Control Engineering

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Lecture – 15

I hope you have attempted the problem that I put before you to draw the complete or composite signal flow graph. For our two basic equations, for the speed control drives the armature equation and the torque equation. So those are the only 2 equations we have, the problem was to draw the signal flow graph which will represent both these equations and I told you that there was one condition that should be kept in mind namely each node represents one and only one equation because at each node, you have some incoming arrows and the equation says that the node signal is equal to the sum of the incoming signals multiplied by the transmittances of the edges that come to that node.

Now in our equations, we had 4 basic variables the applied voltage E_a the load torque T l the armature current I a and importantly the motor speed omega m. So omega m is common to 2 equations it occurs in the electrical equation because of back EMF depends on the speed it occurs in the mechanical equation because the frictional torque depends on the speed. So omega m occurs in both the equations similarly, I a the armature current occurs in both the equation in the electrical equation. It occurs in the form of armature drop I a, R a in the mechanical equation each occur, it occurs in the form of the motor torque K t in to I a the other 2 variables occur in only one equation E a in the electrical equation, load torque in the mechanical equation.

So what are the different signal flow graphs that we could draw to represent these 2 equations. I am pointing out that for a given set of equations or even for a given block diagram one made draw different signal flow graphs because what one will choose is a different thing and that depends on, what the goal is an also on some experience with these things. So what are the alternatives with the condition that I mentioned?
So I can have a signal flow graph in which there is a summation corresponding to the node E a that is of course, very simple E a equal to K b in to omega m plus R a in to I a. So it is quite easy to represent that by in equation the other equation is the torque equation from which I can get T l the load torque equal to K t, I a minus K f omega m and so that is also very easy it to represent so there will be a node T l and there will be a 2 incoming arrows from the 2 variables or signals omega m and I a.

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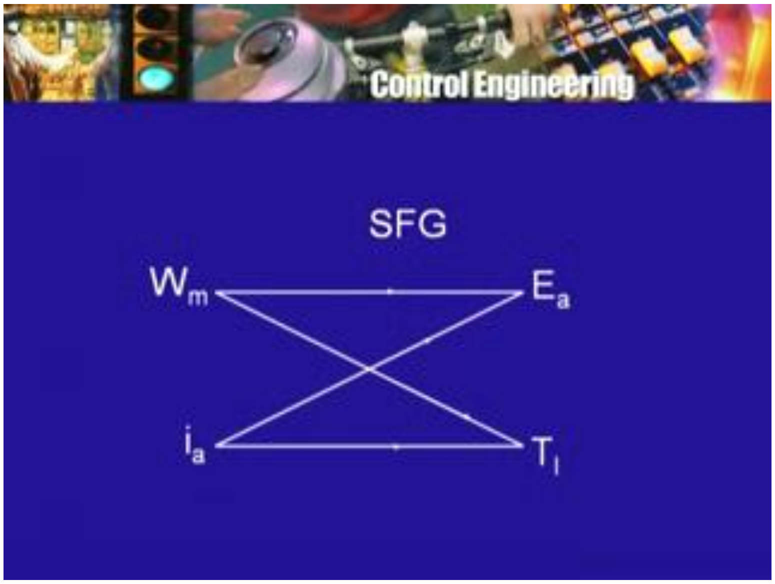

So this will be one signal flow graph which will be quite easy to draw and probably you have already drawn it. So there will be a two input signals because from these 2 nodes there will be only out going arrows, there will be no incoming arrows. So from omega m and I a, one can show arrows going towards a node called E a. So omega m is multiplied by K b the back emf I a is multiplied by R a, the armature resistance and therefore the nodes signal E a equals K b omega m plus I a, R a that is straight forward. The second one requires you to transpose the term T l on one side and the other two terms on the other side but that is all so pretty easy.

So T l equals the generated torque which is K T in to I a minus the frictional torque. So minus K f in to omega m, so here is the complete signal flow graph which represents the 2 equations in which we have armature voltage T a and the load torque T l as 2 nodes which are not input nodes because the have incoming arrows, whereas omega m and I a are the 2 nodes which are input nodes because they have only outgoing arrows. Now what I am seeing is that this signal flow graph is quite correct as it is, one of the signal flow graphs represents the equation for the armature the others signal flow graph the other part of it rather represents the equation for the torque.

Of course, if you compare this with the block diagram in fact, we drew two separate, we drew just one single block showing the motor and the load, we did not really shows several separate blocks but what we will do is will do it as soon as we also look at some other alternatives for the signal flow graph. So what are the other choices and why we might prefer those choices? Well as we saw earlier the load torque T l although it is really physically the output the disturbance comes from changes in it. So we would like to regard T l as input.

Similarly, the armature voltage E a although hopefully it will remain constant but there will be no assurance that the supply voltage will remain constant there will be a disturbance coming from E a also and therefore E a also is profitably regarded as input signal although physically it is also and input signal, it is the think that causes the motor to turn etcetera. The other two signals which appear as input signals in this signal flow graph are from a practical point of view at least one of them definitely of the nature of output omega m. We want the motor not only to provide torque but to provide torque at a particular speed remember the function was to have the motor run at some desired speed constant speed.

So omega m is a natural output variable how about the armature current I a which an input signal in this diagram. Well, it is not really an output in the sense one is not directly interested in the armature current forsake but it is not in input either in the sense, you are not putting in to the armature some current rather, we think of applying to the armature some voltage. So although the signal flow graph is correct it is not a good one in the sense that is it does not enable one to think of inputs and outputs, the way we would like to E a and T l as inputs perhaps changes in them as disturbance inputs and omega l and perhaps I a as outputs or rather change in omega m as the output that is the thinking is changes in applied voltage, changes in the load torque and these are physical signals, these are function of time over a period of time, they may change by small amount or by large amount changes in these will cause or will result in changes in the speed and perhaps also or certainly also change in the armature current. Remember, the distinction between variables variable quantities physical variable quanties, variables or what we call signals all these 4 are signals are variables and parameters parameters of the system such as the coefficients that appear as transmittances in the signal flow graph the armature resistance.

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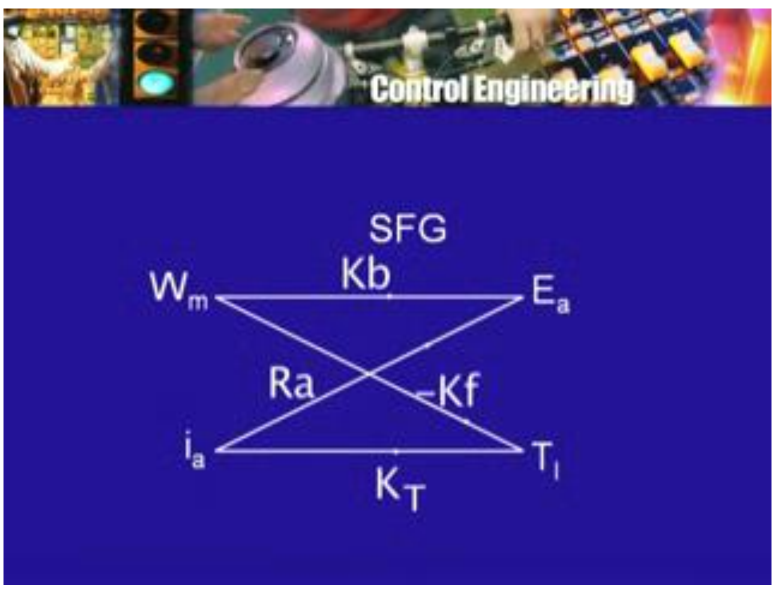

The torque and the back emf constant on the coefficient of friction. For example, these are not signals they are not sort of functions of time in the same sense as omega is or E a is although over long periods of time, they may change the armature resistance because of heating of the motor armature may start increasing the back Emf and the torque constants may change because of change in the field current but certainly, we do not expect those changes to take place as rapidly and in an as unknown manner or we know surprising manner as perhaps changes in load torque and applied voltage so on. This signal flow graph the signals 4 signals appear an also the 4 parameters of the system. This is you can say and advantage the signal flow graph it shows all the things fairly explicitly whereas earlier, we just drew a simple block and a put there the words motor and load an a showed 2 arrows going in and 2 arrows coming out and that time of course I showed E a and T l as the input and omega m and I a as the output.

