Control Engineering
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Lecture - 14

We will derive the relationship between the speed and the load torque in a different way, one way of deriving the relationship is of course, it to write down the equations that relate the various quantities some of them we have already written down, we will note down the equations for the electrical circuit applied voltage E A equal to the back EMF which is K B times omega M plus R A, I A the armature drop then, the torque equation which is the torque produced by the motor which is K T or K_B into I A equals the load torque T L plus K F times omega M. The second term being the frictional torque this is under the assumption of study state the current, armature current is not changing. So there is not inductor drop and the motor speed is not changing therefore there is no inertia term.

So we just had these 2 equations and we solved them earlier, we solved them to get omega M in terms of the remaining quantities except I A and one could find another expression or an expression for the armature current separately. But of course, we are interested mainly in the field current in the motor speed rather than in the armature current. So we looked at only the equation relating omega M to the quantity such as the applied voltage E A, the load power P L and the various coefficients, armature resistance, coefficient of friction and the back EMF or the torque constants but now, that we have decided to change the scheme by using feedback the armature voltage is not directly applied and it is not a voltage which is somehow going to be kept fixed and applied to the armature.

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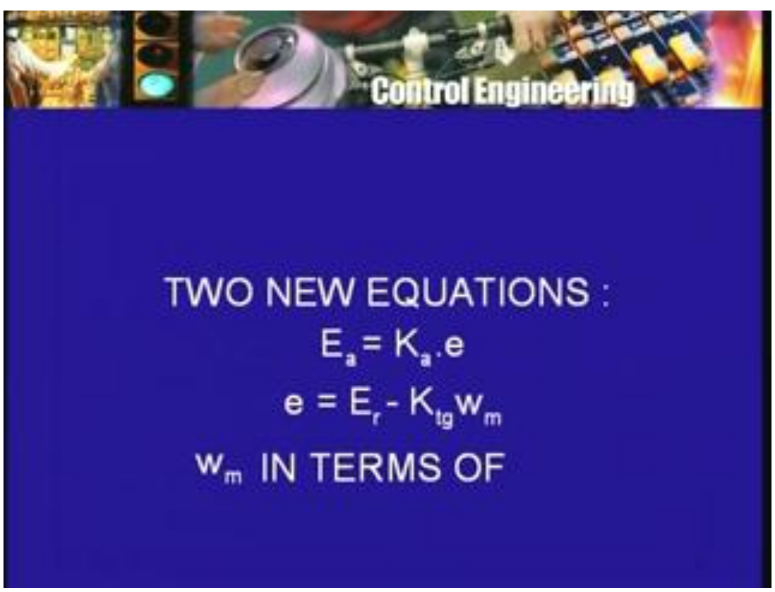

We are going to produce it with generator whose and therefore that voltage in turn is controlled by the field current of the generator which in turn will be controlled by the output of the difference device that we are looked at. So we can now write down a new equation relating the armature voltage E A coefficient K A, so we have E A equal to K A times E and there is one more equation relating the output of the comparator to the reference input E R minus the tacho generator or the feedback voltage, so K tacho generator into omega M. So we have to now consider these two equations in addition to the earlier two equation and then, one can get an expression for omega M, the motor speed the steady state motor speed in terms of what not E A any longer because E A is determine by small e the error, so called error voltage which in turn depends on the reference voltage and the motor speed. So finally, we will have an expression for omega N omega M in terms of the reference voltage E_R and in this expression a what other terms will appear the parameters of the drive will appear that is the armature resistance K B and K F, the coefficient of friction will appear as before but in addition we have two new coefficients.

We have the gain, so called gain of the forward path K A which relates the armature voltage to actuating signal E and the feedback signal coefficient K TG which relates the feedback signal to the output quantity omega M. So we will have two more coefficients K A and K TG and what I ask you to do was to see the effect of changing this K A on the value of omega M, when the load torque changes and the load torque is at the rated value, the motor will run at the rated speed there will be no steady state error that is right therefore, the value of omega M will be the desired value of the motor speed but when the load torque is not at the rated value then, the speed is going to change the question was by how much and in what way will this coefficient K A affect the speed error, the steady state error in speed when a disturbance takes place this disturbance that we are talking about is a change in the load torque.

Of course, the other coefficient also okay tacho generator that is the coefficient which appears in the equation for the feedback signal that also will affect the steady state error. Now, as I said these are all steady state calculations and this is not enough because the conclusion that we will draw from this steady state analysis will be that the greater K A is the less will be the steady state error and therefore, one would be tempted to increase K A to as large a value as may be, practically possible in the hope that this steady state error will be therefore, reduced to a very very small value. But I mention that this K A has an effect on the transient error and we have not looked at the transient performance so far and that is something which has to be looked into and we will do that.

But before we do that, let us take a brief look at a technique which is useful, when we look at some small system problem like the motor case that we are looking at, where there are not too many components and the system although it may have looked more complicated to you especially, when we have introduced feedback, the system is not really that complex, it really does not have too many subsystems or too many devices more over the relationships between the various signals or the various quantities are also fairly simple. For example, the armature voltage that is generated is simply proportional to the actuating signal E. So there is a simple coefficient relating the 2, similarly the back EMF is simply proportional to this speed etcetera.

So the relationships between the various signals or physical quantities are also very simple in this case, they are what are called algebraic or so almost arithmetical not only that they are simply multiplication, additions, subtraction and nothing more complicated than that we do not have any sinusoidal function or exponential function or any square and things like that appearing in these expressions. Of course, no derivatives occur because we are looking at the steady state once, we want to take into account, what is actually going to happen during the transient period. We will have to take into effect a take into account the derivative terms and therefore, the equations will not be such simple equations but they will be differential equations but even for such differential equations some of the techniques that I am going to talk about are applicable.

Now this is a technique or this is a almost a formula and that is what it is usually called which was discovered by Mason in the 1950's and this is, therefore called Mason's gain formula. Now Mason was looking at problem of the sort that I am talking about but he was not looking at control systems only he was thinking of systems which could be of other kinds, in particular one could be looking at an electronic amplifier as a system under study and especially, when the system has feedback in it that when there is some kind of that loop arrangement, you know one signal effects another and signal in term affects the first one and there is some kind of intricate relationship which mean these various signals.

