Power System Dynamics and Control
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Lecture No. # 08
Numerical Integration

We continue with our discussion, on the analysis of dynamical systems. In the previous few lectures, we have been discussing the analysis or the time response of linear systems. Linear systems are amenable for what is known as Eigen value analysis, in which we can understand the properties of the response, just by looking at properties of the A matrix.

Now, in today's lecture, we shall introduce you to a more general technique, which is applied even to non-linear for the analysis of even non-linear systems, that is the technique of Numerical Integrations. So, we will talk about numerically integrating dynamical equations in order to obtain the response.

Now, it should be understood that numerical integration is not limited to the analysis of linear systems however, in today's lecture one or two examples that we will consider; we will consider numerical integration of linear systems. The reason why we do that is, since we know the, what response is, it could be easier to understand the properties of the numerical integration algorithms, which I used in doing numerical integrates, that is also called simulation of a system. Now, before we go on to numerical integration, we have been studying some very interesting topics in linear systems, and how we can actually interpret the response of a linear system.

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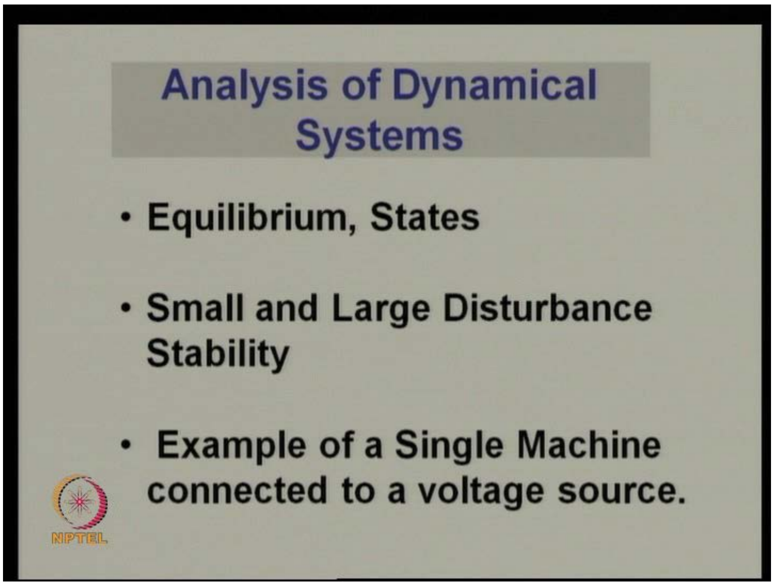

So, will just have a quick review of what we have been doing over the past few lectures, we started with the analysis of dynamical systems. What is equilibrium, what are equilibria of a system, what are states, the issue of small and large disturbances stability, we also considered to illustrate this particular concept of small and large disturbance stability, the example for single machine connected to a voltage source.

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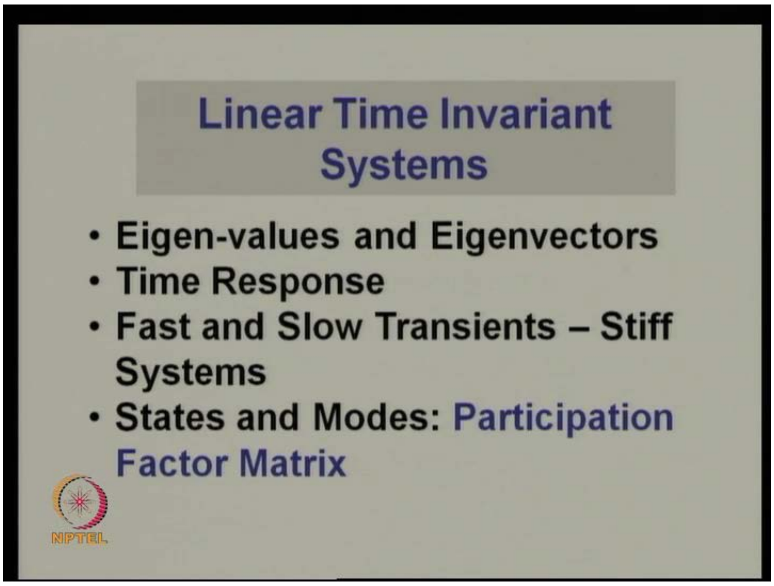

Then we moved on to linear time invariant systems there analysis, and we saw that we could analysis a system a linear time invariants system response, using Eigen values and

Eigen vectors of the A matrix of the systems. That is if (()) system x dot is equal to a x the properties of a, really determine the stability of the system. The response of course, its dependent not only on the Eigen values and Eigen vectors, but also dependents on the initial conditions of the system. Whereas, the Eigen values really determine, the nature of the time response of individual pattern, in this system the Eigen vectors, the right Eigen vectors really give you some idea of the observability of certain patterns in certain stage.

So, that is basically is a physical you know kind of interpretation of Eigen values and Eigen vectors and we have done a fairly detail analysis of the time response of linear time invariant systems. And we can actually write down the time response of course, there there was a kind of cushion, which had you know ask you to bear in mind, that is if your A matrix is non distinct Eigen values, it is possible that you will not be able to get n linearly dependent Eigen vectors, n being of course, size of the A matrix.

So, if you got A matrix, which has got size n cross n and its does not have n distinct Eigen values, this possibility that you may not have n distinct or linearly independent Eigen vectors, in that case you cannot diagnosis the matrix you will get terms like t e rise lambda t and so on, under such situations.

But, the basic point which I want to, you know you to understand that the and behavior of a linear system can be quite properly understood, there is no mystery as for as the functions which appear in the response; the time functions that is e rise lambda t sin omega t, t e rise to lambda t is all are well known and well understood functions. So, that the response of the system can be quite easily obtained and understood.

In addition we can also understand, how the system will behave like, if you got a system x dot is equal to a x plus b u, that is u is an input, then the input the force response for this input also can be written down. So, that is one major advantage of linear time analysis of linear time invariant systems; remember of course, that linear your linear time invariant system, dynamical systems are arise in our day today life and in power system dynamics in two ways.

The system may be inherently linear or you may be analyzing basically originally non-linear system by linearsing it around an equilibrium point to understand its small disturbance behavior. So, linear system arise in these two possible contacts and important point, which we understood in the previous lecture was that, some systems not all systems, some systems have very widely wearing Eigen values or in other words the patterns which are seen in the final response, remember the response is super position of patterns. The patterns which have seen in the response, a time response is made out of patterns, you will find it there is a wide variation in the speed of the response of the various patterns; for example, you may find at is one component of the response, which varies very fast and one component which varies very very slowly, so this modes sort to speed.

Or you know you can say the fasten and slow modes, and in case you have got fast and slow modes, you can actually make approximation in the model which you are using; one of the important things was, that if we can identify the states, which having some way associated with the fast modes. And the states which are associated with slow modes, good actually make modeling simplification, and in the basic point is that it could reduce the order of the system. You could reduce the number of differential equation in the system.

This will be very important, because in power systems you will find that there are transients which you know kind of a very fast, like lighting transients and network transients and some very slow transients associated with the mechanical systems, like the governor boiler and so on, which have very slow. So, when you are studying a particular system, you need not model every, you know you may make modeling simplification depending on the thing you are interested in.

In case you are for example, interested in fast transient you should pay more attention for example, in modeling a transmission line, you may even think of modeling a transmission line by its partial differential equation model. Whereas, if you understand slow transient or perhaps like for example, a loss of synchronism is a relatively slow phenomena, compare to lightening transients.