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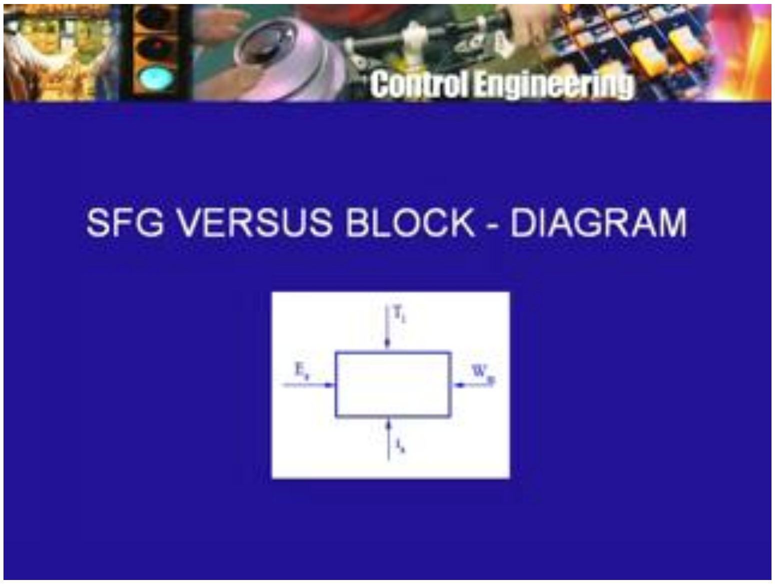

So, if as a draw block like that it only tells me that I am going to look upon these two quantities or variables as my outputs and these 2 quantities as my inputs. So I am would think away in this way that these inputs may change they will in turn cost change in the output how much etcetera, etcetera. The block diagram itself does not give you any further information you are put it down in the form of equations or as I saw earlier, sometimes you can put that information inside the block it is not just writing motor and load I can put down, what did be call it, we can put down the transfer function or various transfer functions associated with the equation and we had seen that earlier. Now what would be another way of drawing the block diagram?

So another choice would be just the opposite we would have omega m and I a as the output variables and therefore in this case we will write down equations or from the equations we will solve for omega m and I a as some of term some 2 terms and therefore they will represent them on the signal flow graph.

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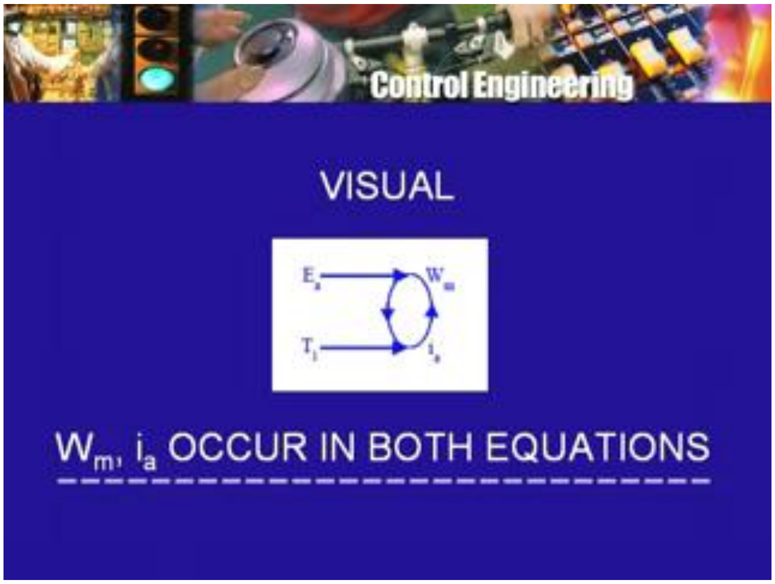

So in this case then E a and T l will be the input nodes, there will be no arrows going towards these 2 nodes omega m and I a will be the output nodes, how shall be relate the 2 unfortunately omega m and appear in both the equations. So we will have to make choice I can either use the Emf equation to get an expression for omega m as a sum of 2 terms that is not very difficult it will be as follows omega m is multiplied by K b, so think of transferring that to one side.

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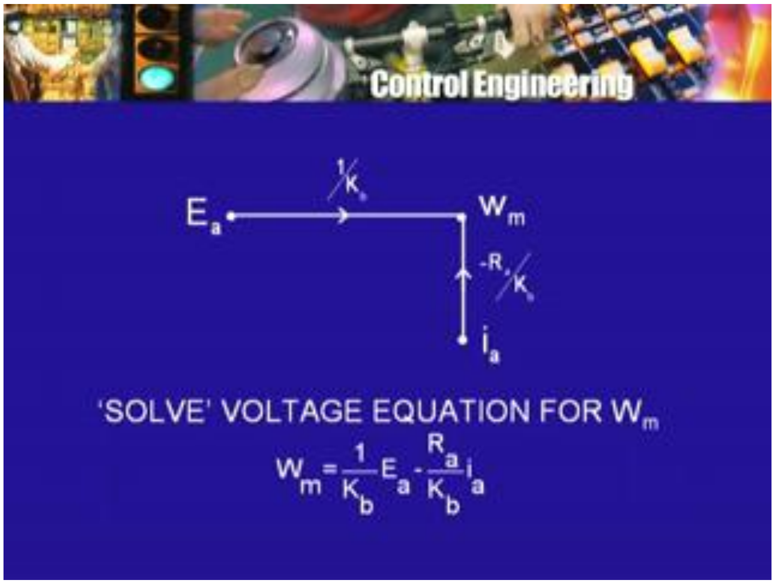

So K b omega m equal to E a minus I a R a therefore omega m will be one divided by K b into E a so there is a transmittance from E a of one by K b. So I have put in an arrow a line with an arrow and the transmittance shown there and the second term is the minus R a, I a divided by K b, so here is the other line go in from I a to omega m and that is minus R a, I a divided by K b therefore R a by K b minus R a by K b is the transmittance. So this represents the Emf equation and we have solved the Emf equation or rather we have rewritten the signal flow graph drawn so far represents that equation in a different form omega m is a sum of two terms there are 2 signals come in towards omega m.

So omega m is one by K b in to E a minus R a by K b into I a just check that this is correct. Now we will look at the second equation however. Now since I have already written down an equation in which omega m is the sum therefore from the second equation now I can not solve for omega m and express that as another sum there is nothing wrong with writing that equation but the way one draws and interprets the signal flow graph, this cannot be done this is the one important condition that one must adhere to. So we have we can solve it for I a because in fact it is very easy to solve I a just divide through by K t.

