Control Engineering
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Lecture - 22

The Laplace transformation is an example of, what is called an integral transformation. As we have seen earlier it is based upon an integral, the integral is known as the Laplace integral. I am not going to go into any great details about the theory of the Laplace transformation, one has to do it really properly although for the purpose of our application, we may not have to go to far into in still as electrical engineers or we should know sufficient about it.

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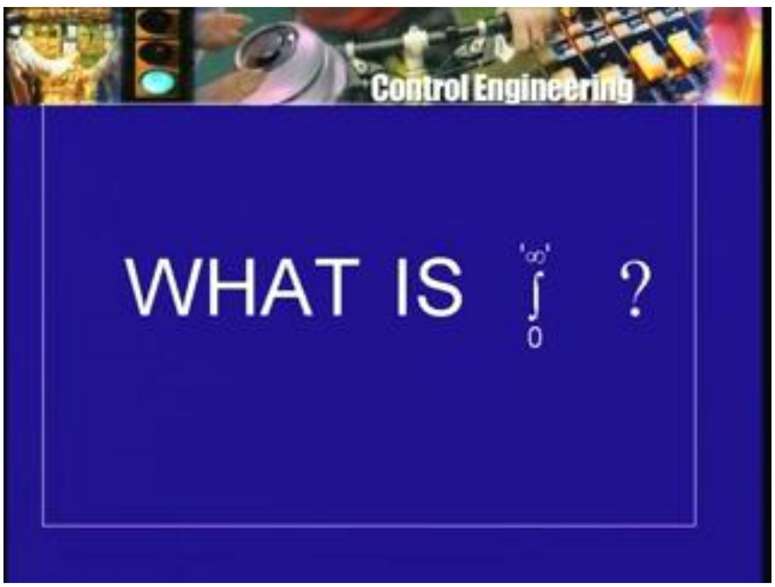

So, as I said earlier it looks like an integral, the only trouble is that it is not the commonly used definite integral whose limits are numbers like a, b or 0, 3 and so on. Here, the lower limit is 0 and the upper limit is infinity and I have put this infinity in inverted comma to indicate that it is not being regarded as a number but actually, what is involved is limit. So strictly speaking, we should be talking about the limit. So therefore limit of let us say, integral from 0 to an upper limit, limit of integral that is capital T and the limit as T increases without any bound or the limit as capital T tends to infinity of the integral 0 to T of the function and for us, the input the function is denoted by u t.

So if we have looking at the Laplace transformation applied to the input function u t and this is multiplied by exponential of minus s t dt, integrated over the interval from 0 to t and the limit of this integral taken as t increases without bound or as we say as t tends to infinity. I pointed out that we have in the exponent the number S, which can be a complex number rather than just a real number and because of that even though the function u t which is a signal, physical signal corresponds to physical signal like may be the applied voltage as a function of time or it can be an output variable like the speed as a function of time and so on, u t is a real number or u is as what is called real valued function. The result of this will in general be a complex number and I had asked you to work out a problem, a simple problem that involves the calculation of this integral for the function u given by the sin function that is u t, given by sin of t and for a particular value of S.

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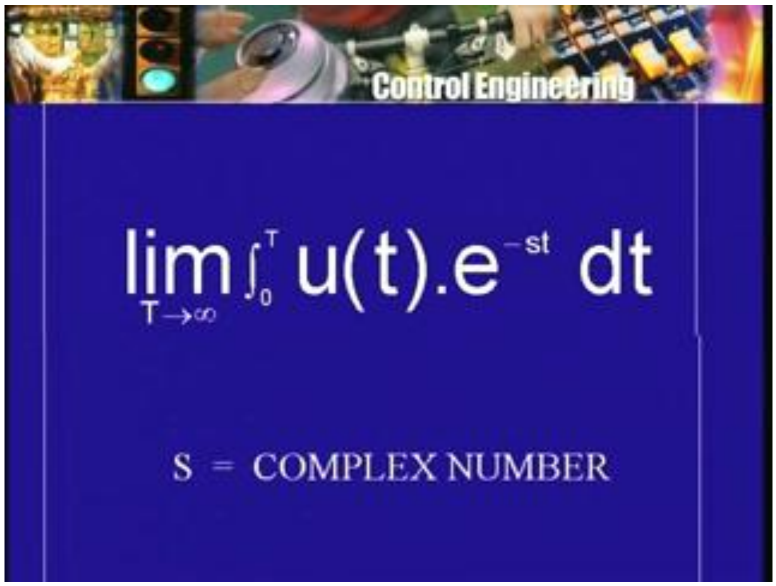

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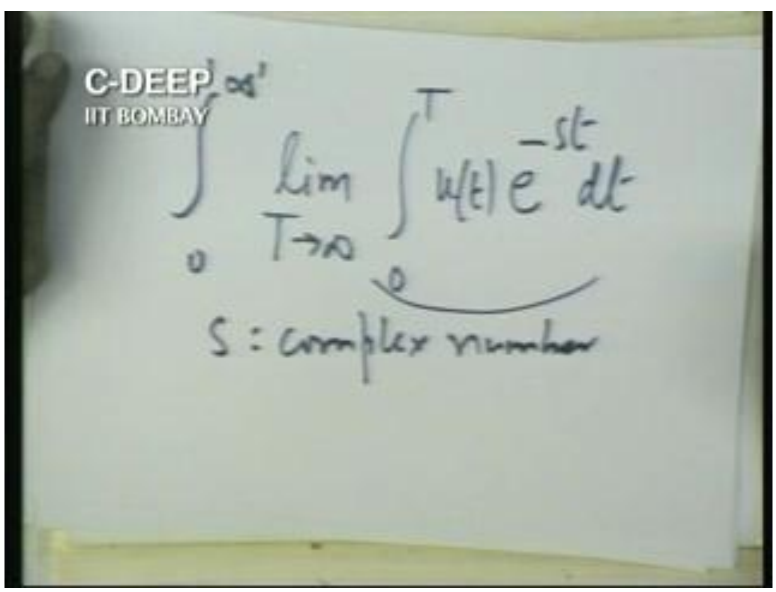

Now, this is where one has to be aware of something although as I said we will not go into any great depth. This integral or rather the limit of this integral may exist as we say or may make sense or be meaningful, only for some complex numbers S and not necessarily for each and every complex number S that you may think of. For example, if you go back to the function sin t then, if I look at the integral 0 to t sin t in place of u t multiplied by e to minus s t and integrated over this interval and the limit of this as t tends to infinity. Then, this I told you will turn out to be1 over S square plus 1 in fact, I asked you to work it out for a value for S which was different for particular value of S rather than for a general S, it turns out that this make sense that is the limit is equal to this number 1 over S square plus 1, only for certain complex numbers S. For example here, the complex numbers for which this integral will indeed as one says converge or have a limit, it is not enough to just write limit there should be a limit, a limit should exist or a limit should be possible.

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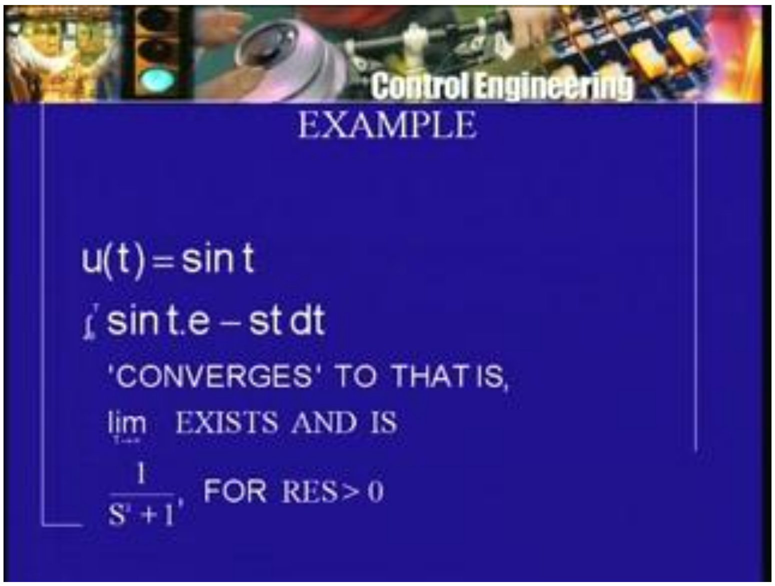

