Power Systems Dynamics and Control

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Lecture No. #07
Stiff systems, Multi Time Scaling Modeling

Based on the understanding of how linear systems behave during transient conditions, we were able to understand the simple linear circuit consisting of capacitors and inductors in the previous lecture. In fact, there was one capacitor and two inductors in that circuit. We could take out a time response. And very importantly, we could take out a few characteristics of the system. In fact, the characteristics of the system depend on the model parameters, in addition to the circuit itself.

Now, one of the most important things which came out of the study was that, there is a combination of fast and slow transients. Now, this is a very fundamental, you know kind of modeling issues which usually encounter in our studies of power systems. So, I would recommend that a great deal of attention to be paid on these; the modeling principles as well as systems in which there are mixture of fast and slow transients. A mixture of fast and slow transients also brings into play several effects, when we try to numerically simulate these systems. That is, when we are trying to understand the system by numerically integrating them. A stiff system or a stiff system in which there are fast and slow transients coming into the picture require some specific tools, numerical tools to solve.

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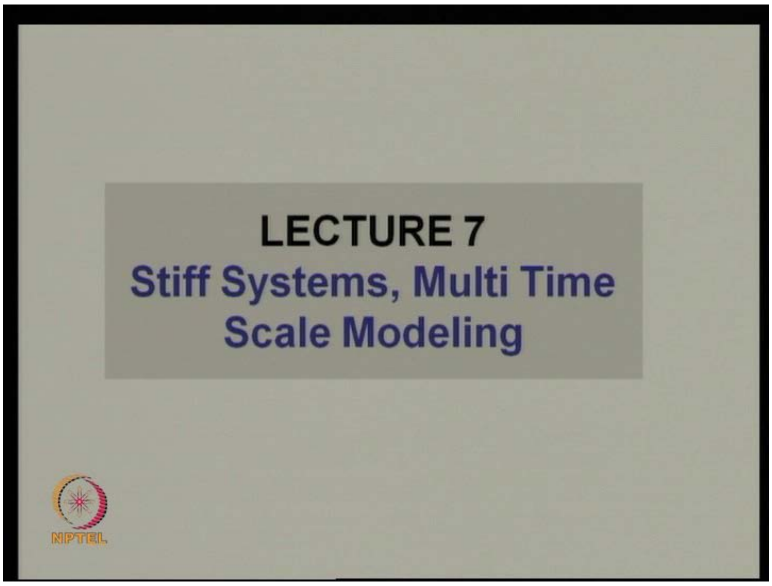

We shall see that, we shall be able to make some modeling simplifications based on the fact that there is a clear time scale difference between the nature of transients, which are seen in certain circuits. Now, so this lecture, we will try to understand stiff systems and multi time scale modeling. We will do that of course by considering again the same example, which we discussed in the previous class. If we find time, by the end of the lecture we will also move on to numerical integration methods. Just a brief overview.

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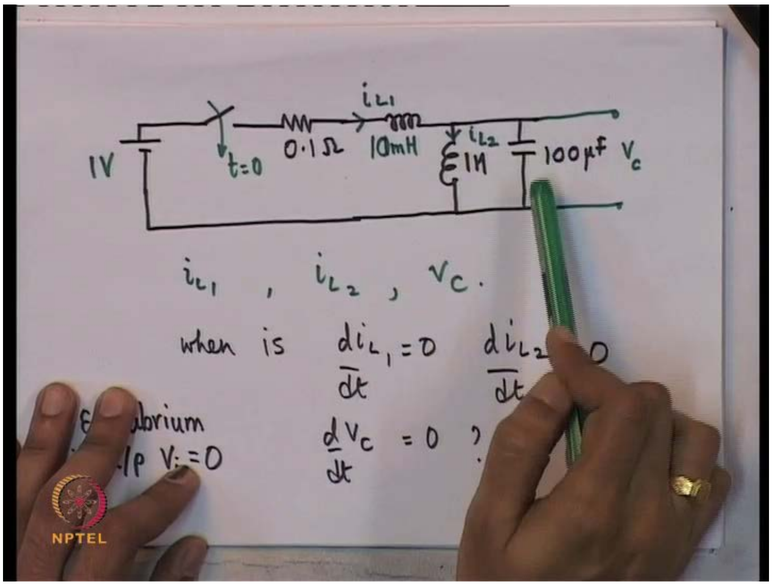

Now, if you recall what we did in the previous class, we consider this particular circuit. It was consisting of a voltage source, which is connected to a circuit consisting of two inductors, a capacitor. The values are of importance. Here, you had a resistance of 0.1 ohm 10 milli Henry inductor, 1 Henry, this is a fat inductor and a 100 microfarad capacitor. And, we tried to analyze the transients associated with this.

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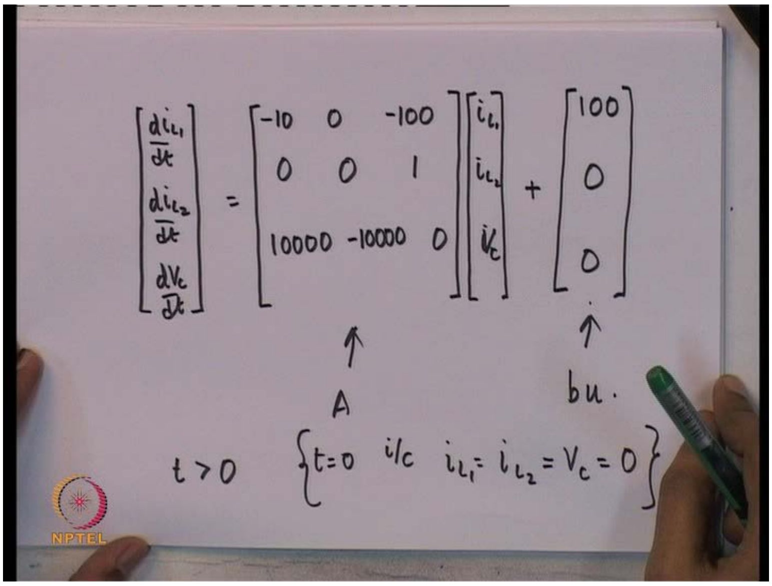

Now, if you look at it from mathematical point of view, it boils down to trying to solve this particular circuit, this particular system. It is a… we write down the state space equations for this system. The states being the current through the inductor 10 milli Henry, the current through the 1 Henry inductor and the voltage across the capacitor. This of course, is the system we derived last time and we try to find out what was the response if the initial conditions are these. I mean, this is the initial conditions before the circuit is energized by an input.

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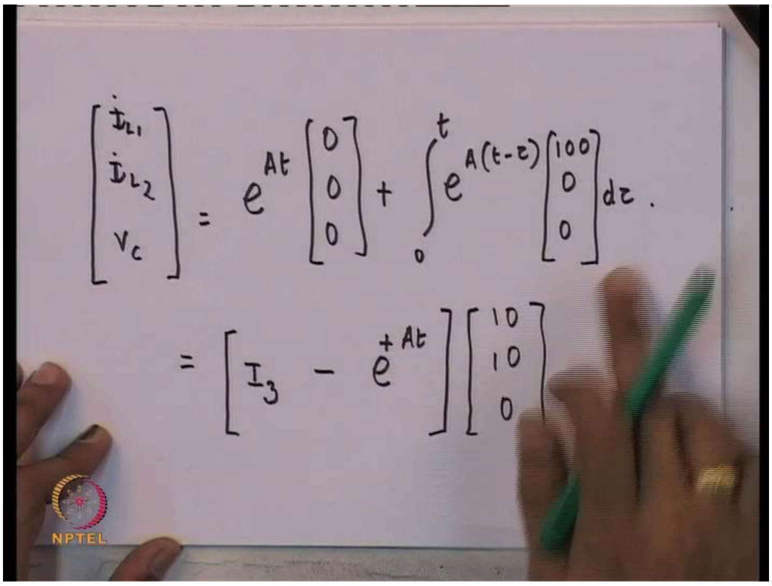

Now what we saw was, the response of course, is given by this formula e raised to the power of A t into the initial conditions plus this convolution integral. Eventually, the response of I l, I 2 and V c turns out to be this, which was evaluated; effectively required us to evaluate this, where P is the Eigen vector matrix and it required us to get lambda 1, lambda 2 and lambda 3 and the Eigen vector matrix.

