Control Engineering
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Lecture - 26

Have you done the homework, what did I ask you, you were to take the 2 equations for our motor control problem, one voltage equation for the armature circuit and that involved derivative of the armature current multiplied by the armature inductance and the second was the mechanical equation, torque equation and that involved derivative of the angular speed multiplied by the moment of inertia.

I wanted you to apply the Laplace transformation to these equations that is as one says, take the Laplace transform of both sides and obtain an expression for the Laplace transform of the speed that is capital omega s, in terms of the Laplace transforms of the applied voltage E a (s) and the torque tl s. Leaving aside the initial condition terms that is the terms that involve the initial current I a 0 and the initial speed omega 0. We will take care of those initial conditions later. I hope you have done the problem, I am going to do it in a different way, I am going to make use of the concept of the transfer function that we have introduced and also the concept of the signal flow graph that I have talked about quite some time ago and the gain formula, associated with it. So with this, let us get back to the 2 equations but what we will obtain from the equations will be transfer functions and side by side with the transfer functions, we will draw the signal flow graph.

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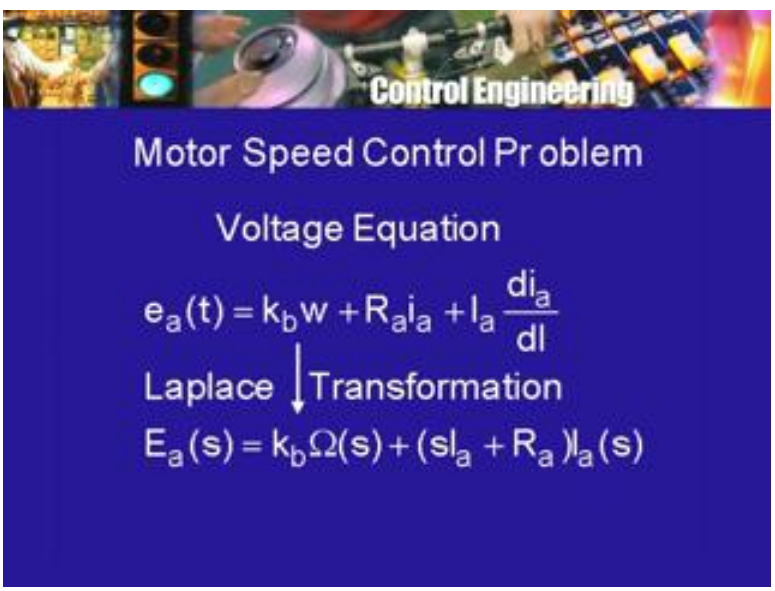

So the voltage equation was the applied voltage and therefore I have its Laplace transform E a of s equals k b times omega s plus R a, I a gives rise to R a times I a of s and l a di a dt gives rise to sl a and therefore I have, I will write this as sl a plus R a multiplying I a of s. So this is the

Laplace transform version of the voltage equation, applied voltage transform equal to k b times the speed transform plus this linear factor sl a plus R a times the armature current transform and as we saw earlier, we can rewrite this equation expressing an output quantity. In this case I a of s, in terms of the 2 input quantities. Well, we cannot do quite do it because we have omega s and E a of s.

So we will write this as a flow graph equation taking I a as an output node, omega as an output node and E a as an input node and with the nodes, we will associate the Laplace transforms of the corresponding variables rather than functions of time which we have done earlier. In fact at that time we are only looked at the steadystate values, if you remember. So, what does the signal flow graph or a part of it look like I have to put down 2 nodes, one for I a of this, the other for omega s.

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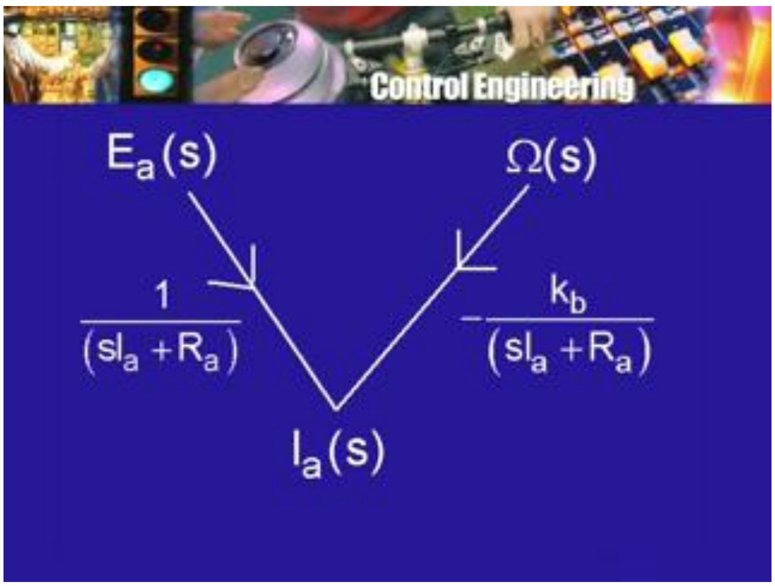

So let me put them down omega s here I a (s) here and I may not write this s now. In order to save time but remember we are dealing with transforms and then, here I have the input node E a of s and so, I have to solve this equation for one of the 2 quantities either omega s or I a (s) in terms of E a and the other one. Suppose, I solve it for I a of s in terms of the other. So I have E a of s minus R a, I a of s minus k b omega g (s).

So I have, I will write sl a plus R a into I a of s, it is always better to write down things rather than try to do them purely, mentally. So this is the equation I have and now, I am going to divide both sides by sl a plus R a to express ia and therefore, I a will be given by E a there will be an arrow going from E a to I a of s and that will have a gain of 1 by sl a plus R a and there will be an arrow from omega 2 I a of s and that will have a gain of minus k b divided by sl a plus R a.

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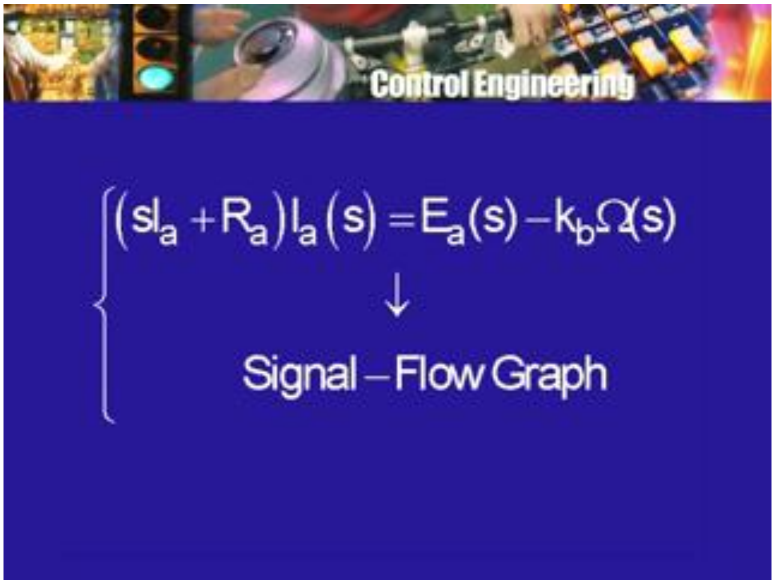

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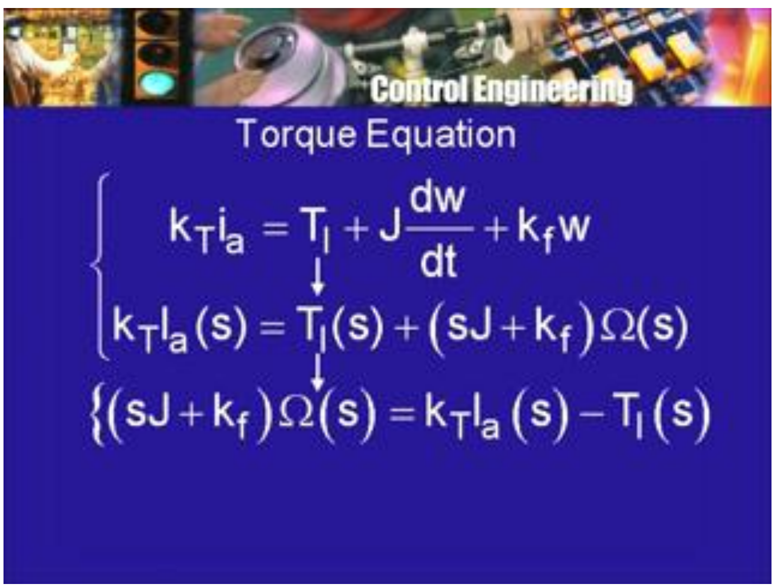

So these 2 arrows represent a the emf equation of the voltage equation after the Laplace transformation. Next let us take a look at the torque equation. So we have k T into I a of s equal to 3 terms, one is the load torque, so T l of s plus friction, so k f into omega s but there is also the moment of inertia term therefore, k f plus s J into omega s. Now the first equation I solved for I a, so the second equation I should solve for omega s and therefore I will rewrite it as s J plus k f into omega s equals k T, I a (s) minus T l of s and therefore I will have that part of the signal flow graph omega s again I a of s here.

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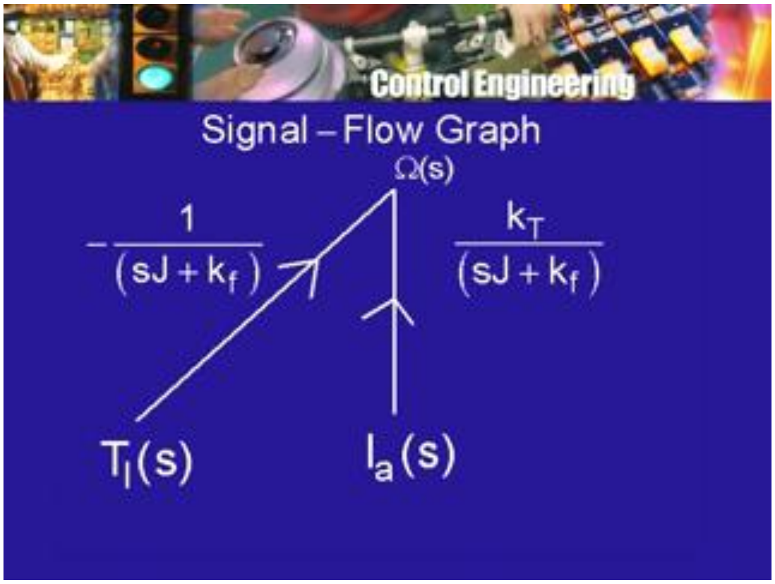

Now, the input other input node is T l of s and there will be an arrow going from, I am solving for omega going from I a to omega s, with a gain of k T divided by s J plus K f and there will be an arrow going from T l to omega s, with a gain of minus 1 divided by s J plus k f. So these will be the 2 additional lines or gain relationships and therefore the full signal flow graph will now, look like combining the 2, what will it look like, draw it I will now drop that is, I do not want to clutter my diagram, here is omega, here is I a, here is E a and here is the node T l. Remember, we are going to look upon omega and I a as none input nodes and E a and T l will be the input nodes.

