Power System Dynamics and Control
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Lecture No. #06
Analysis of LINEAR Time
Invariant systems (Contd.)

In the previous class, we have been discussing the response of linear time invariant systems. In fact, we took a simple second order systems in order to illustrate the response. The basic interesting fact about the analysis of linear time invariant systems is that there is the time response can be written down in terms of well known functions, and because of that one can infer the stability, and the dynamical properties fairly easily. So, the main important result which we got last time was that the response of a linear system is a sum of individual patterns or interior modes of response.

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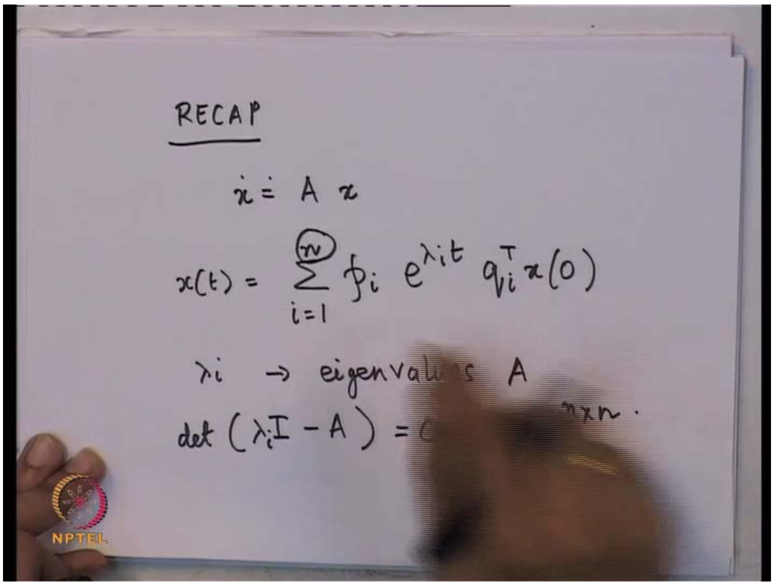

So, to make that a bit more precise, let us take a simple linear time invariant system without any input x is a vector, A is a matrix then the response of the system is given by… It is in fact a summation of several modes of course, this particular response can be written down in case you have got the A matrix diagonalizable, which we will just kind of try to understand once more.

If A is diagonalizable by linear transformation of the variables x in that case, one should be able to write the response in this form. It is a summation of various patterns or modes lambda i s are the Eigen values; they are a property of the matrix A itself in fact, determinant of lambda into an identity matrix minus A is equal to 0 yields the characteristic equations whose solutions in fact are the Eigen values of the matrix A. A remember is of course, an n cross n matrix. This particular characteristic equation will yield us a polynomial of order n and in general, they can there will be of course, n solutions of a nth order polynomial.

Now importantly, this PI are the columns of the Eigen vector matrix. So, if you recall what we did in the previous class, we were taking of a particular transformation x is equal to P y and a P such that P inverse AP yields a diagonal matrix. The columns of P are the right Eigen vectors P of course, has to be invertible and in we discussed this last time in case, A is diagonalizable; that is A has in fact n distinct Eigen values in that case, it is possible to find the columns of P such that P is invertible. So in fact, A is diagonalizable provided we get a P which is non singular, which also means that the columns of P should be linearly independent. In fact, if you have got n distinct Eigen values, A is diagonalizable. And as I mentioned some time back, if A is diagonalizable, you can write the response in this form; the q i here, the q i transpose is a row matrix, is a row vector rather which is in fact, the rows of P inverse the Eigen vector matrix inverse the rows of it are q i.

So the main, this was the main result which we saw last time, and the most important issue which we discussed was depending on the components of P i the components of P i, you can observe or measure the relative observe ability of the pattern e raise to lambda it in the various states. So, that is an important point which we discussed in the previous class, also we saw that depending on the value of the initial conditions, a mode could be excited weakly excited or not excited at all.