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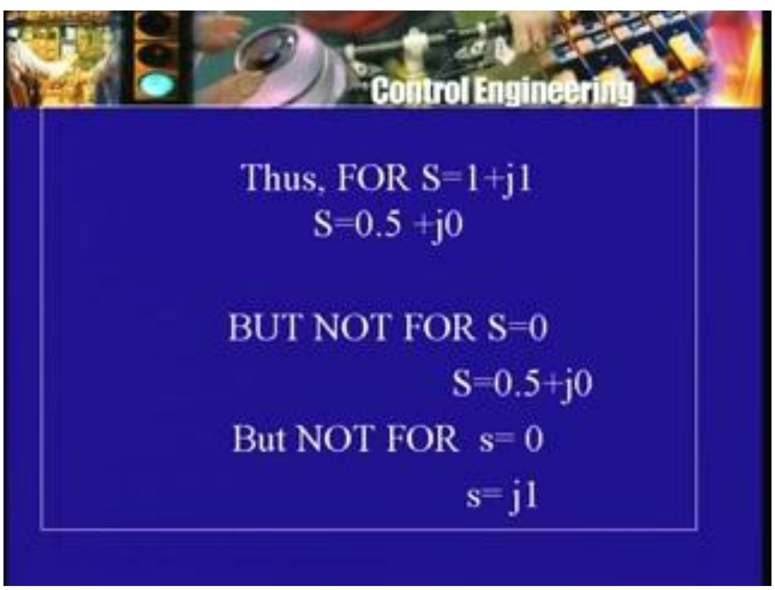

Now that will happen for the function sin t only for certain values of S and these values of S are given by as one writes real part of S greater than 0. If you chose a complex number which does not satisfy this condition real part of S greater than 0 that means the real part of S is positive. For example, what would be an example of a complex number whose real part is positive, well 1 plus j 1 is a complex number, whose real part is 1 which is positive, the number .5 plus j 0 or you can think of it as just the real number rather than a complex number although as a real number, it also a complex number then .5 plus j 0 its real part is .5 that is also positive.

So for such number, for complex numbers says whose real part denoted by real S, real part of S is positive. This integral will converge and the limit will indeed be equal to 1 divided by S square plus 1 not for other S. So, once again as an additional exercise you can put as a very simple and special case put S equal to 0 and find out what the limit is or whether the limit exist at all, that is whether the integral converges then you can try for example, S equal to j or j 1, a purely imaginary number and find out once again, whether the integral converges that means the limit exists or makes sense and of course, you can try it for 1 plus j 1 and check that the limit does converge and the formula 1 over S square plus 1 gives it correctly.

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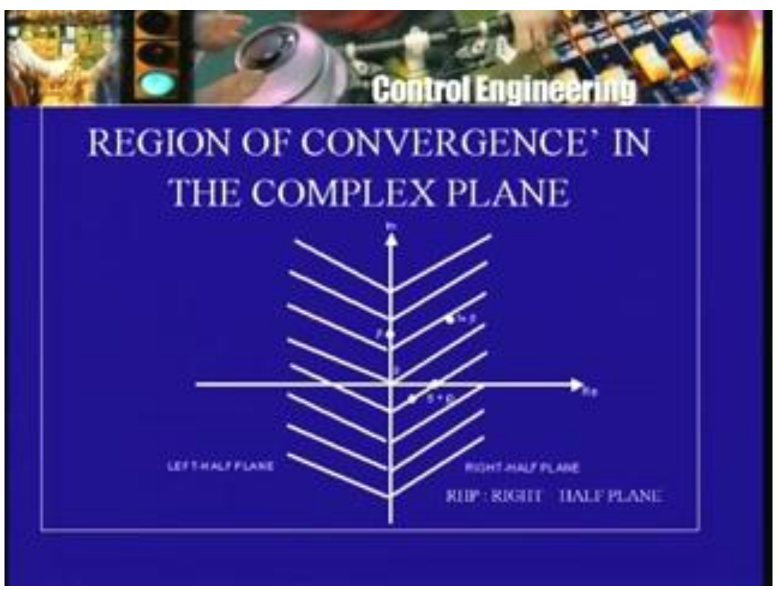

So, this set of real numbers S for which the Laplace integral converges or the limit exists is called the region of convergence of the Laplace transform of the function or the region of convergence for the particular function that we are talking about. So the functions sin t the reason of convergence for its Laplace transformation is real part of S greater than 0 and I had already made use of what is known as the complex plane. So, if one uses the complex plane, here is the real axis of the complex plane, here is the imaginary axis of the complex plane. Then, one can find out what are all the S complex numbers S, for which the integral converges, for example for sin t, here is complex number 1 plus j 1, for which the integral converges, here is the complex number .5 plus j 0, for which the integral converges and there are many more and that is the region of convergence and then, one can show it in the complex plane as a region by the usual engineering method of hatching the region.

So, for example I have done this. The only thing to remember about this hatching is that the imaginary axis is not a part of the region of convergence for sin t, region of convergence therefore is all the complex numbers whose representing point or representative points lie to the right of the imaginary axis. Now, this part of the complex plane is referred to as the right half plane. We will abbreviate it as RHP, right half plane. Now this region of the complex plane will play a role later on when, we talk about stability of the feedback control system.

So, remember this region of the complex plane which is to the right of the imaginary axis. In other words the set of complex numbers, whose real part is positive is called the right half plane and naturally the region that lies on the other side of the imaginary axis, the region to its left, I will show it by a different hatching, this the region to its left that will be called the left half plane, LHP for short and so, one can think of the complex plane as being split into 3 parts. There is the imaginary axis, there is the right half plane to the right of the imaginary axis, the right of course, when we draw the diagram in the traditional way or region were the real part of the complex number is positive. Imaginary axis is a set of points where, the real part is 0, the number is purely imaginary and then, the third one is the left half plane the region were the real part is negative.

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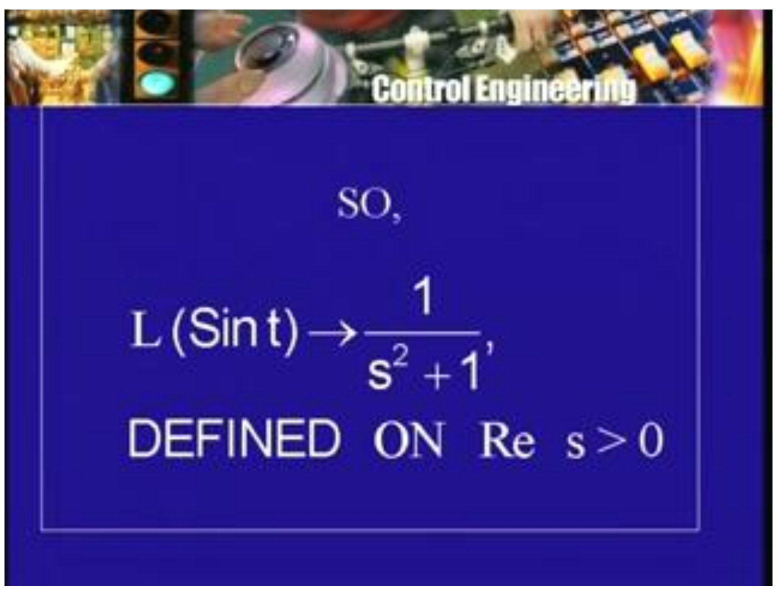

So this is the breakup of the complex plane into the 3 parts which plays an important role in the study of stability of a feedback control system. The left half plane, the right half plane and the imaginary axis. Sometimes, instead of left half plane one says strictly left half plane sort of emphasizing that the imaginary axis is to excluded. Similarly, we can talk about the strict right half plane which means numbers or points which lie strictly to the right of the imaginary axis not on it that is not be included, only those which are to the strictly to the right but usually LHP and RHP will mean strictly to the left and strictly to the right. So one need not always say it.

So the function sin t has a Laplace transform that is after applying the integral transform to it, we get a function which is given by 1 divided by S square plus 1 but this holds only for certain set of numbers S. So in other words the Laplace transform of sin t, we find sinusoidal function t sin t is the function 1 over S square plus 1 but only for the set S given by real part of S greater than 0. So I am writing it here this way the Laplace transform of sin t and one starts using abbreviations. So I will say L, this is called this letter is called script L that is very commonly used in most textbooks, script L of sin t is the Laplace transform of the function sin t. It is a function which is defined for some complex numbers which ones those for, which the real part of S is greater than 0 and at a complex number S satisfying this condition, the value of the Laplace integral is 1 over S square plus 1 and therefore, the value the Laplace transform is 1 over S square plus 1.

So, you can now understands why it is called the transform or the Laplace transform because you start with a function which is for us something called as signal or a variable, time variable, some signal, some variable that changes with time for us it is, the input or output or some other variable, field, pressure, voltage and so on, whereas after doing this business of computing the integral and so on, what results is a function, which is defined on complex numbers and whose value is also usually going to be a complex number and this is called the Laplace transform of the original function or of the time function sin t, t of course wherever we are using t as a significance of time, this S right now will have no significance. Although, in communication theory, one can relate the Laplace transform to another integral transform and what is it, it is the Fourier transform, named after Fourier, it is also an integral transform, it also involves this business of infinity somewhere.

