Control Engineering
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Lecture – 18

Before, we take a look at the other schemes of feedback namely integral control derivative control and even more general types of feedback control. We should look at the transient response of the transient behavior of our speed control system, why because as I said the earlier, in the feedback scheme that we have talked about namely proportional control. It looked as if you could reduce the steady state error e, when disturbances take place to any arbitrarily small amount by increasing the gain K a sufficiently.

So, there does not seem to be any problem. Now, the nowadays of course high gain amplifiers are readily available. So, that is not a big problem, so why not just have a very large gain K a so that the steady state error because of disturbances will be reduced considerably. Reason for not increasing K a to a very large value is because of problems that occur in the transient behavior of the system and so, this is something that we should take a look at before proceeding further there is another reason. When you introduce other kinds of feedback the transient behavior is going to be different and who knows the transient behavior, may be worse than what it is with proportional feedback.

So transient analysis or analysis of the transient behavior of control systems is an absolutely a must and you have already studied to some extend methods, for finding our, the transient behavior or in fact, the behavior of a system, as it progresses in time. Sometimes there is a steady state sometimes there is no steady state, the input may be periodic and so on and so forth. So we will take a quick review of some methods of finding out the behavior of a system. For this purpose then, we have to go back to a description of the system that applies not in the steady state only but applies at any time, that is applies irrespective of whether the signals associated with the system, the variables associated with the system remain constant or not.

So this means that some things which we had ignored earlier cannot be ignored any longer and what is it that we ignored earlier, when we looked at our motor problem because of steady state consideration that is because we said that or write the torque will be constant, the applied voltage will be constant. So after some time, the armature current will be constant the motor, speed will be constant and because of this, we made some simplifications or we simply ignored or deleted some terms in the equations. These terms were arising out of some kind of inertia of the system. In the electrical part of the system that is the armature circuit, we ignored the effect of armature inductance or any inductive effects in that circuit, specifically we ignored the effect of the armature inductance L a or to put it differently, we put it equal to 0.

Of course, you could have similarly ignored the armature resistance but if you ignore too much then, what you will get will be such a poor approximation to the actual behavior that will be of no use and now, that we are going to look at transient behavior that is, when the signals and the variables are no longer constant but might change either because of disturbances changing continuously or because you are not really reached the steady state yet, the motor has been switched on but it has not reached its final speed and so the speed is increasing. So L a cannot be ignored because the armature current may not be constant, the armature current starting from the moment the motor is switched on, will finally go towards its steady state value but during this time it will be changing.

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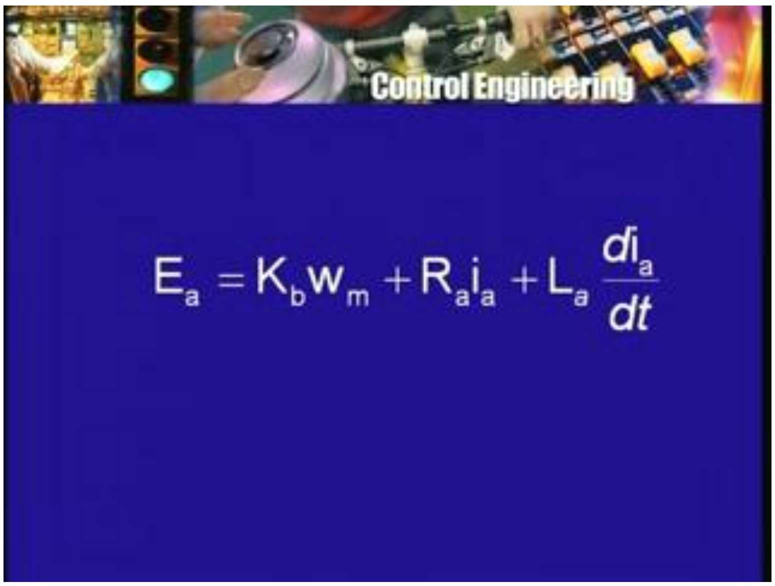

So we have to take account of L a and in what way in the equation that we wrote for the armature circuit, we had armature voltage equal to the back emf of course, that remains as it was K b into omega m plus the armature resistance drop that remains as it is. In fact we called it the armature drop but now, we have one more term associated with the armature and that will be the inductance drop. So, modeled by as L a into di a dt, how to measure this armature inductance etcetera. This is something I am not going to discuss in machines courses one does talk about these inductances and other parameters associated with the machine and in fact, with an AC machine the situation is even more complicated. Let us assume that somehow this L a has been found out.

So many Henries or milli Henries, so now we cannot ignore this term L a di a dt because ia is not constant, so di a dt is not 0, so this term L a di a dt is not 0 and it may not be very small either, it may be comparable with the armature resistance drop or it may be even larger than that. So we are not going to be able to ignore it as a result now, our equation is no longer an algebraic equation and that too it was a very simple linear algebraic equation in which the variables E a omega m and I a were simply multiplied by some constants. It is no longer and algebraic equation but it is still a linear equation only, it is now a linear differential equation, linear differential equation, that is an equation in which derivatives of the variables also appear in addition to the variables themselves.

So in this equation we have not only I a but also its derivative di a dt, the rate of change of armature current appearing in the equation. Of course, we have omega m, the speed, instantaneous speed and the applied voltage also appearing in the equation. The equation is linear, at this moment, I would just like you to think about this word linear and be ready to answer the question, why is this equation said to be a linear equation? There is a simple answer to that, simple in the sense an answer which is good enough for some purposes but not a general enough answer and there is a more advanced or a sophisticated answer as to why this equation is a linear equation. It is a differential equation because there is a derivate appearing in that equation.

This is as for as the armature circuit is concerned therefore, we no longer have a simple linear algebraic equation but we have a linear differential equation, something similar is happening in the mechanical part of the system. We had ignored the moment of inertia of the motor and load combined j, this like the inductance plays a role when the speed of the motor is changing or there is angular acceleration of the motor or the shaft and so it is derivate, the derivative of the speed will appear in the equation and the equation is no longer, what it was earlier?