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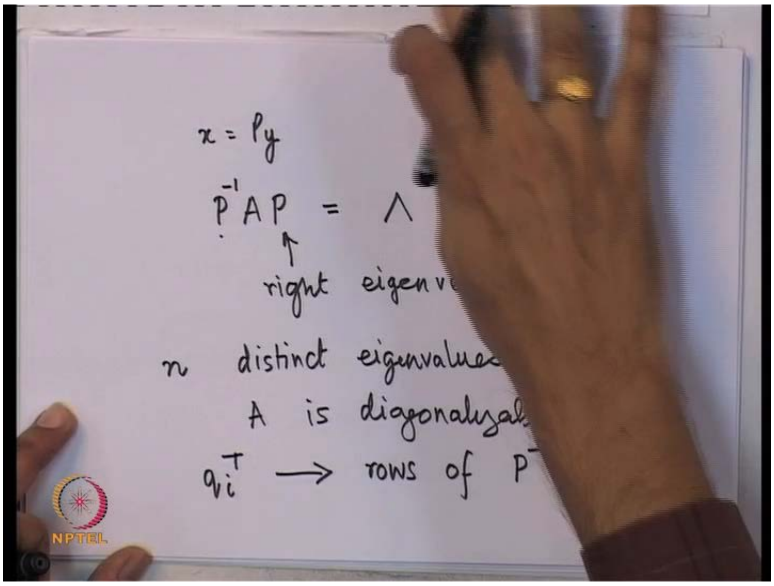

So, this was the interpretation we could give to the layer to the Eigen vectors. In fact, an important point which slipped my mind last time was that q i in fact also satisfies this relationship. So, q i is also known as the left Eigen vector, so q i are not only the rows of P inverse, but they also satisfy this particular property.

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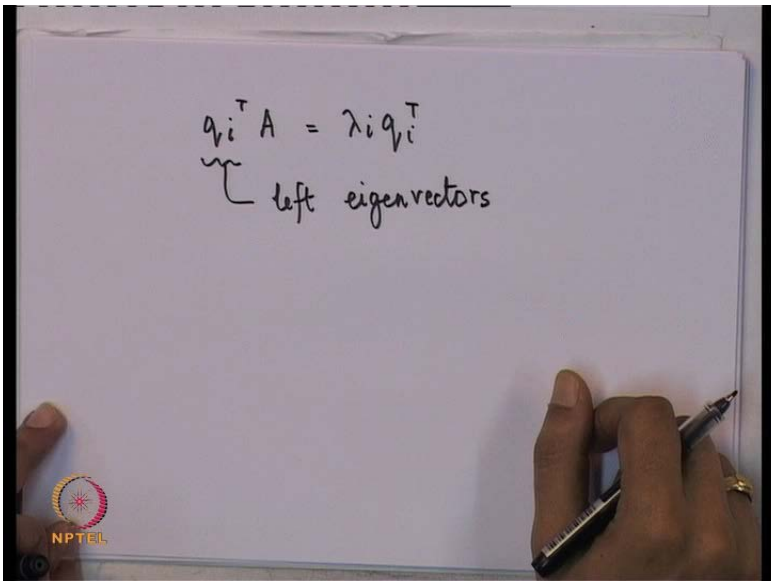

Now what happens if the matrix is not diagonalizable? So, if the matrix A is not diagonalizable, in fact if A has distinct Eigen values it is always diagonalizable, but if A has non distinct Eigen values then, A may not be diagonalizable. So, that is an important point which you should know so for example, the matrix A which is 1; in fact, is non diagonalizable; if you take out its Eigen values which is very difficult to which is not very difficult to find out, the Eigen values are determinant lambda I the identity matrix minus A is equal to 0; the Eigen values are 1 and 1.

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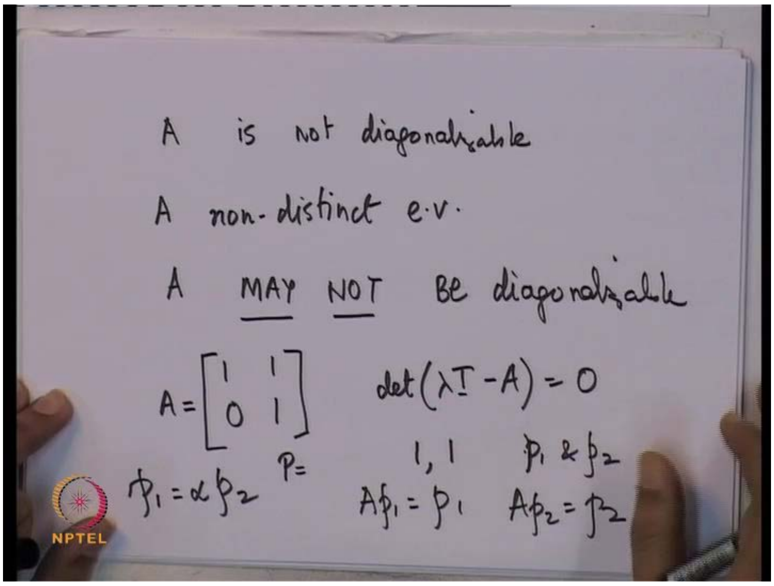