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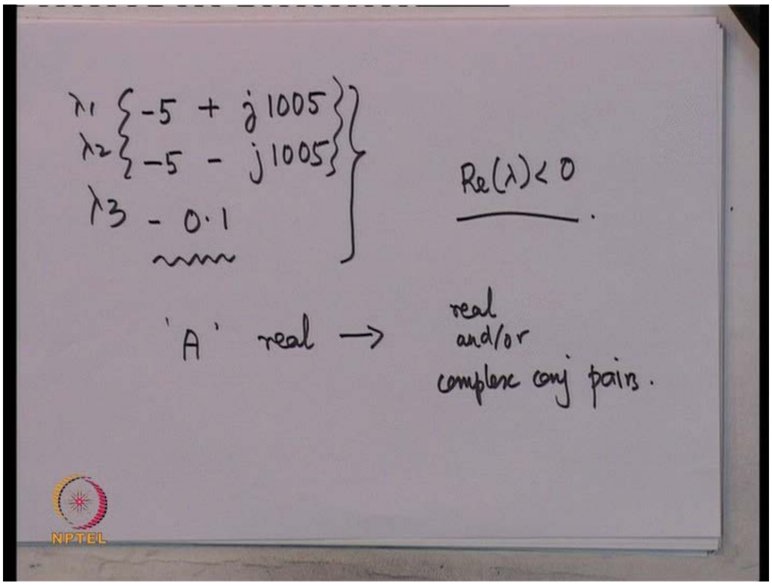

Now, this was got using Sci lab, you can also use mat lab for getting the Eigen vector matrix A. Now, if you recall what A is, A is this. Now the most notable feature here, when we did this study was lambda 1 was approximately equal to this, lambda 2 was this, there was a conjugate, complex conjugate pair and you had a real Eigen value which also had a negative part. So, it was a stable, this is a stable system. Now, these were the Eigen values and the most important feature which struck us then was that the magnitude of this Eigen value. These this pair of Eigen values and this one is vastly different. This is a very small Eigen value. This is a very large magnitude Eigen value.

So, one thing which we had mentioned sometime, a large magnitude Eigen value is associated with very fast rates of change. So, that is one important thing. So, we have got, some part of the system is going to some part of the response is going to move very fast and some of it very slowly. Another thing which we saw when we looked again like the, at the Eigen vectors, the Eigen vectors were approximated by this.

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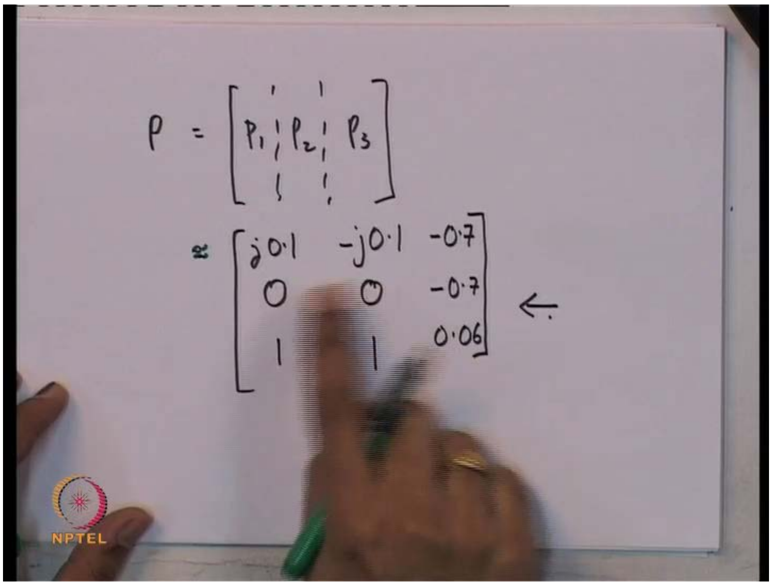

This is of course an approximation. We did some rounding off. These were very small values. So, we have done this rounded off, rounded it off to 0. This was practically point, this is practically 0.1. This is minus j 0.1 and so on.

So, the approximate Eigen vectors were these. And, the most important thing we saw was in the second state in the Eigen vector corresponding to the second state and the first and second Eigen value, these are corresponding to the first and the second Eigen values. These are in fact the complex pair. The second state has no practically zero obser vability. These were of course rounded off, but we saw that these were almost zero. So, you will not observe terms corresponding to e raise to lambda 1 and lambda 2 t in the response. In fact, lambda 1 and lambda 2 t were complex numbers. So, the first and second Eigen values in fact pointed towards oscillated response.

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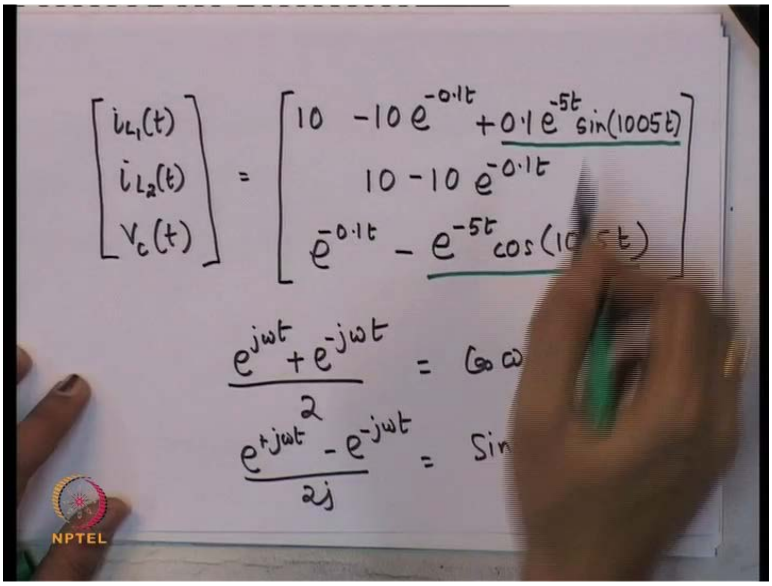

And the final response, of course was evaluated to be this. So, what do you notice here? Of course, here you have got this particular oscillatory response. This comes out because of the complex pair of Eigen values, conjugate pair of Eigen values. You also have this particular mode. This is practically not observable in the variable i L 2. So, this is where we stopped last time.

Now, one of the important things which we can see here is, there is of course, this is a very… this particular transient is associated with high rate of change. You look at the real part as well as the imaginary part; rather the frequency as well as the rate of decay is quite high. So, if you look at, just we will redraw this circuit.

So, what we see is i L 1 is this and i L 2 is this. Now in i L 2, we are hardly seeing any oscillatory component. That is because of course, I mean intuitively, this being a very large inductor, it does not happy with large rates of change. So, you will find that. You know, this large inductor prevents a large fast rates of change of the current. So, that is why in i L 2 1 is not surprised to see that the faster transient is not visible. So, in fact if you look at the response of i L 2, you will find that it starts from zero. Of course, if we and it grows to 10 amperes. So, it starts with 0 and grows to 10 amperes. You can see this; as t tends to infinity the value of i L 2 becomes 10.

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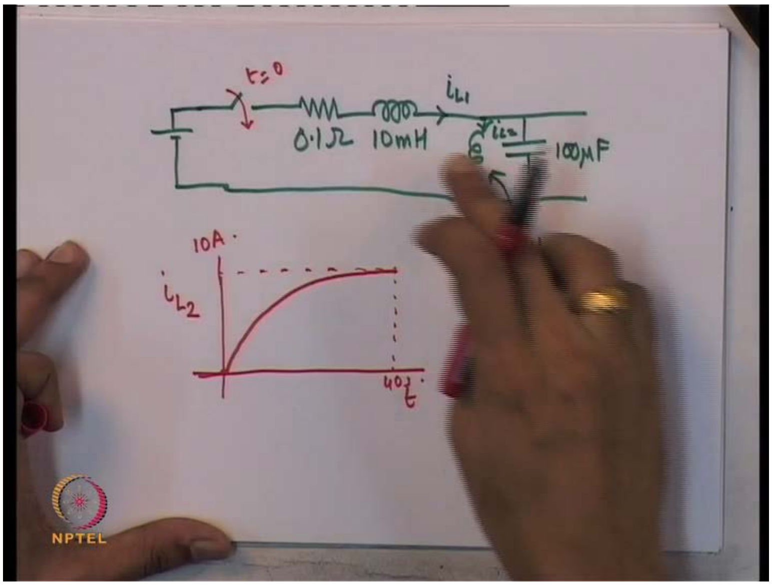

This is roughly four times the time constant associated with this Eigen value. So, Eigen value is approximately 0.1, the magnitude of the Eigen value. So, the time constant associated with this is roughly 10 seconds. So, around 40 seconds you will see that this builds up to near about to the final value. So, this will be around 40 seconds. You hardly see any oscillatory response in this. Now, what about i L 1? If you look at i L 1, the response is primarily consisting of this component. And, a small component which is oscillatory, a decaying oscillatory form. So, what you have got is a decaying oscillation and then again, this settles down to 10 amperes in 40, around 40 seconds.