And oscillation, electro mechanical oscillation which have seen in the grade, in that case you may wish to even represent the network by lumped, you know by lumped dynamical system and some time even neglect the dynamical equation themselves of the network. You may treat the network to be cos sinusoidal steady state of course, this I am jumping steps, you have to recall this, when we come to modeling of transmission lines and other components. The basic point is that depending on what phenomena is of interest, whether it is fast or slow you can make modeling simplification, to make things of bit more precise.

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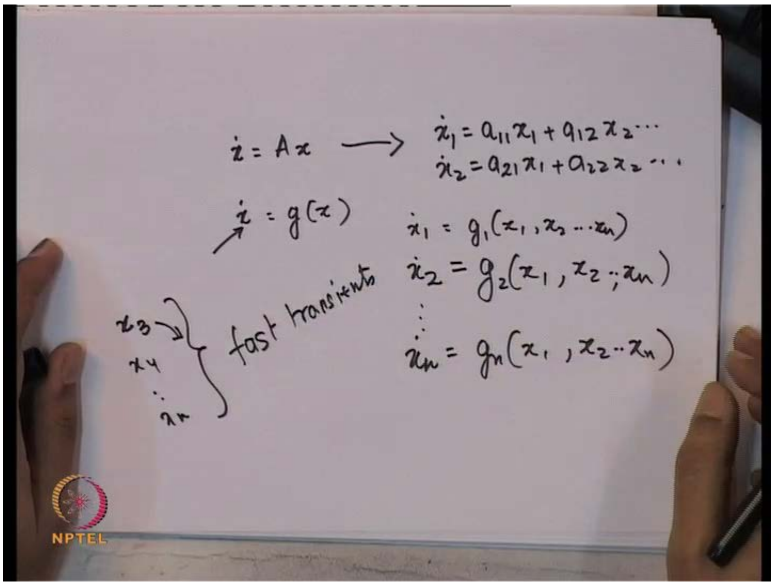

If you got a system x dot is equal to A x or you got system x dot is equal to g of x, where x actually is a set of vectors, I mean this is actually short hand for large number of equations. So, you got in fact, x 1 dot is equal to g 1 (x 1, x 2, x n), x 2 dot is equal to g 2 (x 1, x 2, x n) and so on, or in this particular case x 1 dot is equal to… This is a linear system a 1 1 x 1 plus a 1 2 x 2 and so on, and x 2 dot is equal to a 2 1 x 1 plus a 2 2 x 2 and so on.

So, you can have system, this a linear system, this is non-linear system, if you can identify, if it is possible to identify states, say you know there are certain states say x 3 onwards, that is x 3 x 4 x n, which you can identify as a associated with the fast transients.

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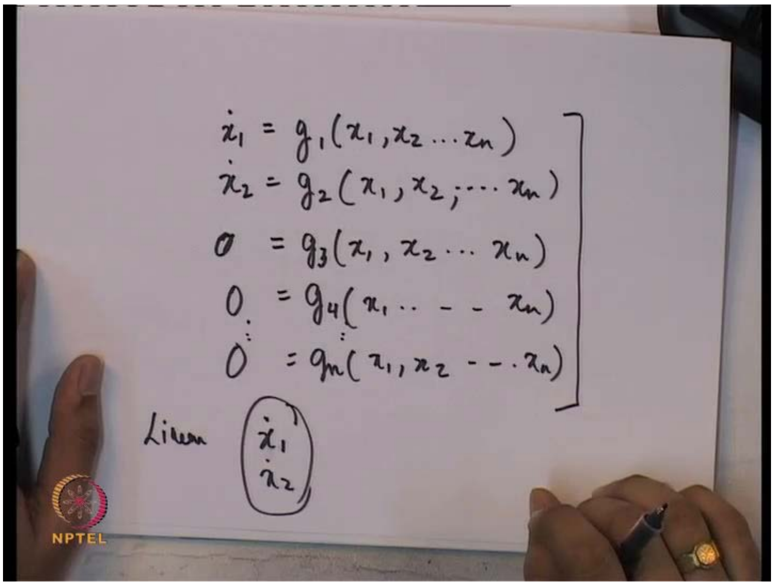

In that case, you would be at least in this non-linear, both in the non-linear and linear system case, you can make this particular approximation this is of course, unchanged (No audio from 10:12 to 10:23) this x 3 dot we just set equal to 0 and replace it by an algebraic equation g (x 1…) Is equal to 0 (No audio from 10:35 to 10:48) this is the differential algebraic model of the system.

And if you take this particular model of the system you if of course, one important point is you know a priory that this states x 3 onwards or in some way associated with the fast transient (Refer Slide Time: 10:58). So, what you mean by association is of course, something we define in the last part of the last lecture, I will just recall what it was in some time. So, what you can do is, if you are interested only in the slow transient, then you can replaced at differential equation corresponding to the fast state variable or the state variables, corresponding to the fast modes of the fast transients of the system, and you can replaced them by algebraic equations.

So, it convert set of differential equation into a set of differential algebraic equations of course, in the non-linear case it may not be easy to you know you could use this algebraic equation to element or rather right x 3 to x n in terms of x 1 and x 2; this is possible in linear system quite easily, in non-linear systems it may not be possible to element in that sense.

So, in linear systems, we can get rid of the fast variables in (()) what I mean by get rid is just write them, in terms of the slow variables. And resulting differential equation, then can be written simple as in case of linear systems, you can have differential equations only in x 1 dot and x 2 dot, that is why we did in the example in the previous class. In fact, there was you know, when we analysis the slow and fast transients. So, if you are interested in this slow response, you can kind of be blind to the fast response. So, this is what we discussed in the previous class.

Conversely if you are interested in this slow response or rather you are interested in the fast transients, for a very short duration of time. So, I am interested only in viewing the fast transient for short duration of time, you can assume the state associated with the slow transient are just fluorescent at the pre disturbance value. So, these are the kind of modeling simplification we can make, it turns out that these kind of system is of course, this resumes that the system can be broken up into fast and slow subsystems.

This may not always be possible, but in case you can do it these modeling simplifications are possible. If shall see that, once you make the modeling simplifications, you may be able to analysis systems in a better fashion. For example, when we understand numerical integration by removing the fast, you know the transient, the state variable corresponding to fast transient by eliminating them, writing them, in terms of the slow variables, you can actually start using some simpler numerical integration methods.

So, of course, this is something will discuss later, when we I introduce you to numerical integration methods, somewhat later in this particular lecture. Of course, the key point which I mentioned last time was to associates certain states with certain patterns, fast transients, you know how you do that? In linear system, I mentioned that, if you take the write Eigen vectors and the left Eigen vectors corresponding to a system.

For example, in the previous class, we studied this particular system and what we did was to find out the association of fast transient since, slow transients, we evaluated the right and left Eigen vectors corresponding to various modes, we find out the right Eigen vector matrix (Refer Slide Time: 14:28). P invest matrix is also as left Eigen vectors as its rows, this is called q, now if I take this P and P inverse and I do this operation, please refer to the previous lecture.

If I do this operation, then the matrix which results gives us the participation of certain state in a certain Eigen value or certain mode in this case. So, this is basically one way suppose the participation of a particular state, in a particular Eigen value lambda 3 is 1, then we say, this state is completely associated with this Eigen value lambda 3. Of course, i l 1 and v c sorry i l 1 and v c both participate in the mode corresponding to this Eigen value, so this is what we did last time.