So, these are non distinct and this particular matrix is non diagonalizable. You cannot diagonalize this matrix, because you cannot get you know P1 and P2 which satisfies Ap 1 is equal to P 1 and Ap 2 is equal to P 2; P 1 and P 2 it will turn out to be will be always in the same direction that is, P 1 will be alpha P 2; alpha is any arbitrary constant. So, what you will find out is that because of this. you will not get you will not be able to construct a P with P 1 and P 2 as its columns so that you can diagonalize this matrix. So you cannot get the P 2 to diagonalize the matrix. So, that is why this particular system is not diagonalizable. Now of course, there are examples in which you do have repeated non distinct Eigen values, but still you can diagonalize it for example, this particular matrix is a kind of academic example.

This is a matrix which is diagonalizable; in fact, it is already diagonal; it also has got repeated Eigen values, but obviously this is a diagonalizable matrix; it is a diagonal matrix in fact. So, there are exceptions so just remember that the correct statement is if A has non distinct Eigen values, A may or may not be diagonalizable; that is the important point which you should note. Now, one aspect which we should discuss is if its non diagonalizable, what is the response; for example, suppose I have a matrix which it turns out that the Eigen values are lambda, lambda, lambda 1, lambda 3, and lambda 4. There are four Eigen values of a four by four matrix A. Suppose, this is non distinct; they are you know Eigen values are repeated. So, A may or may not be diagonal; let us assume that it is not diagonalizable; in that case, I just without really proving it we can show that you can find a P matrix which is just does not get you to a diagonal matrix, but gets you to a nearer diagonal matrix which is also called a Jordan matrix. So, you have got lambda 1, 0 lambda 0, 0, 0, 0. So, if a matrix is not diagonalizable and has a Eigen values as shown, we can actually get it in this form; this is called a Jordan form.

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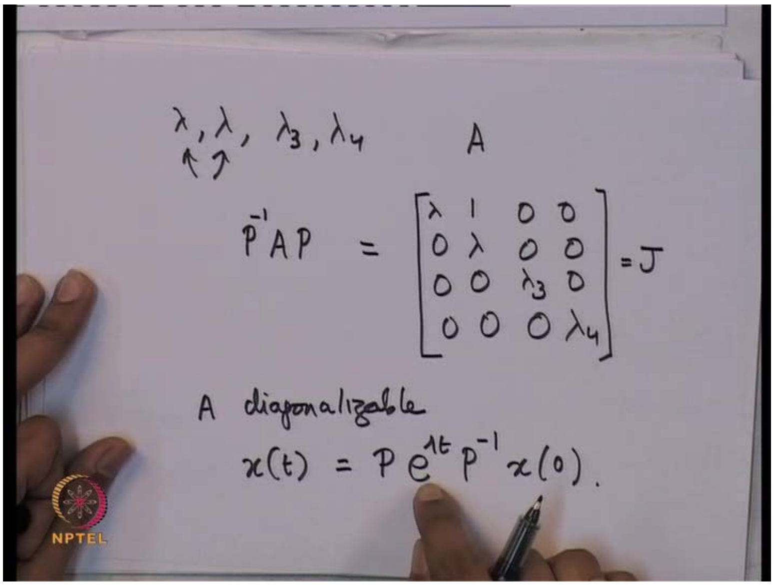

Now, this is a simplified matrix; this is a simplified kind of transformed matrix which of course, one can get the time response quiet easily. So, please recall that in case you have a system in which A is diagonalizable, your response is x of t is equal to P e raise to diagonal matrix into P inverse into x of 0; but incase, your system is not diagonal is not diagonalizable then the response can be different. Remember here, e raise to this diagonal matrix is in fact, this is a diagonal matrix like this. So if your matrix is diagonalizable, your response is as shown here of course, the expanded form of this response is in fact what I discussed some time back that is this; this is the expanded form. If your matrix is not diagonalizable, then your response x of t is equal to P e raise to Jt P inverse where J is e raise to lambda t; te raise to lambda t this taking the example, which we discussed some just a while ago; that is you have got e raise to Jt is this.

