Control Engineering
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Lecture - 16

Let us try applying the Mason's rule to a more complex signal flow graph. Let us a draw graph with 2 input nodes and 4 other nodes this is not necessarily representing some actual physical system, it is just an example to illustrate the application of Mason's rule.

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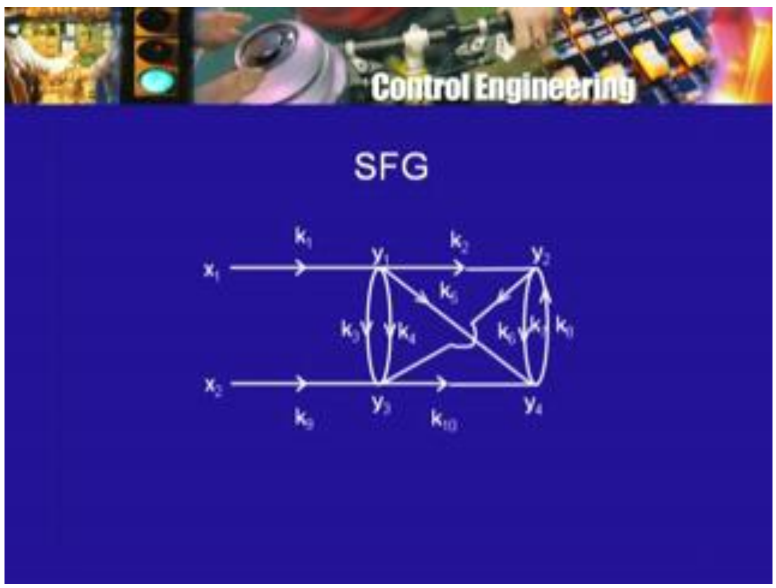

So let us call the input nodes X 1, X 2 and let us distinguish the other nodes which are not going to be input nodes by calling them say Y 1, Y 2, Y 3 and Y 4. So there are 2 input nodes X 1, X 2 and there are 4 other nodes Y 1, Y 2, Y 3, Y 4 and I am going to put transmittances between these nodes as we will see, I will put quite few of them crisscross going this way that way I could have made it even more complicated but this is good enough, I am going to label the transmittances in just some arbitrary order as K 1, K 2 etcetera.

So how many edges do I have here this is K 1, K 2, K 3, K4, K 5, K6, K 7, K 8, K 9 and K 10. So I have 10 edges and 10 transmittances and there are these 4 nodes Y 1, Y 2, Y 3, Y 4 which are not inputs nodes and there are the 2 input nodes X 1, X 2. Let me remind you what all this stands for at the node Y 1 then, we have a summation and what is that summation, we have to look all incoming arrows find out what the transmittances are and multiplied by this transmittances the signals at the other end. So, we have Y 1 equal to K 1, X 1 because that arrow is coming in from X 1 with a transmittance K 1 plus I have this term coming from Y 3, K 4, Y 3 and then I have a term that is all I have just these 2 terms coming in from Y 1.

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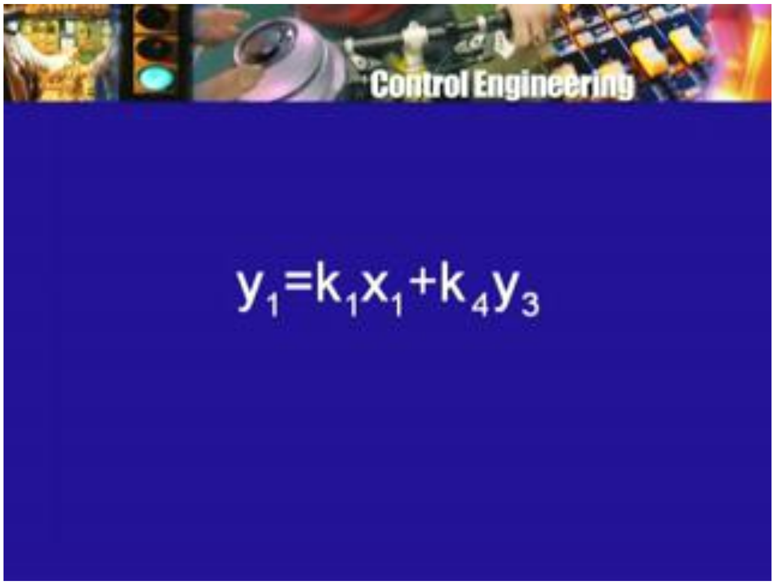

So the equation for Y 1 is Y 1 equal to K 1, X 1 plus K 4, Y 3, the equation for Y 2 it has more terms because there is K2, Y1 and there is K 8, Y 2 no, it also has only 2 terms, how about the equation of Y 3, Y 3 has 1, 2, 3 incoming arrows. So the equation for that node is Y 3 equal to K 9, X 2 plus K 3, Y 1 plus K 6 into Y 2 similarly Y 4 has 1, 2, 3 incoming edges.

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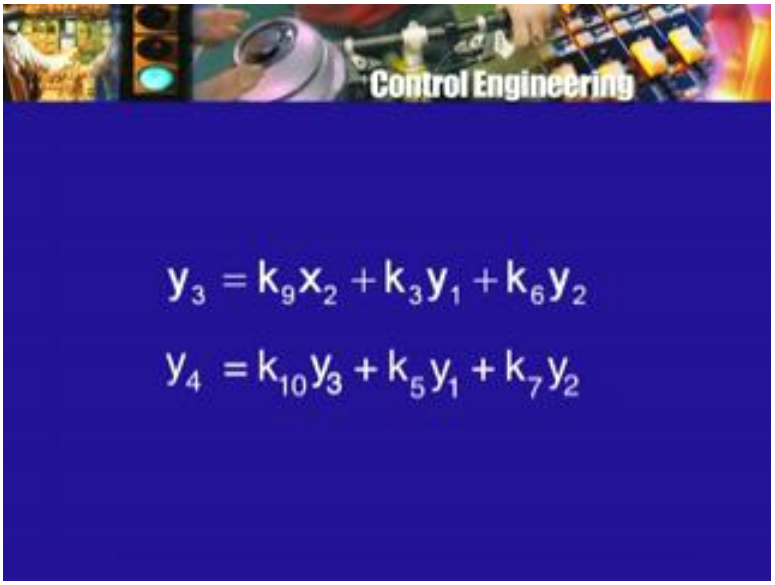

So the equation for that is Y 4 equal to K 10, Y3 plus K 5, Y 1 plus K 7, Y 2. So for each one of the 4 nodes Y 1, Y 2, Y 3, we have an equation and there are the 2 input notes X 1 and X 2 for which we have no equation, they are input nodes, there are no incoming arrows. So there is no equation and now we would like to obtain expressions for the nodes other than the inputs nodes which means in this case Y 1, Y 2, Y 3 and Y 4 the node signals in terms of the input nodes signals X 1 and X 2. So we will obtain expression like Y 1 equal's some quantity which is, what we will obtain using Mason's rule or Mason's gain formula multiplying X 1 plus another quantity or expression multiplying X 2 and similarly for Y 2, Y 3, Y 4. Now how is the rule to be applied. As I mentioned earlier Mason's gain formula or Mason's rule is closely related with Cramer's rule and Cramer's rule if you remember has a common denominator for all the unknowns each unknown is a ratio of 2 quantities and the denominator quantity is the same for all these ratios and in fact they that is the determinant similarly, the numerator quantities also are determinants.

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So in this case also, we will first calculate an expression which is common for all these different multipliers it is the common denominator all of them and as I have told you earlier it is very often called delta because delta is a symbol that is often use for denoting a determinant, how do we determine this delta for that as I told you we have identify all the loops, we have to identify all the loops on the graph. Now if you look at the graph there are some loops which probably standout very clearly there are some others which may not standout so clearly.

So one has to be very careful there is no loop involving X 1 and X 2, why because they are input nodes, there are no ingoing incoming arrows. So there cannot be a loop involving X 1 or X 2 .So you can only have loops involving say Y 1, Y 2, Y 3 and Y 4 and these loops have to be obtained carefully or as one says have to be enumerated or they have to be listed carefully and if you follow a definite procedure then one will not miss out any loop. So for example you can start with the node Y 1 and find out whether there are any loops which will involve Y 1.

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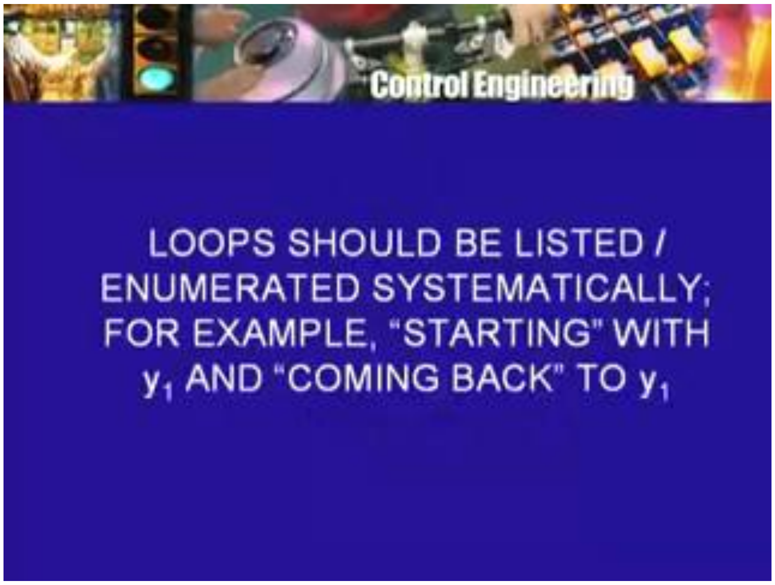

So for example from Y 1, I can travel time Y 3 along K 3 and come back to Y 1 along K 4. So that is one loop and the loop gain for that loop is say we denote it by L 1. So that is going to be K 3 into K 4. So here is one simple loop consisting of only 2 edges starting or involving node Y 1 and coming back to the node Y 1 through only one more node Y 3 we can ask similarly whether we can travel from Y 1, Y2 and then back immediately to Y 1. In this case we cannot because there is no direct edge between Y 2 and Y 1 what about Y 1 and Y 4 and back immediately no because Y 1, I can go to Y 4 but I cannot come back to Y 1 directly.

