Control Engineering
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Lecture - 25

Let us look at our second order example once again, we have the differential equation which describes the system.

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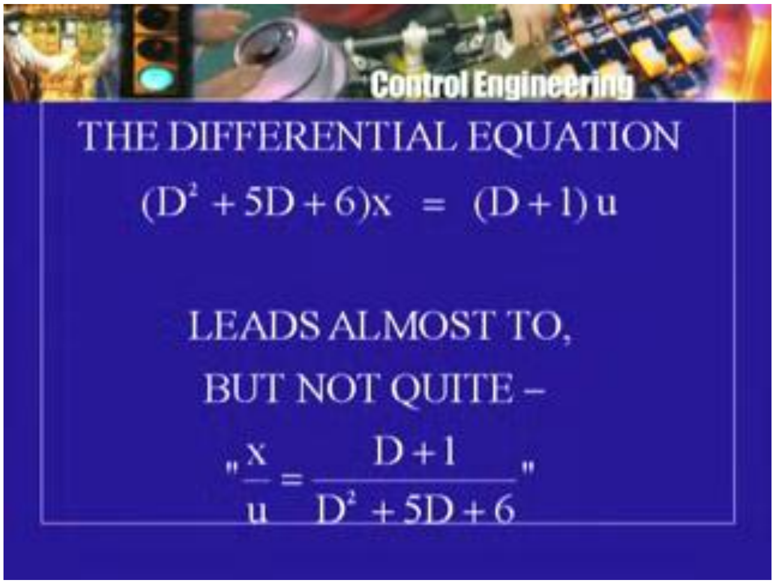

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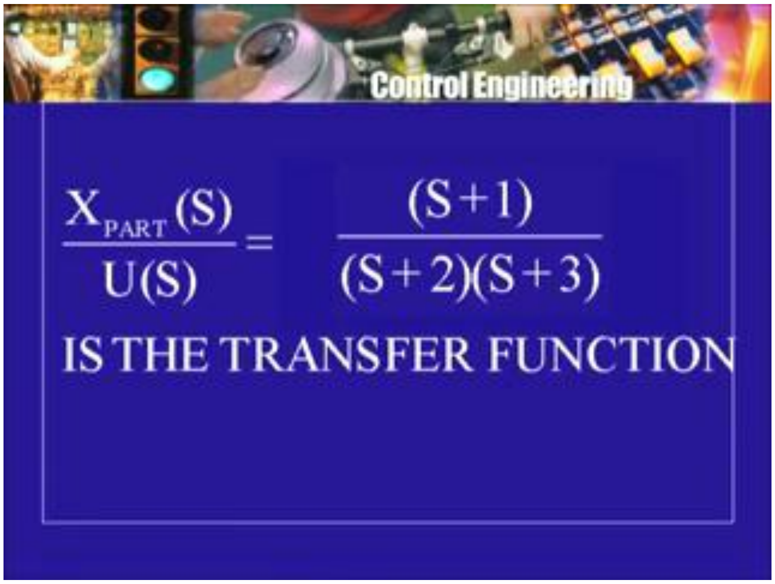

The differential equation is d square plus 5 d plus 6 operating on x equals d plus 1 operating on u, u is the input and x is the response. For this after taking Laplace transformation and looking at only one part of x of s, we got the expression that x s divided by u of s was given by s plus 1 divided by s plus 2 into s plus 3.

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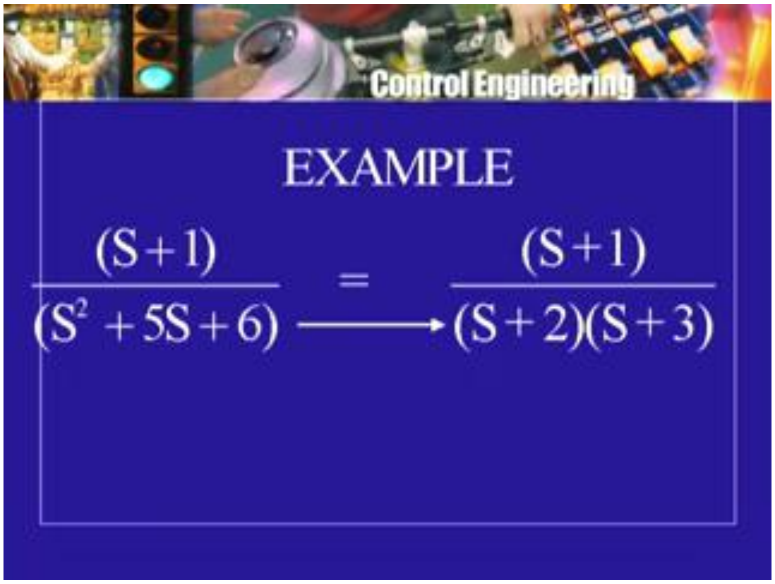

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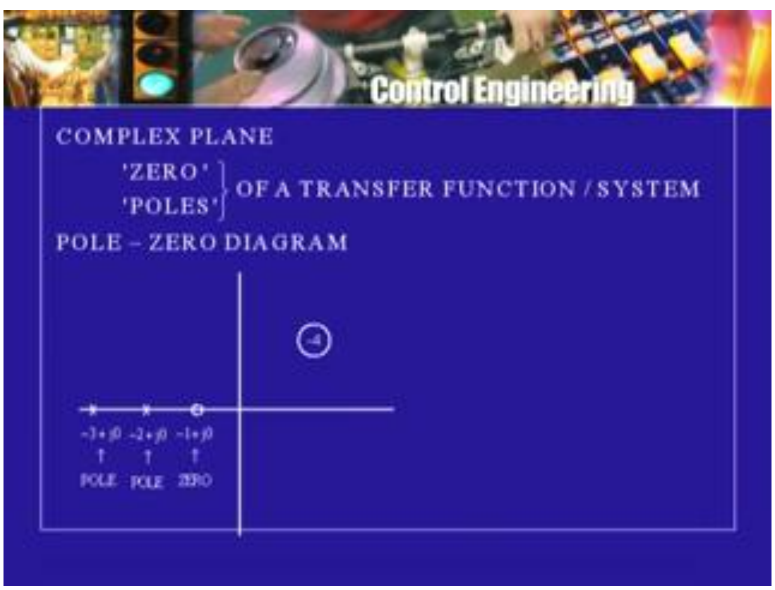

We factorized the s square plus 5 s plus 6 into 2 factors s plus 2 and s plus 3 and this is what I said was the transfer function and it is represented on a pole 0 diagram as shown here and in

general, there may be a multiplying coefficient in front which also can be indicated on the pole 0 diagram.
Now with the help of this transfer function what is it that one can say about the system behavior, without doing computations qualitatively what is it that one can say, the of course what one can say depends on, what one has studied beforehand and upon the experience that you have had looking at transfer functions like this. Later on, we look at what is called the frequency response of a system and we will see that from the transfer function, one can immediately get quite a lot of information about the frequency response of the system but at the moment, we will look at something else.

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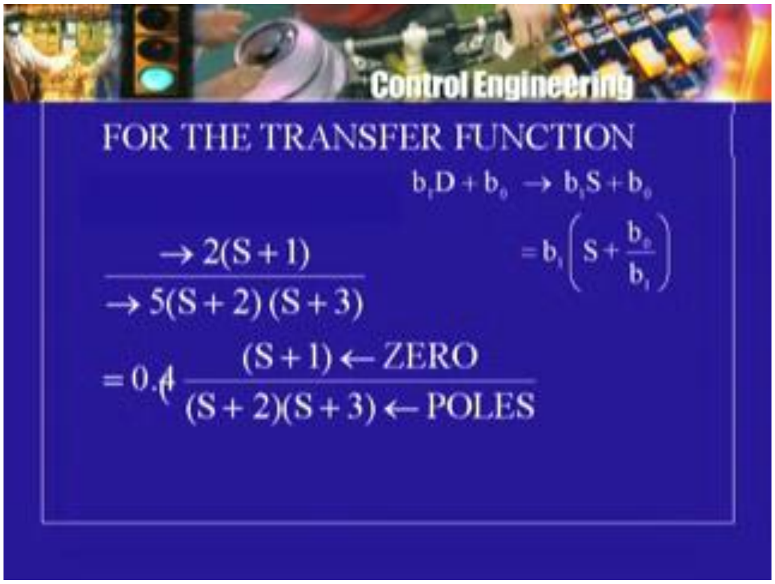

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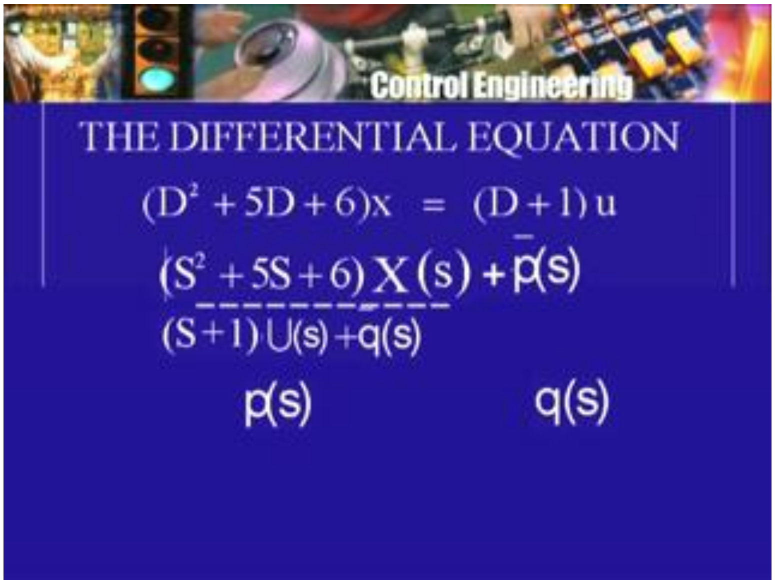

Now you will remember that when we obtained the transfer function, we ignored some terms or alternately, we only looked at one part of the response transform. Let me go through it again, very quickly, we have d square plus 5 d plus 6 acting on x equal to d plus 1 acting on u, applying the Laplace transformation what do we get. On the left hand side we got s square plus 5 s plus 6 multiplying the Laplace transform the response capital X of s plus we got some terms containing the initial conditions of x, x 0 and d x 0, in this case.

So let us write down that those set of terms together as let us say, p bar of s, where the p suggests that it is a polynomial, indeed it is a polynomial of degree 1 as you can check. So this is the thing that we get on the right hand side, the transform of x of s multiplied by s square plus 5 s plus 6. It can be obtain from d square plus 5 d plus 6 by simply replacing d by s and this is because of the derivative property of the Laplace transformation. On the right hand side, what did we get, similarly to the left hand side the d plus 1 gives rise to s plus 1 multiplied by u of s.

So s plus1 multiplies the input transform plus we have a term, which contains the initial values of the input because the input is getting differentiated on the right hand side. So let us write it down as q bar of s and in fact, I am going to write down the coefficient of x of s as p s and the coefficient of u s as q s. So, if I do that then the equation that I get it can be written as p s, x of s plus p bar of s equals q s, u of s plus q bar of s, where p and q are polynomials in s, they correspond to the operators, the operators.