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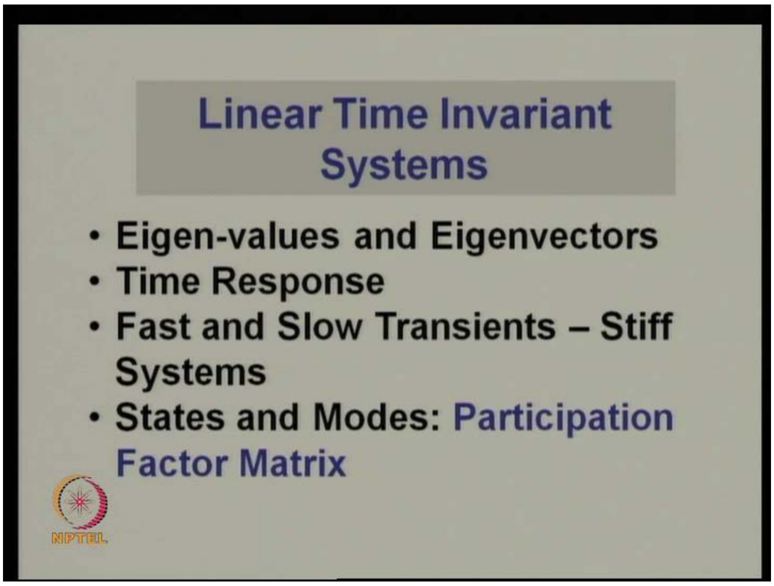

So, this was of course, what I mentioned sometime ago, you could you can use get in association of states and modes using the participation factor matrix, note that in a coupled systems sometime, the participation is distributed among all states. So, sometimes, it may not be possible to this to really make this you know portioning of system into fast and slow transients, you may find it every state in some way or the other way, in some significant sense participates in all the modes. So, that may of course, preclude using this fast and slow transients to make modeling simplifications.

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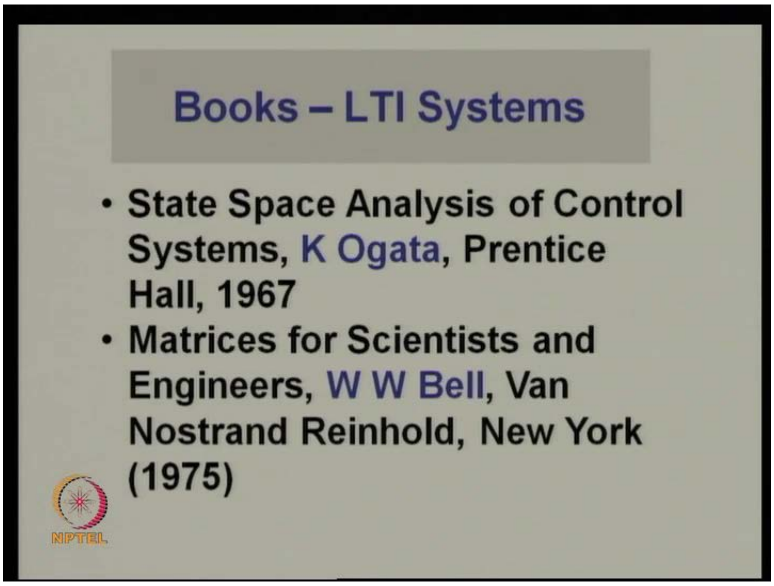

Of course, the aim of this course is study power system dynamics and control, and although the initial part of this particular course, I been concentrating on general analysis of systems. We should soon in may be a couple or 3 lectures, go into domain specific issue modeling issues, but it is to be noted that, you should have some background in the analysis of dynamical systems.

So, I I recommend that you can read as was in linear time invariants systems are concerned, the books by Ogata or some equilibrium book, you will find many. And of course, I would also recommend that you read some books on basic Eigen values and Eigenvectors for example, matrices for scientists and engineers, which are recommended here. But, of course, there are many other equitable books, which are just as good.

There are other many many other topics relating to linear systems, and as we go long we shall also talk about trans a function representation and so on. But, that will be when we understand, excitation and prime mover controller, so for the timing now.

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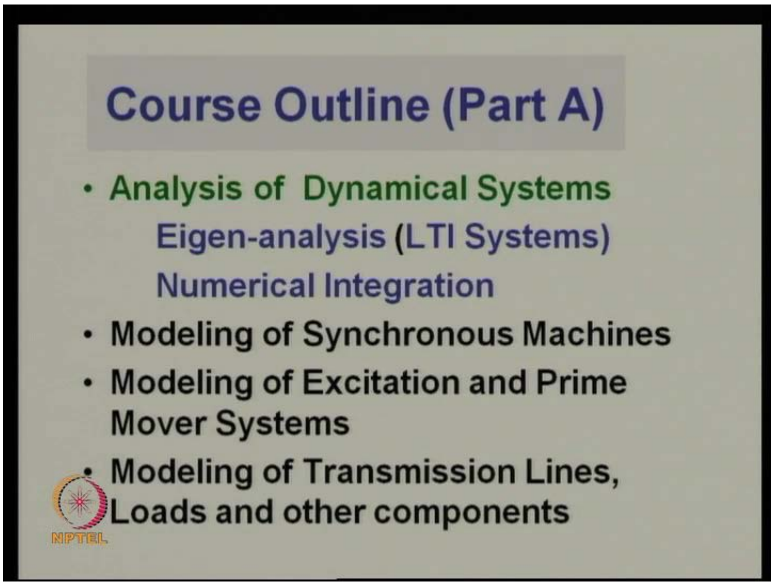

Let us move on to the next part of our course that is the numerical integration, before I really start the topic of numerical integration. Let us look and the course over view again, so that we do not really get lost in what we are doing, remember that the first part of this course, was a basic introduction to analysis of dynamical system, where we understood the Eigen analysis of linear time invariant system.

Now, we shall go on to study numerical integration techniques, what is still to come is modeling of synchronous machines, modeling of excitation, prime mover systems, modeling of transmission lines, loads and other components.

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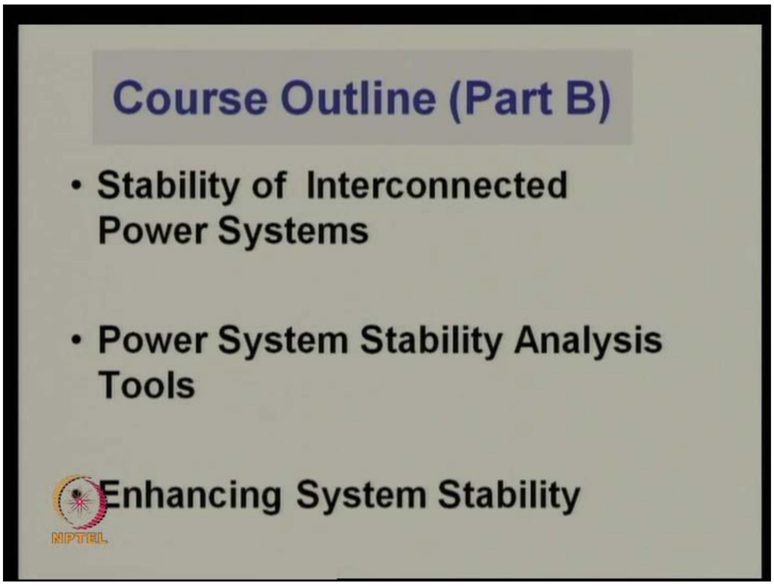

And of course, what is the main important thing in this course is to understand the stability of inter connected power systems. So, we shall use the models which we developed, and analysis tools which are developed in part one of the course, and go ahead and understand the stability of interconnected power systems, power systems stability analysis tools and methods to improve power system stability. So, this is basically, what we shall do in this particular course, now let us move on to the other topic of numerical integration.