So there will not be any 2 edge loop other than the one that we are found out for Y1. Now we can go on to finding out 3 edge loops that involve Y 1 or alternately we can look at other 2 edge loops if any so doing the same thing for Y 2. I can ask whether from y 2, I can go to some other node and come back to get a 2 edge loop and the answer is yes from Y 2 to Y 4 and back. So let us call that loop L 2 and that gain of that loop turns have to be k 7 into or the transmittance of that loop or the loop gain turns have to be K 7 into K 8 from Y 2 to Y 3, I can go but I cannot come back directly from Y 2 to Y 1, I cannot go otherwise I would have included it earlier.

Similarly, from Y 3 are there any 2 edge loops yes Y 3 to Y 1 and back but that loop we already seen so we do not look at it again from Y 3 to Y 2 there is no edge therefore there is no loop involving only Y 3 and Y 2 and from Y 4 to Y 2 is the loop that we already found out. So this exhausts all the loops which will have2 edges and now we can look for loops which will have3 edges and once again one can do it systemically for example I can start with Y 1.

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So I start with Y 1 then go to some other node then go to yet another node and see whether I can come back to Y 1 and this I am saying all the time should be done systemically although when the graph is still not too complex like the one that we have it is complicated more complicated than the one that we saw earlier but it is still not too complicated, one can usually see all the loops and not miss out any one of them. But in general one has to take care and therefore do it systemically for example I can go from Y 1 to Y 2 through K 2 then from Y 2, I have to come back to Y 1 through some edge.

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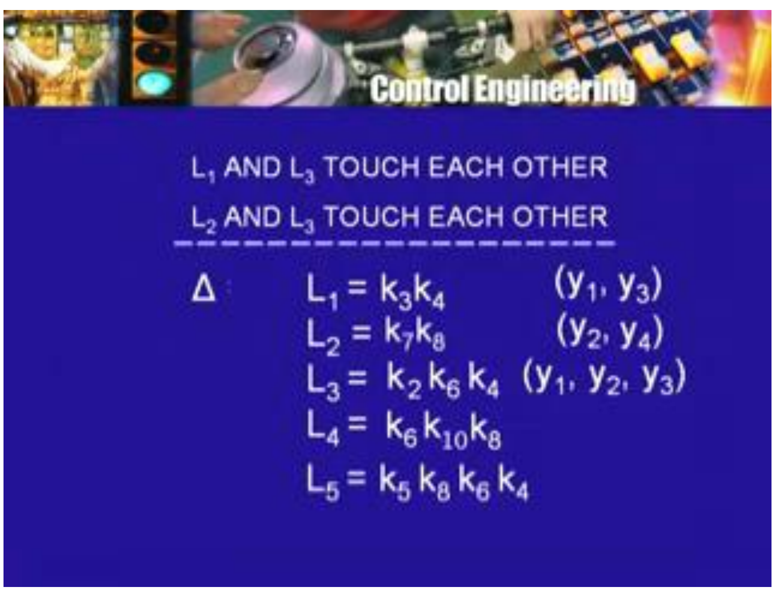

So Y 2 to Y 3, K 6 and Y 3 to Y 1, K 4, so that is a 3 edge loop that involves Y 1. So that is our L 3 and the gain of the loop is K 2 into K 6 into K 4. So that took care of a loop that enable us to go from Y 1, Y 2 then through some other node back to Y 1 . Now we can say okay Y 1 to Y 4 then to some other node and back to Y 1 can I come that way so Y 1 to Y 4 of course I can go there is an edge from Y 4, I cannot go to Y 3 from Y 4 I can go to Y 2 alright but then from Y 2, I cannot come back to Y 1 so there is no 3 edge loop of that kind.

So Y 1 to Y 2 to Y 3 back to Y 1, Y 1 to Y 4 there is nothing there is there is no 3 edges loop that involves Y 1 and Y 2 and what about Y 1 and Y 3 if I go from Y 1 to Y 3 then I already have found out one loop which involves this but from Y 1 to Y 3, I can go to Y 4 but then I cannot go to Y 2, so that is it. This is the only 3 edge loop that involves node y 1. So now we can find out whether there is a 3 edge loop that does not involve Y 1 and therefore it must involve Y 2, Y 3 and Y 4. Now is there such a loop that that involves the remaining 3 nodes Y 2, Y 3, Y 4 the answer is yes Y 2, Y 3 along K 6, Y 3 to Y 4 along K 10 and then Y 4 to Y 2 along K 8.

So that is my 4 th loop I will call it L 4 and the loop gain therefore denoted by L 4 is K 6 into K 10 into K 8. Now I am going in the order Y 2 to Y3 to Y4 may be it is possible to go from Y 2 to Y 4 to Y 3 back to Y 2, I should check. Well I can go from Y2 to Y4 alright but from Y 4 I cannot go back to Y 3. So that loop Y 2, Y 3, Y 4 can only be traversed in one direction namely Y 2 to Y 3 to Y 4 and not in the reverse direction not the same loop it would have been it if it if it if it case it would have been a different loop but there is not any. So there is no way of going along the loop from y 2 first to y 4 then to y 3 and then back to y 2.

So that exhausts all the possibilities, so we have a 3 edge loops now can we have a 4 edge loops when the 4 edge loop must involve all the4 nodes and so we can go through this exercise once again starting with y 1, I have to go to another node then a third node then a 4th node and back to Y 1, is it possible. Well I can go from y 1 to Y 2 then I can go from Y 2 to Y 4 but I cannot go from Y 4 to Y 3 or even y 1 so that is out I can go from Y 1 to Y 4 from Y 4 I can only go Y 2 and then from Y 2, I can go to Y 3 and then from Y 3, I can come back to Y 1. Now this is a node 1 could have missed easily if one was not being systematic and careful.

So we do have a 4 edge loop what was that once again it was Y 1 to Y 4. So the gain was K 5 then Y 4 to Y 2, so the gain was K 8 then Y 2 to Y 3 the gain was K 6 and then Y 3 to Y 1 the gain is K 4. It is a not a straight forward rectangular kind of loop but involves that butterfly kind of figure. In fact one gets familiar with such things and you when you see a loop when were see a particular configuration like a rectangular or a butterfly or shapes of that kind, is there any other loop that is possible. So there is another butterfly that I see here but that does not give me a loop, in any case you should now go over this once again and make sure that these are all the loops that we have. I hope I have not missed out a loop but I leave it to you to check it up and go through it systemically do not simply say yes, these are the loops and nothing else do it systemically the way I told you chosen node say Y 1 then say 2 okay 2 is loop. So Y 1 to Y 2 back Y 1 to 3 back Y 1 to Y 4 back are they loops then y 2 to as loops Y 3, Y 4 then 3 edge loops.

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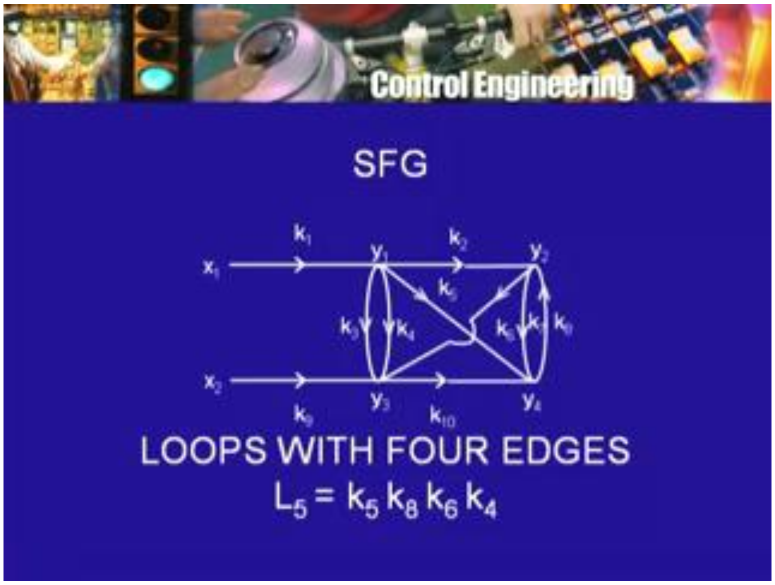

So Y 1 to Y 2 to Y 3 back is there a loop Y 1 to Y 2 to Y 4 is there a loop and so on do this systemically and just make sure that you have get all the loops may be you would have found one more loop notice that all these arrows are one way street is so when1 talks about a loop the arrows have to be taken into the current in there actual direction you cannot go the opposite way to an arrow. So we have found out 5 loops so that will contribute to one term in that denominator delta but there will be most probably many more terms. So what are they going to be so the next thing to identify are pairs of non-touching loops that is 2 loops which do not touch 1 another touch meaning they should not have edge in common they should not have even node in common.

So we have to identify all such node pair, a loop pairs and then take the product of the loops gains of these loops, find all such non touching loop pairs is a product of the case and then add them up and that gives us the next term in the expression for delta. So let us do it here and since it will take quite a while to find out all of them I will just take1 or 2 and then leave to you to find out the rest of the non-touching loop pairs. Here is the loop that we are identify as L1 and in fact now you will see that it would had been nice if I written down what were the nodes that were involved in that loop.

So that loop L 1 which had K 3 and K 4 involve Y 1 and Y 3. So I can put it down in brackets like this some notation to remind me that this was the loop involving Y 1, Y 3 and in fact I could even write down how was the loop traversed. So either Y 1 to Y 3 or to Y 1 or Y 1 to Y 3 back to Y 1 no there is 2 are the same there is nothing but if there are 3 edges then there could be 2 different ways of going around 2 different loops okay. So that is one loop, the second loop L 2 involve K 7, K 8 that involved Y 2 and Y 4. Now while looking at the nodes we can see that they have no node in common we can also look at the transmittance expressions they are K 3 K 4 and K 7 at there is no transmittance in common either.