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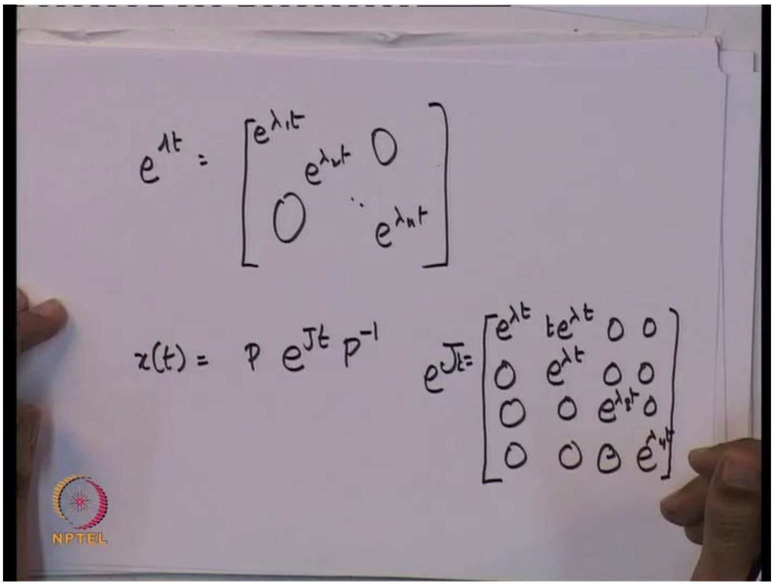

So remember that, in case you do get non distinct Eigen values, you can still write down the response but remember now, you are getting the response in terms of not only exponential functions, but you have got time coming here t into e raise; these kind of terms will come into your response, so that is the important difference. You can of course, have more than two repeated Eigen values, but we will not go further and deeper. Normally in our power system analysis, one will not come across situations there will be there will be more than you know you know you have three Eigen values, which are non distinct and its non diagonalizable. That is very a fairly rare situation. Most cases which we will consider, you will have all Eigen values distinct so, it is important to remember this.

Now one important point which you should note is that, in this particular course we do require you to understand the response of linear systems of course, it is not practicable in this course to give you a full exposition of everything about linear system theory. So, it is good if you refer to some control systems, linear control systems books for more details about the response of linear time invariant systems.

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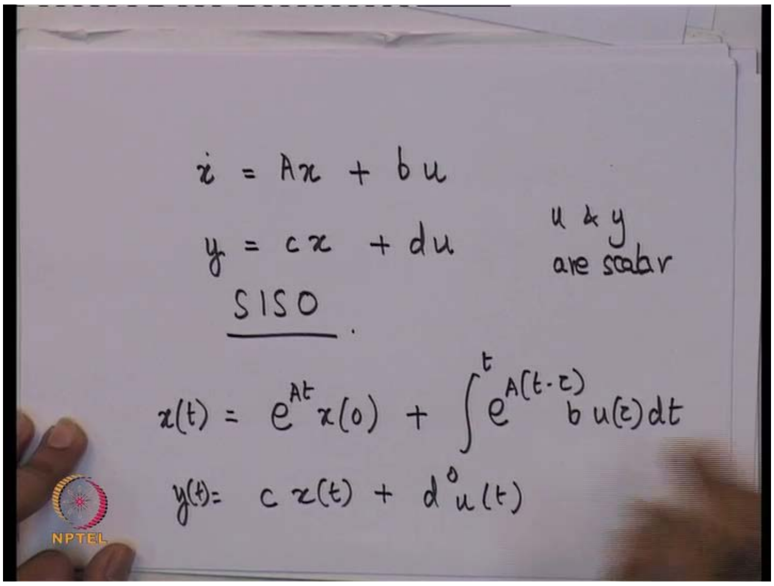

So for example, you can look at the book by Ogata State Space Analysis of control systems in 1967 of course, there are many other good books so I… this is just an example.