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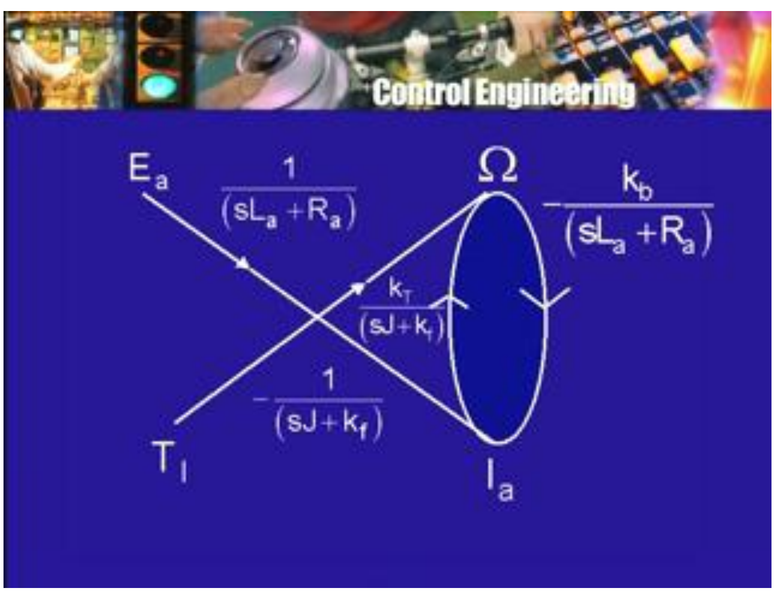

So first I have an arrow going from E a to I a, E a to I a with the gain of 1 over s L a plus R a and there was a arrow going from I a and from omega to I a with the gain of minus k b divided by sL a plus R a that is one. The second one is from T l to Omega and I will show it, this is not a cross over, it is not an intersection, this is only to show on the diagram, I have minus 1 by s J plus K f and there is a gain from I a to omega, which is k T divided by s J plus k f. This is the signal flow graph and now, from this I can write down the desired relationship. Let us say, I am interested in the variable omega in terms of E a and T l therefore, I have to find out the delta what was delta, delta was the discriminant of the set of equations or by Mason's gain formula, it is 1 minus etcetera, etcetera.

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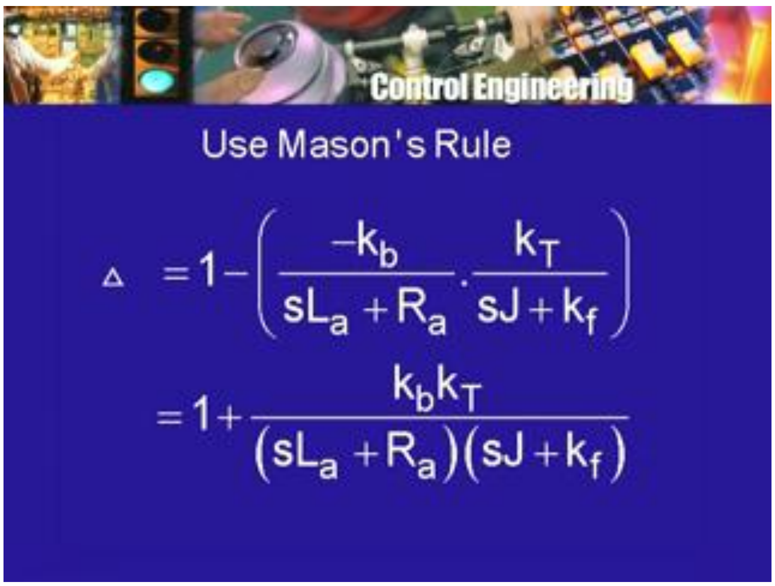

So let us do that, we will have delta there is only one loop here. So delta equal to there is the one term 1 minus sum of all the loop gains, since there is only one loop and there is a minus sign I will write it first minus I will have minus k b divided by sL a plus R a into k T divided by s J plus k f or 1 plus k b into k T divided by these 2 linear factors, s L a plus R a into s J plus k f. Okay, this is delta, this is the discriminant which will appear in the denominator, what about the numerator. I want the numerator in the coefficient that multiplies E a.

So I look at the forward path from E a to omega, the forward path from E a to omega goes through I a and therefore, the gain of the forward path is 1 divided by sL a plus R a into k T divided by s J plus k f, this is the forward path gain. This is multiplied by delta terms from delta that remain when, we consider only loop gains which do not touch the forward path but the forward path touches the loop and therefore, in the numerator I will have only one, so I will have just that.

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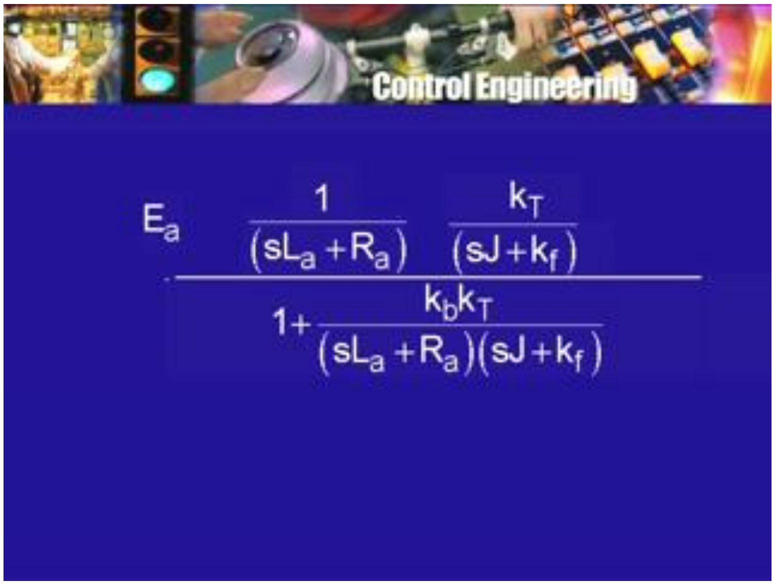

This will be the numerator of the multiplier for E a divided by 1 plus k b, k T divided by that product of those 2 factors. This will be coefficient of E a something similar, I have to do for the coefficient of T l, the denominator is the same and the numerator, is what the forward path from T l to omega that is a straight one that is minus 1 divided by s J plus k f. The numerator delta is simply 1 as before and therefore, I have simply the denominator which is delta and therefore, I can write down the full expression for the Laplace transform of the speed as omega s equal to and I will combine the 2 delta at denominator term. So 1 plus k b, k T divided by sL a plus R a into s J plus k f as the denominator.

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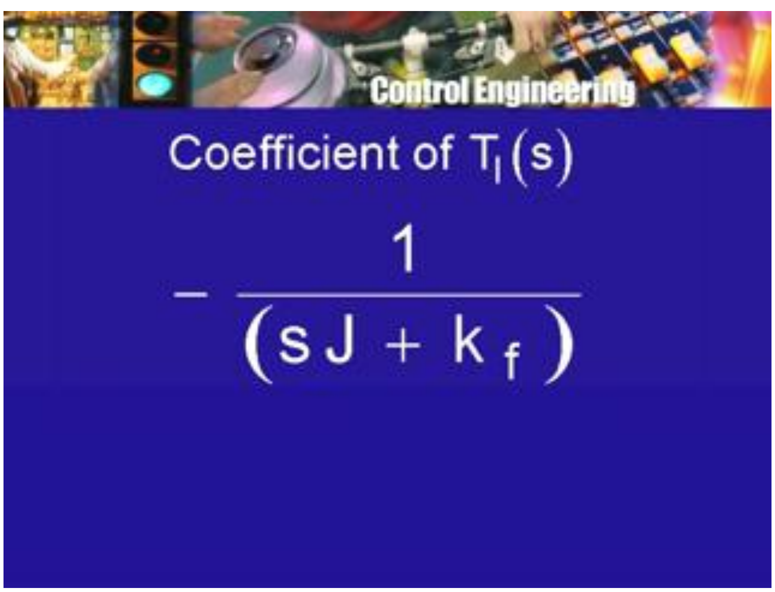

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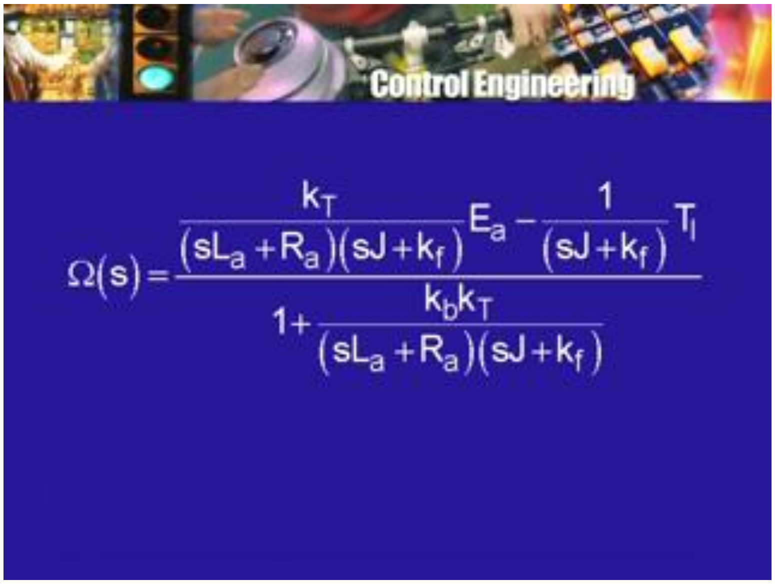

In the numerator I have E a multiplied by 1 factor, so k T divided by sL a plus R a into s J plus k f into E a minus, the torque coefficient is simply 1 divided by s J plus k f into torque. One can always check this set of equations for the dimensionality to check whether they are dimensionally correct. For example, let us look at the fraction k b, k T divided by sL a plus R a into s J plus k f, k b, k b is what, k b is volts per RPM or per radians per second, sL a plus R a, is what, R a is resistance. So volt per ampere, so this is given by you can check whether the dimensionality matches.