So they do not have any in edge in common but we have to check they do not have any node in common. So L 1, L 2 those 2 loops 1 of them involves only nods Y 1, Y 3 the other involves Y 2 Y 4, so there are no node in common therefore this is a loop pair which is acceptable for our next calculation and so we will have to take the product of these loop gains L 1 into L 2 that is K 3, K 4, K 7, K 8. So that will be one term in the expression that we are going to write what about the next loop L 3, I can do the same thing L 3 was K 2, K 6 and K 4. So that involved Y 1, Y 2, Y 3 now obviously if I look at L 1 and L 3 my list tells me that they 2 nodes in common.

So L1 and L3 are touching loops so I should not take that product L 1, L 3 what about L 2, L 3 well there also have a node in common, Y 2 in common. So I should not take the product either so faring have 3 loops L 1, L 2, L 3 of which L 1 and L 2 are non-touching L 1 touches L 3, L 2 touches L 3. So we got only one non-touching loop pair namely L 1, L 2 the product L 1, L 2 from that similarly one can look at L 4 now since L 4 as 3 nodes it will turn out in this case that it will have a node in common with one of the other 3 loops that we already written down and the last loop that we wrote down had all the 4 nodes.

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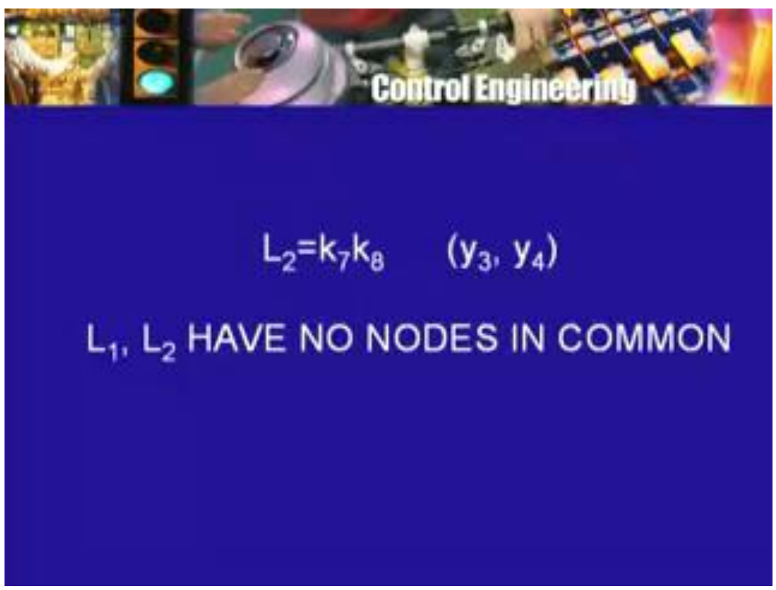

So obviously it cannot have any other loop which does not touch it and so the second term in our expression for delta, we are now ready to write. So let me write the expression for delta so far ,so delta equal to remember the one do not forget this 1 minus some of all the loop gains. So in this case there are 5 of them. So L 1 plus L 2 plus L 3 plus L 4 plus L 5, there is a minus sign which applies to the sum of all this loop gains the loop gains themselves may be positive or negative they may have a positive coefficient or a negative coefficient that does not matter we take them as we are with there sign addition them up.

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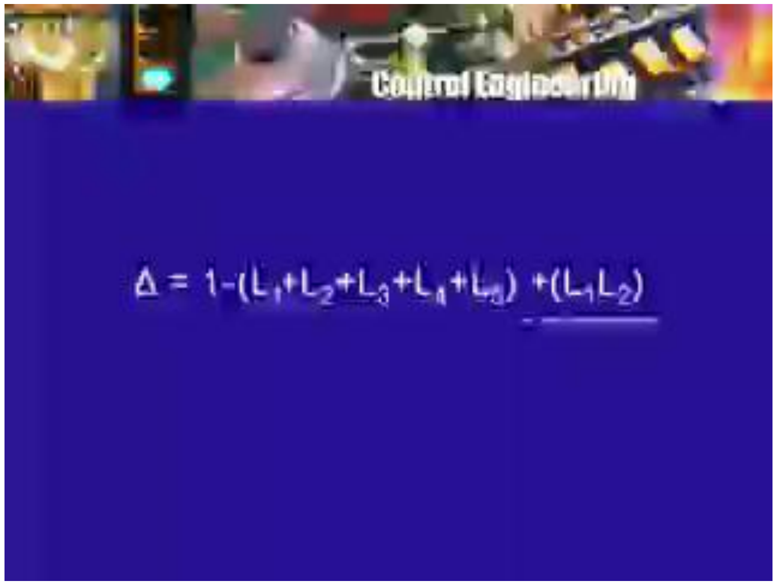

So 1 minus this sum the next term is not minus now but plus and what do we have now we have a sum of product of loop gains of non-touching loop pairs and we have identified or we have find out that there is only one such loop pair. So we have L 1, L 2 that is only one term there is nothing else L 1, L 3 touching L 1, L 4 touching L1, L 5 touching so on L 2, L 3 etcetera. So there is only1 term no other term next we will have to write down minus 3 non-touching loops now can there be 3 in non-touching loops in this case there cannot be because the 3 non-touching loops will involve non-touching loops.

So they must involve L 1 and L 2 but the movement you have L 1 and L 2 both of them all the nodes of the graph are involved and so there cannot be 3 non-touching loops and so there no more term now and there is no need to go any further. So this is the expression for delta then, this is the denominator of all those transmittance coefficients that we are going to find out. This is the determinant sometimes which is called determinant of the are associated with the signal flow graph delta. So that is a major part that a big job and we are done it. Now we have to find out the expression for Y 1 in terms of X 1 and X 2. So first let us concentrate on X 1.

So X 1 is my input node and I want to find out that coefficient which multiplies X 1 in the expression for Y 1. So what is it that I have to find out I have to find out a forward path between X 1 and Y 1 forward path or a path and forward is not necessary if it is understood that you always have to follow the arrow a path from X 1 to Y 1, is there any well of course there is this path K 1 but is that the only one from X 1 to Y 1, in this case is the answer is, yes that is the only one. So the forward path there is only one and it has transmittance K 1. So that is the gain of the forward path K 1 this will be multiplied by what remains from delta when we remove from that delta all the loops which touch this forward path. So this forward path from X 1 to Y 1 has transmittance K 1 and therefore as nodes Y 1 unit what are the terms in delta which do not touch it.

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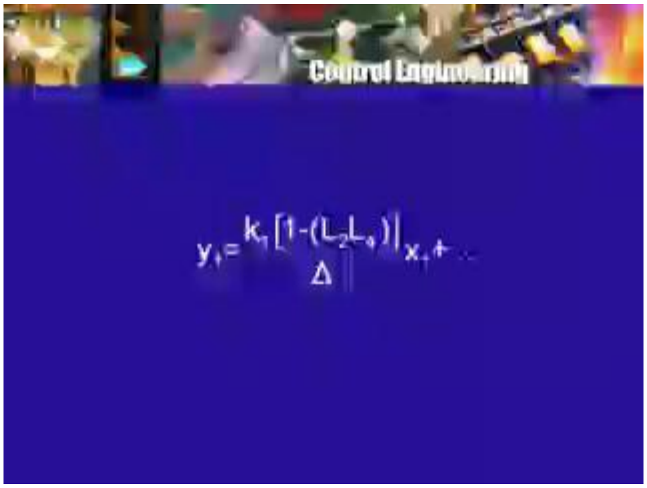

So they are loops which do not involve y 1 and so what will remain from there then of course 1 remains, so do not forget this one minus from that summation L 1, L 1 is out because L 1 had Y 1, Y 3, L 2, L 2 is okay because L 2 does not touch or does not include the node Y 1, L 3 is out, what about L 4, K 6, K 10, K 8. So here is K 6, K 10, K 8 that is okay that does involve not node1, so L 4 is okay L 5, L 5 is not okay because it has all the 4 nodes, so that is out. So 1 minus that plus loop pairs but those which do not include this node Y 1 or in fact any node on that forward path it should not touch the forward path.

Now that is not possible because L 1 itself has node Y 1 in it. So there is no term so that all there is so we have the numerator K 1 into bracket 1 minus into bracket L 2 plus L 4 divided by delta this is the coefficient of X 1 in the expression for Y 1. So Y 1 equals K 1 into whatever that expression divided by delta into X 1 plus the other term which is to be calculated now. So what is the other term that is the coefficient of X 2, the coefficient of X 2 in the expression for Y 1. So what I have to do now is to find out forward paths going from what node from the node input node X 2 to the node which we are looking at for which we are writing the expression namely to Y 1 so we have to identify forward paths from X 2 to Y 1 and now here we may have to be careful because there could be more then one path in fact there is no direct path from X 2 to Y 1, there is no edge going from X 2 to Y 1 but that does not mean there is no path between or from X 2 to Y 1.

So here is a forward path and we can as we go on we can label these paths for example the path that I had identify earlier I could have called it as P 1, P 1 was the only forward path from X 1 to y 1. Now I am looking at forward paths from X 2 to Y 1 there may be more then one. So let us look at them so there is a path that is starts of X 2 then goes to Y 3 and then goes to Y 1, so this is one forward path let us call it P 2. Now what is the gain of that forward path that is what is the product of all the transmittances along that forward path so from X 2 to Y 3 we have K 9 and from Y 3, Y 1, I have K 4. So the gain for that is K 9, K 4, so that is the path from X 2 to Y 3 to Y 1 are there any parts from X 2 to Y 1 well of course when X 2, I can only go to Y 3.

