Control Engineering
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Lecture - 24

I have shown you, how the Laplace transformation can be used to solve a linear ordinary differential equation with constant coefficient. Of course, I only took a simple example of a first order differential equation. We are not going to solve differential equations using the Laplace transformation. So I will not take more examples or higher order examples but I hope the idea is clear to go over it once again.

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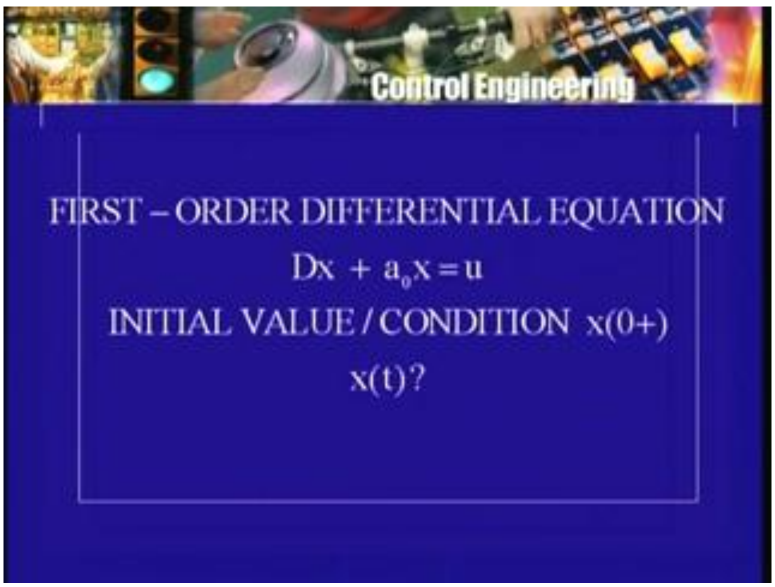

Suppose, we have a differential equation of first order D x plus a 0 x equal to u, u is the input, x is the response, u will be specified as a function of time. The initial value x 0 or x 0 plus will also be specified and the problem is find out x as a function of time x of t. As we saw, if you apply the Laplace transformation to this equation then, from that we got an expression for capital X of s the Laplace transform of the desired response function x of t, just going through it very quickly D x on transformation gives you S x of S minus x 0 plus plus a 0 x of S equal to U of S and therefore, x of S if you remember, was the sum of 2 terms, one for them involve the initial condition x 0 plus divided by S plus a 0 and the other term was 1 over S plus a 0 multiplying U of S.

Now, if you wanted the solution of the differential equation then, of course it is necessary to know what U as a function of time is to know what it Laplace transform US is to be known or to be given also the initial value x 0 and then, do the Laplace inversion of the right hand side here and we saw that this part or this term corresponds to the 0 input response and the second term corresponds to the 0 state response. So if you want to find that the solution X, explicitly as a function of time one will have to do all this, know what U is calculate U of S and then, work out the Laplace inverses of these 2 parts that will give you the expression for x t.

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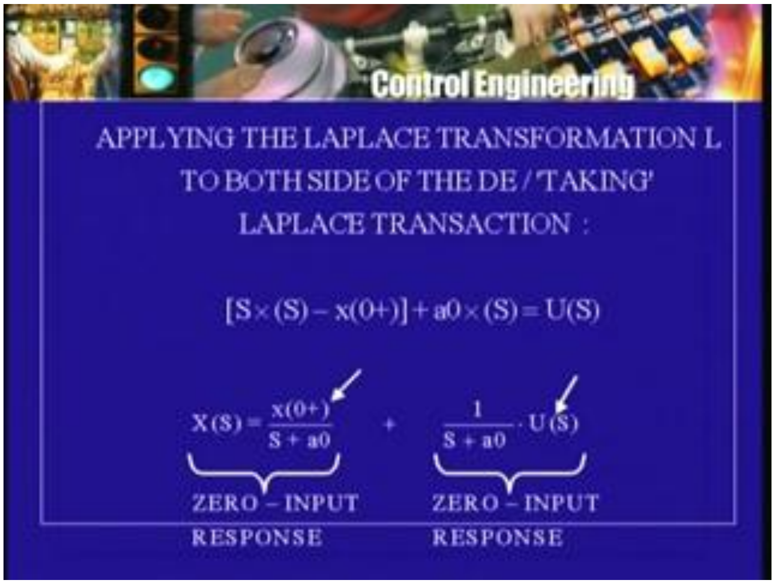

Now as I said, we are not basically interested in solving differential equations but in trying to find out something about their behavior that is how does the response vary for a given input or if I change the input, in what way will the response change and things like that. Now, as we see here the response is already in 2 parts or rather the response is a sum of 2 parts, basically of course, these 2 parts are not separate. You start the system in some non-zero initial condition and there after apply an input and you get the total response it does not come into 2 parts separately.

So what you can observe is only x of t but in theory or when, we do the analyses of it, x of t consists of 2 parts, one is the 0 input response and other is the 0 state response. Now therefore, in what we are going to do very soon we will keep these 2 terms separate. In fact, for some time we will keep aside the 0 input response part of it. In other words, alternately we can say that we are assuming that the initial condition is 0. Now in fact that is not a good way of looking at it because the initial condition may not be 0. For example, suppose the motor has been running for some time and when I start looking at it that is my t equal to 0, I suddenly change applied voltage armature voltage or I suddenly change load torque or some other change perhaps, how will the speed change and behave form that movement onwards.

So the motor may be already running, so if x is my omega the speed angular speed of the motor then, x 0 is not going to be 0 but it will be some non-zero value. So rather than say that we will assume that the initial conditions are 0, it is better to say that we are only going to look at the 0 state response part of the total response. In other words, we are only going to look at a part of the response, we will have to the look at the other part also to get the actual response of the total response but for further moment, we will only look at the 0 state response.

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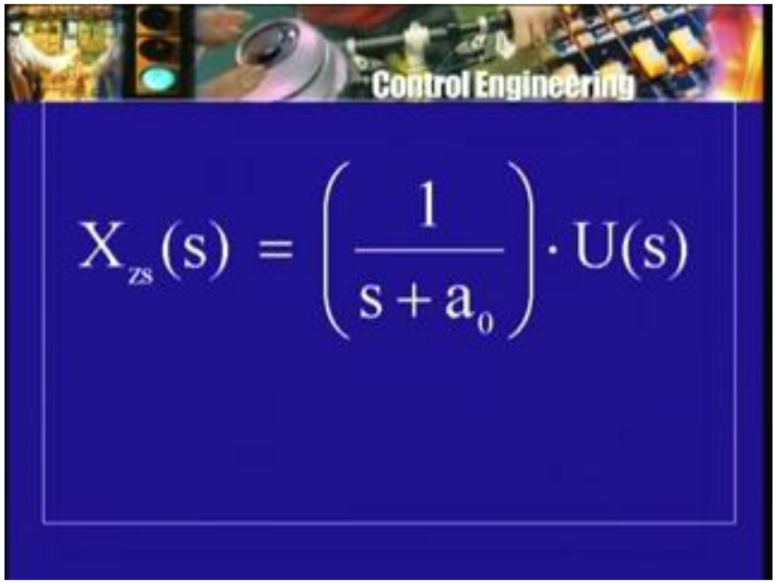

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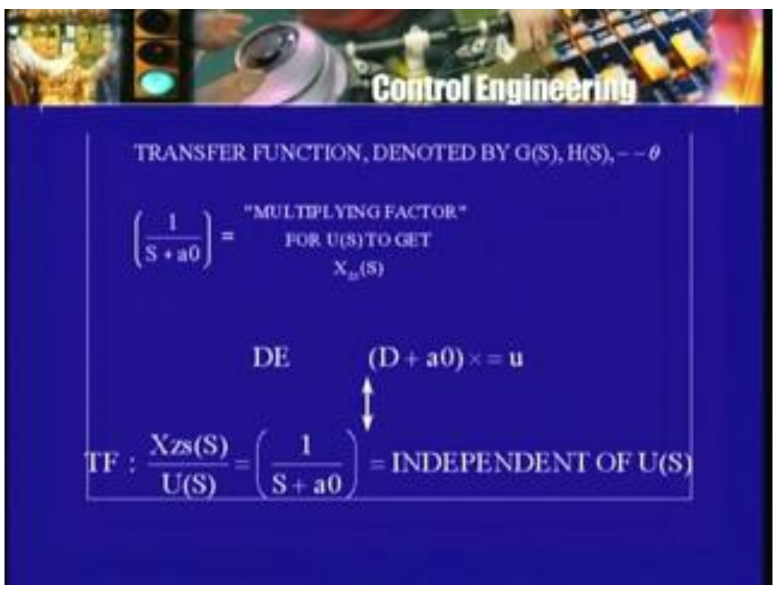

So, if we do that so I can write X, capital X with subscript 0 state. So X 0 state as a function of s is simply 1 divided by s plus a 0 into U of s. Now this is only for a first order differential equation but we will see in that for second order or higher order differential equations also, there will be something which will be like this and therefore, proceed further and introduce a new concept namely that of a transfer function. I have already mentioned the transfer function as something which may appear in the block diagram and in the control system, feedback control system block diagram, you will come across transfer functions G of S and H of S. These are the usually chosen symbols for particular transfer functions in the forward path and the feedback path.