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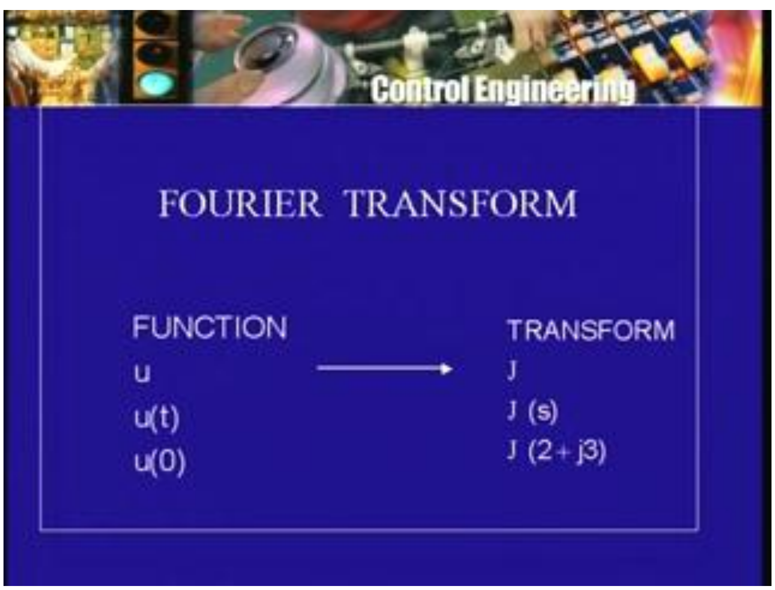

So that also as to be handled carefully, there is a relationship between the 2 transform, the Fourier transform and the Laplace transform. These are 2 major integral transforms which electric engineers have to know. There are few other integral transform which also find the applications in electrical engineering one such is called the Hilbert transform, named after Hilbert. Here, the Laplace transform is a complex valued function of a complex variable and defined only for a particular region of the complex plane called the region of convergence and here, again there is more or less a commonly used notation, if the function is denoted by a symbol small u, the function of time so to speak, this is called the original function that this is what really, we are interested and we start with then, from that we obtain a function which is the Laplace transform it is denoted by the corresponding capital symbol.

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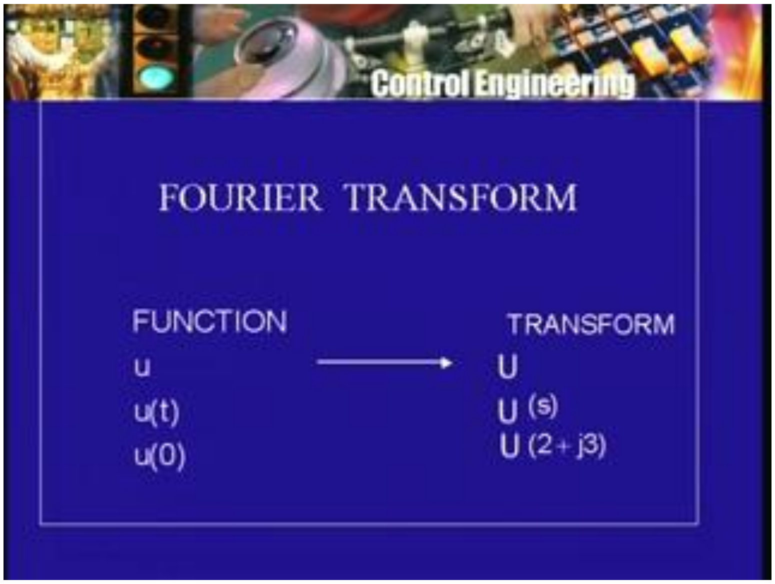

So, one will write capital U and the value of the function U at time t for example, at 0 or phi or 10 raised to phi, the general value at time t will be indicated as u t. So in particular u 0, we denote the value of the function at t equal to 0, the correspondingly for the Laplace transform u, the value at a general complex number will be denoted as capital U of s. For example, we can talk about capital U at 2 plus j 3, this one the u or the original function as it is called is usually real valued, u t is real, it is a function of real variable because t is a real number, time for us is a real number and I told you that as control people, we are interested in usually what happens after or staring with some initial moment of time which about most purposes one denotes by 0.

So the t here is usually restricted to t greater than equal to 0 and then, we obtain the function capital U which is the Laplace transform of the function small u but the number s here or s can be replaced or can, will denote a complex number in general and U of s will also be a complex. So U will be a complex valued function of a complex variable s, this symbols t for time and the symbol s for the Laplace transform integral are fairly standard, in some books some of the older books, you might find the letter P in place of s.

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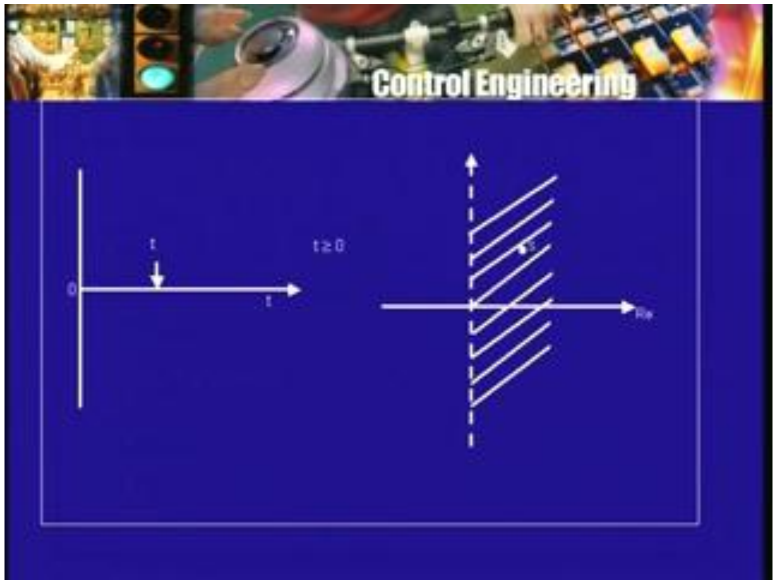

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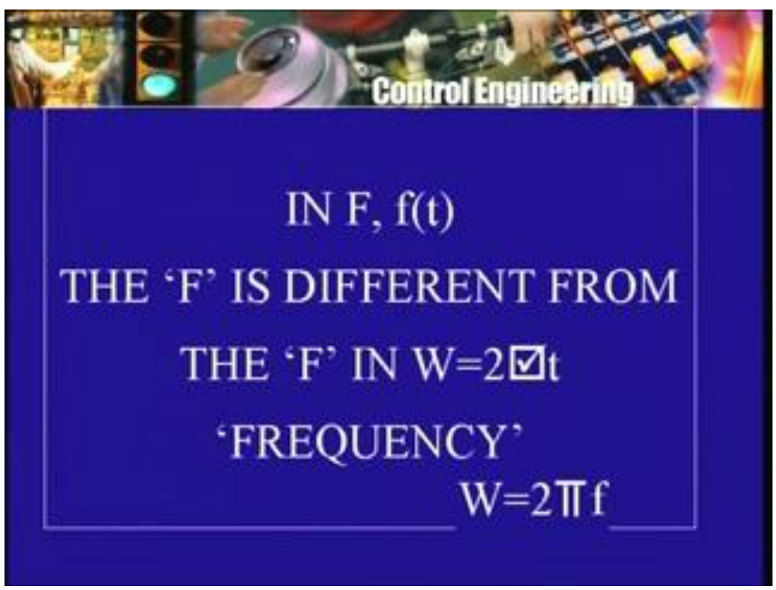

So do not be surprised in fact, some of the people who work with this earlier. One of them was you can call him more or less an electrical engineer, he himself preferred to be thought of as an engineer rather than a mathematician, he made very important contributions not only to what we are talking about, not controlled but generally the study of system behavior, inputs, outputs and so on. But he also made contributions to electromagnetic theory and this person is was Heaviside, Heaviside used the letter P rather than letter s but subsequently, the letter s as become a very common just as in electric engineering practice usually F in communicating theory at least will stand for frequency rather than for to be the symbol for a function whereas, in mathematics one would talk about f of p or f t. In communication theory, the letter f is preferably used for frequency in hertz and as you know, the letter omega in electric engineering is preferably used for what is called angular frequency or omega equal to 2 phi f, in radians per second. However, for our control theory purposes the letter omega is also very commonly used for angular velocity.

In fact, in control theory one usually does not use Omega or f in any significant way because in control theory, we are not interested only in the sinusoidal function. In fact, we are interested in many other functions other than the sinusoidal function and therefore, the symbol omega in control application which usually denotes speed because speed control is one major application of control theory. So do not be surprised if you find other notations but this convention that a capital letter will denote the Laplace transform of a function which is denoted by a small letter is also more or less is very commonly used.