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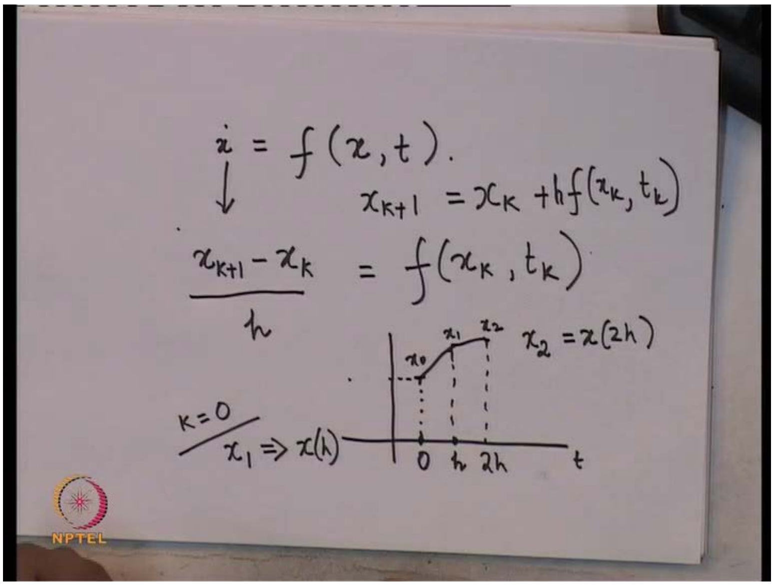

Now, if you studying a system x dot, continues time system x dot is equal to f x of t, you may one of the ways to analysis the system is to numerically integrate the system given the initial conditions. Now the simplest way to do that of course is what is known as Euler's method or equivalently forward Euler's method. So, I shall use of course, Euler's method mean forward, Euler's method interchangeably they mean the same thing.

x dot can be approximated as x k minus 1 minus x k upon h in this fashion, this is known as Euler's method of numerical integration. Now, what we really mean by numerical integration, what I mean is suppose I know suppose this is time t is equal to 0, I know the initial value of x that is x of 0 in some sense, I know the initial value of x I evaluate.

So, k is equal to 0 or you can take the index is 1 does not matter, k is equal to 0 you evaluate this function at this point, using this particular equation you can, since you know what the function is you can evaluated it at this point, k is equal to 0, and get what k plus 1 is, that is k plus 1 is of course, 1 in this case. So, from x 0, this is x of 0, you can get x of 1, so you can say evaluate this, so x k plus 1 is equal to x k plus h times x k t k of course, what is this h this is the value of the variable, the h is essential it time duration after which x k plus 1 is evaluated.

So, x 1 means, actually x of 1 implies x of h, x of 2 x 2 is nothing but, x of 2 h, so I am evaluating this x at discrete points, so this is x of 2, this is x 1, this x 0, so of course, in between I do not know the values, these values are not in fact, given by the numerical integration method, I just interpolated in between (Refer Slide Time: 22:05). So, what I am going to get is a discrete set of points, now by inter polluting between these points, if I can roughly get the actual actual continuous time response of this, I would say my objective as been fulfilled, and I have got the response of the system.

Remember of course, that the basic numerical algorithm will only yield, the values of the variable at discrete points of time. Now, one important point you should notice that, this is approximation Euler's method is an approximation, you will not of course, get the excite value of x by doing numerical integration in all cases, the only some systems which will give you correct answers, if you numerical integrating use using Euler's method. So, what I will do is let us take system, this let us take a linear system, the reason why I am taking the linear system is I know the response of the linear system, in terms of well known functions.

There is no need excepting in very complicated linear system, with lot of switching and then lot of you know intermediate disturbance, there is no need to numerically integrate a linear system, because, you can actually write down its time response, so but the reason why I am studying numerical integration using the linear system is that, I know the response of the linear system. So, I will able to tell how the numerical integration method behaves you know as compared to the correct response, because I know the correct response.

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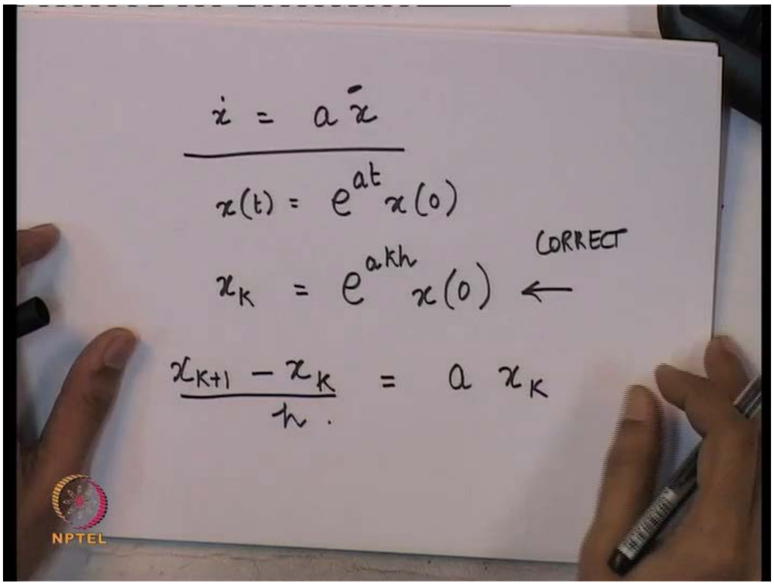

So, if you take for example, a system x dot is equal to a x, so this x dot is equal to a x dot is coming by mistake, so if I try to what is the solution of this, x of t is equal to e rise to a t x of 0 and of course, if I the correct response, if I sample it at discreet points you will get x of k is equals to e rise to a k h into x of 0. So, this is the correct correct sampled response, but if I use a numerical integration method like Euler method, this a linear system.

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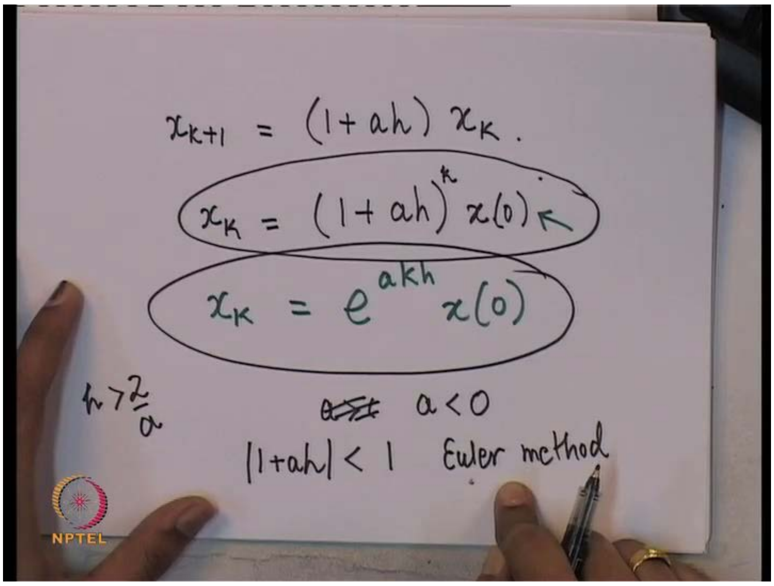

So, what I will get is (No audio from 24:53 to 25:07), this is what I will get, so x of k in this case will be 1 plus a h rise to k x of 0, that is x of 0 is this, so the correct response is (No audio from 25:28 to 25:38) and the numerically integrated response is this, that is if I use forward or forward Euler method or Euler method. So, these two of course, are not the same, the point is is this a good approximation of this, the answer of course, depends on the value of h, if h is very small, then one may except that you are going to come close to the solution, your accuracy will be if h is extremely small.