So, I have already used this word transfer function but I have not defined it or told you what exactly it is. Now is the time to define it. So look at X, z s, the 0 state response but it is transform Laplace transform. On the right hand side we have the input or rather the Laplace transform of the input and the fact is that the 0 state response transform is 1 divided by s plus a 0 multiplied by or multiplying U S or in another words, the 0 state response transform is the input transform multiplied by some expression.

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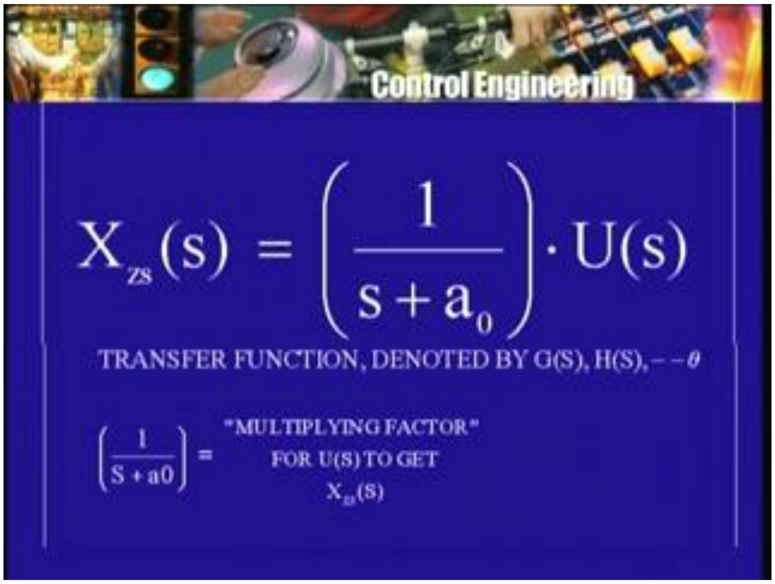

Now, that expression in this case is 1 by s plus a 0, does it depend on what U t is, does it depend on what input we have choosen, no because what appears here is the only transform of the input, the transform of the 0 state response and this is the relationship between the 2. So this 1 by s plus a 0 is a multiplying factor which does not depend on what input function, we going to choose. We could choose a constant input 1, we could choose a ramp, we could choose an exponentially decaying input, we could choose sinusoidal input, anything which is Laplace transformable for which the Laplace transform make sense. We can choose that as U t, no matter what U t we choose, the relationship between X, z s, s for that input and that input transform will be by this factor multiplying back factor 1 by s plus a 0.

So this 1 by s plus a 0 is set to be the transfer function corresponding to the first order differential equation I wrote and I will rewrite it, little differently as D plus a 0 X equal to u, the differential operator D plus a 0 acting on X gives the input U. This is the differential equation, correspondingly the Laplace transform relationship is Laplace transform of not the response but the 0, state response part of it equal to this function 1 by s plus a 0 multiplying U s.

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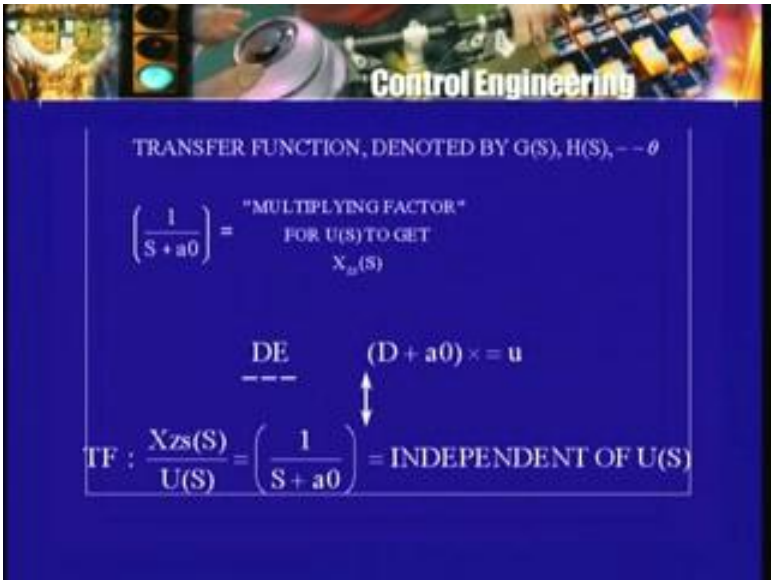

I can also rewrite it as X, z s, 0 state response Laplace transform divided by U of S equals 1 by S plus a 0 and this is how sometimes a transfer function is define. A transfer function of a system is defined as the ratio of the Laplace transform of the response to the Laplace transform of the input. In many text books, you will find the definition in this way, the transfer function of a system is the ratio of the Laplace transform of the response to the Laplace transform of the input that produces that response.

Now, that definition is not quite correct for several reasons. First of all, it is not the Laplace transform of the response that I am looking at I am looking a part of it, a part of it which does not involve the initial conditions but which only involves the input. So this not the whole response, there is the other part which involve the initial conditions and the initial conditions may not be 0 in a particular situation. So cannot ignore them, we cannot say put the initial conditions equal to 0 because we may have to study the problem, when the initial condition is not 0.

So that is one thing it is only a part of the response whose transform we are taking and dividing it by the Laplace transform of the input and therefore, we get 1 by s plus a 0 as the answer. Second thing is for general system not necessarily our induction or dc motor or you know, the tachogenerator, feedback, etcetera. For which the differential equations were linear, for a general system of course I can apply some input, I can observe the response, unfortunately the observed response, we do not know how much of it is due to the initial conditions be non-zero and how much of it is due to the input.

So in general, we cannot from the observed records separate out the 0 state response, as it is it, is only for linear differential equation with constant coefficients that the response can be separated into 2 parts, one part involve only the initial conditions the other part involving only the input. Secondly, for a general system not necessarily linear etcetera. Of course, I can apply an input, I can observer the response and I can even compute the Laplace transforms of the 2, Laplace transforms of the response, Laplace transform the input. For example, the input may be e rest to t

80, no problem Laplace transform is 1 by s plus a. The response I may recognized as a familiar function calculated to the Laplace transform and so, I can write this ratio the actual response transform whatever, it is as a function of s divided by the actual input transform. But the trouble is that will not be something like here 1 by s plus a 0, no matter what the input is, the ratio is 1 by s plus a 0.

Now for a general model, this is not going to happen the ratio will be different for different inputs and therefore, the concept of transfer function does not make any sense or does not have any use. When I can take something and divide it by something else, what use is it, if the result is dependent on what I have chosen as 1 end, what I have chosen choosen as the other thing. So the concept of transfer function is really useful only for systems which are described by ordinary linear differential equation with constant coefficients. No wonder therefore, that directly the transfer function idea is not useful for nonlinear system or for time varying system, linear differential equations with constant coefficients, linear.

So the transfer function idea is not directly useful for a non-linear system. Although one tries to extend the idea and therefore one can do something. In fact, there is a method called describing function approach for studying non-linear systems which tries to extend the transfer function idea to non-linear system. But it is not exactly the same concept, it is a deferent concept describing function that is why it is called the describing function and not a transfer function. So linear, second with constant coefficients or what are called time invariant systems, if the system is not with constant coefficients or in the time varying then, again the idea of a transfer function will not be useful.