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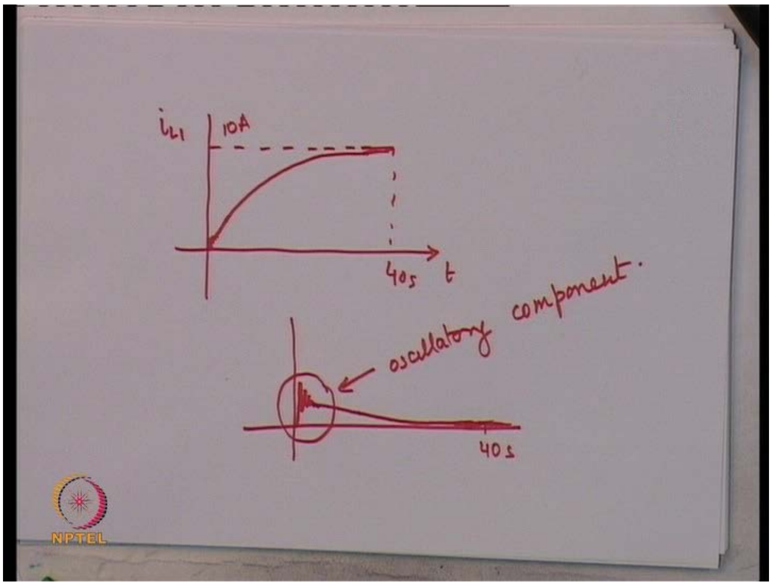

So, this is the response of i L 2. And, of course V c; V c is interesting. We look at how V c looks like. It consists of this as well as this. So, what it has here is exponentially decaying component as well as a decaying oscillatory component of high frequency. So, what you are likely to find is something of this kind; roughly around 40 seconds, it will become a very small value. And in the beginning, there is an oscillatory component. Now when you are studying the circuit, a thing which will get suggested automatically is, can we simplify the analysis of this circuit. For example, if you look at the state space equation of the system, it appears that the response for i L 2, the slowly varying variable will be preserved even if I make certain approximations in this. Just look at what I am trying to say. What I am saying is, as far as the response of i L 2 goes is the 1 Henry inductor goes, I can obtain it simply by trying to studying this circuit. In fact, what is the response of this circuit? It is exactly what we got for i L 2. The time constant is, of course 10 seconds; the lambda for this circuit is 0.1, minus 0.1.

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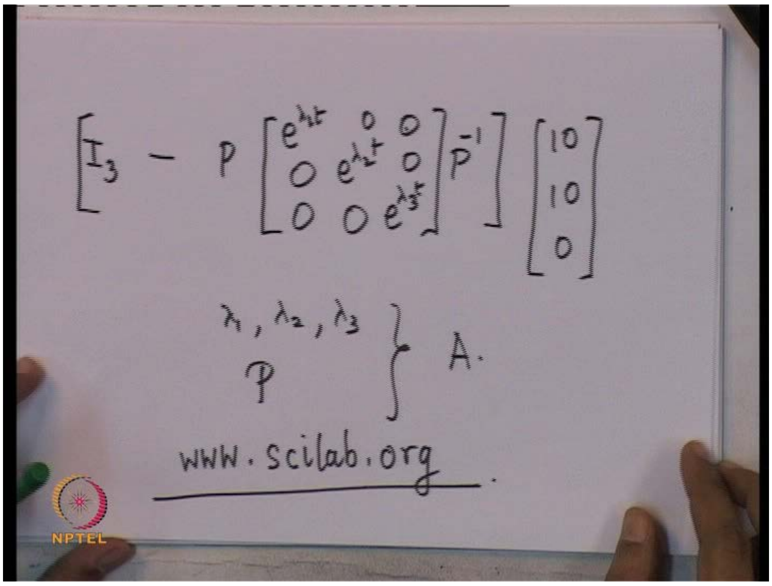

So, what we… you know a very simple, a simplification of the circuit does not seem to affect the response of i L 2. So, it appears that we can actually make a simplification.

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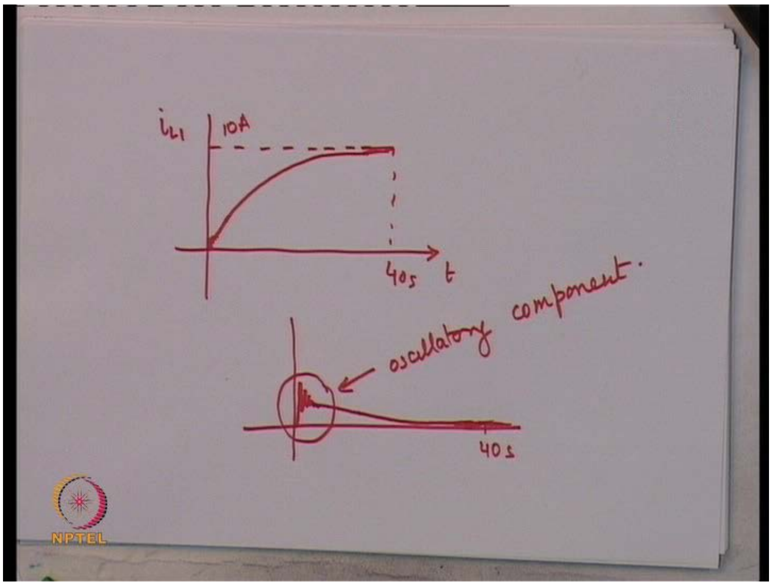

In fact looking at it a bit formally, if I set d i L 1 in this, I artificially set it to be 0. I mean, set this value here to be 0. The rest of the things will remain exactly the same. So, what I have done is just set these two things to 0. In that case, what we have here is only one differential equation corresponding to i L 2 and i L 1 and V c can in fact be written in terms of i L 2. So, what we have done is, we have got now d i L 2 by d t is equal to V

c. This is a differential equation. And, i L 1 is nothing but i L 2. From this last equation, since this has been set to 0 you have 10000 i L 1 is equal to minus 10 plus, sorry, 10000 i L 1 minus 10 thousand i L 2 is equal to 0; which actually gives you i L 1 is equal to i L 2.

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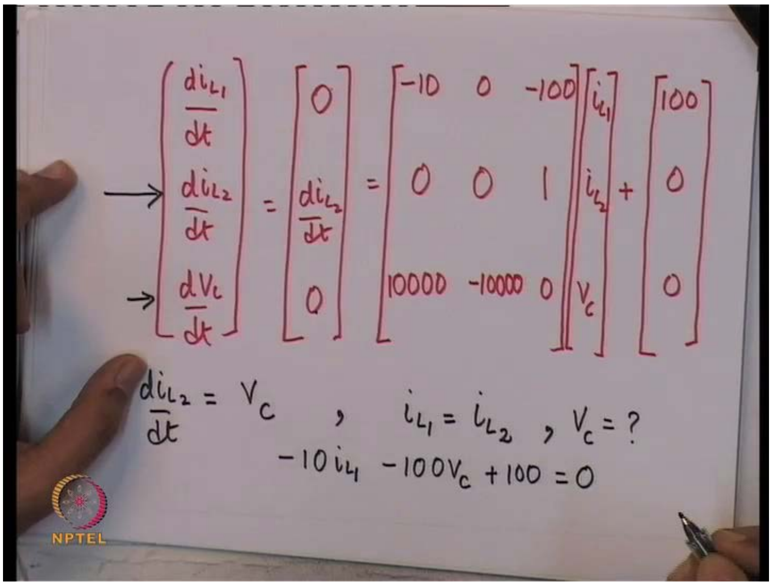

And from this, we will get V c is equal to what? What we have here is minus 10 times i L 1 minus 100 V c plus 100 is equal to 0. So, what we have from that is V c is equal to 100; which is, nothing but V c is equal 1 minus 0.1 i L 2 because i L 1 is equal to i L 2. So, this is what V c is, so it is algebraically, so i L 1 and V c are algebraically related to i L 2. So, what we have is basically a state space equation d i L 2 by d t is equal to V c, which is nothing but 1 minus 0.1 i L 2.

