time constant as we have done earlier. So that is delta and now, for the gains or the transmittances from Ea to omega, there is only one forward path and the gain of the forward path again is very easy to see, simply go through those 3 transfer functions 1 over sL a plus R a followed by k t followed by 1 over s J plus k f that is the gain of the forward path that is going to be multiplied by only one because from sorry, delta was 1 minus that not minus that this is a mistake that students make and I also made just now. It is 1 minus, minus therefore 1 plus this thing this thing with the minus sign was the loop gain and delta was 1 minus sum of loop gains plus sum of products of 2 non touching loop gains etcetera, etcetera.

So this is delta 1 plus that remember, not just minus. So this is the denominator and the numerator is simply the forward path transfer function, similarly the forward path from T l to omega. Now here, we have to take care of the minus sign because T l enters the difference device with a minus sign. So minus 1 into the next transfer function block 1 over s J plus k f that is all in the forward path from T l to omega. So that multiplied by 1 once again, divided by delta is the transfer function from the load torque to the applied voltage E a. The block diagram is a little less cluttered than the signal flow graph but the way, we look at the block diagram and compute the transfer functions is exactly like we did for the signal flow graph. There is not any difference and I told you earlier that the signal flow graph concept evolved out of the block diagram concept. In some situations, the signal flow graph is easier to deal with, in some other situations especially, when you want to get some insight into what is going on the block diagram is the convenient thing to deal with. So this is therefore what we have, right now.

Now, as we saw earlier the disadvantages of this kind of a drive is that, it can be made to run at the desired speed or nominal or rated conditions that is given the rated load torque, in other words the load torque at which or when, for which you want the motor to run at a specified speed say omega 0, one can make calculations and decide, what the value of E a or in our case earlier, capital E a, now small E a should be. In other words, knowing the load torque which to be overcome, knowing the speed at which the drive is to run, one can calculate the voltage to be applied and then we expect that the drive will run in the steady state at the desired speed. But should T l change now then, the drive will not run at the desired speed and by what amount will the speed change.

Now this is something we can find out, we can look at the partial transfer function from omega 2 T l. So, let me right it here as omega by T l but, remember that this is only partial transfer function. So this will be equal to what delta that we had as before in the denominator multiplied by in the numerator, we have minus 1 over s J plus k f and that is all. So this is the expression for the partial transfer function from omega 2 T l and now, what is the effect of a change in T l that is what we want to compute.

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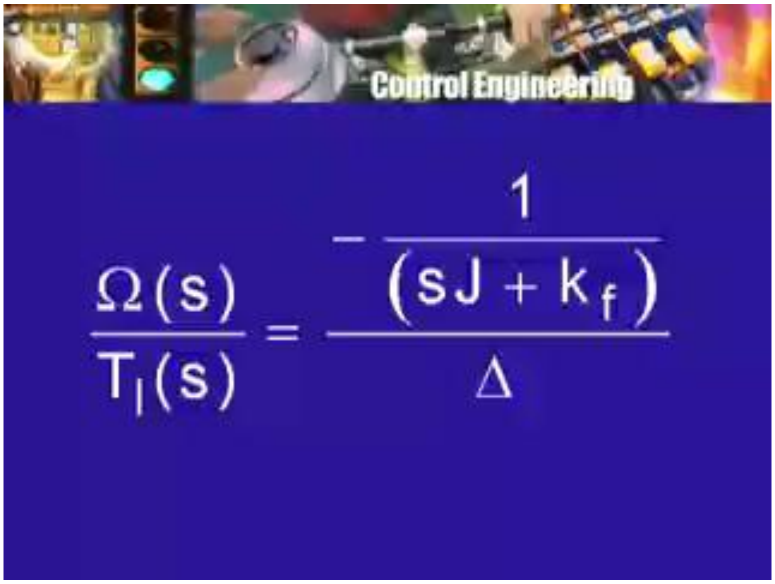

We will look at the change in T l, the effect in the transient state that is as the load torque is changed, let say suddenly from one value to another, what will happen. But if you want find out the effect in the steady state then, this is something we have done earlier with the signal flow graph. We can do the same thing here because we have the same transfer function that is use the final value theorem of the Laplace transform theory and get the value of the speed, as speed tends to infinity and therefore, we can look at the change in torque and find out the corresponding change in the steady state speed and minus sign here shows that if, the load torque is increased, the speed will decrease.

So the drive will not run at the same speed omega 0, when the load torque is changed. So this was one of the defects of the drive that we wanted to over come and we thought that we could do it by using feedback and to that will be coming very shortly. But what if, I want to find out the what happens in the transient period that is when the load torque is changed suddenly, from some value to some other value, very quickly, what happens to the speed. We know that the speed will change and eventually it will come to some new constant value that is a new steady state but we want to know, what will happen in between and the transfer function will enable us to solve that problem and the qualitative study of the transfer function by means of the pole 0 diagram will also enable us to say something about, what will happen in the transient period.

So that is something we will take look at now. So for that let me rewrite the transfer function now, so omega divided by T l equal to, now clearing the denominator that getting rid of the s J plus k f from the numerator as well as the other factor from the denominator. We will have this is equal to minus, minus sL a plus R a divided by, we will have sL a plus R a into s J plus k f plus k t, k b. The transfer function from the applied voltage to the motor speed was different, the numerator was different.

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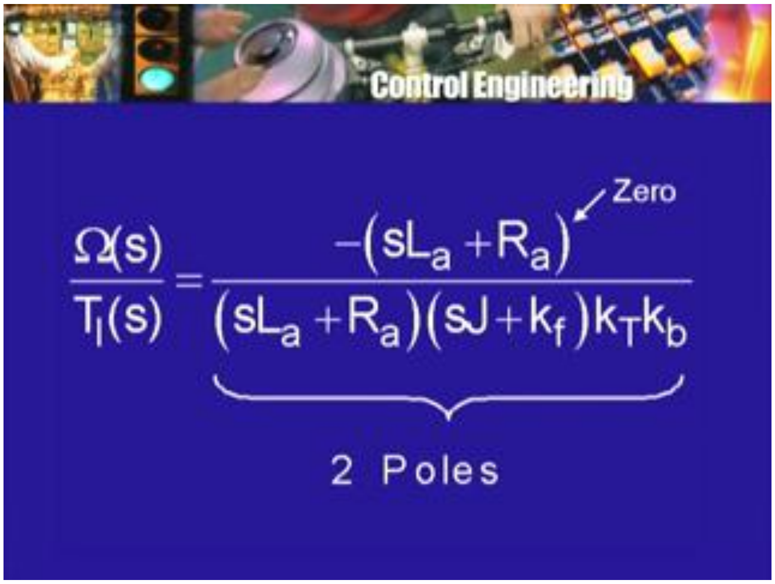

Now, in the numerator I have a factor sL a plus R a, in the transfer function from the applied voltage to the speed, there was no factor involving S in the numerator, all we had was a constant that is difference now, we have factor sL a plus R a with minus sign in the numerator, what does it mean now. This means that this transfer function or this partial transfer function has a denominator, the denominator is a quadratic and therefore I can imagine in the quadratic factorize into it is 2 factors and this quadratic is exactly, what we had earlier for the other transfer function because it is common delta, it comes from common delta. So the denominator is a quadratic which means there are 2 poles, this transfer function has 2 poles but it has something in the numerator which is not a constant but it is a linear or first degree polynomial and therefore, it has a 0.