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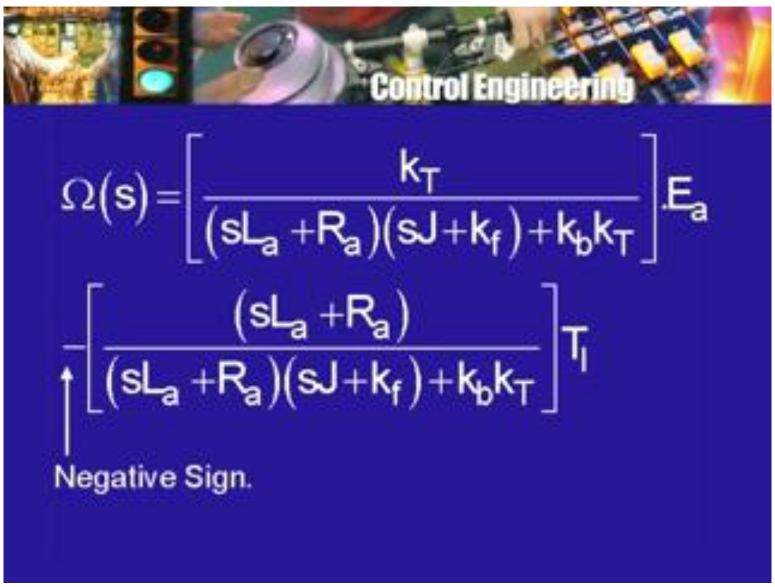

This whole fraction should turn out to be dimensionless and one can sort of understand it very quickly because there is a 1 here. Now the number 1 here, indicates that it is dimensionless therefore this whole expression should be dimensionless. Similarly in the numerator, we will have 2 coefficients which should have the appropriate dimensions. Now at the moment, these expressions look fairly messy and I am going to draw a block diagram later on or when we consider feedback but at the moment, we have to work with the expressions as we have them but we can simplify them a little further by getting rid of this denominator product and if I do that then, what do I get and I will now separate the 2 terms E a and T l.

So I will get omega equal to on the one hand, I will have k T divided by sL a plus R a into s J plus k f plus k b, k T, this whole thing multiplies E a minus sL a plus R a multiplying, nothing divided by the same denominator sL a plus R a into s J plus k f plus k b into k T. This whole thing multiplies T l, again one can interpret that when the load torque is there, the speed is going to be reduced than, if the load torque is 0, if there is no load the motor will run at the higher speed. So that explains this minus sign here and of course, higher the applied voltage, higher we expect the speed to be, higher the torque, the lower we expect the speed to be, so the signs look all right.

Now, we can consider 2 parts of omega and as we, I have been doing it earlier, I talked about the response and 2 parts or even 4 parts of the response. But strictly speaking, the response that you can observe is the sum of all of them, you cannot observe the individual terms separately. It is only for the purposes of study or analysis or our understanding that we look at the various parts of the response. So I will do that once again here in this case. So I will look at the part of the response that depends only on the applied voltage.

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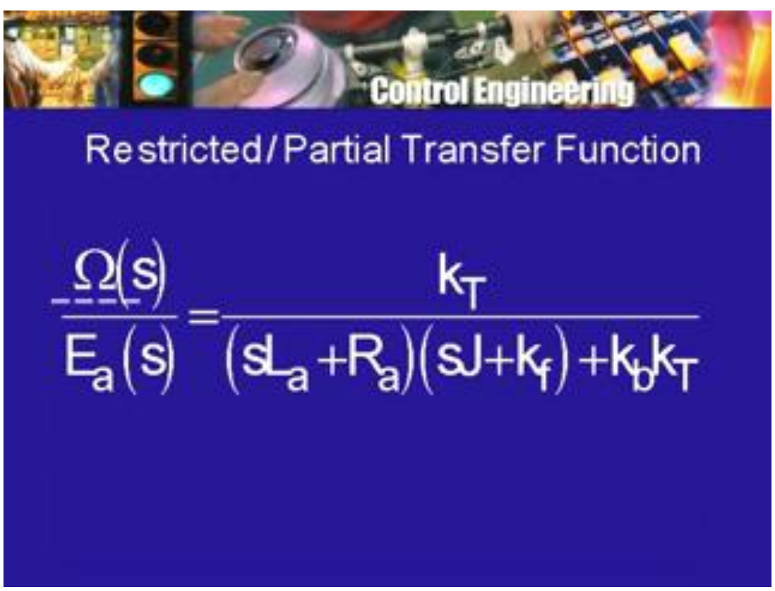

So I will look at only this expression omega I equal to some multiplier multiplying E a and therefore, I can talk about a restricted transfer function from E a to omega that is the ratio of this part of omega s to E a of s and this will be a transfer function, which will look like k T divided by that denominator expression. So this is what may be called a partial transfer function because it does not give you the whole of omega, it only gives you the part of omega that involves E a, the other part of omega involves T l.

So this is the partial transfer function or we say that it is a transfer function of the gain from E a to omega. It is the transfer function relating the applied voltage to the speed,
what can we say about it. Well, look at it in the denominator I have an expression in s but you can recognize, it is a polynomial in s, what kind of polynomial, it is a polynomial of second degree or it is a quadratic. It is a quadratic polynomial because you have 2 linear factors sL a plus R a and s J plus k f multiplied together. So, if I expand it out, I will get something like L a into J into s squared. So it will be a quadratic, what about the numerator the numerator is just a constant or as one may say a polynomial of degree 0 therefore, now go back to what I said about transfer functions earlier. A transfer function can be associated with its poles, its 0s and the gain coefficient.

So in this case, since the numerator is just a coefficient k T, what about 0s of this transfer function, it has none, that is number 0 of the transfer function, the numerator is just a constant and the associated gain, may look like just k T except that in the denominator we have something that I have to take care of, what about poles of the transfer function because the denominator expression is a quadratic. It has 2 poles and therefore, the pole 0 diagram of the transfer function will appear with only 2 poles, no 0 and a coefficient that multiplies the linear factors.

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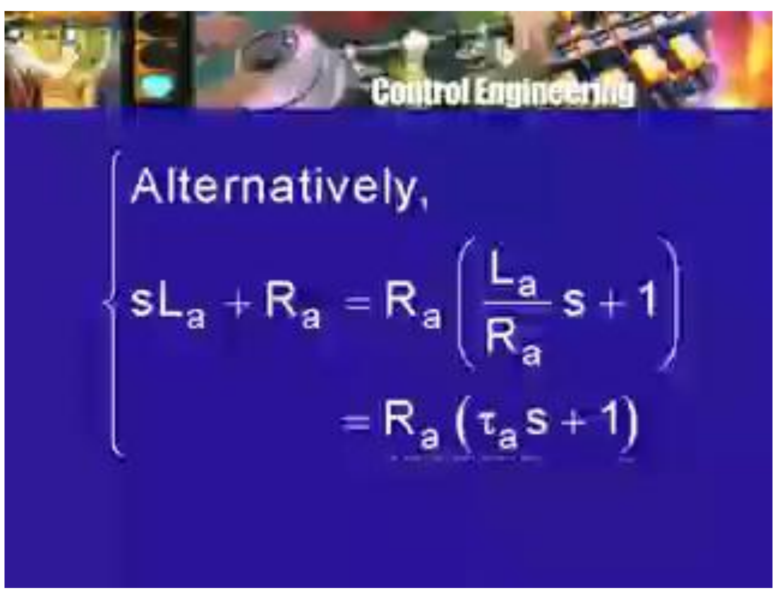

Now, sometimes it is convenient to rewrite some of the expressions that to I have been handling. For example, instead of writing sL a plus R a, I may find it convenient to pull out the factor L a and therefore I will write it as L a into s plus R a divided by L a. This is quite correct and of course, you will see the reason for doing this I can go one step further and write this as L a into s plus. Now, R a by L a can be written as the reciprocal of 1 L a by R a and L a by R a is a time constant. If you remember, your network theory or circuit theory courses then, in a L R series circuit, there is a time constant associated with the circuit which is given by the inductance divided by the resistance and this time constant determines, the time taken for a transient, for the current to build up to its final value or for the current to go down to its final value. As I have been saying all the time, 5 time constants or 10 times constants, it is just about the time required for the transient to be over.

So there is this time constant L a divided by R a and since, it is going with the armature circuit, I will denote it by tau a and so I will have the term L a into s plus 1 by tau a. So this is one way in which I can write that linear factor sL a plus R a, sL a plus R a can be written as L a pulled out into s plus1 by tau a. Remember, the time constant appears in the denominator, there is another way in which this same expression can be written and that is sL a plus R a equal to instead of pulling out L a as a factor, I will pull out R a as a factor.

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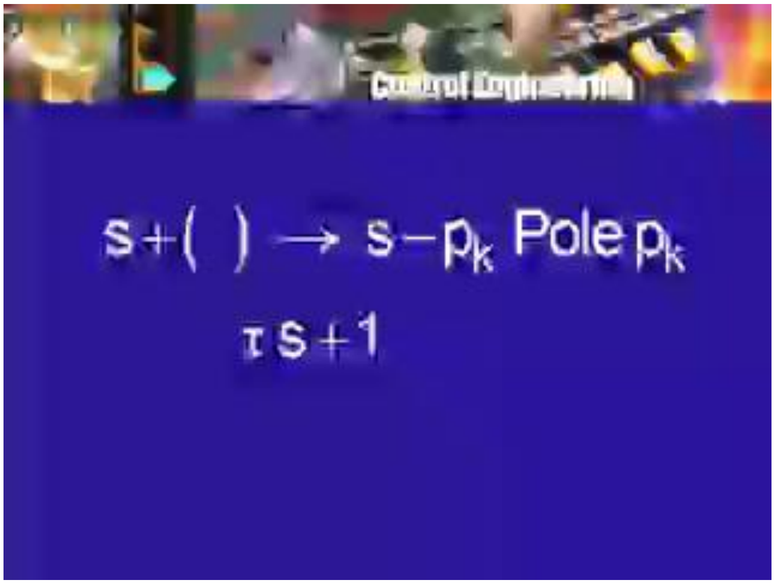

So, if I do that I will write this as R a into what L a divided by R a into s plus 1 and therefore, it is R a into tau as plus 1. You will find in many books, the transfer function terms or factors, written either in the form s plus something usually plus because that number is positive or as I have been writing s minus P k, where P k is the pole a particular pole. Alternately, as tau s plus 1, where tau is a time constant, one has to be familiar with both these expressions and one expression say, s plus 1 by tau will be useful in one application, whereas the other expression tau s plus 1 will be useful in another application and going from 1 to the other is just a question of pulling out a factor and rewriting the expression, nothing really complicated. We will do the same thing with the other factor which involves the moment of inertia and the coefficient of friction.