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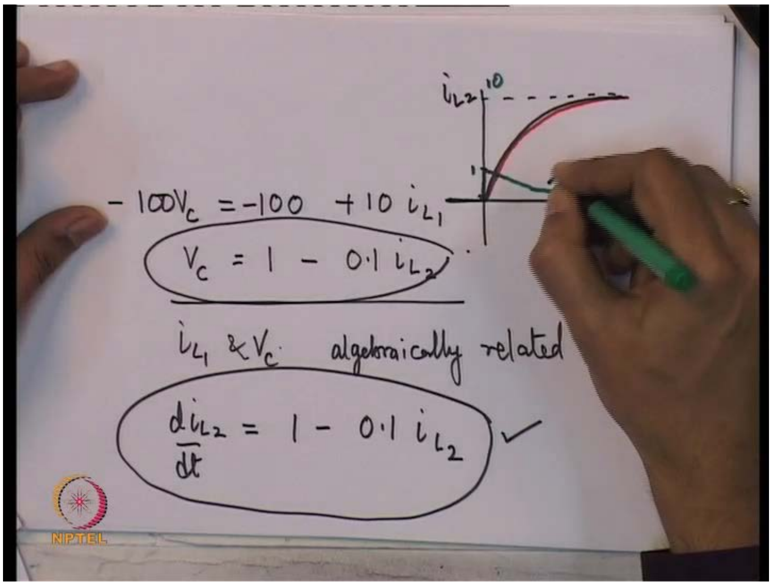

So, this is my one and only differential equation, if I make this approximation. So, if I make this approximation you have reduced your system size to this. And, if you want to get V c and i L 1, you have to use this algebraic equation and this algebraic equation.

So, what we have? If you look at this particular system, if you look at the response of this particular system we have this. It is the same or practically the same responses before what about i L 1. i L 1 also follows the same response because i L 1 is equal to i L 2 and Vc is 1 minus 0.1i L 2. So, if you look at the response of V c, it simply this is one. So, you will have this 10. This is not to scale, like this. So, this is Vc. Now if you look at, we had drawn these responses before. If you consider the whole system in its full glory, you have got this. If you make the approximation which I made, you will get this. So, the basic issue which I would like to point out here is that if you have got a system, please note carefully, if you have got a system in which there are fast and slow transients and your interest is in the slow transients, then you can get the slow transient behavior. Roughly correctly; you know, you will get it almost correct. If you assume the rate of change of the variables, which you think are associated with the fast transients is set to zero.

So, what I have done is that, I have artificially set the derivatives in this equation to 0 and made i L 1 and i V c algebraically related to i L 2. So, you can make a general statement of this kind. So, if you have got some fast variables, then as far as the behavior, the slow behavior of the system can be obtained simply by replacing this algebraic equation. Rather this differential equation by an algebraic equation; that is, I am making x 1 and x 2, x 2 algebraically related to x 1.

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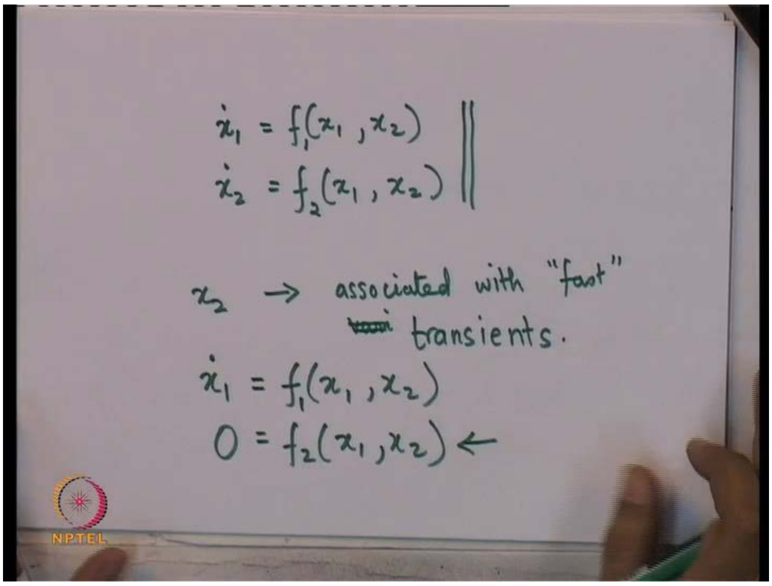

So, in such a case, I will be able to capture the slow transients without making too many errors. So, the basic point is, first thing you need is this is a clear separation of fast and slow transients. There is a mixture of fast and slow transients in the response as is seen in our system here. So, this is a very very important, you know modeling principle or modeling simplification which we are making. Now, this we, in fact this kind of modeling simplification, we seem to making all the time without knowing it. For example, you just take a simple case of a bulb connected to the plug and you are, this is a bulb. And, the basic point is I want to study how this bulb lights up.

Now when you switch this on, the bulb practically lights up instantaneously. It is only a resistive kind of load, but of course a purist may say, well, there are parasitic inductances and capacitance is associated with this, the wire which connects the bulb to the plug point. So, what you somebody may say is, well, if you really want to study how this bulb is going to light up, you ought to model all the inductances, the distributed inductances and capacitance as in a transmission line.

Now the point is usually, you do not really need to do that because the transients associated with this die down very soon. So, that is why we can practically treat this wire as if it is directly a simple resistance less connection to this point resistance and 0 dynamics associated with this. Suppose, of course if you are interested, in fact if one really are looking at a time scales of a few nano seconds or a micro seconds, you may actually see some transients evolving even in this simple circuit. But, usually we may not be interested in those fast transients. Ok.

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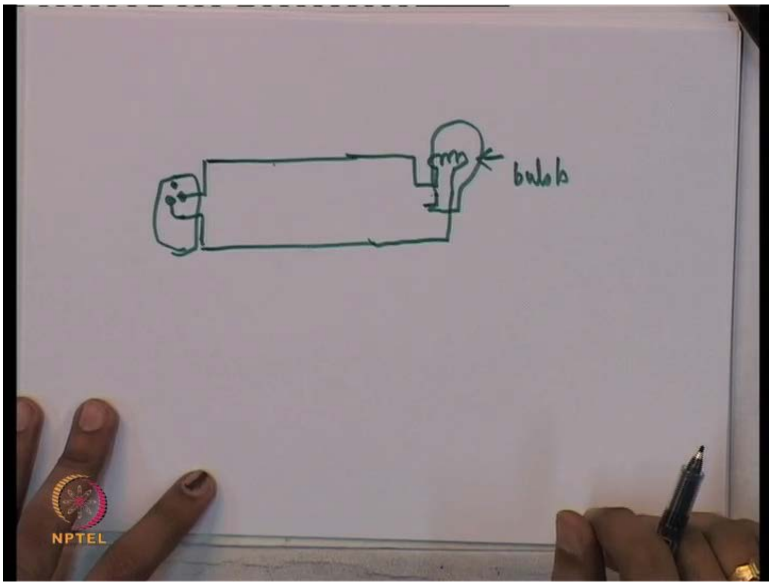

So, in such a case, remember you can make an approximation. You can actually neglect the dynamics associated with that fast transients by simply taking the state equations and setting the right hand side… writing that d by d t is equal to 0 in those states, which we think are associated with the fast transients. Of course, now this seems to be a kind of a chicken and egg story because to know which elements or which states in a circuit are associated with slow or fast transients itself, requires you to do an analysis of the system. So, you have to do full blown analysis and then find out which states are associated with the fast transients and then you can make the modeling simplification.

But, actually engineers do not do that. In most situations, it is fairly obvious to an engineer, what are the things which decay fast and which other thing which are the transients, which really move very fast or which are the transients which move very slowly and which are the states associated with them, even at in a rough way. So, even through engineering, one should be able to find this out. For example, in this particular circuit it is quite clear that the the large inductance, the magnetizing inductance in that particular circuit. 1 Henry is much larger compared to the, you know the leakage; that is the 0.1 Henry inductance and 1100 micro capacitance lightly, you know it looks a large value.