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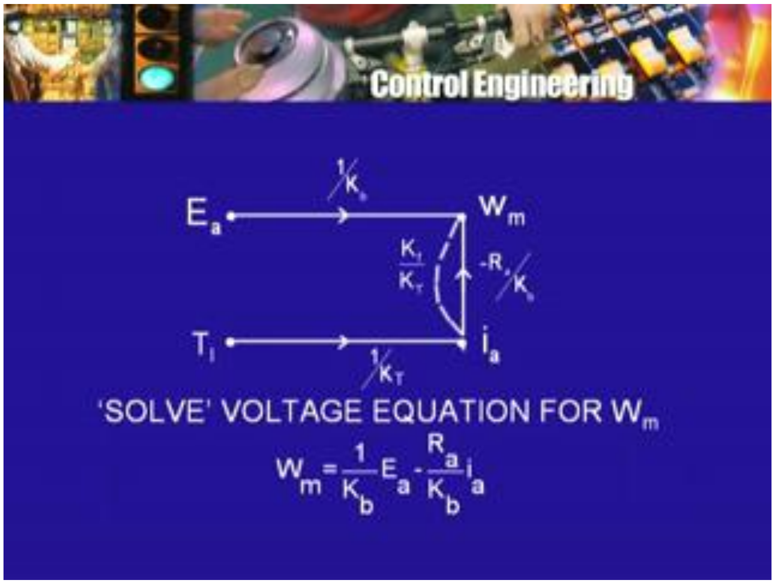

So then what you get we get ia equal to T l divided by K T, so here is one more edge going from T l to ia with a transmittance 1 by K T and we have another contribution which is K f in to omega m divided by K T so here will be an arrow that goes from omega m to I a with a transmittance of K f divided by K T. So this is another signal flow graph which is equally correct for the 2 equations in which now E a and T l are the input nodes omega m and I a are not input nodes because they have incoming arrows of course you notice that there is a difference between the 2 signal flow graphs that we have so far here is the signal flow graph with omega m and I a as the inputs E a and T l as output nodes I do not have to call them intermediate notes because there are no out going arrows from then at all and I have the second signal flow graph in which E a, T l are the input nodes omega m and I a are the output nodes and I can call the intermediate nodes also if I want because omega m has a incoming arrow or 2 incoming arrows and an outgoing arrow the armature current node also has 2 incoming arrows and 1 outgoing arrow.

So there are these 2 signal flow graphs are there any further choices. Of course we can combine one equation solve for one variable with another equation solve for another variable only subject to the restriction that both of them should not express the same variable in terms of the others. So I should not solve each one for omega m or each one for I a, there now way of writing down those 2 equations or representing them on the signal flow graph. So you can think of the various choices that are there from the electrical equation I can either solve for E a or omega m or I a from the mechanical equation I can either solve for I a or T l or omega m but I cannot solve both them for omega m or both of them for I a.

So one could draw some more signal flow graphs for example there could be following signal flow graph which will be a little different the torque equation is solved for omega m instead of solving the Emf equation or the armature equations circuit equation for omega m suppose we look at the torque equation and solve it for omega m. So what will then be the result omega m will be combination of in the torque equation we have load torque and armature current and anticipating there I am going to solve the other equation for the armature current I a. So from T l to omega m what will be the transmittance T l plus K f omega m, so T l goes to the other side and divide by K f so the transmittance is minus 1 by K f and what is the transmittance of the armature current to omega m, K T, I a equal to T l plus K f omega m.

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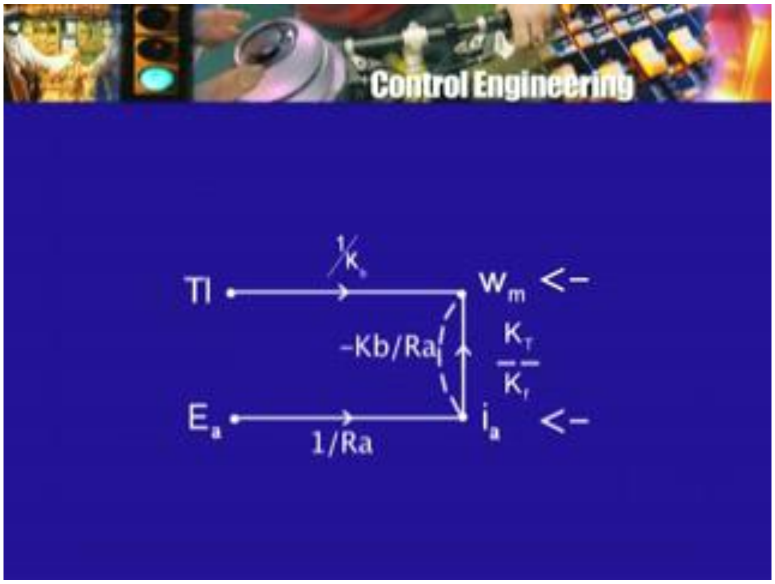

So the transmittance will be K T divided by K f, so this part of the total signal flow graph which we are going to draw will represent the torque equation expressing omega m as a sum of 2 signals. Now if you go back to the first equation I should not solve it for omega m and since view preferred to see I a as an output I will solve it for I a instead what will happen then I a, R a equal to E a minus K b in to omega m and there will be an arrow going from E a to I a and what will be the transmittance, the transmittance will be one by R a because I a, R a equal to E a minute K b omega m and there will be an arrow from omega m to I a and what be the transmittances of that it will be minus K b divided by R a.

So is a different signal flow graph and you can compare it with the graph that we draw earlier same input nodes E a and T l, E a and T l here also same output nodes omega m and I a, omega m and I a here also. So the same choice of input nodes same choice of output nodes but the 2 graphs look different and that is quite okay is nothing wrong about it there is no need to get worried. Although the text book make draw it in one particular way or your teacher may be in the habit or drawing it one particular way nobody can say that either of these or both of them or wrong, both of them correct and in fact there may be a few more signal flow graphs that you could draw, do try to draw those other signal flow graphs to get practice but to proceed further either of these two signal graphs would be good enough.

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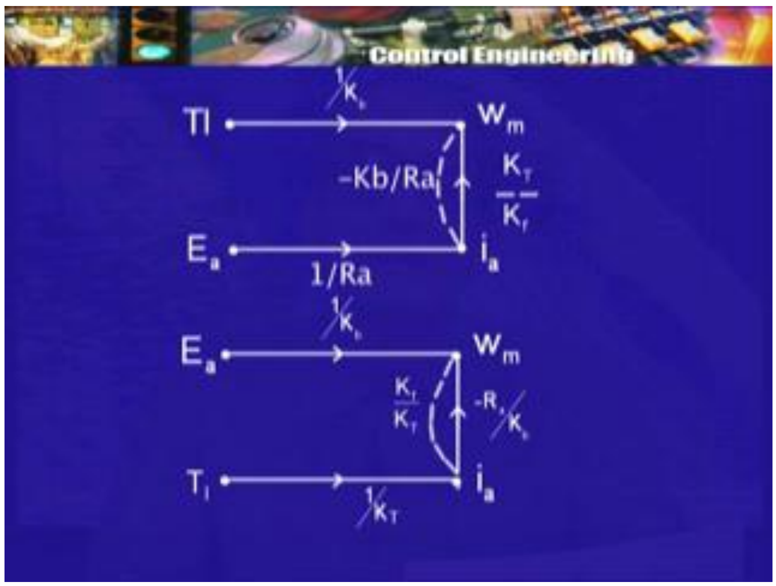

In fact the first one that we drew also will be a correct graph and will see whether it is good or bad or what is the difference between these various signal flow graph and why be may prefer one over the other okay. So now that we have drawn the signal flow graph we have put down in a graphical form all the information that, we now about the system namely the relationships since between the various signals so we can leave behind those equations and concentrate on the graph.

Now Mason as I told you gave a rule or one can write it form of formula and it is called Mason's gain formula because we will see that some terms that appear in the formula look like gains these are the multipliers in one of these blocks transmittances. So we can now take a look ate the statement of Mason's gain formula and apply it to each one of our 2 equally correct signal flow graphs where omega m and I a appear as output nodes and armature voltage and load torque appear as input nodes and we will also take at the earlier the very first signal flow graph which was very simple in which the things where just other way round and see what is the difference.

Now Mason's gain formula or his rule in the beginning may sound fairly complicated and it is complicated because it is meant for not a very simple signal flow graph like the once that we have looked at in which the number of nodes is very small and 2 of them are input nodes only.