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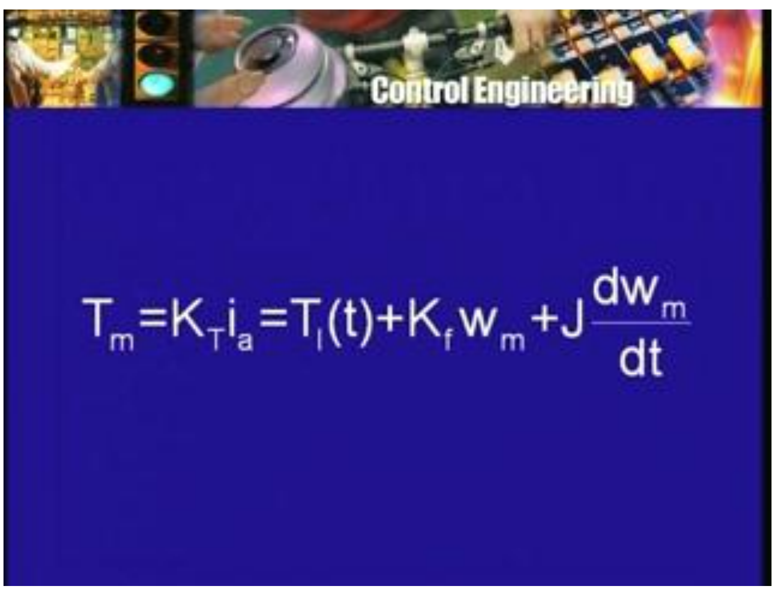

$$T_m = K_T i_a = T_l(t) + K_f w_m + J\frac{dw_m}{dt}$$

But it will be now, motor torque equal to the torque constant into the armature current that remains as it is but now, this is made up of 3 parts, one is the load torque as before the second term is the torque required to overcome friction that also remains as before K f into omega m. But now I have a third term which is the moment of inertia multiplied by the rate of change of the speed or the angular acceleration of the motor, J d omega m dt.
Earlier, we had only 2 terms, the d omega m dt term was not there but once again, if omega m is not constant that is we are not considering steady state then, d omega m by dt may not be small and even, if it is small this product J d omega m dt, may not be small because the moment of inertial can be quite large and so this term, cannot be ignored in comparison with the frictional term or even in comparison with the load torque.

As so we have to look at that term also and as a result our equation is no longer an algebraic equation which it was earlier. It was a linear algebraic equation involving the 3 variables or functions, armature current, motor speed and load torque multiplied by coefficients which were some numbers such as K t and K f and load torque was simply multiplied by 1, if you want to think of it as multiplied by anything. We now have this additional derivate term, so we have a differential equation and it is also a linear differential equation.

So, we have now the behavior of the motor and load, when armature voltage is applied directly described by 2 linear differential equation. In one of them the derivate of armature current occurs and in the other the derivative of the motor speed occur. There is something more that can be said about these equations not only are these equations linear but the coefficients that multiply these variables, they are not functions of time but they are numbers or constants such as K b, K t, R a, L a, K f, J and coefficients one multiplying the applied voltage and the load torque.

So they are not only linear differential equations, they are linear differential equations with constant coefficients. We could abbreviate it by LDE linear differential equations, W with constant coefficients LDEWCC and we have not 1 equation but we have 2 equations. Now the problem that therefore one can consider is the following, the behavior of the motor is describe by these 2 linear differential equations with constant coefficients. Suppose, we apply a some armature voltage and keep it constant. So that is a special of case of course later on, when we think of feedback, the armature voltage that is applied is not going to be kept constant because it is produced by generator etcetera. But for now, let us consider the special case and the applied armature voltage is just a constant, it does not change with time 230 volt from the movement, the motor is switched on, the applied voltage is kept at 230 volt.

Similarly, we could say that the load torque is also kept constant. Let us say 30 Newton meters or whatever it is or we could allow because it is more realistic. For the load torque to change, so we may consider that the load torque, T l will be a function of time and we could consider various ways in which the load torque may change. So what then, is required is to obtain from these 2 equations, how the motor speed omega m will vary as a function of time that is what will be omega m, t the motor speed as a function of time, what is going to be omega m t, if E a is given a constant value and the load torque variation T l of t is given by some particular function.

So T l of t is given find out omega m of t, E a also is given, find out the variation of the speed. Of course, as you know because you have looked at some simpler differential equations in your mathematics courses as well as in your electric circuit or network theory courses that in order to solve the equation, it is not enough to only know the inputs in this case E a and T l , Tl of course is going to serve as the disturbance input, E a is the other input but we also need to know, what are known as initial conditions, certain initial values of the variables, of which variables, the variables whose derivates appear in the equation. They are some time referred to as the dynamic variables, for this reason that is there derivatives appear not just omega m is a function of time but its derivative occurs in the equation not just I a but its derivative occurs in the equation.

So for such variables, we need to know initial values now, what do you mean by initial values, what we mean is we are going to look at the behavior of the system from some particular movement of time or instant of time onwards, what happens thereafter. As you know, it is convenient to choose that moment of time, the starting movement of time as been number 0 that is we call it, t equal to 0, start your clock at that moment when perhaps you have switched on the motor. So that is what is meant by initial now at that time, what will be the values of these dynamic variables I a of t and omega m of t.

Now, of course if the motor has been at rest, if the power supply E a has not been switched on then you do not expect that there will be some current flowing in the armature because after all what will cause flow of that current. So in this case, we expect that ia the armature current at time 0 will be 0. Similarly, I said that the motor is at rest, so omega m 0 is 0. So these will be the 2 initial values the armature current is 0 at t equal to 0, the motor speed is 0 at t equal to 0 that is the initial values of these dynamic variables, motor speed and armature current are both 0.

Now this may not always be the case for example, suppose the motor was running at certain speed, with certain applied armature voltage E a and some load torque T l and suppose at some moment of time, you decided to change the applied voltage or the load torque was change at that moment of time. The motor was in the steady state, the motor was running at a constant speed, with a constant applied voltage with a constant torque, load torque etcetera constant speed, constant armature current, everything was as if because this steady state but then, suddenly or deliberately, Ea was change or Tl was changed.

Now, in that case we start our investigation from this movement. When these changes are taking place so we will say that that will be our T equal to 0 that will be our initial instant of time and we will look at the behavior from there onwards or thereafter and therefore then, the initial values will not be 0 because the motor was already running, the armature already had some current. So the armature current at that movement of time of course, we may choose to call that movement of time 0 once again because it is new investigation. So then I a (0), may not be 0 and of course the motor was running. So omega m 0 is also not 0.

So the initial values may not be 0, they may both be non-zero, what as a special case when the motor was address and the circuit was connected electrically, the armature and the motor speed were 0 and therefore, the initial values will be 0. So the problem is given a system or a set of differential equations. In this case linear differential equations with constant coefficients given this specific input functions, the behavior of the variables E a and T l, may be both of them are kept constant or one of them E a is kept constant, T l is varied in a certain way or both have changed in some specific ways knowing, how E a and T l are going to change with time knowing the initial value obtained or calculate or find out omega m as a function of time and perhaps I a also as a function of time.