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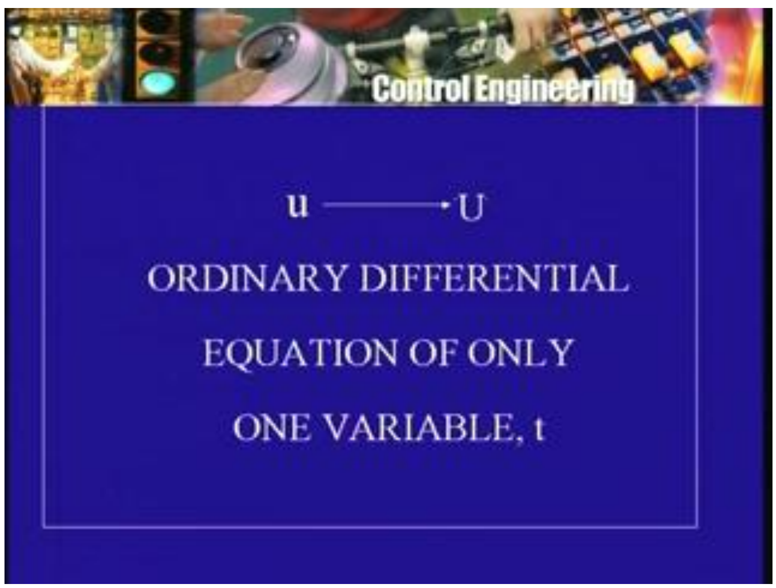

So, this the Laplace transform or Laplace transformation. We say that the function small u is transformed into the function capital U and therefore, we talk about the Laplace transformation, the transformation applied to u results in a different or a new function capital U. Now, what is the use of the Laplace transformation as I told you the Laplace transformation began to be used for the study of differential equation, essentially not more than 100 years ago or may be a little over 100 years ago, I mentioned Heaviside just now, Heaviside was one of those who started using the transform idea for studying differential equation Laplace gave his integral around 1800, almost 200 years ago but his purpose was a little different, he was more interested in problems in mechanics.

So, how shall we use a Laplace transformation then, to solve a differential equation and why is it useful. Well, the Laplace transformation is a useful technique for solving differential equations but not all kinds of differential equations. The Laplace transformation is a useful technique for solving differential equations only of a particular kind and what is that kind, first of all it should be an ordinary differential equation. In other words, there should be only one independent variable fortunately, in the control theory situation usually there is only independent variable namely time. Although, there are occasions where in place of time you may have a single space variable as the independent variable indeed there are some applications in what are called flexible structures, where one may instead of time think of distance as an independent variable.

So, the differential equation should be an ordinary differential equation not a partial differential equation, only one independent variable and for us usually most of time, it will denote a time. Dependent variables of course, there can many, there can be the angular speed, there can be angular position voltage, applied voltage, current, torque whatever any number of independent variable, dependent variables. You can have but they should all be functions of one independent variable t. So ordinary differential equation not partial so, the derivatives that occur will be only ordinary derivatives not partial derivatives.

So things like the d u d x partial derivative of u with respect to x, will not occur in our work. The derivative will be what then are called sometimes ordinary derivative or it is a function of only one variable. So that is one, secondly these are differential equation must be of the kind that I mentioned earlier, namely linear, liner differential equation but not only just linear with constant coefficient, liner differential equation with constant coefficient. The Laplace transformation is useful technique or the use of the Laplace transformation is useful technique for solving ordinary linear differential equations with constant coefficient.

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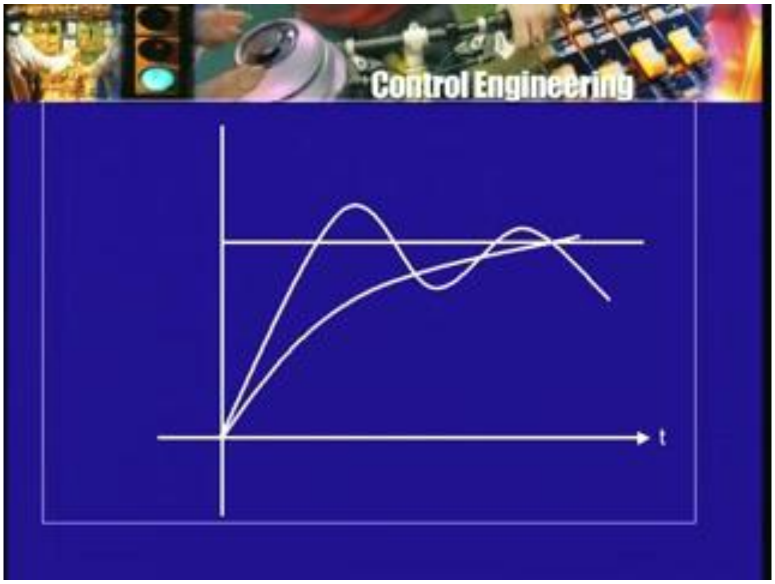

Now, once again with the development of computers, with their becoming comparatively cheap, very fast, with very large memory capacity. The use in a Laplace transform is restricted more or less or if at all you want to solve a problem, for solving lower order problem, may be first order differential equation or second order or may be third or 4^{th}. If I had a 100 th order differential equation ,you may not use the Laplace transformation method but that is not the only use of

Laplace transformation. As I told you earlier quoting having, the purpose of computing is inside not number.

So, similarly by the use of the Laplace transformation, we can get some inside about the behavior of the system. We can find out using the Laplace transformation, if not exactly how the system will behave and it is the output signal at 10 seconds will be precisely this and so on, that of course we can calculate but something qualitative about the behavior of the system or of the variable. For example, we can say that the speed of the motor as the motor starts and builds up to its final speed will keep on increasing steadily or the speed will overshoot, the steady state value that it will increase but unfortunately, it will cross the steady state value. So it will overshoot then it will stop increasing then, start decreasing but on the downward turn it will again cross the steady state value and so, there will be what is called an undershoot and then, again an overshoot and so on.

So, whether the speed will behave like this build up steadily to the final value or will have decaying oscillations with overshoot and undershoots. Qualitatively, the use of the Laplace transformation plus something more will tell us, actual computation of the peak value etcetera will require numerical work associated with Laplace transform or the Laplace transformation and as I told you there are packages today, which will solve differential equations and plot the results for you. So it is not necessary really do it, if you wanted to calculate with some precision the response to use the Laplace transformation but no amount of computer program will give you this insight about what kinds of behavior to expect and so on.

So, for us the Laplace transformation is a tool mainly for getting inside into the behavior of system rather than for doing lots of computation, keep that in mind. So do not underrate it, at the same time do not try to use it for everything okay. So, how is the Laplace transformation to be used for solving or finding out, what happens to a system described by say, first order differential equation or a second order differential equation and so on. Now, the Laplace transformation one can show has some properties, it has quite a few properties in fact and your textbook of control theory will probably give you, a list of properties of the Laplace transformation, if it does not certainly a mathematics textbook which talks about mathematics of engineering or engineering mathematics will have a chapter on the Laplace transformation and we will use, quite a few properties of Laplace transformation.

For the immediate purpose of trying to solve a linear differential equation with constant coefficient, we require only a few properties of the Laplace transformation. In fact, you require only 3 properties of the Laplace transformation and I will show you that with the help of these 3 properties not only can we make some progress towards the solution of the differential equation. But we can also almost calculate the Laplace transform of a few function, not all a few functions. We will see how, but first the properties of the Laplace transformation that are going to be useful.

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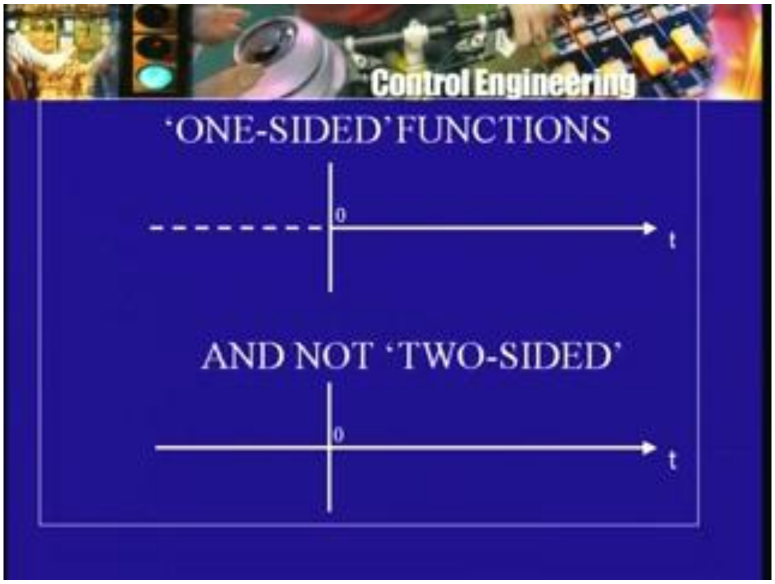

Now, these properties have to understood properly. First of all, recall once again, that the Laplace transformation transforms one function usually regarded as a function of time variable into other function and one more thing, I mentioned that in control, we are interested in the values of time greater than equal to 0 and therefore, such functions are called one sided function. We are interested only in one half of the time axis starting from 0 and going on towards the future, I have being showing like this this is 0, we are not interested in the other half. But it is possible in fact, in communication theory, one likes to think of functions whose past behavior also may be useful or important.