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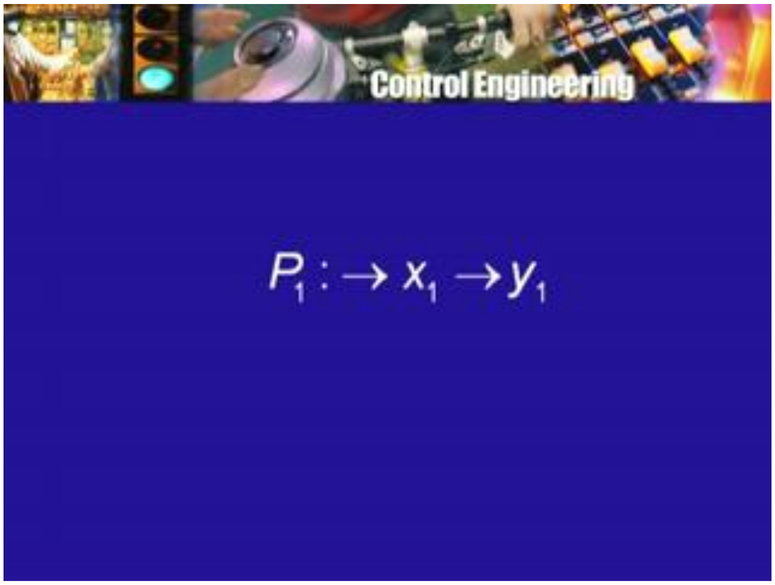

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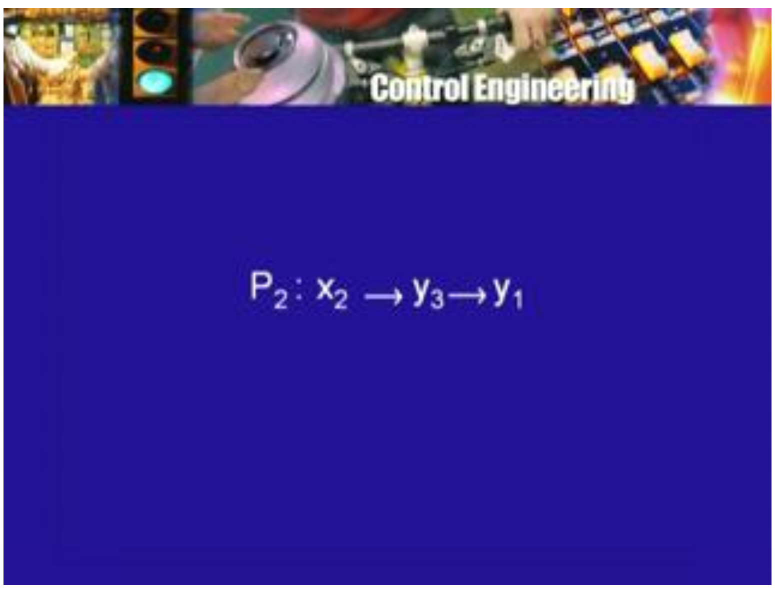

So if I want to get to Y 1, I have get to y3 first but then from y 3 can I get 2 Y 1 in some other way well from y 3 I can go to y 4 but it is seems there is no way of going to Y 1 unless I come back Y 3 from Y 3 I cannot go to Y 2. So that is it so there is only one forward path from X 2 to Y 1 and that forward path has gain K 9, K 4. Now this forward path gains K 9, K 4 is to multiplied by what remains from the delta when we remove all the loops which touch this forward path.

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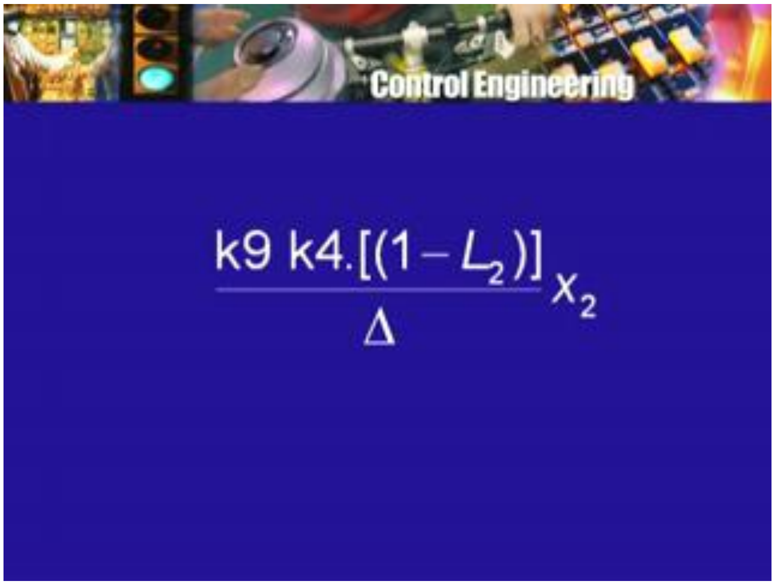

So once again that 1 is to be kept as it is minus what now here again I have noted down the nodes that occur in the forward path namely Y 1 and Y 3. So I should only look at those loops which do not involve these 2 nodes Y 1 and Y 3 and there is only one such loop and that is L 2. So I will have L 2, so I will have 1 minus L 2 there is no other loop that i need to look at plus are there any loop pairs such that neither of them involves any 1 of the nodes that occur along this path namely Y 1, Y 3 the answer is, no.

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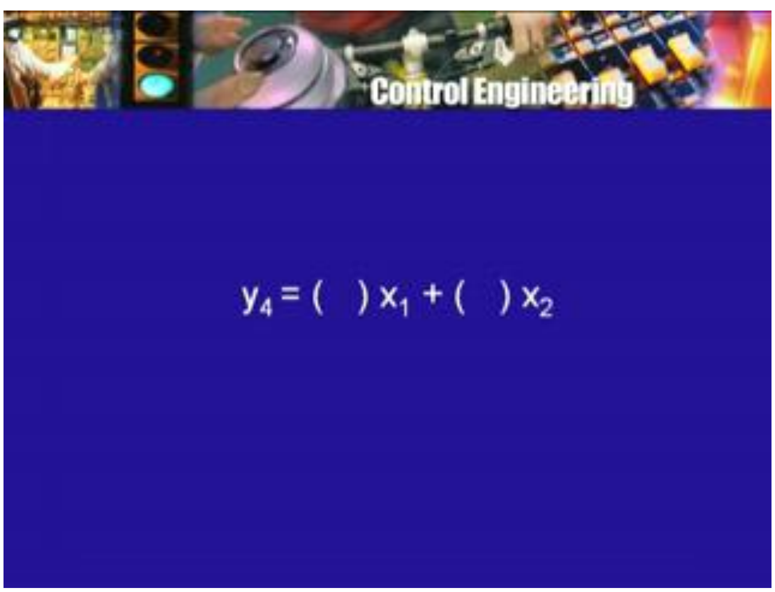

So that all divided by delta this is the coefficient of X 2 in the expression for Y 1 and so we can now put the 2 together and write down the expression for year 1 as some expression or coefficient whose denominator is this common delta multiplying X 1 plus some other ratio whose denominator again is the same delta multiplying X 2 and in this problem for this case namely even writing the expression for Y 1 there is only one forward path from X 1 the input node X 1 to Y1 and only one forward path from X 2 to Y 1 and therefore in the numerator we had only1 forward path gain multiplied by what remains of delta when we remove some of the terms.

Now there can be in some other cases more then1 forward path in that case in the numerator we take each forward path gain multiplied by whatever remains of delta and addition of these products. So for example let us say we want to look at the expression for X 1, Y 4 in terms of X 1 and X 2, so I want Y 4 equal to some expression into X 1 plus some other expression into X 2, Y 4 I want in terms of 2 input signals X 1 and X 2. To find the coefficient of X 1 then I have to identify forward path from X 1 to Y 4 there may be more than one in fact there is one for example X 1 to Y 4 I can go from of course X 1, Y 1 are no alternative and then I can go over to Y 4 the gain of that forward path now we have got so far may be P 1, P 2.

So we will call it P 3, so the gain of that path is K 1 into K 5 and it involves the nodes X 1, Y 1, Y 4. Now from X 1 to Y 1 I have to go but from Y 1 may be one I can go to Y 4 in a different way in fact from Y 1, I can go to Y 3. So X 1 to Y 1 to Y 3 to Y 4 that is a path P 4 and what is the gain of that path X 1 to Y 1 is again K 1, Y 1 to Y 3 is K 3 and Y 3 to Y 4 is K 10. So there is one more path is there any other X 1 to Y 1 again, now instead of Y 3 I can go to Y 2 and then from Y 2, I can go to Y 4 and what is the gain of that path let us call it P 5, K 1, K 2 into K 9.