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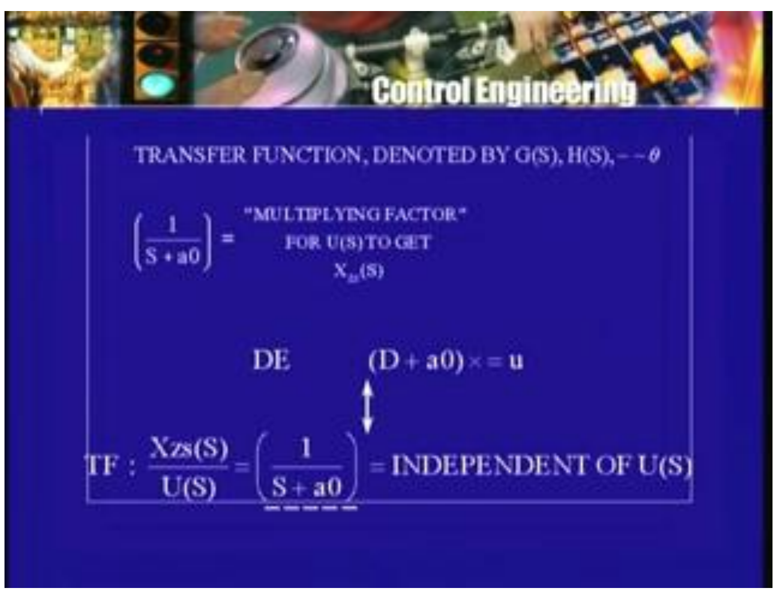

Again, in that case one can try to push it through but then some different concept, we will have to be applied. In our course, we are not going to look at time varying system models or non-linear system models. So this will not be of any use to us right now. Now as you can notice the system differential equation, I wrote in this operator from D plus a 0 operating on x, the response the equal to the input U, the transfer function I written in the form 1 by s plus a 0 or alternately, X 0 state S equals 1 by s plus a 0 into U S and I suppose, you can notice some connection between the operator and the transfer function. In the operator, I have D plus a 0 in the transfer function I have s plus a 0 of course, have it in the denominator.

So may be from the differential equation itself, I could directly write down the transfer function from D plus a 0, I go directly to the transfer function as 1 over S plus a 0. Now this is correct of course, we have to prove that it is correct. So we can take a second order differential equation and see, what happens in that case and I will take a second ordered differential equation in which on the right hand I just do not have the input but I have a derivative of the input of the also.

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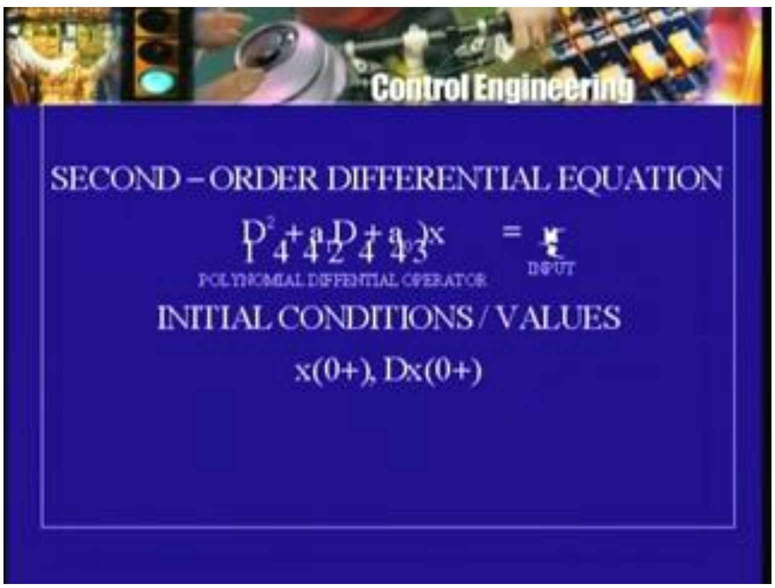

So, first let us take a second order differential equation, so D square plus remember the notation a 1 D plus a 0 operating on x equal to U, this is a second order operator, input is U, response is x. If I have to solve this I will have to know what U t is, calculate its Laplace transform U S, what about initial conditions, I will have to know 2 initial conditions x 0 plus and D x 0 plus, the value of x at 0 limit form the right and value of the of a derivative of x at 0 are rather the limit from the right, knowing these 2, knowing U t if I can compute U of S, I can calculate x of s and then, if I know Laplace inversion techniques I can calculate x of t. Let us see, what x of S will look like.

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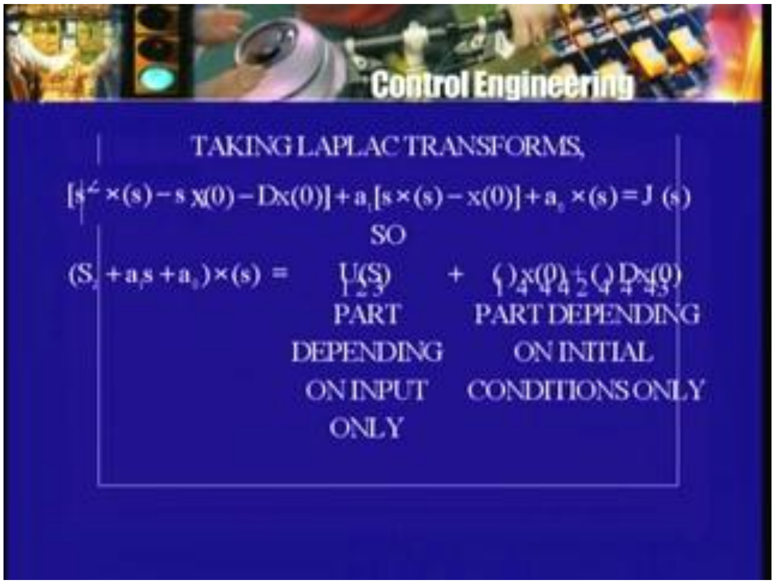

Here, we have D square acting on X, so that if we remember by the derivative formula. For a second order derivative will be S squared x of s minus s x 0 minus D x 0. So this bracketed expression is the Laplace transform of D square x its value at s, the second term is a times the Laplace transform of D x is s x s minus x 0, the third term is simply a 0 x. So the Laplace transform is a 0 x s equal to U s. So if I take the terms involving only x of s, keep them on the left hand side, transfer the other terms involving the initial conditions to the right hand side, what will I get, I will get x s multiplied by s squared x s multiplied by a 1 s and x of s multiplied by a 0.

So, I will have s squared plus a 1 s plus a 0 multiplying x of s equal to what do I get on the right hand side, let me write down U of s first plus x 0, D x 0 terms transferred on the right hand side. So I will write here as some multiplying expression for x 0 and some other multiplying expression for D x 0. So x s is a the response transform it obeys this equation. Now like, in the first order case you can see that there are 2 parts here, one part only involves the input , the other part involves the initial condition or conditions in this case x 0 and D x 0.

So, if I am only going to look that the 0 state response of the response under initial conditions 0 or strictly speaking, if I am going to look at the part of the response that only depends on the input then, what will I get? I will have x 0 state of s divided by U of s equals 1 divided by S squared plus a 1 s plus a 0. So I am only looking at part of the response which depends on the input and therefore, it is what will remain if the initial state was 0 that is why it is normally called the 0 state response divided by the input transform equal to this ratio, 1 divided by S square plus a 1 S plus a 0 and now, one can immediately see that S squared plus a 1 S plus a 0 occurs here in the denominator, whereas the polynomial differential operator is D square plus a 1 D plus a 0.

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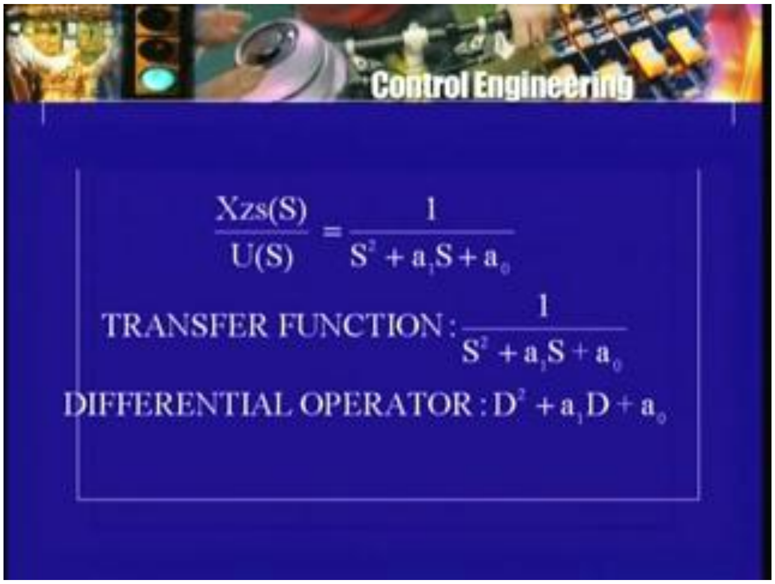

So, simply replaced D by S, I get the denominator in the transfer function hopefully, this will happen for higher order differential equations and you can actually show for yourself that it will happen. So you can try this as an exercise, write down third order differential equation, write it as D cube plus a 2 D square plus a 1 D plus a 0 acting on x equal to U. Take the Laplace transformation or the Laplace transform of both the sides, use the derivative formula appropriately, write down an exasperation for x of S multiplying some multiplied by something equal to a sum of a number of terms, pick the term that only involves the input and get the transfer function.