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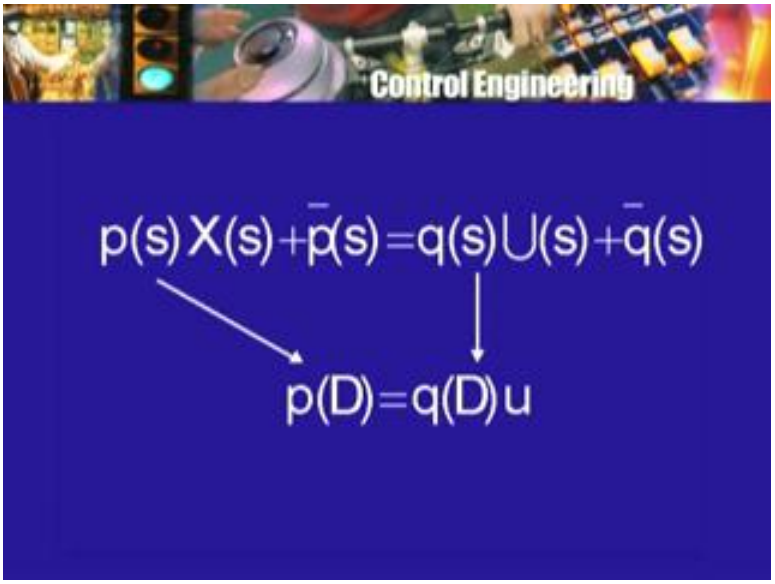

So the original differential equation, if it was p d on acting on x equal to q d acting on u, then this p s appears here multiplying x s, q s appears here multiplying u s and then, you have these 2 additional polynomials p bar and q bar which involve or which depend on the initial values of the response and the input respectively and therefore, solving for x of s, we get 3 terms, 1 term is simply q s divided by p s multiplying u s, the second term would be let us say plus q bar of s divided by p s and the third term will be minus p bar of s divided by p s.

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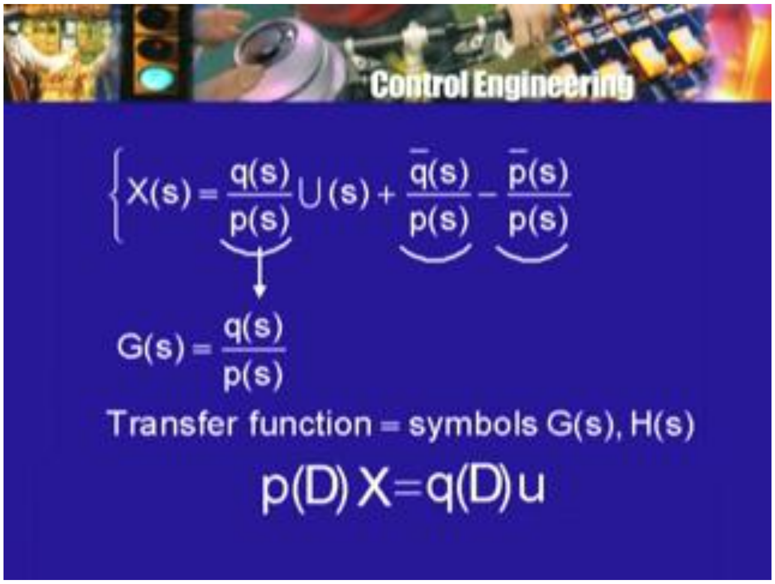

Now what you have been looking at or what we look at or what we consider and we talk about the transfer function is only this part q s by p s multiplying u s. We are not looking at the other 2 parts there is this part q bar s divided by p s, which not 0, if initial values of the input are not 0 and there are derivatives of input on the right hand side. The second term that we are not looking at is p bar of s as numerator p bar of s, p bar of s is 0, if the initial values of the response are 0. So we can say that we will get only this part namely q s by ps into u s, if initial values of the response and the input are 0, but that will not be the case in general.

So it is better to say and think that we are only looking at a part of the response transform then, will be the part which does not involve the initial values of the response and the initial values of the input but involves only the Laplace transform of the input and that is multiplied by a ratio of to polynomials, q s by p s and this q s by p s is what we call the transfer function of the system and as I told you earlier in the control literature, the letter g of s, capital G of s is very often used, to denote a transfer function of, what is called the forward path of the control system. Of course if it is the feed back path then, the transfer function is labeled as h of s usually.

So then we have the transfer function of the system g of s equal to q s by p s and you can say that it gives you one part of the response when it is multiplied by the input transform. So remember that that the transfer function does not give you the complete information about the response transform. It only gives you one part of the response transform of course, you can think about this and see that if I know p s then, I can find out the expression for p bar of s, of course p bar of s will involve initial values of x but knowing the polynomial p d, I can obtain p bar of s similarly, knowing the polynomial q s or q d, I can obtain the polynomial q bar of s.

So in that sense, if I have given the transfer functions like this as a ratio of 2 polynomials from that I can write down the 2 parts of the 2 terms which are not looked at or which you are not looking at, when we talk about the transfer function and as I told you earlier from q s by p s as transfer function, we can easily go back to the differential equation p D x equal to q D u. So we can recover the differential equation from the transfer function.

Now what information can we get about the system behavior from the transfer function that is what you would like to look at for a little while? Remember, we are not going to look at frequency response that will come later. Now to get that information from the transfer function as I told you it is useful to factorize q s and p s in to linear factors and that is what I have done earlier for our numerical example, we factorize q s of course, q s was a first degree.

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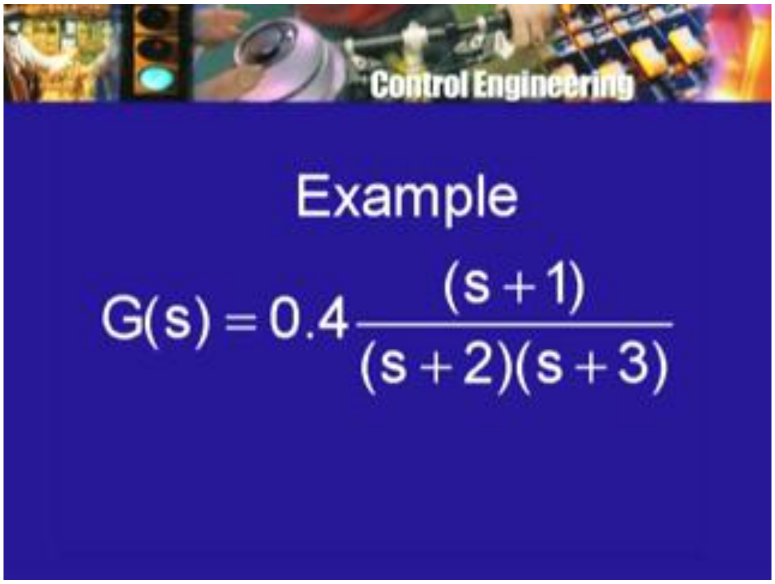

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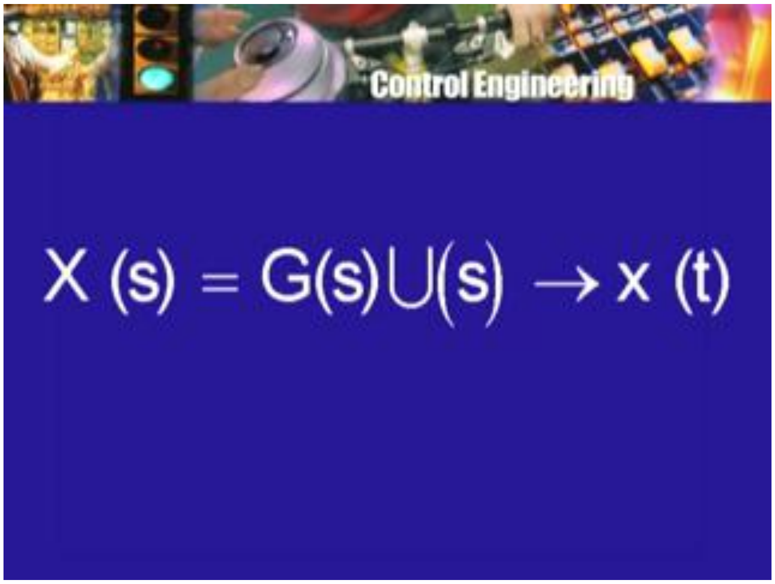

So it came already in factorial form only one factor whereas the denominator was of degree 2, so it required some factorization. Now, so let me take some example, now select, suppose g of s equal to, I will put that multiplying factor in front. So I will write as I did earlier .4 into s plus 1 divided by s plus 2 into s plus 3, this is g of s. So what kind information can I conclude from this g of s. Now we do not know of course, what u of s is until you know about u of t is that is until you are given a specific input function. Let us say a constant functions, sinusoidal function, exponential function, t square or ramp or whatever or a sum combination of these until, we are actually given u t, we cannot find out or calculate capital U of s and therefore, I cannot calculate x of s because x of s is simply g s into u of s, I know g of s but until I know u of s, I will not able to calculate x of s, with the intention of finding from it x of t which is the response or a part of the response that we are looking at.

However, the following can be said now, for most of the functions input functions that one comes across certainly for the constant function for the unit ramp or a ramp with a non unit coefficient, for the exponential function, for the trigonometric functions sign and cosine and even for they products we saw something like this earlier, a product like some power of p multiplied by some exponential multiplied by either sign or cosine of omega t. For an input which is as complicated as that and in fact, more complicated even for an input which consists of a sum of terms of that kind, one can show that u of s will also be a ratio of 2 polynomials, what is it that such a function called a ratio of 2 polynomials, I told you earlier that is called a rational function. It is called a rational function and so u of s can be written as I would write n for numerator, a numerator polynomial n of s divided by a denominator polynomial d of s.

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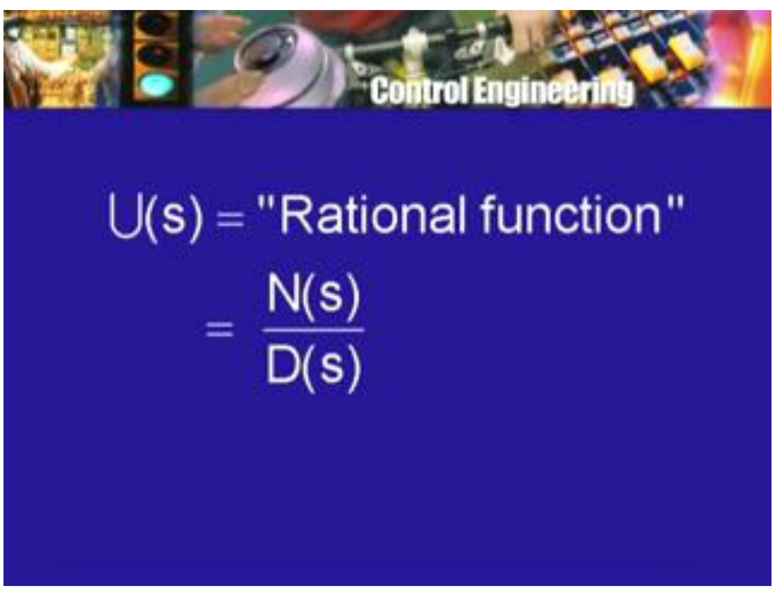

So N s by D s, for many inputs U s can be found out to be a ratio of 2 polynomials or a rational function. Let us quickly run through familiar examples for the constant function one u of s is 1 by s that is a ratio of 2 polynomials the numerator polynomial is just the constant polynomial 1, the denominator polynomial is s, for the ramp function whose value is t for t greater than equal to 0, the Laplace transform is 1 divided by s squared again a ratio of 2 polynomials, for e raised to minus alpha t what is the Laplace transform one divided by s plus alpha, for sign omega t what is the Laplace transform, omega divided by s square plus omega square, for cos omega t, it is s divided by s square plus omega square and I have asked you to calculate a few Laplace transforms earlier.