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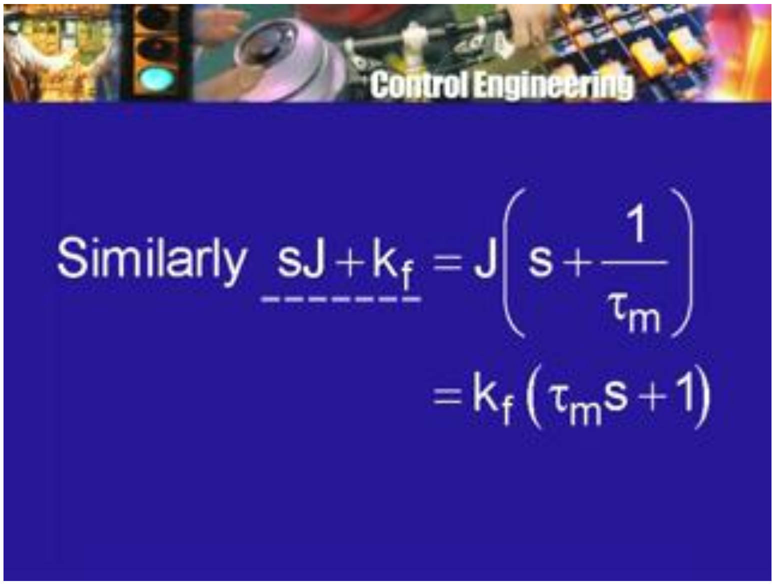

So, I have again the term s J plus k f pulling out J, I will write it as J into 1 plus 1 divided by and I will write down tau m, where tau m is the motor time constant but I can also rewrite it as k f pulling out k f as tau m s plus 1. So, we have these 2 alternate ways of writing the linear factors. One advantage of writing it this way is that, the time constant appears explicitly in one case, the motor time constant in the other case, the armature time constant.

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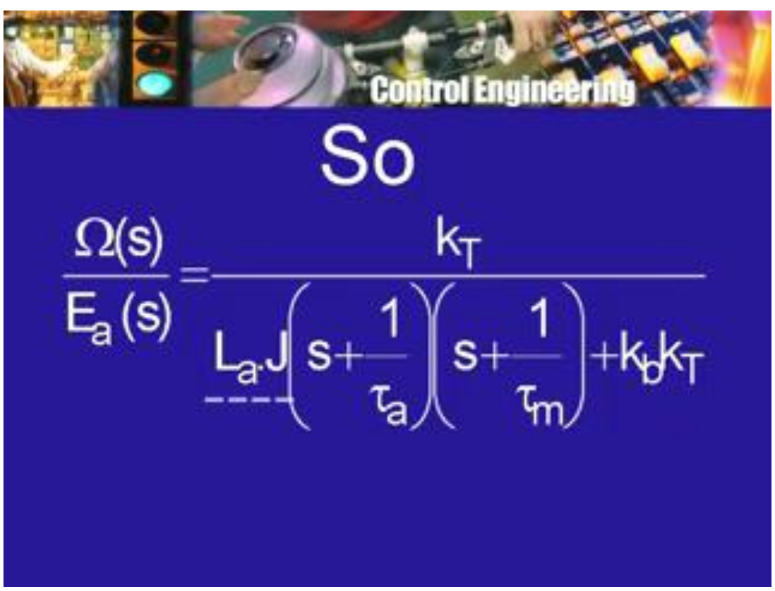

Now, the next step is to look at the transfer function once again, using this rewriting of the terms and therefore I will have the partial transfer function, omega divided by E a given by k T divided by, I have the product and now pulling out L a, I will write it as L a into s plus 1 by tau a, I also pull out J. So I will write down L a into J into s plus 1 by tau a into s plus 1 by tau m, this whole thing plus k b, k T. So that is how it looks. Some further manipulation is required the idea is that I want the denominator to look like a quadratic with coefficient of s square equaled to 1 plus the way usually we like to look at the quadratic, the coefficient of s square is reduced to 1 by pulling out a factor. It is not necessary but it is convenient to do so because then, one can talk about a poles of the transfer function rather easily.

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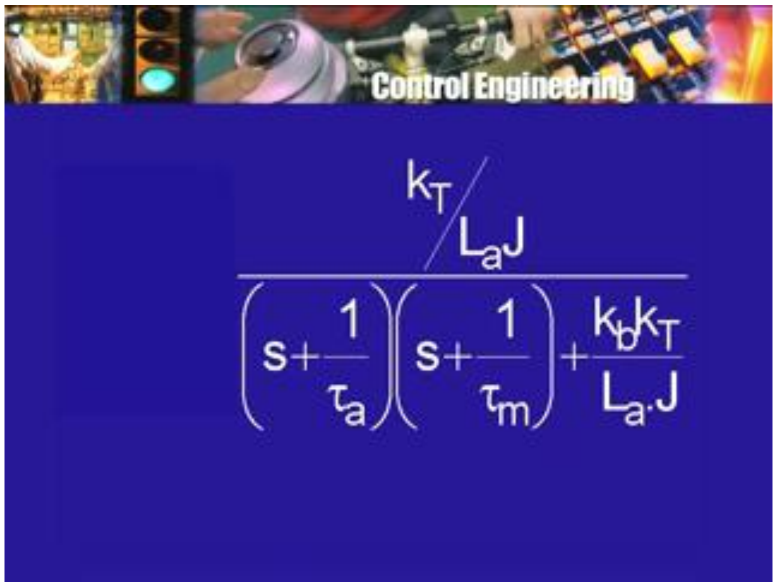

So, if I do that I will rewrite this once again as now, what I will do is I will divide by that L a J. So I will write down k T divided by L a J in the numerator, in the denominator s plus 1 by tau a into s plus 1 by tau m plus k b into k T divided by L a into J, what have we achieved by doing this. Well, let us see in the denominator I have a quadratic, what about its roots, which are going to be the poles of the partial transfer function, transfer functions on the applied voltage to the speed, what about its poles, it is a quadratic. So there are 2 roots and because the coefficients are all real, the 2 roots will be either both real and different or coincident or repeated or they will be complex and in the complex case, of course there is a possibility that the roots may be purely imaginary or the roots will be complex and not purely imaginary, which of the 4 possibilities is going to hold here.

Now immediately, you may not be able to figure out what is going to happen, you may have to write down the discriminant delta, unfortunately there is a delta that occurred earlier that was also a discriminant but that was for the set of equations which are represented by the signal flow graphs. Here, this discriminant is the discriminant of the quadratic equation, one will have to look at that and consider cases, as we saw earlier, delta greater than 0, delta equal to 0, delta less than 0. But one thing is immediate, I can rule out one possibility, I can rule out the possibility that the roots are purely imaginary that is of the form plus minus J into now here again, is a clash of symbols. Usually, we refer to the real part of the complex number by the symbol sigma, the imaginary part of the complex number by J omega that is the without the coefficient J, it is omega.

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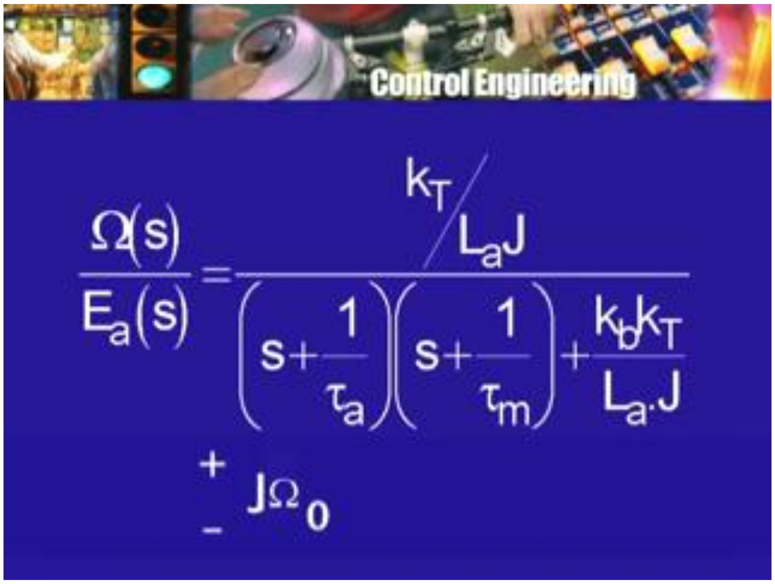

We use omega also to denote speed or angular velocity. So there is going to be a clash of symbols hopefully by context, we can tell that, what is what. So I am going to write here s a plus minus J omega k or J omega 0, if you wish because there is only one pair of poles for this, is this possible? Now we saw earlier that if this happens then, the corresponding polynomial was to look like s square plus omega squared.

Now, is this polynomial going to look like that what do you think for some values of tau a tau, m motor time constant, the armature time constant, the back EMF, the torque coefficient, the inductance and the moment of inertia, for some specific numerical values if I substitute and simplify, will it become an expression like s squared plus omega square
the answer is no, why, because if you expand out this term, the product to the to s containing terms then, the coefficient of s will be 1 by tau a plus 1 by tau m that is not going to become 0, because both are positive numbers. The reciprocal of the armature time constant and the reciprocal of the motor time constant both are positive numbers.

So when they add up, they are not going to add of to 0. So this quadratic will have a s term. In other words, this quadratic will look like s square plus some coefficient non-zero coefficient multiplying s plus another coefficient, not only that this non-zero coefficient multiplying s is exactly 1 by tau a plus 1 by tau m. So it is not only non-zero, it is actually positive. So we have a situation where the coefficient of s is not 0 and the coefficient of s is positive.