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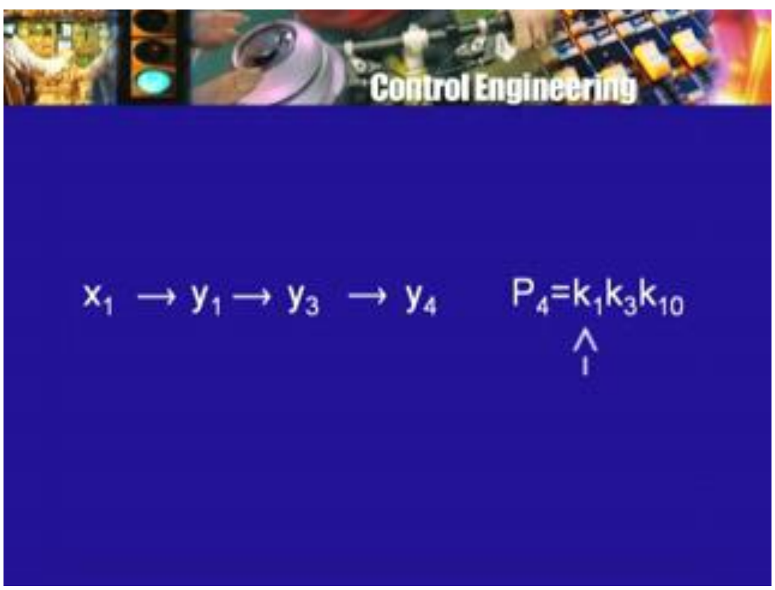

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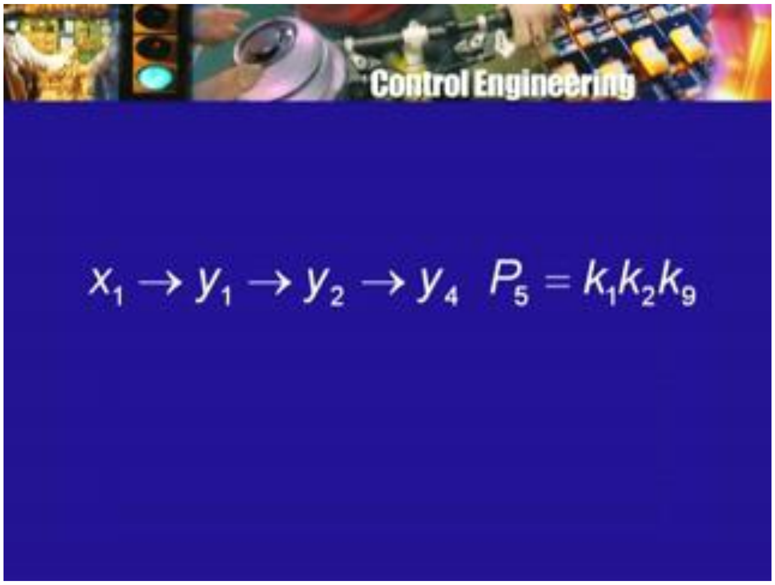

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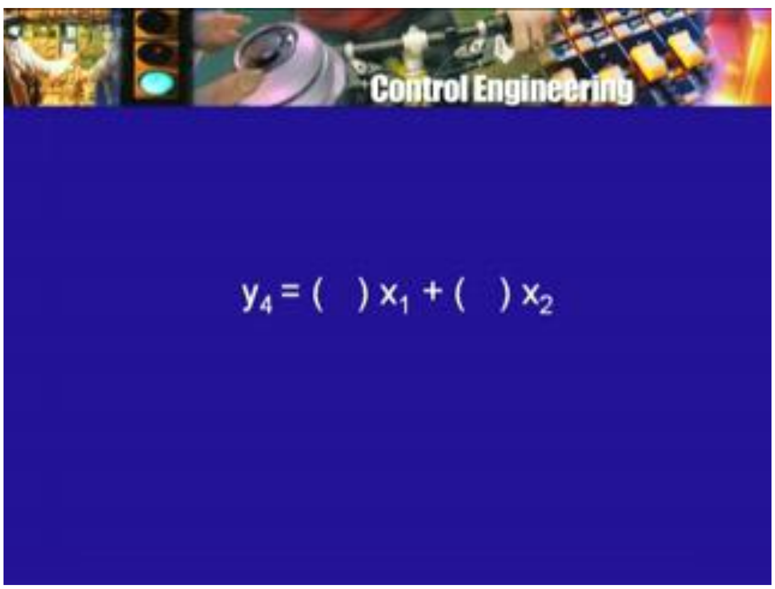

So that is a third forward path, so now when I write down that expression for Y 4 in terms of X 1 in the numerator there will be 3 terms each one of this forward path transmittances will multiplied by what remains of delta when we remove certain terms. So the numerator now has more than1 term and so on. So this is how Mason's rule or Mason's gain formula, as you can see it's not quite a formula in the sense it is not a readymade formula it is a rule more therefore correct to call it a rule and we can apply the rule to obtain these expressions for non-input signals in terms of the input signals. As you have realized it requires great care to identify all the loops then identify loop pairs which are non-touching, loop triples which are non-touching and so on then to identify forward paths from an input node to a non-input node identify all the forward paths then check a forward path and a loop to find out whether they touch each other or not because that is what is to be used when writing down the numerator expression.

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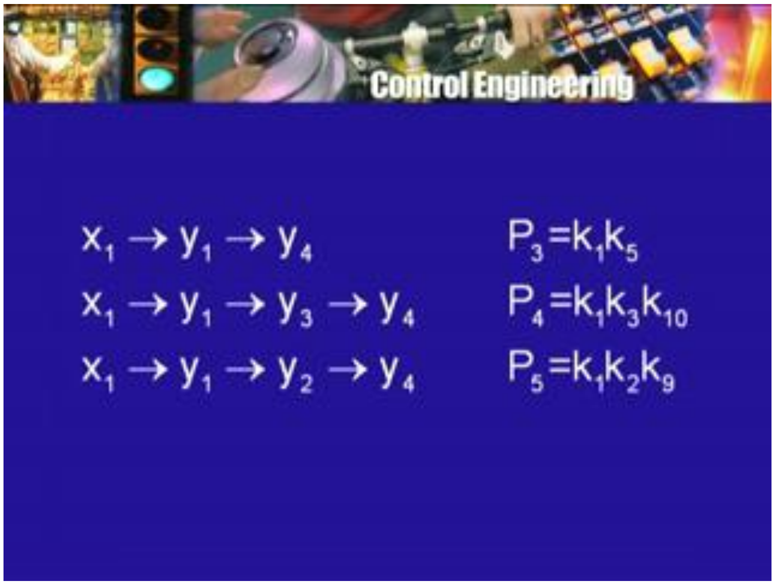

We only select those loop or loop gains or gains of those loops which do not touch the forward path at all a for the numerator expression. So all this has to be found out carefully and then of course one can write down the final expression expressing a non-input node signal as a summation of input node signals multiplied by certain coefficient or effective transmittances from a particular input node to a particular non-input node. This is Mason's gain formula or the other Masons rule applied to a signal flow graph, you can compare this with what you would have to do, if you use Cramer's rule or if you solved simultaneous equations in some way or the other going back each one of the nodes has an equation association with it right. For node Y 1, I have the equation Y 1 equal to K 1, X 1 to plus known K 4, Y 3 that is all. For node Y 3, I have a more complicated equation Y 3 equals K 9, X 2 plus K 3, Y 1 plus K 5, Y 2 similarly, for Y 2 and for Y 4.

So I will have 4 equations for Y 1, Y 2, Y 3, Y 4 in which the input signals X 1, X 2 will also appear. Now I can solve these these4 equations for 4 non-input node signals unknowns Y 1, Y 2, Y 3, Y 4 using Cramer's rule or using some other techniques of solving simultaneous linear equation, will it involve less work or more work. Well that really depends it will involve work which would not really require looking at a graph that is for sure because when I have the equations I do not have to look at the graph. So it involve work of a different kind altogether in does not involve looking at any picture or looking at any graph and as I said earlier and I repeat the signal flow graph technique or the Mason rule is really useful when it is applied by human beings by you and me for small problem to get some understanding or some inside into what is going on.

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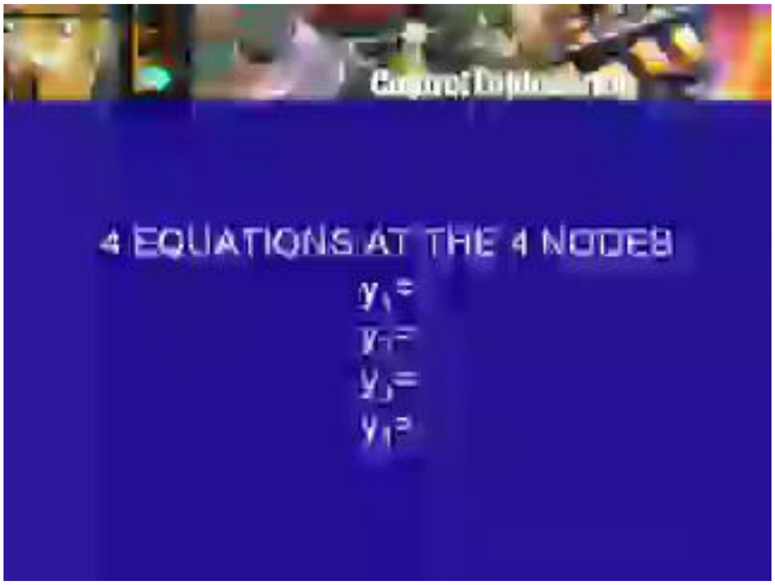

For a large number of signals and a large number of equations drawing the signal flow graph and then looking at it is not a very good method because as human beings as you must say already notice with our 6 node graph there are so many loops that one can easily get confused and one can miss out loops and make mistake and realize that these 2 loops are touching but include them there product in delta or a loop touches the forward path but include that loop in the numerator expression and so on, such mistakes are very easy to commit when the graph as a large number of nodes and the large number of edges.

So it is not really something which is to be used no matter what the problem may be and of course it is to be done by using a computer program then there is a no question of drawing a graph because as yet we do not have programs which will take a figure as a input especially a figure that may be drawn by hand like I have done then I can trace a figure on this computer screen alright but I should have an intelligent program. So that it will transform this information which is in the form of the signal flow graph into a set of equations and then use some other appropriate program to solve the equations which are which may be symbolic equations.

Finally to obtain expressions which are also symbolic involving these coefficient letter like K 1, K 2 up to K 10 and the 4 symbols for the non-inputs nodes Y 1, Y 2, Y 3, Y 4 and the 2 other for the inputs nodes X 1, X 2 and therefore finally display on the screen and expression like Y 1 equals with that equality sign some stuff multiplying X 1 plus some other thing multiplying X 2 etcetera. I am not aware of any program package that will do this do this job at the movement although1 can think of a computer program which is smart enough to accept an input for an input hand drawn or a hand traced signal flow graph, this signal flow graph is traced on the screen with the help of a mouse perhaps with any one of the graphic programs and then the rest of the work is done by a program. So that you have to display of the final result in any case this may not be the best way of doing yet because of equations can be solved in other ways and especially if there are numbers rather then symbol then all this is really not required people have been solving simultaneously in a equations by computers for the last 50 years or more and they have developed a very good efficient methods for doing that.