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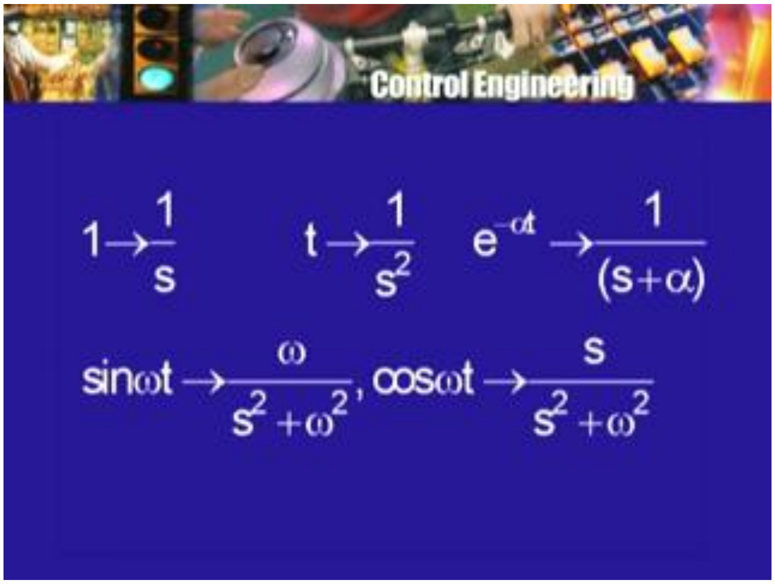

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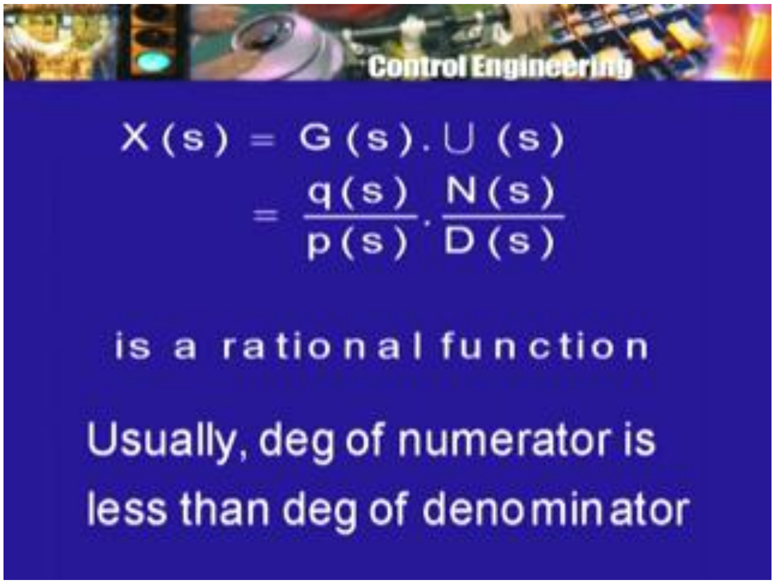

So in many cases, the Laplace transforms of the input is such that its Laplace transform is ratio of 2 polynomials n s divided by d s therefore, let me write down the expression for the part of x s that we are looking at x s is equal to G s into U s but G s is already a ratio of 2 polynomials q divided by p and U is also ratio of 2 polynomials N divided by D and so, x s is a ratio of 2 polynomials again the numerator polynomial is q s into N s, the numerator of G in to the numerator of U of s and the denominator polynomial is p s into D s, the denominator of the transfer function G and the denominator of the input transform U of s.

So this is also a rational function or a ratio of 2 polynomials. Now remember, what I told you to look up from the algebra books earlier, that a rational function can be expressed as a sum of partial fractions. Remember, partial fraction expansion now, there is only one condition which must be satisfied, if it is not satisfied then, you have to add something to what one is saying and that condition is the degree of the numerator is less than the degree of the denominator.

If the degree of the numerator, so I will write down here degree of the numerator n r for numerator is less than the degree of the denominator then, a rational function can be split into partial fractions each one of which is a rational function, indeed and not only that we can factorize the denominator into its linear factors. In other words, find of the roots of the corresponding polynomial then, the partial fractions will look like what, each term or each partial fraction is a rational function and it so happens that the degree of the denominator for this part will also be greater than the degree of the numerator but the denominator is a linear polynomial. So, let us say it is s minus pk why p because p is to remind you have poles, in general roots of the denominator polynomial are called poles roots of the numerator polynomial are called 0's.

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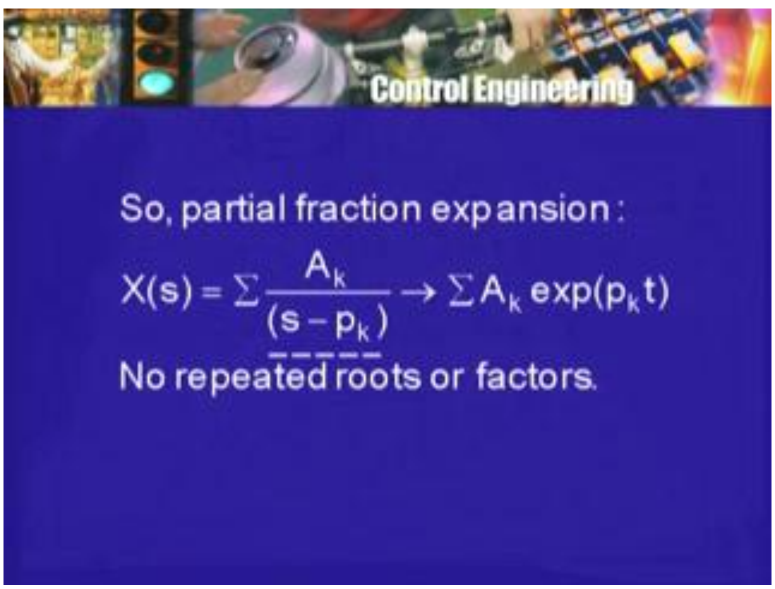

So I have s minus p k in the denominator and in the numerator polynomial of degree 0 and therefore, only a constant and so my rational function can be expanded as a sum sigma of A k divided by s minus p k. Of course, this is not really 100 percent root this is true provided the factors of the denominator are all distinct. So, what are the things that one needs to look into what if, the denominator polynomial has repeated roots or has identical linear factors, several identical linear factor or repeated root or root of a certain multiplicity that is thing number 1. Secondly, what if the degree of the numerator is not less than the degree of the denominator that is equal or greater than.

Now that can create certain problematic situation. So we will keep that the side for the moment we will assume that the degree of the numerator, it is less than the degree of the denominator and to illustrate the point, we will assume that the factors of the denominator are all distinct that is there are no repeated factors. So let us consider that special case is not all too special because it is not in, it is in very few cases that really we get repeated roots exceptional cases really in most cases, the roots of the denominator polynomial will be distinct, there will not be any multiple roots. So let us look at that is situation, I have X of s equal to qs by ps into N s by D s. Now think of the partial fraction expansion for this.

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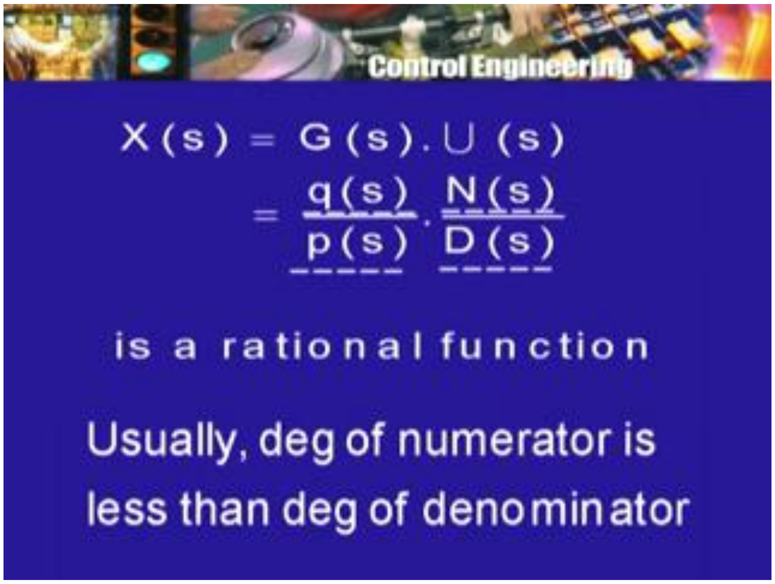

Now of course, what are the factors of the denominator of this X of s the denominator is p s into D s. So the factors are either factors of p s or factors of D s, factors of either the denominator or the transfer function and what is the corresponding root called pole of the transfer function or factors of denominator of the Laplace transfer of the input and we can call by analogy that also as a pole of the input. There is a difference in one case, I am talking about a pole of a transfer function which is a ratio of 2 polynomial, in the other case I am talking about the ratio of or rather the pole of the input but what I mean is really, the root of the Laplace transform of the input.

Now there is on more assumption that I am going to make right now and if, I do not make that assumption the situation once again is a little more involved that p s and D s, do not have any common factor that is there is no root of p which is also root of D and conversely, roots of p are distinct from roots of d. Suppose that also happens then what then, think of the partial fractions that come from the denominator polynomial p of s that is come from the factors of p of s, they will of the kind A k divided by s minus p k.

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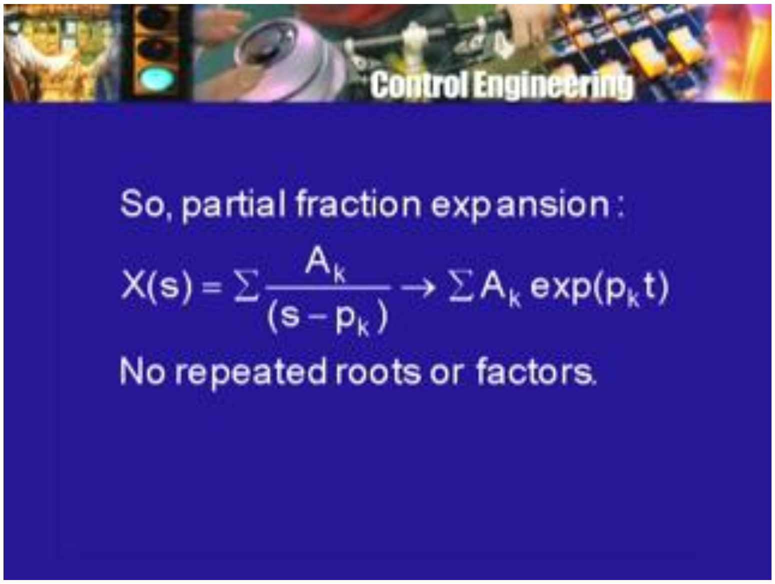

Therefore, if I take the Laplace inverse of that part of x of s, what will that look like, what is the Laplace inverse of A k divided by s minus p k, it is A k into e exponential of p k into t that is, it is an exponential function because in general, p k may be complex and therefore, this becomes a complex exponential but that should not worry you because for each complex root, there is also conjugate root. So for each term like this there is also a conjugate term, so together they give you a real function of time. So finally in your final answer you do not get any complex numbers or complex functions at all. So a root of p s or a pole of the transfer function, if it is not a pole of the input, it will give rise to a term like this A k into exponential of p k into t, if p k is the pole, we get the corresponding exponential p k t.

