

Control Engineering
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Lecture - 12

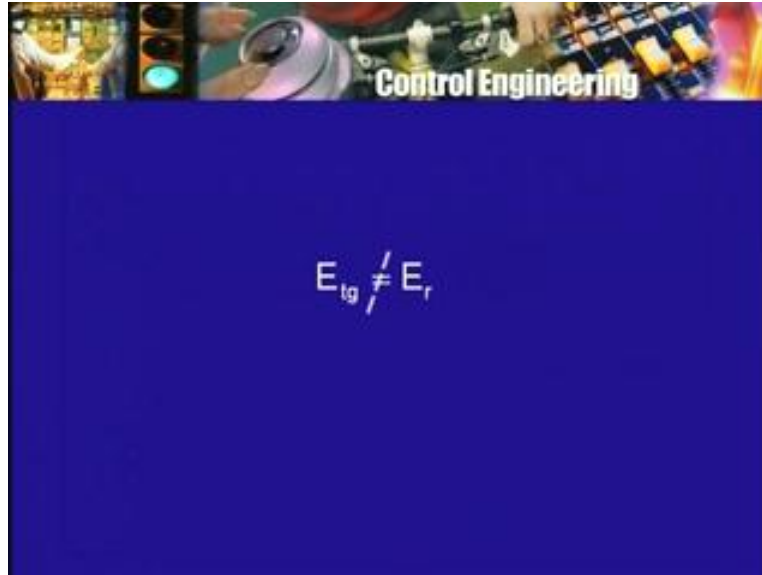
So, the desired speed preferably it could have been called ω_d is going to be physically represented by a voltage because it is convenient to have a voltage rather than a shaft running at that speed and this voltage is usually called a reference voltage or a reference signal. So we have E_r the reference voltage or the reference signal which represents the desired speed in our case, let us say 1500 RPM at which speed the tacho generator would have produced an output of 20 volts. So the reference voltage would be 20 volts.

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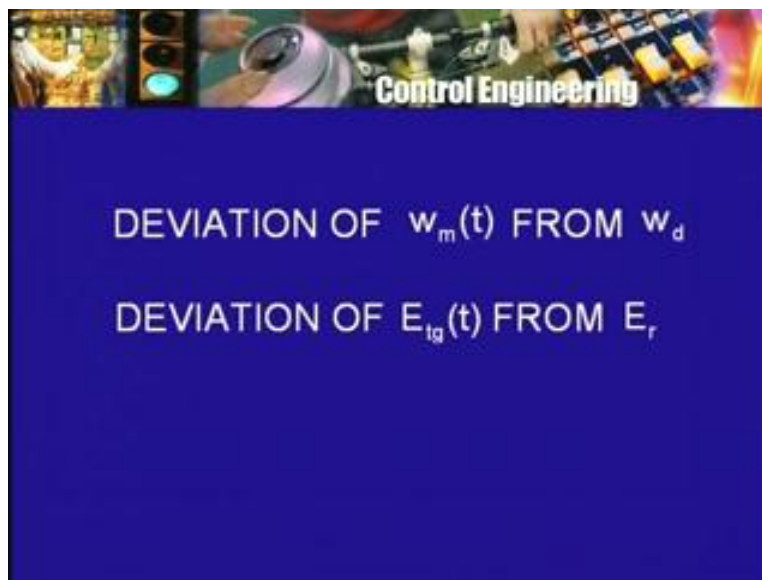


Now, when the system is actually in operation the speed may not be equal to the desired speed and so the tacho generator voltage $E_{\text{tacho generator}}$ will not be equal to $E_{\text{reference}}$. So, there will be a difference between the 2, this difference is also referred to as a deviation, deviation of the output quantity. In this case the speed of the motor from the desired value which is represented by deviation of the tacho generator voltage which is proportional to the output quantity, the speed from the reference voltage which is proportional to the desired speed in the same proportion. So in place of the actual speed ω_m at any moment of time t and the desired speed ω_d , we have the tacho generator voltage at any moment of time and the reference voltage and if the two are equal then the motor is running as expected at the desired speed. But if they are different then it is not and so there is a deviation in that case or the system is in error.

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Now, unfortunately in control theories somehow the word error and the related word correction are used in a sense which is just the opposite of what it is during our normal course of things in engineering practice or in other places. Let us look at the word correction very often, correction refers to something an amount which is to be added to what is there to give you, what is required or what should be the case. So in this case therefore, suppose that the reference voltage once again is 20 volts but the tacho generator output voltage was 19 volts.

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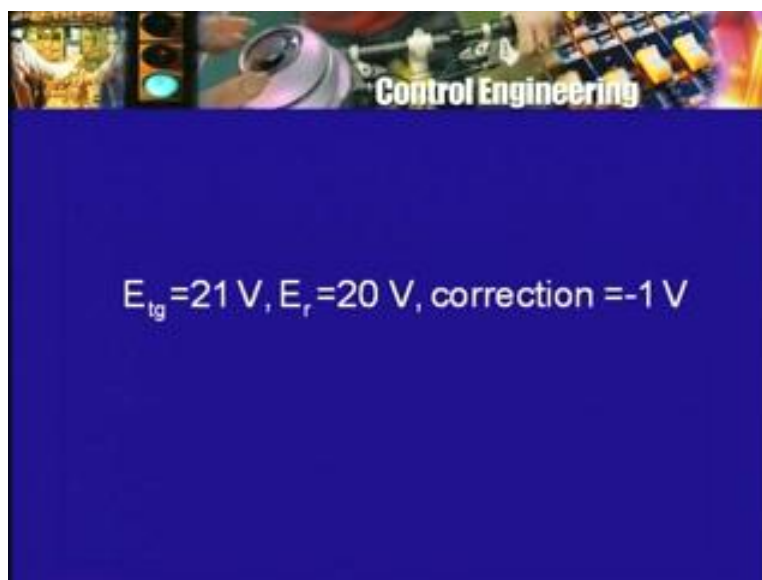


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$$E_{tg} = 19 \text{ V}, E_r = 20 \text{ V}$$
$$\text{correction} = 1 \text{ V}$$
$$19 \text{ V} + 1 \text{ V} = 20 \text{ V}$$

So in this case, where would we one volt refer to a correction of one volt that is when one volt is added to what is actually the case you will come to the desired value. So the desired value is obtained by correcting the actual value by this amount of one volt on the other hand suppose, the actual voltage was 21 volts whereas the desired or the correct voltage is 20 volts then the correction would be minus 1 volt or alternately or that is minus 1 is to be added to 21 to give 20 or from 21, one is to be subtracted to give 20.

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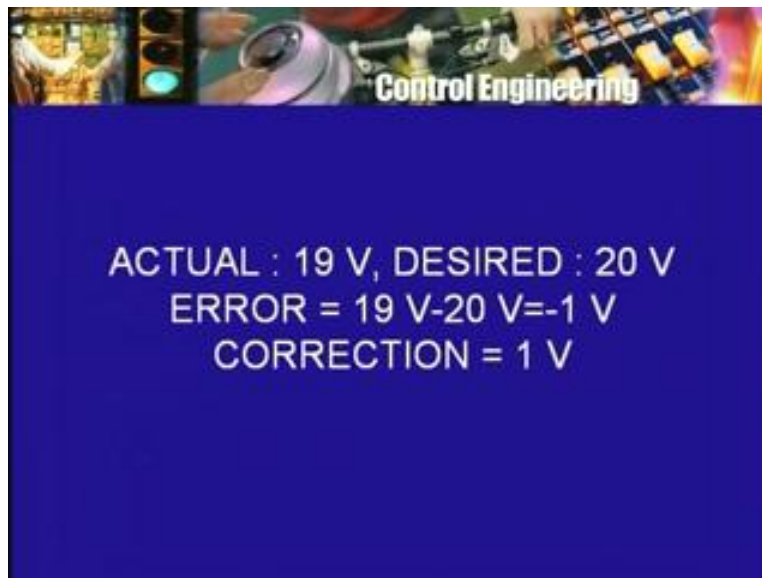
$$E_{tg} = 21 \text{ V}, E_r = 20 \text{ V}, \text{correction} = -1 \text{ V}$$

It is usual practice, when talking about instrumentation errors and accuracy and so on, to talk about plus and minus correction rather than, say that correction is to be added and correction is to

subtracted in other words correction is added and if it is positive, it actually adds for example, from 19 volts correction 1. So, 19 plus 1 equal to 20 where as if its 21 volts correction is minus 1 therefore 21 plus minus 1 equal to 20.

So this is the normal use of the word correction at least in instrumentation practice, when one refers to readings of instruments various instruments or other quantitative measures of various kinds, one talks about taking corrective measures to improve an economy and whatever other things. The word error in this context refers to just the opposite that is therefore error in this context it is usually the negative of correction.