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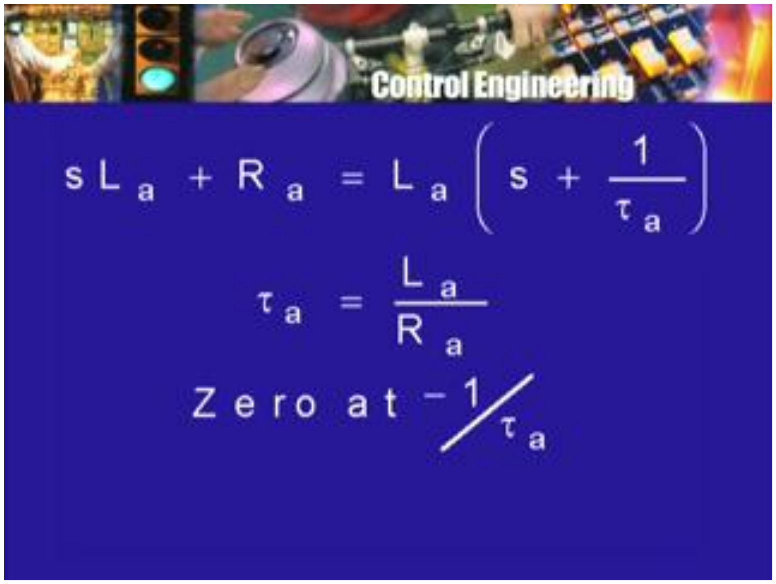

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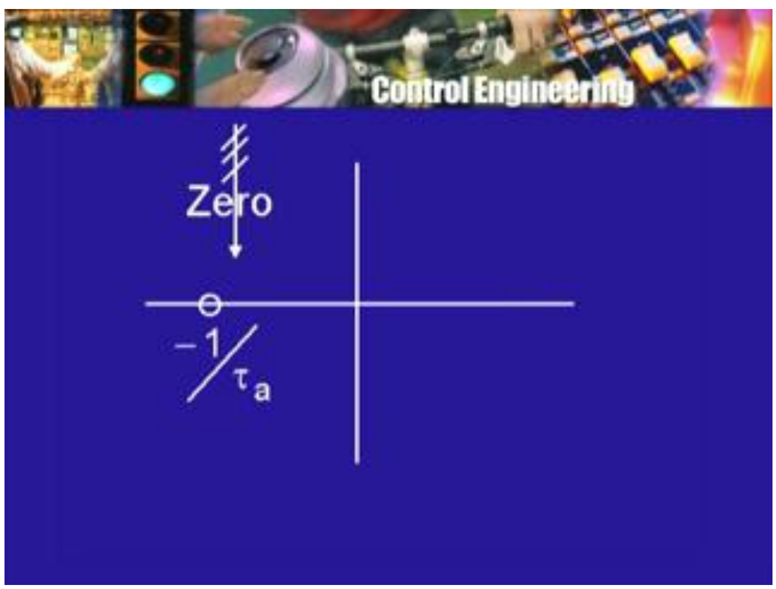

So the situation is different now, for this transfer function has 2 poles but it has also 1, 0 and the 0 is given by what sL a plus R a. I told you, it can be rewritten by pulling out l a as plus 1 by tau armature as of the 0 is at minus1 by tau armature. If tau armature is the time constant then, minus1 by a is the location of the 0, tau is of course 0 positive. So minus 1 by tau a is real and negative and therefore, the 0 is in the left half plane not only that it is on the real axis, what is called the negative real axis and the 0, I told you earlier is indicated by putting circle like this and so, I will put the circle here and write down minus 1 by tau a.

So this is the location of the 0, what about the locations of the 2 poles and I told you last time, we looked at this situation that depending on various values k t, k b etcetera. We can have a number of different situations, we can have 2 distinct real poles, we can have 2 coincident real poles and we can have 2 complex and conjugate poles. But all of them lying in the left half plane and so different situations are possible as far as the 0 is concerned the 0 is, what I have put down here but the poles could be one of those alternative locations. There could be a pair of poles on the negative real axis, there could be that is distinct poles, there could be a repeated pole that is a second order pole or a second order of factor corresponding to a second order factor on the negative real axis or there could be poles out in the complex plane, but all in the left half plane.

Now what will be the effect on the response. Now for that to find out the actual expression for the change of course, one has to get all the numbers substitute the numbers then, find out the roots and then go further but right now, I am only talking about qualitative understanding. So let us assume that the denominator polynomials are such that it has 2 real linear factors therefore, the transfer function omega divided by T l will look like minus sL a plus R a divided by and I have to pull out the factor l a and s J out and I get a quadratic S square plus a s plus b is what I wrote last time and therefore, I will rewrite this finally as minus and that sL a, l a can also be cancelled.

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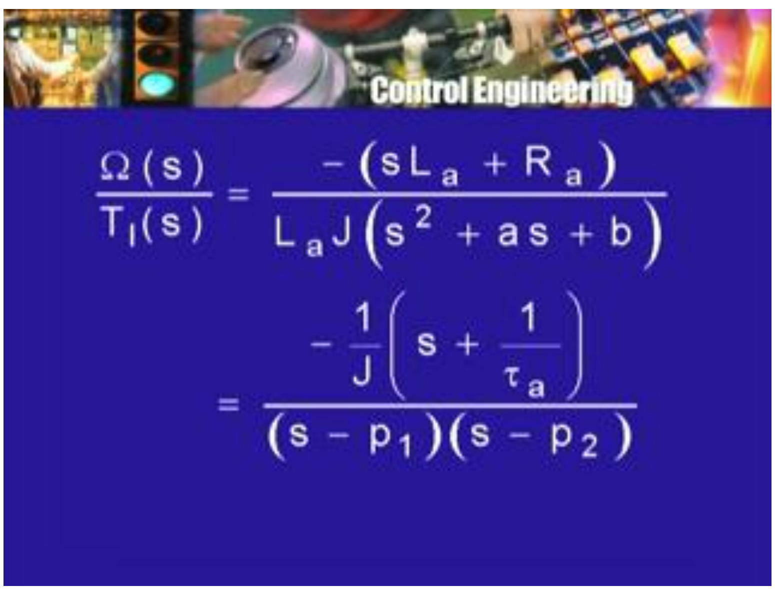

So I will get minus1 by J into S plus1 by tau a divided by I will write this as s minus p 1 into s minus p 2, if there are 2 real poles both of them negative p 1 and p 2, if this is what is the case that is the numerical values of the parameters are such that the quadratic has, 2 real factors like this, with both the roots p 1 and p 2 negative and real then, this is what we have. Well, now I want to find out what will be the change in speed when I change the torque suddenly by a certain amount. Now a sudden change in torque can be represented by means of a constant function. Of course, it is called a step function because we think of it, as being 0 before p less than 0 and we think of the thing suddenly becoming non-zero at and following t equal to 0 and that is why it is called a step function. So, now instead of considering T l, I am only going to consider the change in T l.

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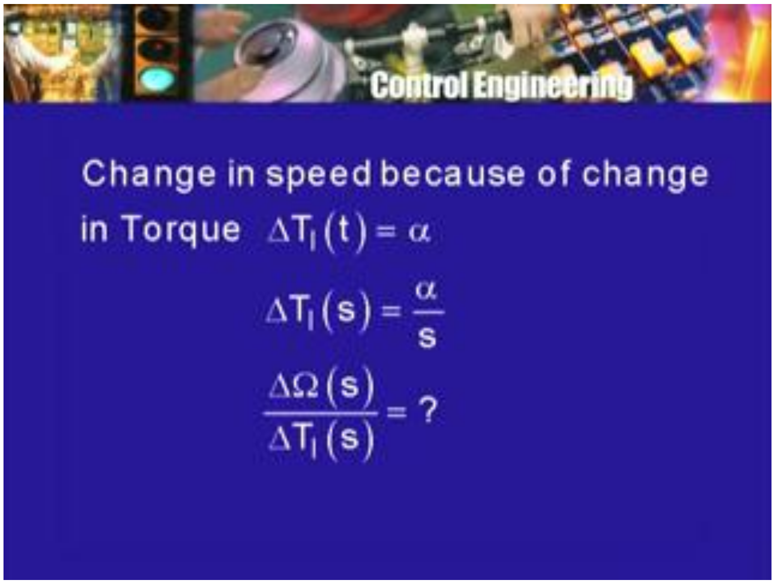

So I will write down in this sense then delta omega divided by delta T l. Remember, this is not the ratio of 2 numbers it is the Laplace transforms for the change in speed divided by the Laplace transform to the change in load torque. This is the expression that I am writing and that expression will be exactly the same transfer function that I have but now, I can think of applying the partial fraction expansion idea to the transfer function not only that my delta T l will be what, my T l as a function of time as a step function. So the Laplace transform of that is 1 by s multiplied by the value of the step or the amount of the step function amount of the jump.

So, let us call that amount by again there is going to be a conflict of notation as I told you small t for time and capital T for torque or transform. So this confusion is little unavoidable unless we introduce some more symbols, may be I will avoid the confusion right now by saying that okay, let the change in torque in Newton meters b equal to what say alpha, alpha is the change in torque, say the load torque changes suddenly by amount alpha Newton meters, alpha may be positive, the load torque is suddenly increased, alpha may be negative, the load torque is suddenly decreased and we are looking only at this particular kind of change, when the load torque changes sudden instantaneous and therefore, it can be represented by a step function.

In that case, delta T l s the Laplace transform of the change will be simply alpha divided by S and therefore, the change in speed, the Laplace transform remember, will be given by alpha divided s into the transfer function. Now, that will be what that will be 2 factors in the denominators s minus p 1 into s minus p 2 and something in the numerator. Now, this you will recognize is a proper rational function, it is a ratio of 2 polynomials and degree of the numerator is less than the degree of the denominator. The degree of the denominator is 3, the degree of the numerator is only 1, such a rational function is known as proper rational function or a proper rational fraction.