So, I could have guessed that I could have probably made these assumptions. It was an engineer, I would not have waited. I would have made this assumption and in so far as a slow transients are concerned, I would have made very little error, even if I had just taken this circuit. So, if I wanted to represent, only represents my slow transient correctly, it was adequate to have a circuit of this kind. Now, what we really have done is there was an inductor here, but it was a very small value. So, we assumed that it goes into steady state right away. It kind of is algebraically related to the current here. And, we also have a capacitor here, which you have basically said it is open circuited.

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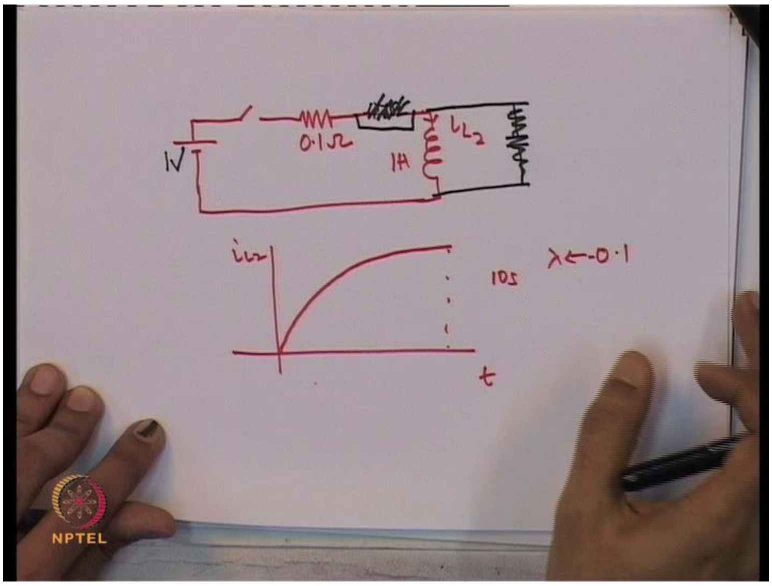

So, we have basically got this particular circuit which is marked in red by making the d by d t s associated with these states equal to 0. But remember, if you do not want, if you want to make this approximation without actually doing the analysis of the circuit, then you have to rely on engineering intuition. Now, you can also make a converse kind of analysis. Suppose, you have got a system which has got a mixture of slow and fast transients and you are interested in, what happens just after the transient has occurred. You do not want to; we are not really interested in the fast rates, the slow variations or even the final steady state. But, you are interested in… what happens? Just a few, may be just for a short while after a transient has occurred.

For example, in this circuit, remember that your V c varies like this. This is around 1 and this is around 40 seconds. Now, if my aim is to study this fast transient here, so, of course there could be situation you are interested in the fast transients. So, if you are interested in the fast transients, can you guess what has to be done? Well, one thing is, you need to know the states which are associated with the fast transient. Again, this is an intuitive thing. How you know which states are associated with the fast transients? You need to really look at something more than what we have studied till now. I will just mention that in a few more minutes from now.

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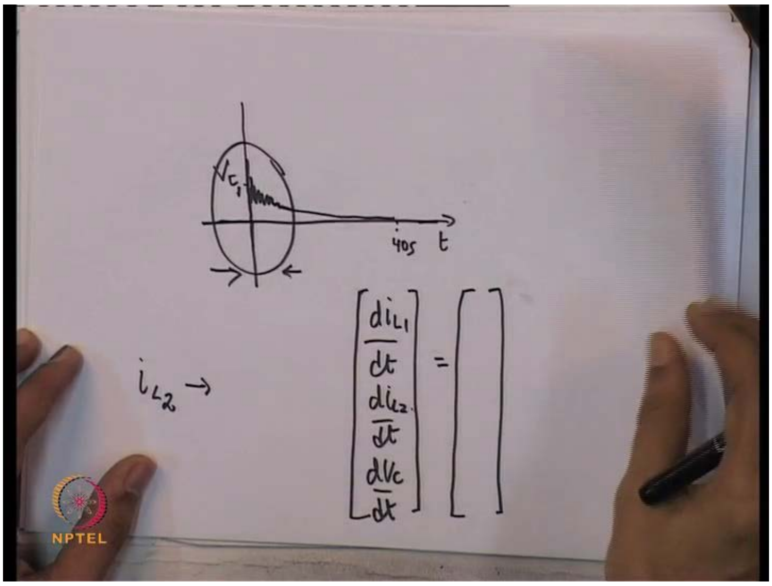

But, right, now let us try to do it intuitively. So, if I want to study the fast transients, probably if I assume, we will just first write it down. These are the… if we assume that this particular state remains where it is during this extremely short time period, you know that this is a very fast transient. So, it will decay very fast if it is stable. So, rather I should say that I want to see the transient in a very small window. In that window, i L 2 does not change at all. So, what I will do is, I will rewrite my circuit in this fashion. I will just rewrite it on another page; d i L 1 by d t d i L 2 by d t and d V c by d t is equal to… what I will do is, this is the original state space equation. What I will do is, I will set this equal to 0. This is also as zero.

So, what it means is that, i L 2 d i L 2 is equal to 0. The rest of the thing is, of course remain the same. So, what we have is i L 2. See, if I set d i L 2 is equal to 0, it implies that i L 2 is equal to as a function of time is nothing but i L 2 0. So, what I am going to do is make this approximation.

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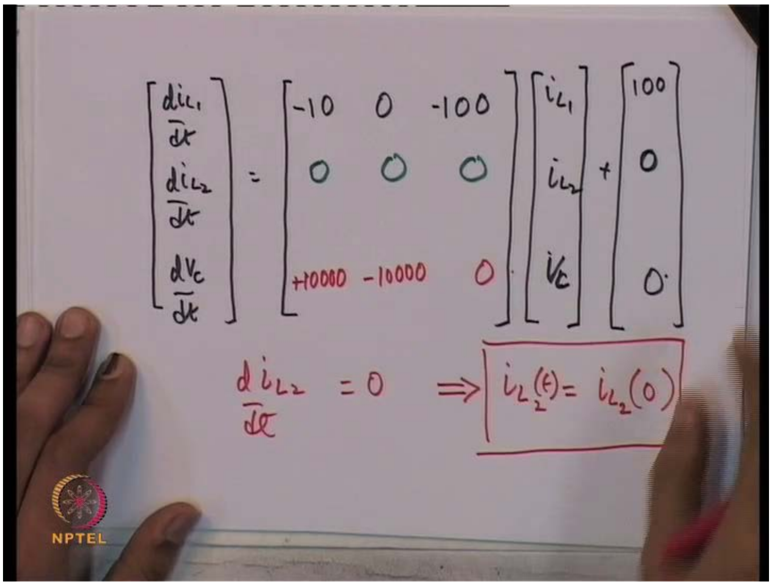

So, when I am interested in, what happens for a short while? After a particular disturbances or the initial period after a transient has occurred or some disturbance occurred, then I can make this assumption. If I know that this variable is associated with slow transients only. So, if I know that, this is what I can do. So, in such a case, you will have d i L 1 by d t d V c by d t. you know, what i L 2 is because I have assumed it to be 0. So, you will have simply the state space equations as… this is of course 0 because I have assumed that i L 2 does not change in the short period which we have.

So, this is the kind of a, converse kind of approximation which we can make. Now, this particular circuit or this particular set of equations, in fact this describes if this is set to 0, then in fact it describes this.