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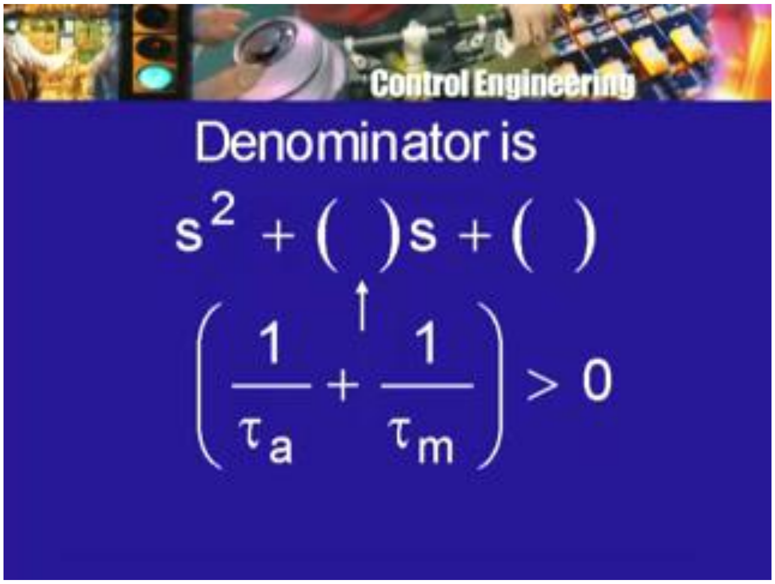

Now because of this, what can we say about the roots of the quadratic now, because of this we can say that the roots of the quadratic will not be purely imaginary that is ruled out, will the roots be coincident that is will there be a repeated root, will the roots be purely real, will the roots be complex, they cannot be purely imaginary. Now there is one more thing that we can conclude, now this requires a little bit of familiarity with the quadratic and the location of its roots but as control theory people, it is good to become familiar with this.

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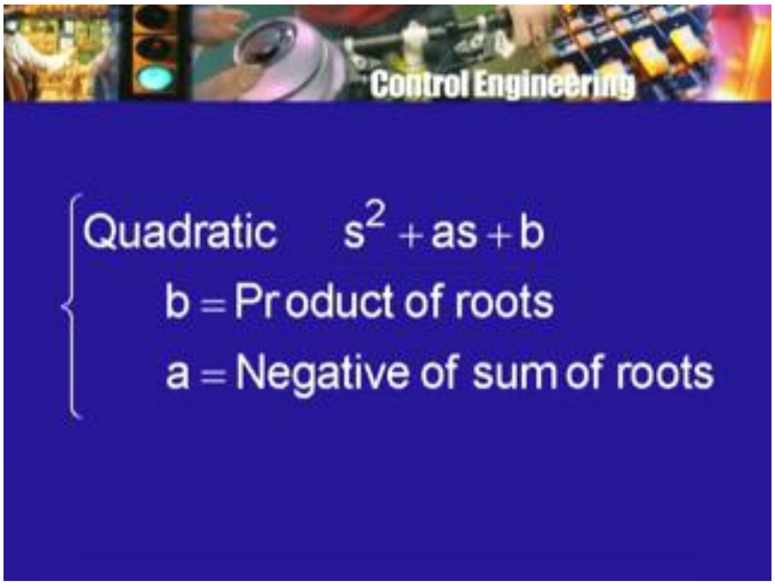

You may recall from algebra that when you have a quadratic that looks likes s squared plus as plus b then, there is something that you can say about these coefficients a and b, coefficient b is what, it is equal to the product to the 2 roots. So, b equal to the product of the roots and what about the coefficient a, the coefficient a is the negative sum of the roots that is you add the 2 roots and take its negative that is equal to a or the sum of the roots equal to minus a. In this case the coefficient of the s is 1 by tau a plus1 by tau m.

So the sum of the roots will be minus of that sum and therefore, it will be negative. So the sum of the 2 roots will be negative. Now, from this and the fact that the product of the 2 roots which is b, in this case is a positive number. So the product of the 2 roots is positive. From this, we can draw some conclusions without actually calculating the roots. For example, if the roots are purely real then, what can happen can the roots be both positive the answer is no, why because if the roots were both positive then, the product will be positive, which is okay but their sum will be positive, which is not the case.

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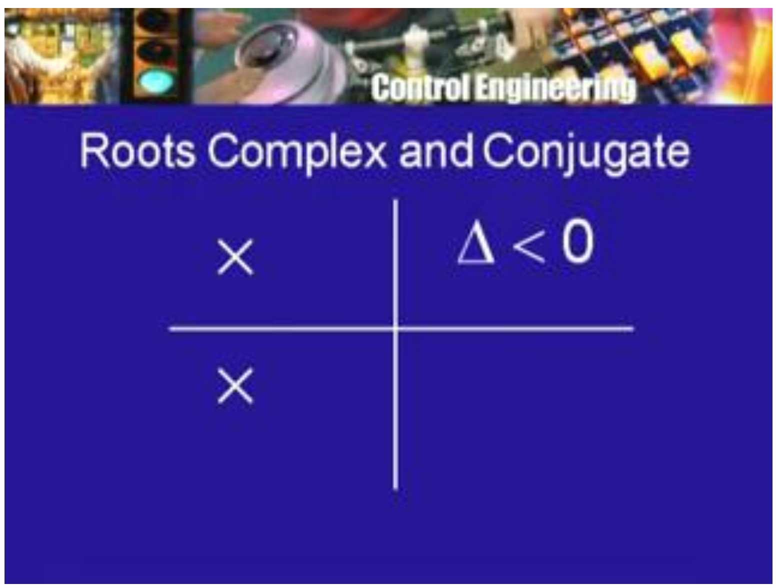

So the roots, if they are real cannot both be positive, can one have one positive root and one negative root, no because in that case the product of the 2 roots will be negative where as the product of the 2 roots is positive because that is the coefficient b, which in this case is the positive number. So there remains only the third possibility that if the roots are real then, both the roots must be in the left half plane, both the roots must be in the left half plane, if they are real. I am not saying they are real, I am saying, if they are real they must be in the left half plane. Now that is good is not it because we saw earlier that left half plane poles are good, if we had a pole in the right half plane that would have caused trouble, that could have caused something to build up, to infinity as time passes.

So fortunately, for our motor problem the 2 poles, if they are real are both in the left half plane and strictly in the left half plane, one of them cannot be 0 because if, one of them is 0 the product is 0 but the product is not 0, the product is positive.

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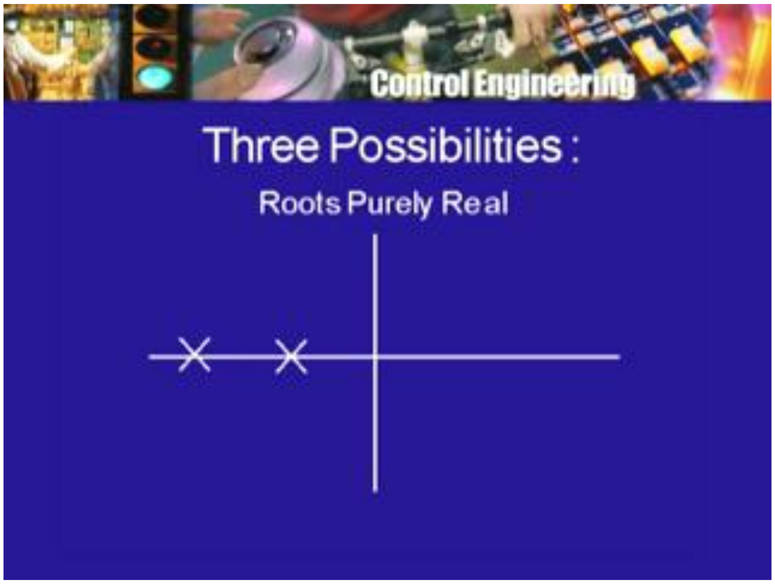

So there is no pole at the origin, the 2 poles are in the left half plan. This is one possibility, what about the other possibility of complex conjugate roots that also we can consider. So if the roots are complex conjugate can they be both in the right half plane, what about their sum then, when you add the 2 numbers which are conjugate, the sum is simply 2 times the real path and therefore, the sum will be positive. But, we know that the sum is negative. So this is ruled out because I have already ruled out J omega axis roots because in that case, the sum is 0 that is also ruled out.

So the only possibility is that if the roots are complex, both of them are in the left half plane or in other words, they have their real part negative and therefore that also gives you a good pair of poles. So fortunately, for our motor transfer function, partial transfer function from the applied voltage to the speed, the transfer function has its poles which are in the left half plane and therefore, they are good poles therefore, we will not have any problem of omega going to infinity as time increases.

Of course, we do not expect it either, nobody has seen the motor simply speeding up without any limit, with a given voltage applied, it has not happened. We can now see, why it cannot happen and you will see the reason, why I wrote that s factor with the a multiplier s the multiplier pulled out. So that I could think how the quadratic as s squared plus the coefficient s into plus another coefficient. Now, what about their decision between these 2, either the roots are real and negative or the roots are complex with negative real part, which one of the 2 will hold.

Now, this will depend on the discriminant of the quadratic and therefore, it will depend on the relative values of the various parameters of the motor, what is the discriminant of the quadratic in this case, delta, this is not the Mason gain formula delta, this is the for the quadratic our quadratic is s square plus as plus b. So the discriminant is simply a square minus 4 b, our a is 1 by tau a plus 1 by tau m.

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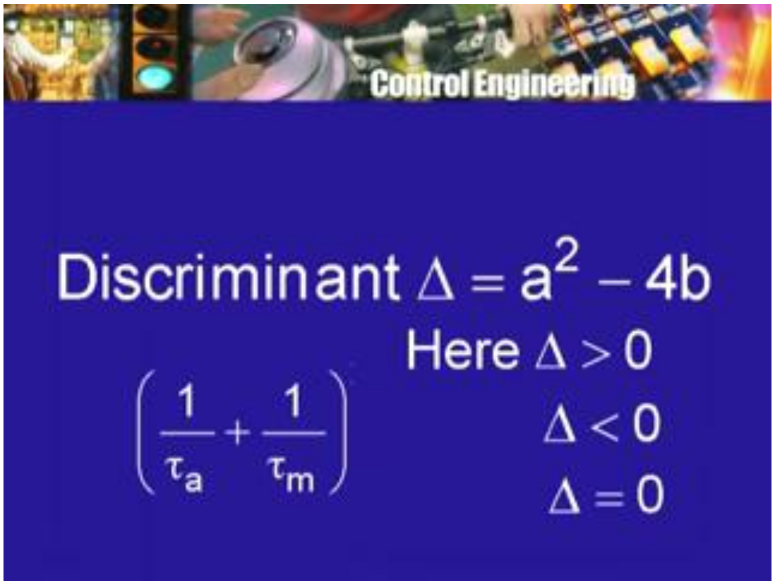

Now instead of squaring it, now this is something again as engineers we ought to be doing, we ought to be simplifying expressions. So that they can be handled more easily. So instead of writing a square minus 4 b but what I am really interested is in finding out whether delta is greater than 0, in that case the roots are purely real or delta is less than 0, in that case the roots are purely imaginary, I of course we know that delta is coincident, yes.