Now these, this is only one term in the total partial fraction expansion, what will be the other terms there will be partial fractions corresponding to the other factors of p s and partial factors corresponding to the other factors of d s. So this is not the only term but if I assume that p k is a root of p and not of D and it is a simple root of p then, this is one term that I will get and there will not be the another term with exactly the same p k, exponential p k other terms will have different exponents.

Now what about this exponential p k t, we have looked at the graph of the exponential function earlier and we saw that, if p k is greater than 0 then, the exponential function simply keeps on growing without limit for example, e raised to t, e raised to 4 t and so on. So this means that the response will have a term like some number multiplying exponential which is growing and therefore, the response will keep on growing indefinitely as time passes. Now this is not exactly stability as such, we will see that the word stability of course can have different meanings, this is one meaning of stability but it is one of the simplest meanings of stability that is the response will go on increasing indefinitely or as one says, it will become unbounded or a 1 should rather say, it will not be bounded, it will keep on growing as time passes.

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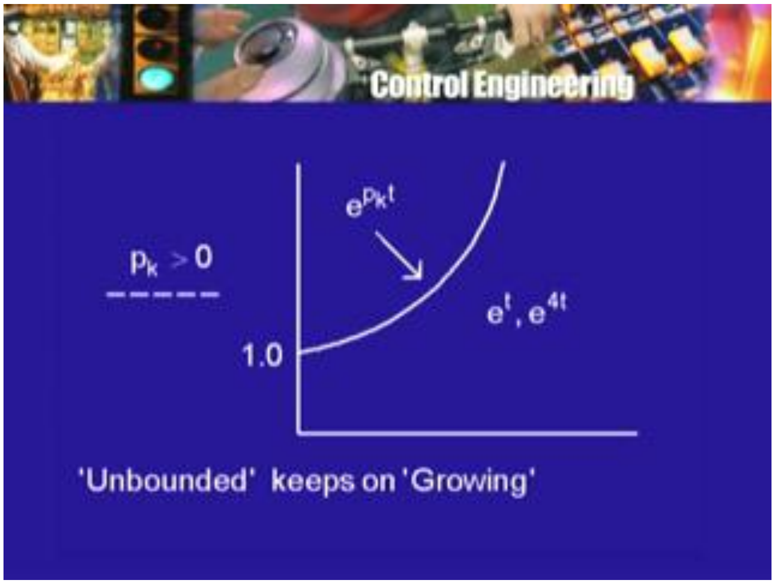

Now certainly we do not want that to happen in a physical system, why not because we are afraid that the voltage in the circuit will build up from 1 volt to 10 volts to 100 volts to million volts, why are you not afraid because in practice, you will before you reach that stage of even may be 1 million volts or 1000 volts, probably your circuit will burn out or another possibility is the differential equation that we assumes to hold for the system will no longer hold because of effects like saturation and non-linearity's and so on.

Remember, magnetic circuit is saturation, I told you earlier that as you increase the field current the field circuit may get saturated that is a flux will not keep on increasing proportionately to the field current and it will saturate, it will almost become constant. So because of this this unboundedness or growth exponential growth is not going to take place at all but because of it we will no longer be able to say anything about this system the model that we had assumed, will no longer hold, the system will no longer be described by that model, when the variable x of t is large. This is true for most practical systems, no model is valid for very large values of the variables, whether is as simple a thing as a resistor and ohm's law, v is equal to ri is okay provided the current is not too large, a very large current in the resister will simply result in heating and the heating will go on accelerating or cascading and the resistor will finally burn out, it will no long remain as a resistor obeying the law v equal to ri.

So a pole of the transfer function which is positive is bad from this point of view. On the other hand if p k is less than 0 then, what happens then we have the exponential function but this time here the exponential decaying function and we know already about this that although this function never actually becomes equal to 0 but there is something called a time constant which we can associate with this p k and in this case, this time constant will be the reciprocal of the absolute value of p k, as we saw earlier.

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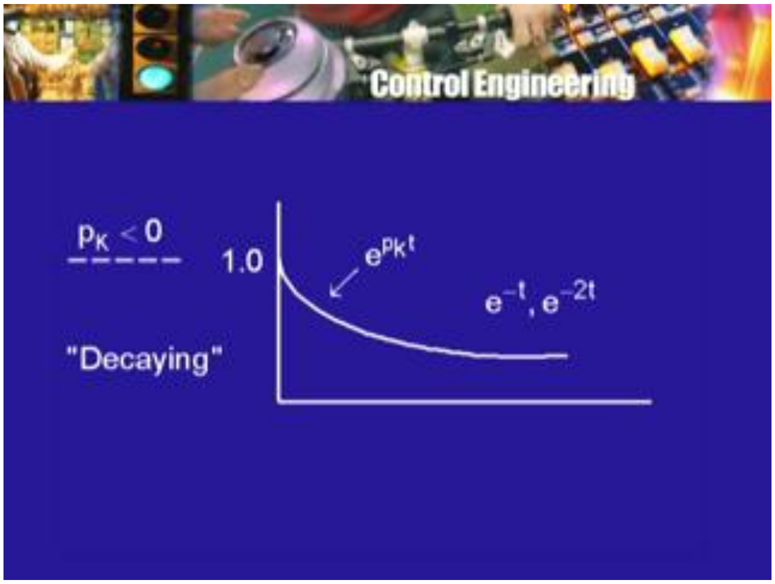

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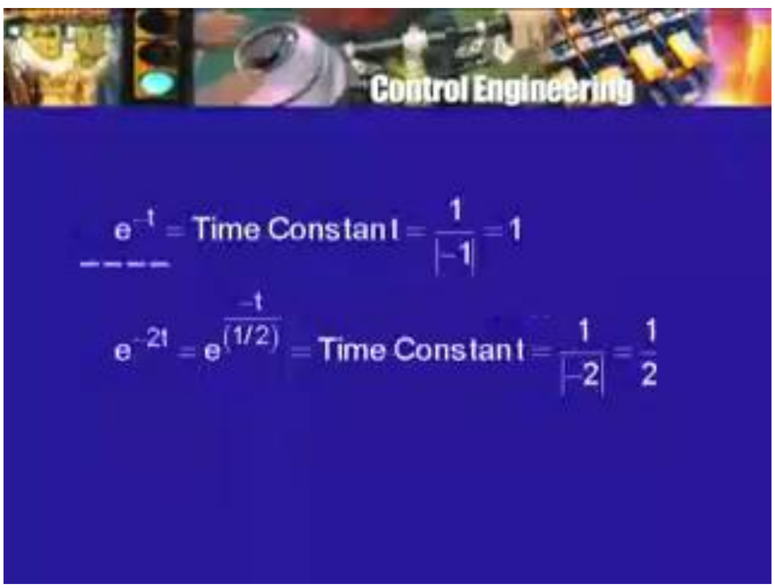

So if it is e raised to minus t, the time constant is 1 divided by absolute value of minus 1 or 1 second, if it is e raised to minus 2 t rewrite this as e raised to minus t divided by1 half and so, the time constant is 1 half and so on and I have told you that after 5 time constants or 10 time constants, may be 20 time constants if you wish, the function e to the minus 2 t is almost 0, it is 10 raised to minus 5 or something of that sort which can be ignored because all these variables are never known to precision which is very much greater than that any way.

So exponentially decaying is no problem at all, it only says then that part of the response goes to 0 but what we can find out from the root p k is the time constant that is the decay takes place with a certain time constant that corresponds to the root p k. Now this is for the case, when the factor s minus p k is such that p k is real and positive or p k is real and negative, what about one more situation, what if p k is equal to 0, if pk is equal to 0, the corresponding term in the partial fraction expansion is A k divided by s then, what is its Laplace transform it is Laplace transform is simply the constant function A k and therefore, this part of the response will simply remain constant but because it remain constant, it will keep on surviving as time increases.

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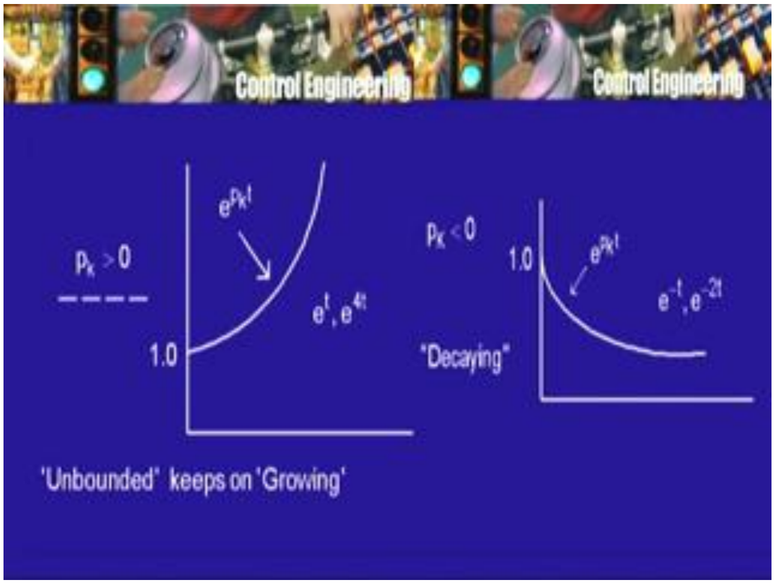

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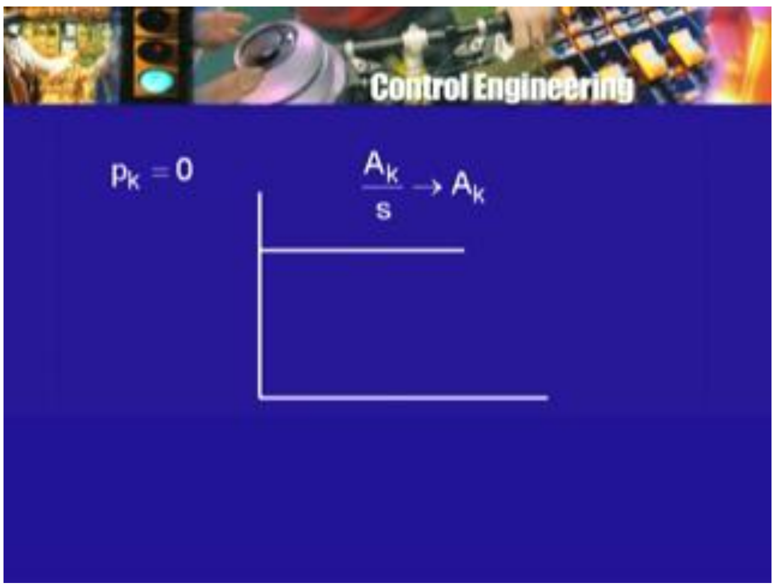

So it will contribute to this steady state that is it will remain even when 20 time constants are have elapsed or 100 times constant have elapsed, it will remain there, till so long as this is thing is operating. So it contributes to this steady state and will see later on that, this can contribute to the steady state error that is this kind of situation can not necessary always will but, can contribute to a steady state error. Therefore, a pole which is the number 0 or 1 says the system of the transfer function has a pole at the origin because the complex number 0, in the complex plane is the origin of the complex plane and so, the language used is there is a pole of this system at the origin. If there is a pole of the system at the origin then, in the response there will be a constant term and we will have to watch out because that could contribute to steady state error.