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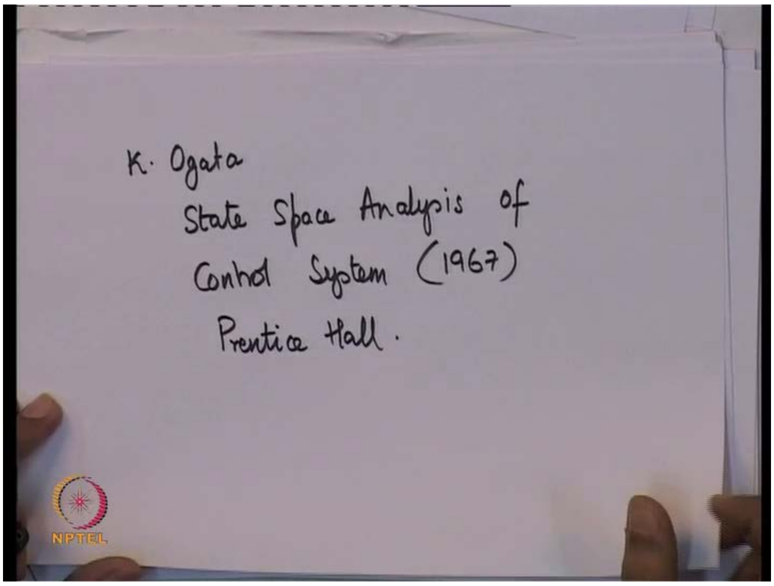

So this is an example of a book you could refer to. Now of course, an associate problem which we will come across is what happens when you have got a system which is forced that is it has got an input. So, if you have got a system x dot is equal to A x plus b u and you are not observing all states, but you are observing one output which is a, which is basically a combination of the states. So, such a kind of a system has got both an input and an output. So I will just write it down x dot is equal to A x; let us assume there is only one input u and you are also observing a certain variable you know which could be a combination of the states c is a c is a vector; x is our state.

So, you could have a combination of states is a multiplication of a row with a column. We will assume u and y are scalars; so this a single input, single output system. In such a case, what is the response of x? We had written down the response when A was a scalar some in some lectures before. The response of the system when A is a coupled matrix is x of t is equal to e raise to At x of 0 plus 0 to t e raise to A t minus tau b u could be a function of time. In the convolution integral, dt is should actually be d tau so, there is this some minor error here; y is simply c of x t y of t plus d of u, u of t.

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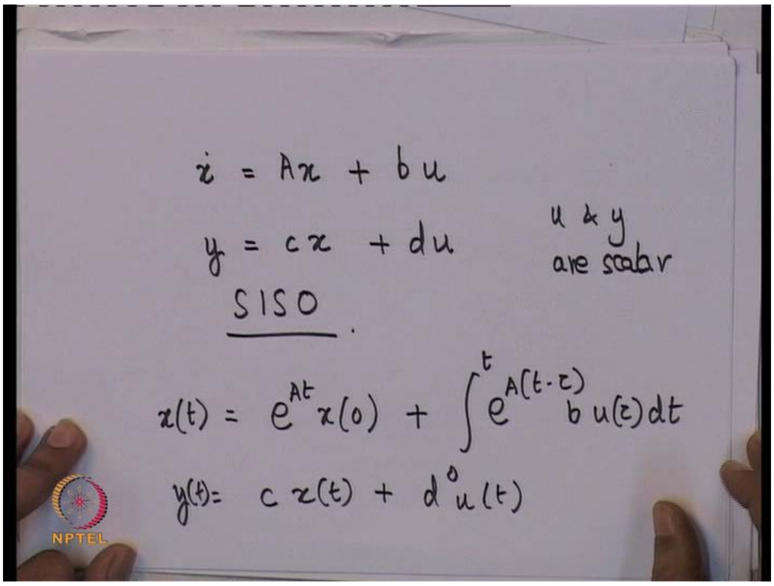

So, this is our response of the system. Remember, e raise to At is a short hand for P e raise to P inverse we have already discussed what this is. In case it is diagonalizable or it is equal to P e raise to Jt P inverse if it is non diagonalizable; this is Jordan form matrix. So, just remember this formula which you have for the first response. So, it is quite easy to take out the first responses as well, it is not too difficult. Especially, if you are a simple inputs like sinusoidal inputs or step inputs this is trying to evaluate this is not a really a major problem. Now before, we will just write this particular equation and this particular equation in full form. So if I expand this, let us assume that A is diagonal and I expand this, what you will get is y of t is equal to c times i is equal to 1 to n; this is the response of x of t plus of course other terms. If input u if the input u is 0 these other terms are 0.