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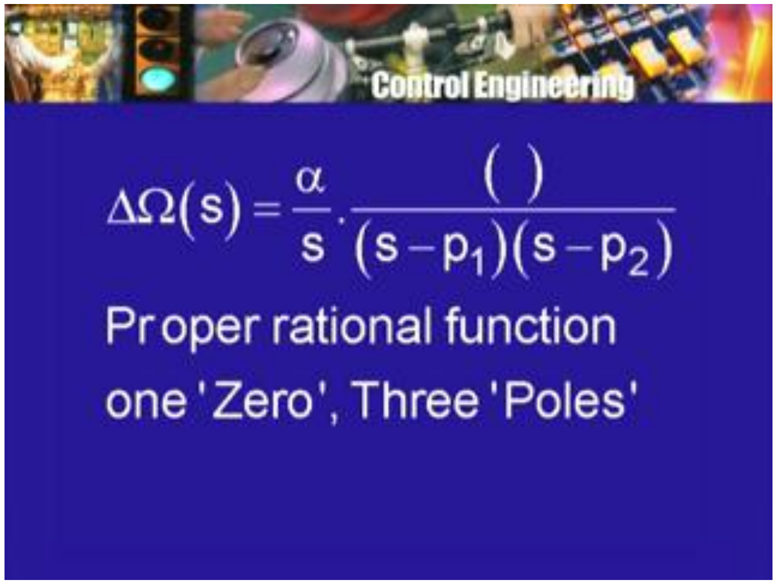

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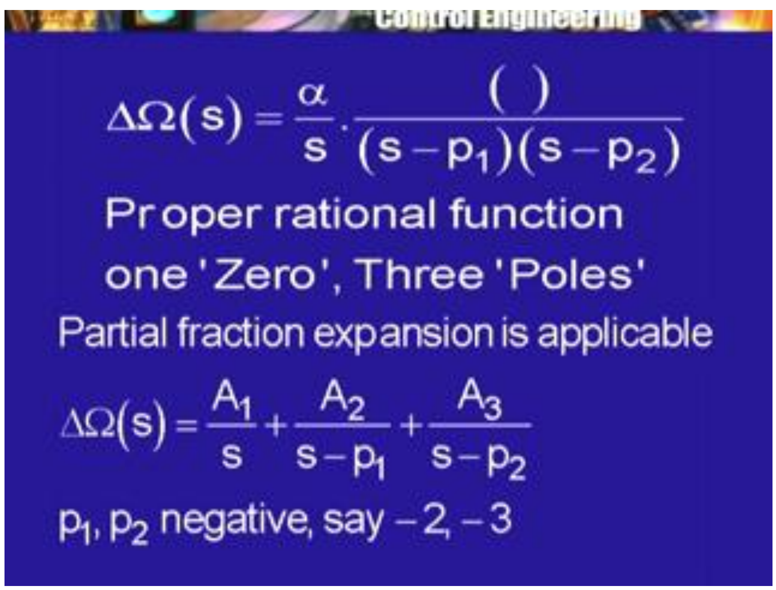

Now as soon as we see that we can apply the partial fraction expansion idea to this and therefore, what will we have, we will have a partial fraction expansion, which will have 3 terms. There is one factor, so there will be 1 s into denominator plus, there is another factor s minus p 1, so there will be s minus p 1 in the denominator plus there is a third factor s minus p 2, so there will be third terms s minus p 2 in the denominator. So this ratio of the 2 polynomials can be written as a sum of 3 proper partial fraction, each one of them has only a first degree polynomial in the denominator s, s minus p 1 s minus p 2. In the numerator, I will have some constants A 1, A 2, A 3

So delta omega s is given by A 1 by s plus A 2 by s minus p 1 plus A 3 by s minus p 2, where p 1 and p 2 are negative. So for example p 1 may be minus 2, p 2 may be minus 3. So the denominator factors look like s plus 2, s plus 3. Fine then, what can we say about the inverse Laplace transform or what can we say about the change in speed. The change in speed delta omega t with a small letter omega and a function of time will therefore be what, corresponding to A 1 by s. We will have just the constant A 1 corresponding to A 2 divided by s minus p 2, p 1, we will have A 2 into e raised to p1 t plus A 3, e raised to p 2 t.

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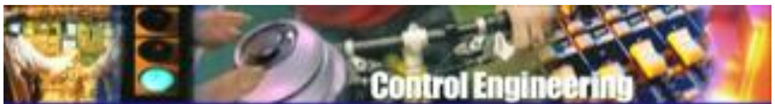

$$\Delta w(t) = A_1 + \underbrace{A_2 e^{p_1 t}}_{\to 0} + \underbrace{A_3 e^{p_2 t}}_{\to 0}$$

So as a function of time, it will come to consist of 3 terms, a number or a constant a 1 and 2 other terms which are changing with t. But remember, p 1 and p 2 are negative, so these 2 terms are decaying exponential and therefore after enough time has elapsed now, each one of them corresponds to a time constant, we can take the larger of 2 time constant and if you wait for say 5 times that or 10 times that duration then, both of these would have become nearly 0 and what would remain is A 1.

Now this A1 using the partial fraction expansion approach can be calculated fairly, easily and as a result the one can find out the change in speed rather quickly to find out this A 1, go back to the partial fraction, the transfer function. So let me rewrite the transfer function once again as delta omega divided by delta T l equal to, now I am going to use the letter G once again, but this G is different from what I may have written earlier. as I told you that the preferred letter for a transfer function in control theory literature is capital G and capital H, if it something that us to do with feedback or in the feedback path.

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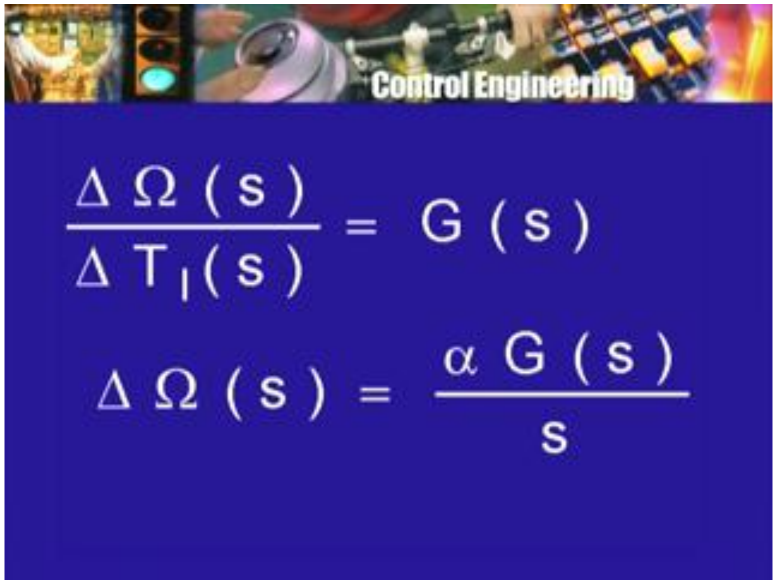

So I will write it G of s, this G of s is of course an expression that we had written earlier. So I have delta omega divided by delta T l equal to G of s delta T l once again, was simply alpha by s. So delta omega by s was equal to alpha G (s) divided by s. Remember, the G (s) is a ratio it is not simply a polynomial it is a ratio of 2 polynomials, in fact that is why we had 2 factors in the denominator and we had one factor in the numerator. Now this A 1, the coefficient that goes with that partial fraction expansion term 1 by s can be obtain by this following simple device. Now this is something which is proved in your algebra textbooks which deal with partial fraction expansion.

So look up a good algebra textbook that discusses partial fraction expansion but the way to obtain that A 1 is very simple, all that you do is you look at this expression, suppress the factor s and then put s equal to 0. Suppress the factor s because if I do not suppress and if I put s equal to 0 then, I will have 0 in the denominator which indicates some trouble, no but the rule says that suppress the corresponding factor in this case s from the denominator and put s equal to 0 that is the root to which it corresponds to the term, s would correspond to root 0 because it will be 0 at s equal to 0.

So put s equal to 0 that will be A 1 therefore, immediately we have A 1 equal to alpha G 0 and so we have an expression for the change in speed. In the steady state, when the load torque is changed suddenly and kept constant thereafter, the amount by which the load torque was changed was alpha and we have G 0, the transfer function from the load torque to the speed evaluated at s equal to 0 that is, in the transfer function simply put replace s by 0 and whatever number you get that number is G 0. It will turn out as you can check for yourself the G 0, is not 0 in fact it is positive no, it is not positive, it is negative, why because if alpha the change in load torque is positive, if the load torque increases I expect the motor speed to decrease. So just check that this is negative because there are the minus sign speaking out in front.