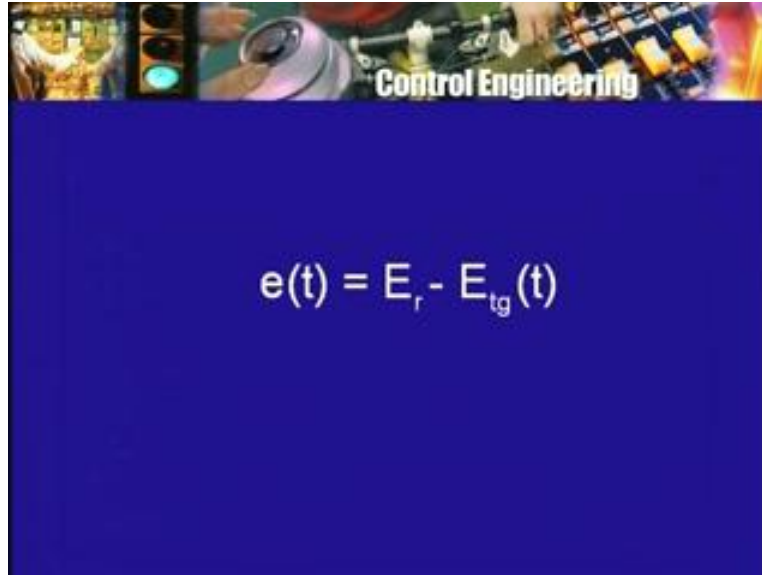
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So because of this when the actual quantity is 19 volts the desired quantity is 20 volts, the error will be the negative of the correction the correction is 1 volt, the error will be the negative therefore minus 1 volt or therefore the error will be 19 volts minus 20 volts equal to minus 1 volt. So to repeat when the actual quantity is 19 volts and it is to be corrected to 20 volts, you talk about a correction of one and an error of minus 1 or you will say, that the error is on the negative side meaning that the actual is below the correct one whereas, if the actual was 21 volts then the correction would be minus 1 volt and the error would be plus 1 volt.

So this is the practice with respect to instrumentation and other cases but somehow in control systems work, the significance of these terms has become just the opposite. In fact, the one does not use the word correction as much as one uses the word error and so in our case, one would refer to error or one would refer by error to what error will be the reference of the desired minus the actual rather than the other way which it should be for error usually the letter e, small e is used capital E is usually reserved for voltages especially dc voltages or rms values.

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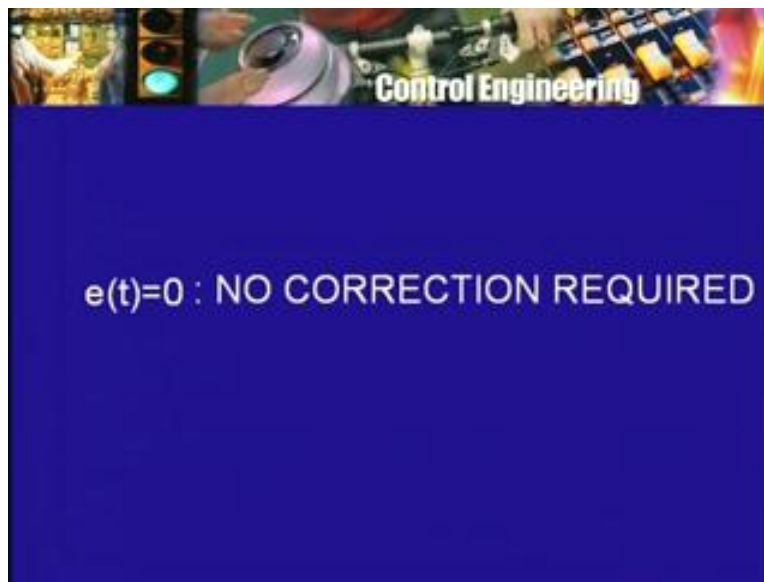
So small, e is the error then this will be equal to the desired one, the desired one in our case is going to be represented by the reference voltage E_r which for our problem, we have assumed it is 20 volts corresponding to 1500 RPMs minus the actual speed which is represented by the tacho generator voltage. So, E_{tacho} generator at any moment of time t , so this in control theory work is referred to as the error in instrumentation work, the error would be just the opposite. So what the operator was doing was actually not really subtracting something from the other but we can say he was simply comparing the reference voltage which was a mark on a dial with the actual voltage which was the position of the pointer and if the two coincided then the system was doing all right.

If they did not there was an error, the actual speed was different from the desired speed. The pointer could be on the left of that mark or could be on the right of that mark and as far as the operator has was concerned all he had to remember was something like the following if the pointer is to the right then move the slider may be to the left and if the pointer is to the left then move the slider to the right, he is not going to look at numbers calculate the difference and so on and so forth. As human beings of course we learn to do all these kinds of things we are driving a vehicle and we feel its going a bit slowly.

So, we press on the accelerator or we turn then we twist the accelerator control if it is on the handle and then you increase the speed. So you make this adjustment and you know which way to do it sometimes of course, one goes wrong and when one is trying to actually brake one may press on the accelerator pedal and hit the car ahead of you, but these are very rare situations. Now in our case there is no pointer, there is no dial, there is the voltage of the tacho generator which is available to you on a pair of wires, there is a reference voltage source which is also available to you on a pair of wires and we are not going to look at these voltages take readings and so on and so forth.

So whatever, we want to do we are going to do electrically and what is convenient to do electrically is to take a difference between the 2. This difference could be E_r minus E_t , in other words combine these 2 voltages in such a way that a resultant voltage E is produced which is equal to the reference voltage minus the tacho generator voltage. You can think of some very simple electrical ways of producing this difference of voltage and you should try your hand at it. Of course, do not think of a difference amplifier that is if really sophisticated way of doing it although today it is really very cheap and a very simple solution but there can be some really very simple ways of obtaining a voltage which is the difference of two voltages.

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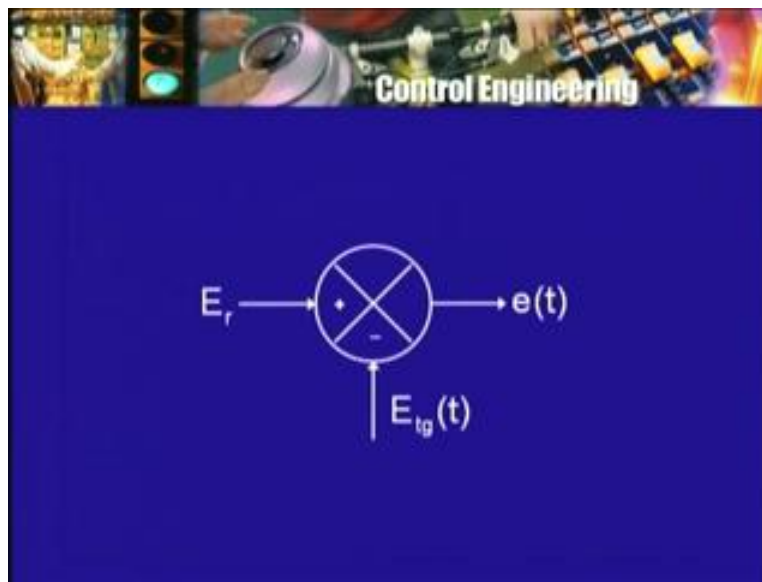
So think about it, see what can be the problems with that way of obtaining a difference of 2 voltages and then maybe, you can graduate to the stage of the difference amplifier or a difference circuit which produces a signal or a voltage, which is difference and in this case the reference voltage minus the tacho generator voltage. This device is referred to also as a comparator actually it is usually normally called an error detector, which is a very correct name because it detects whether the system is in error, it detects whether the actual speed is equal to the desired speed or not, of course in order to detect it, it is enough to look at e and see whether e is 0 or not.

If e is 0, the error is 0 then, there is no error or the system is operating correctly where as if e is not 0 then there is error. The system is not operating correctly and therefore a correction will be required some adjustment will be required but sometimes it is also referred to as a comparator, but that word is really not a very appropriate one because in the case of a comparator and you will recall a comparative circuit in electronics. A comparator compares two signals and produces an output which is of one value, if say the first signal is greater in value than the second signal and it produces an output of a different value, if the first signal is less than the second signal.

So it not only checks whether the 2 signals are equal or not in fact, normally that 2 signals will not be equal but when they are unequal depending on which one of the two is greater than the other, it will produce a corresponding output of one value or another. So it also detects the sign

of the difference of the error in the case of control system then one can naturally think of an error or an error detector. So, we now have a small arrangement whereby with the reference voltage and the tacho generator voltage as the inputs to it, we get an output voltage which is the error signal E_t and therefore, we can show this by a simple small block on our diagram and the symbol for this is a circle with a x inside it and the 2 signals are traditionally shown in this particular way. The reference voltage E_r is shown entering from the left side and since, the feedback voltage was already on the bottom of the diagram, the feedback signal comes from the bottom of the circle.