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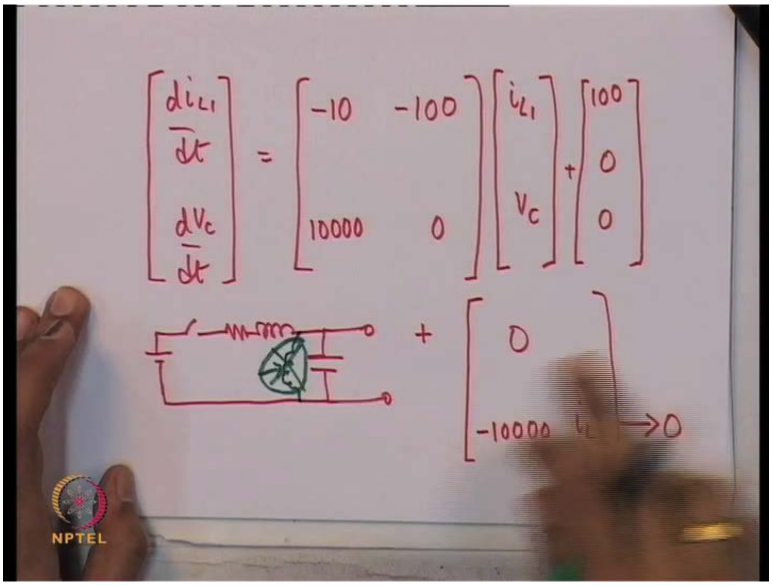

So, in the fast period, when we can say that this, the current in this does not move at all. So, we have for all practical purposes, disconnected this. The current through this stays at 0. It does not change at this in that case; you have got this fast dynamical equivalent of the circuit. And, the Eigen values of this will have to be found out. In fact if you do a study of this system, the Eigen values you will get are or I will just write down the directly the response of this circuit. It will turn out to be i L 1 is equal to 0.5 e raise to minus 5 t sine 999.9; that is almost 1000 and V c of t. This is the final time response for this circuit. I am just writing it down directly, you can take out the Eigen values and Eigen vectors using Sci lab, Cos plus 0.005 sine…

So, this is our response. So, what you see is that i L 1… V c is this. If you look at… actually find out what V c is, if you evaluate this it is this. Whereas, what we had got using the complete system was something like this. So, actually only in the initial portion this gives the correct result.

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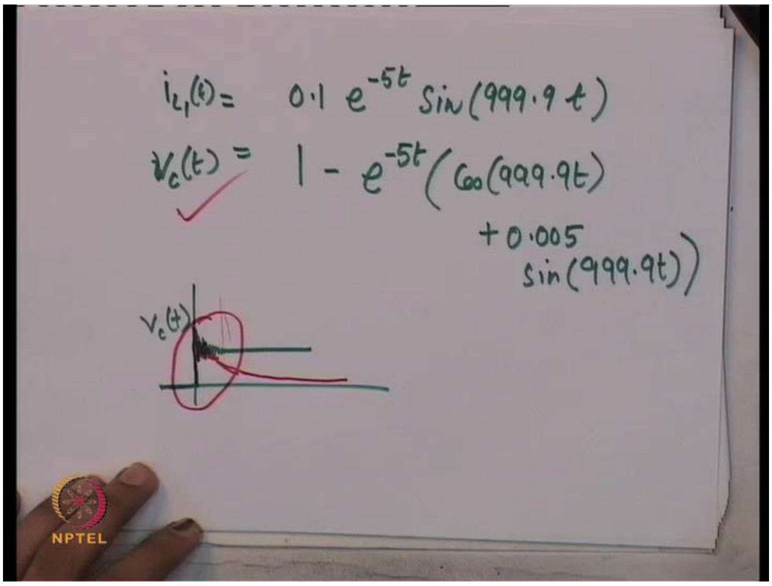

So, if you are only interested in the initial behavior; the very in a very short time period, if you are interested in the behavior, you can assume that the slow state or the state associated with the slow variable it just frozen at its pre disturbance value. So, that is an important modeling point, which you should appreciate.

Now, well, one of the simple reasons why we would need to do this modeling approximation is that, if you have a system consisting of fast and slow transients you can actually reduce the size of the system you want to study. In fact, the differential as we have in this; of course simple example, we did not get much of a improvement as far as the size of the system is concerned. in the… we manage to get rid of two variables, two state variables and we are left with one state variable d i L 2 by d t, when we wanted to understand the slow transients. And, if you wanted to understand the fast transients, you could assume that state associated with slow transients i L 2 was frozen at its pre disturbance value.

So, in this particular example, there seems to be no very great advantage computation, otherwise of making this modeling simplification, but when you come to large systems, you know consisting of very many diverse components, it does not make sense to make these modeling simplification. So, one important thing you should note, which is true of any engineering modeling is, you do not have to model the system in its full glory. You do not have to start from Maxwell's equation and model everything in terms of p d s and so on. You can make some approximation, get some simplified models. You can even neglect the transients d by d t is associated with some elements, which we know from engineering judgment, you know they are associated only with the slow transients. So, you can actually make this kind of time scale distinction and the modeling can be the approximate.

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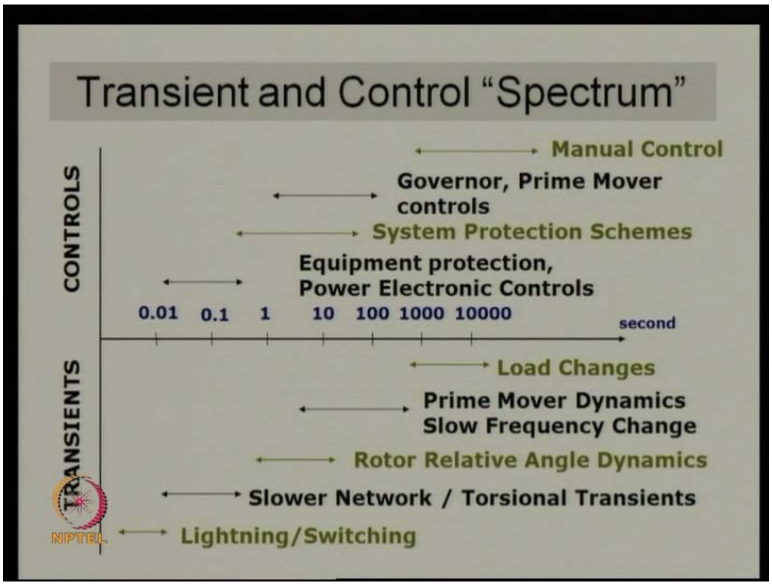

Even in a power system, since we have many diverse components, the huge variation in time scale. So, you can shift your attention to this particular slide. So, I have drawn a kind of a transient and control spectrum. So, if you just look at the kind of transients, you are likely to study, in fact you will be studying some of these transients in this particular course, as the fastest transients, the fastest transients in a power system are those associated with travelling wave phenomena on the transmission lines, in fact in such a case, you will have to model transmission lines in a great amount of detail. You know, you may have your model m s p d e's partial differential equations.

But, of course when you are studying lightning and switching transients, you can assume some things like the speed of the synchronous generators, etcetera, are not going to vary much in 0.01 seconds or 0.0001 seconds. So, you can make modeling simplifications. So, if you are only interested in looking at lightning and switching transients, you can assume that the slow transients are frozen at the pre disturbance values. As you move towards the right in the spectrum, after lightning and switching transients you have slower network transients as well as some torsional transients associated with the shaft of turbines and generators.

Even at a slower time scale, say from half a second to around 10 seconds, you have what are known as relative angle dynamics. The dynamics associated with machines staying in synchronism. So, if you give a… for example, a push to a synchronous machine which is connected to another synchronous machine. You are likely to excite oscillations of around 1 hertz or so. And also, if you give a very large disturbance, you may have loss of stability. All these phenomena take place in a period of around half a second to 10 seconds. So, all these things are visible in that if you take a snap shot in this period. In fact if you are studying slow relative angle dynamics, you need not model a transmission line by; you know the partial differential equations. In fact, you do not even, you can even neglect the d by d t s associated with the transmission line inductances and capacitances.

At even slower scale at the prime mover dynamics… slow frequency and load changes. These are slower, much slower. In fact, although it is true that I can cause a load change by simply going and switching off a bulb, on an aggregate sense, you know if you look at a substation, extra high voltage substation or the power system as a whole, the over all power, you know power being consumed at a given time does not vary very dramatically. I mean, you can almost predict how the load changes. In fact, you know at unearthly hours like around 1 or 2 am, this load is very low. And, after 10’ o clock in the morning, it starts rising. There is a kind of peak at around 10:30 or 11. And, may be after 8’ o clock in the night there is another big peak that, is the highest peak in the system.

This is a typical kind of loading scenario. So, these of course things, these load changes can be considered as very slow changes. There, you cannot even call them as disturbance unless you have a sudden load trip of say, you know 100 mega watt in a in a, say 20 giga watts system. So, this kind of thing can be considered as a disturbance. But, generally load changes slowly and it is not really changing at a very fast rate.