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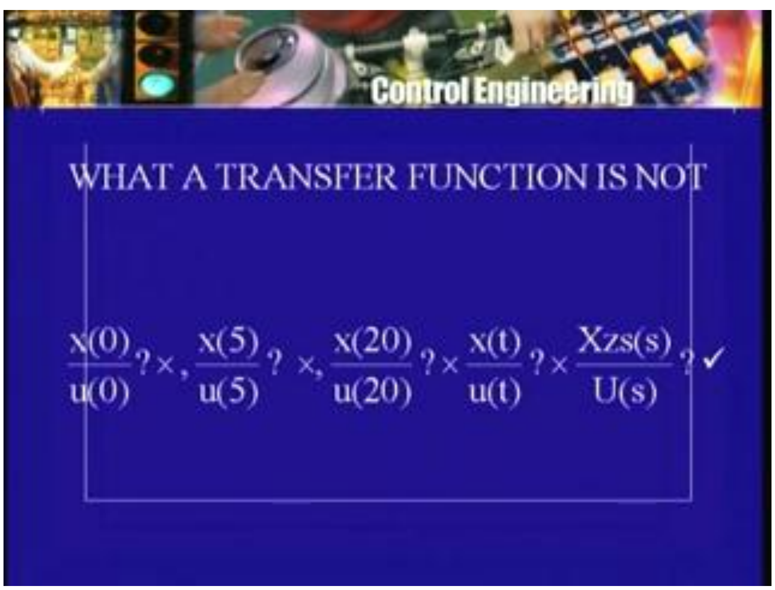

We will find that this rule replace D by S is indeed correct in this context, let be mentioned that transfer function idea. As some students mistakenly remember it, is not the ratio of the output value at any given instant of time to the input value at the same instant of time. So it is not a ratio of the output value at say 0 second or 5 second or 20 seconds to or which, the input value at 0 or 5 or 20 seconds.

So the transfer functions is not X 0 divided by u 0 or x 5 divided by u 5 or x 20 divided u 20, not even the 0 state response part of the response, no this is not true. The transfer function is a ratio of transforms of Laplace transforms, it is not a ratio of output divided by input, very often students will write in the examination paper or when I conduct interviews and I ask, what is a transfer function? This is usually the first response, it is the ratio of output to input , student does not bother to think, what he is saying, it is not what he means is or what he should mean is and therefore, we should say that the transfer function is the ratio of the Laplace transform of the response.

In fact, not that that is not quite true the Laplace transform of a part of the response that depends only on the input or that remains when the initial conditions are put equal to 0 to the Laplace transform of the input and secondly, which also the number that this concept is useful only for linear time invariant system models or those which are described by ordinary linear differential equations with constant coefficients.

So, keep this in mind, transfer function is not the same thing as gain output is 20 volts, input is 1 millivolt therefore, the gain is output voltage divided by input voltage, 20,000 or whatever is not. The transfer function is not gain in this sense, it is not output voltage divided by input voltage, the transfer function would be Laplace transform of the output voltage divide by the Laplace transform of the input voltage and the Laplace transform is not the same thing as the value of the function, 5 volts is deferent from Laplace transform of a signal which has the value 5 volts at some particular moment of time, The two are quite different.

So do not do that mistake transfer function is a ratio of Laplace transform, it is not a ratio of output to input. Now, when the differential equation was simple like this the first order differential equation or the second order differential equation, where there is only U term on the right hand side. The transfer function, I told you could be written down immediately without going through this business of applying the Laplace transformation to the 2 sides, what if the differential equation is a little more complicated.

Let us take for example, the second order differential equation D square plus equation a 1 D plus a 0 operating on x but that is no longer equal to the input U for some region, it is not U but it is a combination of a derivative of U and U. For example, it may be now here I am using a notation which is actual some books use it, others do not I will write here b 1 du dt plus b 0 u that is the right hand side is not just u but it is a multiple of u added to a multiple of the derivative of u and of course, I do not want to write D on the left hand side and D by dt on the right hand side.

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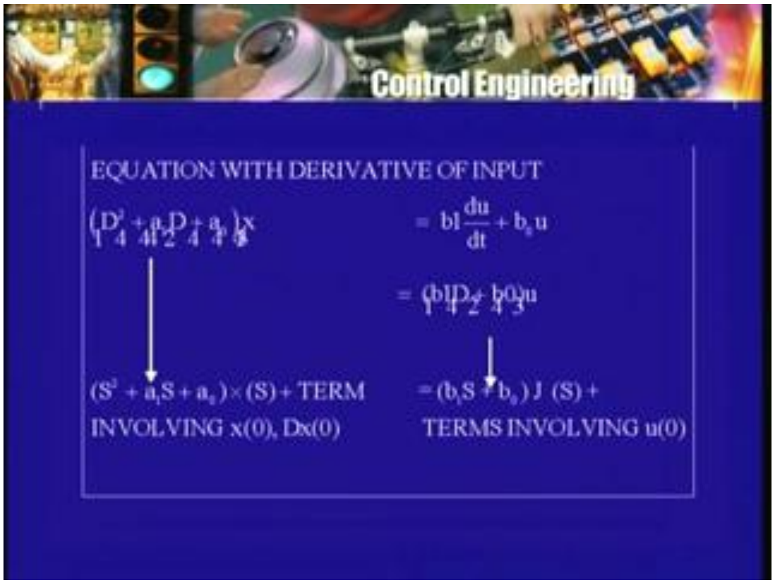

So I will rewrite the right hand side as a polynomial differential operator operating on u that is very easy, I will simply write b 1 D plus b 0 operating on u. Now that is a second order differential equation in x but it involves first order derivatives of u also, first order derivatives u also. It is a little different from the one, where we have just u on the right hand side. However, we can still apply the Laplace transformation to both sides and what will we get on the left hand side I have D squire plus a 1 D plus a 0 x.

So, applying the Laplace transforms to that I will get as I did earlier s squared plus a1 s plus a 0 multiplying x of s plus I will write terms involving, the initial conditions x 0 and D x 0, that is what I will get on the left hand side. When I take the Laplace transform of the left hand side or the apply Laplace transformation to the left hand side, which is the second order differential operator, polynomial differential operator, operating on x, what about of the Laplace transform of the right hand side. The right hand side is b 1 D plus b 0, operating on u and using the derivative formula, it will turn out to be b 1 s plus b 0 into U of S but just as there are the initial values of x on the left hand side because there is a derivative of U on the right hand side, their will be an initial value of U on the right hand side.

So plus term involving U 0, in this case. So that is the full equation for x of s and therefore S square plus a 1 S plus a 0 multiplying x of S equals 3 terms, one is b 1 S plus b 0 into U of S this involves the input, input transform plus terms involving x 0, D x 0, the initial values of the response plus a third part is terms or in this case only one term involving the initial value of U, U 0.

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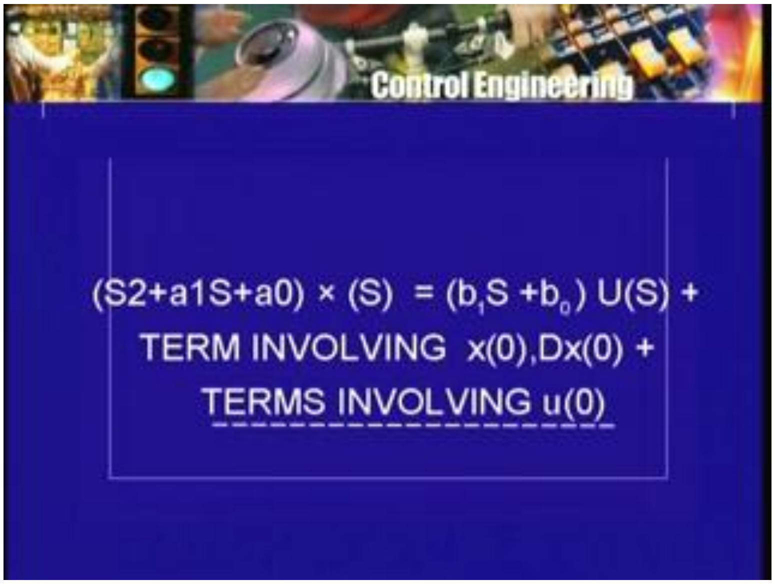

Now, as we did earlier we will only look at this part which involves the transform of the input, it is not that the other parts are 0, they may not be for example u 0, may not be 0, if it is a constant function 1 then, u 0 is 1, if it is the function e to the minus t u 0 is 1, if the function cosine t u 0 is 1. So I cannot say u 0 equal to 0 because it is not 0 similarly, x 0 may not be 0 I cannot say I will ignore it as I told you earlier, I may be I am looking at a problem when the motor is already running and at t equal to 0, I make some changes.

So we cannot say x 0 equal to 0 and I cannot solve the problem, if x 0 is not 0 that is not true we can solve the problem because we can find out what the terms involving x 0, D x 0 and u 0 are but if, we choose to look at or if, we restrict as says to only this term then, the other terms are not being considered right. Now not that they are being neglected or they are can neglected of they can be ignored, many books give you the impression that the other terms do not the matter.