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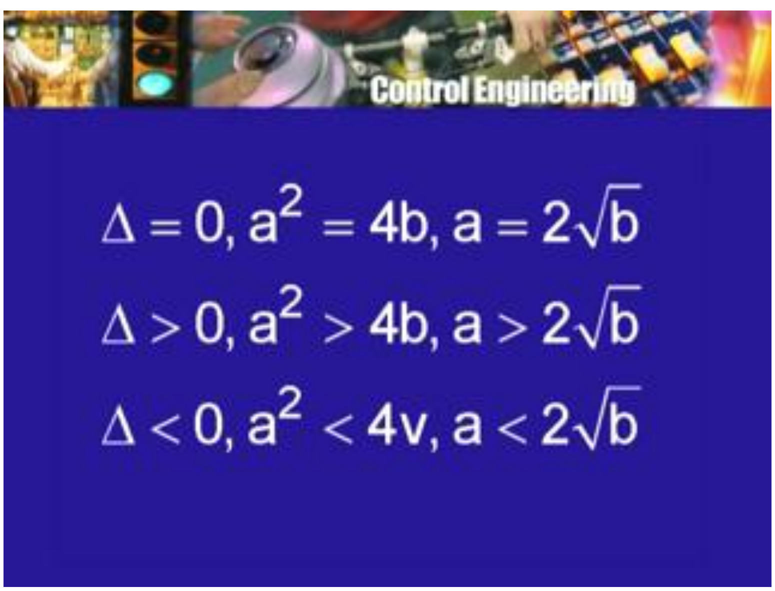

So, I have to look at the third possibility delta equal to 0. So, then I have the 3 conditions which I can write down delta equal to 0 gives a square equal to 4 b or a equal to 2 root b delta greater than 0 gives a square greater than 4 b. So taking square roots a is greater than 2 root b and delta less than 0 corresponds to a square less than 4 b and therefore, a is less than 2 root b. So depending on which one of these relationships holds, if a, the coefficient a equals 2 times root b then, the roots will be coincident or there will be repeated root of multiplicity 2, if a is greater than 2 root b the roots will be purely real, of course lying in the left half plane and therefore on the negative real axis whereas, if a is less than 2 root b the roots will be complex but lying in the left half plane. Before, actually writing any complicated expressions, let me give the numerical example, to see whether there is really a possibility that any one of these conditions may occur or is it that only one of them will occur.

Now as I told you all that depends on the actual parameter values. So in general one cannot say anything but if you look at the form of the quadratic, is there something that one can say. So, let us make an attempt, so suppose I have a quadratic that looks like s plus 2 into s plus 4, is looks like that s plus 1 by tau a into s plus 1 by tau m term plus a constant term.

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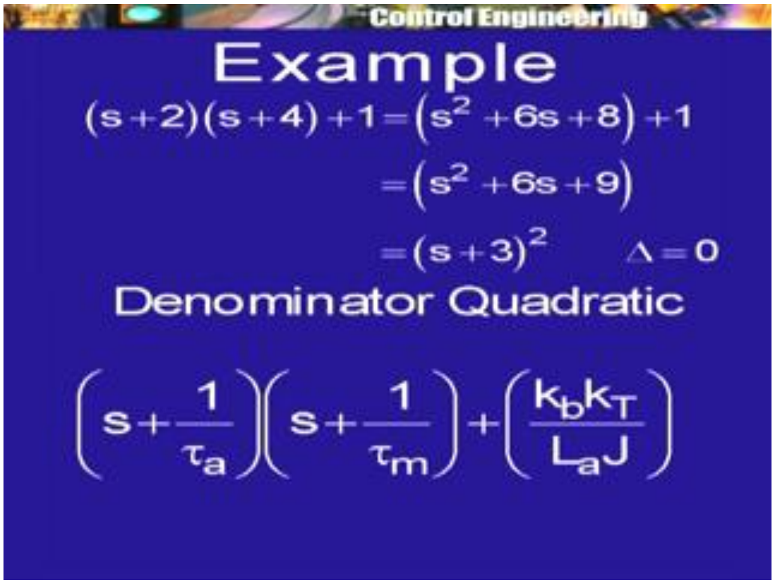

Now depending on what that constant term is, is it possible that the roots will be coincident and real that is the case delta equal to 0 distinct and real and distinct and complex. Now, I have here s plus 2 into s plus 4. So this looks like simplifies to s squared plus 6 s plus 8 and so, what should I add here. So that this becomes a perfect square. Well, does not take much time to realize that if I add 1 here then, this becomes s square plus 6, 8 plus 1 plus 1 or s square plus 6 s plus 9, which is s plus 3 square.

So the quadratic just becomes the square of a linear factor. In other words it has 2 roots which are coincident each one is minus 3 and that is, if this number which is being added is 1. So now what will happen, if this number is not 1. Now it is not difficult to see that if the number is less than 1 then, one thing will happen, if the number is greater than 1 then, another thing will happen, if the number is less than 1 then, in the discriminant a square minus 4 b the b is smaller. So a square minus 4 b will be positive. So if to this I add a number less than 1, positive number less than 1 then, I will get real distinct roots whereas if I add a number greater than 1, I will get complex distinct root whereas if I add just 1, I will get roots which are coincident.

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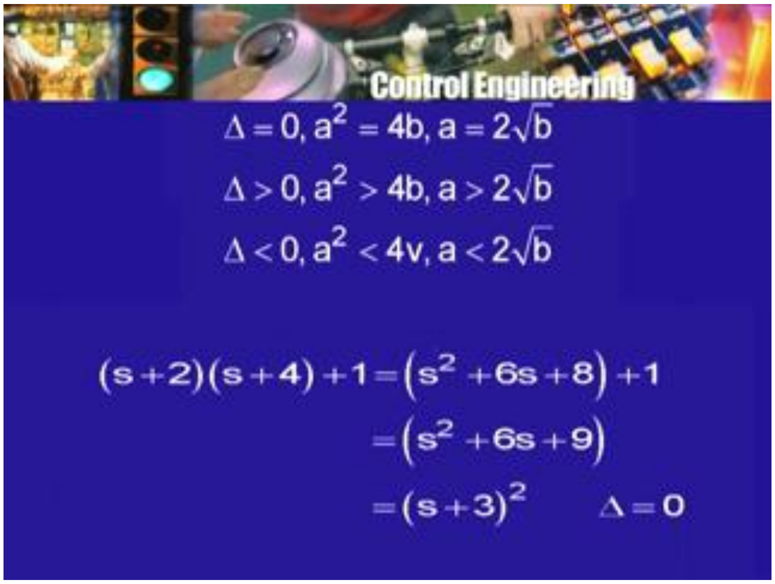

So any one of these 3 possibilities could actually occur depending on the relative values of the motor and the drive parameters. So, this is not just a fictitious or so you know a waste of time to locate the various possibilities. The possibilities may actually arise depending on which one of the conditions is satisfied a is equal to 2 root b, a is greater than 2 root b or a is less than 2 root b.

Now, this can be carried out further using some additional symbols and so, I will do it only to some extent because too much of symbolism sometimes obscures, what one is thinking, sometimes numbers are useful, sometimes symbols are useful. What is the relationship between the various parameters, which will enable us to discriminate us between the 2 situations. This is the question that one can ask and by looking at this numerical example, I can take a hint or queue and try to figure out, I have the quadratic which looks like s plus1 by tau m into s plus 1 by tau a into s plus 1 by tau m plus a coefficient which is k b, k T divided by L a into J.

Now from this example, we see that the relative value of this. With the product of these 2 is what is going to decide, what is going to happen, something of that sort, the product of these 2 and their sum that determines the coefficients of s and the constant in this part. To that we are adding another constant now, that can make the roots coincident or that can make the roots even complex.

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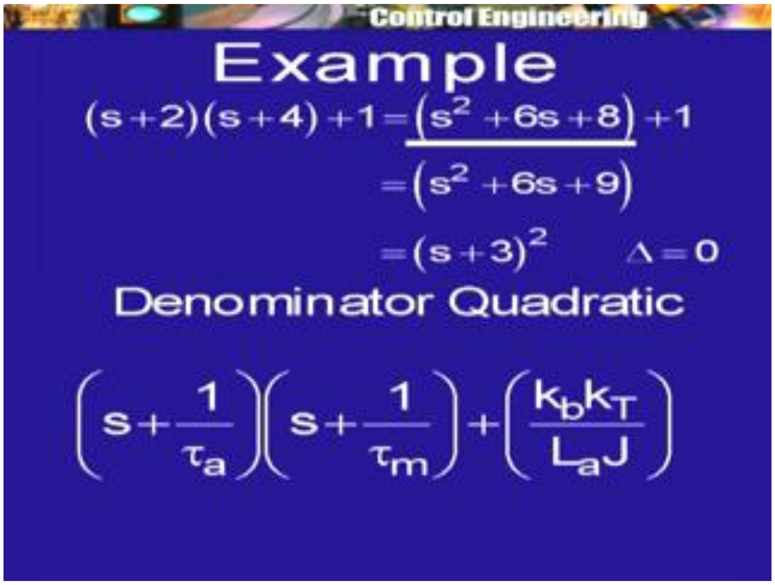

Now, we will be anticipating here a method which is known as the root locus method and although we will be looking at the root locus method in connection with the use of feedback. The root locus method can be used in order to tackle this problem. Now, I am not going to explain the root locus method right now, because we are not here to introduce feedback, we have not looked at the case when feed back is introduced to improve the drive. I will discuss the root locus method at that time but I am going to use some ideas from that method to illustrate this case.