So Mason was looking at that problem and he discovered this gain formula and that was almost 50 years ago. The formula is really suitable for people for you and me to understand to get some understanding of what can happen if we make some changes. So it is a very useful tool for qualitative understanding of small systems the formula of course can be used for large systems and will give exact quantitative relationship between the various signals. So the formula is applicable in a very large number of cases, where such relationships are simple enough. But today one would find it advantages to use a computer program to do all these calculations especially.

When we are looking at the situation in the transient state and not in the steady state only and therefore, when we are going to look at differential equations then today, certainly it is possible to use not only write computer programs, on your own but use readymade computer packages which enable you to handle the problem at hand and therefore Mason's gain formula is not necessarily to be used in such calculations. But for conceptual understanding for getting some feel for what can happen, if we do some changes Manson's gain formula is a very good tool and therefore we will take little time to look at it

I will not prove anything I only state the result or the formula and we will not really go into the full generality of it. We will right now, look at whatever is enough for our purpose at hand. The gain formula as I told you earlier arose out of Manson's study of signals that is various physical quantities that are associated with the a system which consists of a large number of interconnected subsystems at that time already the idea of block diagram was commonly, we being used. So that was not anything new engineers have at started using block diagrams a such as we have already used and as I told you the block diagram really enables us to concentrate on only a few signals associated with what is represented by that block.

For example, when we represent the motor and the load by a block perhaps the only relevance signals as we saw were the armature voltage, the speed, the load torque and perhaps the armature current, if it might be of some interest or use other quantities we do not a write explicitly also with the ultimately, we will simply associate an equation or two equations which express relationships between these signals. Now the technique introduced by Mason goes a little further, one can look at the simplest case where you have only two variables or only two signals or only two quantities associated with some device or with a system, you can think one of them as the input quantity and as the other as the output quantity depending on what particular significance those quantities have. But if you are only interested in the relationship between these two quantities then sometimes it does not matter which one is regarded as input and which one is regarded as output, if you do not mean by input and output literally that input is what you are going to change in order to make the output change in certain way.

So in the simplest case, we will have block or a system which has two variables or signals associated with it with a very simple relationship between these two signals and the simplest relationship that one can think of in our context is just what is called a linear relationship or more specifically one signal is simply a multiple of the other for example the amplifier block or if you think you of the so called error signal or actuating signal somehow effecting the field current and the in term then giving rise to a certain armature voltage if you only look at these two signals, the error signal E and the armature voltage capital E A then already I have represented that by means of single block, a small block single block K A showing e as the input and capital E as the output.

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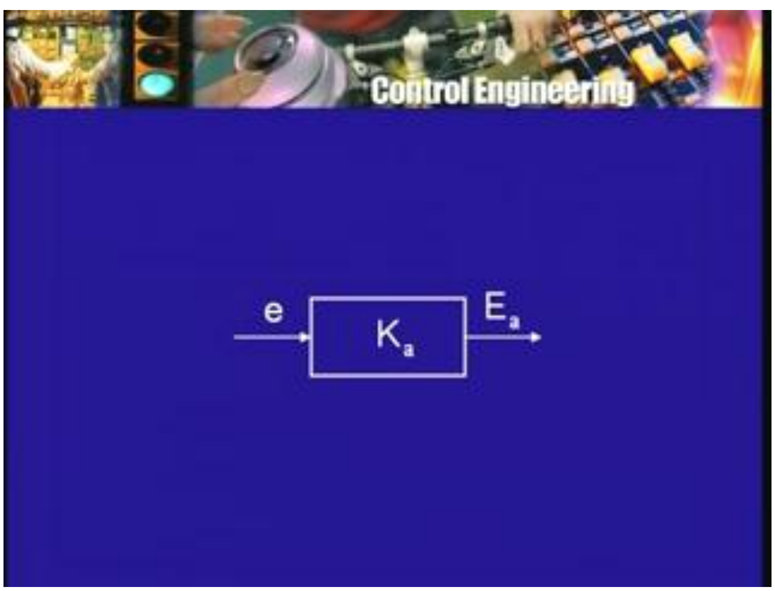

So I have not shown in this the motor drive that drives generator the field current of the generator and the may be the armature current of the generator and all that kind of thing I only shown only two variables which are of importance here the armature voltage produce by that arrangement and the error voltage which is spread to that arrangement to produce that armature voltage and so there is the simple block K A sitting between E and E A, Mason goes one step further and replaces this by the following and this is done not only for a block like this but is done for the various blocks to get something which is different from block diagram but as we will see and as you can see its going to be very closely related to the block diagram.

So it is a different way of sort of conveying the information in the block diagram and the advantage of doing it this way will become obvious as we go on. So instead of using that block with that K A inside the block E going in and E A coming out we can denote it by means of a single line, this line of course sometimes this is called an arc although it is not arc of circle or anything, it is a line the two ends of it, the ends points of it are to be associated with signals and on the line I am going to put an arrow and along with the line I am going to write down a number or symbol like K A and now, if I complete this new diagram, this is what it will look like. This point at the two ends of the arc those points are called nodes because what we are going to draw almost looks like what is called a graph of an electrical circuit or an electrical network and in that context one uses the terms like node and arc or sometimes one uses the term branch, one talks about branch of electrical network and the nodes are the points where the branches a number of branches are connected together, there are connection points or junction points of the electric network.

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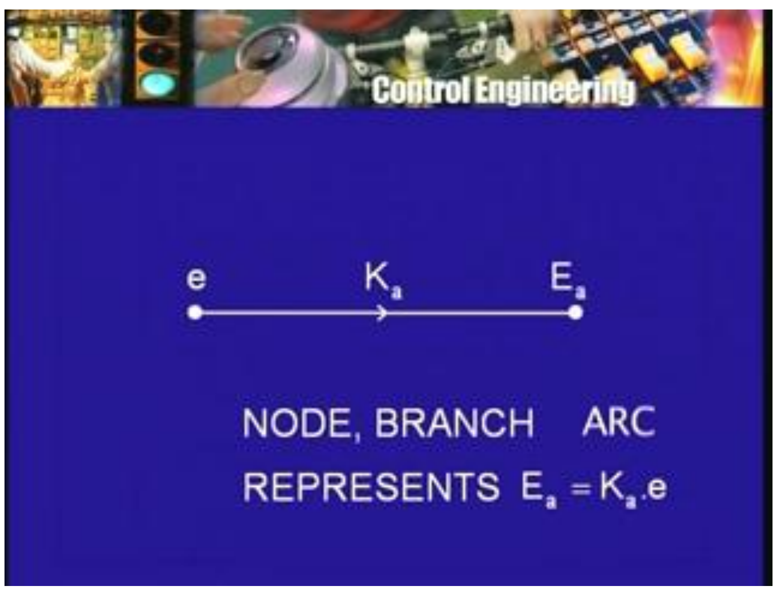

So what we have right now is a single arc or a single branch with two nodes, one is labeled e small e, the other labeled capital E A there is an arrow on the branch and alongside there is the symbol K A. All this is supposed to represent the relation E A equal to K A into E the block did the same thing in a different way E A equal to K A into E on the block diagram. This new method if one knows it and therefore understands what it means gives exactly the same information E A equal to K A into E that you can have the block diagram you can have the single branch and you can have the equation they all are ultimately related to one another.