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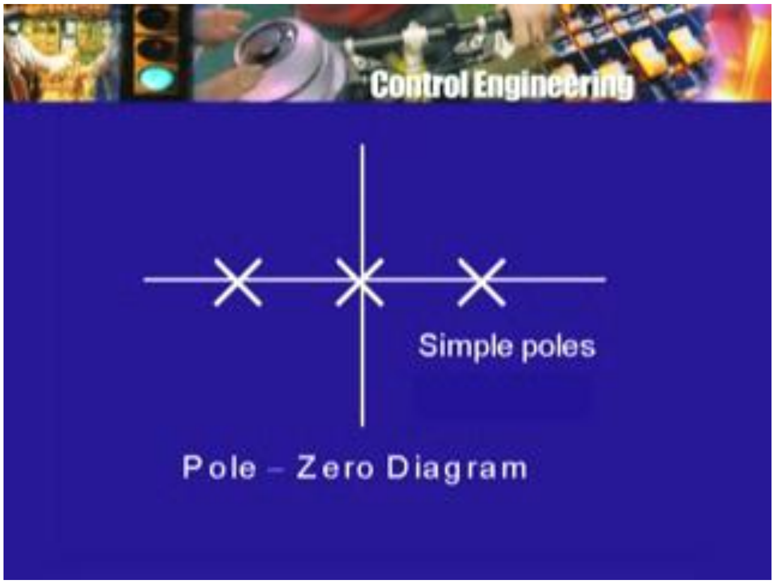

So the 3 cases when the pole is real and again on the pole 0 diagram, what will they look like, if the pole is in the right half plane then, p k is positive s minus p k, pole is p k. So this is bad, if it is on the left half plane that is okay, if it is at the origin then we have to be careful. So if the pole or the root lies on the real axis of the complex plane then, these are the3 possibilities at we have to consider. Remember, I am only looking at the case of non repeated factors or what are called simple poles.

So there is only 1 pole at the origin and not 2 or 3 that is not repeated. The next case to consider will be of course, when the pole is complex but before we consider the complex case. Let us take care of the purely imaginary case, so what if there is a pole on the imaginary axis therefore, the pole let us say is s j omega k or the factor looks like s minus j omega k. I told you not to worry by the j because there will be ak correspondingly a pole on the other half of the imaginary axis, in other words there will be a factor which is s plus j omega k and therefore, in the transfer function really, I would have had a factor like s squared plus omega k squared.

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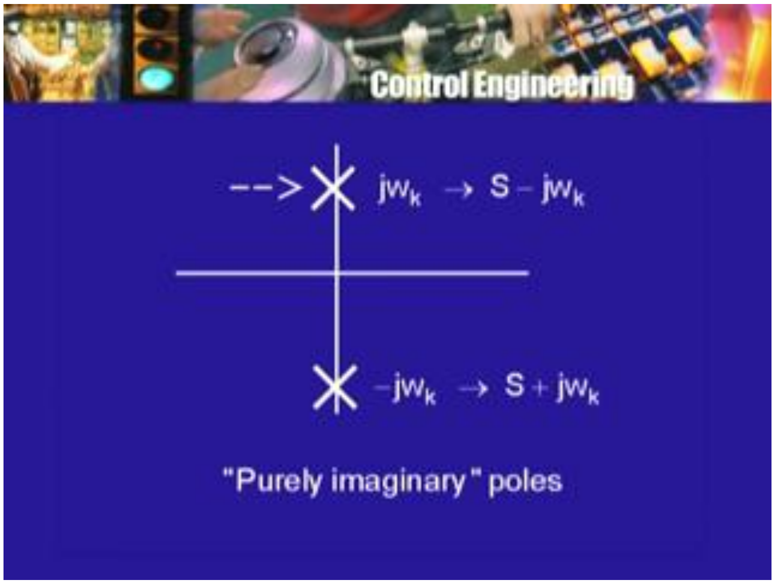

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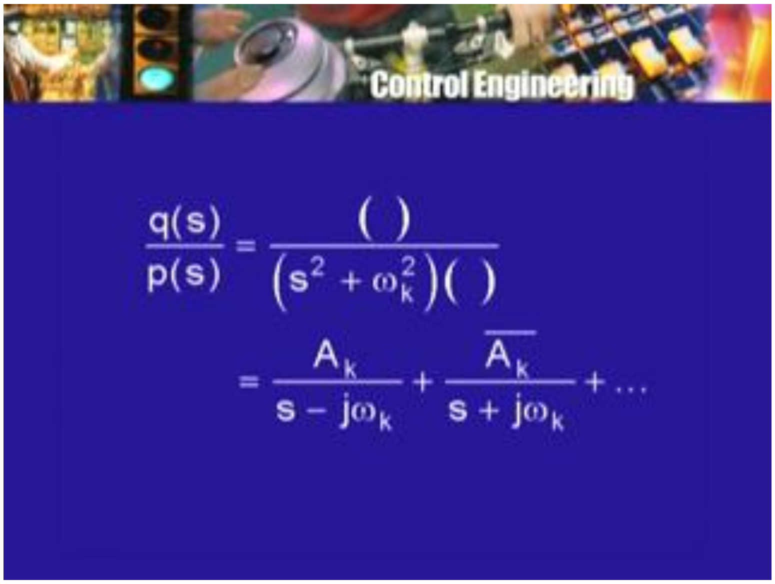

So going backwards, if in the transfer function q s by p s, if in p s I have a factor s square plus omega k squared, a quadratic factor with no s term then, it will correspond to a pair of poles like this on the imaginary axis. Therefore, in the partial fraction expansion I will have 2 terms like A k divided by s minus j omega k and A k, one can show that this will be A k bar divided by s plus j omega k, where A k will be a complex number.

Now this thing looks like a complex expression indeed it is, in fact it is not complex, it looks like complex, when I take the Laplace inverse I will therefore get A k into e raised to j omega k t plus A k bar into e raised to minus j omega k t. Now all of you have studied a little bit of communication engineering and therefore, you have we have seen this kind of factor, e raised to j omega t and e raised to minus j omega t before and you know that when they occur in pair like this, in pair and with coefficients which are conjugate, this whole thing will reduce to what, it will reduce to a sinusoidal function, it will reduce to a function like a cosine omega k t plus some angle theta k.

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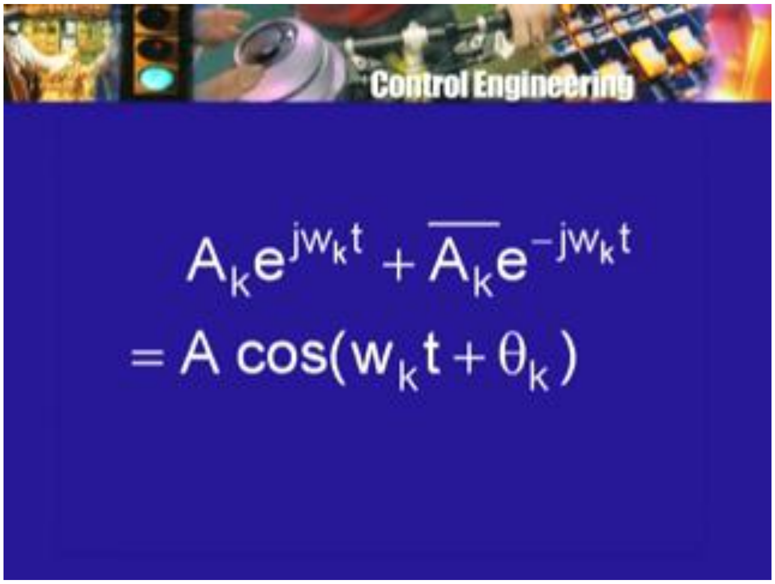

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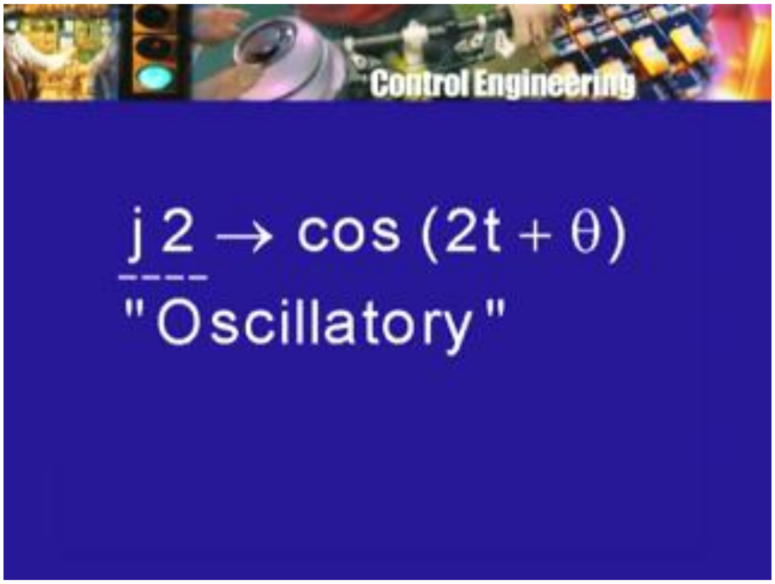

So in this case when the poles are on the j omega axis, the Laplace inverse of this part or therefore, the response will have a term which looks like A cosine omega k t plus theta k, is it good or bad. Well, it is not too bad but it may be bad because this means what, this trigonometric function is going to be there for all time to come, the trigonometric function does not become 0 either. So there will be a part of the response which will go on oscillating, will go on oscillating till as long as you have a system on. The angular frequency of the oscillation is given by omega k, if the pole is at j omega k, the angular frequency of the oscillation will be omega k t.

So, if the pole is as j 2 then, I except terms like cosine 2 t to occur therefore, the angle of frequency will be 2, correspondingly you can find out the period of the oscillation. So in the response then, there will be a term which goes on oscillating forever and that may be bad in some applications. It may not be steady state error which is constant but it may be steady state error which goes on oscillating and therefore that also may be back. Now we have 2 remaining cases and we can dispose them of pretty quickly, one of them is a bad one, when there is a pole which lies in the right half plane.

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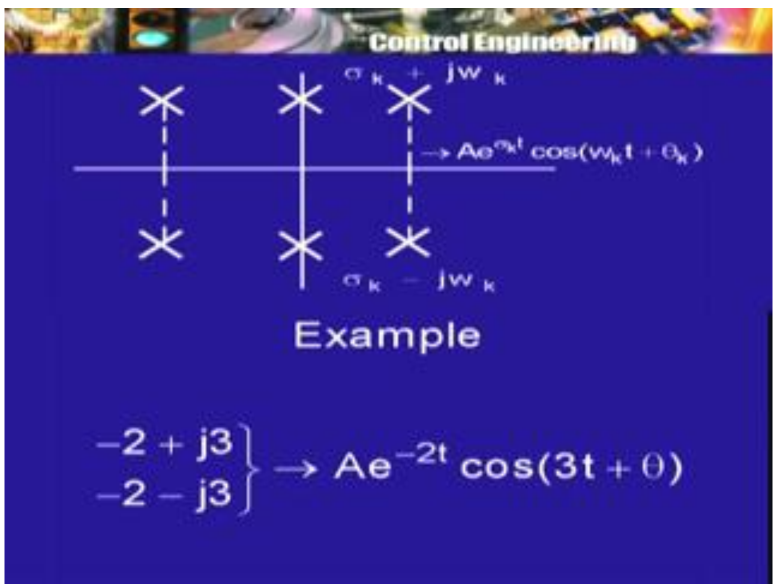

So let me write it as sigma k plus j omega k and for that you know that there is a conjugate pole, a sigma k minus j omega k. So we have 2 linear factors of a p of s which corresponds to 2 poles like this both of them are in the right half plane, both of them are complex then, from this what do you expect to get in the response. In the response there will be a function which will be first the j omega part give rise to the oscillatory function but the sigma k part give rise to an exponential multiplying it and therefore, you will have a function which looks like e raised to sigma k t multiplied by cosine of omega k t plus theta k.