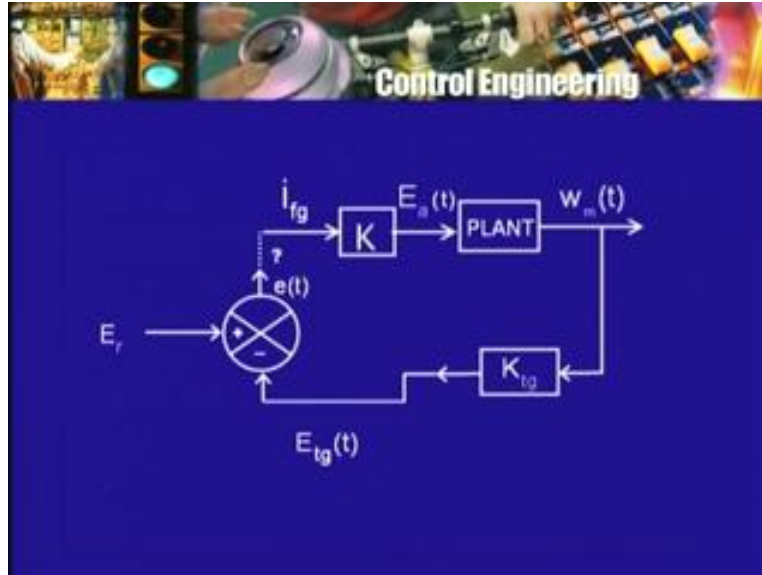
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So here is E_r , here is E tachogenerator and to emphasize that we are taking the difference E_r minus E tachometry generator, a plus sign is put in one of those quadrants and minus sign is put in the other quadrant and there of course, is the output e of the error detector.

So this simple symbol with the 3 signals, 2 arrows going in and 1 arrow coming out will really tell us or correspond to the equation E and the error just as e tacho generator will change with time. Therefore, the error at any time t will be the reference voltage E_r and it is expected to remain constant minus E tacho generator of the signal that is being fed back and this is the language of course, one uses feedback therefore, the signal that is fed back on the feedback signal E , tacho generator t , the output of the error detector is this signal or difference E_t and therefore, our block diagram will now, look like the following. Here is the plant and then, we have the tacho generator which is shown by a coefficient K tacho generator.

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'LINK UP' ERROR $e(t)$ WITH $i_f(t)$

The plant produces an output ω_m which is an input to the tacho generator, the tacho generator produces an output which is a voltage E_{tacho} . The plant in turn gets a voltage E_a of t the armature voltage which is coming, I have assumed from a in the Lenard system of control from a generator whose field current is being controlled. So in effect, we can think of a block before this E_a with the coefficient K and the input to this is the field current I_f and then we have this error detector with E_r as one input to it. The tacho generator voltage as the other input to it and the error as the output from this error detector and now, what we have to do is we have to link up this E_a , the error which has been detected with, if the field current which is the means of adjusting the armature voltage E_a . So we have to connect or link the error to the

field current. Now the field current is in turn coming from some power supply and therefore, one has to have an arrangement where by the field current can be varied once again. You can think of various alternative arrangements of the varying the field current, a serious rheostat would be a very simple one but not a very practical one because the rheostat requires someone to move the slider and that is not a very convenient thing to do, there are other ways of doing it.

So you should think a little bit about it and then may be look up the solution in a textbook as to how exactly the field current adjustment can be effected. So the error voltage is to be related to the field current, so that the loop is closed and the feedback is then use for making the adjustment in the hope that the system will sooner or later run at the correct speed. We have noted that there will be a transient period, when there will be a error that is E_t will not be 0 but hopefully after a short while E_t will become 0 the motor will run at the desired speed.

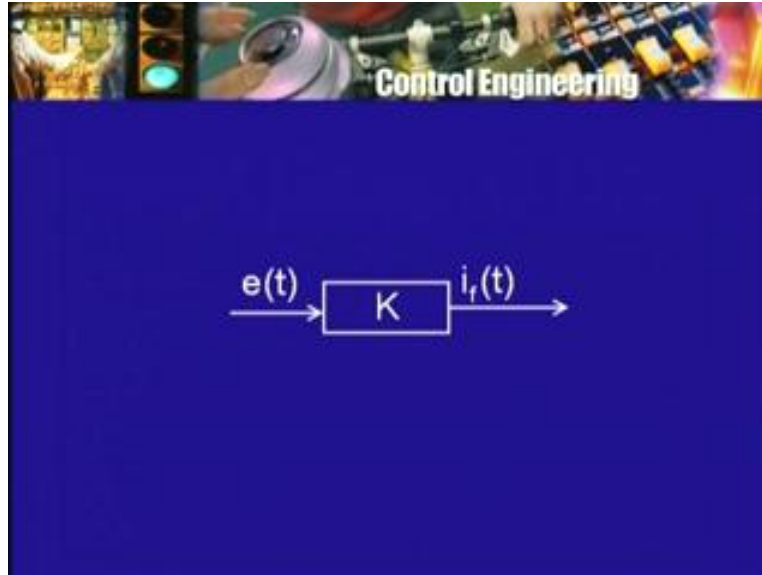
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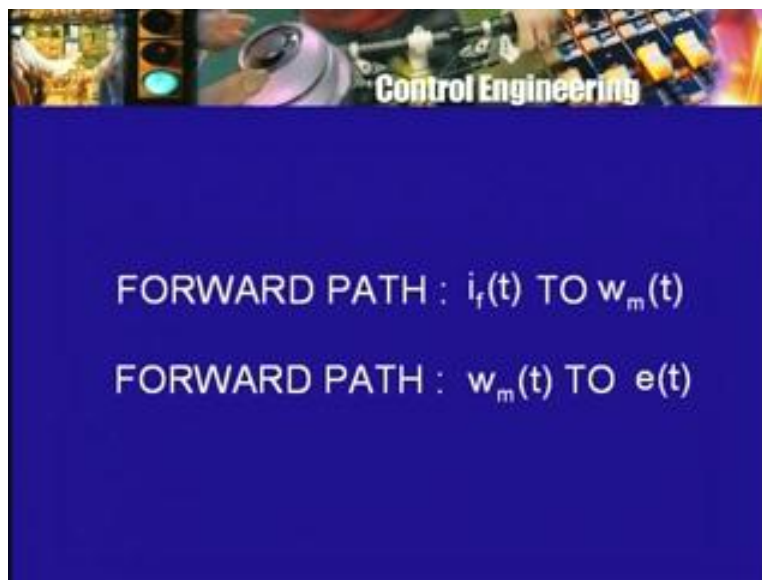
Now in many books when this problem where, this problem is discussed the following scheme is suggested, how is one to link up the error E with the field current which is the input or which is the quantity the variable, which is being manipulated or which is the variable which is providing the adjustment, how are the two to be linked up. Now in many books the assumption is made that they are simply linked up through some simple block.

So I will show is as one more block whose output is field current and whose input is the error voltage and this block therefore, will have some coefficient. There are all kinds of K is you already we had K_t , K_b , K_f , we have K tacho generator, we have K going between the field current and the applied voltage. So we can one more K that relates the error to the field current instead of that of course, we may think of single K which relates directly E at the error voltage to the voltage to be applied to the armature capital E_a and therefore, this coefficient I am calling K_a and this coefficient is very often refer to as a gain specifically a gain of the forward path.

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You can see from the diagram, the reason for talking about 2 paths and already mentioned this from the armature voltage to the speed is one kind of thing, the motor and load from the speed or the rotator shaft to the tacho generator voltage or the error detector, output is something else some other physical system namely the tacho generator some difference device. So that is referred to as feedback path and therefore the path on the diagram from the applied voltage to the speed is referred to as the forward path and this is therefore referred to as the gain in the forward path and in a very simple minded way we can think of it as simply an amplifier.

I told you earlier that in the during the second world war period. In fact even before that the amplifier people, telephone people in particular Harold Black had discovered the use of feedback for a different purpose for stabilizing the amplifier against parameter variations and so on and therefore, the word gain came from there and therefore, thinking of it as some kind of an amplification factor also came from there because in amplifier practice, the K a block was actually an amplifier and electronic circuit and that time a vacuum tube circuit which functioned like an amplifier and it amplified the input voltage to produce a larger output voltage not only that the arrangement was, such that at the output you could also provide more power, where as an input you took very little power.