Now, will just take a simple example to understand this point, but before we do that let us look at a qualitative issue, when is this correct solutions stable, when it is the system stable, the system is stable if a is less than 0, if a is real number it should be less than 0 then the system is stable. However, here this particular system, when it is stable, you will note that, if the modules of 1 plus a h, 1 plus a h of course, is a constant if you choose your h, if it is fixed h, in that case this particular 1 plus a h is always a constant in your numerical integration rise to k.

So, the point is if mode of 1 plus a h is less than 1 then Euler method will say that the system is stable. Now, what is the important thing is that, the actual system is stable when a less than 0, but Euler method says that it is stable only if this true, these 2 conditions in fact, are not cinnamons there slightly different. In fact, if h greater than 2 by a then even, if is negative this may be greater than 1. So, the stability of Euler method will depend on its time step, it does not not only dependent on a, but it also dependents on the time step. So, it is obvious that Euler method can give qualitatively wrong picture of the stability of the system, if you do not choose a h appropriately. So, what we shall do is numerically integrate a system x dot is equal to a x using Euler method, so for this purpose, I will use (( )) as ours doing in the previous lectures.

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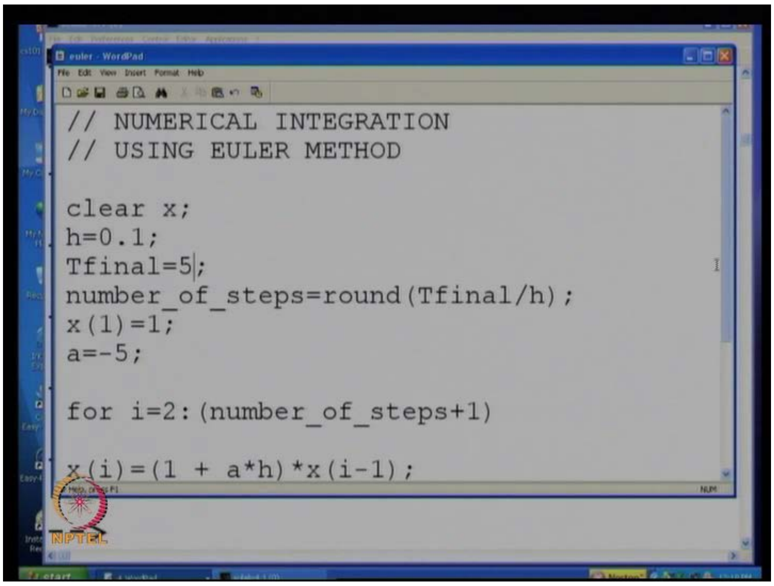

Now, this is the basic window of (( )) in which I will run a programmed, now instead of typing of all the commands 1 by 1 I have done it already in separate file which I will show now. So, we shall study numerical integration using Euler method, what will do is this is the variable x, we start with clean slate with clears variable, let say that a is equal to 5, a is equal to minus 5 is it is stable system and unstable system; it is stable system.

Because, the x dot is equal to a x if is negative, is a negative real number you will find at the system is actual stable; now let us try to integrate the system x dot is equal to a x using Euler method for that I should define, what my time step is let say right now, I choose the time step h is equal to point 1. Let us simulate the system for say I am sorry, let us simulate the system for 10 seconds, actually need not simulate for 10 seconds, let us do one thing, let us simulated for 5 seconds.

So, the number of steps to simulate from 0 to 5 would be given by this formula, so you have got T final is five h is point 1. So, the number of steps is you will get by rounding of T final by h.

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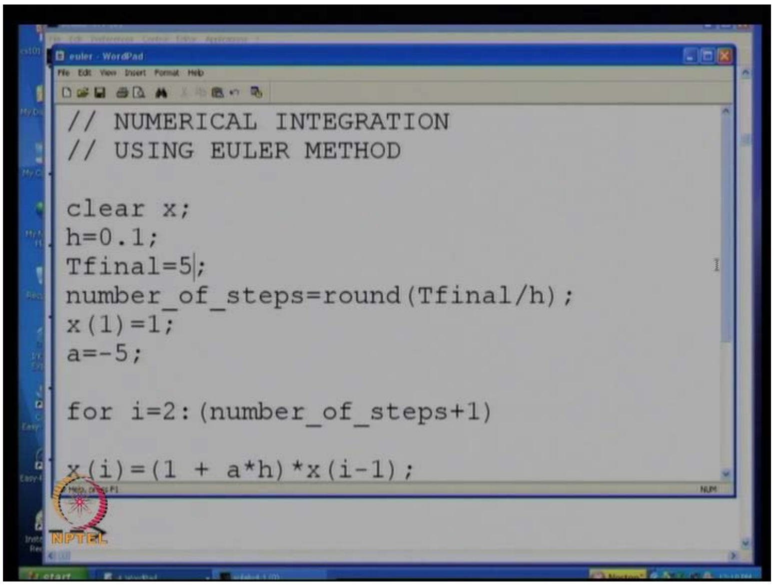

So, that is of course a integer, so that is why we have to round it off, suppose the initial condition remember, we are solving the dynamics of the system for a initial condition which is not the equilibrium, for x dot is equal to a x the equilibrium is x is equal to 0. So, we are giving an initial condition x 1 is equal to 1, incidentally I am not written x 0 here, because (()) does not rather, (()) does not normally permitted to give index, which is 0, so that is why its x 1, so we shifted all the indexes by 1.

As I mentioned sometime back, if it a is equal to minus 5, so what is the response of the system it is e rise to minus 5 t into x of 0. So, you know we except about in a second or so the system should settle down, it is a stable system; the time constant of the system is 1 upon 5, that is point 2 seconds. 4 times this time constant is roughly, the time it will take to settle down to steady value which is 0, that is a equilibrium value, this is the stable system.

So, if I want to numerically integrate, I will use this particular command, for i is equal to 2 to the number of steps, i is the index; instead of using the variable k and I am using the variable I here, where x of i is equal to 1 plus h into x of i minus 1. And then I plot time, these are the time steps, individual time steps I plot the time steps and the corresponding x values. So, of course, if I execute this (No audio from 32:01 to 32:10) this is what I get as a response.

So, if you look at this response, I slightly expanded, you see the response is fairly well captured, this remember the time step I have chosen is x is h is equal to 0.1, point yeah was it 0.1, yeah it is 0.1. So, for this particular system, h is equal to 0.1 seems to give you this response, the system settle down in around 1 second of course, this seems to be a continuous graph, that is because that because (()) is doing the inter pollution between the individual points, so that is the not very big surprise.

Suppose, I changed the time step from 0.1 to 0.2, so first what will do is redo this with 0.1 and then we do this with 0.2, we see that our response is slightly changed, you see here it is gone slightly different, is a different response here. On the other hand, if I changed into 0.5 things start looking very very different, the response is completely different, this is the response and of course, it is not correct response, it is in fact, going unstable the discreet time system is going unstable. So, what we see is that, if you have got Euler method and I choose a time step, which is not compatible.