So there is no need to draw a signal flow graph and then apply any Mason's rule or whatever to it. It is main purpose will be to for small systems or even for larger systems perhaps to get some inside into what can happen and so on and so after this little excursion into signal flow graph and Mason's rule and so called Mason gain formula. We will get back to our feedback control system problem and there we will apply the system being small will apply Mason formula or may Masons rule to get some result. We will also do it using block diagram to show that one can do it with block diagram one not necessarily have to have signals flow graph.

So let us get back to our example and there we had decided that the 2 equations that we have let me reminds you what are the 2 equation one was the armature circuit equation involving the applied voltage E A the back EMF which was dependent on omega M the armature R A, I A that equation we solved for 1 or the 2 variables either omega m or I a in terms of the other 2 and correspondingly started constructing a signal flow graph. So let us suppose that we had solved that first equation for omega m.

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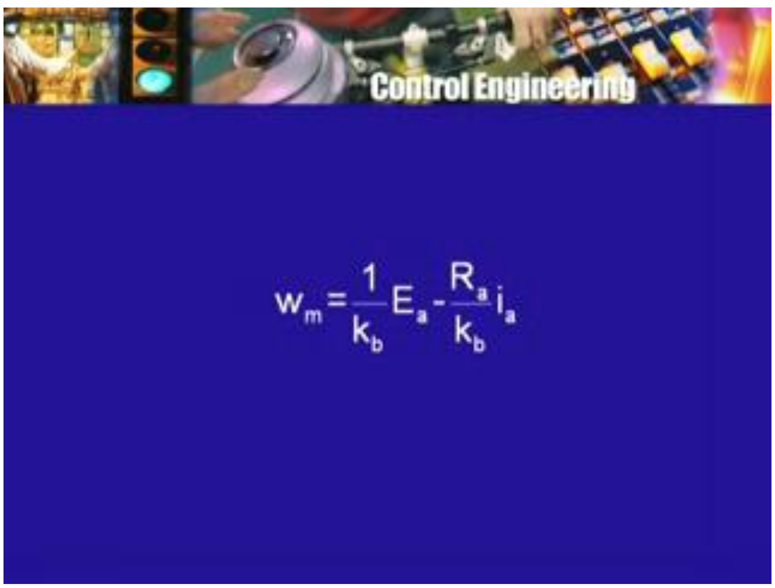

So what with be the solution omega m will be 1 dived by k b into E a and the other term is transpose on the other side so minus R a divided by k b into I a. So that is the equation first voltage equation for armature circuits solved for motor speed in terms of armature voltage applied voltage and armature current and that resulted in a part of the signal flow graph that we are going to construct namely with E a as 1 node omega m as 1 more node and I a as a third node 1 arrow going from E a to omega m with gain or transmittance 1 by k b and another arrow going from I a to omega m with transmittances minus r a by k b. So that takes care of that 1 equations the other equation is the torque equation and what is the torque equation once again the torque produce by the motor k t into I a equal to the load torque plus the torque to overcome friction and solving it for I a that was easy we get I a equal to1 by k t into t l plus k f by k t into omega and therefore on the signal program. Now I can put 2 more edges to stand for this equation this equation is for node I a, I a equal to 1 by k t, t l m. So I put one more node t l here is the edge from t l to I a it is transmittance is 1 by k t that is okay and then I have the other edges from omega m, k f by k t.

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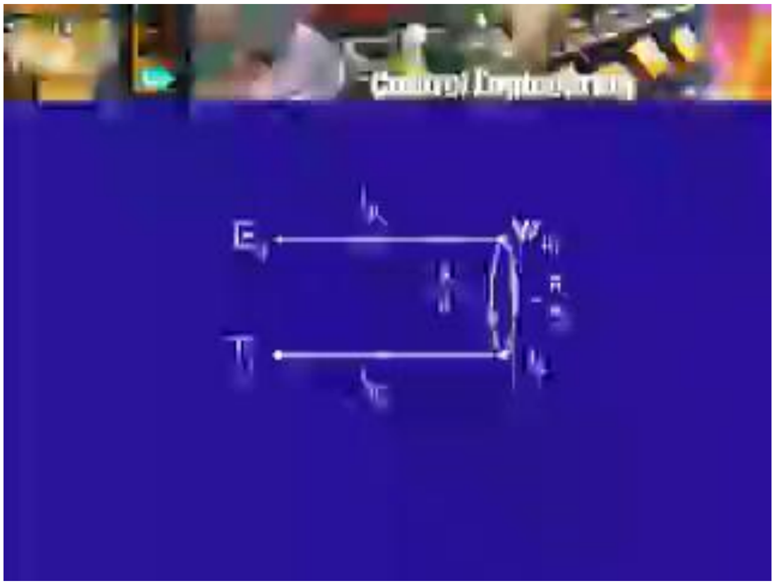

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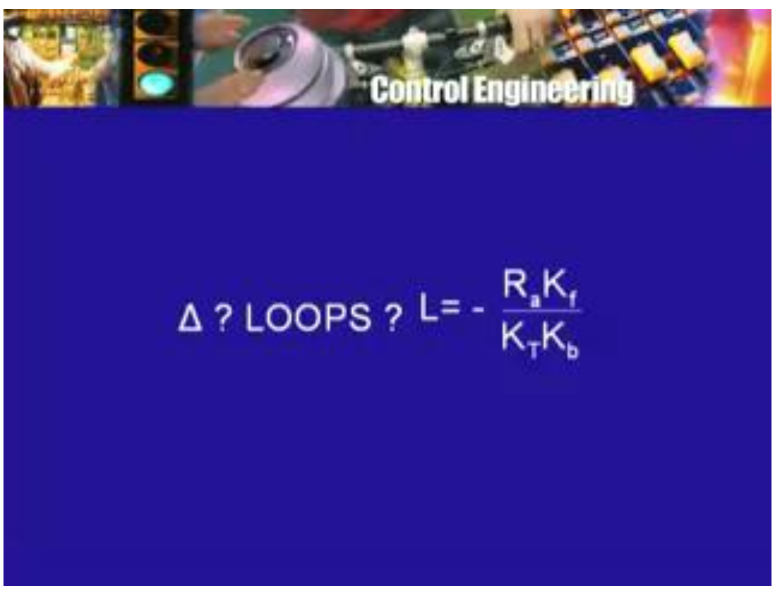

So I have k f by k t, so that is my signal flow graph for the armature speed control problem where we had only applied armature voltage and no feedback, the scheme was really very simple. Now what is the goal of all of this we wanted expressions for omega m basically and perhaps I a also in terms of the applied voltage and the load torque to find out how the speed will change if the applied voltage changes or if the load torque changes from the nominal or the rated value. Now will use Mason's rule for this purpose as you can see the graph is very simple. So this application will be very easy first business is to find out that expression delta which requires us to find out all the loops that there are in the signal flow graph. In this case the graph being very simple I can immediately spot 1 loop and only one loop. So let us called that loop from the loop 1the loop gain l is simply the product of those 2 transmittances.

So it will be minus r a, k f divided by k t into k b that is the loop gain l there is only1 loop. So what is delta? Delta is simply delta equal to1, do not forget this 1 minus some of all the loop gains there is only one loop gain therefore l plus loop pairs but there are not any loop pairs of course when one says non touching they have to be this thing. So l and l do not constitute a loop pair in that sense and therefore there is nothing else so delta is simply 1 minus l, we already have an expression for l so delta will be1 plus r a, k f divided by k t into k b. So that is the common expression delta and now we can write down the expression for omega m in terms of E a and t l, to find the coefficient of e a we need to find out the forward path from E a to omega m fortunately for us there is only one forward path that is 1 by k b.

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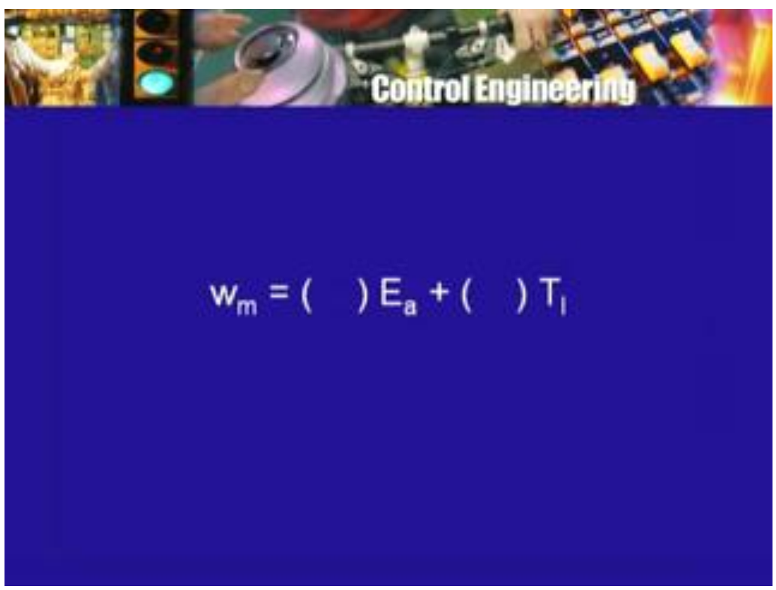

So omega m equal to that expression multiplying E a, the denominator of that will be delta, the numerator will be this forward path gain1 by k b, this forward path gain will be multiplied by what remains of delta when we remove non-touching loops. Now the only loop we have as both omega m and I a therefore it touches this forward path E a to omega m therefore what remains of delta is only one. So there is nothing there to write do not forget that1 if you forgot that one you would have got 0 there and probably you will get a 0 for your answer also. So that all there is for that expression 1 by k b forward path gain multiplied by only one that remains from delta divided by delta into E a plus the other term which multiplies t l the load torque for that we need the forward path from the load torque node t l to the speed node omega m is there a forward path yes and there is only 1 from t l to I a and then to omega m and what is the gain of the forward path then1 by k t multiplied by r a y k b that is the gain of the forward path only one then I have to multiply it by what remains of delta, delta has 1 minus l I have to only choose loop which do not touch this forward path but this forward path involves both the nodes omega m and I a, whereas loop involves both the nodes omega m and I a.