Similarly, you can even, you know these are the inherent, you know elements in the system, the power elements in the system. There are elements which we put into the power system. These are usually control system designed by us. For example, protection systems are a kind of special controls which are put in a power system, which kind of isolate faultier equipment. So, the equipment protection can be in the range of around 1 cycle to around 1 second. So, this is typically the equipment, range of equipment protection the transient, rather the time at which equipment protection acts.

Power electronic controls on the other hand also, rather they also are very fast. I mean, you are trying to control. For example, a rectifier in a h V d c station. That also is a very fast kind of moving system and the controls associated with them are also fast. You are trying to control the firing of the electronic wall. So, basically you find that the control systems, there also are fairly fast. You also have other control systems like, for example, the excitation in a synchronous machines is also controlled by what is known as automatic voltage regulator. That also, you know, you consider as acting in the range of 0.1 to around 1 or 2 seconds.

So, that is a typical. You can say this response. I am I am deliberately not giving too much precision into what I am trying to say. When we come to the topic later in this course, when we model a V r's automatic voltage regulator excitation systems, you will come to know about the physical elements concerned with these systems. You also have system protection schemes like; you know, I mentioned that if a system looses synchronism, you have got synchronism machines loosing synchronism with other synchronous machines. You may wish to do islanding; that is controlled system separation of the system or you can do under frequency load shading and so and are different circumstances. For example, when under frequency load shading is done, when there is a sudden, very gross kind of load generation imbalance.

So, you may, these are all called system protection schemes, prime mover controllers and governors, they act between the range. The actuators associated these things required around 1 to 100 seconds. And, of course manual control is much slower of the order of minutes. You do have continuous monitoring of the system and manual control actions. But, usually manual control actions are associated with changing the set points of various automatic regulators in the system as well as in some, you know unusual cases, you may even want to, you know actuate your protection system manually.

So, manual controls, of course are much much slower. And, you know, they can take a longer longer time. Now, of course one of the important points which we have come across is that a very important thing infact is the chicken and egg story. You know, when do you really say that a particular state is associated with a particular Eigen value or a mode. See, the basic point which we really try to tell you today was that if you have got a slow transients, find out the, you know state variable which are associated with the slow transients and the fast transients. Then in the fast transients, you set the d by d t equal to 0. I mean, set the right hand side rather the left hand side of your state equation corresponding to the derivatives of those fast variables equal to zero.

So, I have converted your algebraic equations, rather the differential equations into algebraic equations. Eliminate them if you can and just work with the slow subsystem with the x 2 the faster variable, simply algebraically related to your slower variables. So, this is basically what we have learnt. But, the whole point is how do you, which state is associated with, you know the faster and slower transients. I told you, you have to do it intuitively. But, that is not a very, you know comforting answer for students who are learning an Engineering subject for the first time.

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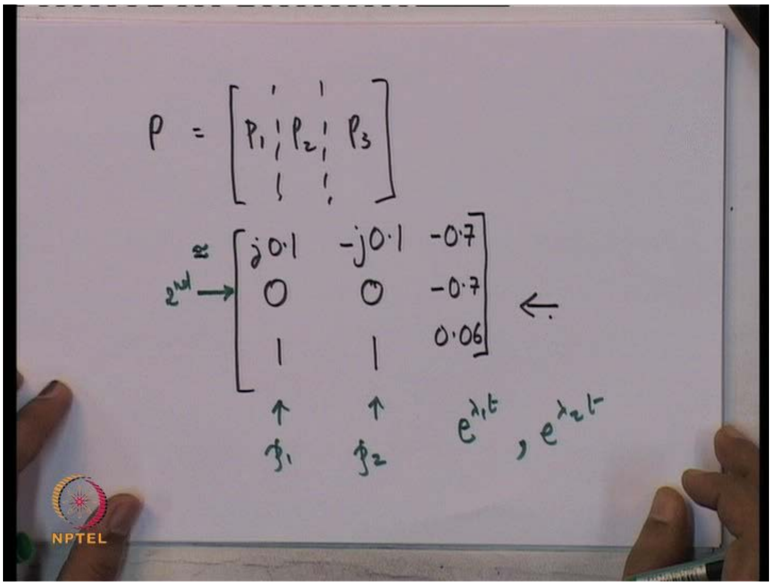

So, let us just first quickly look at the Eigen vectors associated. Perhaps, they will give us a clue. So, if you look at the Eigen vectors associated with the slow and fast transients, remember lambda 1 and lambda 2. Lambda 1 is minus 5 plus or minus plus or was it minus, it was plus j, lambda 2 was minus 5 minus j and lambda 3 was point one, minus 0.1. What you see here is, in the second state variable you do not observe lambda 1 and lambda 2. There is very little observability. So, perhaps it is practically zero.

So, if such a situation occurs you can say, may be you can say that i L 2 is not associated with the fast transients. But, there is a pit fall. In just looking at these Eigen vectors to come to this conclusion, let look at the converse thing; which are the variables are associated with the slow transients. So, you will directly say, oh i L 2 was associated with the slow transients. But, look at the components corresponding to i L 2, corresponding to lambda 3, the Eigen vector components corresponding to the three states and the Eigen value minus 0.1, this is a slow transients. You will see that, in fact i L 1 also is associated with it. You will, you will observe this slow transient is not only in i L 1, but also observe it in V c.

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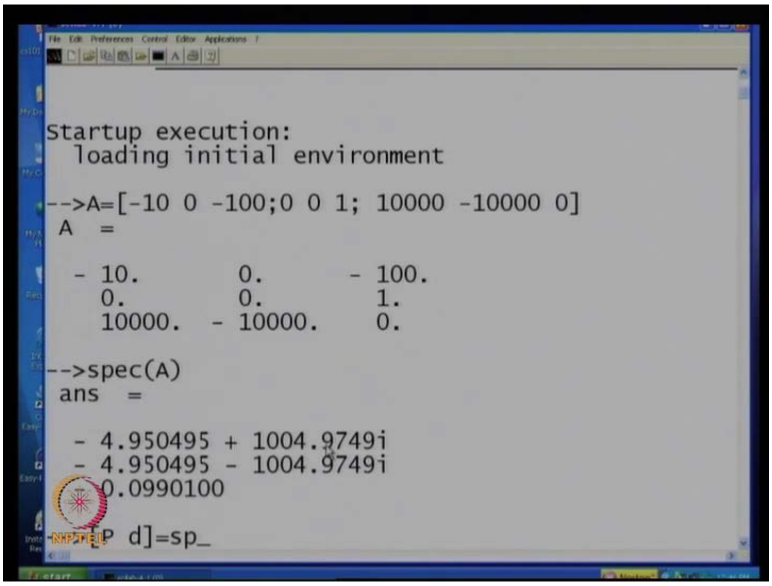

So, just looking at these Eigen vector, components of P will not tell you that i L 2 is associated with the slow transient. So, what you need to do here really is, look at both P and P inverse. If you look at both P and P inverse, so we will do this computation using Sci lab. So, I will just write down the the A matrix first. The A matrix was minus 10 0 minus 100; 0 0 1… So, this is A. so, the Eigen values of A are obtained from this command. This is what we got. So, this is the Eigen value 0.1. We rounded off it to 0.1, minus 0.1. This is minus, approximately minus 5 plus or minus j 1005 j or i is i or j are square root of minus one.

No, if you want to get the right Eigen values, right Eigen vectors, you have to use this command. And, P gives you the right Eigen vectors. In fact, you see that this is roughly j into 0.1. This is a small value. This is an extremely small value. So, we have approximated it to be 0. In fact, if you look at what I have wrote here on the sheet, these are just approximations of what you have seen there. So, just just rounded it off, you know, so this is approximately 0.