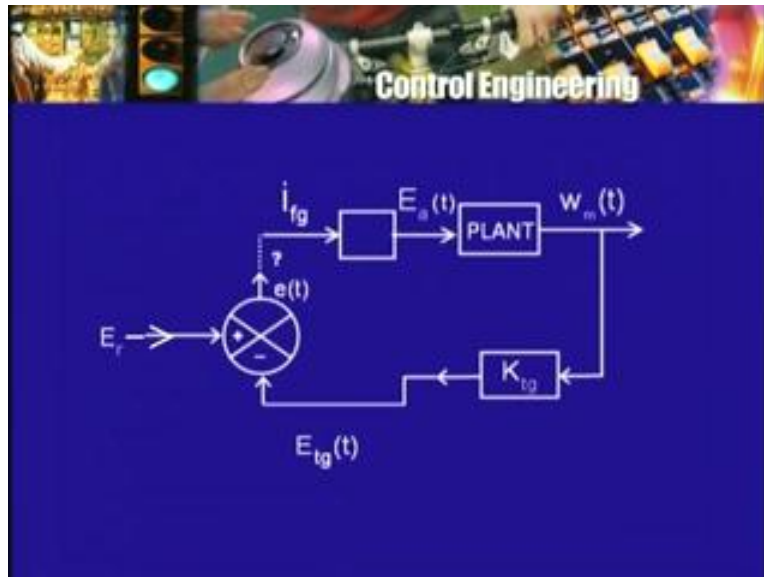
So it was both a voltage amplifier and power amplifier, of course all that is not shown when you are just put a block there the electronic circuit needs its own power supply and what not all that is not shown, all that you show as I mentioned earlier, all that you show on the block diagram is simply the variables of interest and our attempt is meant to show the relationship between the two. So in this case then, from the error detector output voltage some how you have to arrange for the field current to be influenced by it. Of course, the field current influence is the generator output voltage which is the voltage to be applied to the motor armature E_a all this is shown by a single block K a between the error voltage e and the applied voltage, the voltage to be applied capital E_a .

So this is referred to as a gain in the forward path, so our block diagram is now, almost complete in the forward path now and again there is something which happens here. The forward path is now normally regarded as the path not from the armature voltage to the motor speed or even the field current to the motor speed. In fact, that could be the correct thing to do because the field current in the generator I_{fg} results in a certain voltage being produced at the armature of the generator, this is voltage generated by the generator which is then applied to motor, which results in current flow torque generation rotation of the shaft.

So the action begins with the field current and ends should be rotation of the shaft. So the forward path strictly speaking is from the field current or whatever is causing that field current to change. Let us say some voltage which can be which is changed to change field current, if that is how it is happening to the quantity which is being controlled the output quantity. The speed of the motor that should be really the forward path and the reverse path of the feedback path should be from the speed of the motor to the output of the error detector E because the speed of the motor of course, the reference voltage is also needed the speed of the motor represented by E tachogenerator and the reference voltage E_r produces the difference E which reflects the actual condition of the drive whether, the motor is operating at the correct speed or not whether, the speed is greater or less than, what it is supposed to be depending on the sign of E .

If the motor is running at the lower speed, the error will be positive although traditionally in instrumentation along the error would have been negative, if the speed is greater than the desired speed then the error would be negative because we have defined e as E_r minus e tachogenerator. So the feedback path should be really from ω_m to e . It is common practice in control theory to think of E_r the reference signal as some kind of an input to the whole thing and therefore one thinks of the forward path is going from E_r .

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Of course, E_r is followed by the error detector then to the error then through the gain K_a to the armature voltage E_a etcetera. All the way to ω_m and then the return path or the feedback path goes from ω_m to the other input to the error detector. So one begins to think of E_r as an input to the system rather than the field current which has in fact, disappeared from our diagram because we have absorbed it E , error voltage to field current to armature voltage. We have taken care of that by a single block K_a and because E_r is the reference represents the desired thing, it is referred to as the reference input, it is called the reference input to the whole control system but strictly speaking, this is a misuse the input is not E_r , chemical engineering people use a slightly different terminology and they use the word set point for this E_r .

So E_r is referred to as the set point for the system which is a good expression because you set it, **it is** let us in this case, you set E_r at some value which represents the desired quantity. If the desired value is different you will set E_r to a different value. So it is called the set point, all set point value rather input a such although in the block diagram it does look as though E_a is something that is coming from outside, which is producing ω_m as the output and the other signals e , E_a , e tacho generator are all in between the control system has input E_r and output ω_m this is what it appears from the diagram and this is how one normally speaks about it. If one wanted to change the speed at which the motor should run constant speed at, which the motor should run then the set point will be changed, which means that the reference voltage will be change. If instead of 1500 RPM which was producing or which require or which was represented by reference voltage of 20 volts, you wanted 750 RPM of one half it then perhaps the reference voltage would be also one half of 20 volts namely 10 volts.

So, if you wanted to change the speed of the drive the constant speed of the drive what we will do is change the reference voltage. Now this is the simple minded approach which one thinks is quite okay, the problem was, if you remember the error is to be some how linked with the field current that is when the error is present, you need a change in the field current, you need an

adjustment of one way or the other either increase or decrease the field current depending on may be, the sign or the direction of the error, think of the operator. Once again, the pointer is to the right, I will move will the slider this way the pointer is to the left I will move the slider in the other direction.

Now this adjustment is being reflected or it requires a link between the error e and the field current and through the field current to the applied voltage. So it is quite natural in the sense, this the simple assumption that the field current is simply proportional to the error e and therefore, the generated voltage E_a will be proportional to the error e with a constant of proportionality which is the coefficient K_a and therefore, such an arrangement is refer to as a proportional control P for short capital P, for short some of you have perhaps heard of other kinds of controls, I for integral and D for derivative and so, besides proportional control one could have integral control, one could have derivative control and one could have a combination of these.

So, every general control will be proportional plus integral plus derivative that is the order, in which it is normally mentioned and therefore, it is referred to as P I D controller or scheme is refer to as P I D control. So on the face of it, this arrangement looks all right we have the plant, we have the feedback element which is the tacho generator in this case, we have the error detector which produces a difference between the reference value and the tacho generator voltage, it represents the actual speed. So e can be referred to or e is really does correspond to error of the system that error amplified produces the voltage E_a in this case, the input to the plant and so things should be all right.

In fact, they are all right except that there is bit of confusion around this point and this is what needs some care because many text books commit this mistake and so, I can call it really an error, regarding error. I mentioned earlier that usually there is long section or a chapter called error in control systems, error constants or error coefficients and so on.

In that chapter or when taking an example like this, one in many books this mistake is committed now, how does this mistake arise, the mistake arises in the following way and if you think a little more, you are find out what is the mistake in the argument. But let us first go through the argument. Remember, once again our normal values, so I want 1500 RPM, so the armature voltage let us say is mean pre calculated to be 230 volts, the 1500 RPM produces a tacho generator the voltage of 20 volts.

So I choose a reference voltage of 20 volts okay. So I set up this actual arrangement, I have this reference voltage source of 20 volts produced somehow, I have this what is represented K_a and the error detector as something which starting with the error voltage results in the generation of the voltage to be given to the motor. Now, one can argue that in this situation the speed will not be 1500 RPM but will be less than that. We want the speed to be 1500, so you want the applied voltage to be 230 volts at 1500 RPM, the tacho generator will be 20 volts and so, we chose E_r as 20 volts and now I am saying that if you actually do connect up things likes this then the speed will not be 1500 RPM but will be less.

So, in other word there will be error a steady state error remember, 230 volts was pre calculated for 1500 RPM producing a certain torque knowing the parameter values and back EMF coefficients and what not. So, in other words, if everything was okay 230 volt should produce

1500 RPM. Now, why is it that it is not going to produce it is something like this this K_a which connects or links the error voltage through the field current to the generator voltage will have some value. Let us say, a field current of one ampere in the generator winding generates a field winding gives rise to a generated voltage of 230 volts if the field current is not very high. So that is there is now saturation in the field circuit of the generator then the generator voltage is nearly proportional to the field current, this is a very good assumption.

In fact, in the laboratory one does this kind of test open circuit voltage of generator verses field current at constant speed of the drive of the prime mover up to a certain range of field current values one obtains a split line graph. Of course, as field current is increased considerably then the increase in the generated voltage is not any longer proportional or with the same proportionality. So the curve begins to bend you call it, the knee of the curve and then eventually it might become almost constant and that is said to be saturation. So if the generator field is not being operated in that region is going to be operated in the linear region then, the generated voltage is proportional to the field current and I said that, the link between the error and field current is also such that it is constant, the field current is constant multiple of the error voltage therefore, armature voltage E_a which is produced by the generator equal to this constant K_a the gain of the forward path as it is usually called into e .

Now whatever is the value of K_a it must be some number for the purpose of our argument, we can again choose some rounded number. Let us say I choose it something very simple it is, I choose it as 200, so this gain K_a is 200, what are the units of it gain K_a well, there are no units because the input is volts the output is volt. So K_a is 200 means error voltage of one volt produces a generator output of 200 volts. I am just taking some numbers to get a feeling for the problem, so this a these are not actual figures of any particular drive keep that in mind numbers one can always find out but that does not mean that you understand what is going on, in order to understand what is going on sometimes you use numbers but you do not worry about them too much. You do not look at them up to the third for fifth or tenth place of decimals there is time to do that later on okay.