So there are no incoming arrows it is a rule which will applied to a signal flow graph which may have larger number of nodes with all kinds of transmittances, with no restriction on transmittances. In fact Mason allowed something which is very peculiar which is called self-loop because it is looks like a loop all right.

At the moment we do not have at in our signal flow graph but when Mason introduced this idea in his very first paper what a did was he suggested or he gave some simple rules for manipulation of the signal flow graph that is transformation of the signal flow graph. You have the signal flow graph at looks fairly complicated and so you make out what is the relationship between the variables other then the equations at the nodes. For example what we did earlier, I do not see it here that omega m in terms of load torque and armature voltage or armature current in terms of load torque and armature voltage because I have the 2 equations but each of the equations the other quantity appears.

In the armature equation both omega m and I a appear in the torque equation booth omega m and I a appear whereas I want and expression for omega m only, in terms of E a and T l that is with I a eliminated. Similarly one may need an expression for the armature current in terms of the armature voltage and the load torque with the speed eliminated from the equations and in fact this is how you normally solve algebraic equation do not you, if you have what are called simultaneous linear algebraic equations, may be 3 equations in three variables or larger number. You use this method of elimination of variables and essentially end finally getting one equation in one unknown and do this in different ways. So that you get expressions for all the unknown and one of the rules associated there is of this short, what is it called whose it named after you may remember a rule called Cramer's rule in connection with determines or simultaneous linear equations. It is not surprising that Masons's gain formula is a sort of a way of using Cramer's rule for solving a system of simultaneous linear equations.

So only thing is Cramer's rule requires you to write down the equations in particular way for example in the form for matrix and a vector and so on nicely arranged set of equations and then you looked at some determinants and so on in Mason's rule you do not have equations written in that form at all as you do not write equations you write them symbolically or graphically in the form of the signal flow graph and there are no determinants to be calculated, there are some quantities to be calculated or obtain from the graph. So it is a sound of a graphical or pictorial version or Cramer's rule. So the first thing that Mason was doing was to give you rules of transformation of signal flow graph and at one time that use to be an interesting topic.

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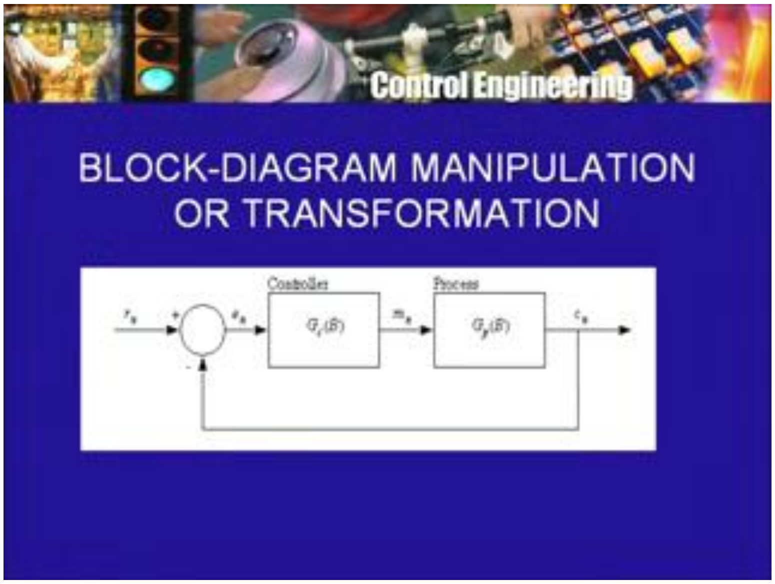

In fact, it is still a good topic for getting some practice what is called signal flow graph manipulation or transformation or you can do the same thing with block diagrams then it is called block diagram manipulation or block diagram transformation and when I studied my a subject many years ago, I spent some time looking at these things block diagram transformations and signal flow graph transformation. But today of course there are so many other thing that we have to learn so you do not have time for everything also as I told you signal flow graph is really to be use by human being for a small problem.

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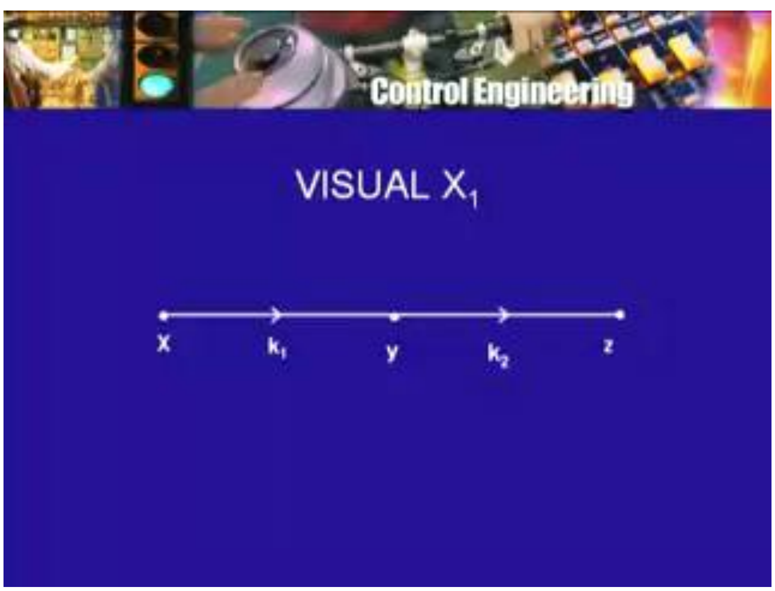

For a big problem we have do things differently and today of course be make use of computer program or computer packages but just to illustrate this idea of signal flow graph transformation or manipulation. This one simple system that I can look at not system really or simple signal flow graph it is so simple that you do not really have draw graph a signal flow graph for the equations that are involved. Suppose, we have 3 signals x, y, z and there are 2 relationships which are put down on the signal flow graph like this.

So what are the relationships y equal to k 1 (x) and z equal to k 2 (y) there is only one input node x and there are 2 other nodes y and z, each one of them as incoming arrows so it corresponds to an equation. Of course y has an incoming as well as an outgoing arrow. Now suppose you have this simple signal flow graph then what can you do with it one can see immediately that the variable y can be eliminated that is the signal y can made to disappeared from the signal flow graph to draw new signal flow graph because you may now be interested in this y. In fact truly this y may be what I have called earlier and intermediate variable y may be what I have called earlier and intermediate variable it is not a variable directly of interest to us it is neither input, neither output that is easy to eliminate it because even high school student can do that y equal to k 1 (x), z equal to k 2 (y).

So substitute in for y in z equal to k 2 (y) will get z equal to k 1, k 2 in to x. So here a simple transformation rule for signal flow graphs that is if you have a configuration like this that is there is a intermediate node y to which there is an incoming arrow exactly one. So there is very simple equation y equal to k 1 x and there is an outgoing arrow in this case exactly 1 then that intermediate node can be eliminated. So this is a very simple rule indeed and if Mason stop with this then what would not appreciate his work but you can think of a more complex situation where you have a signal flow graph which is little more complex.