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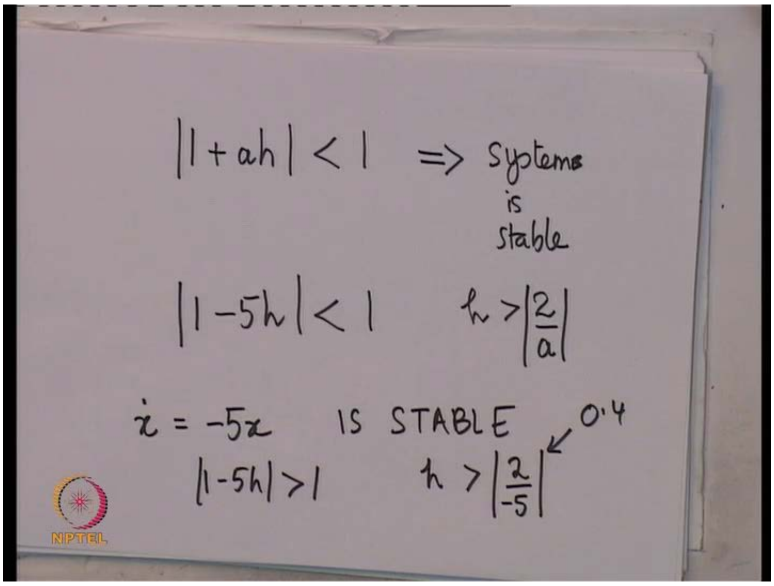

In fact, if you look at my condition which have written down, you can focus on the what I am writing 1 plus a h less than 1 implies system is stable as per Euler's method, the discretize system is stable (No audio from 34:39 to 34:53). So, in this particular case, 1 minus 5 h less than 1 implies the discreet time is system is stable remember, the original system x dot is equal to minus 5 x is stable. The discreet time system which is obtained by using Euler method gives you this condition; this is not the same as this condition. In fact, if I choose h which is greater than 2 times a in fact, I should I should write modules of 2 times of a, then I should basically get this condition will not be satisfied and will get an totally wrong response. So, actually this happens when h is greater than 2 by minus 5, if this is nothing but, 0.4 yeah, so if h is greater than 0.4 1 minus 5 h is greater than 1 and Euler method gives you completely wrong information about stability.

So, obviously, whenever you are using a numerical integration method, we need to be careful about this stability of the numerical method, whether it gives good qualitative understanding of how the system behave behaves. So, we shall now consider other methods obviously, Euler method under certain circumstance does not give you good response. In fact, we shall see later that in dealing with stiff systems, Euler method is particularly unsuitable; especially one is interested only in the slow response.

So, this is something which will of course, understand in a few moments from now, perhaps in the next lecture. Now, this takes us to other methods of numerical integration, whenever we talk of numerical integration, there is several terms which you will come across, one is what is known as the order of the method.

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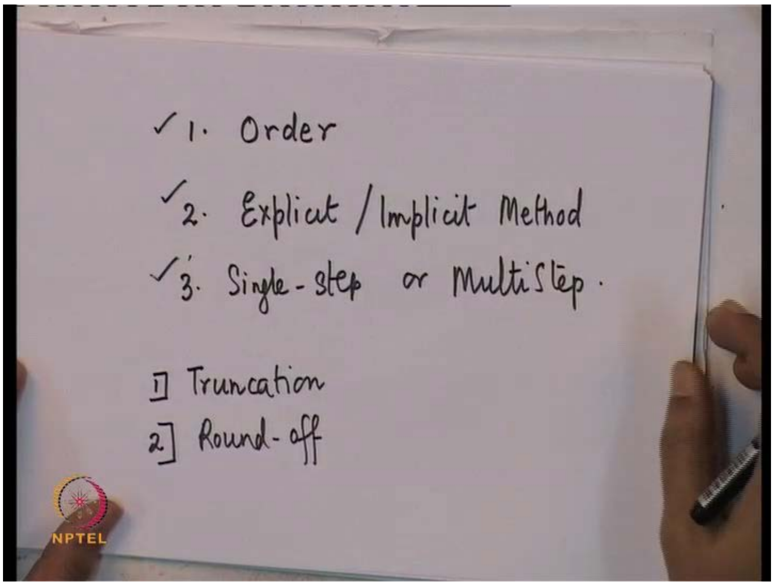

So, one of the thing which you will come across is the order of method, second whether it is what is known as explicit or in implicit method; the third thing which you will come across is whether it is a single step or a multi step method.

Now, there are two issues, which we or two errors which result, whenever we use numerical integration method, one of the error is because of fact because, of the fact that the numerical integration is in fact, the discretization is almost always and approximation, like Euler method is an approximation of the original continuous time system. So, that error which is introduce, because of the approximation is called truncation error, the second thing which causes an error is because, we will be of course, doing this numerical integration on a computer, and a computer will introduce another error which is called a round of error.

Because, any number can be limit represented only to a finite precession in a computers, but of course, round of errors in normally you will not have problem of round of error, because these days you can specify a very high precession in computers. So, you have to normally bother about the approximation which is made, that is are you going to use the Euler method or some other method. So, let me first introduce you to some other methods, and then we will try to understand what order is, what is explicit or implicit method, and what is the single step or multi step method.

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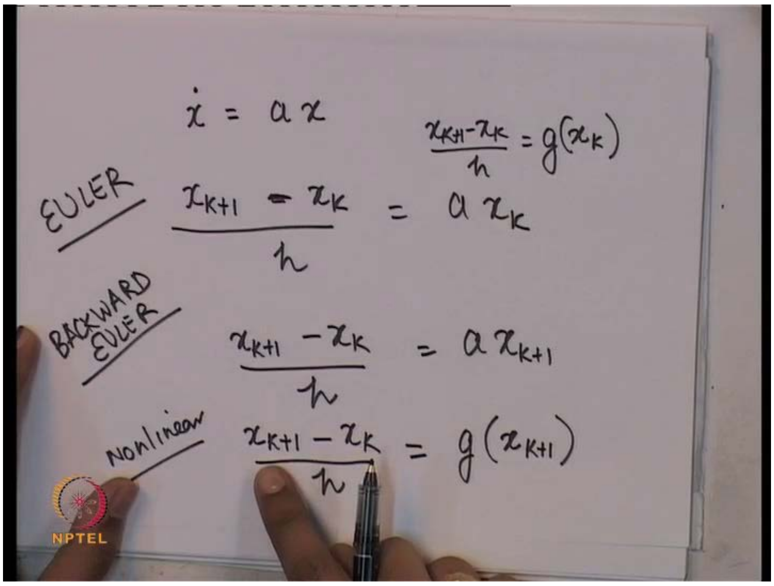

First thing, suppose you have got system x dot is equal to a x x k plus 1 is equal to or rather x minus x k upon h is equal to a x k this is Euler of forward Euler, backward Euler on the other hand is x k plus 1 minus x k upon h is equal to a x k plus 1, so this is an approximates. So, you are using the value of the function a x at the point x k plus 1, now this is you may see you are using x k and you are using x k plus 1. What is the significance of this, in fact if you got a non-linear system backward Euler would look like this, this is function g which you are discuss some time earlier, so for a non-linear system backward Euler will look like this. So, backward Euler requires, x k plus 1 to evaluate g x k plus 1 and then you have to solve this, now do you notice what the problem is this, is as compare to this see in this case, if even if it was a non-linear system you would have got x k plus 1 minus x k by h is equal to g of x k.

So, typical you will know what is x k is, so you can evaluate g of x k, you know what is x k is 2, then you can get x k plus 1, in contrast look at this problem have you got x k plus 1, you know what x k is, but you do not know what x k plus 1, x k plus 1 is obtained from this. In fact, what you get is an algebra you know x k, so you have got algebraic equation in x k plus 1, you need to solve this implicit equation in order to get x k plus 1.

In a linear system this would imply that of course, you will have to do a division, but here you will find it you will have to, in fact use a numerical method if g is non-linear function, you will have to use numerical method for solving algebraic equation in order to get the solution of this. So, backward Euler involves a bit of complicity, forward Euler or Euler method is easy to evaluate, this x k plus 1 is a obtain explicitly from x k, x k plus 1 here is obtained implicitly from x k.