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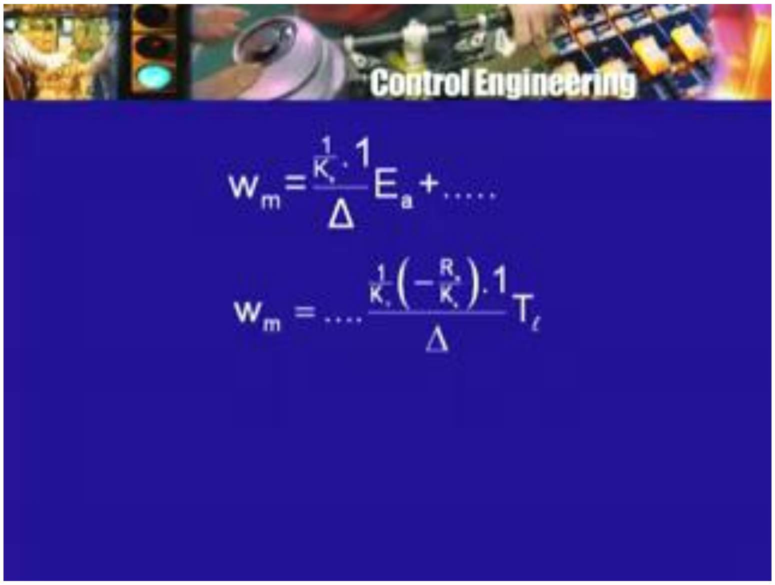

So what remains of delta once again only one, so I will write one dived by delta and that is the complete expression for omega m in terms of E a and t l, omega m equal to 1 by k b divided by delta into E a minus r a by k b, k t into delta into t l. With this expression then one can look at what happens when e a changes or t l changes or any one of parameters happens to change. So for example if E a changes by such and such amount and we can talk in terms of percentages but we can also talk in terms of absolute value. In fact this expression is such that it easier to find out in terms of absolute values but then one also thinks of percentage change as what is normally expected it is not enough to say that something has change by 50 volts, change from what value if it has change from 50 volts by 50 volts it has become 100 from 50 that is a drastic change where has if it has change by 50 volts from 1000 to1050 that is not such a big change, although it is still is change of fifty volts. So that is why talking in term of percent change or per unit change is also useful but that is very easily related to the absolute gain.

So now suppose my E a were to change from 230 volt by10 percent so by an amount 23 so 230 volts plus 23 volt my E a as become 230 plus 23 right now what will be my omega m. Now this expressions tells you that omega m will change by certain amount if omega m was previously 1500 r p m because that is the way the drive was designed then now the new omega m will not be 1500 because this part a has changed by how much by 23 volt and so how much will the speed change the speed will change by 23 multiplying this coefficient whatever this coefficient turns out to be putting the actual numerical value.

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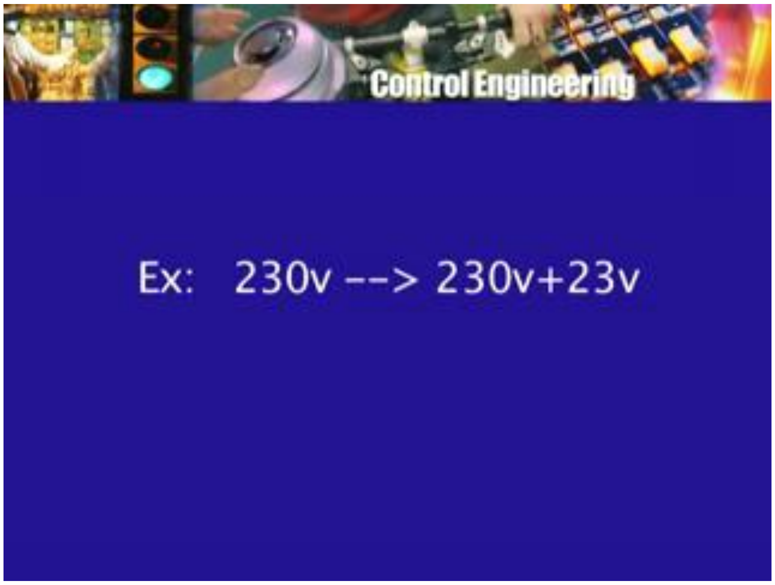

If it let say 2 then the speed will change by 46 rpm then I can find out what percentage it is in term of 1500 r p m and say whether that speed changes very large or its acceptable, say 46 rpm from 1500 r p m the speed is increased that just about 3 percent and it may be acceptable for a particular case. So with 10 percent change of armature voltage I will have 3 percent change of speed that may be acceptable, if it is acceptable no problem we do not have to do any redesigning think of feedback or what not something similar for t l if t l changes by such an such amount by how much will omega m change that can be figured out easily the change in omega m will be simply this coefficient that multiplies t l into the change in t l.

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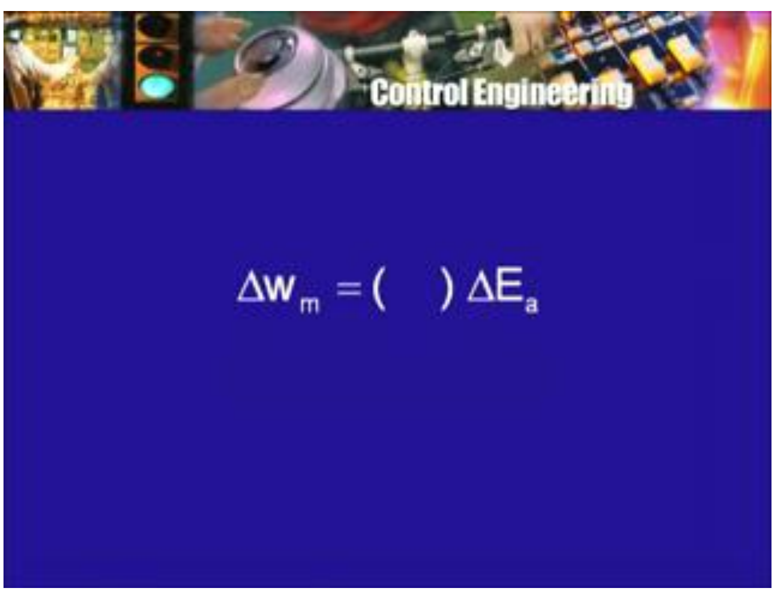

So in fact one can write things like delta omega m the change in speed if E a changes by certain amount t l will remain in the same will be that coefficient of delta into delta E a. Similarly, if t l changes e a remaining the same then delta omega m will be the coefficient of t l multiplying delta t l and Mason's rule enabled us to obtain those coefficients rather easily in this case although solving those 2 simultaneous equations also I am sure you would agree is very easy for you would not get bogged down with solving 2 simultaneous equation for 2 unknown but this was to illustrate the applicability of Mason's rule that we did it this way but with the larger system it may be advantages to use Mason's rule or it may be advantage to write down equations and solve them that depends on what the problem is how large things are etcetera.

So we have now a way of figuring out whether the drive is satisfactory for such an such in e a only what is the change in speed and is it expectable, for such and such change in load torque what is the change in speed is it expectable. One can think of what is called worst case scenario that is E a changes and t l changes. For example, one can see because of the minus sign that multiplies t l that is E a increases and if t l decreases then the speed will increase much more then it would have if only one of these to thing have taken place in other direction if the armature voltage goes down and at the same time the load torque increases then delta omega m will be negative, the speed will decrease and the decrease will be more than what would have taken place if you have only one of those 2 things happening.

So the effect of disturbance input on the behavior steady state behavior and their performance can be found out and one can decide whether it is good or not if it is good then there is no need to do anything more the drive as it is okay, if it is not then we have to think of something and of course what we thought of was the feedback mechanism of the feedback control system, what about changes in parameter? If parameter values change rather than the signal or the variables like armature voltage and load torque changing then what will happened to omega m that also can be found out from these formula for omega m the only thing is the formula is not in a very nice form, it does not show omega m some coefficient multiplying a particular parameter like say the back EMF constant k b because the parameter does not occur in a linear fashion as you will noticed from the delta has k b in it and the numerator in the multiplier E a also has k b in it.

So it is quite possible that k b occurs in a not a linear fashion and therefore omega m cannot be written as some expression which multiplies only k b plus something else. So that we can look at change in k b, so10 percent change in k b or k b changes by this much what will be change in the speeds. So this the expression is such and we cannot help it because the dependence of the speed onto parameters is not linear, is not in our hands, it is it turns out to be none linear we cannot do anything about it whereas the dependence of the speed on the armature voltage and on the load torque does turn out to be linear and therefore the signal flow graph analysis could be done because we simply had multiplying coefficient of transmittance and therefore we finally got coefficient multiplying E a and t l. So, so much for our drive or without any feed back.