Now this is bad, the cosine part is oscillating it will be a multiplier in front perhaps, the cosine part is oscillating but then, it is multiplied by the exponential part which is growing. So this is again a bad situation, this means that after some time the model of the system will cease to be applicable and therefore, you cannot say much about what will happen to the system. So right half plane complex poles are also bad, what about the remaining case. There is a pole here sigma k plus j omega k, where sigma k is negative and conjugate pole there. For example, minus 2 plus j 3 is one pole minus 2 minus j 3 is another pole, what about the corresponding Laplace inverse, corresponding to j 3 there will be this cosine function cosine 3 t plus theta and corresponding to this minus 2 t, there will be the exponential multiplier but this time the exponential multiplier is not going to be bad it will be A e raised to minus 2 t cosine of 3 t plus theta. The cosine part will go on oscillating but the exponential part will go on decaying into 0 and so, after 5 or 10 time constants this will be virtually 0.

So this means that it will not contribute anything to the behavior of the system after sufficient amount of time has elapsed and therefore, one need not worry about it. So we have the following situation that if you have the poles of the transfer function such that none of them are also poles of the input transform and the poles are all distinct, no factor is repeated then, poles which are complex and in the right half plane are bad poles which are real and still in the right half plane on the real axis are bad, poles on the imaginary access are bad, a single pole or a simple pole at the origin may not be bad but may have to be taken care of or may have to be examined because it may lead to steady state error. But poles in the left half plane, whether they are real lying on the negative real axis as it is called or in the left half of the complex plane, complex and lying, in the left half of the complex plane, there all okay and they would either cause oscillatory behavior which is damped or simply exponentially decaying behavior, more over one can find out the time constant of the exponential decay by knowing the locations of the poles.

So simply by looking at the pole 0 diagram, if these conditions are satisfied that is none of the poles is a pole of the input then, one can immediately figure out this information that some parts of the response will behave in such and such way or because some parts the response will behave in a bad way, the situation asks for some change. The system model will not remain valid and therefore, one may have to do something about the system to go further or to study it further.

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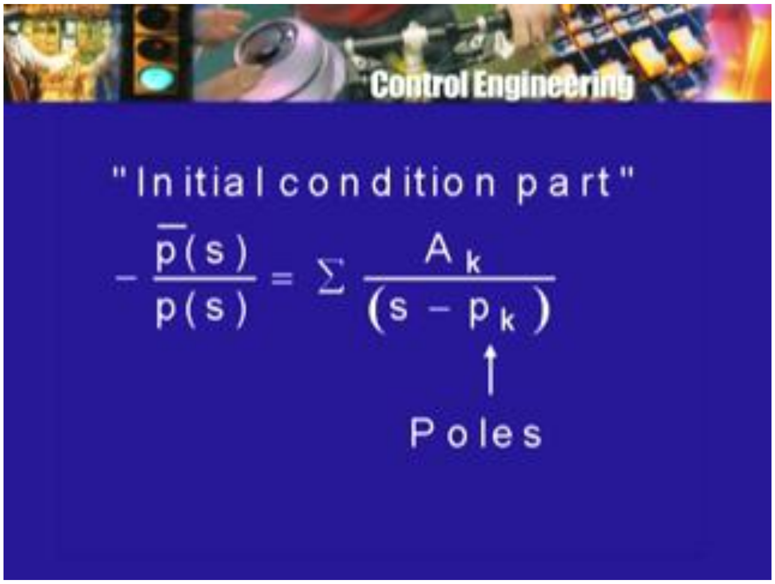

So this is where the knowledge of the transfer function can be useful and I am not looking at a particular input , I have not said that be this then the response will look like this. I am not looking at that part of it at all, I am looking at only a part of a part of a response that is determined by the poles of the system. But there is something else of what which is also we can say right away that this is going to be the case and that is the following there is one part of the response which look like minus p bar of s divided by p of s. Remember that p bar of s involved the initial condition of the response have the not input, input is q bar of s p s is the denominator of the transfer function, what about this part.

Now this part is such that the degree of the denominator is greater than the degree of the numerator and therefore, for this a partial fraction expansion is certainly possible and again if I assume that the factors are distinct then, partial fraction expansion will be like this and so I can say something about the Laplace inverse of this part that is again another part of the response and what is it that I can say exactly the same as I said earlier about the part coming from u s in to p q s by p s, mainly it will depend on the location of the pole of system p k, is it in the right half plane, is it on the imaginary axis, is it on the left half plane, is it on the real axis or it is out in the complex plane. Accordingly, there will be a component of the response which will be behave in such and such way either it is a bad behavior. So we have to do something about it as we saw doing something about it will involve usually feedback or making some changes in the system or we can say something about the response which is okay but we can say something specific like this is the time constant, this will be the frequency of oscillation and so on.

So there is something like that we can say. There is an interesting case that can occur here and that is the following. It may happen that a factor of p s for some specific choice of initial conditions is also a factor of p bar of s. So for example, in our transform function the denominator had s plus 2, s plus 3 and the numerator therefore was some polynomial minus p bar of s of degree 1.

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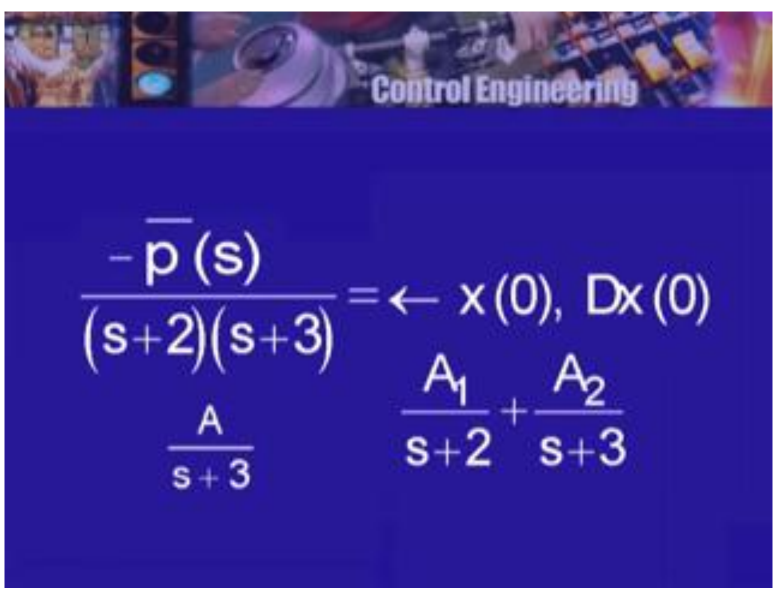

Now, I will leave it to you to figure out that one can choose values of D of x 0 and D x 0. These are the 2 values which determine P bar of s, one can find out a pair of values x 0 and d x 0 not 1 but many such that the numerator minus p bar of s is simply say s plus 2. So if that is s plus 2 then, what is that we are to do. Well, the answer is you can cancel that factor s plus 2 that has to be justified because I am looking at a transfer function, I am not just writing some algebraic expression in school algebra and then, cancelling a factor. Remember, these are all Laplace transforms of function, they not just algebraic expressions or they are functions of a complex variable s which arrives out of Laplace transformation.

So one has to take care of it, but one can justify the cancellation and so, if I cancel the common factor, the numerator will be of degree 1, the denominator is s plus 2 in to s plus 3. Suppose a numerator is just some multiple of s plus 2 then, what remains will be something like A divided by s plus 3, where as if this had not happened I would have had 2 terms like A 1 divided by s plus 2 plus A 2 divided by s plus 3, in place of that I have only one term A over s plus 3, what does this mean? In this case, in the case when there was no cancellation A to the minus 2 t with the pair in the response as also A to the minus 3 t.

So there will be 2 exponentials, so with 2 different time constants in the response, where as if there is a cancellation if the initial conditions are such that this happens then, one of the exponentials will not appear. So one say some times that the exponential will be suppressed and these exponentials are also referred to as the modes of mode of the system and they are in fact called is free modes of the system, why free modes because going back this is what will remain, when the input is 0.

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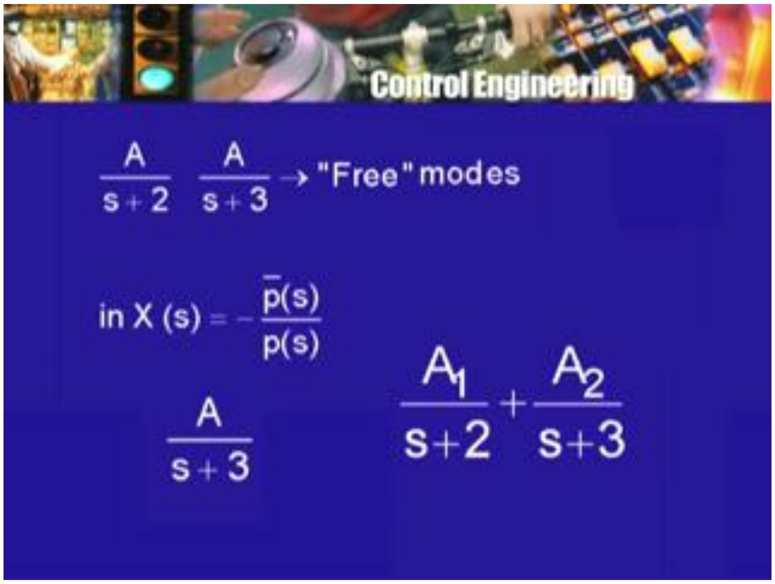

When the input is 0, x of is simply minus p bar of s divided by p s, is nothing else the input is 0. So the input terms have dropped out, it is only because of non-zero initial conditions that you have some initial response and therefore, such a response is called the free response or the response is 0 input and therefore, these terms like A to the minus 2 t, A to the minus 3 t which correspond to the poles of the transform function are called the free modes or some times they are called the natural modes of the system. The word natural is supposed to tell you that it is under 0 input.

So having a non-zero input is in some way is unnatural otherwise, this is nothing natural about it they are as much part of the a system for inputs and non-zero and 0 input. So some sometimes the free or natural modes of the response may get suppressed and that will depend on the particular values of the initial condition. So this is one thing that can happen.