Of course, I am ignoring some other things and these are the assumptions which are made for example, the generator has an armature, the generator armature has resistance and so can I neglect that resistance of the generator armature, the generator is connected to motor armature. So can I neglect the resistance of the generator armature will think about it. So I neglect it do I have to neglect it no because the generator armature and the motor armature are connected in series, one is generating the voltage be other is absorbing the voltage and so, I can add the generator armature resistance to the motor armature resistance and think of that as a single resistance.

So in place of r_a which was only the motor armature resistance I can now add to it the generator armature resistance. So I do not have to assume or neglect or ignore the generator armature resistance I can account for it this way. Of course, we are assuming that there is something like a constant armature resistance and I told you earlier that there is phenomenon called armature reaction which we are ignoring certainly but in the absence of that this armature resistance business is okay, the generator and the armature resistances can be combined and thought of as a

single resistance. So I am not neglecting that of course, I am assuming know saturation which is reasonable assumption.

So 1 volt of error voltage signal produces 230 volts generator output which then drives my motor at 1500 RPM at under given torque conditions okay. If the generator has to produce 230 volts for the motor to run at 1500 RPM then, the error voltage must be what the error voltage must be 230 volts divided by 200 or 1.15 volt, what is important to know is not the magnitude or a value of this voltage, if the rate is 1.15 or .23 or .06 that is not the important thing, what is the important thing, if I want to produce 230 volts I needs some non-zero error voltage because I assume that this arrangement is such between the error signal and the generator output. The arrangement is such that one proportional to the other that is what proportional control is that is why it is called proportional control okay.

So in order to produce some generator voltage I need a non-zero error. In this case to produce 230 volts I need 1.15 volt because the gain was 200, if the gain was say 460, I would still need a non-zero error voltage, it will be less it will be .5 volts. If the gain was 2000, I would still need an error voltage, it will be one-tenth of what I have one .15, one-tenth of that that is 15 volts but it is not going to be 0. Now such a system or the chain from the output of the error detector or the let us say, in this case the field current or the error detector to the field current and the field current with the generator into the motor and final the speed is referred to as type 0 system which of course implies that there are systems which are type 1 and type 2 and may be higher types.

So what is this type well, we will talk about it later I told you that there will be a discussion of error and error constants and error types of types of systems and so on. We will go into that discussion later. Right now, it is enough to note that the system is type 0 and a simple way of remembering, it is that for it to produce a constant output quantity in this case a constant output quantity of 1500 RPM or 230 volts of the armature voltage applied to the motor whichever way you want to look at it, you need a non-zero voltage and the input or a non-zero input quantity, the DC motor when you locate it from armature voltage as input and speed error output you also a type 0 system.

You cannot apply 0 volts to the motor armature and expect the motor to run. For the motor to run at a constant speed you have to apply a constant voltage to the input the armature of the motor. So the motor itself can be said to be a type 0 system if the motor speed is the output and the armature voltage is the input. So this means that when one talks about that type of system one should really talk about what is the input and what is the output. We are looking at speed control our job is to make the grinding wheel rotate at high speed, if problem was different one if the problem was one of position control, positioning a gun.

So that it tracks an enemy air craft which is being detected by means of a radar. Then the situation is different in that case the output of the system will not be speed of the gun or the turret on which is the gun is mounted. The rate at which the turret it turning that is not of interest, what is of interest is the angle of position of the gun. So then the output will be position and the input will be perhaps the armature voltage, the voltage being given to the armature of the motor which is driving turning the turret on which is the gun mounted. So, if now then I look upon the system as armature voltage to gun position then it not of type of 0. In fact, if I give a constant input

voltage to the motor the motor turret, the motor will go on rotating the turret will go on rotating and the gun therefore, will go on swinging round and round that is not what you want to happen for the reason, it will be seen that system will be seen to be type one system because the constant armature voltage.

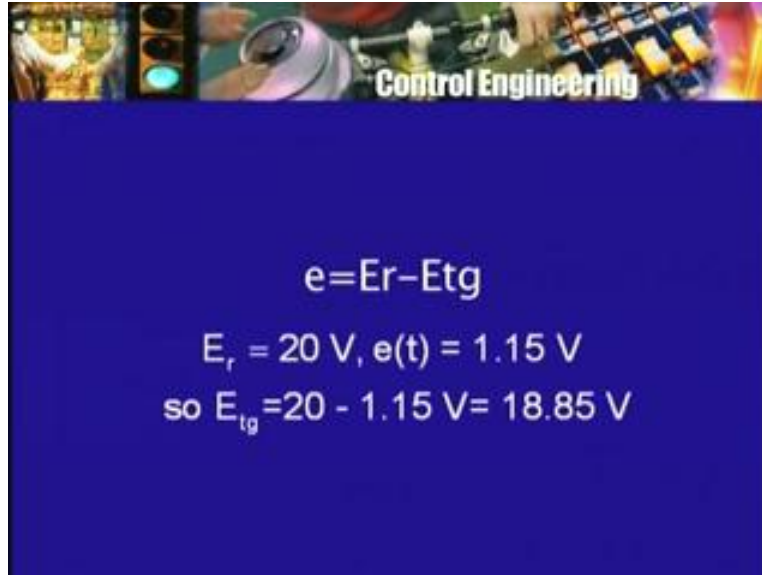
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We produce a constant speed only motor and therefore, the angular position of the motor will go on changing continuously and although it comes back to its original position, one can have this fiction of 0 degrees being equal to 360 degrees. Therefore, the amount or the angle through which the gun would have turned will go on increasingly linearly. So constant input produces a linearly increasing output therefore, the system would then be thought of as a type 1 system. So, what is the type of a system depends on, what is the input and what is output? This is a very important thing to remember simply because you have a motor you cannot say, it is type 0 or type 1, if it is a motor with armature voltage as input p speed as output then, it is type 0, if it is motor with armature voltage as input but position as output then, it is not type 0 it is type 1.

Now, we are looking at speed controls so we have type 0 plant for a type 0 system non-zero constant speed output requires a non-zero constant input, in our case 1.15 volts. So for the system to operate for the motor to run at 1500 RPM, I need an error voltage of 1.5 volts. Now go back to the diagram and look at that difference device or the error detector one input, it is E_r which were came constant and 20 volts as the reference source. The output e , so must be the other input e tacho generator because e small e the error is reference minus the tacho generator voltage. So the tacho generator voltage must be less than the reference voltage by 1.15 volts in other words in tacho generator voltage must be equal to 20 minus 1.15 or 18.85 volts.

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The slide features a blue background with a decorative header at the top showing various engineering components like a motor, a light, and a circuit board, with the text "Control Engineering" overlaid. The main content consists of three lines of white text on the blue background:

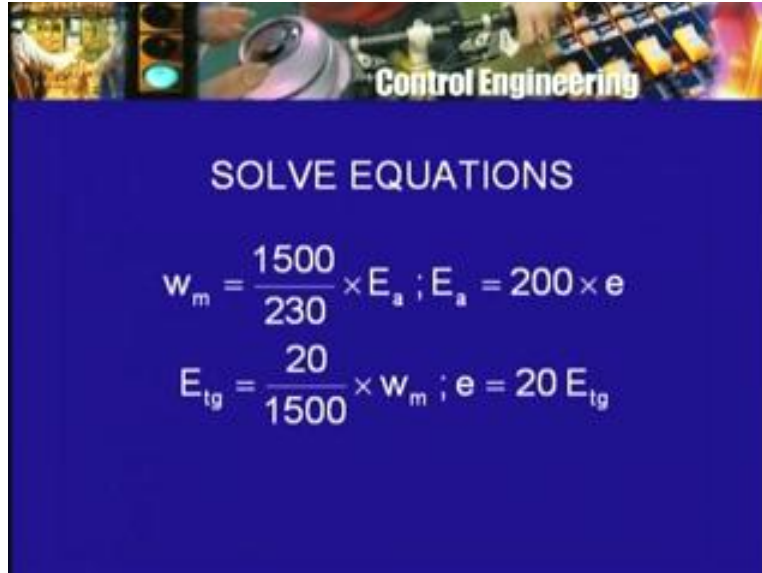
$$e = E_r - E_{tg}$$
$$E_r = 20 \text{ V}, e(t) = 1.15 \text{ V}$$
$$\text{so } E_{tg} = 20 - 1.15 \text{ V} = 18.85 \text{ V}$$

So for the motor to run at a constant speed of 1500 RPM, we need a tacho generator to produce an output voltage of 18.85 volts but, we have assumed that our tacho generator produced a voltage of 20 volts, when the motor speed was 1500 RPM. So there is this inconsistency, if the motor runs as 1500 RPM its output will be 20 volts but if the motor has to run at 1500 RPM then the tacho generator output voltage must be 18.85 volts.

Now, the tacho generator voltage cannot be simultaneously both 18.85 volts and 20 volts it will be something, it will not be 20 volts because, if it is 20 error is 0 armature voltage is 0 motor will not run, if it is 18.85 volts then the speed must be less than 1500 RPM but on the other hand an error of 1.15 produces armature voltage of 230 which produces a speed of 1500. So the speed will have to be both 1500 and less than 1500. In fact by a proportional amount 20 volts, tacho generator voltage corresponds to 1500 RPM therefore 18.85 volts of tacho generator voltage will correspond to what speed and you can calculate that roughly may be, it will be or the order of 1350 or there about not 1500 RPM certainly.