In that case, it is possible to talk about a different Laplace transformation that is, it is like the Laplace transformation but not quite, what we had and then, we have therefore a 2 sided function and a 2 sided Laplace transform. If you remember, the Fourier transform is a transform for which the function is supposed to be define for all time t whether, t equal to 0 or t greater than 0 that is future time or t less than 0, that is past time. The Laplace transformation that I am going to talk about and we are going to find useful is a one sided Laplace transform. So functions defined only for t greater than equal to 0 the Laplace transformation one says changes or transform or converts this function of time into a different kind of function into a complex valued function of a complex variable usually denoted by s.

Now, this transformation, this action has some special properties, the first property is what a called additivity, we will say or one says that the Laplace transformation is additive, is an additive transformation, what does that means. Suppose, I have 2 functions of time one denoted by u 1, another denoted by u 2, u 1 denotes the function of time, u 2 denotes another function of time, let us say u 1 is some signal u 2 is some other signal. Then, I can certainly think of a function which is obtained by adding these 2 functions. In fact, we most often talk about such functions.

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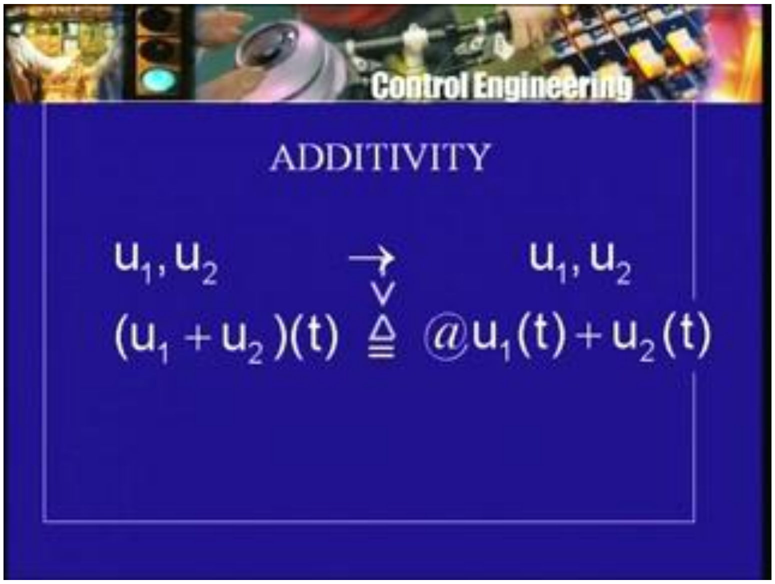

The sum or the result of adding the 2 functions is of course denoted by u 1 plus u 2. If I want to show the time argument explicitly then, I will write u 1 of t that is value of u 1 at t plus the value of u 2 at t. This sum is the value of a new function which is denoted by u 1 plus u 2, its value at t is u 1 of t plus u 2 at t, this symbol delta tells you that the function u 1 plus u 2 is being defined this way and this is just notation but one has been writing sums of function of time. For example, if I writer t minus e raised to minus t or maybe you see a minus sign so, you may not see a sum right away.

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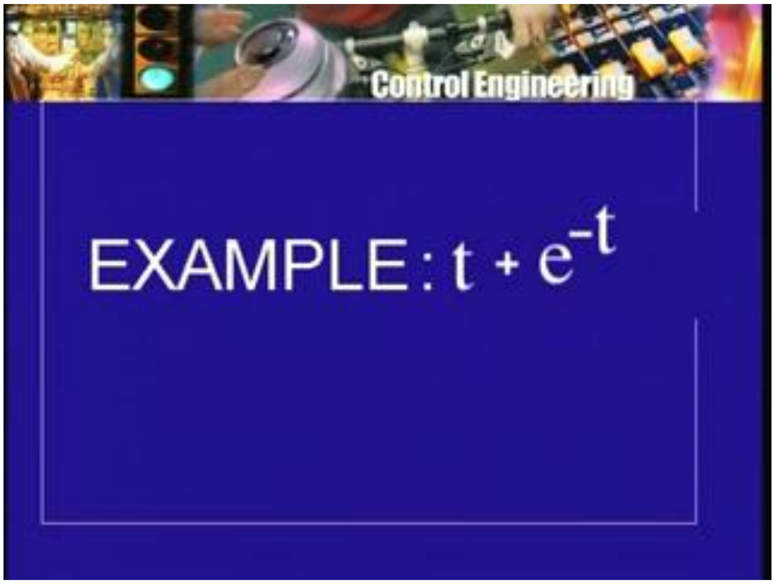

So, suppose I write t plus e raised to minus t this is a sum of 2 function, one function was value at t, t itself, another function whose value is e raised to minus t. So this is a function, function which is sum of 2 function. So, if we have 2 functions u 1, u 2 physically there could be 2 voltages, 2 currents then, we can think of there sum U 1, U 2. Now, we could as 1 puts its apply the Laplace transformation to the function u 1, if I apply the Laplace transformation to the function u 1, the resulting function as were agreed will be denoted by the corresponding capital letter.

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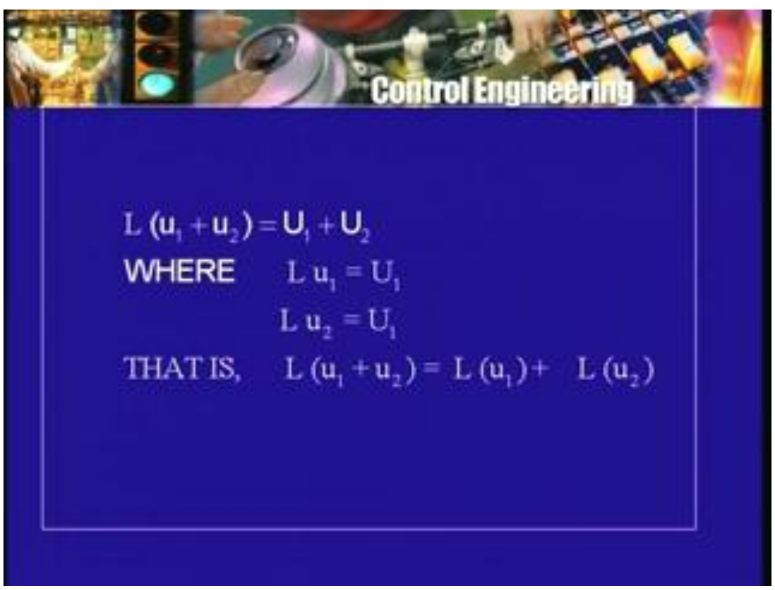

So, we will read this is as L applied to u 1 results in capital U 1 or the Laplace transform of the function small u1 is the function capital U 1. Similarly, I apply Laplace transform to u 2 to get the function capital U 2 then, I can ask the question, what will be the result of applying the Laplace transformation to the function u 1 plus u 2, just one thing to remember here, going back the Laplace transform usually has a region of convergence which means that L u 1 is capital U 1, but the function u 1 is defined only for certain complex number s, namely the region of convergence of u1 similarly, L u 2 or capital U 2 will be defined or will make sense only for certain complex number s.

Now, if I want to talk about L of u 1 plus u 2 then, again I must talk about the region of convergence of U 1 plus U 2. So the result that I am going to write has to be understood correctly that it is only for numbers complex number s which belong to the region of convergence of all the functions that are involved. For such complex numbers s, what is L of u1 u 2 is a function which happens to be the sum of U 1 and U 2. So one reads it as the Laplace transform of the sum of 2 functions is the sum of their Laplace transform or if, I write it more explicitly L of u 1 plus u 2 equals L of u 1 plus L of u 2. On the one hand I add 2 functions of time then take the Laplace transform, on the other hand I apply the Laplace transform first and then add them.