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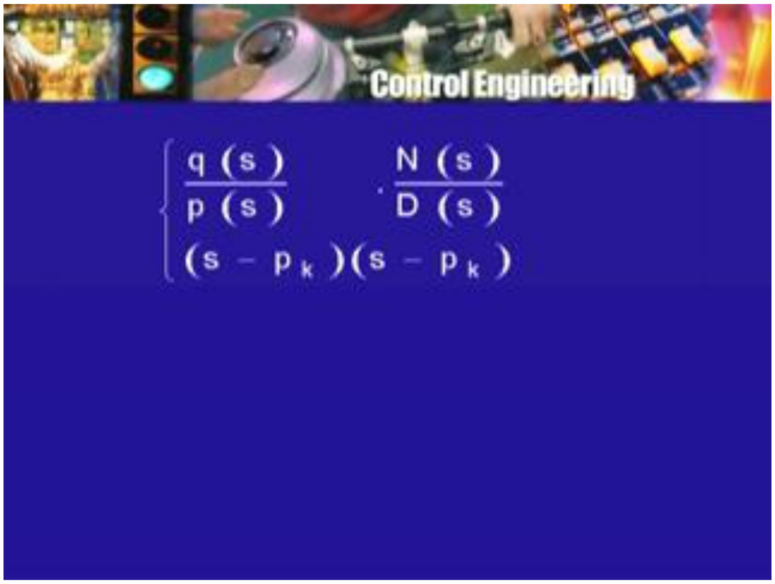

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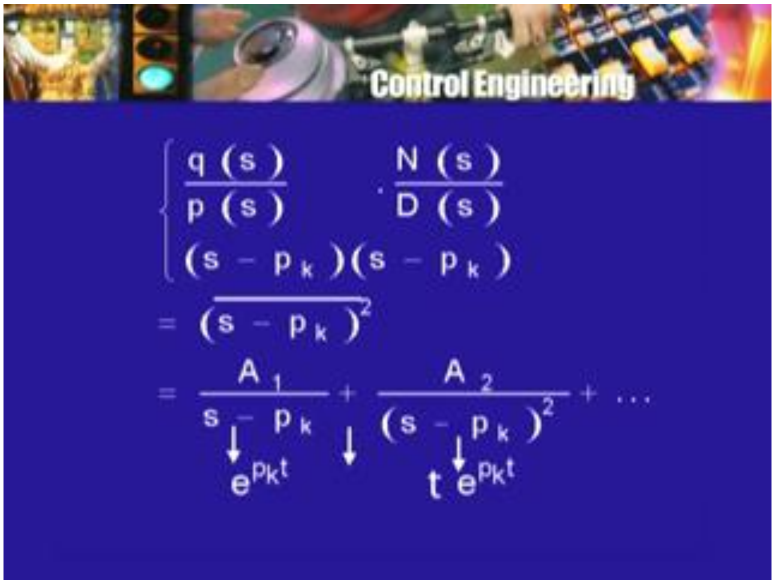

So let me say that I have q s divided by p s multiplied by N s divided by D s and this is only one part of the response that I am looking at. Suppose, P s has a factor s minus P k and Ds also has the same factor s minus P k then what ,that is a pole of the transfer function is also a pole of the rational function which is the Laplace transform of the input suppose, this applies for the particular input. For example, the input may be is simply e raised to P k t, if the input is e is raised to P k t, the Laplace transform of it is 1 divided by s minus P k and so, I will have the situation that a factor of P is also factor of D.

Let us say, the factor of P simple but it is also of factor of D. Now what now, when I do the partial fraction expansion I cannot consider the 2 factors separately. I have to take account of the fact that there is a factor here which is also a factor here and so, I should really think of the denominator as containing the term like s minus P k square and one can show from partial fraction expansion method that this will give rise to 2 terms not just A divided by s minus P k as we had earlier but also and let us call it therefore, A1 divided by s minus P k and the second term will be A 2 divided by s minus P k square.

So these will be the 2 partial fractions corresponding to a term like or a factor like s minus P k squared in the denominator of x of s. So in when a pole of the system is also pole of the transfer function, we will get factors like this, what is the result of this the Laplace inverse of the first term is simply e raised to P k t which is bad or good depending on P k is in the right half plane or in the left half plane etcetera. But the second part is the Laplace inverse of A 2 divided by s minus P k squared and what does it look like. Well, it looks like P in to exponential of P k t, so it is no longer just the exponential but t times the exponential.

So is it bad or good, well turns out that it is not really too bad, if P k is positive then this the growing exponential and this multiplication by t increases its growth further. So it makes it from bad to worse but if P k is negative that is the pole is real and on the negative side of the real axis then this multiplication by t does not influence things as t tends to infinity because it requires some work and you have perhaps calculated such limits in your calculus courses, if the limit of t in to e raised to minus 2 t, the limit of this as P t tends to infinity is fortunately 0.

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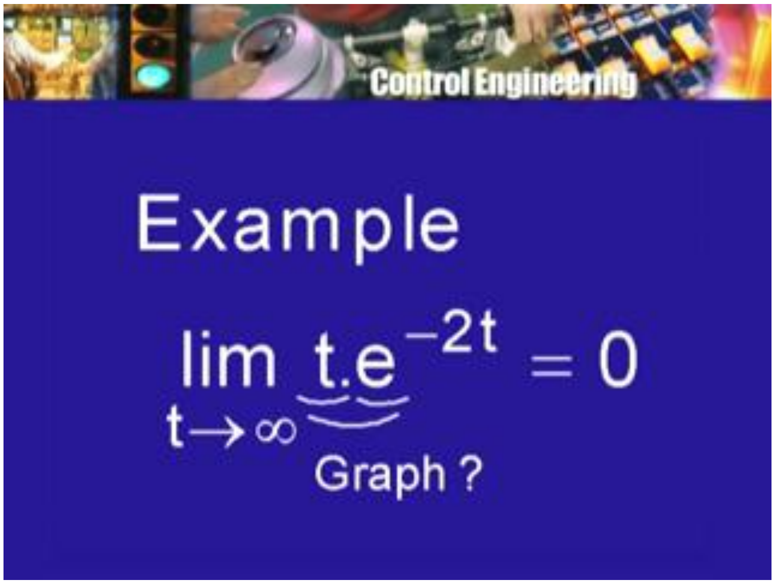

So this signal although one part of is D k and the other part part factor of it grows, the product goes to 0. However, it is not simply a pure exponential now, so I cannot talk about the time constant and this function has a particular graph which I had asked you to draw long time ago. So if you have not done it at that time, sketch the graph of t, e to the minus 2 t as a function of time for various values of t starting from say 0.5, 1, 1.5 up to may be 5 and then, see the trend you will see that the function does go to 0 but before it goes to 0, it can exhibit some interesting behavior, it can become large, e raised to minus 2 t is 1 and then, goes to 0 but this product can exceed one, not in this particular case but try to figure out, when it can exceed one.

However, as far as long time behavior is concerned or long term behavior is concerned or this steady state behavior is concerned, this is no problem. The t is swamped over by the exponential D k, so it does not create any problems. Similarly, when we have oscillatory behavior then the t will cause a problem, if there is no exponential damping but there is a problem already, if there is no exponential damping either oscillations or growing oscillations.

So t will only make it worse, so this is not a terribly thing bad thing to worry about a factor which is common to P s and D s, what if P s itself has a factor of multiplicity greater than 1 P s itself has s minus P k square. Well, what you have done will convince you that if the pole is good one then s minus P k square is, no problem, if the pole is bad 1 then, the s minus P k square just makes the problem from back to worse and so, the conclusion that we can draw is that on the pole 0 diagram, look at the poles of the system. If the poles of the system are all strictly in the left half plane then, there is no need to worry about this part of the response, things are all okay but if the poles lie on the imaginary axis including the origin or lie in the right half plane then, that is cause for worry. The system requires to be changed or some modification requires to be introduced may be some parameter needs to be changed or some feedback element, this to be introduced or feedback element is to be changed and so on.

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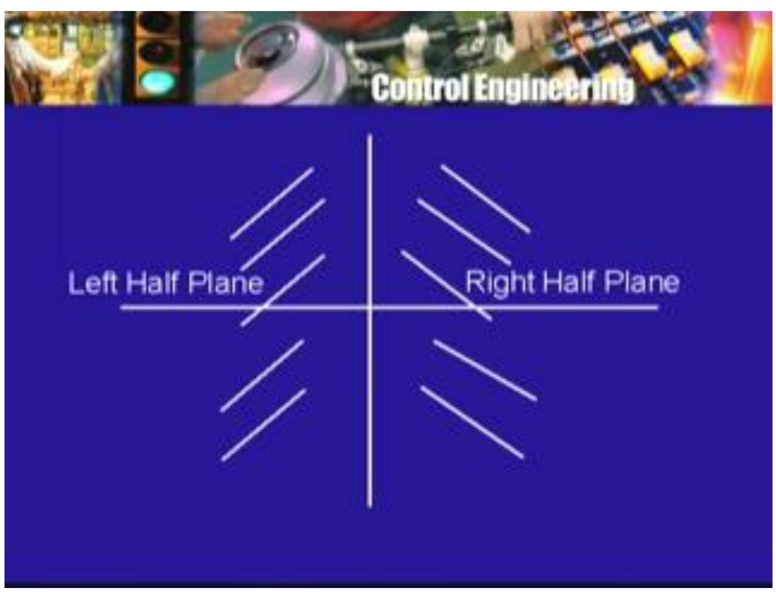

This phenomenon of things happening because of poles in the right half plane or on the j omega axis is vaguely referred to as some kind of instability and it is something connected with the boundedness or the response going to 0 or staying constant or being a sustained oscillation or growing indefinitely. This is what is connected with the poles of the transfer function. So just by knowing what the transfer function of the system is you are able to say something about at least one part of the response.

In fact, 2 parts of the response because there is a part of the response that involves the polynomial t bar of s that also goes with P s in the denominator and the part of the response that comes because of the inputs present, N s by D s q s by P s and in fact, there is one more part of the response which is the other polynomial, which was q bar of s divided by P s that also has terms contributed by the poles of P and therefore, the same thing will apply for that part of the response.

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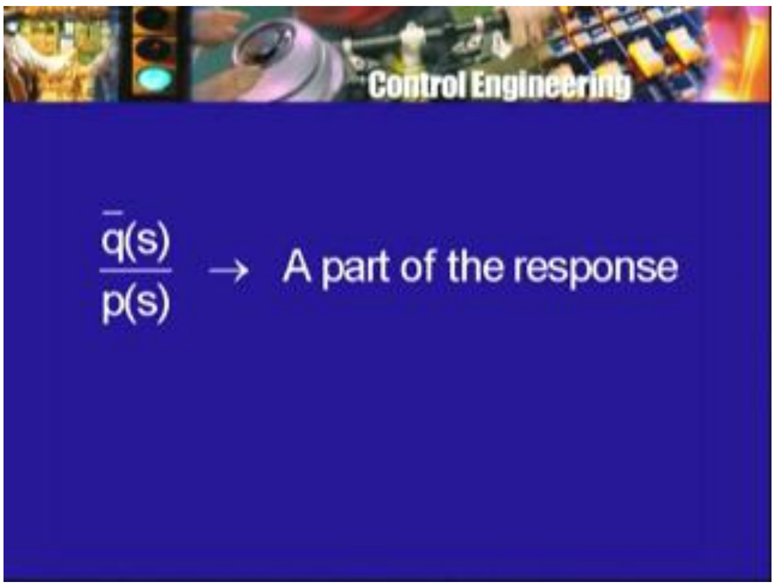

So poles of the system can you give a lot of information about the behavior of the system as time increases without limit that is after sufficiently long time or in the long term or particularly, in the steady state, if there is a steady state and therefore, the transfer function is a very important tool for this study of systems. We are looking at a part of the response mind you, I have told you time and again that I am only looking at a part of the response. To find the actual response, I need to know U of t, what is the actual input I need to calculate its Laplace transform U of s and then, do all these calculations, use partial fraction expansion and get an expression for the response as a function of time after inverse Laplace transformation. But, before I do all that I can find out from the poles of the system that this investigation is worth it or not worth it. If the poles are bad there is no need to go further, you need to change the poles, you need to change the system to introduce some changes.