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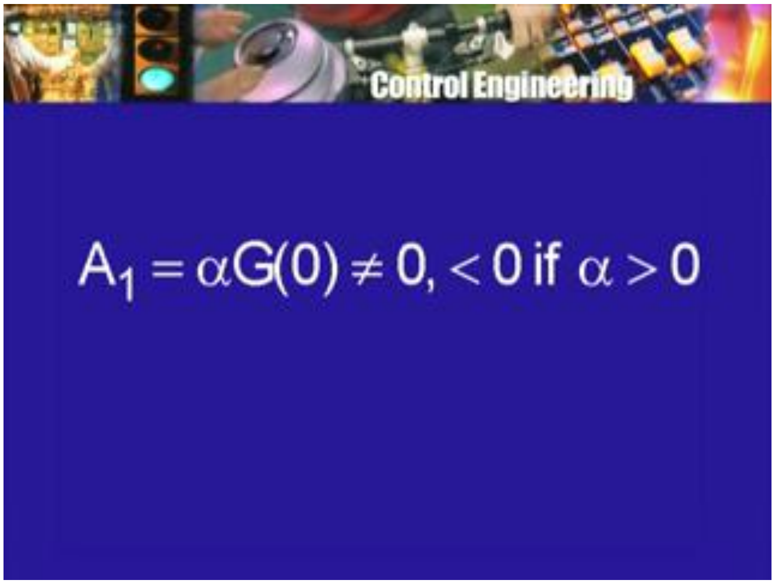

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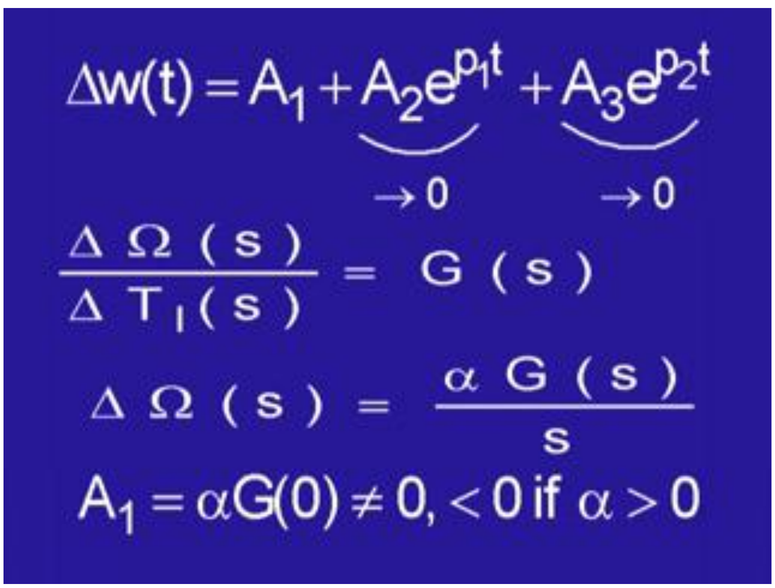

So, G 0 is not 0 it is negative, so an increase in the load torque results in decrease speed, decrease in load torque results in a increase speed and so for a instantaneous or sudden change in load torque, we can find out the change in speed as a result of that change in the long run that is in the new steady state, I am assuming that the motor was running at some constant speed with a constant torque. Suddenly, the load changed it is nature say the job was changed or some friction additional friction was encountered on the job or whatever. As a result, the load torque suddenly increased the motor speed will then reach a new steady state value but that is not all that we can get from this, the 2 terms which go to 0, tell you how the speed changes because if, I write down the expression for delta omega T then, it has A 1 this is the final value of the change but we in between that is before that final value is reached, you have these 2 exponentially decaying terms.

Now of course, e raised to p 1 t, e raised to p 2 t are exponentially decaying why, because p 1 and p 2 are negative, the poles are in the left half plane therefore corresponding we will have exponentially decaying functions but their multipliers a 2 and a 3 may not be both positive. So one them could represent an exponential decay but which is with a positive sign, the other could represent an exponential decay with a negative sign. One has to look at the actual numbers and workout what A 2 and A 3 are going to be that thing is getting added to the change that will survive as p tends to infinity namely, this change in the steady state and one can say now, that how long will the motor take to reach the new speed beyond the 2 poles p 1 and p 2 correspondingly, now the 2 time constants. The time constants are the reciprocals of the absolute value of the poles the poles are negative therefore I have to take the absolute value and take the reciprocal that is the time constants.

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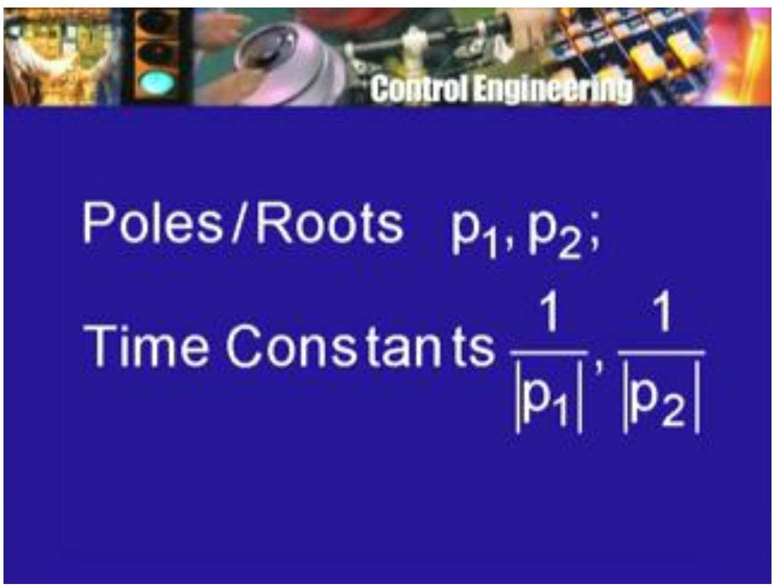

So one of the time constants is 1 by absolute value of p 1, the other time constant is 1 by absolute value of p 2 and therefore, whichever is the larger of 2, say 5 times that or 10 times that is the time by which the motor would almost reach it is new steady state value and so, by drawing the block diagram and by using the Laplace transformation, by working out the various transfer functions or put have the expressions that we associate with the block and by applying the gain formula for the block diagram, as we did earlier for the signal flow graph. For a special change in input namely the step change in input, we can calculate the change in speed as a function of time exactly from that we can also find out change in the final value but we can also find out, how fast the change will take place or how long it will take for the speed to go to its new value that requires knowledge of the 2 poles of the corresponding 2 time constants.

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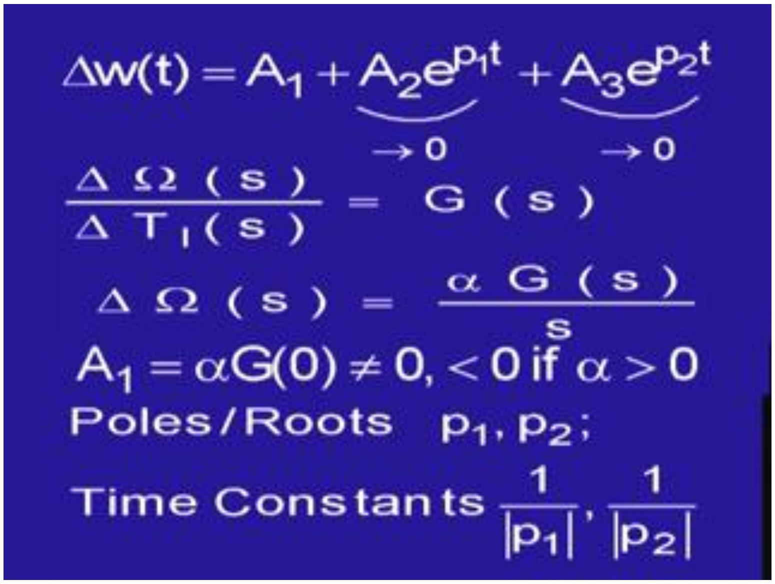

Now, this was for the case, when the quadratic had 2 distinct linear factors or the partial transfer function had 2 distinct poles in the left half plane lying on the real axis. Now, we have to consider the other 2 cases remember, we are only doing some kind of a qualitative steady I am not actually calculating anything here but I am trying to figure out before hand what could happen. So let us consider the second case the denominator factor is repeated, as a result now, what I will have is delta omega s will be something in the numerator that polynomial first degree polynomial will remain divided by for a step change in load torque s will be there once again, but multiplied by say s minus p 1 square because there are 2 factors, which are the same.