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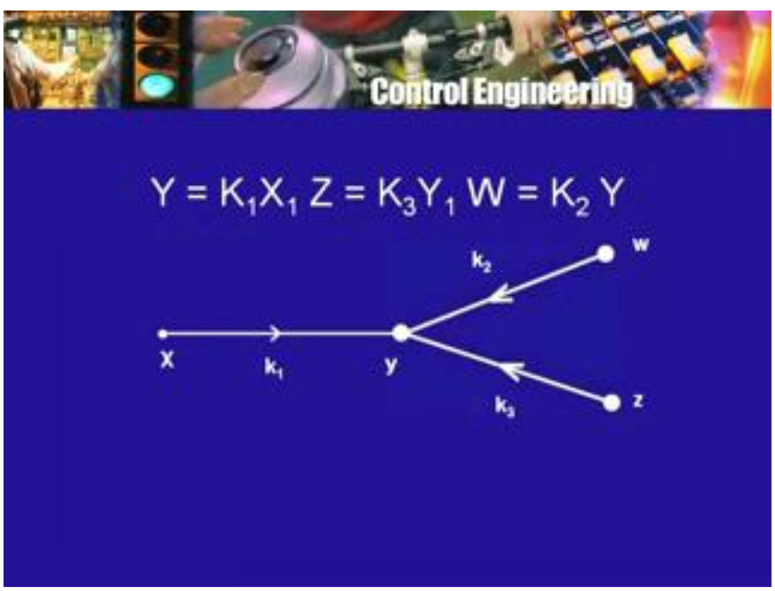

So I have x input node then there is intermediate node y which is connected with x through a transmittance k 1 but y feeds in to 2 different signals. Let say some signal w with a transmittance k 2 and some signal z with a transmittance k 3 okay. This is different from the earlier one earlier one we had all most straight transmission x going to z through y. In fact that kind of a connection is given a name if you thought of that k 1 and k 2 as some physical blocks are systems that kind of connection is given a name, what is called it is called tandem connection one after another it should not be called a series connection because series can be confused with series connection of electrical components. We are not talking about series connection of components, we are talking about connection of blocks or systems.

So tandem connection or some it is also called the chain connection like a chain you have a chain of these blocks or systems one coming after another and so that rule that simple transformation rule applied to a tandem or a chain connection of transmittances but the this situation is little more complex. We have an intermediate node however y now can be get rid of it, yes and as we can see it will be just one more step beyond the earlier rule because now I have a w equal to k 2 (y) and y equal to k 1 (x). So w equal to k 2 in to k 1 (x), so I can directly draw an arrow between the x and w with the transmittance k 1, k 2 but I should not forget about the other node z the other node z is also a signal which may be important or useful later on.

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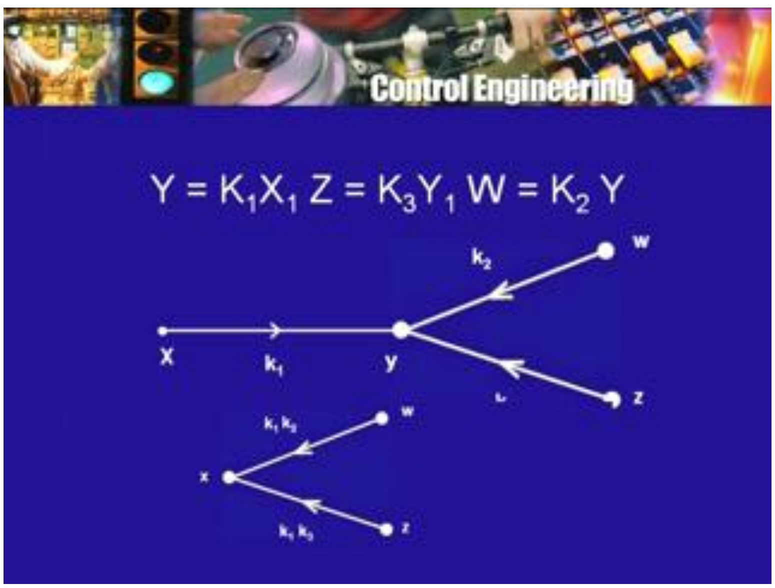

So I cannot leave that node alone in fact if you leave a node alone which means you have no equation for it, you can have a convention that he equation say z equal to 0 is nothing coming in to it, so the signal is equal to 0, what we have z equal to k 3 (y) and y equal to k 1 (x) therefore I must put another transmittance from x to z of value k 1, k 3. So I can eliminate they intermediate node y but now I have to replace 2 new edges or transmittances. Now what is the point doing this transformations because why are these transformations to be done or thought of at all because the simplify the graph. Here for example the graph for three nodes after transformation it has a 2 nodes but there is a flip side two it that in doing this we are removed the signal y, it may happen the that signal y is also useful or required later on and therefore what one does is one then goes back after your work with x and z without y then you go back to y and that is easy because y is simply k 1(x).

Similarly, in this case we had 4 nodes we have eliminated one, so we have only 3 nodes now. So we can go on may be carry out some further transformation simplification elimination and finally of course once we have some relationship we can find out y, you can always go back to y (x) , k_1 (x) this is no problem what is ever. So there are a number of such rules transformation rules that Mason looked at and there all there in his very first paper on this signal flow graph representation of system equations. Mason must have realized while doing this transformations that something interesting could come out of it and that is what perhaps led him to give his gain formula or rule and today we can see what is it that could have given rise to Mason's gain formula made him to short of persist and go ahead and try to get something interesting not very obvious from the signal flow graph.

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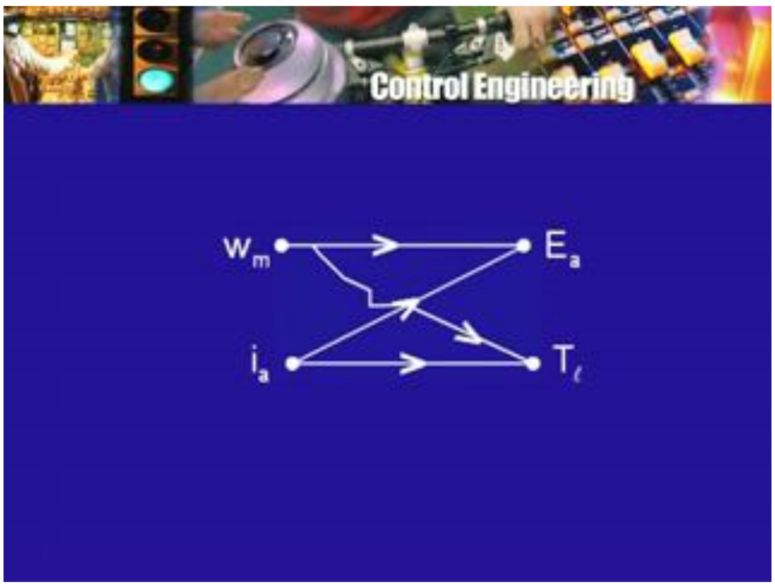

Let us go back to our very first simple minded signal flow graph with omega m and I a as the inputs, T a and T l as the outputs. As you can see from the signal flow graph there is a hardly any work to be done in the sense E a is equal to omega m in to some transmittance plus I a in to some transmittance, T l is equal to omega m in to some other transmittance plus I a in to some other different transmittance. So I already have expressions for E a and T l in terms of omega m and I a is that is what wanted then there is nothing more to be done. In fact I may not have a drawn the signal flow graph at all whereas the other 2 graphs that we have got there are little more complex or they are different I look at the node omega m, I get a relationship expressive omega m in terms T l and I a, I look at the other node I a, I get I a in terms of omega m an E a.

So omega m involves I a, I a involves omega m and how does this appear on the graph it appears in the graph in the form of loop or a closed path for example here is a path if I want to trace it starting with omega m through the transmittance minus K b by R a, I will go to the node I a and from the node I a through the transmittance K T by K f, I come back to the node omega m. So omega m to I a back to omega m that is referred to as a loop or close path and of course I must look upon it the arrows as one way speeds. So I should go in the correct direction so when I trace the close loop or path I should go from omega m to I a through minus K b, R a divided by R a and from I a back to omega m through this transmittance K T divided by K f. Of course I could start going around the loop from I a so from I a, I go to K T by K f to omega m and then from omega m, I come back to I a through minus K b by R a.