One interesting point here is that if cp i is equal to 0, then that the mode or the pattern will not be visible in y t. So, if c is such that cp i is equal to 0 is not visible. Although, it may be visible in the states, but it still may not visible in y; so y is actually a combination of states. These other terms of course are dependent on u. So this is an important point; so not visible that is a proper technical term called observable. So, if cp i is equal to 0, then the i th mode is not observable in this input output y.

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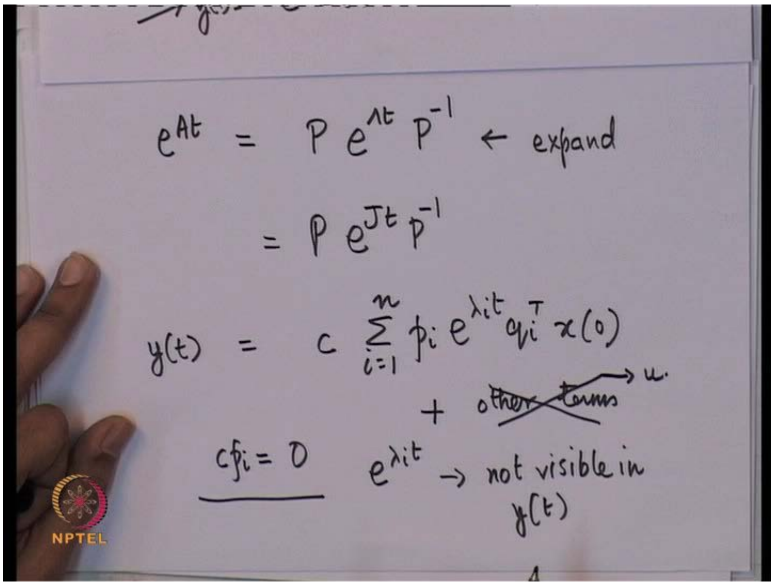

Similarly, since x of t is equal to sigma of i is equal to 1. I will just continue here, so what we have we can just write this a bit differently. In fact, we can take out Pi e raise to lambda it common and then in the bracket, we have got 0 to t; this whole thing is summed; qi you will have e raise to lambda minus lambda i tau q i transpose bu tau d tau. It is a bit messy here, but I have just since the variable which is getting integrated is tau, I have kind of taken out this e raise to lambda it outside the integral. So what we see one critical point, if q i transpose b this is 0 for some i; in that case term corresponding to lambda it in this response is dependent only on initial conditions. If such a situation occurs we also say, that this particular mode i is not controllable by the input u. So please remember these important concepts. In case, q i transpose b is equal to 0; the e raise to lambda it term in the response of the states is dependent entirely on the initial conditions that cannot be changed by the input u; so that is one important point which you should note.

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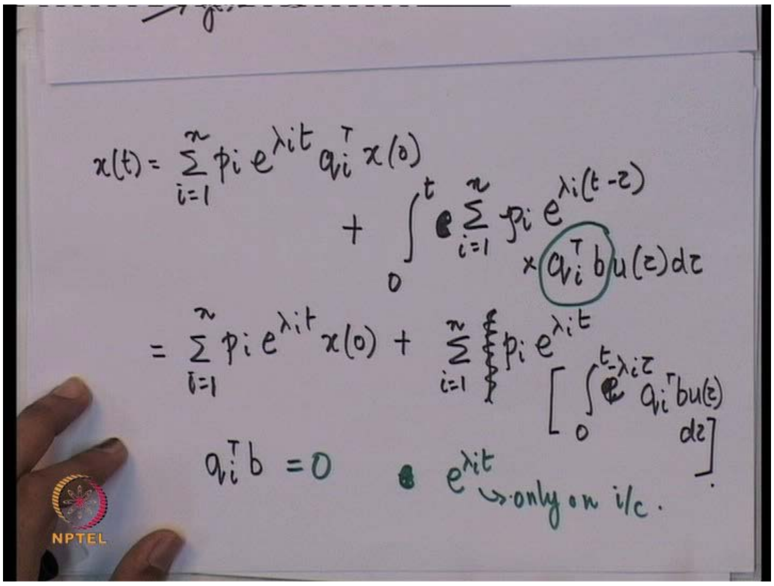

So, we have through this rather straight forward analysis given you a very intuitive kind of notion of observability and controllability. We shall actually come back to these concepts in a multi-machine system where we shall see that, there are many many modes and certain modes are not observable at certain places; certain modes are not controllable by controllers which are located at certain locations. So, this is a very… these concepts are pretty important; the concept of observability and controllability in linear control systems of course, our analysis of the same has been very intuitive and sketchy hopefully, we will revisit these things and it will become clear at that point of time. That will be some time of course, much later in the course.