So what is actually going to happen, if I connect up the system like this what will be the actual speed of the motor? Now, this is something I would like you go ahead and calculate, it is a very simple exercise all I need really is this relationship that the gain K_a is 200 and error voltage of 1 volt produces an armature voltage of 200 there whereas, the motor like another gain or something which produces 1500 RPM for 230 volts. Next the proportional relationship between the output speed and the input voltage and the third device, which is the tacho generator it is output voltage is proportional to the speed.

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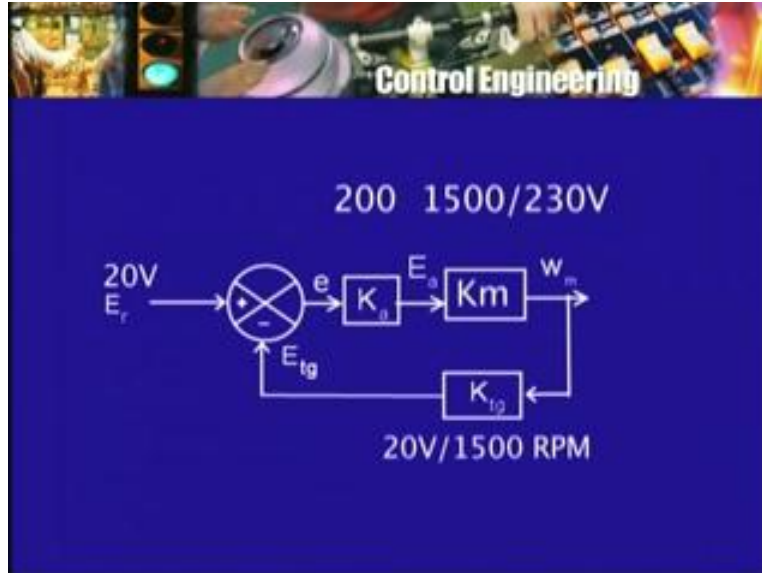
The slide features a header with the text "Control Engineering" and a background image of various electronic components. The main content is a blue box with the text "SOLVE EQUATIONS" and two mathematical equations:

$$w_m = \frac{1500}{230} \times E_a ; E_a = 200 \times e$$
$$E_{tg} = \frac{20}{1500} \times w_m ; e = 20 E_{tg}$$

We have assumed that 20 volts is produced, when the tacho generator runs at 1500 RPM and lastly of course, you have to error detector which is simply taking a difference of 20 the reference voltage minus the tacho generator voltage. So look at all these equations and find out what is going to actually happen to the motor, one conclusion is already obvious, that is the conclusion that we have reached that is the motor will not run at 1500 RPM. The motor cannot simultaneously run at two speeds and therefore, a motor will not run at 1500 RPM because, if it did the tacho generator voltage will have to be 20 volts on the other hand, if the motor run at 1350 RPM which corresponds to tacho generator voltage of 18.85 volts, we know because in that case the error voltage is 1.15 which produces or gives arise to motor speed of 1500 RPM.

So you need to go through these calculations, you have to the find out, what will be the actual output quantity or motor speed, when all this is happening and this can be done by using the block diagram. In this case, ignoring that may be have motor, we have this armature, we have tacho generator and so on by simply putting up a block diagram with these very simple blocks of gains or coefficients motor speed related to armature voltage, armature voltage related to error detector output, tacho generator output related to motor speed and the error detector output is the difference between the reference speed and the tacho generator output.

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So, we can look at this very simple situation, we will have one block K_a which in our case is 200, the motor can be thought of as another block. So we can call it say K_m which is, what 1500 RPM divided 230 volts. So I say dimensional quantity RPM in the relation to volts but it is 1500 divided by 230 and that produces ω_m , on the feedback side we have this ω_m going to this block K_{tg} and what is this value of K_{tg} , the value of K_{tg} is 20 volts divided by 1500 RPM. So it is volts per RPM and value is 20 divided by 1500 whatever, it is and then, we have the error detector for which we have the reference signal of 20 of course, it is volts I will shown it as volts and this it.

So what is the value of ω_m in this case? So essentially we have to write 2 equations or we have to write some equations from which we will get this value ω_m and we can write 2 very simple equations and later on, we will be doing this for a more general situation and therefore, we can do it right now in a very simple minded way. One is the forward path from the error detector output to be output quantity of interests. This is referred to as the forward path. So what is the relationship there e is amplified 200 times and then it is changed 1500 by 230 times to get ω_m . So I can write ω_m equal to 200 into 1500 divided by 230 times e .

So ω_m is some number into e this number 200 into 1500 divided by 230 can be referred to as the overall gain of the forward path from the error detector output to the output of the system dimensionally, it will be RPM per volts, the input e is 200 volts, 200 was dimensional less 1500 is volts 230 is volts. So this will be volts per RPM per volts and will this number. Now this is one equation which must hold, if the motor is running, if the generator is functioning, if the parameter values are known exactly, if the torque is at the expected value then this is what must be true after all, we do believe that we have modeled the motor fairly accurately.

So this equation must be true is nothing wrong about this equation. This is an equation for the forward path which relates the output ω_m , in this case to the output of the error detector e in this case. So this equation is correct there is another very simple equation that we can right

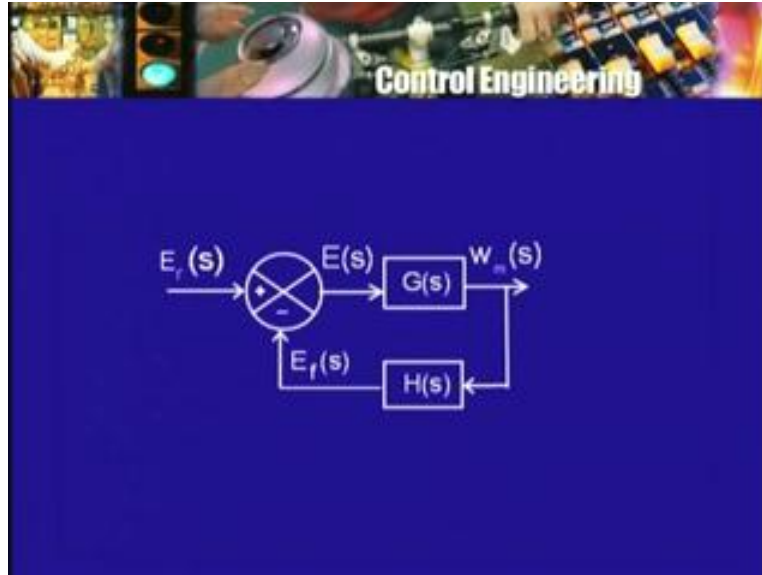
and although it is not necessary to write that equation in the sense, we can avoid it and write some other equation directly will be good to write it because in a general case, it will be worthwhile the signal which is feedback, which is the tacho generator voltage is e_f , f for feedback and e in this case because it is voltage.

So e_f the signal of that feedback equal this constant K tacho generator which is 20 divided 1500 into ω_m and this equation also is correct, there cannot be anything wrong with it the tacho generator is mounted on the motor shaft life. It was not if it got disconnected in some way or some other thing. But if the tacho generator remains connected with the motor shaft or is mounted on the motor shaft the tacho generator is not malfunctioning, if it is a permanent magnet generator then unless the permanent magnet has somehow gone bad, it will be functioning producing an output voltage and so this equation also must be true. This is an equation for the feedback path which gives you the signal that is fed back into the error detector in terms of the output signal ω_m in this case, the tacho generator voltage related to the output voltage ω_m .

So this equation also is correct, we are not making any mistake in writing down these two equations and there is one more equation which we are to write and that is equation for the error detector which is the equation $e = 20 - e_f$, the feedback voltage. So all these 3 equation are correct, if the error detector is functioning then, when the input and we assume that the reference voltage can be maintained constant, if it is a cell or you know some standard voltage source or something obtained and kept constant. So this is an assumption that we are making this takes the place of a operator having a mark on the dial that is 1500 RPM, we need something like that for the automatic system. If we are not able to produce a constant voltage source of 20 volts or whatever value then we cannot go ahead and use automatic control.

Of course, we may think of 1500 RPM producing 1500 RPM then you need that, so if the error detector functioning correctly this equation is also true, so all the 3 equation are correct. In the general case, this not still very general but fairly general, we can draw the following block diagrams and anticipating, what we are going to do later on I am going to put some symbols and talk about them the reference signal E_r I will now put is s in addition do it. So this s those of you who have studied already will know that this is the Laplace transform variable.