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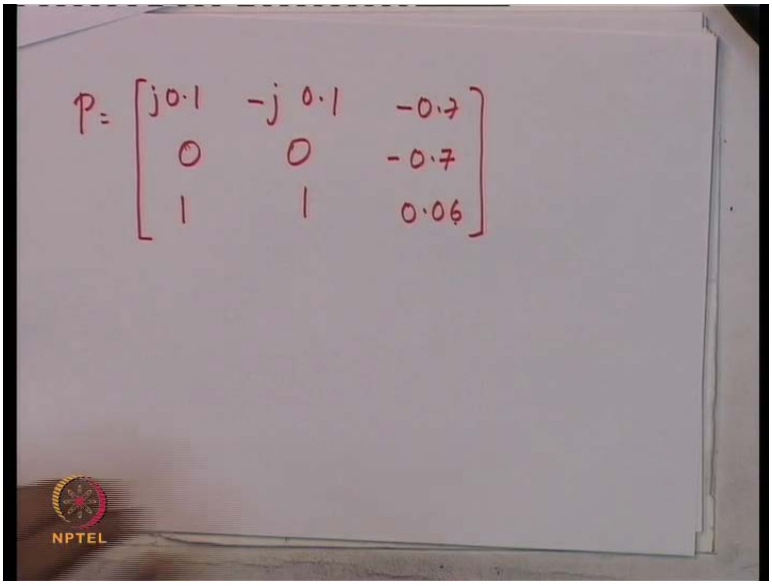

And, this is what we get is P. So, P is, I will just rewrite it. P is… now, what is P inverse? P inverse is what we called as Q.

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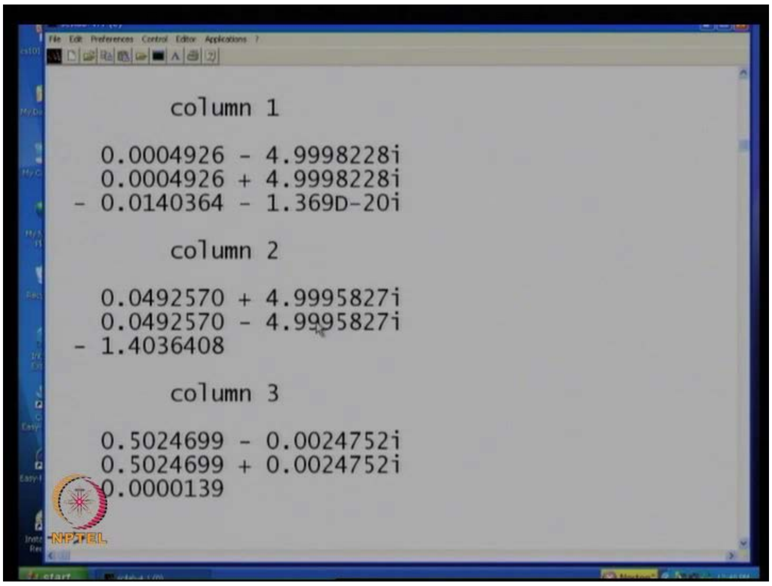

So, if you look at Q which is nothing but P inverse, this is just roughly, I will just write it down roughly. So, it is roughly, I will just write down while you can note it down also. It is minus j 5, plus j 5 approximately. And, this is roughly 0.015 and this is j 5 minus 5. It is not surprised that this Eigen vector, this is the complex conjugate of this, sorry, minus 1.403, this is a complex conjugate, 1.043. I encourage you to download Sci lab and just try out these example, try out these particular example yourself and this is almost zero.

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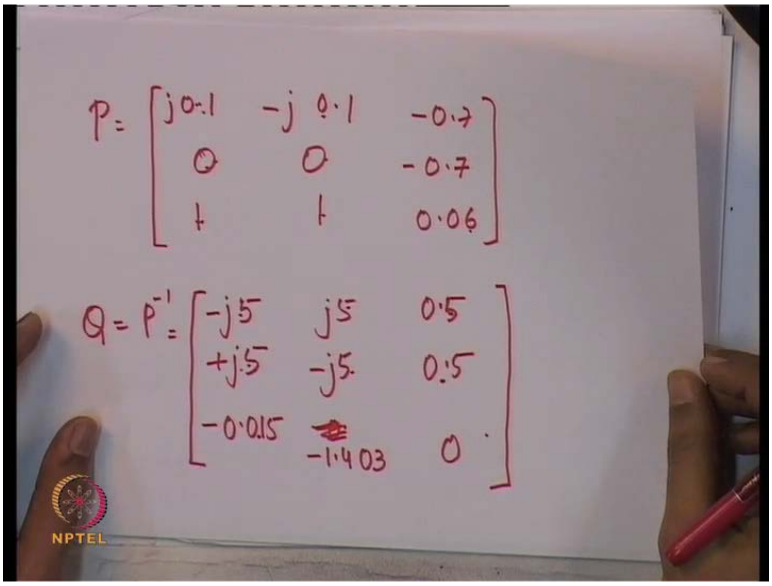

So, what we will do is, do an element by element multiplication of this kind. We will multiply this by this, this by this, this by this and we multiply this by this, this by this, this by this and so on. So, what we are doing effectively is a kind of an element by element multiplication of P and P inverse transpose. So, what we will do is use this command; P dot star inverse of P dot transpose. This is a transpose. So, what we will get in such a case? So, what we have done is P dot star into P inverse transpose. This dot star implies the element by element multiplication.

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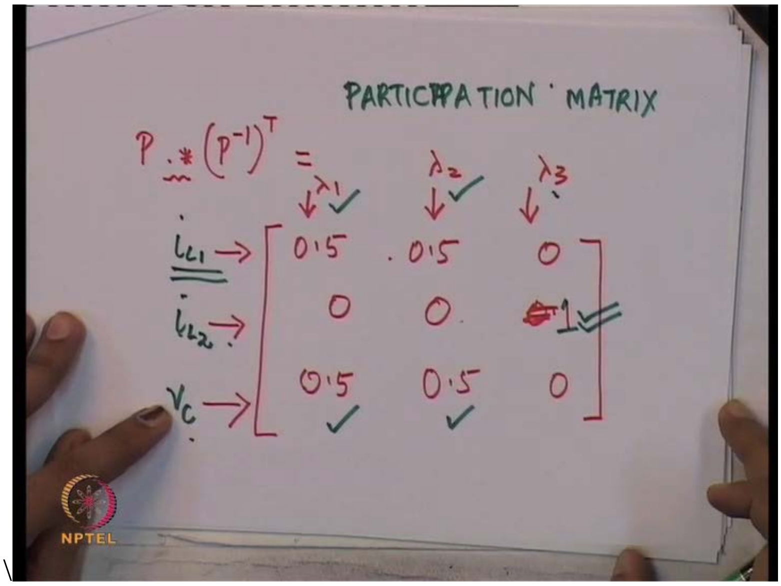

So, this is given by 0.5, 0.5, roughly 0, roughly 0, roughly 0, 0.99. So, this is almost 1 and you have got 0.5, 0.5 and 0. So, if I compute P, I compute P inverse and I do the computation P dot star; that is the element by element multiplication of P and P inverse transpose. This is what I get. What we see here is, this is of course corresponding state 1, state 2 and state 3; this is Eigen value 1, Eigen value 2 and Eigen value 3. This is I L1, I L 2 and V c. What we see is that, this particular thing is called a participation matrix participation matrix. And, it gives you the participation of a state in a mode, for example, i L 1 is associated with the complex pair of modes. V c is also associated with the complex pair of mode and i L 2 is associated fully, it is 1. It is a normalized measure is completely associated with lambda three.

So, actually this particular matrix, P dot star P inverse transpose. This is the element by element multiplication of these two matrices, actually gives you an interesting matrix called the participation matrix. And, this participation matrix, in fact gives you correctly the association of various states to various modes. So, if the participation of state number 3 in Eigen value, Eigen value 3 is 0. It means that this state is not participating in that particular Eigen value.

So, if you relook at this participation matrix, what you see is i L 1 participate in the complex mode, complex pair of mode, Eigen values and i L 2 participates in this. In fact, remember that i L 3 is the slow mode and lambda 1 and lambda 2 correspond to the fast transient. So, i L 1 and V c correspond to the fast transients and i L 2 corresponds to the slow transient.

So, we can actually, if you take out this participation matrix, we are able to tell which states are associated with which modes. Of course, there may be situations in which many states are associated with a certain modes like, for example, i L 1 and V c are both associated with the fast transients. So, you have to consider them together, whenever you do fast modeling, fast transient modeling. Fast transient modeling means the slow transients are assumed to be frozen at the pre disturbance state.

So, this is a very important concept called participation. We will, I will give you a reference for this in the next class again, when we recap this particular part of the subject again. In the next class, I shall, we shall move on to the next part of this course; that is, numerical integration techniques, for you know numerically integration integration techniques for dynamical systems. Numerical integration techniques will be required in systems which are too complicated to handle by linearized analysis of the kind I have shown you today. So, with this, we will end today's lecture.