Now if this was the only thing then there is hardly any point in even drawing a block diagram or this kind of a graph with only one relationship and only two signals one might as well right the equation itself. But of course in a practical case we just do not have one block already in our motor control example we had a number of blocks and therefore we will have a graph which will have not just one branch but a number branches and may be not one node but or two nodes but more than two nodes such a graph because it represents really something that has to do with signals was called by him a signal flow graph. The word flow is not to be taken literally its not that you know along that branch something is flowing from one end of the branch to the other.

So do not interpret it that way the symbol K A that is associated or put alongside the branch, it is not something that is flowing through the branch at all neither or the 2 labels that are associated with the nodes are flowing through that branch in any sense of the term but in particular situations for example if you had a hydraulic network or if you had an electrical network then as we know in an electrical network we like to talk about charges flowing or even currents flowing and the arrows very often represent the direction of the flow of current or the reference direction of flow current or positive charge and so this word flow has been used. So let us use it, so we will call it a signal flow graph.

So what we will do is now for the block diagram of the motor we will construct a signal flow graph and then I will state Mason's gain formula then I will apply it and we will obtain the relationship between the motor speed and the other parameters and the reference voltage so instead of writing equations and then manipulating them substituting solving etcetera. We will do it this way as I said this is as you can see also this is certainly really for human consumption I have to draw a graph then I will look at it then I will do some operations or some mental operations on the graph and then write down using that formula some expressions.

So it is going to be useful only when the signal flow graph is not very large, when the graph is large, when the system large the relationships are more complicated not any longer linear then one will have to use computer programmers or computer packages. Now these two nodes e and capital E_A there is a narrow one of them obviously e is going to be called the input node and the other node is going to be called the output node. The arrow points from the input to the output node and as I said earlier these terms are not to be taken literally that is something is being put in at node e and something is coming out of node E A it is not that which is communicated by a this terminology of input and output. In fact I once I have that arrow there I do not have to call e input and call E output or something I can simply say e or quantity associated with that node that is e and quantity associated with the other that is E A the relationship is E A equal to K N to E.

Now which it has to be written that is I have written E A equal to K A into e and I have not written e equal to K A into capital E A. Now that is told by this arrow that is the only function of the arrow here the arrow tells you that the signal at the what is called the head of the arrow E A here equal to this symbol K A and this is called the transmission coefficient unfortunately there is no transmission taking place but as I said there are analogies or a other situations where there may be some transmission or flow taking place you can simply call it the gain of the branch the branch gain but sometimes it is called the transmission coefficient or transmission multiplied by the signal and what is called the tail of the arrow.

The arrow has a head the arrow has a tail, the signal associated with the node to which the arrow points or on the head side of the arrow equal to the transmission coefficient or transmittance or gain of the branch multiplying the other signal. The signal at the tail end of the arrow or the signal associated with the node and the tail end of the arrow. So this is the simplest block in the closed loop system that we are looking at armature voltage is just gain times that error signal E. Now let us look at the other parts of the system and simultaneously you can look at the block diagram and see, what are things that we are going to do?

Now the block that represents the motor is little complicated although earlier we just showed one single block with the E A as the input omega M as the output perhaps the load torque also as an input and armature current as another output and that was because there were two equations that we could write relating these various quantities and it was enough to think of them together as one single block but now since we are going to use this idea of signal flow graph we will have to break it up into parts and then build up to the graph from these simple parts. So for example look at the armature circuit equation E A equal to the back EMF coefficient into omega M plus R A into I A.

So it says that some signal armature voltage is a sum of 2 other signals something which is called the back EMF which cannot be really measured when the motor is in operation and something called the armature drop which also cannot be measured separately when the motor is in operation. So E A is sum of these 2 signals, now we did not represent this in the block diagram at all because the block just stood for the whole thing motor and load but now if you want to represent this relationship between these signals then how shall we do it. Mason's idea was that one single is a sum of 2 signals so device some means whereby this information can be put down graphically, some signal is sum of two other signals. Now this indeed by simply using the following the convention I am going to write down I am going to put down three nodes one of them I am going to label K B omega M the one more I am going to label as R A, I A and the third node I am going to labeled as E A notice that the nodes are not labeled as 1, 2, 3 or a, b, c or whatever the nodes are label directly with this name of the signal.

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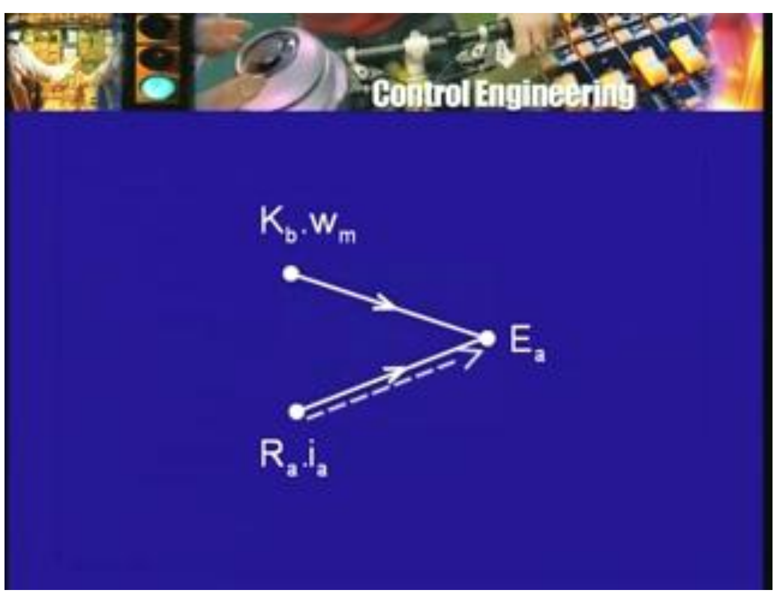