So whichever way I want to do it I can trace the loop but I must trace it in the correct direction. So there is nothing like starting node because I come back to that node. So in a loop every point can be a starting point where you in finish. So that we there is no difference between the 2 but there is a loop in this loop goes with the fact that for omega m, I have an expression in which I a is involved for I a, I have expression in which omega m involved whereas as we saw and we did this earlier we want an expression for omega m in terms of E a and T l alone we do not want armature current if possible we want an expression for the speed in terms of the armature voltage and the load torque only which therefore means that if I wrote down the equation I will have to eliminate I a as the unwanted variable.

Now if we look at this signal flow graph I can not eliminate either omega m or I a very simply because it is not like the earlier case were we had tandem connection and an intermediate node which could be eliminated easily or I connection were from a node, we branch of to two different nodes. So that intermediate node also could be a removed very easily that is not the case here. We have this loop situation the other signal flow graph that we drew also has exactly the same feature there is a loop occurring there of course if you look at it carefully you will see that the transmittances are different in one case I a to omega m it is K T by K f in the other case omega m I a it is K f by K T.

Similarly, from omega m to I a it is minus K b by R a from I a to omega m it is minus R a by K b. So the transmittances are just the reciprocals they are not the same for that has happen because we solve the equation for one or the other depending on that one transmittance appears or it is reciprocal. But in either case we have a loop now what Mason notice therefore is that this is the situation where we have to eliminate one of the node and therefore he thought of a rule for elimination of the node. Of course that requires redrawing the signal flow graph just as we redrew the signal flow graph by removing the intermediate node y, I like to starts with the one of the signal flow graphs and do this some operation on the signal flow graph to remove one of the nodes. But eventually Mason probably saw that we can do it in such way that we do not have to remove anything from the graph but of course we have rule which is a little more complicated and as I said no wonder it is complicated because a signal flow graph could have a large number signals or nodes and they could be connected in various crisscross ways.

In our two signal flow graphs each one for them as only one loop there are signal flow graphs which you can draw for actual control systems which will have more than one loop and I mentioned feedback control systems, we have talk to about this feedback loop and I had said in chemical process situation where there are large number of variables which are to be controlled there are large number of feedback signals and therefore there is more than one loop. So if you draw the signal flow graph for a chemical process example not a very simple or to may be when we practical one you will end of getting quite a number of loops. So the rule has to be complicated because it has to account for or it has to take care for such complex situations. But once you understand what are the various terms or words that are used in the rule what do they mean in terms of the signal flow graph and take care with some of these signs that have to be looked at the rule is not that difficult at all and so I will get started with the description of the rule.

Now what the rule says or the rules enable you to write down is the following. To start with the rules tells that you can obtain an expression for node signals which do not correspond to input nodes in terms of the input node signals that is you can write down expressions for each one of the non-inputs nodes in terms of the input node signals. So for example omega m and I a are the non-input node signals E a and T l are the input node signals. So I can write expressions for each one of these non-input node signals in terms of the input signals and what do I mean by write in terms of what I means is I can write down an expression like omega m equals some expression and Mason's rule tells you what this expression will be were, how to write it down multiplying one of the 2 input node signals in this case E a plus another expression multiplying the other input node signal in this case T l and that is exactly what you want.

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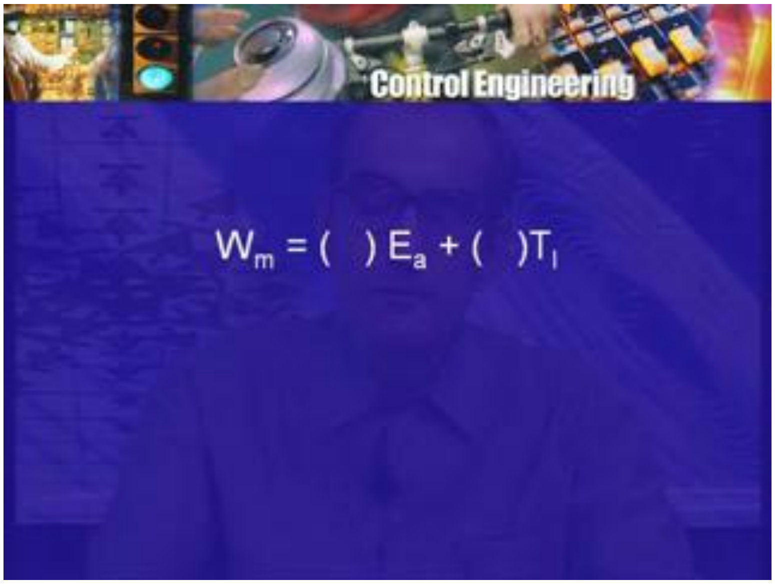

We want an expression for omega m in terms of E a and T l, these 2 expressions or what eventually make all coefficients that multiply E a and T l and therefore you could even think of them some kind of transmittances from E a and T l to omega m, Mason's rule or Gain formula enables you to write down these 2. In fact when I filled in the expressions I have a formula and that is why it is called the gain formula although strictly speaking it should be really call Mason's rule.

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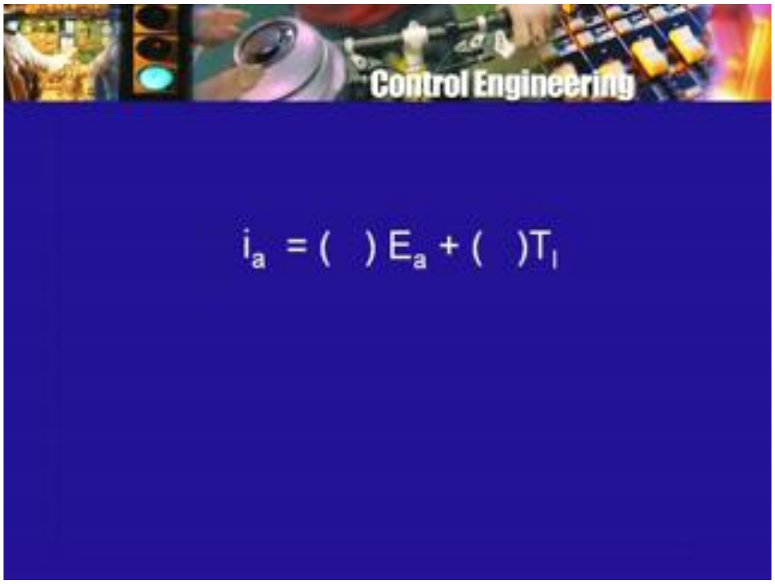

Similarly, for the other non-input node I a I will be a write down some expression in to Ea plus some other expression in to T l. So that if I need to look at the armature current I can also use this expression. Of course if I am only interested in the speed I am have not worried about the armature current because speed is the main thing of important to us then I need not write down the other equation but the rule will enable you to write down the other equation as well.

So that is how Mason's rule or that is the final outcome of a applying Mason's rule you will be able to write down an expression for a non input node signal equal to a somehow various input node signal multiplied by some coefficients or transmittances and the rule tells you how to write down of the transmittances. You will see if you remember your school algebra that there is a similarity between this and Cramer's rule because Cramer's rule also enables you to do the same thing if X 1, X 2 to X n are the n unknowns in the equation, linear equations and on the right hand side of the equations as we usually write them are some quantities say b 1, b 2, b n then we have x 1 in Cramer's rule of course x 1 is ratio of 2 things but those two things can be written as b 1 multiplied by some coefficient plus b 2 multiplied by something plus and the last term is b n multiplied by some coefficient.

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So it is very much like Cramer's rule although not in the usual form in the usual form Cramer's will be simply x 1 equal to on determinant divided by another determinant but if we expand the numerator determinant then you will get a summation. So in fact these bracketed expressions that the rule tells you how to write are in way determinants of some kind and the rule tells you really how to write down these determinants without going through the usual procedure of Laplace's expansion or whatever. Mason has done that work for you and has produce this rule. They are of course other earlier attempts which are made to handle simultaneously linear equation Cramer's rule was satisfactory all right.