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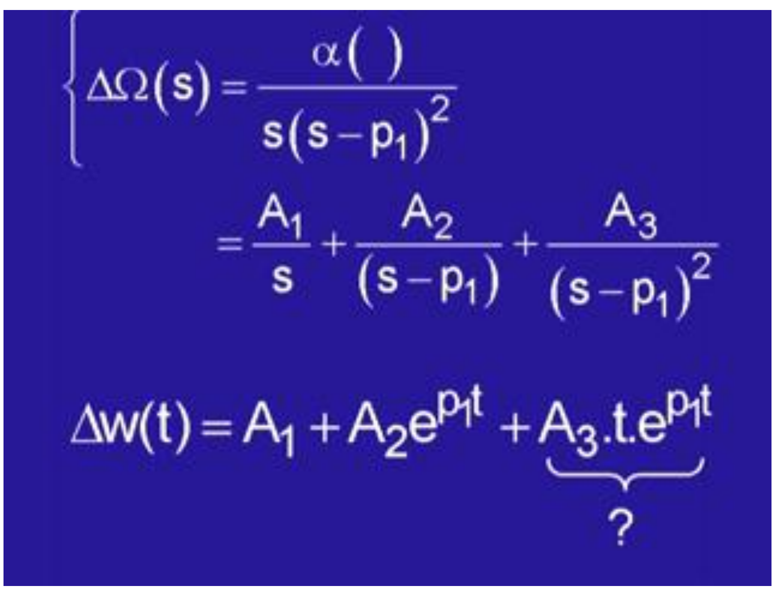

So instead of s minus p 1 into s minus p 2 has s minus p 1 squared, this will be expression for the change in speed the Laplace transform of it, when there is step change in the load torque and the step is of the value alpha as we have talked about. So I will put that alpha in front here, so it will be alpha multiplying some expression divided by s into s minus p 1 squared. Now the same thing once again, apply partial fraction expansion to this unfortunately, the partial fraction expansion is not as simple as it was earlier however, one of the terms is exactly as before one of the terms is simply A 1 divided by s, corresponding to the factor s in the denominator. But because I have the factor s minus p 1 square in the denominator, I have now 2 partial fractions, one of them will have s minus p 1 as the denominator and the other will have s minus p 1 squared as the denominator.

The numerators will be A 2 and A 3, again some constants which have to be determined. The partial fraction expansion result or theorem tells you that this expression can be rewritten like this. To calculate A 1, A 2, A 3 you have to do some work, we have to know what all the numbers involved are and do some computation. But qualitatively, we know that this can be rewritten like this and therefore, we know that delta omega t will consist of 3 terms and what will they be corresponding to A 1 by s, I will just have the constant A 1 corresponding to A 2 divided by s minus p 1 what will we have, A 2 into e raised to p 1 t and remember p 1, again is negative. So this is the exponentially decaying term, so there is no difference but the third term has s minus p 1 square in the denominator because of that the Laplace inverse of that has to be used and Laplace inverse is a little different. So it is A 3 multiplied by e raised to p 1 t but that is not all multiplied also by p.

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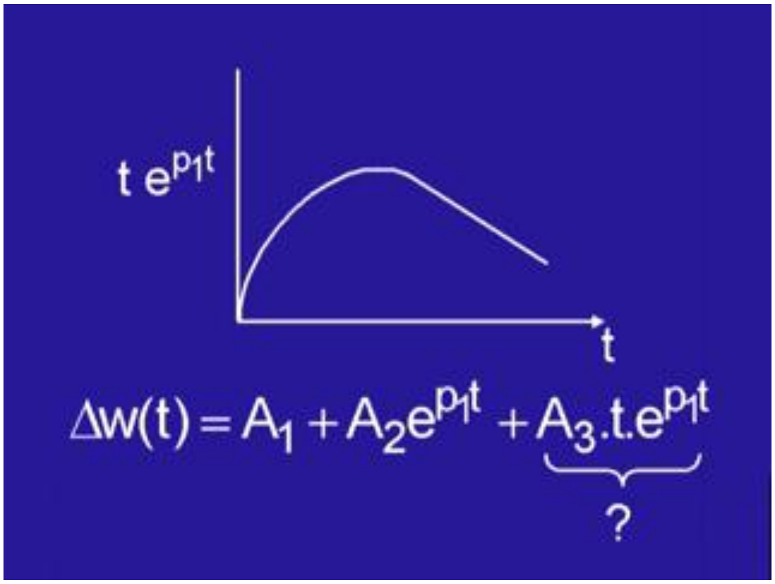

So there is this additional factor t that has slicked inhere, it is exponential but multiplied by t and I told you earlier to plot such a function, product of the exponentially decaying function that is the exponential which decays multiplied by t and see that this thing does not decay like the exponential does but it starts off from 0, builds up to a value and then goes to 0. The graph of that t e raised to p 1 t for a negative p 1 because at t equal to 0, it is 0 starts of here, in the long run it goes to 0 the exponential negative exponent dominates the growth of t.

So it goes to 0 but in between it reaches some kind of a maximum value and so this is the graph of t, e raised to p 1 t. However, as far as this steady state is concerned there is nothing to worry because e raised to p 1 t is going to 0 as t tends to infinity, even this factor of this term t into e raised to p 1 t goes to 0 as t tends to infinity. Now luckily, you can verify how long it takes for a term like this to become sufficiently small, if it was only the exponential I would talk about 5 time constants or 10 time constants. You can find out for a product like this, when does this become sufficiently small, this will be homework for you, by sufficiently small of course we may say it becomes say 10 raised to minus 3, 1 by 1000, if that is sufficiently small or 1 over 10 raised to 4,1 by 10,000, if that is sufficiently small that is .1 percent of .0 percent depending on what you mean by sufficiently small find out the time t by which t e raised to p 1 t becomes 10 raised to minus 3.

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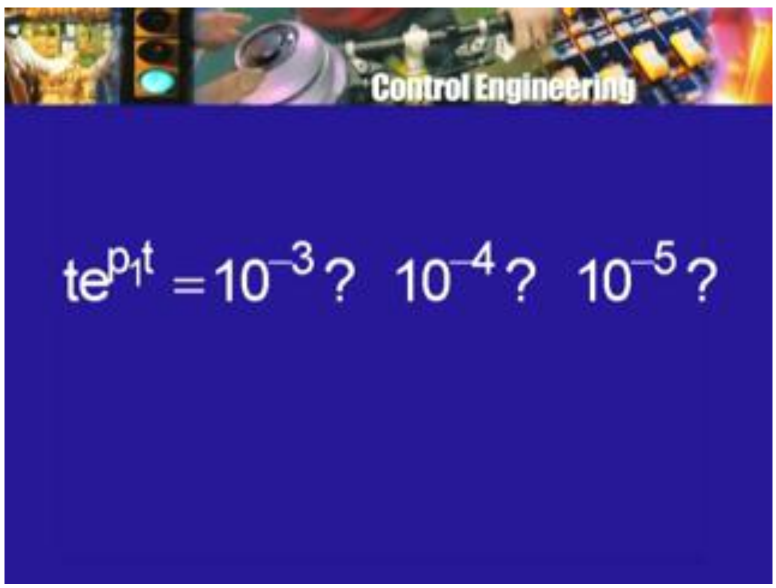

So let me write this term, find time t such that t e raise to p 1 t, for given p 1 negative becomes 10 raised to minus 3 or becomes 10 raised to minus 4 or becomes 10 raised to minus 5, how many time constants, should be lapsed before you can say that the function has become know 1 by 1000, therefore it is negligible or 1 by 10,000 therefore, it is negligible and so on. So from this transfer function manipulation, we can say once again that in the steady state there will be change that will be given by A 1 as before and this A 1 will be obtained by exactly the same technique that I indicated to you earlier, suppress the factor s and then look at put s equal to 0 you will get what you have.

So it will be again alpha into G 0, will G (s) is the transfer function from the load torque to the angular speed. So there will be a steady state change in speed and in the transient period, the behavior will be governed by the time constant corresponding to the exponential e raised to p 1 t but also this additional, exponential multiplied by t which can make little difference to the time constant computation. If instead of a sudden change in load torque, we think of a more gradual change in load torque then, we have to find out the Laplace transform of that change and then, work it out in detail, find out what is the Laplace transform of the speed change take the Laplace inverse and get thus, an exact expression for the change in speed as function of time. Usually, one works with step changes in some cases one thinks of what is called the ramp input that is changes which correspond to ramp.