So immediately from the diagram or from the signal flow graph we can see that this is armature voltage this is back EMF this is armature current. So one can see signals immediately there is no need to write down number the nodes as 1, 2, 3 and braches as a, b, c, d or when you use some network solver program or network analysis program you have to label all the components and devices and so on and so forth. There is no need to do that here, so as is 3 signals E A, K B omega M and R A, I A and I want to show this relationship E A is a some of these 2 Manson choose this following simple idea just draw an arrow from K B omega M to E A and another arrow from R A, I A to E A. Now what does this say we will suppose that R A, I_A thing was missing that branch was not there then you would have simply E A equal to K B omega M. On the other hand suppose the K B omega M term was missing that is that branch was missing then you will have only E A equal to R A, I A but now we have both of them present.

So simply interpret E A as the sum of these 2. So at the node when there are incoming arrows you have to assume that a summing action takes place or sum is formed and one can describe it by saying that the signal associated with the node is the sum of all incoming signals, in our case E_A equal to K B omega M plus R A, I A is that quite correct not really because on these two edges along with these arrows I have not indicated their transmittances or gains. So if I want to be really quite correct about it I will have put down one and one alongside each of these branches. So the node K B omega M is connected to E through a branch with gain one, the other node is connected also to E A with gain one the arrows are as shown. Therefore, E A is the sum of the signals which are associated withered those two branches whose arrows point towards E A multiplied by the gains in this case one and one each.

So in this way one talks about incoming signals for E A, K B omega M and R A, I A are the incoming signals and this node not only stands for E_A but it also stands for this equation E A equal to K B omega M plus R A, I A that is it also stands for a operation of summation or summing of adding it is a nice and simple way of representing therefore addition on a diagram. It is a convention and therefore if you understand it and if you remember it then only you can use it but that is the part of life you know if you want to go ahead there are some things which you have to understand which you have to remember and then you find that they are useful. I can make a slight change in the flow graph I can keep the node E A as it is but now the other two nodes I will label them as omega M and I A the motor speed and the armature current because I am not really interested in the back EMF, I am interested in the speed.

Similarly, I am not interested in the armature drop I am interested in the armature current. So how do I draw the graph now well from omega M to E A draw an arrow and the gain of this branch will be simply K B similarly, from I A to E A draw an arrow and the again of this branch will be R A. So I have 3 nodes now 3 signals E A omega M, I A and I have 2 gains, gains of 2 branches I have 2 branches one with gain K B the other with gain R A and all this represents that simple equation E A equal to K B omega plus R A, I A. So that takes care of one of the relationships that we have used earlier.

Now let us look at the second relationship which is the mechanical relationship between the various torques this equation was what K T into I A was the torque produce by the motor equals T L, the load torque plus K F into omega M omega M being the speed of the motor this is the frictional term. Now I would like to represent this equation in the flow graph manner so how shall I do it well if we go by what we did earlier with the armature equation maybe I should take K T, I A as one node that is that is one signal and that is equal to T L so show that as coming in and K F omega M as the other signal that is coming in and certainly that is possible, it is possible to do it that way.

So I will show it K T, I A is the node to which these 2 signals are going T L with a gain of one and omega M with the gain K_F, nothing wrong with this this stands for the equation K T, I A equal to one times T L that is just T L plus K F times omega M which is exactly the equation that we had notice that T L and omega M appear as some kind of input signals in this signal flow graph and I A appears or K T, I A appears as an output signal. Now as I said earlier the load torque is it an input quantity or is it, is it an, is it an output quantity it is certainly an output quantity in a practical sense the purpose of the motor is to produce load torque, the output of the motor is the torque or motor torque is not the load torque that is the output of the motor.

So I am showing here T L as an input but as I said these words input and output do not have to be taken laterally perhaps when you have an line with an arrow then one signal is called the input and the other is called the output and all that matter is the relationship between two signals and the arrow tells us which way the relationship is to be written that is but one can go, I can go little step further and instead of writing K T, I A because K T, I A is the torque produce by the motor. But again I am not interested in the torque produce by the motor as such and so I could get rid of that coefficient K T by dividing that equation by K T.

So in other words I take the equation as I_A the armature current is equal to T L divided by K T plus K F omega M divided by K T and therefore I can draw a graph where I A is the output node there are two input nodes load torque and omega M these are the two input nodes and with the corresponding gains the gain from the load torque is one divided by K T and the gain from the speed of the motor is K F divided by K T. So this represents now the second equation and now I can combine these two separate parts that I wrote or drew earlier there was this equation for the armature voltage and there was this equation for the armature current and I will try to I can combine them.

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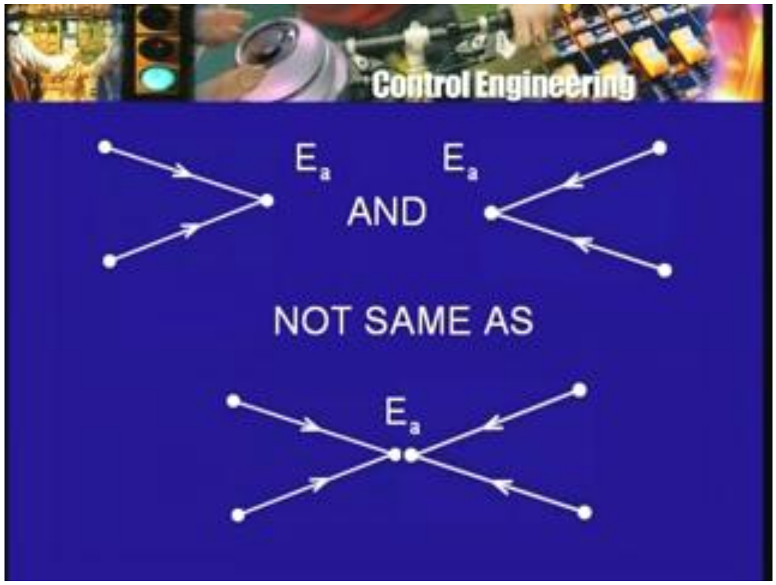

I can combine them by simply superposing except there is one condition that has to be observed or one restriction that has to be followed and this is one of the if you say feel it is a limitation of the signal flow graph because each node will not only denote a voltage or current a signal in general each node will represent or denote a stand for a signal but because node may have more than one incoming arrow each node also stands for a some or summation therefore each node can only represent one equation each node can represent only one summation equation.