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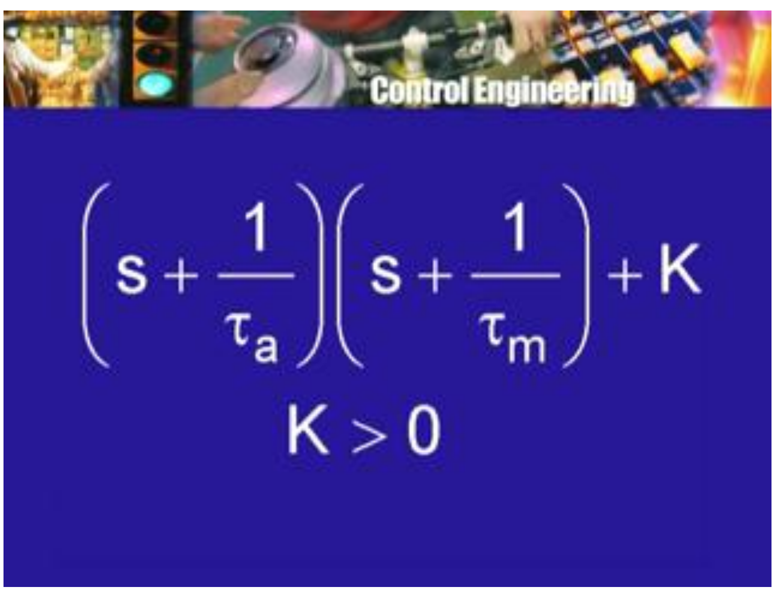

So, what I am looking that looks like s plus 1 by tau a into s plus 1 by tau m plus a coefficient which I will call k. So I am looking at the roots of this kind of an expression a quadratic s plus 1 by tau a into s plus 1 by tau m plus a coefficient k depending on the value of this coefficient k in relation to tau a and tau m, I will have one of the 3 situations, the root locus method enables you to find out, what is exactly the case.

Now what I am looking at is the partial transfer function from the applied voltage to the speed, you remember that and I am going to look at the poles of that transfer function.
The root locus method enables us to handle this kind of a situation. When you have a polynomial in which some coefficient of it, can be considered to be varying and we want to study the effect of what happens to the roots of the polynomial as this coefficient k is changed. In our case of course this 1 by tau a and tau m terms are fixed, in fact all the coefficients are fixed.

So I am only saying that we want to decide which one of the 3 cases is holding and that is why, this whole exercise. So for this just anticipating what we are going to do later and you will appreciate it, when we go ahead and look at the root locus method but the following I can say at this moment that corresponding to these 2 terms 1 by tau a and 1 by tau m, I will draw, if looks like a pole 0 diagram, this is not the pole 0 diagram, this is not the pole 0 diagram for the transfer function that we are looking at. For the transfer function, this is the quadratic and therefore I have to look at the roots of this quadratic whole thing. But I will draw the pole 0 diagram as it were corresponding to only this part that is when k equal to 0.

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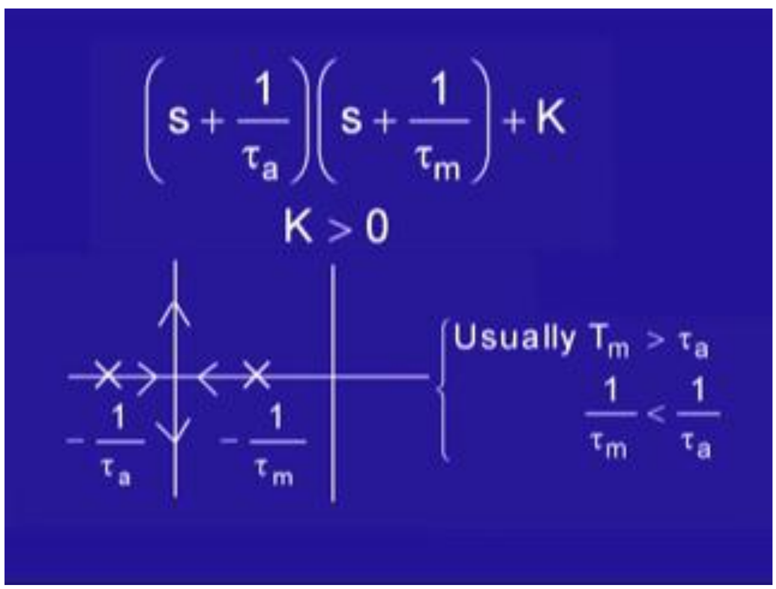

When k equal to 0, what do I have I have 2 poles 1 of them is minus 1 by tau a the other is minus 1 by tau m. Now depending on which one of them is greater than the other, their relative positions will be determined. Let us say, we have this equation where minus 1 by tau a is here minus 1 by tau m is here. So the armature time constant pole is to the left of the motor time constant pole or so, which of the time constant is greater of the 2, the motor time constant is greater than the armature time constant, tau m is greater than tau a therefore, 1 by tau m is less than 1 by tau a and therefore, the poles are as shown. Now, if k is 0 then, these would be the 2 poles.

Now, the root locus method tells you that when k is not 0 but the k is positive then, the roots will not be these at these 2 points but will be somewhere else and anticipating the root locus method, what we can conclude is that as this coefficient k is increased from 0, the roots move as it were towards each other or in other words, the roots change their values till for some value of k, the 2 roots become coincident that is the critical value and the discriminant is equal to 0 and when this coefficient is increased further, the roots move out into the complex plane and therefore, depending on the value of this, the coefficient k we will have either 2 real roots, 2 coincident roots, real roots or 2 roots which are complex.

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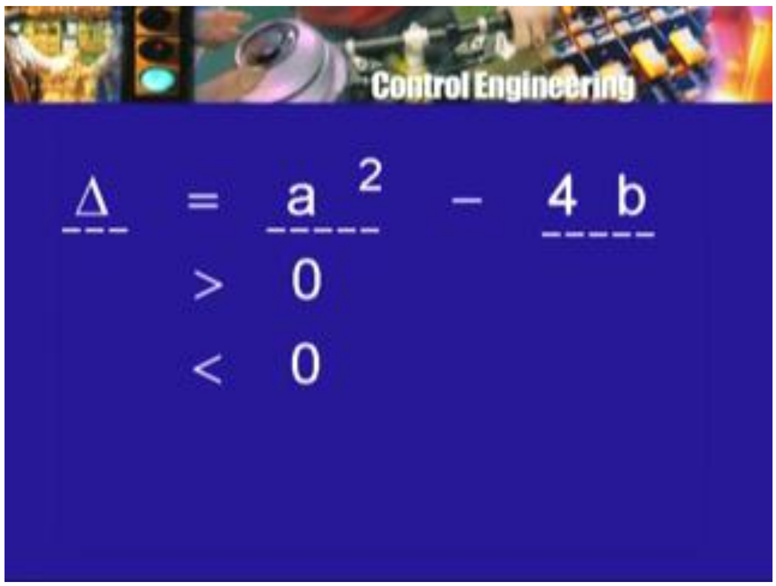

So the 3 situations that we are looking at can indeed arise depending on the relative values of the coefficient and one can work out the exact relationship between the relative values of the coefficients and let me, write that down and you can look at it, by after some simplification. So our discriminant was delta equal to a squared minus 4 b, where a is the coefficient of s, the coefficient of s is 1 by tau a plus 1 by tau m. So whole squared and we are looking at delta equal to 0. So this should be equal to 4 times b, b is the constant coefficient which is 1 by tau a into 1 by tau m plus that k b, k T divided by L a into J.

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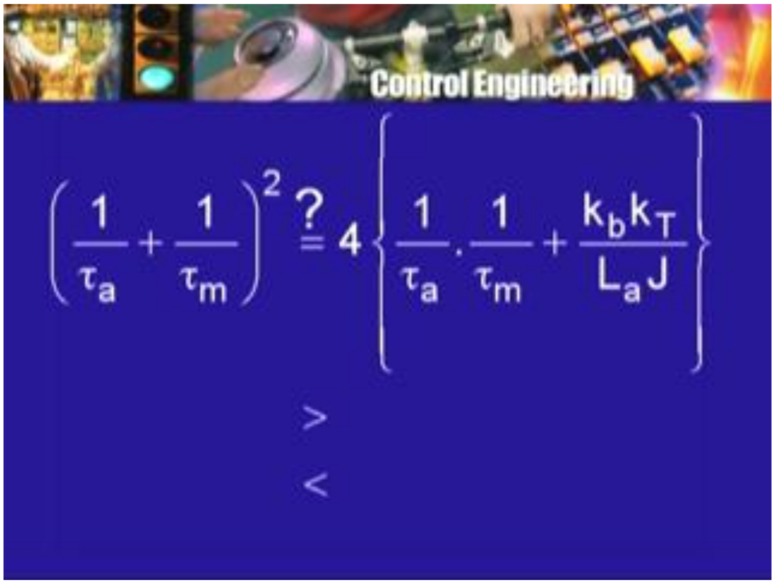

So depending on the values of tau a which is the ratio of armature, inductance to armature resistance, tau m which is the ratio of moment of inertia J to coefficient of friction k f, k b which is the back EMF constant, k T which is the torque constant and the armature inductance and the moment of inertia itself. If this relationship is satisfied, the roots will be real, if this relation is not satisfied but if any quality like this holds then, the roots will be real non coincident and if the any quality holds this way, the roots will be complex but in all cases, we are safe because the poles of the transfer function are in the left half plane, they are all good pole.

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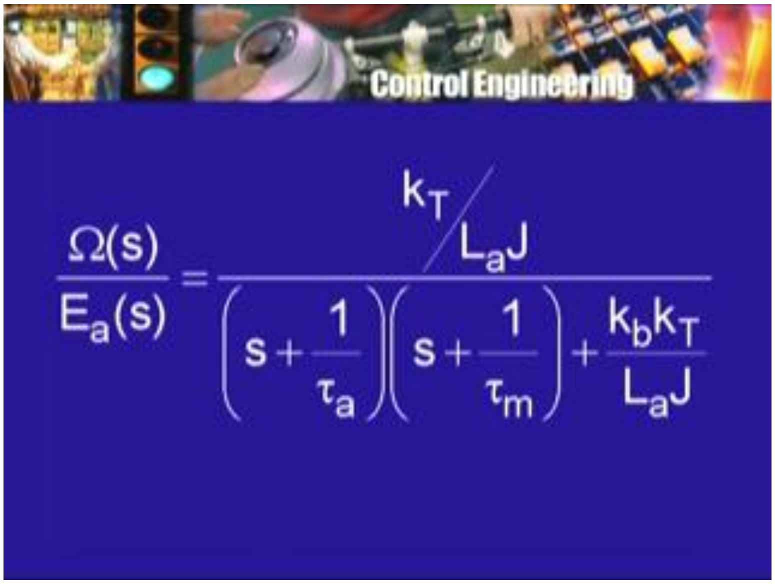

This is something we can conclude because in this case we simply have a quadratic and so the work was easy. The root locus method enables you to draw some such conclusions, when there are not just 2 poles but there may be more than 2 poles and there may also be 0 that is there may be terms in the numerator. I will tell you more about it, when we come to that but for the moment this will suffice. So we have looked at the transfer function that relates omega to E a, let me write it down once again, omega divided by E a as a transfer function was given by this messy expression k T divided by L a into J in the numerator divided by s plus 1 by tau a into s plus 1 by tau m plus k b, k T divided by L a into J.