So, in fact, without going to the any formal definition I hope your getting what, we mean by explicit and implicit method, if method requires you to know x k plus 1, in order to compute this function g, in that case you would call this method and implicit method.

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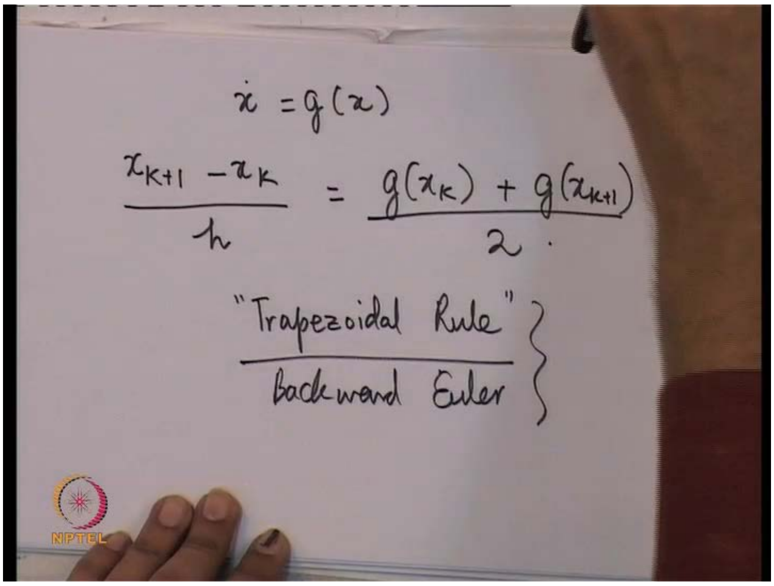

In fact, would you call this an explicit method or in implicit method, suppose you got a system x dot is equal to g of x this is what we are considering. Is this explicit or is this implicit, another approximation, this a another way of doing a numerical integration, in this call trapezoidal rule, trapezoidal rule is actually in implicit method, because to get x k plus 1 from x k will require to solve this non-linear algebraic equation, in case its non-linear system.

And that itself is an implicit algebraic equation is an implicit algebraic equation which will probably require numerical method, so in fact, in every time step if it is a non-linear system, you may have to iterate in order to get x k plus 1 remember, however that the numerical method in order to get x k plus 1 from x k is a method for solving numerical algebraic equation. Rather non-linear algebraic equations, like (()) or (()) method, so trapezoidal rule backward Euler are known as implicit methods.

Now, why should we use trapezoidal rule or backward Euler, if its slightly more difficult to solve as compare to Euler method, one of the problem this is the properties of Euler method, Euler method unfortunately for systems stiff systems is not a very good idea, will do an example in the next class, a few examples in the next class show this. Whereas, backward Euler and trapezoidal rule in fact, a quite suited for numerical integration of stiff systems. Now, whenever I say suitable what do I mean a numerical integration should be easily fast, it should not require you to use very trans, small time step other wise to sibilate or numerically integrate over interval which is large, it will take you very long long time.

So, numerical method to the extent possible should be you know able to use time steps, which are not too small compare to the interval which one wants to simulate, now in a stiff system as you may imagine since, you got fast and slow transients, there will be an issue about choosing your time steps is of course, something will discuss later in the may be in the next lecture or so.

So, the second thing, the thing which we discussed of course, was implicit or explicit methods; now let us talk about order of a method, whenever we talk about an order of method, what we really mean is suppose the response of x of t, suppose the actual correct response of x of t can be written down as a polynomial.

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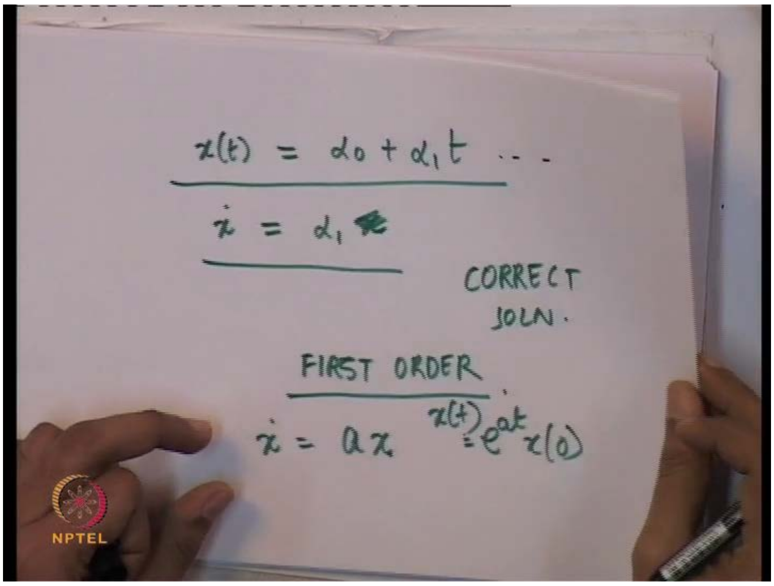

For example, if x of t is can be written suppose is alpha 0 plus alpha 1 of t, suppose this is the time response in fact, all the time response we have been talking of involved exponential function, but suppose this is the time response, then x dot of t is nothing but, alpha 1 of x sorry alpha 1 that is all. So, x of x dot of t is nothing but, alpha 1, so what this means is of course, this particular system, if I numerically integrate with back with Euler method or backward Euler method, I will get the correct solution, without any truncation error.

So, backward Euler and forward Euler will give you the exact solution, now here truncation error, in case a numerical integrates a system of this kind, specifically of this kind. So, in fact, backward Euler and forward Euler or simple Euler method, they have what are known as first order method methods of course, if you got system x dot is equal to a of x is not alpha alpha 1 here is a constant, if x dot is equal to a into x, in that case your response is remember e rise to a t x of 0, this is the x of t.

In such a situation of course, you know the expansion of e rise to a t is an infinite series in t, so obviously, the response is going to have many many more terms. So, whenever you use Euler or backward Euler to simulate this system you are bound to get some truncation error. So, remember first order method means, if your response is can be written down simple as a first order polynomial of rather the first order polynomial in t, in that case backward Euler and forward Euler which are first order methods will give you correct response.

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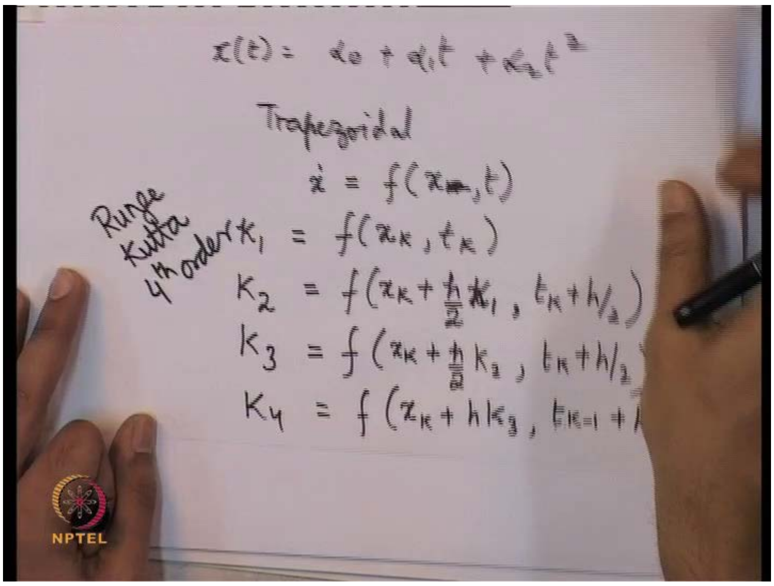

You can show that if your system response can be written as, then method like trapezoidal rule will give you the correct, no truncation error, so this kind of system is known as a second order system. So, trapezoidal rule will not give any truncation error for a system, which has got response of this kind, again trapezoidal rule will always give some truncation error for a system which has got this response, because this is the infinite sires in t. So, it is not just second order polynomial in t.