So you cannot have 2 equations, let us say showing E A as a sum of 2 signals and another equation showing E A also as sum of 2 other signals because then the same node will be doing this double job and if I have node with not 2 but 4 incoming arrows then by this convention E A is the sum of all those 4 incoming signals and not P equal to sum of 2 incoming signals on the one hand and also E equal to sum of 2 incoming signals on the other hand. So because of this dual use of the node node denotes a signal and node can also denote as summation or represent the operation of summation this is something we have to take care of so with this in mind now can I combine the two signal flow graphs that I drew earlier the graph for the load torque omega M as a input nodes and in term they determining I A, I A is the output node and omega M, I A as the input nodes and they in term determining E A as the output node.

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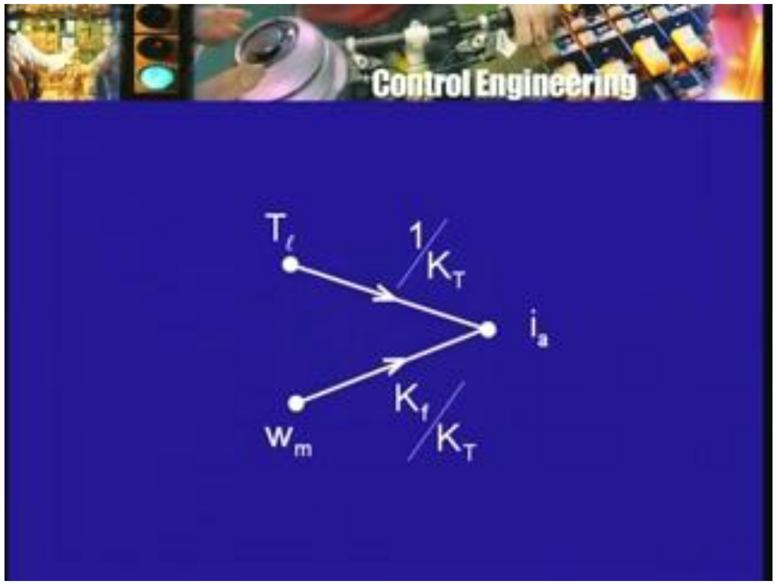

Is there going to be any problem in superposing these two diagrams? Well, there is none although we will see that that is not going to be that useful and we will therefore change the diagram but one should know the reason for making these changes its not simply that the book draws that diagram in a particular way and therefore all you have to do is copy it in that way you should understand why diagrams are drawn in a particular way and why not in some other way. So let us combine the two diagrams.

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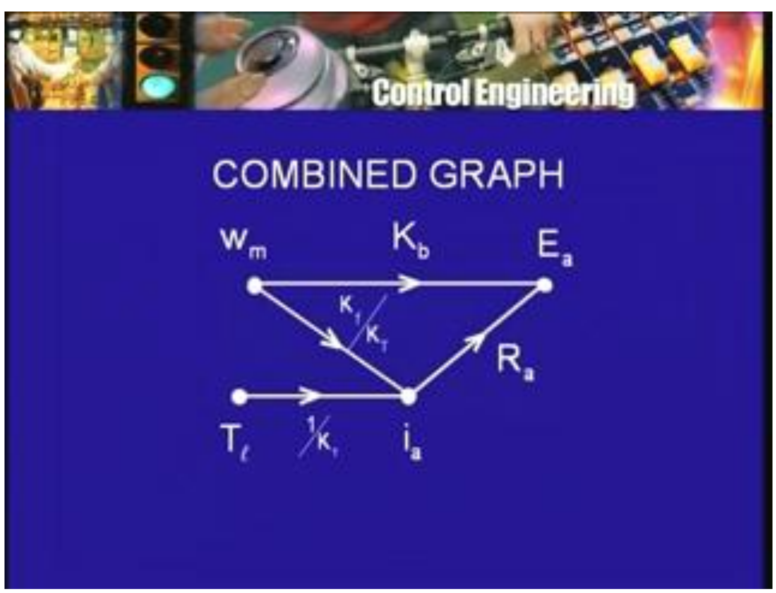

So I have T L as one node omega M as another node I A is the armature current there is that equation I have I am putting down the gains one by K T here and K F by K T there. So that is one equation I A output node signal is the sum of two input nodes multiplied by the transmittances that is the torque equation then this I A in turn is an input node affecting E A through a transmittance R A and there is omega M also and therefore I will draw another arrow from omega M to E A with a transmittance of K B.

So I have now a diagram in which there are 2 nodes, T_L and omega M from which there are only outgoing arrows there is no incoming arrow for these 2 nodes and usually such nodes only are called input nodes of the whole graph. If I talk about the whole graph that I have now and it has 1, 2, 3, 4, 5 nodes and 4 variables then T L and omega M set to be input nodes because there are no incoming arrows, there are only outgoing arrows whereas E A is set to be an output node because there are only incoming arrows and no outgoing arrows and I A is then set to be what an intermediate node because it has arrows coming in and it has arrows going out, what is the difference between these 3 at an input node like T L and omega M there is not equation that is only a signal associated with the node and nothing else. At other nodes both the intermediate nodes and the output node there is an equation the out put node because it has incoming arrows only therefore it stands for an equation the intermediate node also because it has incoming arrows it also stands for an equation.

So at each of the intermediate and output nodes corresponding to each one of them we have an equation whereas for an input node there is no equation. So that is what distinguishes the input nodes from the output and the intermediate node and from the diagram or from the graph the way of finding out which are input and output and intermediate nodes is by looking at the arrows. Now strictly speaking the distinction between the intermediate nodes and the output nodes is not that important because as we will see the gain formula which Mason introduced applies for both these intermediate nodes as well as output nodes.
In fact that is a good point about it, so one need not worry about this distinction between intermediate nodes and output nodes but what is an input node must be understood very correctly. It is a node from which there are only outgoing arrows no incoming arrows. So in this diagram or graph T L and omega M of the input nodes and there the other nodes are not input nodes. Now given a graph like this one can apply what is called the gain formula Mason's gain formula this of course if you remember is the speed control of the motor without using feedback. So E A the armature voltage is going to be kept constant and we let the motor run no matter what happens to the load torque or parameter changes and what not and therefore omega M is not going to remain the same.