So that the poles are no longer bad in other words we check whether the poles are bad and if they are bad then do not analyze any further change them to good poles and we will illustrate this a little later, when we will go back to feedback and how feedback can change the behavior of the system. In particular, feedback can change bad poles to good poles or the system was unstable in this sense after using feedback it becomes stable. Unfortunately, feedback can also do just the opposite feedback can change good poles to bad poles in some cases and therefore the system may be stable to start with but because of improper feedback, feedback which has not been correctly selected the system may become unstable.

So we cannot just use feedback blindly that is say that no matter what feedback is always good no, we have to watch out there also, how much feedback or what kind of feedback is very important to find out and go in to because it can convert or remove bad poles or change bad poles in to good poles but it could also change good poles in to bad poles and therefore, further analysis is required of the effect of feedback on the poles of the transfer function.

Now, I am going to look at our motor model once again and now, look try to apply all of this to the behavior of the motor or it is speed as a function of time, not in the steady state, it is not after sufficient time has elapsed, the motor is running at a constant speed and so on. But in the transient state what happens after the motor is turned on, switch is closed, the voltage is applied, the motor is initially at rest, it builds up its speed, in what does it build up its speed that is the transient behavior or transient analysis that we are going to do and in doing that we are going to use the idea of the transfer function and therefore, as homework before we actually look at it, let us look at the situation once again.

We had written down 2 equations for the motor speed control problem. Remember, them one was the armature circuit equation or voltage equations, the other was the torque of the mechanical equation. Now since I am going to use Laplace transformation or Laplace transforms, I will change the notation a little bit remember, that in Laplace transform work it is convenient to let the function of time be denoted by small letter like f of t or u of t for control people.

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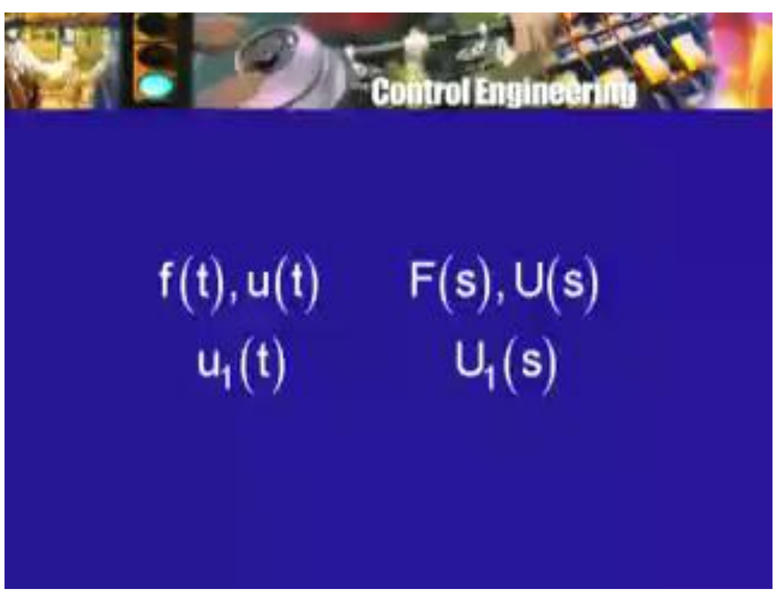

The corresponding Laplace transform is then denoted by the capital letter f t, u of s or if there is an input u 1 of t then, the corresponding transform will be capital U 1 of s. Of course, in the English script capital U and small u are some times difficult to distinguish in hand written script, so one should be careful but the presence of s tells you that I am talking about the Laplace transform and not the function of time. Fine, therefore I will change my notation, the applied voltage had been denoting earlier by capital E for EMF or voltage and armature a of t, this is what I had been writing.

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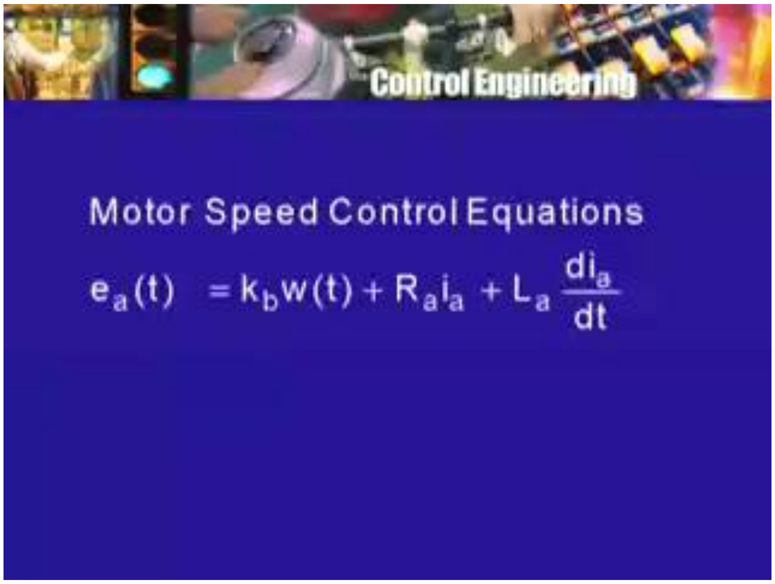

So I wrote E a of t equal to denominator 2 terms, back EMF in to omega t but I am not writing omega motor omega m. So understood that we have talking about the motor. So the applied voltage is the back EMF term plus the armature drop R a I a but that was only in the steady state and therefore, now I have to consider L a d I a dt. The only change that in the notation that I will now introduce is instead of capital E a of t on that left hand side I will write small e a of t. This is the applied voltage as the function of time may be the applied voltage is the just kept constant after the switching moment at 230 volts, may it is changed, may be it is increased, decreased or whatever it is will see all these things later on.

So small e a of t will denote the applied voltage that equal to the back EMF plus the armature drop plus the armature inductance term La d I a dt. So there is a derivative appearing here. Now, so the home work is apply the Laplace transformation to the 2 sides of this equation then from it getting the equation involving transforms and functions of s. So that is what will happen to the voltage equation then, we have the torque equation, what are the torque equation. The generated torque and now there is a conflict of the symbols here, I cannot resolve it so easily, I need a symbol for the torque, I had used capital T but then, if I use small t this small t is our preferred the symbol for the time.

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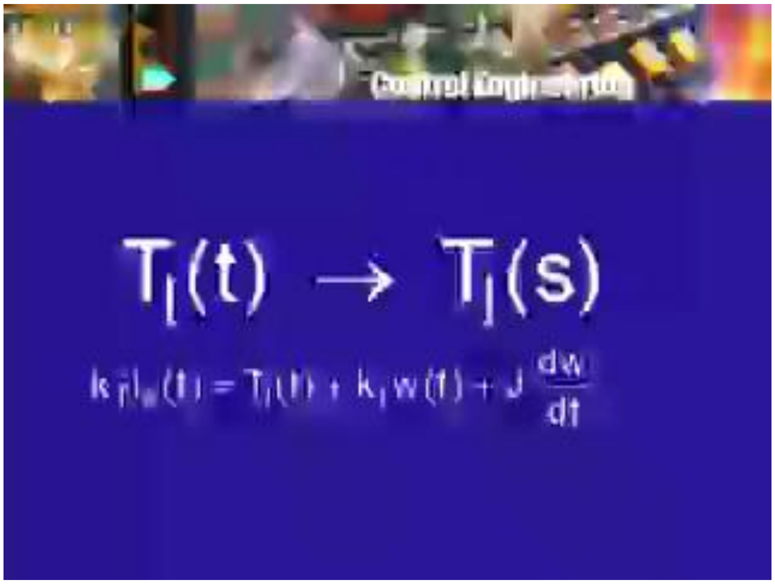

So what do I do, so let me stick with capital T with the understanding that if I write t of t or l of t then, this makes the torque as function of time there is if I write T l of s then, I am talking about its Laplace transform. Alternately, I can use some other letter other than t but it is good to have t reminding us of torque okay. So we have T l of t, no that is not the equation the equation starts with the generated torque, what was the generated torque K t in to I a, this is the torque produced by the motor because of the motion of the conductor carrying current in a magnetic field K t in to I a equal to, there are2 terms earlier, what are the 2 terms, one is the load torque of course T l of t plus the torque to work on friction which was k f in to omega but now, because the speed may be changing I have to consider the moment of inertia term and I told you the moment of inertia is usually denoted by j.

So I have j in to d omega dt. So now there is a derivative appearing in this equation also like the armature equation take the Laplace transform of the 2 sides of the equation or apply the Laplace transformation to the 2 sides and get any equation involving functions of s. Remember, there are derivatives here d omega t dt and d I a dt therefore there will be initial values which will appear. From these 2 equations then, try to solve, for what try to solve for omega s the capital will denote the Laplace transform of the speed as a function of time, omega s in terms of what, in terms may be T l of s and also E a of s. We have done this earlier for the steady state, the steady state speed in terms of the load torque and the applied voltage.

Now we will have to find out the Laplace transform of the speed in terms of the Laplace transforms of the torque function and of the applied voltage function and of course, there will be the initial values, what will be initial values initial value omega 0 of this motor. If the motor is starting from rest omega 0 is 0, if the motor was already running, it will be the that value and what is the other one, the armature current I a 0. Once again, if the motor was running the initial value of the armature current will not be 0 but if the motor was at rest then, the initial value the armature current is find out whether it will be 0 or not.

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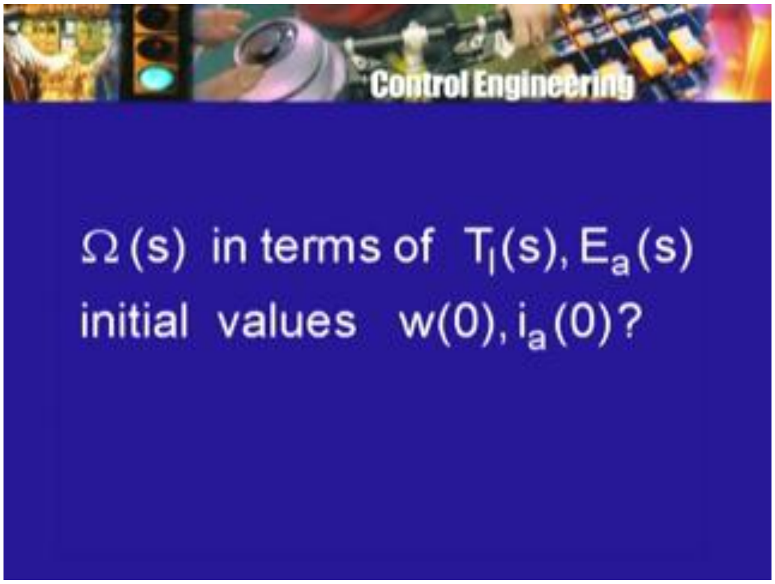

So these 2 will appear in the equation also you find the expression for omega s in terms of load torque transform, applied voltage transform and the initial values of the speed and of the armature current, this will be your home work.