This is the problem of solving differential equations with initial conditions or with initial values. Now there are many techniques for solving differential equations especially equations which are linear with constant coefficients. So for linear differential equations with constant coefficients there are a number of special techniques available, of course in the last analysis one can always use a computer program and a package of computer programs to solve a system of equations, numerical analysis procedures and programs based on them, numerical analysis procedures like the Runge Kutta technique or the Milne technique and so on, there are many that are available. They are numerical methods for obtaining an approximate solution of a differential equation not necessarily linear, not necessarily with constant coefficients but for the moment, we are only looking at a very simple problem.

We have systems of equations which are linear with constant coefficients. So it is fortunately possible to do something about them and get some idea of what the solution will be like without submitting it to a computer program and then, waiting for whatever comes out. As I said the purpose of all the study is not just to compute some numbers but to get some insight into what is possible, what is going on and so forth. One applied mathematician said by name Richards Hamming said this very nicely, the purpose of computing is insight not number by that what you meant of course was the purpose of computers is not only to get numbers but to get some insight into what is going to happen or what is happening and to consider alternatives and make changes and so on.

The purpose of computing is not just to calculate or solve a particular problem and that is it that is of course important. But many techniques have to very studied, so that one can get some insight into what is possible what is expected and so on. So we will look at some of the techniques that are applicable for linear differential equations with constant coefficients and this will be basically a review of what you have studied already in your mathematics course and also perhaps in your course on networks or electrical circuit.
One of the techniques which is used and which we will use and control theory people have been using it for almost 60, 70 years now, is based on the use of what is known as Laplace transformation.

So this is one technique the technique of Laplace transformation of the technique that uses or applies the Laplace transformation. This is a very popular technique with control theory people, so certainly we will look at it and all of course should become familiar with this technique therefore that means that, we must know a little bit about what this Laplace transformation is an as a technique, how it is use to solve or to help solve linear differential equations with constant coefficients.

So this is one technique, the technique that uses the Laplace transformation but there are techniques which are little simpler than that and in fact, some of those techniques probably you are already familiar with from your mathematics courses especially, when you have a single differential equation but which may involve not only first order derivative but higher order derivates of the unknown function. For such an equation that technique is applicable of course that technique has to be use with certain modifications and certain care and so on.

Let me give an example, we can of course make a beginning with a simple equation, the simplest differential equation will contain, what it will contain linear differential equation with constant coefficient. Since, it is a differential equation the derivate must appear, so what is the simplest derivate of a function, it is the first derivate or the derivative of the function itself.

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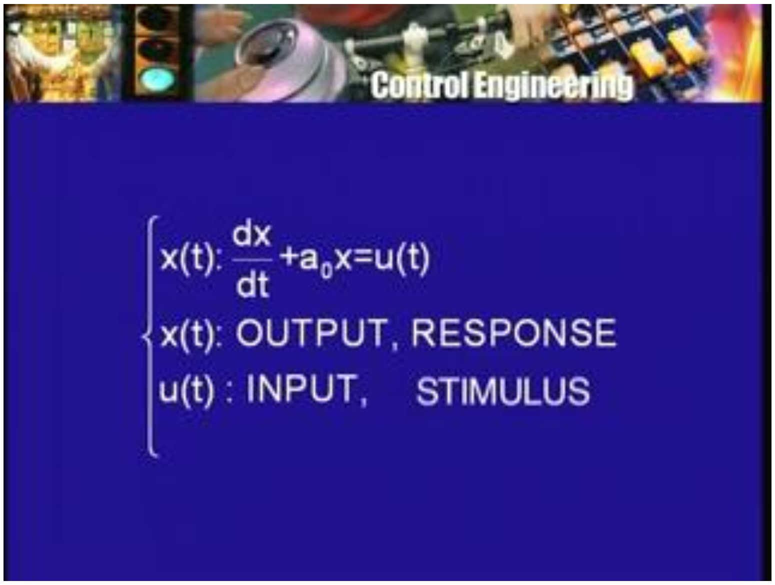

So suppose the unknown function of the system response or output function is x then, its derivative and for us these are function of time because we are interested in what happens as time passes, our variable are functions of time. So x is, x of t really function of time, we have its derivative dx dt in the equation, the derivative must appear and so a general first order and that is why, it is called a first order linear differential equation because on the first order derivative occurs in it will be something like dx dt plus let us say, some coefficient and I will call it a 0 and you will see why I am going to call it a 0 plus a 0, x equal to a function on the right hand side which is regarded as input.

In mathematics one may use for it the symbol y, but in control theory practice for the input a general symbol that is used is u for some reason, not u is not from English but it is from another language namely German that the letter u has been selected. So the input, we call it u of t of course in a particular case, it may be armature, it may be something else, it may be a force or what not but it is a function of time which is the input. We have the response or the output x but its derivative also appears in the equation and I could have put a coefficient with the derivate but that coefficient must not be 0 otherwise, there is no point in writing the derivative. So, if that coefficient was let us say a 1 then, I could divide the equation through by a 1 and therefore I can imagine that dx dt has only coefficient 1.

So I have the simple first order linear differential equation with a constant coefficient for only one function x, x is the output or the response function. It may be the response of a system 2, an input function or sometimes one uses the word stimulus, this stimulus response comes from physiology and psychology. In control practice, we use the words input and output. So u is the input, x is the output of the system and the system behavior is describe by this simple first order linear differential equation with constant coefficient and now, we want to solve this, one technique of course as I mentioned is the Laplace transformation technique but is there any simpler technique. You will recall that there was a simpler technique, in fact that technique starts off with a differential equation which is even simpler namely dx dt plus a 0 x equal to 0.