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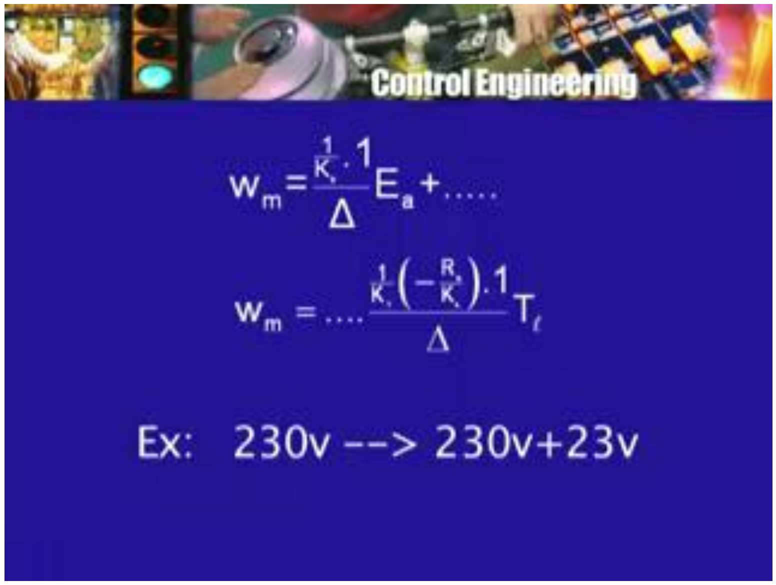

Now suppose you find out that for the motor that we have chosen for the armature voltage that we selected and so on the performance of the drive is not adequate that is per say10 percent increase in load torque the speed drops by a amount which is not acceptable. So now we have to use some other scheme, we have to adjust make some adjustment in the system to compensate the change in the load torque from the rated or the design value and would like to do this adjustment or compensation automatically rather then manually. If it is time be done manually if we remember that is not a big problem of course we have to provide for it the operator if the load torque increases simply increase the applied voltage.

So that the 2 cancel of each other and therefore the speed remains the same but that requires 2 things 1 is of course there must be an operator who is looking at the speed who knows what is to be done and does it secondly we have to provide in the means by which he will do it that is the armature voltage has to made variable because armature voltage is the supply voltage coming from the supply and you have no arrangement of changing it then the operator can only watch helplessly the speed will change he cannot change the armature voltage. So it is not enough to have an operator we have to give in the means we have time enable in to change the apply voltage in other words you have to bind something more now so that the operator will change the applied voltage.

Now instead of a operated doing it instead of it being manually, we wanted to done automatically and therefore we wanted not only feedback whether an operator watches the speed or whether you have a tacho generator which gives you an indication of the speed that is feedback in one case it is manual, in the other case it is automatic. But we want the adjustment or we want the compensation or we want the change to be done also automatically and not by human being therefore we have feedback which is going to result in a system which is an automatic feedback control system not a manual feedback control system and we have already thought of how to do it we take the voltage from the tacho generator and connect it as one input to a device which we are called the comparator or the error detector to which we give a consistent reference voltage as the other input this device produces therefore the difference of these two E r, the reference signal or r, p in general minus the feedback signal in this case the tacho generator voltage and the difference is then amplified that is how it appears on the block diagram. In practical terms that difference voltage will somehow change the field current of the generator which supply the armature of the motor. Remember, Leonard we are following his drive that is the motor is not given the fix voltage supply but it is given a variable voltage supply coming from a generator that generator in term is driven by another motor or it may be driven by some other prime mover which has an IC engine or steam engine or whatever may be at hand.

So that is the fee back scheme that we are we had decided to use and therefore we have now a few more equations that we have to write and so the next business will be to modify the signal flow graph to take care of these equations. So what are these equations well what are the signals that we introduced the additional that we have introduced, we have introduce the error signal e, the error signal e, this error signal e was the difference of the reference signal E r minus the tacho generator voltage and we could write it directly now as k tacho generator that is some coefficient multiplying the motor speed. This negative sign is important here as we will see and that sometimes is referred to as negative feedback that is we are taking a difference rather than a sum e equal to E r minus k t tacho generator omega m. So we have one equation now which we dint have earlier and we have 2 signals e and E r which we dint have earlier. So we have to add to our signal flow graph 2 more nodes 1 E r and the other node e and represent this equation on that.

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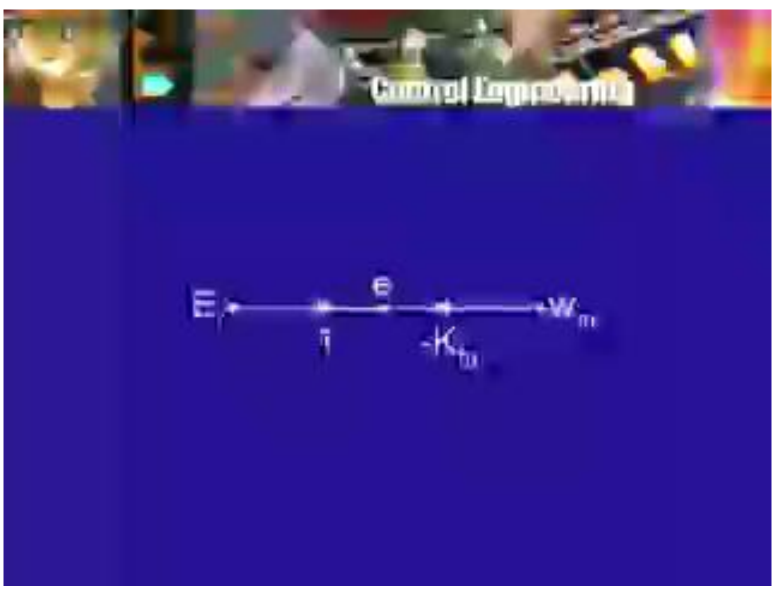

So I will have E r, therefore there will be transmittance from e r to e of one and then I will have omega m there will be a transmittance from omega of minus k t g. So this is an addition to signal flow graph that i have to make I have added 2 nodes and I have added 2 more edges, this is one equation but there is one more equation and what is that equation, that is the equation which represents that motor which drives the generator which feeds the armature and the generator field current is then controlled by this or determined by this error signal e and therefore we are simple written armature voltage E a equals k a times e that is a very simple equation because that simply says that put between the node E a and the e node E a transmittance k a. So I have to add to the signal flow graph now one more edge no node is required to be added because e and e are there on the on the signal flow graph already I just add one more edge and so my signal flow graph will now look like this. This was my earlier signal flow graph for the 2 equations without any feedback. Now what is it I have had to introduce I have this error signal e from there to E a I have a gain k a, so there is that gain k or transmittance k a from E a to e that is one simple equation and then p itself comes from the reference signal e r through a transmittance of only one and feedback signal tachogenerator voltage from omega m.

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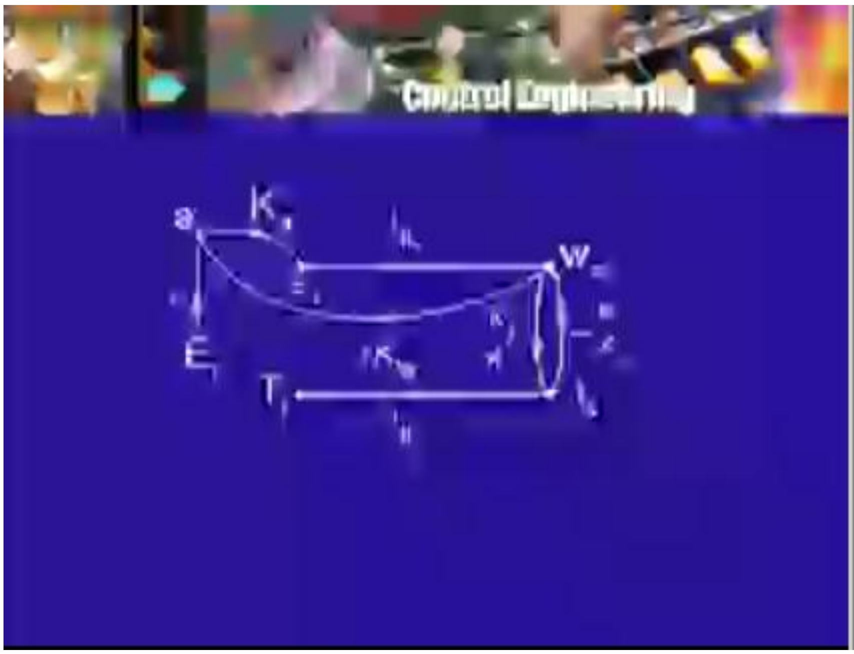

So I will have that edge with a gain of minus k tacho generator okay. So my signal flow graph now has got modified by the addition of these 2 equations and by the addition of a few more nodes, the 2 nodes that I have added are E r and small e as a result now what has happened of course the signal flow graph has become more complicated, it has more edges it has more node. So when I work on it I will have to be more careful secondly you will notice that the node E a which was an input node in the earlier signal flow graph is no longer an input node and that is correct because the armature voltage E a is now going not going to be kept fixed, it is going to depend on what is happening, it is determined through e, it will be k a times e, e itself is determined by what is happening because e is E r minus k tacho generator omega m and that is the meaning of feedback that is earlier E a was not dependent on the actual speed, now we make it depend on the actual speed through this scheme.

The scheme of course involves expense it involves additional motor generator set or a motor generator set or a prime mover and a generator and what not tacho generator difference device may be an amplifier and so on and so forth. Now E r appears as an input to my signal flow graph input node to my signal flow graph and as we saw earlier it is called the reference input it is not really an input to the system but it is what you may call as a set point input that is now if I want the drive to run at some speed other than the one for which I had designed it then I will have to change er so er is the set point input and there is still one input node and what is that input node that is the input node T l that is the load torque because that can still cause disturbance. So we have now2 input nodes e r and T l, E r is the reference input node and T l is the disturbance input node or the load torque input node.

Now we can go back apply Mason’s rule and get an expression for what in terms of what, we are interested basically in the speed omega m, what are the input nodes now E r and T l. So I want an expression for omega m in terms of E r and T l. So something like omega m equals some expression and by this time we know that it would be a ratio of 2 things by Mason’s rule denominator will be a delta for the signal flow graph etcetera. So some ratio multiplying E r set point input plus some other ratio multiplying T l the load torque input.

Once we calculate those coefficients then we can find out the effect of load torque change on the speed and the set point change on the speed. So we can even figure out what should be the set point voltage for the speed to be changed to such and such a value. So do this and will get back to that confusion about steady state error and we will see what is wrong and make changes accordingly.