Now that is not true, we will see later on that these 2 terms matter. They cannot be ignored they are not negligible it is only that we chose to look at 1 term at the moment, you cannot look at the whole world at any 1 movement of time. So, we will look at this first then we look at this then, we see if both are present what is the effect and so on that is what we will do. So, if we chose to look at the term that involves only the transform of the input then this is the term remains and now, I have X s multiplied by something equal to US multiplied by something and therefore, I can write the ratio, for this part of X there is no name, for it this is the part that involves only the transform of the input does not involve initial conditions of either the response or the input divided by U of S equals in this case, now in the numerator I do not have1 but I have b 1 s plus b 0 divided by S squared plus a 1 S plus a 0.

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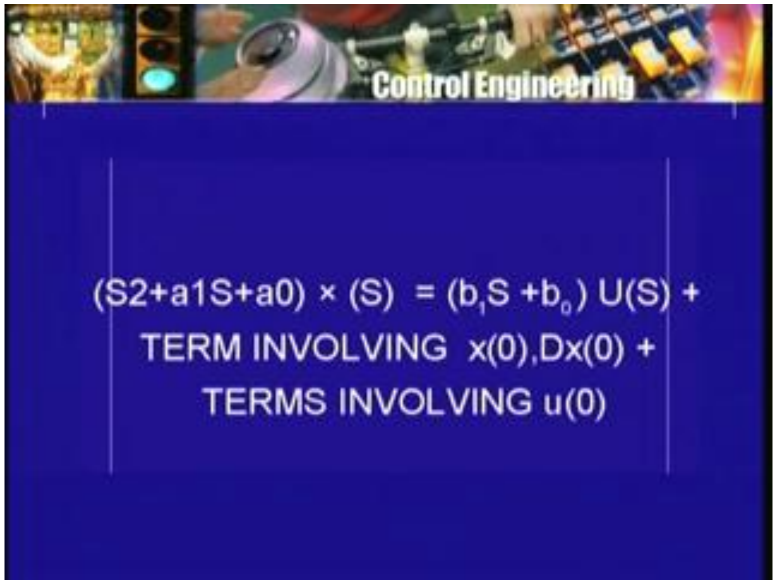

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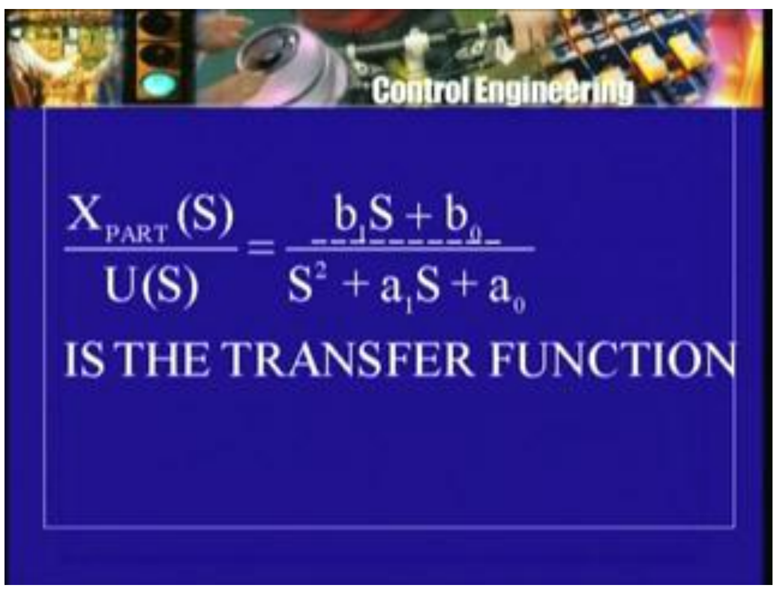

Now look at the differential equation, could I have obtained this transfer function form the differential equation. In the differential equation have an operator acting on x on a left hand side, I have operator acting on u on the right hand side and look the operator which was acting on u, the corresponding S thing appears in the numerator of the transfer function, whereas the operator that was acting D, D replace by S appears in the denominator of the transfer function and so, we can go ahead and formulate a rule for a writing down the transfer function for a differential equation which need not be of second order, second order derivative of x, which may have higher order derivatives of u also and so, I will write this equation in the following symbolic form, p D, where p D will stand for a polynomial differential operator.

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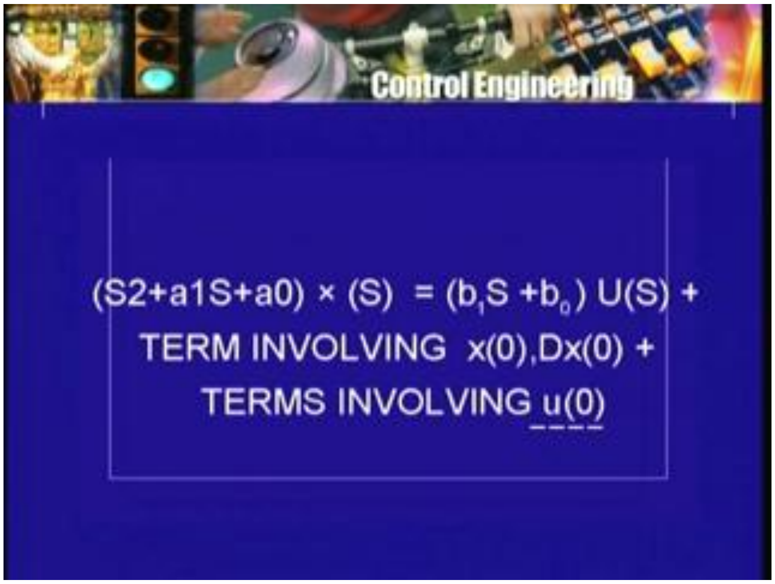

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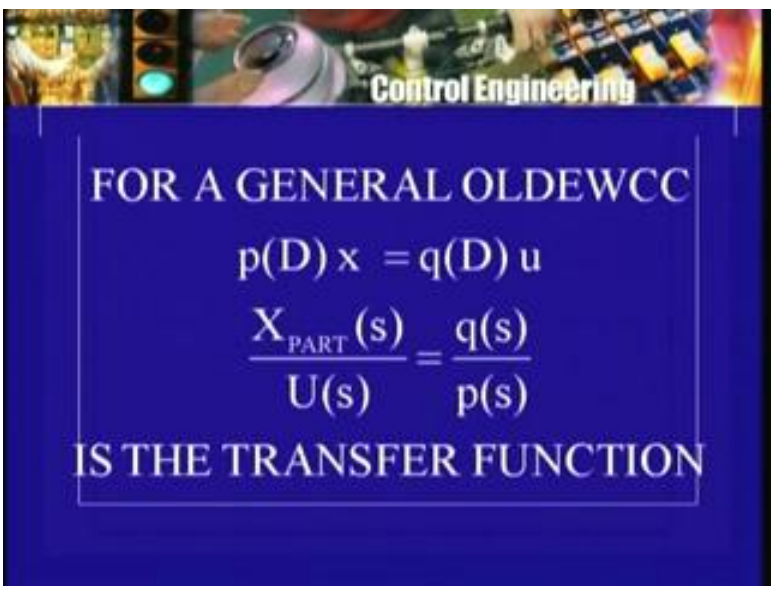

It may be of degree 1, it may be of degrees 2 that is differential equation as of first order, second order, it may be of higher degree, it may be of degree 6, p D acting on x, x is the response equal to q D, q is another polynomial differential operator acting on the input u, if this is my differential equation model for my system then, if I look at not X of s all of it but a part of it that only depends on the input transform then, the ratio of that transform of that to the transform of the input will be equal to q s divided by p s or in other words, the transfer function corresponding to this differential equation will be q s divided by p s.

We have to remember that the multiplier for u q D is corresponding q s is the numerator, the multiplier or the operator acting on x, the corresponding p D becomes ps in the denominator of the transfer function, do not interchange the numerator and the denominator, in a hurry sometimes students make this mistake. But there is no reason to make this mistake, you can think of applying the Laplace transformation and almost see that okay while x square D square x rather on the left hand side, I will have X square x of s plus some stop.

So that D is replaced by s but that multiplies x of s and therefore, in it will appear in the denominator when I write the transfer function, where as if there is a derivative of U there will be a corresponding s thing but that is on the right hand side therefore, when I write it as a x divided by U, it will be in the numerator. So the polynomial differential operator which operates on the input is in the numerator and the polynomial differential operator which operates on the response or output is in the denominator of this transfer function and the transfer function looks like a ratio q s by p s and q s and p s are actually 2 polynomials.