Remember, that this is only a part of the response this is not the whole of the response, this is the only the part of the response that depends on the applied voltage, also it is a part that does not involve the initial value of the armature current and the initial value of the speed. Now, can we conclude something about long term or steady state behavior from this sort of a expression or power transfer function like this, the answer is yes, in fact when I talked about the Laplace transformation I mentioned a particular theorem known as the final value theorem and one can use the final value theorem here as follows.

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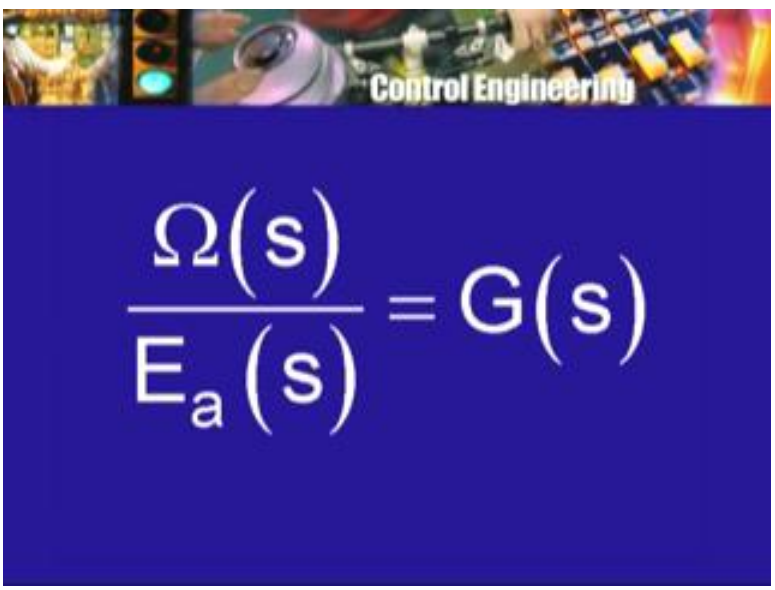

Let us denote this transfer function by simply G (s), so that I do not have to read out the whole expression all the time. So I have omega divided by E a equal to G (s) or I will write it as omega s equal to G (s) into E f (s). So let us now, consider the special case when the applied voltage is constant. So the applied voltage let us say, E a of t is equal to some number E capital E, constant. So what is the Laplace transform of it E a of s is E by s therefore omega s becomes G (s) into E by s. Now, let us apply the final value theorem, what does the final value theorem say, that the final value of the function that is a limit as t tends to infinity of the function in this case omega t equal to the limit as what happens to s, s tends to 0 of s times omega s but s times omega s, in this case because omega s is here is therefore limit as s tends to 0 of E into G of s or therefore, it is E into G (0).

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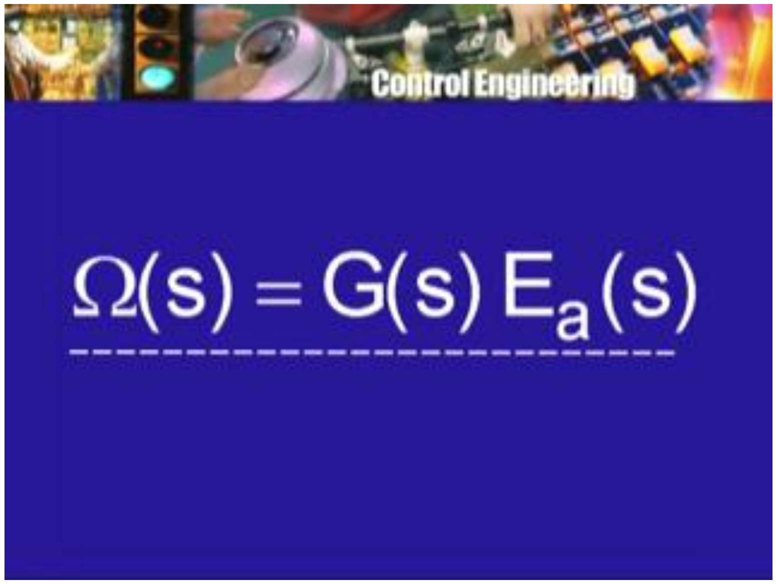

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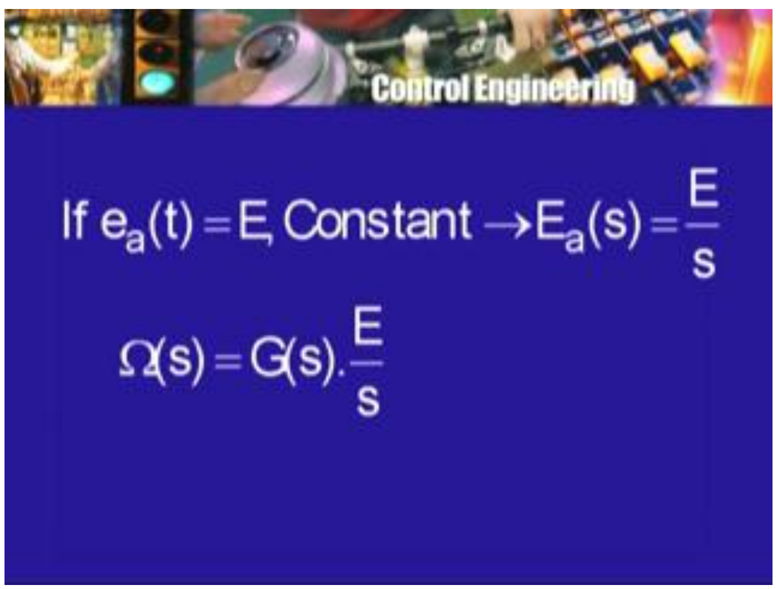

So it is simply the value of the transfer function for s equal to 0 multiplied by E that will be the final value of the speed, once again not the actual final value of the speed but the part of it that depends only an the applied voltage. There is a part of it that depends on the torque, so it is as if we are only looking at the 0 torque after load torque situation or we are looking only at a part of the speed. So what does the G (0) become now, if you look at the expression for G of s this is the expression for G of s.

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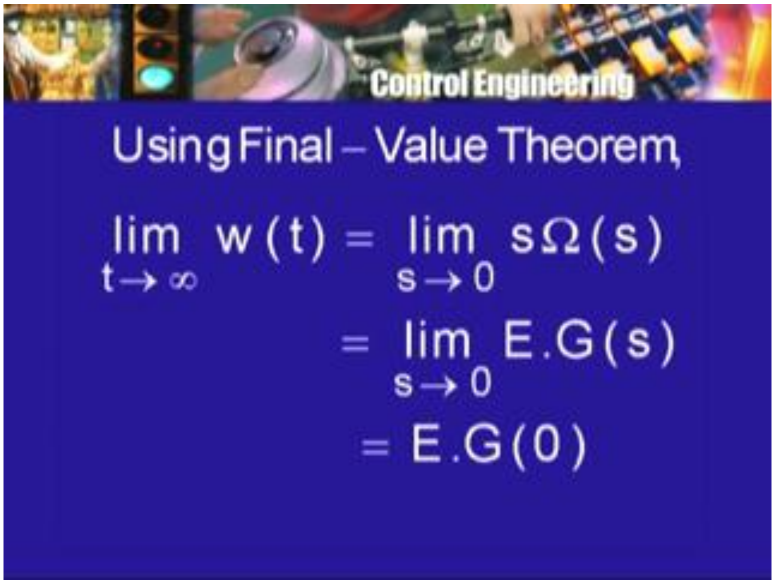

So, when I put s equal to 0 this s cancels this s cancels and I will get some expression remaining and therefore, it is that coefficient multiplied by E which will give me these steady state speed or once again the part of the steady state speed that depends on the applied voltage. So, let me write down that expression and you can go back and compare it with the expression that we had earlier.

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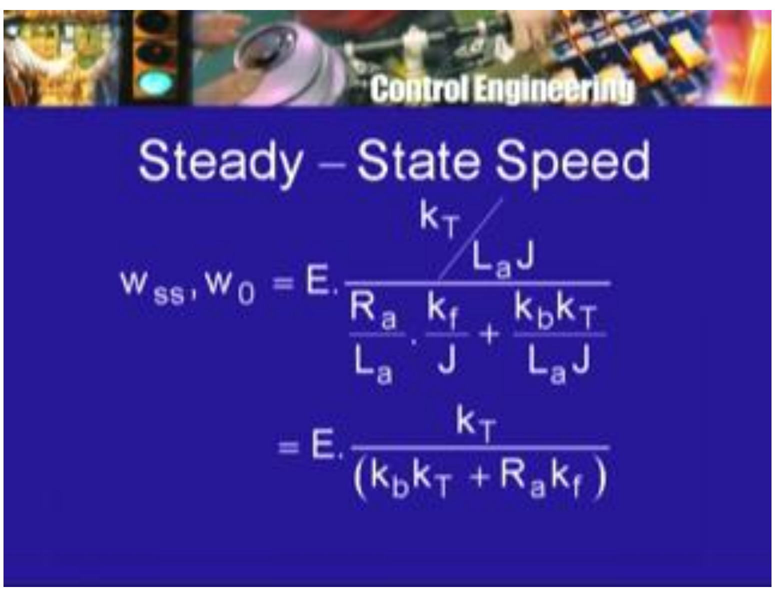

So I will write it as omega steady state, I am going to write it as omega ss equal to from what I have said just now the applied voltage E multiplied by G (0). So which is k T divided by L a into J into1 by tau a and I will rewrite tau a as R a by L a into 1 by tau a is k f by J plus k b, k T divided by L a into J and so getting rid of L a into J, I will have it as E into k T divided by k b, k T plus R a into k f. This is the value for this steady state speed, check whether it tallies with the value that we had obtained earlier that is omega ss equal to this coefficient into E minus some other coefficient into the load torque.

So check whether this is the same coefficient that we got earlier. Once again, you can look at the dimensions and check whether the dimensionally the 2 are the same, roughly you can see it as follows there is k T here, there is k b, k T here. So if I cancel k T, I get k b in the denominator. So this looks like voltage divided by back EMF constant but that is dimensionally speed. So it tallies same thing of course should hold about the other term R a into k f divided by k T, I will leave it to you to verify that dimensionally this equation is correct.