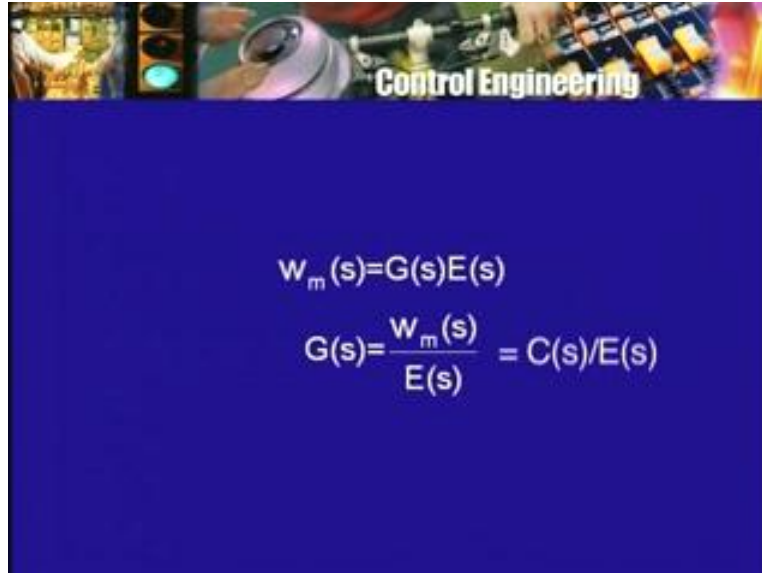
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So $E_r(s)$ will be the Laplace transform of the reference input function. The output of the motor instead of ω_m as function of time, we may, we will talk about its Laplace transform and in this case, I can use the corresponding capital Greek letter capital $\omega_m s$. So this is the transform the Laplace transform of the output speed, output quantity speed as a function of time in its place, I have the Laplace transform of it. In this case $\omega_m s$ the motor generator load all that is amplifier or that part of the error detector further beyond the error detection, this will be represented by transfer function and this is the usual or standard symbol for it is capital G and this as we will see, will also be a function of s although, it is not the Laplace transform of any signal and this is an important point capital $\omega_m s$, s is not just some function of s , it is the Laplace transform of the output signal of, the output quantity, the output variable the motor speed as a function of time, as it changes or does not change in time. The reference input in this case of course is a constant thing and a constant function also has a Laplace transform.

So, it is also the Laplace transform of constant signal namely the output of the voltage of that reference source it produces 20 volts for all time to come but this $G(s)$ is not the transform of any physical signal, what is it it is something like this constant that we had relating ω_m and e , it is the transfer function relating the error detector output to the output of the system and so, I will put one more symbol here capital E of s , this will stand for the Laplace transform of the error signal. This error signal also can be physically measured, if necessary although we vary many applications, one may not want to measure it but it does correspond to an actual physical signal e of t corresponding to it is Laplace transform is capital E of s and so, the equation that we can write is similar to the other equation in which motor speed constant number was some number into the error voltage, another constant number. In this case, we have to write ω_m of s equal to $G(s)$ into $E(s)$ or in other words we can rewrite this as G of s is a ratio of 2 things ω_m of s divided by e of s . This is the transfer function of the forward path of the whole system.

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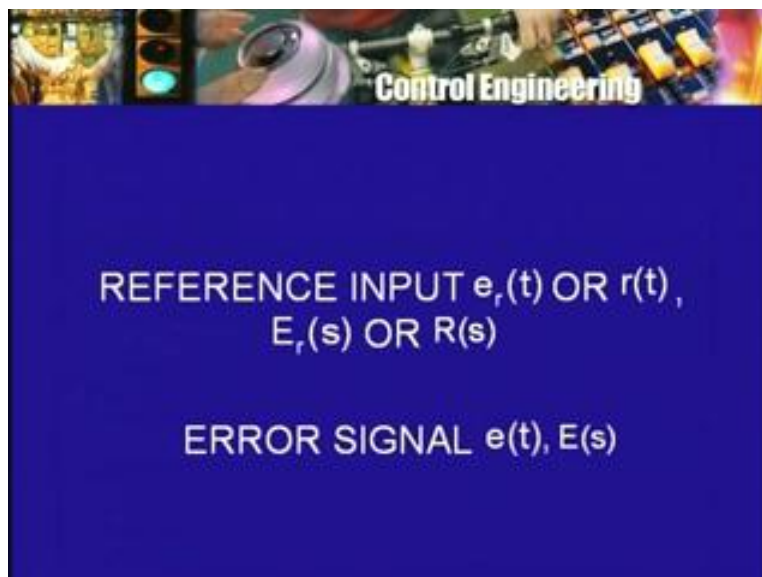


Control Engineering

$$W_m(s) = G(s)E(s)$$
$$G(s) = \frac{W_m(s)}{E(s)} = C(s)/E(s)$$

Of course, I am using omega because I am talking about speed again in control theory literature some symbols are used when talking about the general situation. The output quantity since it is the quantity being controlled very often one uses the letter small c for it. So when talk about C t the output quantity as function of time and therefore its Laplace transform will be the corresponding capital letter, capital C of s. When this is done the reference it may not be a voltage in as another application. So then it will be for some reason or the other one uses the letter e still because it is related to the error as E r, r for reference.

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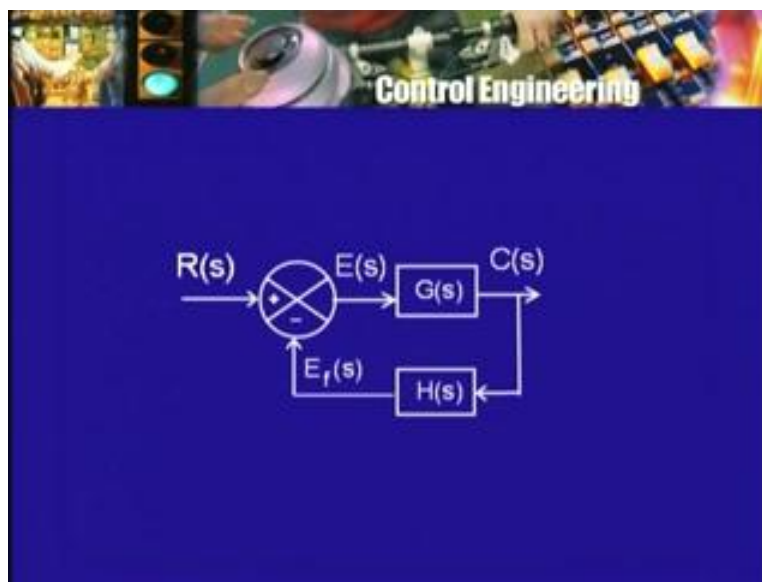
Control Engineering

REFERENCE INPUT $e_r(t)$ OR $r(t)$,
 $E_r(s)$ OR $R(s)$

ERROR SIGNAL $e(t)$, $E(s)$

So it is the input as it were for the whole thing although, I repeat it is not really an input in the strict physical sense of the term. So this is E_r and its Laplace transform will then be capital E_r of s the error detector output is e of t . So its Laplace transform is e of s and therefore the transfer function of the forward path G of s is c of s divided by e of s . In fact in many books in place of e reference the letter r itself is used for the reference signal which is also a natural way of doing it and so, instead of E_r of t , I will have simply r of t the reference input and therefore, its Laplace transform will be capital R of s then, what about the feedback element, the feedback element has an output which is feedback signal e_f of s here certainly the letter e is used and therefore, its corresponding Laplace transform will be E_f of s and this will be equal to some other function H of s into the output transform in the general case C of s .

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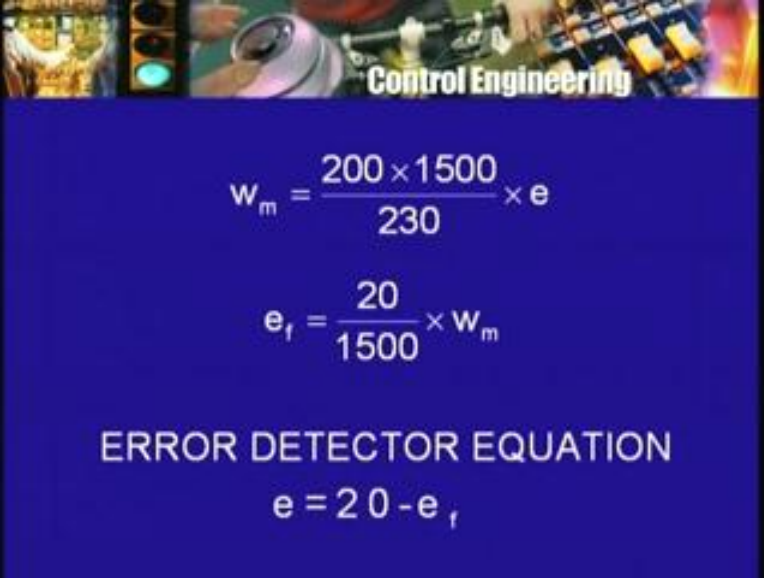


In our case, it will be the tacho generator voltage as function of time its Laplace transform equal to in this case of course ideally it just the coefficient K tacho generator into C of s . So we have a transfer function in the forward path, we have transfer function in the feedback path and so, the block diagram will then look like this where in place of E_r of s . In many books, you will find capital R of s and similarly, because it may not be a speed control. So it generally one may use the symbol capital C of s and this is a very basic diagram of a simple feedback control system output $C(s)$ not the actual time function or signal but its Laplace transform. The reference think input call input again not R of t or r of t but its Laplace transform $R(s)$ the transfer function in the forward path capital G of s .