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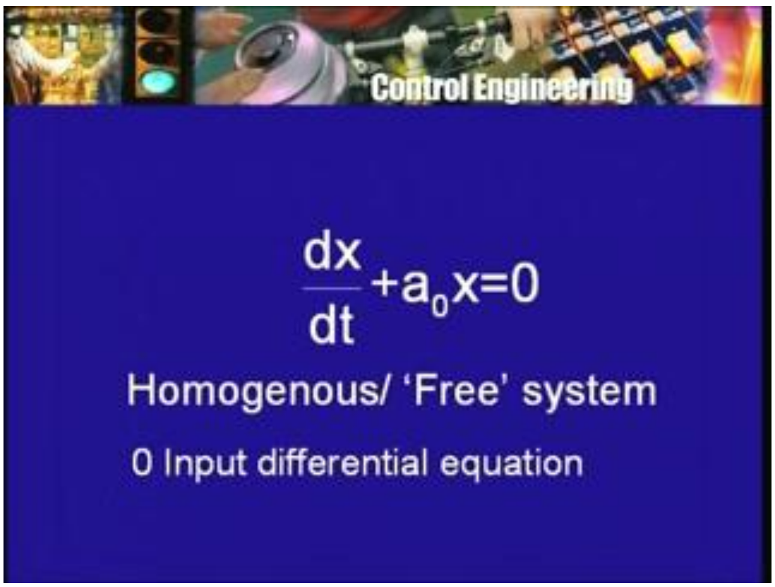

The input has been put equal to 0, what is such a differential equation said to be, it is said to be a homogeneous differential equation or we could call it in control theory language as a 0 input differential equation or we can say that the system is free, the system is not subject to any force or any external action. It is a homogeneous differential equation, when the right hand side is not 0 or the input term is present then, one talks about the system being forced and this of course, this is a term that comes from mechanics that is the actual force applied to a mass spring system or a mass spring dash pot system and therefore I that appears as in input in the differential equation. So one uses the word force and you say that the system is forced.

So, a simpler case is when you have a homogeneous system or a free first order linear differential equation with constant coefficient. We just have dx dt plus a 0 x equal to 0. Now what is the solution of this equation now, of course everybody ought to know the solution of this equation because the solution of this equation is what is called the exponential function and in fact one way of defining the exponential function is as the solution of a differential equation like this.

So, what is the solution of this equation, in fact I have to be careful, I should not say the solution because this differential equation dx dt plus a 0 x equal to 0 has not one but infinitely many solutions. So, I cannot talk about the solution unless I am able to select or specify one of them that is why one talks about not the solution but the general solution of this differential equation, by that one means the whole set of solutions. Fortunately, the all the solutions can be describe by one single formula and what is that formula, all of you must know it, if not then you have go back to your first year and read up about it, no electrical engineer can a afford not to know about this function and about some other functions. These are sort of bread and butter in whichever branch of electrical engineering you peruse.

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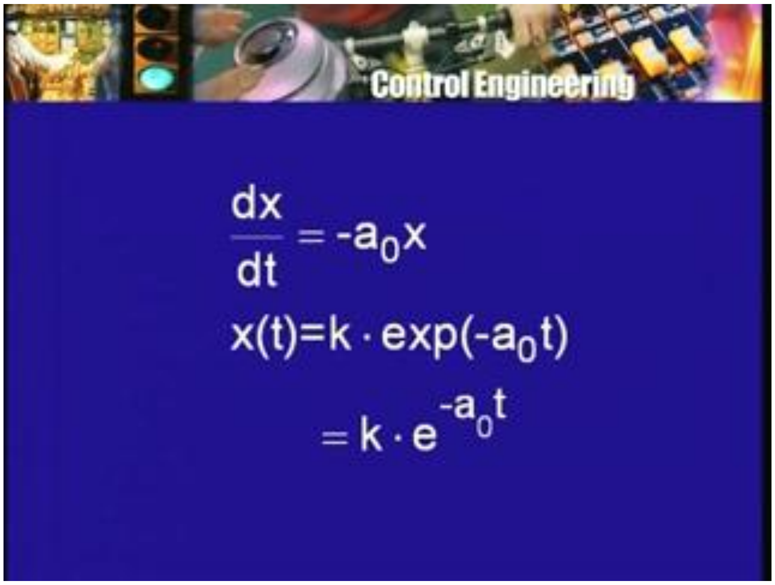

So what is the general solution of this differential equation. As you know it involves the exponential function and so, the solution can written as x of t, the solution at any time t, a particular solution among that class of solutions is given by x of t equal to some coefficient or some number k multiplied by x exp, this exp stands for the exponential function, exp of what, what is the argument of the exponential function.

Now this is where one can make mistake. In order to avoid the mistake, I will write the equation as dx dt equals minus a 0 x. When that is done perhaps1 cannot make a mistake and therefore the solution x of t will be k into exponential of minus a 0 into t or writing it in a different notation using a different notation k into e raise to minus a 0 t. This gives you the general solution, in the sense for any number k it gives you a solution and all solutions look like this choose k equal to1. I will get one solution e raise to minus a 0 t, choose k equal to 10 ,I will get another solution, choose k equal to minus 10000, I will get a third solution.

All of them satisfy the differential equation dx dt equal to minus a 0 x or equally well, the original differential equation dx dt plus a 0 x equal to 0 and so that is why this is said to be the general solution and by that we mean, this gives you the set of all solutions and to use a slightly better notation I will put it like this, k exp minus a 0 t where, k is any real number. This set is the set of all solution of the differential equation. As I said, we can take this as a definition of the exponential function or alternately, if the exponential function is defined in some other way which is what is normally done then, one can prove that this is a solution and some more theory is required to show that these are all the solution, there is nothing else. There are no other functions in the world which satisfy the differential equation.

So then, one can call this is a complete solution or these complete class of solutions of the differential equation. Now when I have a particular system which is governed by this differential equation of course, I expect the system to behave in particular way. So one of these is the solution that is applicable that is one of these will describe, what is going to happen, so how do you get that, how do you get that particular behavior of the system
and this is where the initial value comes in. For example, if you not only know that the differential equation is satisfied but you also know that x 0 is equal to 3 that is, you know the initial value of the response or the output then, from this class of solutions there is only one that will meet this condition. This therefore, a sometimes knows as a boundary condition, again the term is a more general1 from physics. In this case, it is only an initial condition, initial meaning at time t equal to 0, this initial condition then tells you which of these is the correct solution and what is you expect to find namely 3 into exponential of if a minus a 0 t.

So the linear first order differential equation with constant coefficient which is homogeneous, its solution, the general solution is consists of multiple of an exponential function and the particular solution, for a particular system under particular conditions will be determined by the boundary condition or the initial condition of the system namely the value of x at the time 0 that we have chosen as the initial time. We are going to look at the system from that time onward.

We are not worried about may have happened in the past, the control theory person usually looks forward to the future because he wants things to happen in a certain way in future, not to say that past is not important, the study of the past what happened during the lab, what happened during earlier runs of the system, all that is of course useful. But we are faced with the problem of what to do from this moment onward to make certain things happen. So that is the idea of this initial time or the initial condition.

So this is the solution of the simplest case, you should all be familiar with the graph of this function, what it looks like. Let me just draw it very quickly such graphs as I said and such functions are the bread and butter of electrical engineering and so one should be able to draw them with a proper shape, being engineers we cannot be very sloppy. Our drawings as far as possible should be neat and nice wherever possible one should use a scale to draw straight line, one should use a pair of compasses draw a circle. I am not doing it here, because if I do that it will require much more time. But I hope, my straight lines do look reasonably a like straight lines. Anywhere my exponential function will look reasonably like an exponential function and yours also must look like exponential function.