Now because omega M for us is the output quantity that is we are really interested in finding out how omega M is effected by these other things. Therefore, the way we wrote down the equations and the way we represented them on the signal flow graph is not so helpful. For example, the load torque T L appears as an input node and that is okay because in a way that is a can be thought of as a disturbance input change in T L essentially a disturbance but omega M is not an input variable it is not a physically an input variable of the drive. In fact E A is the physical input variable to the drive and so we have sort of turned upside down the input and output quantities E_A should be the input that is now appearing as not an input but as an output node omega M is the output that is appearing as an input node I_A is an intermediate quantity yes, because I A is neither an input nor is it an output quantity it is not of directly of interest to us.

The load torque T L however appears correctly as an input although from practical point of view it is the output of the drive or it is what the drive produces but when looked upon the point of view of disturbance changes in T L or disturbances which I can act like inputs unwanted inputs and unknown inputs to the system. So having T L as an input node is okay but this thing having E A as output and omega M as input is not quite satisfactory and therefore one can try to change the diagram or the signal flow graph as we will but let me repeat the graph as we have drawn is quite correct there is nothing wrong with it, the graph because there are two nodes which are not input nodes and therefore which are incoming arrows the signal flow graph represents two equations and what are those two equations look at the node E A it has 2 incoming arrows therefore E A equal to this sum of the incoming signals.

Now one of the arrows at the tail of it is I_A, so I_A into transmittance R A that is one term in the sum, the other arrow has its tail omega M, so omega M into K B is the signal that is coming along the other path has it way and therefore E A equal K B into omega M plus R A into I A. So this node not only represents the voltage E A but it also represents an equation that is the equation that we are written earlier nothing wrong about it therefore as far as the diagram or graph is concern, what about the graph second node I A which is also not an input node. Well remember, when you are thinking of an equation associated with the node you are only to look at the incoming arrows so that there is an arrow going out from I A is not to be thought of when writing down the equation. In fact that is what made it an intermediate node there are incoming there are incoming arrows as well as outgoing arrows.

Now what is the equation associated with this node I A look at the graph, what are the arrows which are incoming what are signals at the tails of these arrows? what are the transmittances or gains of these two branches which are pointing towards to the node I A, one of them is from T L multiplied by 1 by K T the other is from omega M multiplied by K F by K T and so at the node I A we not only have the signal I A associated with it the armature current but we also have an equation I A equals T L into 1 by K T plus K F by K T into omega M and this is almost the equation that we wrote except we are divided both sides by K T. So instead of K T, I equal to something I have I A equal to a sum of 2 terms so as far as this signal flow graph is concerned it represents the two equations for the steady state operation of the simple DC motor drive.

You will not find this diagram perhaps in your text book simply because as I said here omega M appears as an input quantity and E A appears as an output quantity whereas I may if wish to see them in the opposite and their opposite rules E A as input and omega is an output but the signal flow graph is quite correct it represents the 2 equations 100 percent correctly, there is nothing wrong with signal flow graph. But can we make some changes or draw another graph in which E_A may appear as the input quantity omega M as the output quantity and this is what I want you to remember and realize that if I have an equation I can represent it on the signal flow graph in one particular way but I can represent it in a different way also and there is nothing wrong about doing it the only thing that you have to remember is that at each node there is only one equation.

So you cannot have a node variable equal to some particular summation and also equal to some other summation and represent this on the diagram or on the graph by all arrows coming in because then by convention that signal will be the sum of all incoming arrows. So that is the only restriction in drawing signal flow graphs and therefore when drawing signal flow graphs one has to take care of this fact that at each node we will have only one equation. Nowhere should it happen that you are trying to represent two equations by single a node all right.

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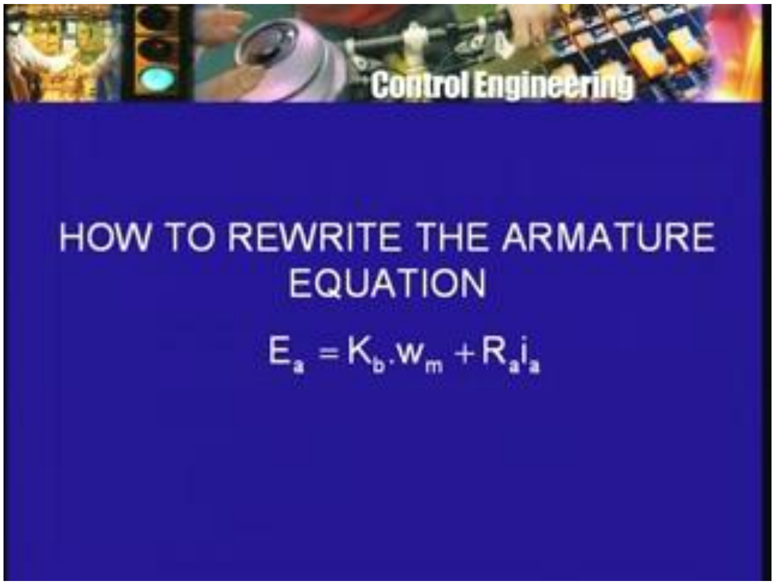

So let us look at the armature voltage equation E A equal to K B into omega M plus R A into I A how can I rewrite this equation and then draw the corresponding branch of a signal flow graph. The way it was we just took omega M, I A as the incoming nodes E A is the output node and therefore drew that branch two branches going from omega M and I A though transmittances K B and R A to the node E A. But now if I decide that I would like E A to appear as far as possible as an input node because physically when I want to think about the drive I want to think about the armature voltage as an input variable if I want to do that then I would like see E A as input node and therefore that will not be the node with which the equation will be associated then which other two signals now we have a choice, we have omega M and we have I A.