The transfer function in the feedback path capital H of s and the error detector or the difference device producing E_r of t equal to or rather E of t equal to E_r of t minus E_f of t or capital E of s equal to E_r of s or $R(s)$ minus the output of the feedback device E_f of s or H of s into C of s . This is of course anticipating, what we are going to do later on in a very general analysis I going getting back to our numbers then we had those 3 equations relating ω_m to e that is the forward path relating e_f to ω_m that is the feedback path and relating e to the reference signal and if feedback signal e_f . From these 3 equations there are 3 equations and there are how

many unknowns are there in these equations there are 3 unknowns ω_m the speed the error signal that will be produced e and the feedback signal that will be produced e_f .

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The slide features a blue background with a collage of control engineering images at the top, including a tachometer, a motor, and a control panel. The text 'Control Engineering' is overlaid on the collage. Below the collage, two equations are displayed in white text:

$$\omega_m = \frac{200 \times 1500}{230} \times e$$

$$e_f = \frac{20}{1500} \times \omega_m$$

Below the equations, the text 'ERROR DETECTOR EQUATION' is written in all caps, followed by the equation:

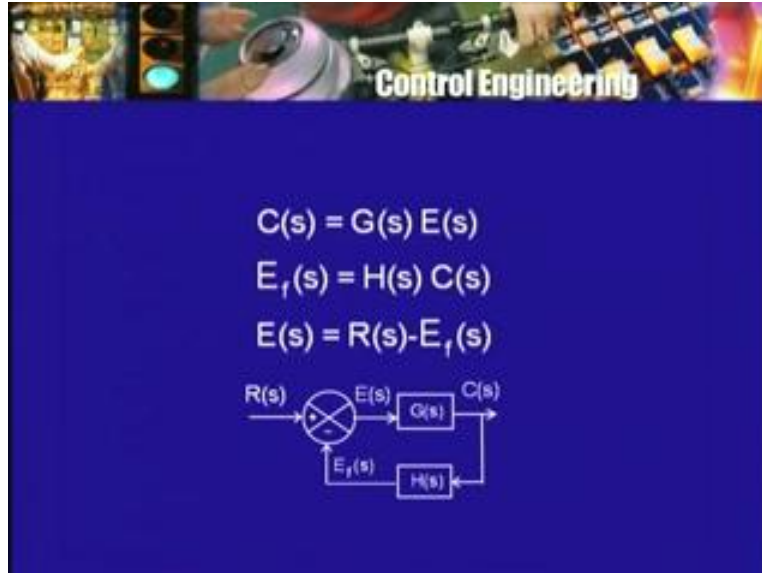
$$e = 20 - e_f$$

So from these equations find out what ω_m will be and check that it will not be 1500 or since it is speed it will not be a 1500 RPM but it will be less than that but it will not be such that the tachometer output is what we have calculated earlier when doing some qualitative explanation or understanding namely 18.85 volts which corresponded to roughly 1300 and 50 volts but it will be somewhere in between.

So do this calculations, a hint perhaps I can give here because that is what is we are going to do later on for the general control system for which add this diagram involving forward path G of s , feedback path H of s the error detector producing output E of s etcetera. There is a relationship between the output transform and the reference input transform which is referred to as the overall transfer function, overall transfer function or sometimes referred to as a transfer function of the feedback control system that is of the whole thing not just motor or the tachometer that is part of it is going to be called the controller or it is the part controller but this whole works. The overall transfer function, this overall transfer function can be shown to be and this is something also you can do G of s divided by 1 plus G of s into H of s .

So it will be also function of s and it will be given by this expression G over 1 plus GH and this is a formula which every control theory student learns to remember by heart and if you have been a good student then, you will not forget it as long as you live. This is based on the simple diagram that I have drawn earlier and that we looked at earlier here. It is $G(s)$, $H(s)$ and the error detector $E(s)$ is $R(s)$ minus E_f of s and I told you that this is not really the instrumentation engineers practice of error calling something error, he talks about an error and correction in just the opposite sense but somehow the control theory people have been talking about this as the error.

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So let be, let it be, so but it is difference of $R(s)$ minus $E_f(s)$ and just those 3 equations that is we can write $C(s)$ is $G(s)$ into $E(s)$, for the forward path $E_f(s)$ equal to $H(s)$ into $C(s)$, for the feedback path $E(s)$ is equal to $R(s)$ minus $E_f(s)$ that is for the error detector element from these 3 equations, one can eliminate the other 2 unknowns and get the relationship which relates $C(s)$ to $R(s)$ that is the ratio $C(s)$ to $R(s)$ will be $G(s)$ over $1 + G(s)H(s)$. This is referred to as the overall transfer function and therefore one may use a symbol like capital $T(s)$ or sometimes because capital T is already being use to denote torque G reminds you of gain but then H is not gain.

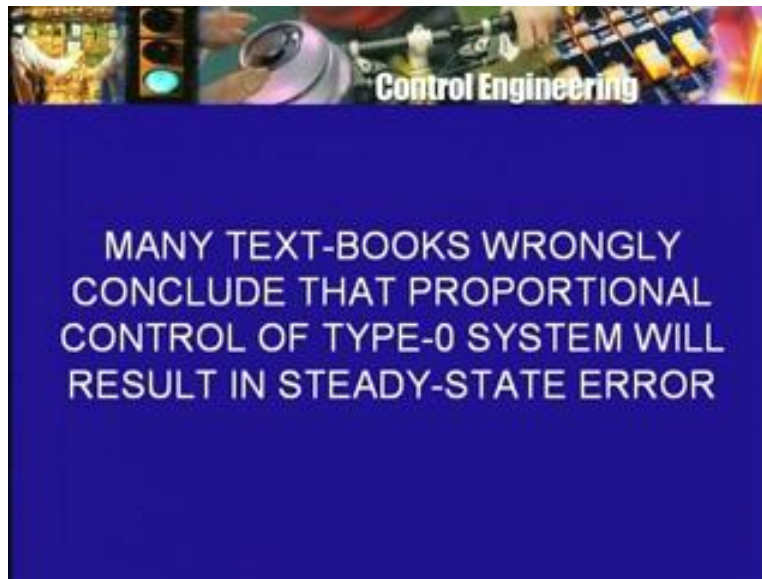
So there are all kinds of notational tricks but G may be when one can say G closed loop that is the whole thing which is the closed loop system, what is the G the gain of the overall system, the overall transfer function G_{cl} , G closed loop and in that case the $G(s)$ which goes from the error detector output to the quantity being controlled is refer to as the open loop transfer function. So we have the closed loop transfer equal to the open loop G divide by one plus $G(s)H(s)$ in our particular example of course we did not look at transfer functions, we just looked at numerical equations and one of them was known or was actually know as 20 volts.

So that then one could actually calculate the speed as so many RPMs in the general case one can only calculate a ratio or a transfer function. You will not know what the output is until, you know what the reference signal is what you can find out however is the ratio of the 2 which is called the transfer or in this case, the overall transfer function. So use the equation which is analogous to this equation for our numerical case. So in the numerical case then it looks like well the $G(s)$ the role of that is played by that coefficient 200, error voltage is multiplied by 200 to produce armature voltage and that at the rate of fifteen hundred RPM per 230 volts gives rise to this number.

So this $G(s)$ could be replaced by this number then in the feedback path, we have the transfer function $H(s)$ that in this case will be replaced by the number 20 divided by 1500, 20 volts

tacho generator output for 1500 RPM motor speed. So that 20 divided by 1500 RPM that takes the place of H and the place of R of course, no transform is taken by the number 20 volts. So then my C (s) would correspond to the motor speed that will actually result. So you can just take that formula and substitute and get in get omega m this way all look at these 3 equations and calculate omega m from those 3 equations.

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You can also calculate the error signal e and feedback signal E_f , but the outcome of the whole thing will be that this drive will not run at the desired speed 1500 RPM and so many books wrongly conclude that such a drive must have a steady state error that with this kind of proportional control of a type 0 system therefore, I must have a steady state error.

Now, the homework for you is first of all of course calculate the omega m but then reflect upon, what is wrong with this argument, if this is argument correct then, what is wrong with the arrangement, something as to be done. You should be able to think about it and to show that we can still use this idea of proportional control. So I do not have to go to integral control and derivative control and so on, you do not even know or I have not told you what they are, but even using proportional control but with a slight difference, the motor will run at the desired speed of 1500 RPM when a reference signal of 20 volts is applied how, think about it.