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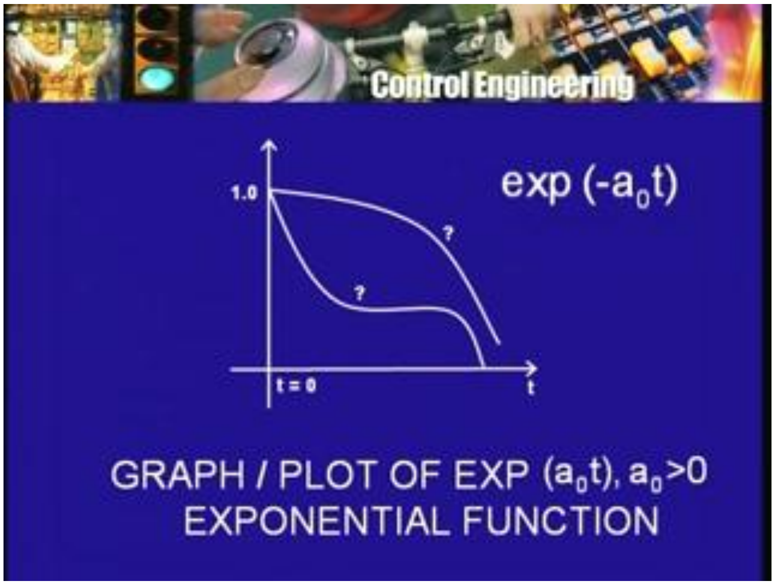

Now, these are not details which we can discuss in this course but they are very essential. So revise your graph techniques, graphs of elementary functions, how to draw them reasonably accurately in things like that. For example, if I want to draw the graph of exponential of minus a 0 t, what will it look like. Well, one thing is certain at t equal to 0 and this is my t axis shown as symbol t there, this is t equal to 0, if you says as just 0, the value of the exponential function at t equal to 0 is just 1, so this is1.0.

Everybody must know that that the exponential function when the argument is 0, exp 0 is 1, e raise to is 1, what next, what will be the graph of this function for t greater than 0. Do not rush to an answer because that depends on the number a 0, if the number a 0 is a positive number as it is in many cases, if the number a 0 is positive then what is the function going to look like as one says the function is going to decay, it is going to decrease continuously. In fact such a decay is known as an exponential decay for this reason because the exponential function for the reason that the exponential function turns up in many places.

So what does the graph look like. does it look like this, no, does it look like this, no. So what does it look like, it looks like some thing which on decreasing continuously but does not become 0. So if you draw graph where it shows crossing the t axis, I will send you back to first year or even perhaps to your 11 th and 12 th standard, no electrical engineer can draw a graph like this and say that is the graph of exponential of say minus 3 t, no simply not allowed go back and redo your earlier work.

So it cannot be like that also it cannot have any shape of this kind, it cannot have this kind of an oscillatory behavior, it must have a behavior which is continuously decreasing.

But the rate of decrease also goes on the reducing in absolute magnitude. So it is something like this, do not show it as intersecting the t axis but do not also show it very far away from the t axis. Of course I cannot draw the whole graph because t equal to 10 second, 100 seconds, 1 million seconds. I will need a very very long graph paper and tends to infinity cannot be accommodate any way, but this is what the rough shape should be like and associated with this is something called time constant and that is something you must also be aware of, associated with the decaying exponential like e raise to minus 3 t, let us say, is a time constant.

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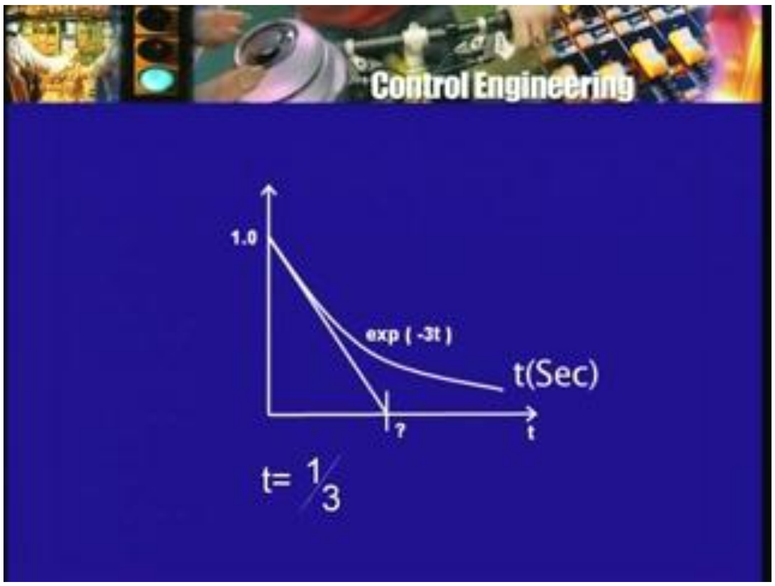

So, if the function is exp minus 3 t or e raise to minus 3 t, what is the time constant and what is the meaning of the term time constant. It is not enough to know that is the time constant, you must also know why it is called the time constant, what is the time constant for exponential of minus 3 t or e raise to minus 3 t, is it 3, no, what is it? It is 1 by 3 all right, it is1 by 3 but why is it 1 by 3 and why is it called a time constant. One reason and in fact, the main reason for using this expression time constant and talking about it, is the following look at the graph of the exponential function, it starts decreasing from the value 1.0. So the tangent is down words but the tangent becomes more and more horizontal as time proceeds that is the rate of change of the function goes on decreasing in absolute value, the slope is of course negative but the absolute value the absolute value of the rate of change goes on decreasing as time passes.

So the slope of the tangent goes on decreasing in absolute value but if you draw the tangent to this curve at t equal to 0 then, that tangent, if this is the tangent intersects the t axis at a point which therefore, can be interpreted as the time and this time is the time constant and that is why, the time constant can be then, defined as follows. It is that time at the end of which the function would have become 0, if it had decrease at its initial rate of decrease, initially it was decreasing at a certain rate, initially the tangent at a particular slope. If that slope instead of changing had remains constant, at what time it would have become 0, when at this time called the time constant.

So look at this function, think of it as say y of t or x of t equal to e raise to minus 3 t, look at its derivate which gives you the slope of the tangent then, write down some appropriate equations and verify that act one third of a second is a unit of time here t is in seconds then, at one third second this tangent would intersect. At that point this tangent would interest the t axis this is one interpretation of the time constant that is if the initial rate of decrease had be remain constant the function would be 0 at the end of the time equal to, duration equal to the time constant.