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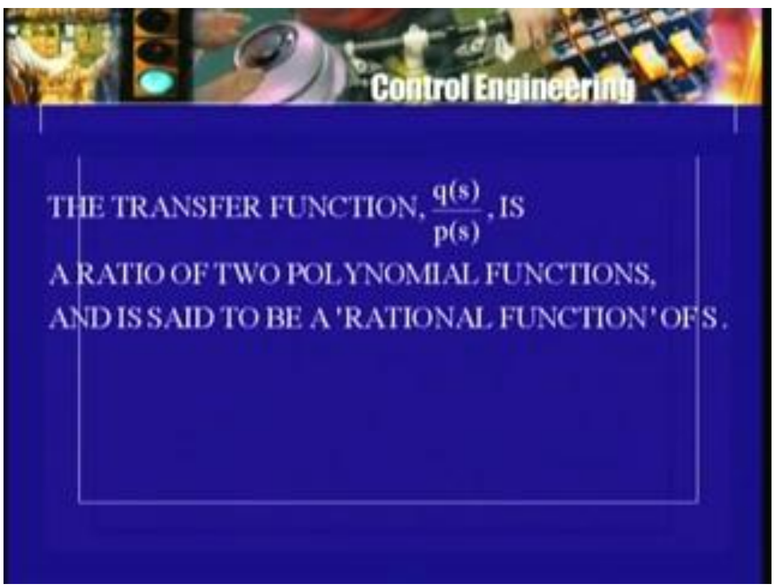

So such a function is called a rational function, in the theory of functions of a complex variable such functions are known as rational functions. They are ratios of 2 polynomials as an example such a rational function may be the numerator is S plus 1 and the denominator is S square plus 5S plus 6, if I ask you, well what kind of differential equation or what differential equation could be for which you obtain this transfer function.

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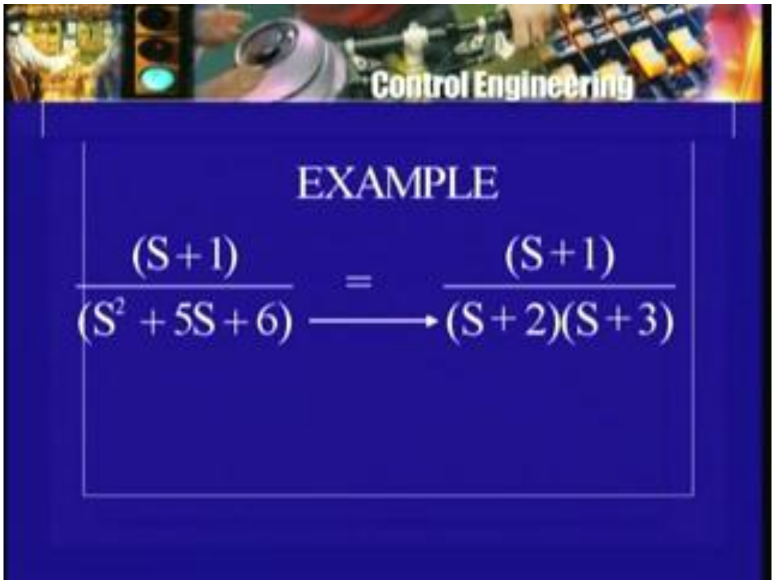

So inverse it, given the differential equation, find out the transfer function with little practice, you will be able do it without any delay, simply replace D by s kind of thing appropriately. You will get the transform now the inverse question given the transformation to what differential equation it may correspond that is for, what differential equation could it be the transformer and the answer is not difficult to the opposite replace S by D only use the operators properly something operates on u, something else operates on x,

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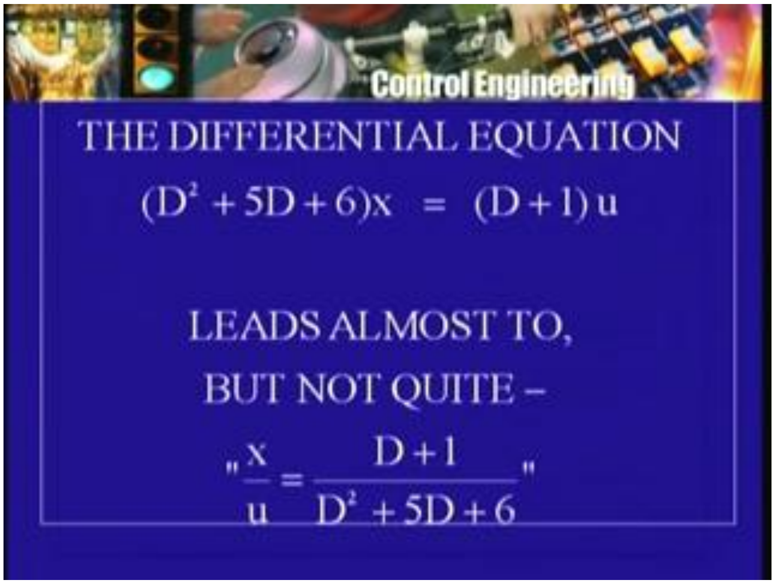

So in this case, what will be the differential equation S plus 1 is in the numerator. So D plus 1 operating on u, S square plus 5 S plus 6 is in the denominator. So S square plus 5 S plus 6 operating on x and therefore, the differential equation will be D square plus 5 D plus 6 operating on x equals D plus 1 operating on u. If this started with this differential equation by applying the Laplace transformation and by choosing to look at only a part of the response u will obtained precisely this transfer function. So it is not difficult to go back from the transfer function to the differential equation is as simple as that.

Remember, that we are not looking at the response, we are only looking at a part of the response, so part of the response that depends only on the input transform, it does not involve the terms or it does not look at the terms that involve the initial conditions or initial values of the response nor the initial conditions or initial values of the input. It is only looking at 1 term which involves the input transform and so happens that that part divided by u s is something, which does not depend on the input transform not on the input initial conditions, not on the response initial conditions. It only depends on the operators that involved in the description of the system, if this is the differential equation for the system then, for this system, this is the transfer function.

Now of course, there is a way of so to getting this which is not really completely justified and that is to rewrite this equation as x divided by u equal to D plus 1 divided by D square plus 5 D plus 6. So I will put it in inverted comma, why because first of all as I told you the transfer function is not x t divided by u t at t equal to 0 or 5 or 20 or 10 is to 3 seconds or whatever, it is not a ratio of output or response value to input value, no it is the ratio of Laplace transform.

So on the left hand side x by u makes, no sense I can of course take x t divided by u t but that is not the transfer function. But I am committing another crime, on the right hand side I have writing D plus 1 divided by D square plus 5 D plus 6. Now I know what D plus 1, is it is an operator acting on you, does something or the other operator acting on x does something to it but what is this D plus 1 divided by D square plus 5 D plus 6, what is it then, I am supposed to be doing. At this stage, it is not immediately clear that this can be interpreted as you operator but in advance mathematics courses one can give some meaning to this. So that this becomes an operator which is an operator, which is not a polynomial differential operator but it is a rational differential operator but at this stage of our course, we are not going to look at that at all.

So we will say that this just does not make sense but to remember, how to get the transfer function this is okay, as if I write x by u equal to D plus1 by this and then, replace fully D by S, it is not the justification but it is the short cut to just remembering, what the transfer function can be or how I can obtain it from the differential equation. So long as you, no what is doing it is okay, do not make the mistake of thinking that this is way in which you show that the transfer function is S plus1 divided by S square plus 5 S plus 6, no. This is only a way of remembering or a shortcut to writing down the transfer function. So that you do not make the mistake of putting the wrong operator in the numerator or the corresponding S polynomial in the numerator and something else in the denominator.
The numerator and denominator polynomial should not get interchanged in their positions.

So just to remind ourselves or to keep you know enables us to do it quickly, this is okay but this is not correct, this is non-sense on the other hand, I have operators and I do not know what the ratio means, on the other hand I have put and response values but that is a number and this is operator, if at all it can be interpreted just does not make sense all right. So we have the concept of transfer function, for a system differential equation which is p D x equal to q D u, a linear ordinary differential equation with constant coefficient in which not only derivatives of the response may appear but the derivatives of the input may also appear and we will see soon. When we use feedback of various kinds that you can indeed have an equation in which the derivative of the input appears in the differential equation.