In this case of course, it is unlikely because the load torque, I can very well conceive can change suddenly from some value to some other value. But I cannot think of situation, when the load torque will just go on increasing without any limit and it that does happen there of course ,I expect that my motor will finally come to a standstill, the motor will simply not run because it will not produce enough torque. In fact, there is an upper limit on the amount of torque that the motor can produce before we look at the third case that is when the poles are in the complex plane, let me draw some wave forms or graphs of the behavior of the speed as a function of time. So, here is omega and here is the time axis t, I am plotting omega t versus t.

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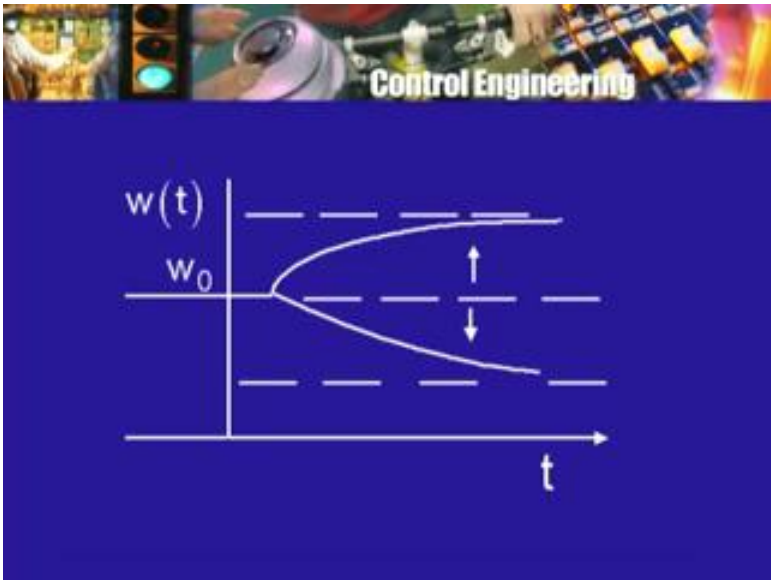

Let me assume that before t equal to 0, the motor was running at some constant speed. So I am showing it here on the left hand side of the t equal to 0 line and then, I make a sudden change in the load torque. If the load torque was increased then by what I just now discussed there will be a new steady state speed which will be less then, the older one and therefore I am showing it as this dash line and of course, for the sake of clarity I am showing the speed change as being very significant and that could very well happen, the load torque may get double for all you know.

So the speeds starts of here at t equal to 0, there is a sudden 0, the speed is ultimately going to end up at the steady state value and in between, it is going to make a transition that is the transient and in the first case, when we have the exponentials most probably, the transition will be like this that is the speed will start decreasing and go on decreasing till after 5 or 10 times the larger of 2 time constants, the speed would have virtually become constant at the new and lower value. On the other hand, if the load torque was decreased then the speed would start increasing and keep on increasing till after a certain time equal to say 10 times the larger of the 2 time constants, the speed would have become almost constant at the new higher value.

So with increase in load torque, the speed drops, with decrease in load torque, the speed increases and this is precisely the problem with our drive that is the drive is speed, if we are not making any adjustment of the applied voltage cannot remain constant as the load torque changes. Remember, this was not the only reason for thinking of feedback, load torque thinking of it as an disturbance, something which is not under our control that was one thing and what was the other, the other was lack of certainty about the parameter values or the parameter values also changing over a period of time and so on. But we are not looking at those effects right now.

We are only going to look at the effect of a disturbance like signal such as the load torque okay. In the second case, when the roots coincide so when the 2 poles coincide or there is second order pole or repeated root of second order or multiplicity 2, these are all alternate ways in which people talk about it, multiplicity, order of the pole, repeated roots, one should be familiar with these various terms. In that case again, there will be a change from the initial constant value to a new constant value but the change may not look as simple as I have shown here because of that t term, the t term can make a slight difference and after you have plotted t e raised to p 1 t, you will realize the difference okay. So this is what this speed versus time curve will look like, when the load is changed suddenly from 1 value to another, load torque is changed suddenly from 1 value to another.

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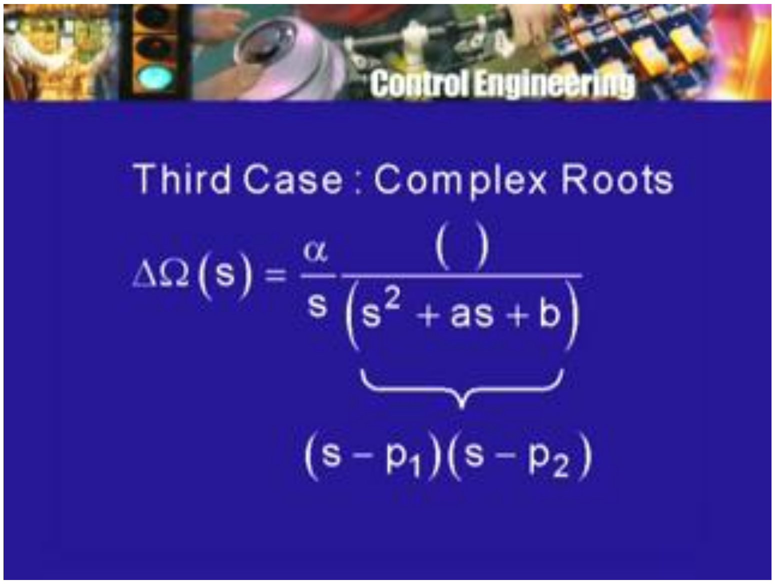

Now we are ready to look at the third case. Now what happens in the third case, in the third case then, I will have delta omega s once again equal to alpha divided by s that is the Laplace transform of the change in load torque as before multiplied by, once again there is a polynomial in the numerator of degree 1 and in the denominator now, I have a quadratic. So let me write it once again as s square plus a s plus b, whose roots are not real but are complex.

Now, one could as before factorize this into s minus p 1 into s minus p 2 carry out the partial fraction expansion, apply the Laplace inverse, the only trouble so to speak will be that this p 1 and p 2 are going to be complex and therefore, the numerators in those partial fractions will also be complex numbers. So, one will have to do some complex arithmetic. Of course, I suppose you are already familiar with complex arithmetic and how to do it using your calculator but we can avoid the complex calculations in fact, we are not going to do any calculation at all. As I told you, I just want to get some understanding what is likely to happen. So, we can avoid that entire thing as follows.

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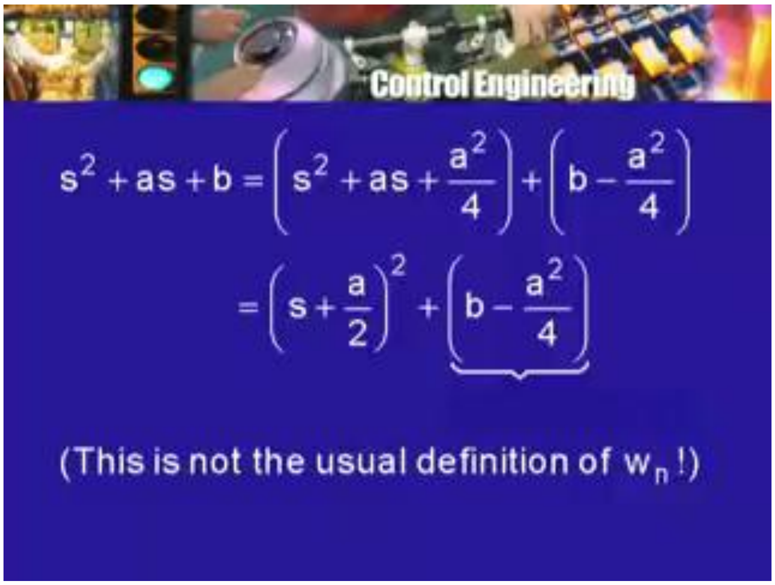