In this case, one third of a second but there is something is that also one can interpret and that is what the exponential function e raise to minus 3 t is not 0, when t equal to one third in fact, I told that this exponential function is such that it never becomes 0. Of course, practically speaking this function may be so close to 0 that practically you may consider that the motor is off, the current is 0 and go away that is a different thing. But the mathematical function exponential function never becomes 0, the function that we are looking at such as e raise to minus 3 t with exponent which is negative that is minus a 0, a 0 is positive, so minus a 0 is negative such a function never becomes actually equal to 0. It only tends to 0 as a limit as t tends to infinity therefore at 1 third second after it is the initial time, the function will not be 0 but it will be what, e raise to minus 3 into 1 third or it will be e raise to minus 1.

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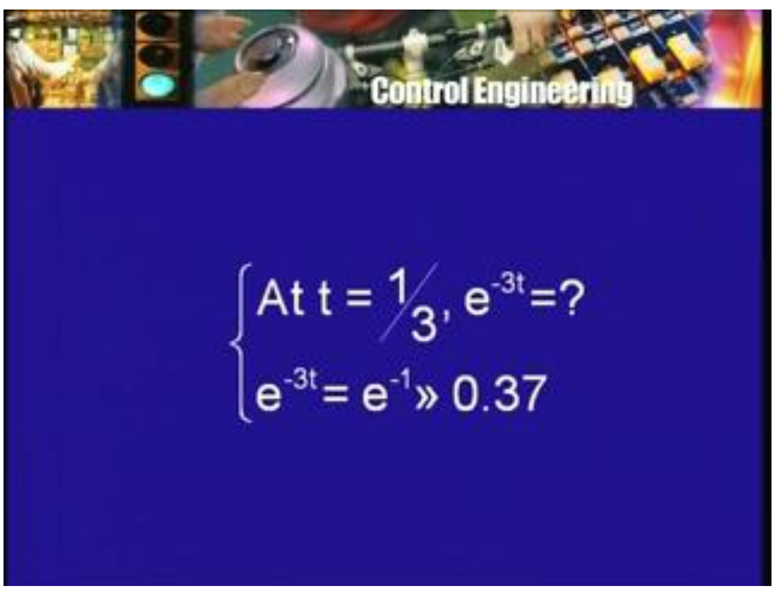

So how much is e raise to minus 1, e is what, e is some 2.7 something at least that one should know roughly and so, e raise to minus 1, 1 by e, look at your calculator and find out what e raise to minus 1 is up to maybe, 2 or 3 places of decimal. It will be something like say .37 perhaps check. So at the end of one time constant that is after at duration of one time constant, the function would have become about .37 or it would have become about 37 of its initial value, initial value in this case is 1.

So the time constant is also happens to be the time at the end of which the function becomes about 37 percent of its initial value. But the basic interpretation of the time constant is not this, but it is that rate of change at t equal to 0. The slope the initial rate of decrease and the time that that tangent intersects the t axis at time constant. So, if you want to remember this using this idea then e raise to minus 3 t to find out what the time constant is, think of it as e raise to minus t divided by 1 third.

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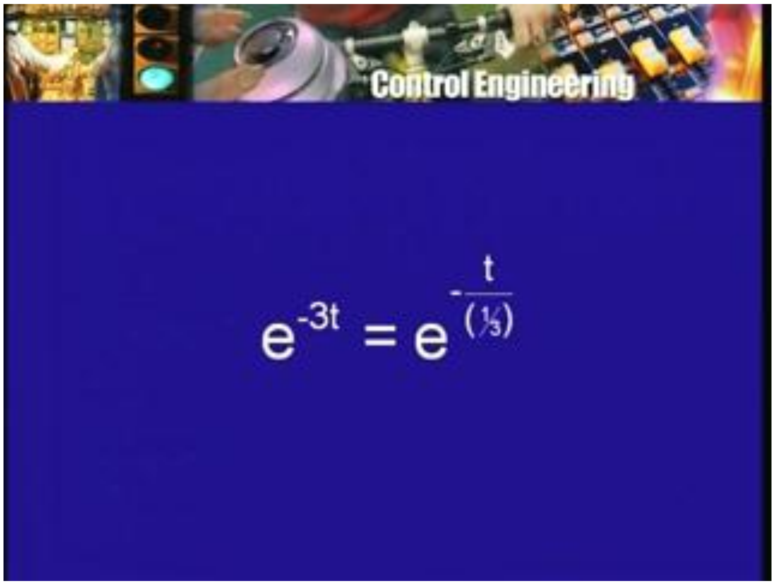

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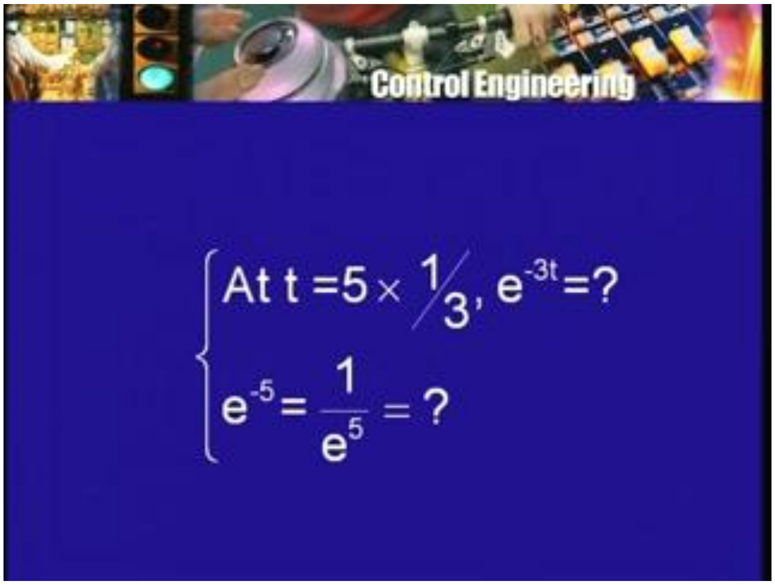