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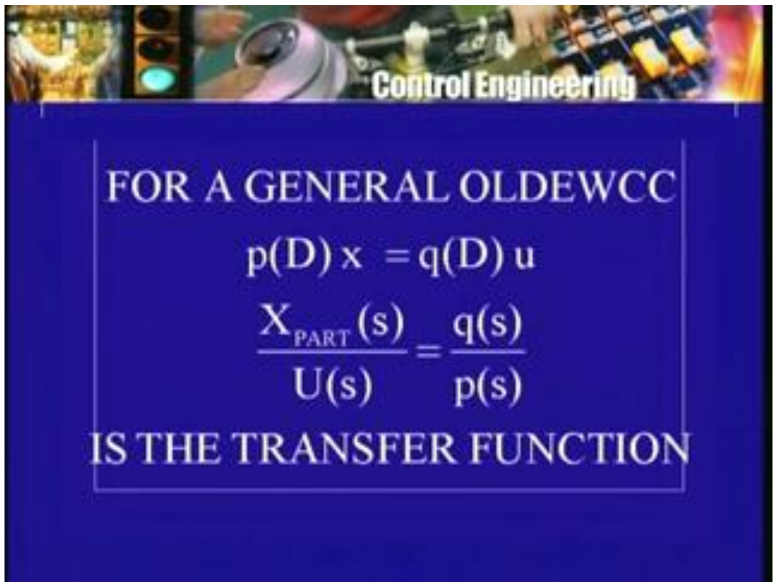

So, it is necessary to look at a differential equation in which, what you have on the right hand side is not just u but q D operating on u. So it is not just for the sake of generality that I am doing it. We will come across situations where this will happen, there will be derivatives of the input occurring on the right hand side of the equate, their will be system that our very motor control problem where, this will appear. As I told you earlier the main purpose of having this concept of a transfer function and in fact using the Laplace transformation defining the Laplace transform and so on, is not really to compute responses or solve differential equation.

In fact, from the beginning I have been telling you that today, 2004 we are progressed quite a bit beyond what the situation was 10 or 20 or 50 years ago. The Laplace transformation method was evolved sometime in the first and second decade of the 20th century 1910, 1920, a formal definition of Laplace transform, of the Laplace transformation and then, its use for solving differential equations or studying differential equations and so on. The primary aim is to get some idea, to get some inside in to the behavior of system into what will happen, if I change this or if I change that. It is of course, necessary to do some computations but today we have computer programs or software which will do the computations for us. If I have a differential equation like p D x equal to q D u, for say p or third order or a polynomial degree 3, may be q of polynomial of degree 2 and I know the input function u t say, sin t or e to the minus t. There are programs in which all I have to do is simply, to input this data that is this differential equation because I cannot simply put this into the computer.

There is no way, I can only push in a floppy or a cd into a computer, I cannot write a differential equation on a piece of paper and then, push is some, put it in somewhere. We do not have smart enough computers sometimes, we may so all I do is scribble on a piece of paper. The differential equation write down u t equal to sin t write down x 0 equal to this D x 0 equal to this and just push it into some a box or some slot of the, on the computer or on the terminal or whatever I see in front of me and the computer program or the computer takes over and finally, shows me on the screen, the response or gives me a printout, no today we are not able to do that but if, I can input the data about the equation in a proper format, input the function also that is says something about the input function and input the initial conditions then, these programs will compute the response and they will not be taking Laplace transform by applying some laws of or the rules of the Laplace transformation at all and they will also in fact, not compute the exact solution but they end of computing only good approximations to the solution.

So they will not only compute and then, give you print out they can also plot. So for solving differential equations to get the solution numerically, numerical values or plots I do not have to really use the Laplace transformation method and do it all by myself. There are programs available to do it, of course if I do not have the programs then, I may have to do it myself like I showed you for a second order differential equation, how I might obtain the transfer function and then, of course if I know u t, I can find out us etcetera. I can actually calculate x s get inverse and so on.

So, if no program or packages are available no software is available then, of course even I may have to do it all by ourselves. But the purpose is not there for only computing or it be may not even compute, the purpose to repeat Hamming's admonition is purpose of computing is in sight not number. So, what kind of insight can we get from this and we will see that we will indeed get some insight, if I have a transfer function which is S plus 1 divided by S square plus 5 S plus 6 then, if I want to find out or if, I want to compute the response then, of course I have to know u t I have to calculate u s, when I have to multiplied us by this and then, find out the Laplace inverse of this, which will involve a lot of work. Of course of I have to first find out what us is it will depend on u, u what u t is I may have to look up a table of transforms, if I forgotten or if I do not remember then, I will get a messy expression involving S.

I have to find out its Laplace inverse I may use a technique which I mentioned earlier, the partial fraction expansion technique or use some other technique use the complex inversion integral to find the original function knowing the Laplace transform and so on.
But is there something that I can do without or which will not involve actual computation
and the answer is, yes and that is what I would like to tell you about.

So I have the transfer function which looks like this, what can I say about the response or can I say something about the response by looking at the transfer function, without even knowing what the input may be. Of course, let me repeat once again what we are looking at is only a part of the response, it is not the response or if is not the whole response, I am not looking at some other terms which are important, which cannot be neglected all the time therefore, we have to consider them also. Right now, we are only considering a part of the response keep this in mind all the time but what can I say about this part of the response.

In fact we will see that we can say something not only a not about this part of the response but actually even a further part of it but we can say something, but in order to say something it is necessary to go 1 step further and that is why I have chosen a particular transfer function in which the numerator is S plus 1 but the denominator is a quadratic is a S square plus 5 S plus 6.

Now, in order to get some in sight out of this I should not leave it just as S square plus 5 S plus 6, what should I do with it. I told you that one of problems which one encounters in control system is factorization of a polynomial or finding the roots of a polynomial. This you will find is a problem which no matter, what technique you use unless you are just giving something to a computer program therefore, you do not have to do anything except inputting the data properly, factorizing a polynomial or finding out of the roots of a polynomial is something which is required, you may not do it, the computer program will do it.

In this case or for the example that I have before you, I will choosen the denominator polynomial S square plus 5 S plus 6. So that it can be factorized quite nicely in fact, you should be able to immediately find out its factors. The 2 factors are S plus 2 and S plus 3, I chosen the example as a very simple one. So the factorization is also very simple or easy we can use the roots of a quadratic formula or you can just do it somehow, you are done it in school algebra S square plus 5 S plus 6 factorizes as S plus 2 into S plus 3 yes, verify that it does correctly.

So now, I have a transfer function in which the numerator is already in factorized form because there is only one factor. The denominator was not in the factorized form but I have now its 2 factors S plus 2 and S plus 3. Now this is something which one can think of doing for any transfer function which looks like a ratio of 2 polynomial. As we have seen here q s divided by p s, we can imagine that q s is factorize, we can imagine that p s is factorize if we do that then, in the numerator we will have a number of factors, linear factors. In the denominators, we will have a number of linear factor the information about the transfer function is therefore presented in a different manner in the form of factorized polynomials and from this, then one can go one step further and represents this information in a graphical or in a pictorial way and we will find that this way of representing the transfer function is going to be useful to us.

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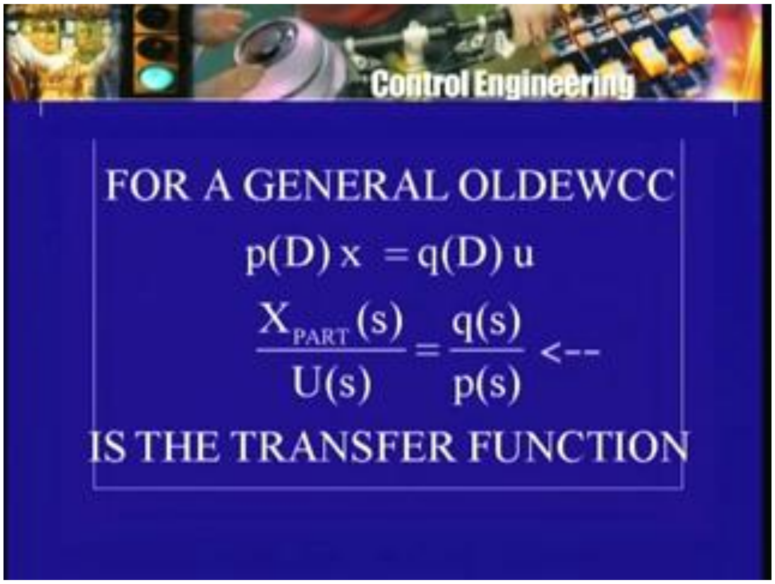