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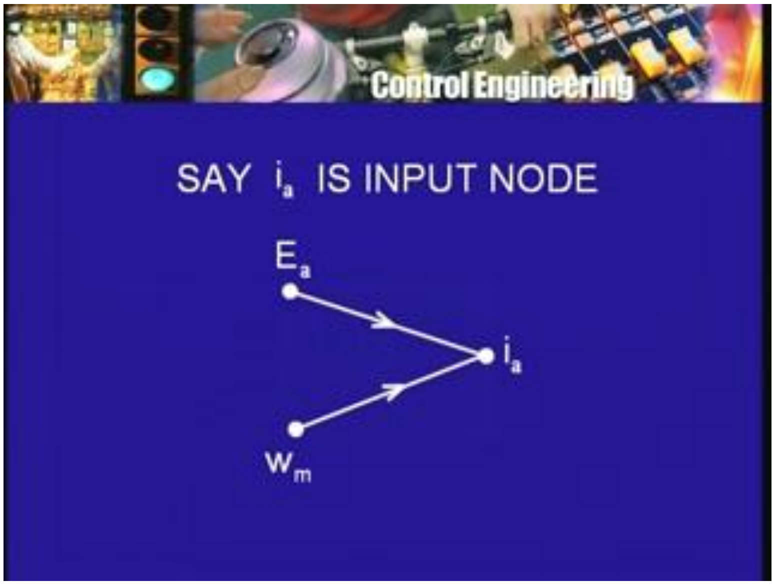

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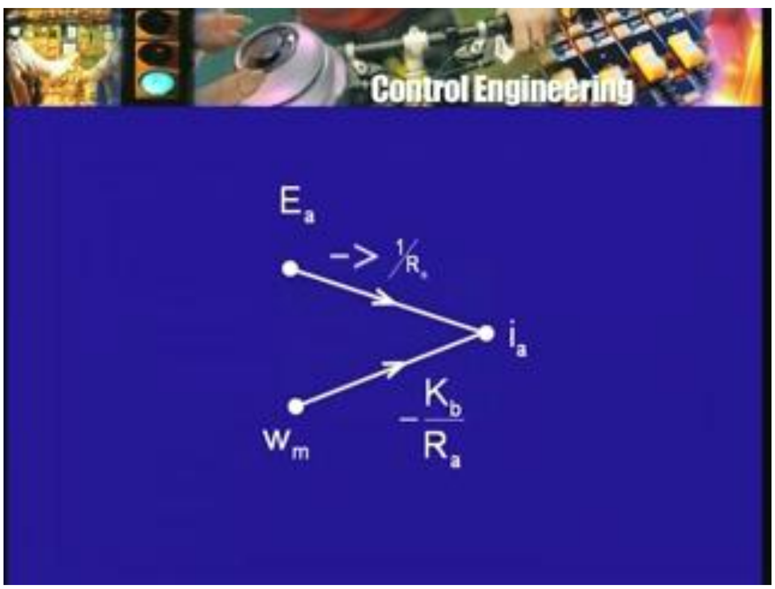

We have speed and armature current which one of them shall we choose therefore as the output node or the node with which we will associate the equation and therefore one of the two omega M and I A will then become an input node. Now as you can see there are two choices possible and so it can be done in either way at this point we may not know which one is going to be more useful of the two. So one way will be of course therefore is start doing it start with E A as an input node omega M as another input node and therefore I A will be my node which corresponds to this summation and therefore what will be the transmittances going for E A to I A what will be the transmittance, what I have to do is take this equation solve it for I_A in terms of E A and omega M and then draw or show the gains of the transmittances on the signal flow graph.

So you can see that for I A, a transmittance coming from E A will be one by R A, why because if I take this equation and I do try to solve it for I A, let us say by taking I A to one side. So that I get R A, I A equal to E A minus K_B omega M and now I solve it for I A in the sense I express I A in terms of the other 2. So I will get I A equal to 1 by R A into E A minus K B by R A into omega M the gain or transmittance from E A to I A is plus one by R A.

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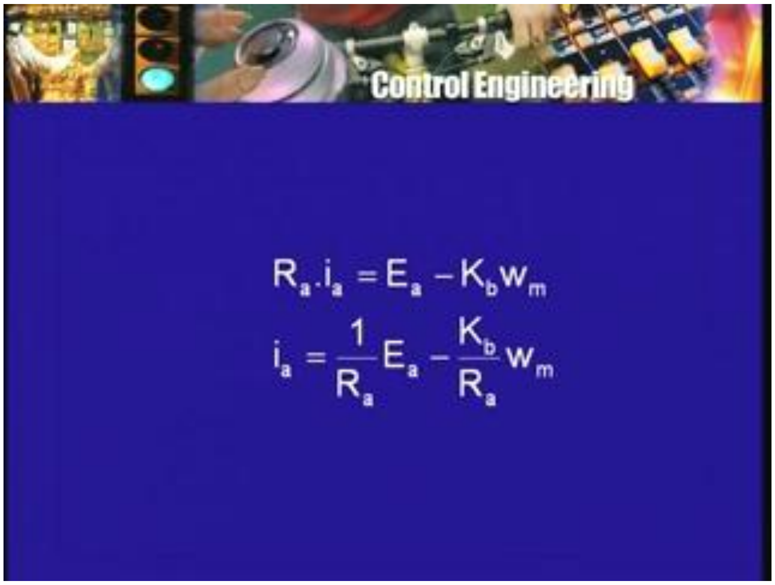

So it is good to do it slowly rather than look at that equation then then try to draw to this signal flow graph directly. The other coefficient however has a negative sign it is minus K B divided by R A so that armature circuit equation I can represent by this signal flow graph in which E A and omega M are 2 input nodes and there is a single output node I A and this there for that single equation I A we can do it choosing omega M as the output node and E A and I A as the input nodes, why do not you try? What is to be done? I have to solve for omega M in terms of E A and I A.

So transfer some other term to the other side and get an expression for omega M as some coefficient into E A that will be the transmittance of that branch plus some other coefficient into I A that will be the transmittance of the other branch so I could also have a signal flow graph with E A and I A as the input nodes, omega M as the output node and with two transmittances appropriately chosen. So for example from that equation the transmittance from E A to omega is K B and the transmittance from I A to omega M once has a negative sign and it is minus R A divided by K B.

Now let me emphasize once again that both these are correct ways of representing that equation as was the third way of representing the equation namely E A as the output node omega M and I A as the two input nodes. So all the 3 ways are equally correct let nobody tell you that this is not right and that is right. Of course what is more useful what is not so useful that will depend on what happens as we proceed further but at this point what is important to note is that an equation can be represented on a signal flow graph in this particular way. An equation expressing one variable one signal as a summation and by summation by means plus sign all right but a coefficient which may be negative of a number of other signals multiplied by their transmittances and so from those nodes draw arrows pointing towards this signal which is the sum show the transmittances along the various branches.