So, the trapezoidal rule is this something you can show, I will not you known prove it here you can just try to do at home, trapezoidal is in fact, a second order method there are other methods for example, Runge Kutta method. So, if you got x dot is equal to f of x f of x t sometime, t may explicitly in this function.

Then Runge Kutta method uses first it evaluate function k 1 which is nothing but, then it evaluates k 2 which is nothing but, (No audio from 49:12 to 49:31) sorry k 1 then it is evaluate function k 3. Rather it is evaluates k 3 where k 3 is nothing but, and then k 4 nothing but h k 3 (No audio from 50:07 to 50:25), so its evaluate its k 1, k 2, k 3, k 4 is Runge Kutta 4th order method 4th order.

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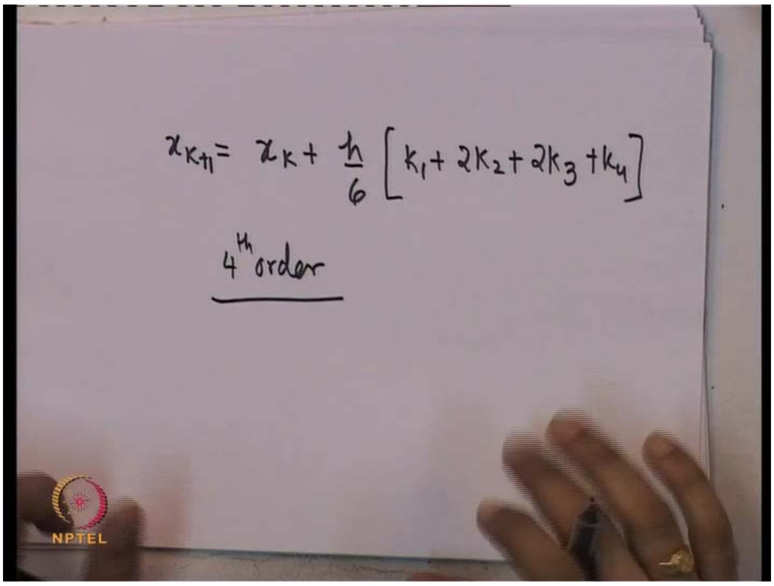

So, Runge Kutta 4th order method eventually calculates x k as rather x k plus 1 as x k plus h by 6 into k 1 plus 2 k 2 plus 2 k 3 plus k 4. So, this is basically what Runge Kutta method does, it is basically evaluating k 1, k 2, k 3, k 4, which I in fact, intermediate points, the intermediate values of this function between the interval t k and t k plus 1 and then using this particular formula.

So, this is Runge Kutta 4th order method, first question is it an explicit or implicit method, the answer it is an explicit method, because it requires the evaluation of x k plus one, but it does not require x k plus 1 itself. So, it you can evaluate all this function explicitly, you do not require x k plus 1 in these evaluation, so this is an explicit method. And something which of course I will not prove here, but this is what is 4th order method, so if your response can be written down as a 4th roughly as a 4thorder polynomial in t, then you will have no truncation error. So, this is in fact, a Runge Kutta method, so we have of course, discuss now order of the method, whether it is an explicit or an implicit method, we have really discuss this without going into any great amount of (()); I seen in the I can refer you to a few books at the end of the lecture, you can go through them for more detail analysis of this.

There is a another point which we missed out, that is multi step and single step methods, now I will not write down the various multi step and single step method, but all the three methods or four methods which I have told you Euler, trapezoidal, Runge Kutta and backward Euler all requires simply x k to get x k plus; you do not require anything more. You could in principal use for example, x k minus 1 also in your calculation to get k plus 1. So, when you are trying to get k plus 1 your discretization uses values, which is two times step before or three or you know even a time step ahead in time.

In that case of course, it will like t k plus 2 or t k plus 3, in that case it become in implicit method, so such methods which use not only the previous time step, but time steps other than the previous time step, values of x at time steps, other than the previous time step. They In fact, are known as multi step methods, so we have gone to order of the method, whether it is explicit or implicit and whether its single step or multi step method.

In most of what we are going to do in this course, we shall restrict yourself to single step methods, explicit methods of course, are easy to evaluate, because they do not require to solvent algebraic equation, especially this is the problem, when you are talking of non-linear systems. Because, the algebraic equation is non-linear and you will have to use gross side or (()) at every time step to evaluate iteratively, what x k plus 1 is given x k.

So, that creates a bit of you know kind of problem, when one tries to numerically integrate a non-linear system using an implicit method. So, you should have really some nice tangible benefits, when you are trying to use an implicit method, these in fact, implicit methods are known to be straightly better, when it like backward Euler or trapezoidal rule are usually better suited, when you have got stiff systems and you want to study the slow response.

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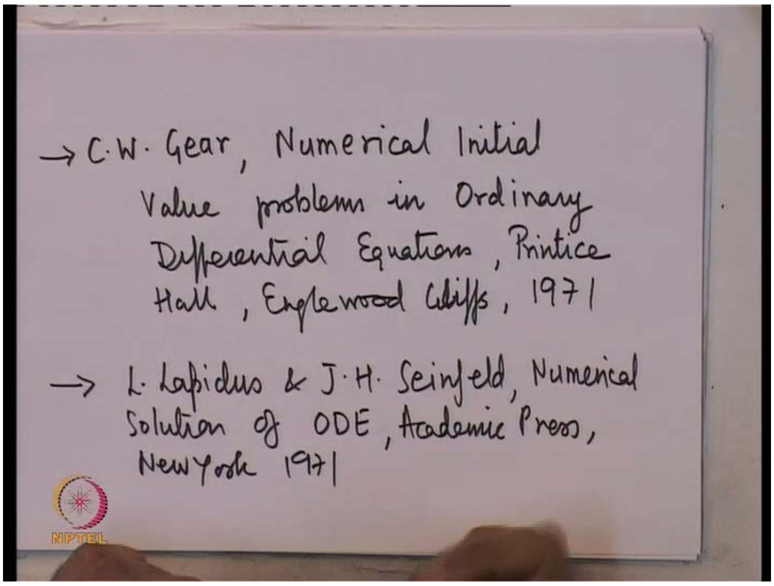

So, this something of course, we shall do in the next lecture, before we stop this lecture, let we just give you a few references for example, for numerical integration method, there lots of books, but some of the classic books are one by Gear, ordinary differential equations by Printice Hall, I am sure it will there in a well stock library, Englewood Cliffs 1971.

And there is another one that is L. Lapidus and J. H. Seinfeld, Seinfeild numerical solution of ODE, ODE is of course, is a ordinary differential equations, this is academic press, New York 1971. These are what I would say is the classical books in a in a, this field numerical integration methods, in some cases they may be fairly regress more regress then we will ever be in this particular course.

Therefore, you can also look at other books which are somewhat simpler, which may be available, I am sure there many other books which are available in our library, but these are the classical books in numerical integration methods. In the next lecture, we shall understand the numerical integration of stiff systems in more detail. And also understand some of the stability properties are the you know the properties general properties of the numerical methods in particular Euler method, backward Euler method and trapezoidal rule.