But it involved computation of determinants and therefore naturally a rule which enabled you to do compute the determinants in some shortcut way would be desirable and I found out that one of the persons who was associated or who really introduced the concept of a matrix, the English Mathematician Cayley had something to say about this. In fact a gain way of evaluating determinants which amounts to writing a signal flow graph corresponding to the determinant and then doing some work on that signal flow graph as the same kind that we are going to see he was of course more interested in evaluating determinants rather than in solving systems of equations are any block diagram or signal flow graph manipulations, he was not looking at physical system equations as that he was only looking at the problem of solving simultaneously linear equations but Cayley gave a rule which looks very much like Mason's gain rule or formula okay.

Okay so now let us choose this particular signal flow graph I will express omega m in terms of E a as omega m equal to sum coefficient multiplying E a plus another coefficient multiplying T l. So suppose I look at only the first part of it how does omega m depend on E a that is in that omega m equal to what multiplied by E a plus the other term that involves T l, what is the coefficient that multiplies E a. Now to obtain that coefficient from the signal flow graph one introduces the almost obvious idea of a path form the input node in this case E a to the node whose signal you are a evaluating or you are trying to find an expression for.

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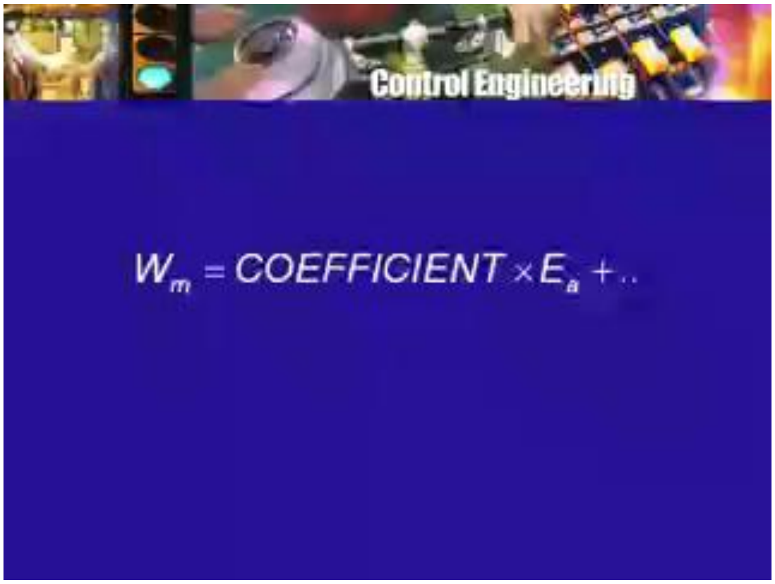

So we look at the path from E a to omega m this almost looks like our forward path in the control system block diagram. In fact when he get back to the control system problem with feedback we will see that it is all most close to that but at the moment we need not worry about forward and backward because from an input node you can go only in one direction. So it is only a forward path anyway so for this first we need to find out path from the particular input node that you are looking at to the particular non-input node that also your are looking at namely in this case path from E a to omega m what are the paths from E a to omega m because that is this path E a to omega m through that 1 by K b is there any other path.

Now this is where you have to be careful, for example if I were to sort of think of this as some road network and I wanted to move around the city of course the roads are all one way roads there are arrows. Now I wanted to go from E a to omega m, how would I go while I can go of course directly along this transmittance one by Kb along that road I will reach omega m. But let us say I was absent minded or I just wanted to room around a little bit so then instead of stopping there I go on to I a and then come back to omega m. So E a to omega m to I a back to omega m does that count as a path or not in practical terms it may of course if I do it this way a policeman who might be watching might get suspicious as to why this person is going from omega m junction to I a and then back to omega m, has he lost his way or something or is he up to some mischief. In this case we do not allow such a thing the rule says that by path we mean something where the same node will not be visited again.

So E a to omega m is okay but E a to omega m to I a to omega m is not to be chosen that is way the rule is because that is way the determinant expansion will have it so we are not to do it that way. So then what are all the paths from E a to omega m there is only one namely this edge 1 by K b and so with that forward path then or with that path from E a to omega m we have a single transmittance 1 by K b. So that is one thing that we have first of all work out find out all the paths from an input node that we are looking at two the other node that we are looking at in this case I want to have an expression or part of an expression from omega m in terms of E a.

So I am concentrating on E a as the input node and omega m as the node for which I want to write down an expression in this case there is one only path but of course in a very complex signal flow graph with a large number of nodes with large number of transmittances connected various way, in various way there could be more than one forward path and just to make sure that they are all together I will take an example just a numerical example not representing any physical system where there is more than one path between a node which is an input node incidentally some books might use the word source node, in fact Mason use that term and a node which is an output node is called as sink node for control theory people input and output is more appropriate choice of terms than source and sink.

So path or all the paths have to be found out. The second thing that we have to find out is all the loops all the loop or closed paths in the signal flow graph, in the signal flow graph that we are looking at either of them the loops sort of stands out immediately and this is were the advantage of the signal flow graph technique is that as a human being if I look at a graph like this and the graph is not too complex I can immediately spot the closed loop or the closed path. So there is this closed path which involves omega m and I a omega m to I a through some transmittance and then back to omega m through another transmittance. So this is a loop with that loop is associated what Mason call the loop gain that is the product of the transmittances along that loop.

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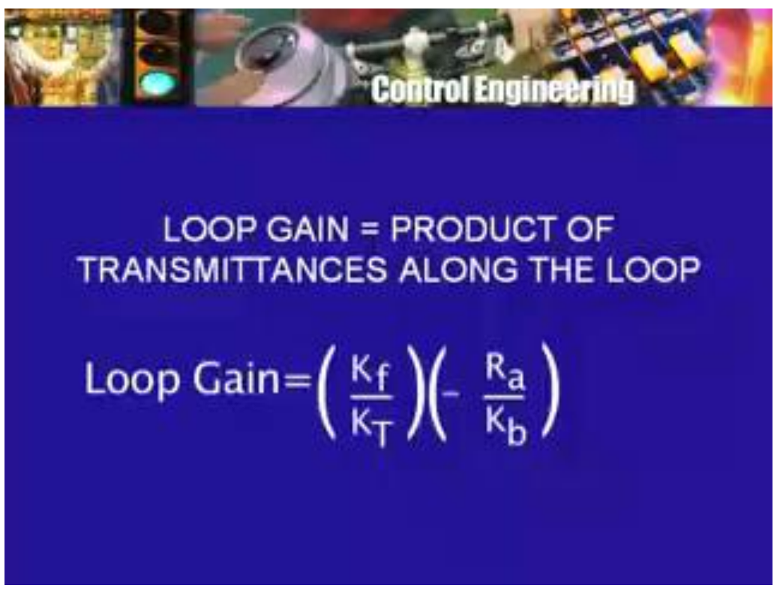

So in this case K f by K t multiplied by minus R a, K b this is the loop gain there is only one loop here and the loop gain therefore is K f by K t, this is one of the transmittances multiplied by the other transmittance, do not worry about the negative signs minus R a divided by K b. So this is the loop gain associated with that loop. In general you are to identify all the loops and there may be more than one loop. So for each one of the loops you are to find out or you are to calculate the loop gain.

Now when the signal flow graph is complex there is more than one loop one should not be over confident and say that I found out all the loops that they are there are in the graph one as to make sure that you are not missed out any loops and so it is good to do it in some systematic way for example, I can ask is there any loop that involves a node E a, no the answer is no because E a is an input node and arrow only goes out of E a and arrow never comes back to E a. So E a cannot be in the any loop but omega m could be because there is an incoming arrow and there is an outgoing arrow similarly, I a could be in a loop but not T l.