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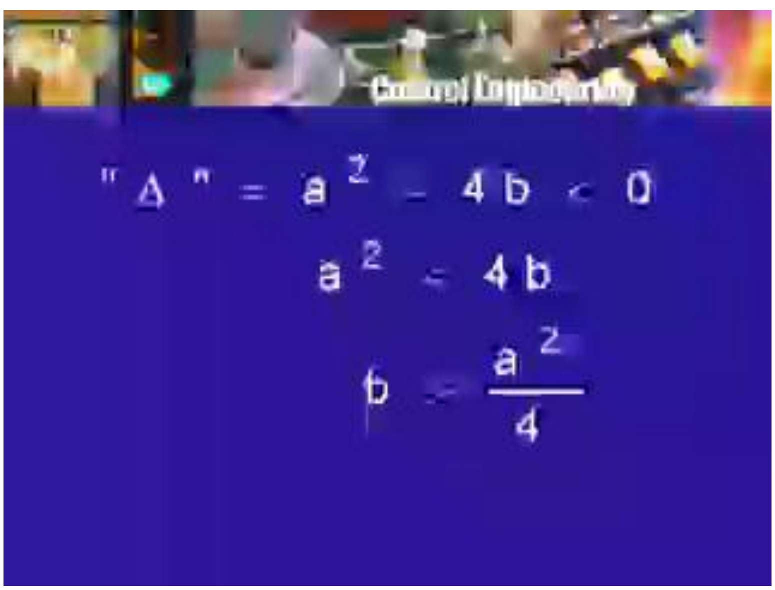

Now this requires that the quadratic s squared plus a s plus b be rewritten as follows, what we do is, we take first 2 terms s squared plus a s and do what is called completing the square. So I write back as s squared plus a s plus s square by 4 but then, I have added this a square by 4. So I will subtract from that that from b, so b minus a square by 4. So I will have s squared plus a s plus b quadratic written a s, s squared plus a s plus s squared by 4 plus b minus a square by 4. Now this first polynomial is simply s plus a by 2 whole squared plus b minus s square by 4. Now, what is the discriminant of this quadratic, discriminant of quadratic delta is a square minus 4 b. Now the case of complex roots is when this discriminant is less than 0 or when s square is less than 4 b or when b is greater than a square by 4.

So this term b minus a square by 4 is positive because it is positive, we will rewrite it as the square of some number and we will write this as omega m square. So, I will rewrite this as s plus a by 2 square plus omega m square and I will rewrite this further as I go on but, let me remind you of something which probably you have done in your circuit theory or network analysis courses. When you look at the RLC series circuit with AC or DC voltage and a switch, the problem of what happens, how the current builds up, how the voltage across the capacitor builds up or the current decay rather, the voltage across capacitor builds up, when the switch is closed. This is the problem which is one of the basic problems in network analysis and I hope you have solved that.

Now that involves a similar situation, in fact I have told you a long time ago that one way of getting a differential equation is in network theory that is an electric engineering, the other way is in mechanical engineering by having a mass spring dash pot system and remember, we talked about analogy between the two. So, probably you have already looked at such a thing from that course, if so you should look up that material. So the denominator polynomial s square plus a s plus b, I want to write as s plus a by 2 whole square plus omega n square.

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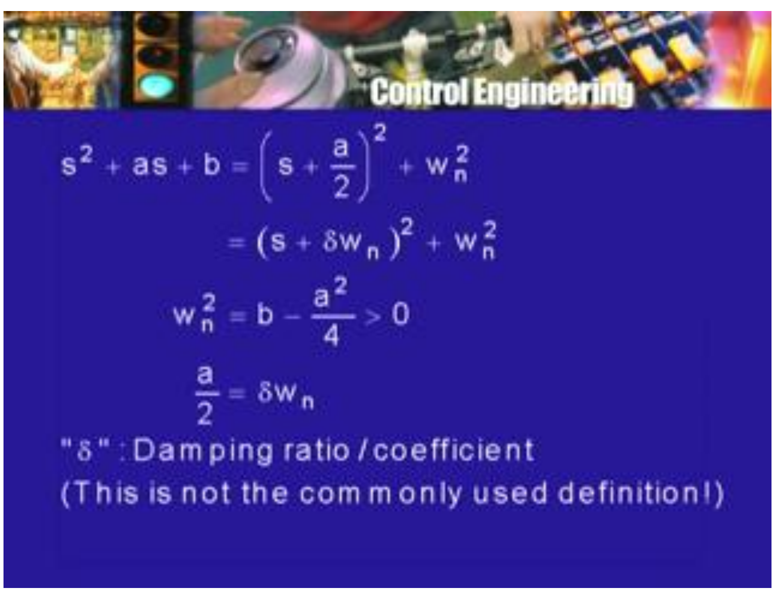

Now, I am going to do some further manipulation here, which will not be clear right now but as we go on, it will become clear, I am going to write this as s plus that a by 2 term, I am going to write as delta omega n square plus omega n square. So, I have introduced omega n and delta these 2 new things, so what is this omega n, omega n was defined from omega n square equal to b minus a squared by 4. This was omega n squared, so the positive root of that gives you omega n, so if I know b, if I know a, if the discriminant is negative then, this indeed will be positive and therefore square root of that will be a real number, I take the positive square root think of that or call that omega n. So that is my omega n.

Then, I have this a by 2, this a by 2, I can write as delta omega n and therefore I will write here, a by 2 equal to delta omega n, I know omega n, I know a. So from that I can calculate delta. So the quadratic will be now represented in terms of omega n and delta. I have used omega n earlier, if you remember n for natural, so in a sense it is called then a natural frequency associated with that quadratic factor omega n, n for natural. The delta, the letter delta is used because it should remind you of damping and we will see, how that damping arises. So it is called the damping ratio or the damping coefficient. So that is the damping ratio or the damping coefficient that one can calculate.

Let me illustrate this for a very special numerical case and show that it is not really that difficult. It is of course possible to write down formally for example, here will be formula of omega n there is formula of delta and then, substitute and solve but as I will saying all the time. It is not enough to just remember, formally and substitute and get correct answers because you must understand what is going on. So, let us see how this can be applied very simply, for a very simple numerical case. So let me take the polynomial s square plus 2 s plus 2. Now of course, I know this polynomial earlier, I have thought about it earlier therefore I can go ahead with it.

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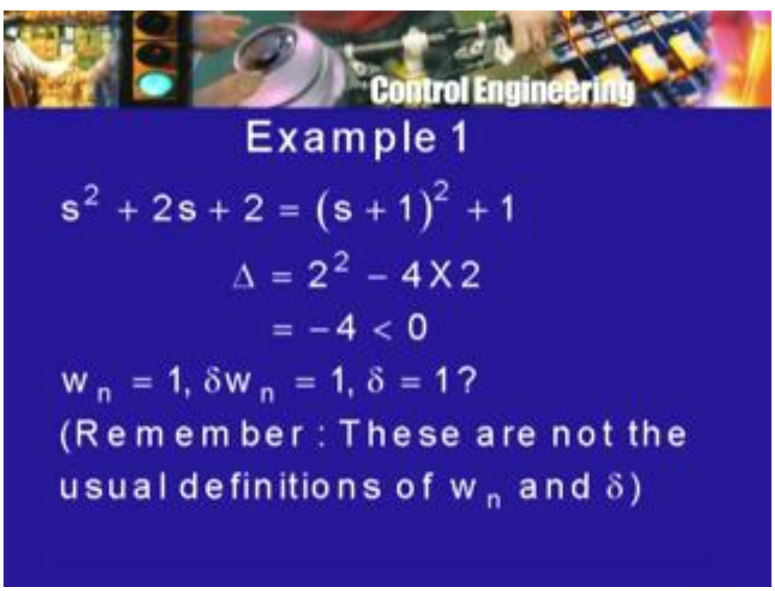

Let us take that discriminant is indeed negative. So what is delta, the may coefficient of s is 2, so 2 square minus 4 times 2, so that is delta is minus 4 that is less than 0. So this is indeed a fit case for the complex root situation. Now instead of substituting and getting omega and delta, I will complete the square as I showed you just now take s square plus 2 s complete the square. So, I will get s plus 1 squared that is squared plus 2 s plus 1 but I have 2 here therefore plus 1, it is as simple as that s squared plus 2 s, so this 2 from that I borrow 1 to complete the square and the remaining part is 1.

Now what is omega n then, omega n is 1 and what is delta, will this thing delta omega n, this is delta omega n and therefore delta omega n is also1 therefore, delta equal to1 right. I can do it with another numerical example, let me show you so let us say s squared plus 4 s plus, what think of completing the square. So I have s squared plus 4 plus 4 and therefore, let us say s square plus 4 s plus 5, if I complete the square, this will become s plus 2 whole square plus 1 therefore omega n is equal to1 and my delta omega n in this case will be 2. Right now, do not worry about the fact this delta omega may turn out be greater than 1 or less than 1, we will see what exactly this delta is doing. But, I am simply telling you that this delta you perhaps encountered earlier while looking at the RLC series circuit the resistance produces a damping in the circuit and the capacitor voltage and the inductance in the circuit current, if there is no resistance then, we can have a sustained oscillations but if there is a resistance then, there is no sustained oscillation and that is what is called damping.