So when t equals 1 third this becomes e raise to minus 1 or 37 percent. So the time constant is 1 by 3 seconds and not 3, many students commit this mistake, exponential minus 3 t, time constant is 3, no, you ask them, what is the significance, what is the meaning, why do you want to consider this at all, why do you call it time constant. They have forgotten, do not forget it. It is very important because a lot of control theory work is based on this concept of time constant something happening within one time constant or within 2 time constant and so on. As an exercise find out what will be the value of exponential function e raise to minus 3 t. After a time interval equal to 5 time constants has elapsed that is for t equal to 5 times 1 by 3, what will it be, it will be e raise to minus 5, e raise to minus 5 is 1 divided by e raise to 5.
So what sort of a number is it while it will be quite a small number, just find out what it is is it about .01 or there about calculate and find out. So at the end of 5 time constants then, the function will become about 1 percent of its original value, if e raise to minus 5 is approximately .01, is not necessary to remember these numbers to any number of decimal places but one must a.36, .36 or third place of decimal or what not or 5 time constants make it is the function nearly become 1 percent or less than 1 percent of its initial value and therefore, it is almost 0. Well, if not wait for 5 more time constants then, look at e raise to minus 10, find out from your calculator what e raise to minus 10 is and then, you will get the answer as to what this e raise to minus 10 is. Is there a quick way of finding out what is e raise to minus 10, rough idea, is it 10 raise to minus 2, 10 raise to minus 3 10 raise to minus 10, how much is it. Yes, again something which1 can remember without too much difficulty is the logarithm of the e to the base 10, what is the logarithm of e to the base 10. It is not a very difficult number to remember if I remember it correctly it is .4343, it is .4343.

So what is the logarithm of e raise to minus 10, it is minus 10 times that 10. So what is the number whose logarithm is minus 4.343. So first convert it into the appropriate form therefore I will have to write it as 5 with a bar, this is the mantissa multiple by 657. So it is the antilog of .657 multiplied by a 10 raise to minus 5. Now what is the antilog of .657, what is the logarithm of 2 to the base 10. Now this is a number which we will come a cross later and again control theory people ought to remember that much logarithm of 2 to the base 10 is, look up your calculator up to 3 places of decimal or even 4 places of decimal 3010 or certainly .301, up to 3 places of decimal, .01 logarithm of 2 to the base 10. This will come across, later on it has something to do with, what is called 3 db per octave. This is something we will come across later on and that 3 db per octave is related to this .3010, it is almost .3 not quite but almost .3.

So what will be the logarithm of 4 to the base 10, 4 is 2 square so logarithm of 4 to the base 10 will be double of this. So that will be .6020 that will be logarithm of 4 to the base ten now what I have is the number whose log is .657. So it is greater than 4, what is the logarithm of 5 to the base 10 where, it is the compliment of the logarithm of 2 to the base ten because 5 into 2 is 10. So it will be .699 almost .7, our number is .657. So it is a number that lies between 4 and 5 multiplied by 10 raise to minus 5, so e raise to minus 10 is about something 4. some 1 or 2 digits multiplied by 10 to the minus 5.

Now the initial voltage was 100 volts then, it will be 100 volts into 10 raise to 5. So it will about 1 millivolt therefore, it may be considered as virtually 0 or negligible. So in that sense, after 10 time constants the function would have nearly become 0, nearly not exactly equal to 0. The exponential function does not become 0 for any time t, I am saying all this because the exponential function is extremely important in control theory work. The sinusoidal function also will come across soon and these basic functions one must know them very well. You cannot do good control theory unless you have some idea about these functions are, as functions of time, what their graphs look like and so on.

So going back then, so for the first order differential equation the solution is an exponential function and the exact they solution, the solution will be determined by the initial condition and it will be therefore, some multiple of an exponential function with a particular exponent. If the exponent is positive for example, if a 0 is negative then, the behavior of the exponential function is quite different and what is that called as instead of exponential decay one talks about it as exponential growth.

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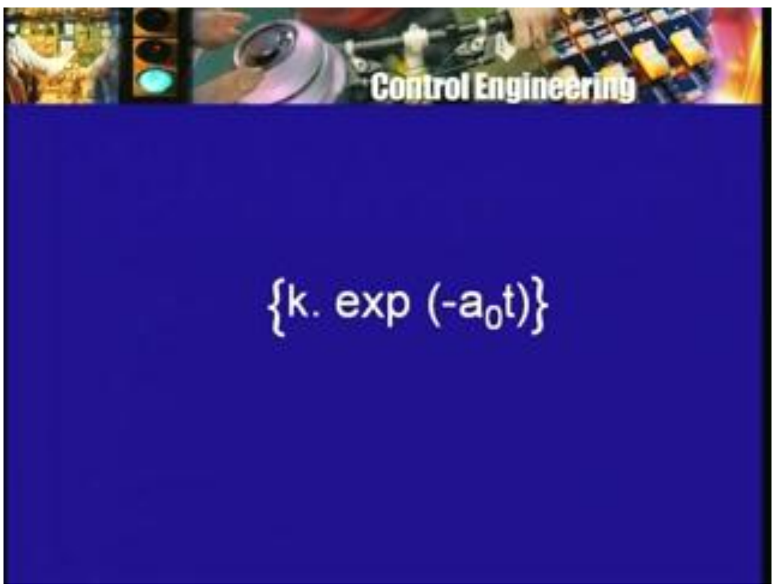

Now these are 2 terms which one must not simply use, one must understand what they mean and one must be able to draw the correct graphs for these functions. So what does exponential growth look like. Again, let us say we plot this time e raise to 3 t instead of e raise to minus 3 t. Now once again, a mistake where in place of e raise minus 3 t, you write e raise to 3 t, is not acceptable, the 2 functions are drastically different. If I get e raise to 3 t in my control theory work then, I will be very very suspicious and unhappy but if I see e raise to minus 3 t that is no problem.

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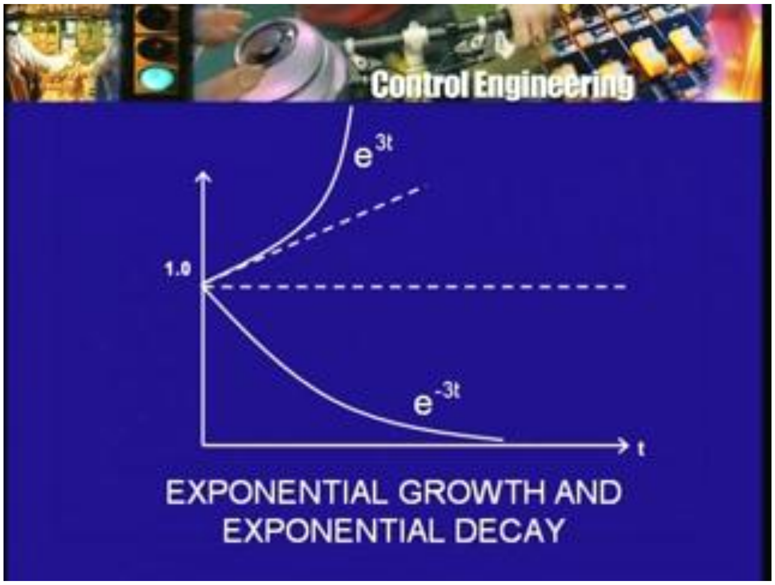

So this function x 3 t or e raise to 3 t, what will the graph look like. At t equal to 0 it has the same value 1.0, there is no problem. But now, as t increase the function keeps on increasing. The exponential decaying function kept on decreasing but never becomes 0, the exponentially increasing function keeps on increasing and its rate of increase keeps on going up so the graph the slope of the tangent becomes steeper and steeper and the what is the limit of it, as t tends to infinity, it is infinity. That, this function can become as large as you wish, if you wait for sufficient time. Instead of e raise to minus 10 you can look at e raise 10 and what will that be e raise to 10 will be 10 raise to 4.3 something.