So this can be done systematically that given a signal flow graph first of all input nodes are excluded from loop they do not lie in any loops because there are only outgoing arrows and the other nodes you can check whether there are any nodes which have both incoming and outgoing arrows, there may be some nodes which have only incoming arrows and as I told you earlier they can be introduced fictitious nodes can be introduced to ensure that that is that happens otherwise many of these nodes will have both incoming and outgoing arrows and then one tries to trace the closed paths or loops carefully 1 by 1.

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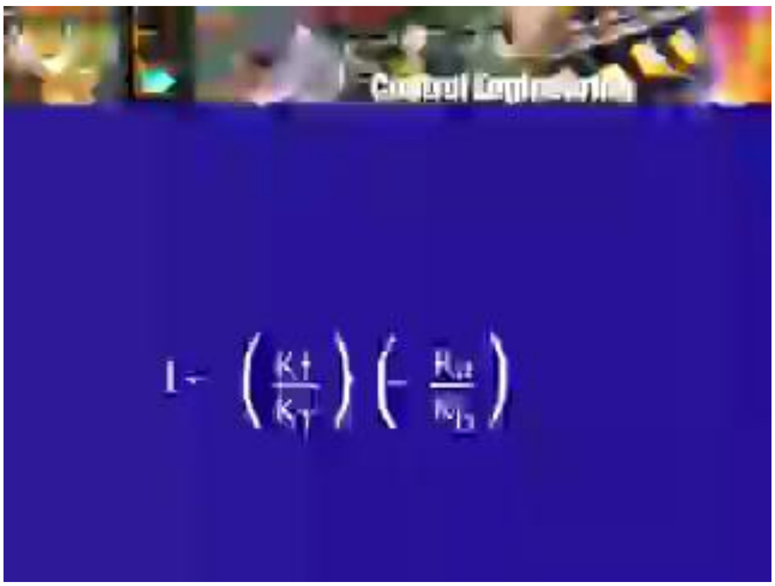

In our figure there is only one loop, so there is only one loop gain to be calculate and then that is what I have calculated. Now the next part is a little harder and we will take of an example to illustrate that but let me first state it and then we will look at the example where there will be more than one loop when there is only one loop as in this case then just write down the expression 1 minus that loop gain 1 minus the loop gain. So in this case it will be one minus K f by K t in to minus R a by K b therefore that minus minus will cancel if you worried by the that minus sign and you will get one plus K f, R a divided by K t into K b and if I assume K t equal to K b which is usually the case then I will get K b square in place of K t in to K b.

So if there is only one loop then the rule is as simple as simply one minus loop gain that is the next thing to be calculated. But if there is more than one loop present in the signal flow graph then some more work is to be done, what is work to be done? First thing you select the loops in pairs, select a loop pair, a pair of loops there may be one loop here there may be another loop here but select them in such a way that the two loops have no nodes in common at all and such a situation is referred to as non-touching loops. So I look at a pair of loops and make sure that they are non-touching, they do not have any node in common, they will not have an edge in common certainly but they should not even have any node in common.

So we need non touching loops, so 2 pair of non touching loops what do I do with them then for a such a pair I take the product of the loop gains when I take all such non- touching loop pairs take there per loop gain products 2 by 2 and add them up. So I have one minus if you have more than one loop then it is not just one minus loop gain as I said earlier for our problem but it will be 1 minus the sum all the loop gain, all loop gains whether the touch each other or not all the loop gains plus the sum of loop gain pair products but non touching, so sum of product of 2 non-touching loop or their gains. All possible loop pairs which are non-touching take the loop gain products and add them off that is next term.

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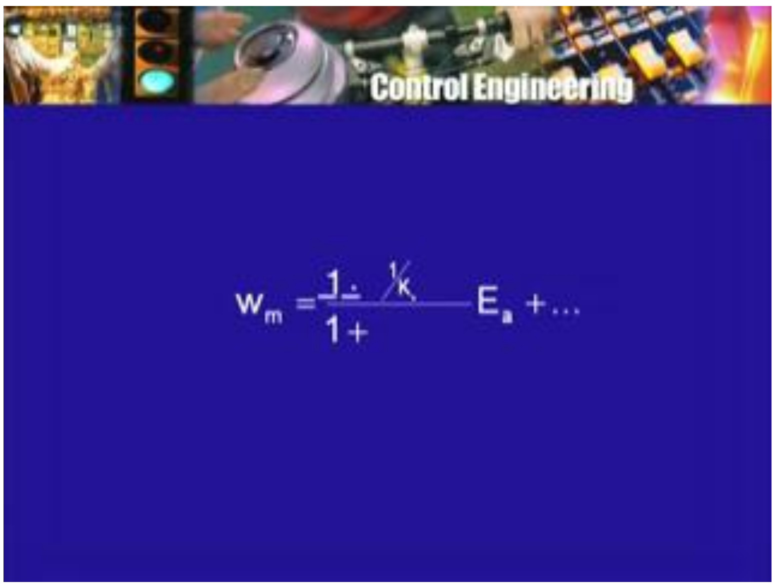

So the sum all products of two loop gains subject to this non touching condition but that is not all if there are more than two loops I will have to look at a possibility that there are three loops which are non-touching pair wise each one of them does not touch the other two then I have to take the negative of the sum of product of such 3 loop gains and then plus again the sum of product 4 non touching loop gains and go on till I have exhausted as large a collection of loop gains as I can find may be I end up having 6 loops finally which are non touching to each other but the moment I look at 7, I find that some two of them are going the touch each other. So I stop there with the non touching loops as many as possible.

So I have to calculate this expression this expression will be the denominator in my multiplier for that input signal. So for our problem fortunately the calculation is very simple I simply have one minus loop gain and therefore I have one plus K f, R a divide by K t in to K b that is going to be the denominator of the multiplier for E a and this denominator because it is really related to determinant is sometimes called delta, delta for the signal flow graph. In fact the same delta we appear in all the terms in this case this delta was simply one minus loop gain.

But it could be one minus loop gain or sum of all loop gains plus sum of 2 non touching loop gains minus sum of products of 3 non-touching loop gains etcetera going on till as many as you can get non touching loops that is the denominator part and the numerator part is obtained a little easily, you have found out the a forward path or path from the input node to the node under consideration. Look at the denominator expression and get rid of all those terms which involve a loop which touches this forward path. For example our forward path from E a to omega m touches this loop which involves omega m and I a.

So I remove that term so what remains on that only one so the numerator in this case will be only one and this one is to be multiplied by the transmittance of that path. So this one is to be multiplied by 1 by K b and therefore my expression for omega m in this case as far as the term containing E a is concerned will be omega m equal to one by K b divided one plus K f, R a divided by K t, K b this whole expression multiplying E a but there is the other term that depends on the load torque, so plus something else multiplying the load torque.

Now if I have followed what I have said so far you should be able to write down that second multiplier in the same way as you wrote the first multiply. The denominator which is one minus sum of loop gains plus sum of products of two loops gains etcetera, etcetera. The numerator is the transmittance of that forward path multiplied by the denominator expression only choosing those terms that do not touch the forward path that is only keeping those loop gains which do not touch the forward path but do not forget about that one that term one remains in the numerator also. So do not remove that term one only the loop gains which touch the forward path are to be removed and the rest of the loop, loop gains are to be retained.

So in other words you are looking at loops which do not touch the forward path and also do not touch each other. So this the way to write down the expression or the multiplier for each one of the input signals. So apply this for our signal flow graph to complete this problem and get the expression for omega m in terms of E a and T l and then compare it with the expression that we have obtained earlier. It is arranged differently but it must be the same formula finally then you can repeat this with the other signal flow graph that we drew of the same system choosing a somewhat different arrangement but both omega m and I a as the output nodes apply it to that signal flow graph and check that will get the same expression. So which are we you draw the signal flow graph the final answer will be the same and that is the way it should be because you do not expect that from that will get two or more different answers.