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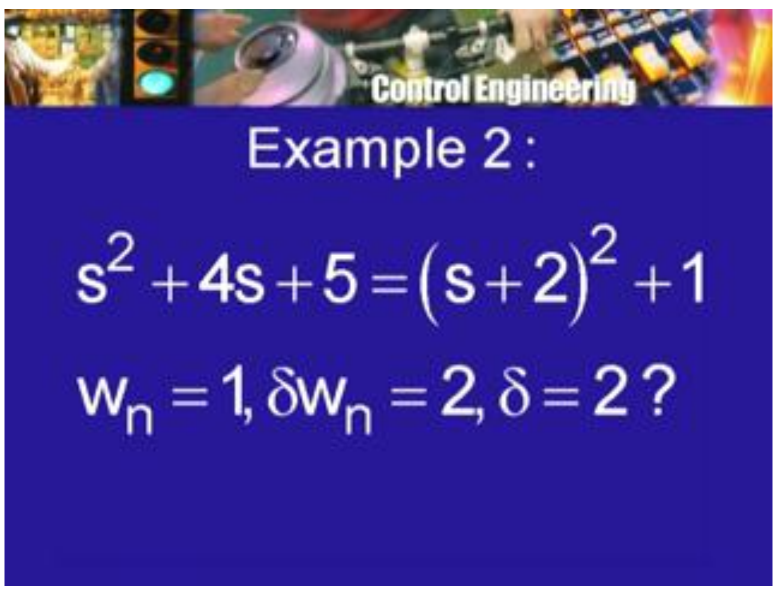

So this delta is to remind you about that damping that is all. So this is the situation now, therefore when I look at the Laplace transform of omega s change delta omega s that will be given by alpha by s into an expression in the numerator and in the denominator, I have s plus delta omega n square plus omega n square. This is what I have, now one can show the following this I can split into2 partial fractions. In other words, I keep the quadratic as it is, I do not factorize it into linear factor if, I do that I get complex numbers I have to do complex arithmetic but I can keep the quadratic as it is and I can split this into 2 partial fractions, one will be as before a 1 divided by s and that is obtained by exactly the same technique.

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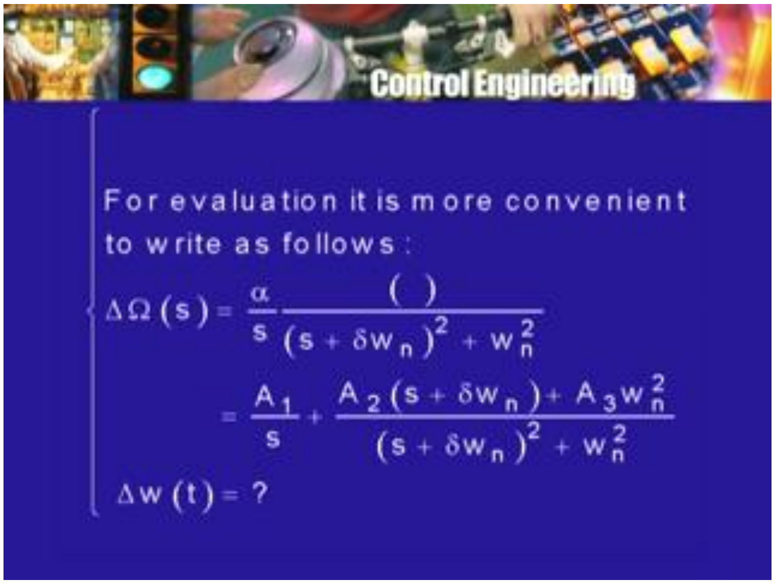

Suppress the factor s and put s equal to 0. So that will be alpha G 0 as before, the second term when add the denominator which is already there s plus delta omega n square plus omega n square, right and in the numerator I will right the expression as a 2 times s plus delta omega n plus a 3 into omega n square. This is one way of writing it, there are other ways in which one can write this expression. So I will break it up into 2 parts, one part as s plus delta omega 1, the other part as omega n square appearing there.

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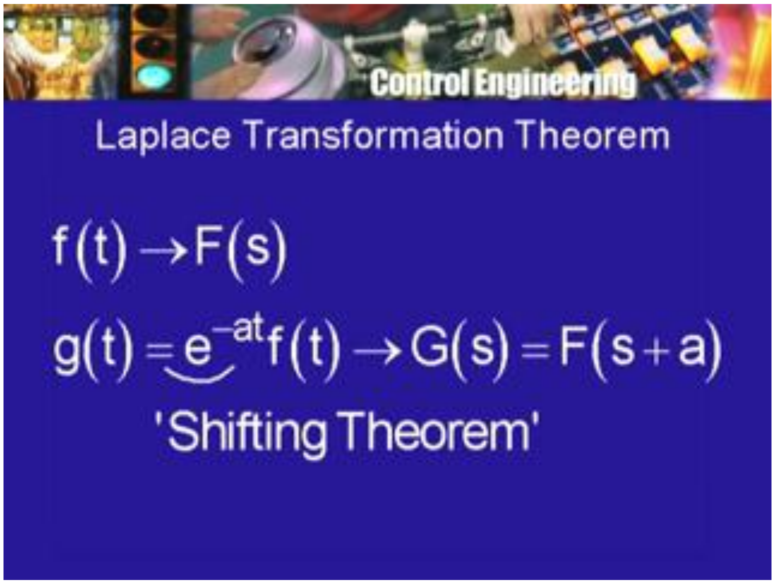

Now why I am doing this because there is a theorem of Laplace transformation and I would like to look up your book, textbook either the mathematics textbook or the control textbook, to read look up this theorem or result about the Laplace transformation and that is as follows. If I have function f of t, let us say signs Laplace transform denoted by capital F of s, in keeping with our notation. If I multiply this function by exponential function, if I multiply it by say e raised to minus at f t remember, this a may be positive or negatives. So this multiplying factor may be decaying or growing does not matter which one it is, I multiply it by and exponential and you see I am writing minus a here then, what will be the Laplace transform of this function, let me call this function G of t.

So the original function f t is now changed by multiplying by the factor e raised to minus a t. The original function has Laplace transform F of s, the new function has Laplace transform G of s, what is the relation between G and f, the relation is a simple one, G of s equals F of s plus a and so one easily can remember, what this result is multiply the function by e raised to minus a t. This is an operation in the time domain, in the s domain replace s by s plus a that is in the Laplace transform for f, whatever expression you have replace s, wherever it occurs by s plus a. You will get the Laplace transform of this new function G, this is theorem from the Laplace transformation theory, look it up from your textbooks make sure that you have understood the theorem correctly, you can apply it for few examples to get some practice. It is this theorem that I have in mind, when I have written the numerator expression in this peculiar way because you can see that I have s plus a kind of thing appearing here, s plus delta omega n appearing here and both in the numerator and denominator.

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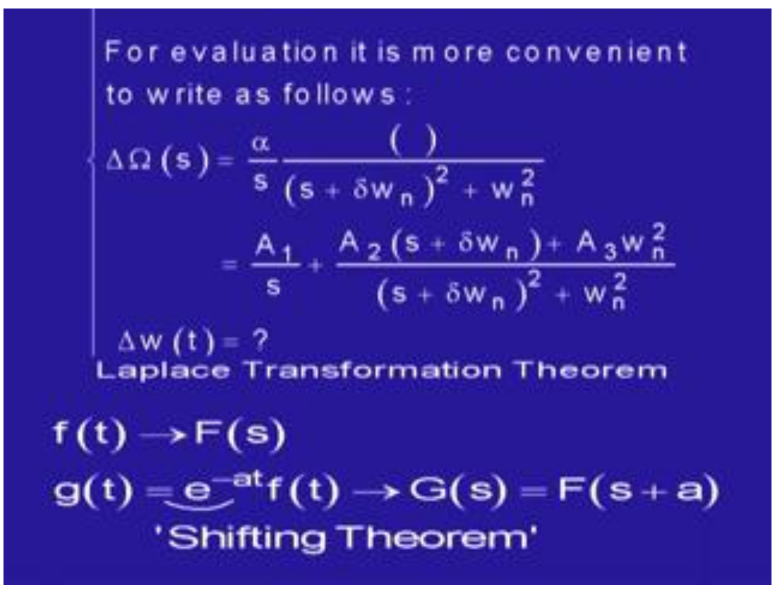

So, I am going to use this theorem in a backward direction as it were to get something about the behavior of omega as function of time.