These are function of time, these are function of s but I am adding here because I can add 2 real numbers, I am adding here because I can add 2 complex numbers, L of u 1 plus u 2 is L u 1 plus L u 2. This is property of additivity and one can immediately, see a use for this because if I have 2 functions whose Laplace transforms I know then, the Laplace transform of their sum also I will able to calculate almost immediately. For example, if you look up your textbook of mathematics or control theory, you will find actually a table of Laplace transform that is you will find a list of functions, they may be denoted by f or u, it does not matter and there corresponding transforms capital F S or capital U of S. The function whose value at any time t, is t remember only for t greater than equal to 0, its Laplace transform is the function whose value of certain complex numbers s is given by one by s square.

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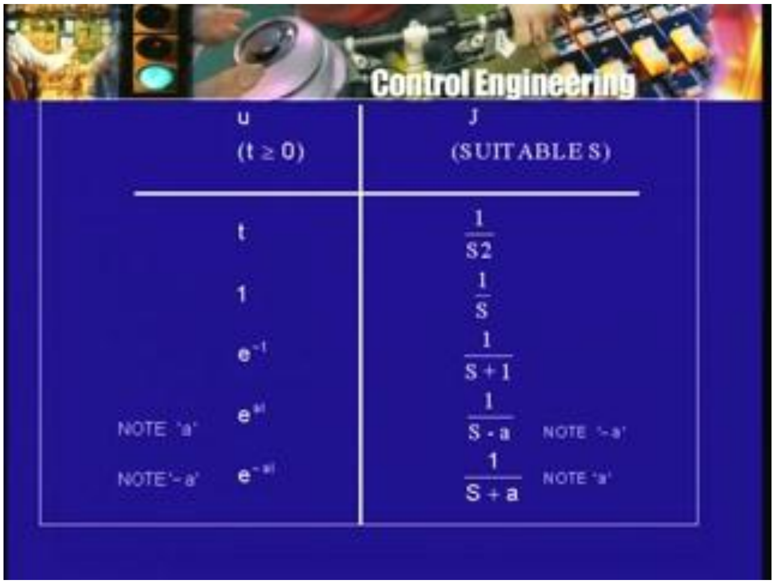

If the function is simply the constant function one, it is sometimes called the step function, the unit step function. The function whose value is just one for all time t greater than equal to 0, its Laplace transform is given by a function whose value for some complex numbers in the region of convergence is 1 by s. For sin of t, I have already told you that the Laplace transform is 1 over s square 1 for e raised minus t, the Laplace transform is simple 1 divided by s plus 1. For a general exponential e raised to e t, the Laplace transform is 1 divided by and this where, one as to take care if there is at there is a minus sign here. The Laplace transform is 1 divided by s minus a or if write e raised to minus at then, the Laplace transform is 1 divided by s plus a. I am almost writing a part of a table of Laplace transform.

So, let us use this for calculating the Laplace transform the function t plus e raise to minus t, the Laplace transform of t is given by 1 divided by s square, for certain complex numbers s, the Laplace transform of e to the minus t is given by 1 divided by S plus 1 again for certain complex numbers S therefore, those complex numbers for which this holds and this holds, for those complex numbers, the Laplace transform of the sum is given by the expression 1 over S square plus 1 over S plus 1.

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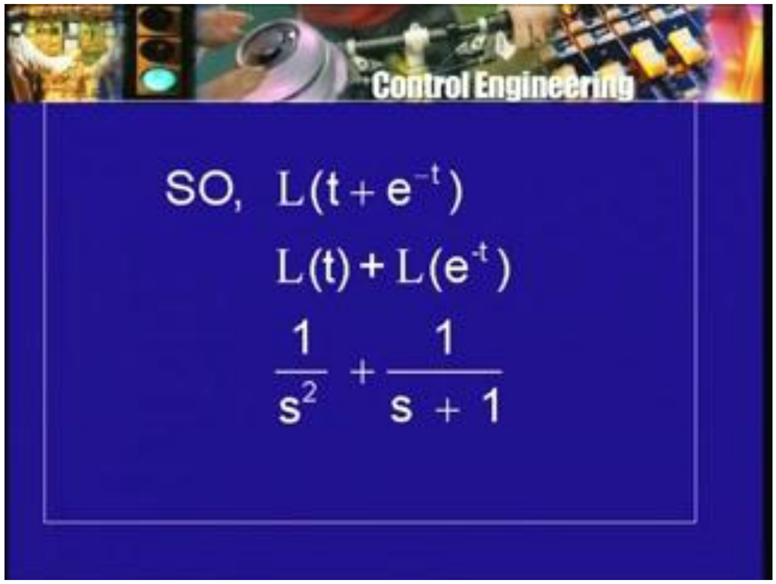

So, if you know the Laplace transform of 2 functions, you can immediately state what the Laplace transform of their sum is going to be. This is the property of additivity, the next property which is used problem solving LDEWCC is the property which is known as homogeneity, homogeneity, only it has something to do with scaling of a function. I have a function u, let us say it is some speed or whatever and I multiply it or scale it by a scale factor usually for such multipliers or scale factor as you know one uses letters from the Greek alphabet like alpha, beta, lambda and so on or letters like k etcetera as coefficient.

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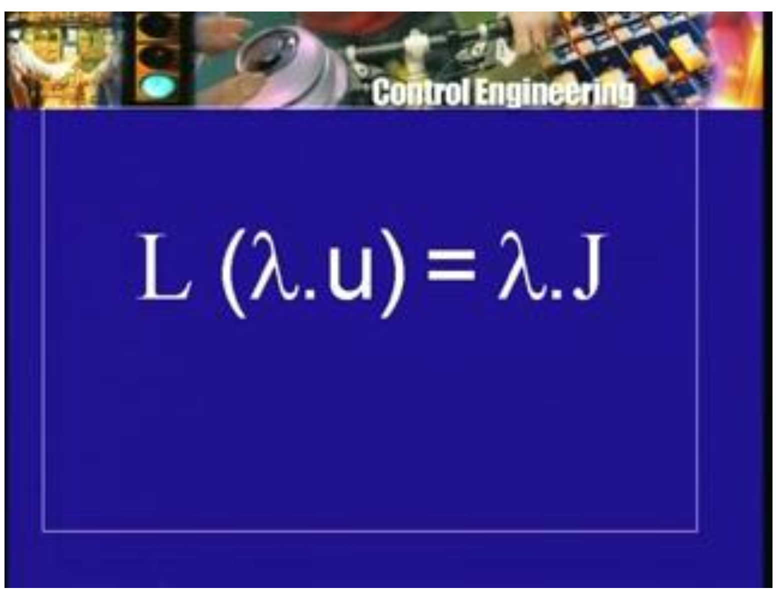

So, if I have a number lambda, real number lambda, I can scale my u by this scale factor lambda for example, lambda may be 10. So I have the function which is ten times the original function u or lambda may be 10 raised to minus 3. So I have a function which is an attenuated version of the original the function u. So this is referred to as scaling u is scaled by lambda by the factor lambda to get a new function lambda u, what will be the Laplace transform of lambda u. Well, using the integral you should be able to show that the Laplace transform of the integral will be simply a Laplace transform of the multiple scaled factor, lambda multiplying u will be simply a multiple of the Laplace transform of the original function.

So it will be lambda times US and it turns out clear that the s is the same for both u as well as its scaled version that is the region of convergence of u and lambda u, the 2 functions, the original function and its scaled version. The region of convergence is the same, so for whatever S capital U of S is defined for the same S, the Laplace transform of the lambda U also is defined and its given simply by the same multiple.

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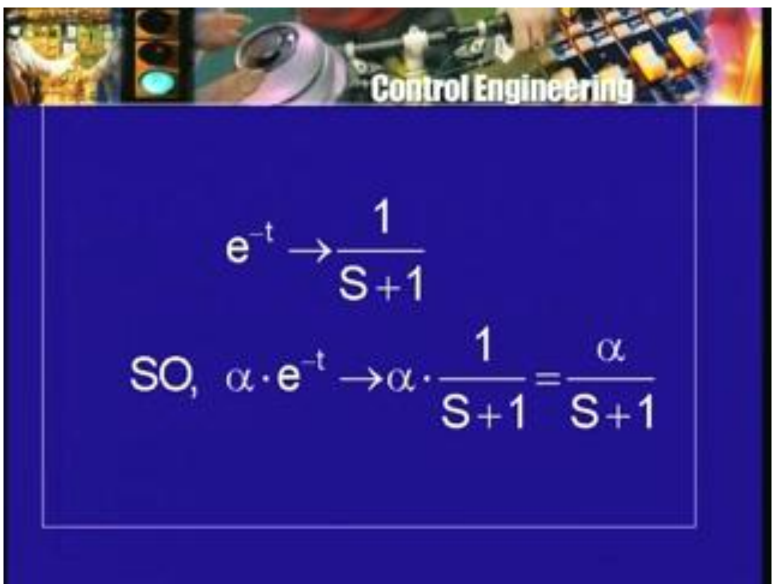

So what is the use of this, well I know that the Laplace transform of e to raised to minus t is 1 divided by S plus 1 and therefore, if I have the function alpha into e to minus t and changing from lambda to alpha because one should get use to these various symbols then, what will be the Laplace transform of alpha times e to the minus t. It will be simply alpha times 1 divided by S plus 1 or as you know one prefers to write alpha in the numerator like this. So its alpha divided by S plus 1 this is the property of homogeneity. So the Laplace transform is a homogeneous transformation or it is a transformations which has the property of homogeneity. The Laplace transformation is also an additive transformation, it is a transformation which has the property of additivity. Now, because it has both these properties one can go further for example, I have the function which is given by 3 t minus 4 e raised to minus t, what is Laplace transformation or the Laplace transform of this function or the result of applying the Laplace transformation to this function.