This is the way of representing that equation on or by what is called a signal flow graph and there can be more than one way of doing this and its only experience or what it means is that you try this try that. When eventually you see that this is better than that and then you can say to yourself why did I not think of it earlier and so on but that is not the point of it one should know that there are alternatives they are all equally correct and of course one should learn that this works better than that and so on. Now as homework, I would you to look at the torque equation and then write it in two different ways we wrote it for I A in terms of the other 2 quantities. Now you can write it for each one of the other quantities in terms of the remaining two quantities and draw the corresponding signal flow graph okay.

So for example, T L as the output node armature current as an input node omega M as another input node that is the graph that can be drawn and this next second one or really the third one will be omega M as the output node I A as the input node T L as the input node. So for that second equation also we will have a choice of 3 graphs for the first equation we have a choice of three graphs. But now we do not want these two graphs to remain separate why because there are some common signals. In fact almost all the signals are appearing there omega M our crucial signal is appearing in both the graphs the armature current also appears in both the graphs although it is not of direct importance or use to us. The other signals E A appears in only one graph T L appears only in one graph.

So of the 4 signals omega M, I A, T L and E A omega M and I A appear in both the graphs one corresponding to the electrical equation and the other corresponding to the mechanical equation whereas the armature voltage appears only in the electrical equation. The load torque are appears only in the mechanical equation and that’s quite natural because the armature current because it is flowing through an armature which has a resistance makes that contribution R A, I A in the armature circuit. But on the other hand this armature current because it is flowing in the presence of a magnetic field produces a torque therefore it appears in the torque equation what about the speed of the motor omega M, why does it appear in both the cases, both the equations. It appears in the mechanical equation because of friction there is a shaft rotating may be there is a fan mounted on the shaft, so there is friction in the bearings, there is this windage air is to be moved therefore there is torque required for that which is proportional to omega M.

So omega M occurs in the mechanical equation and why does omega M occur in the electrical equation this is because we have a conductors which are moving in a magnetic field and therefore going back to Faraday an EMF is produced in that conductor and therefore omega M appears in the electrical equation as well. Now our problem is of combining these two signal flow graphs one corresponding to the electrical equation the other corresponding to the mechanical equation, I want to combine them in a single graph but when doing this I want to take care of this fact that I mentioned earlier that is a node of the graph should be associated with at most one equation there can be a node which is not associated with any equation, what is such a node called? It is called an input node an input node is a node for which there is no equation there is only a signal associated with that node no equation.

So there can be some input nodes in the final diagram with which there may be no equation associated but with any particular node there should be at most one equation and not 2 equations or more than one. So combine these two signal flow graphs. So that this condition is satisfied find out whether there is only one way of combining these 2 graphs that is actually we have a choice of 3 graphs for the electrical equations, 3 graphs for the mechanical equations and so I could think of 9 combinations which one of them will fulfill this condition that when I put the 2 graphs together literally I superpose them. I will not find that one node is doing the job of two nodes that is one node stands for one equation and it also stands for another equation because when you combine them a given node stands only for one equation because you can only see incoming arrows even if there outgoing arrows, the node associate the signal associated with a with that node is sum of the incoming signals that is the signals associated with those nodes multiplied by the transmittances added up this is the way we are chosen or Mason's suggested that we represent equations and so one you.

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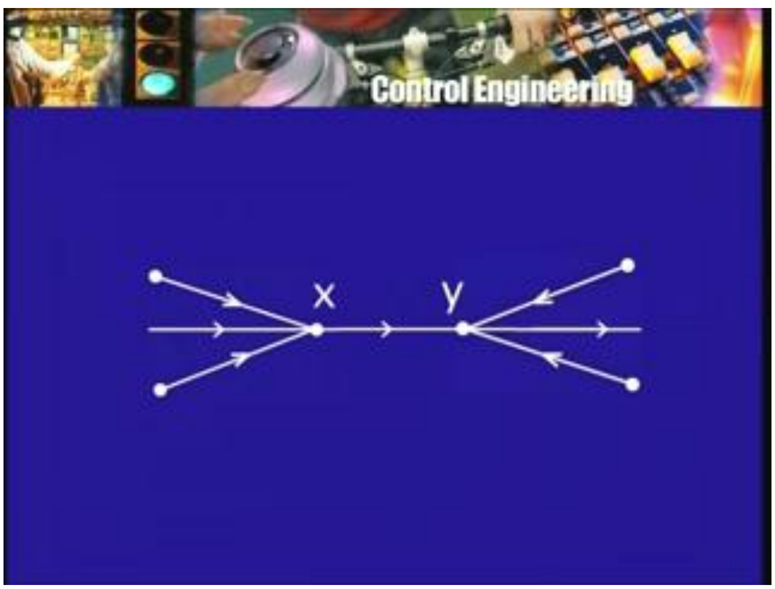

Once you make that convention you have to stick to it is a logical requirement because how do I distinguish between one node representing only one equation and one node representing two equations although there are some tricks that one can play and therefore one can do some of these things which are not obvious at first glance. For example, I can always put a what may be called a useless branch like this one which has a transmittance one and it has only at both of its nodes whatever signals are associated with them if whatever are the signals coming at the other end it does not matter what they are if that signal associated with that node is X and the other signal is Y and whatever branches go out it does not matter.

So, if we have very simple situation like this the branch between 2 nodes X and Y with the transmittance one and Y has only one incoming branch and X. Of course could have more than one incoming or outgoing branches that does not matter then what is the equation that corresponds to this node y, it is simply y equal to x it is simply y equal to x that is the is there any point in writing down that equation or therefore putting that branch with transmittance one there is really not much point except this condition that we are insisting upon if that is to be satisfied something like this can be used or may be required.

So do this homework and we will then look at what we can do with the composite signal flow graph we will look at statement of Mason's gain formula apply it to that composite graph and get a nice formula rather easily by looking at the graph and applying the gain formula of course when I say easily it means after you have understood and remembered Mason's gain formula correctly and you can apply it to the particular signal flow graph correctly and of course after you have drawn the signal flow graphs correctly. So it is only when you draw a graph correctly first of all therefore you must write down correct equations to start with then represent them correctly by the signal flow graph then you must remember what the Mason's gain formula is and then you must apply it correctly. If you do all this then hopefully, you will not go wrong and you would have found out the desired equation or relations that we are looking for.