So it will be of the order of 10 raise 4, a multiple of 10 raise to 4 greater than 10 raise to 4. So e raise to 10 will be about 10,000. So at the end of 10 time constant the function would have a value which is about 10,000, if the initial value was 1 volt then, the value at the end of 10 seconds or 10 time constants, 10 in this case, 10 time constants will be at the time constant is just one, it will be 10,000 volts and that just may not be possible. The system will have blown up a fuse would have blown somewhere. So this is the exponential growth and the other one was the exponential decay. With the exponential growth, there is no such initial slope kind of thing conveniently defined.

So the time constant idea comes basically from exponential decay but it can also be applied to exponential growth. That is, the same coefficient plays a role in the rate of growth as it plays in the rate of decay when the coefficient is negative, instead of positive. So this solution of the first order LDEWCC homogeneous is an exponential function multiplied by an arbitrary coefficient, that is the general solution, what happens in the particular problem depends on the initial condition of that problem and if, we know the initial value then, that is that multiplying coefficient.

So the first order differential equation was very easy to solve, once you know what the exponential function is and then, you are able to actually show that the exponential function satisfies the first order differential equation and in mathematics it is shown that it is the only solution of that equation. Now, we can go on to the second order differential equation , so what will be the second order LDEWCC once again, why is it called going to be called second order because the second order derivative will appear in it. We are still going to look at linear because of that there will be something more and with constant coefficient.

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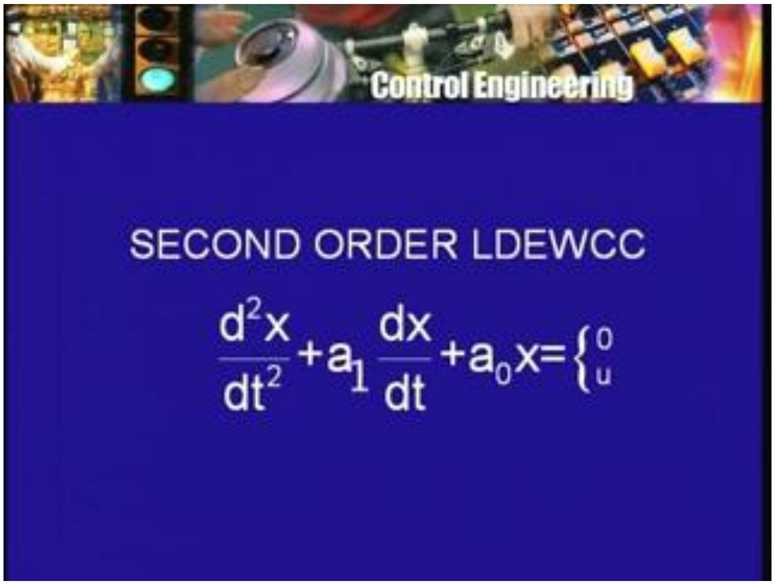

So the second order LDEWCC will look liked square x dt square now or the second derivate of x plus a 1 dx dt plus a 0 x, if it is homogeneous on the left hand side I will have 0, on the right hand side I will have 0, if it is not homogeneous, if it has an input or a forcing function. Then, it will be that function u which is our generic symbol, our standard symbol for an input. So this will be the second order differential equation. Now you will see, why I wrote that coefficient as a 0 first and now, I am writing 2 coefficients a 0 and a1, a1 multiplies first derivative a 0 multiplies the function itself or if you wish, I can call it the 0th derivative and the second order derivative is multiplied by just 1 therefore, I do not write it if you do not multiply by some number, I would have divided the equation by that number and therefore made that coefficient 1.

So this is the second order LDEWCC, now how to solve this. As I said you know it, so we will look at it very quickly. But before we do that a slightly different way of writing this equation which we have used earlier, I will use it again because it is very convenient and compact will denote this operation of differentiation by the symbol capital D. So capital D, so will stand for the operation of taking the derivative of anything. Therefore, we will rewrite this equation as D square x and I will, I am saying D square although I do not really by that I mean some number squared but D applied twice, take x apply the operation D to it that is, take its derivate and apply the operation D again that is take the derivative of that derivate that is exactly what d square x is d t square is.

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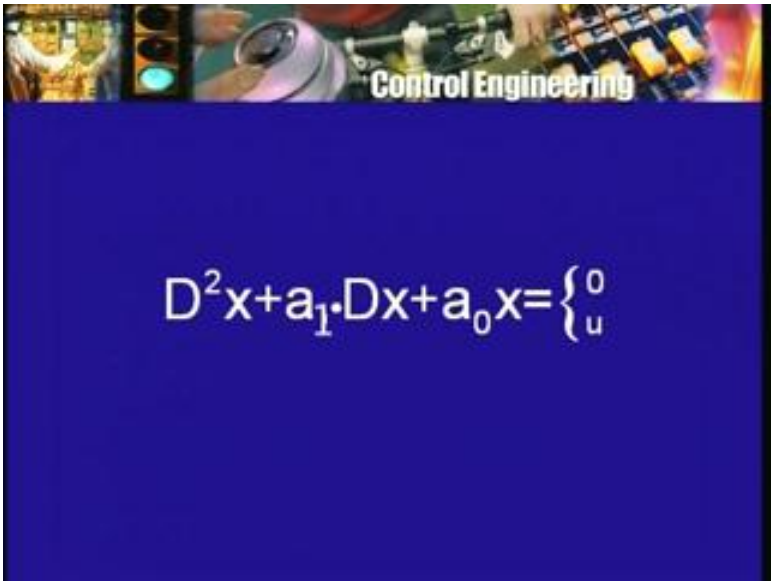

Ao D square x plus a1 times D x, D x is simply the derivative of x plus a 0 times x equal to either 0 or the input u. We will see why it is advantages to write it in this form okay.