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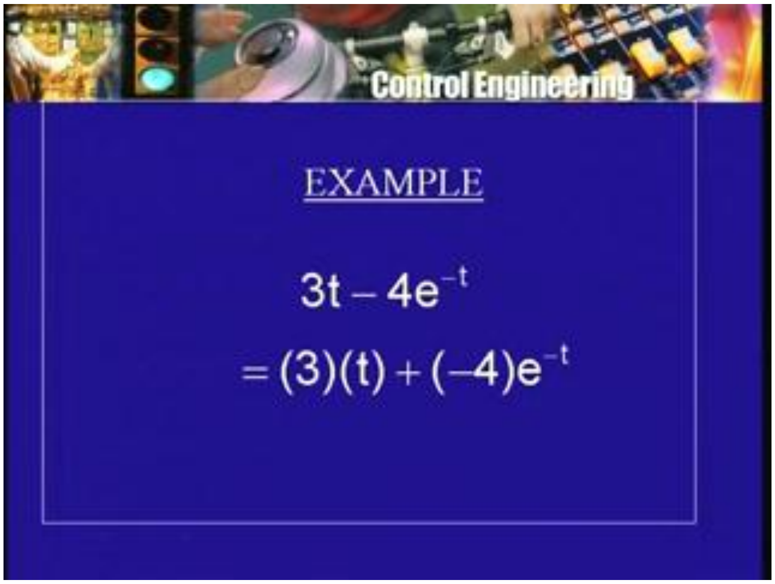

Now one can immediately see that, this is a sound and the minus sign need not worry us because I can write this or think of it also as 3 t plus minus 4 times e to minus t, either I multiply e to the minus t by 4 and subtract it or I multiply it by minus 4 and add it, the result is the same. So, what will I have I have a function which is a sum of 2 functions. So my additivity can be used but each of them is multiple of another function 3 t is the multiple of t, multiplying factor 3 minus 4 e to minus t is a multiple of e to minus t the multiplying factor is minus 4.

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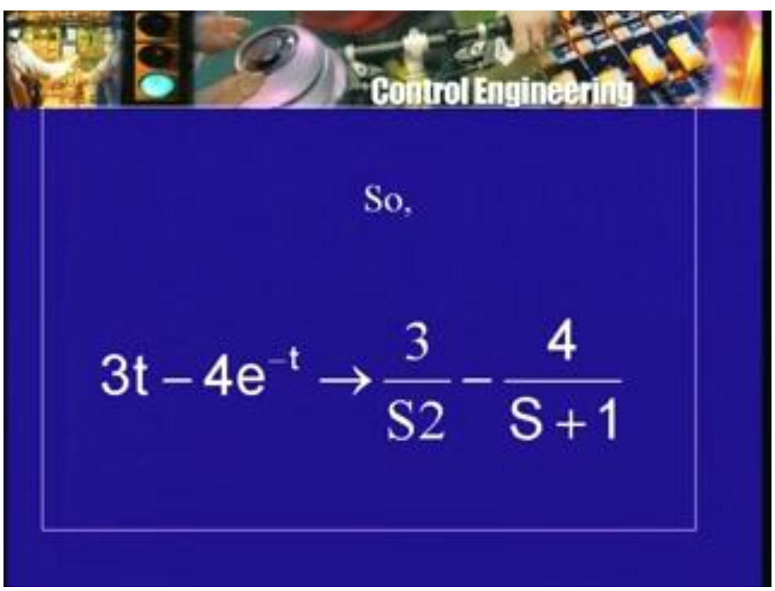

So what is going to be the Laplace transform of 3 t minus 4 times e raised to minus t and I am showing like this way that is this function is transformed into, what function of S, for t is was 1 by S squared but now, I have 3 times t. So I will have 3 divided by S squared for e to minus t it was 1 by S plus 1 but now, I have minus 4 times that and therefore, I will write it as minus here and 4 here or I can write alternately plus and with minus 4 in the numerator of the fraction.

So the Laplace transform of this function 3 t in minus 4 e to minus t is the function 3 divided S squared minus 4 divided by S plus 1 and of course, what applies to a sum of 2 terms here, will apply to sum of 3, 5 50, 100, whatever number of terms with different multipliers here 3, 4 or minus 4 and so on. I do not have to state separately a result that is Laplace transformation of u 1 minus u 2 equals the Laplace of u 1 minus Laplace transform of u 2 because u 1 plus u 2 can also be regarded as u 1 plus minus 1 times u 2.

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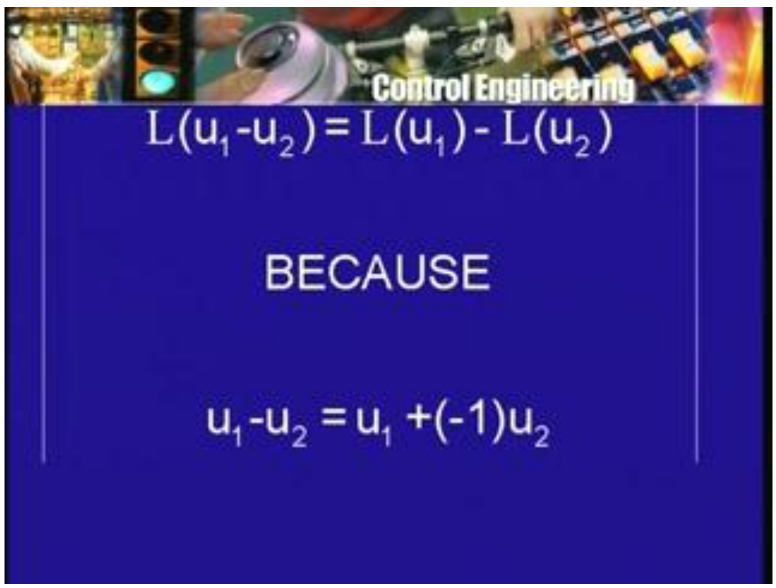

So, we have 2 properties of Laplace transformation so far and we can see how there are going to be useful. If you know the Laplace transform of some function, we easily get the Laplace transform of their multiples, we get also the Laplace transforms of their sum or sums of different multiples of them. So starting with say only t and e to raised to minus t, I can build a whole class of functions. For example, alpha t plus beta into e raised to minus t this is not just one function for each choice of alpha and beta it gives me a function.

So it gives me infinitely many functions parameterized by alpha and beta then, I can find out the Laplace transform of each and every one of them because I know the Laplace transformation of t, the Laplace transform of e raised to the minus t and because I know and remember, these 2 properties of the Laplace transformation. If you did not remember those properties then, you would struggle with this multiply by e raised to minus t integrate from 0 to t, find out integral take the limit of results as t tends to infinity and all that but, if you remember the 2 properties of Laplace transformation and if you remember the Laplace transform of the 2 functions that are occurring here then, the Laplace transform of all such sums, each one of them can be found out quickly.