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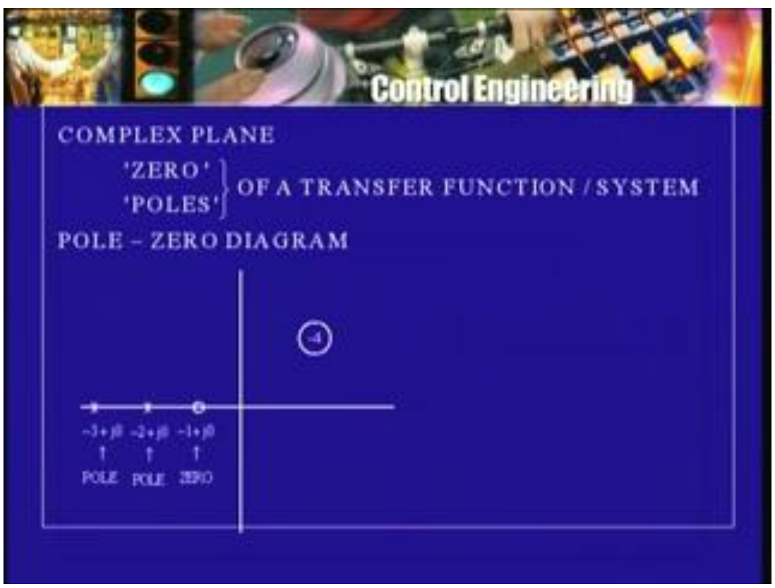

We will look at a method known as the root locus method which is based on this idea of starting with a graphical or pictorial representation of a transfer function. It is not very difficult to write down this representation but again, you must be careful. So first of all I am going to represent the information which is present here, what do I see here, I see factors linear factors S plus 1 in the numerator S plus 2, S plus 3 in the denominator. We will transfer this or represents this information in a different way by going back to the concept of the complex planes in which we represent complex number. Of course complex includes real numbers, of course complex includes real numbers, so real numbers as well, how do we do it if there is S plus 1 in the numerator that S plus 1 factor corresponds to root of minus 1 value minus 1, S plus 1 is 0, when S is minus 1.

So this is a bit of information about the numerator factor, there is only one factor that is only minus 1. Now this root is called a 0 of the transfer function, this is the terminology which control people have developed and have used. The word 0 has a slightly different meaning in mathematics, the word 0 has this specific meaning in control theory or when you talk about the 0 of a transfer function, this is what we mean. The transfer function is a ratio of 2 polynomials I will looking at the numerator polynomial, I factorize it and I find out its roots show them on the complex plane or on the Argand diagram and how do I show it.

Now again one makes a convention, it is in the numerator so it is roots are shown by means of small circle. So, I will put a circle here and I write down here minus 1 or minus 1 plus j 0. So this is referred to as a 0 of the transfer function and it is shown by a small circle not by dot but a small circle. This is a only a way of representing it, this circle does not mean it is 0 or whatever I have to write here of course, the actual root value minus 1 plus j 0 as I done here. There is nothing else in the numerator. So there is only one circle or there is only 10.

Now, let us turn to the denominator the denominator has not 1 but 2 factors S plus 2, S plus 3, for S plus 2 the root is minus 2 for a S plus 3 the root is minus 3. So there are 2 factors, there are 2 roots. I will show them now, on the diagram by using a different symbol not the circle but by using crosses. So I will put a cross here and I am not showing this on scale because otherwise, my diagram will get cluttered. So I put 1 cross here and I will put down the location of the value minus 2 plus j 0 and I will put down 1 more cross corresponding to minus 3 plus j 0.

Now these crosses or these roots which correspond to the denominator are called poles. They are called poles of the system or poles of the differential equation or poles of the transfer function. So what do have so for, I have a transfer function where is there is only 1, 0 corresponding to minus 1 which means there is only 1 factor S plus 1 and that is in the numerator mind you. For the denominator I have 2 factors s plus 2 corresponding to this pole at minus 2 and S plus 3 corresponding to the pole at minus 3.

So the denominator is S plus 2 into S plus 3. So instead of writing down a transfer function as S plus 1 into S plus 2 divided by S plus 3 as a rational fraction or even S plus 1 divided by S square plus 5 S plus 6. I will represent this transfer function by means of this diagram and for an obvious reason is going to be called what, what do you have on the diagram I have poles, I have 0s of course, I can say 0s and poles, the standard terminology for this is pole 0 diagram.

So such a diagram is known as the pole 0 diagram of a transfer function or of a differential equation or of a system. It shows the 0s which correspond to the numerator factors, they correspond to the differential operator operating on u, the poles correspond to the factors in the denominator, they correspond to the polynomial differential operator operating on the response. This is the pole 0 diagram of course one may have to have something more because for a particular differential equation, when I factorize it the factorization may be as follows instead of S plus 1 in the numerator, I may have S plus1 multiplied by some coefficient of some number for example 2 and in the denominator of course I may also have a number, let say 5.

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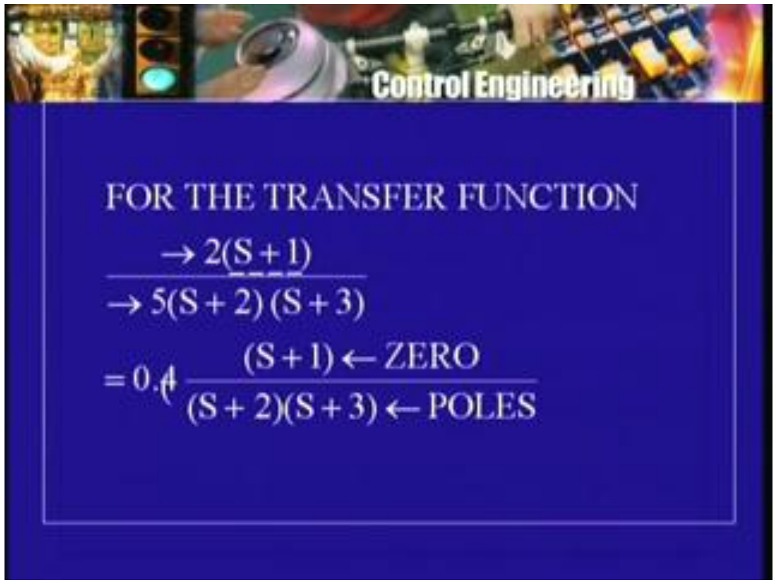

So I could have a multiplier in the numerator, I could have a multiplier in the denominator but instead of keeping separately because I could write this as .4 into S plus 1 divided by S plus 2 into S plus 3. So, if I agree that the polynomial will be written in this form that the numerator factors and the denominator factors will be of the type S plus something, it is that something could be positive, negative number, real or complex or imaginary whatever, it is and this whole thing will be multiplied a number then all I have to specifies, in addition to the 0s and the poles of the locations of the 0, in this case minus 1 location of the poles minus 2 and minus 3. I have to specify also this multiplying coefficient .4 and I can put it down as a number on the pole 0 diagram somewhere it a put here .4 in brackets or in circle it like this. This could be called the gain of the system but the word gain is used in, so many different ways that is better not to call it gain.

Let us reserve the term gain, for what is normally called gain in fact decibel gain, gain in decibel. But remember, that this coefficient may not be 1 and so it may necessary to be told what exactly this coefficient is. Remember that we had a differential operator b 1 D plus b 0 and so, we got b 1 S plus b 0 in the numerator and so in order to write it as a, in the factor form and I will have to pull out that coefficient b 1 and write this as b 1 into S plus b 0 divided b 1.

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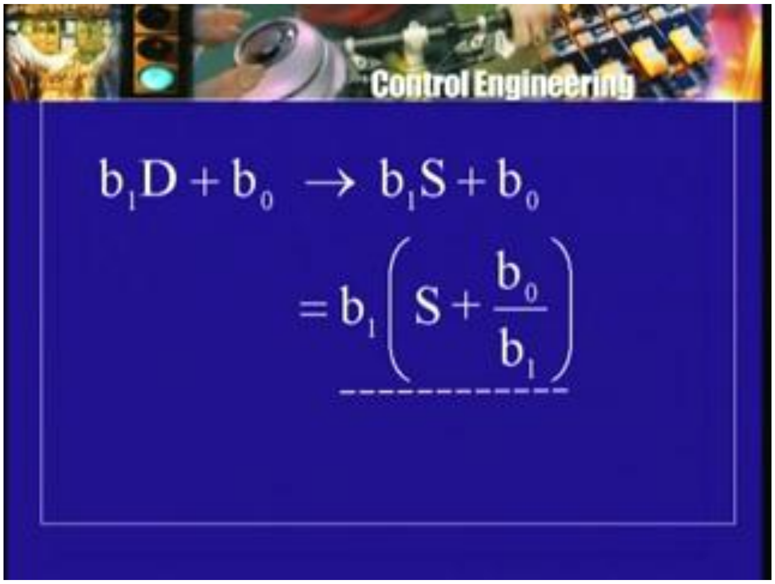

So, I may get a non-unity coefficient multiplying therefore, I have to write it if it was1 I can suppress it, if it is not1 I have to write it like .4 and so this can appear on the pole 0 diagram or I can put a statement saying that this coefficient will be .4 or whatever it is. So the transfer function information can be shown as a pole 0 diagram and we will see that this diagrammatic representation is going to be useful, as we proceed further.