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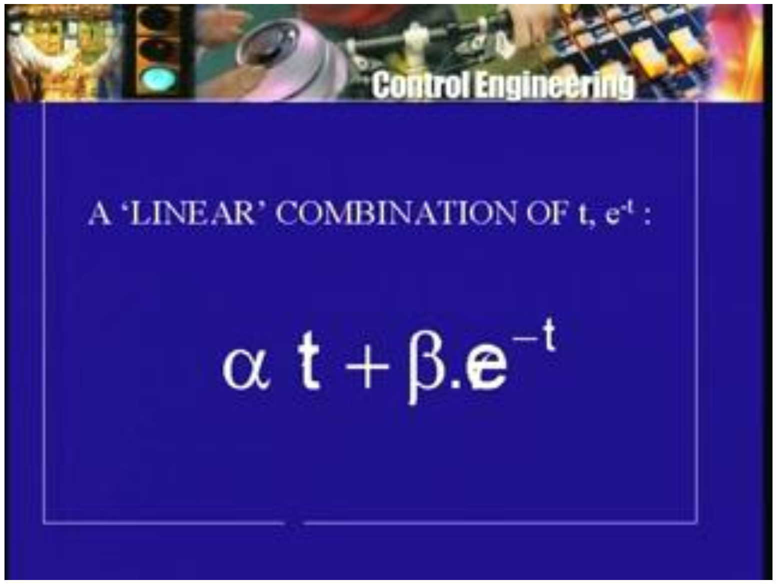

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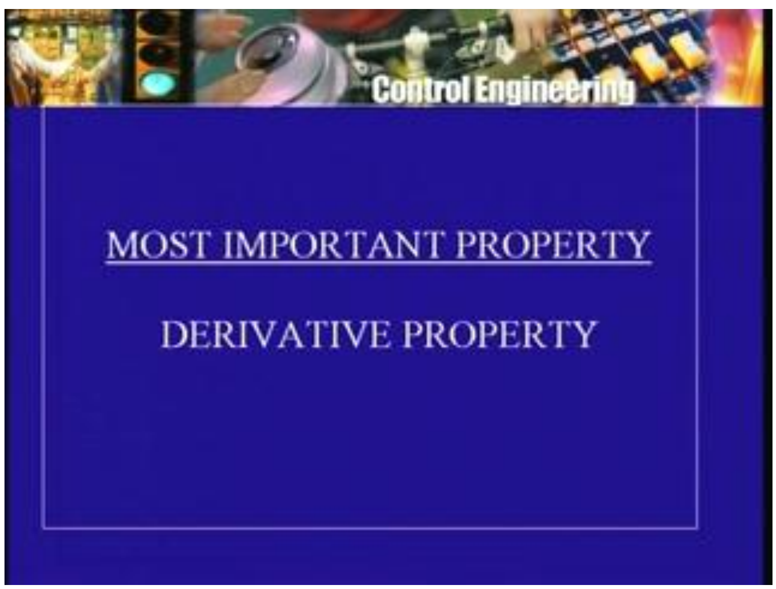

Now, the third property of the Laplace transformation which is the property that makes it useful for the solution of differential equation and the third property, therefore involves a derivative and therefore I am going to call it, the derivative property of the Laplace transformation. It is property that involves derivative and it is a property that is makes it useful at for the solution of differential equation. The differential equations are derivatives, the Laplace transformation has this nice property as we will see and therefore, we can use it for the solution of differential equations. What is the derivative property? We have to a little careful with this and as I told you there is always the region of convergence etcetera.

So I am not going to state the result in its fully rigorous form that is in its fully correct form, but its good enough for our purpose. So let us say, I have the input signal u or I have the signal u of the function u and this function u is differentiable that is it has a derivative. Unfortunately, not all function are differential although, most of the functions that we come across as engineers are differentiable. But there are functions which are not differentiable and there are functions which can even occur in an engineering application which are not differentiable or at least not differentiable at some points as we say. Incidentally, there are some functions in fact, there are many function for which the Laplace transformation does not apply at all because the Laplace integral does not converge at all, no complex number S that integral u t, e raised to minus s t dt from 0 to t, limit of that as t tending to infinity may not exist. For any complex number s what so ever there are such that bad function but there are mostly perhaps good functions, which occur in our practice which are one say Laplace transformable that is for which the Laplace transformation can be applied in the sense, the Laplace integral converges for at least a large set of values of S.

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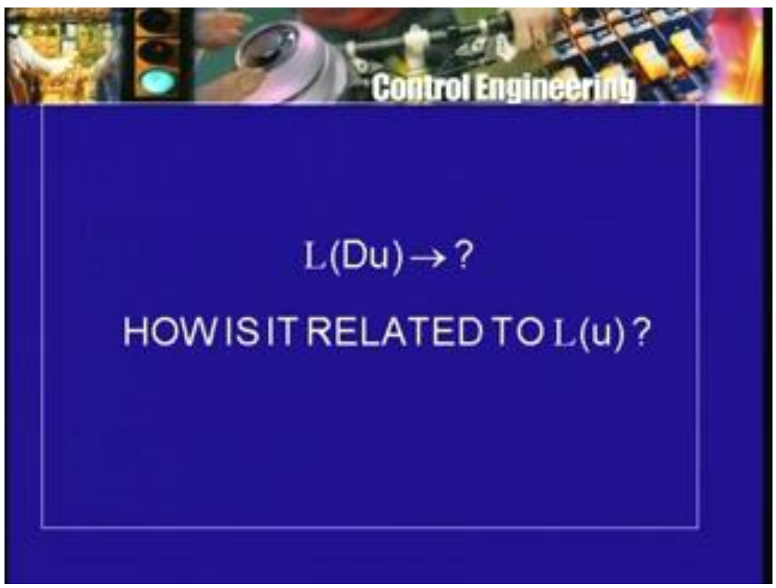

So, here is the function u I am going to assume that is Laplace transformable that is for some s, capital U of s is well defined then, I am assuming that the function is differentiable then, I am going to assume that the derivative and by now, you are familiar with this notations, I am not going to write D u by D t and I am just going to write capital D u, meaning the derivative of u or in fact, the derivative operating applied to the function u produces another function. The derivative operation applied to the displacement function produces the velocity function. So u the derivative of u, I am going to assume that the derivative of u is also Laplace transformable.

So, I can talk about L of that, so that is a function of s. On the other hand I can talk about L of u that is another function of s, what is the relationship between these 2 functions, the Laplace transform of the derivative and the Laplace transform of the original function or to put it the other way the Laplace transform of a function and the Laplace transform of its derivative. There is a property and that is what is called, I am going to the derivative property of the Laplace property of the Laplace transformation. However, that derivative property requires k f to apply and is a little more difficult to remember than the additivity and homogeneity. I am just going to write it at the moment and we will look at it more detail by taking specific example. The derivative property can be stated as follows, L u and L D u, what is the relationship between the two?

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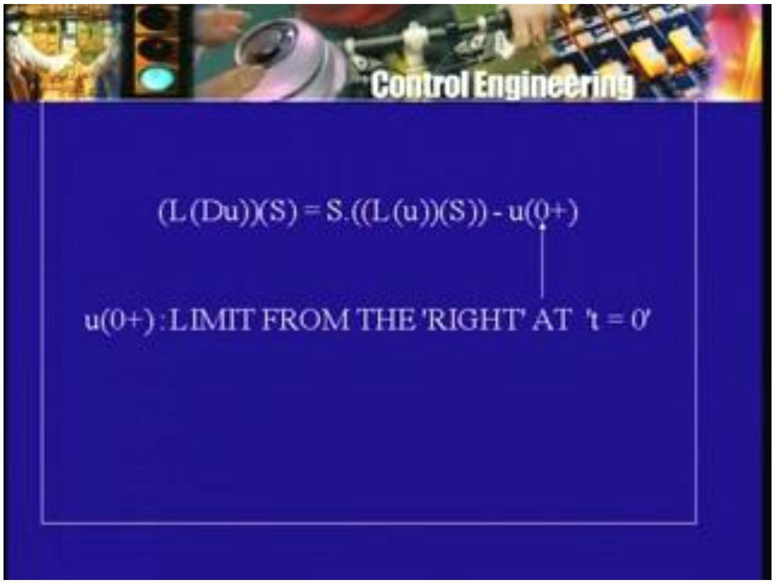

Well, the relationship is given by L Du, the Laplace transform is a derivative function its value at a complex number s such that this Laplace transform exists for that complex number s is given by s times, the Laplace transform of u its value at s minus a number, which is written as u and I am going to write here 0 plus. There are 2 terms here on the right hand side, there is the Laplace transform of the original function but that gets multiplied by then, a complex number S but that is not all, there is something that gets subtracted namely, minus u 0 plus, what is this u 0 plus?

This u 0 plus is what is called the limit of the function u at time t equal to 0 but this 0 plus, tells you that it is the limit from the right and that is because of a technical mathematical problem that I have to talk about the limit from the right, rather than u 0 or many function of course in place of u 0 plus, I can simply write u 0 and the result will be correct. But the precise result, the correct result of the Laplace transformation theory will have like this. The Laplace transform of the derivative its value at S is S times the Laplace transform of the original function, the value of that at S minus u 0 plus. The limit of the original function at t equal to 0 but the limit from the right.

Now, you can see how this might be useful because on the left hand side, I have the derivative, derivative of u but on the right hand side, I do not have the derivative. So then, I have got rid of the derivative by taking the Laplace transformation or by applying the Laplace transformation and we will see that this is what makes it useful for the solution of what kind of equation? Ordinary, linear, differential equation with constant coefficient. We will take some examples to illustrate this. First the derivative property and then, its use for solution for resolution of differential equation of that kind.