Now although, we have been doing the analysis of linear time invariant systems. There are few computational issues which we should worry about because eventually, the power systems which you are going to the realistic power systems or systems we are going to study are going to be fairly large. So, one or two simple points and important points which you should bear in mind are that, if I want to compute Eigen values I had mentioned that, one of the ways of doing it is solve this equation determinant lambda I minus A is equal to 0; it turns out that this is not very easy to do if your size of A becomes larger. In fact, if you have got for example, a matrix of size 20 by 20, you will find that it is becomes almost next to impossible to actually solve it by actually computing the characteristic impedance, characteristic equation. In fact, computing the characteristic equation the coefficients of the characteristic equation themselves become pretty tedious.

So, there are in fact iterative methods like Power method or the Q-R method. We will of course not be discussing all the methods or in fact, we will not really be going into Eigen value computation when A is a large system; I can recommend that you look at any book on matrix computations where this particular aspect is covered. But remember, one small point that whenever you need to compute the Eigen values of a matrix and if that matrix is large, you have to take recourse to some iterative methods; it is not feasible to do it by actually computing the coefficients of the characteristic equation by actually evaluating this that to be very computationally intensive. Of course, we shall take a bit of the easy way out in this particular course otherwise, we will get our hands will get totally occupied in trying to understand all the computational methods and the theory behind linear systems and so on. We lose track of what we really wish to do that is understanding a power system.

So from now onwards, I also take help of mathematical software to do the computations. So one some of the software which can help you to do this quite widespread and used often SCILAB, this is of course freely downloadable software and also MATLAB which is quite popular.

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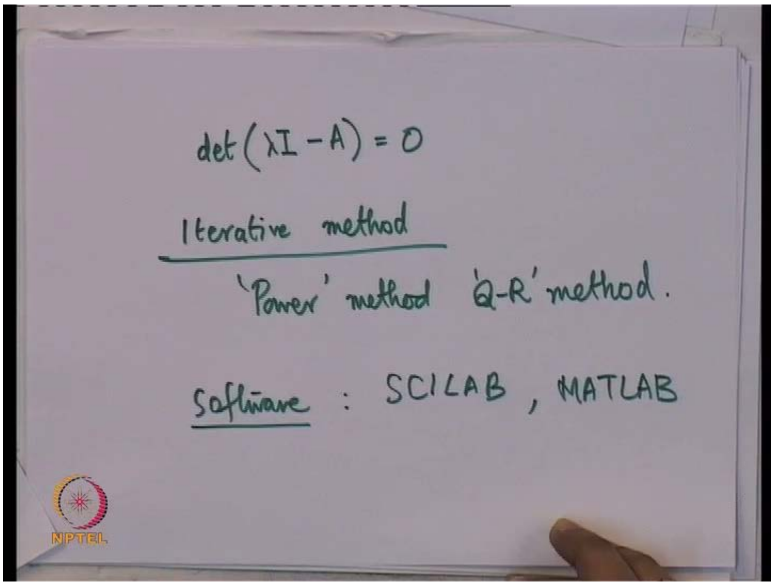

So, if you use these one of these two tools or probably any equivalent tools, it should be fairly straight forward to do most of the computations which we will do in this particular course. So, rather than spend more time on trying to give you more and more results and mathematical results and so on. Let us instead take a more pragmatic view; let us take a simple system and numerically compute the Eigen values. We will take a slightly larger system we have been always considering second order systems which are the Eigen values Eigen vectors are easy to compute.

We will take a slightly more complicated example it is a third order system not really very complicated, but we would require some computational help otherwise, it will become very tedious to do. So, rather than just talking terms of a you know abstract some random A matrix, we will just take a simple engineering example which will bring out not only how the method forgetting the response, but will also tell us some interesting ways of interpreting the results, you know all the Eigen value and Eigen vector information has to be interpreted correctly.

So let us consider an example. There is a voltage source connected in fact you can look at this as some kind of equivalent of a pulse transformer of course, I am not given very realistic values of a transformer; it is a kind of this is a kind of leakage reactance, the resistance of the wire, you can consider this as a magnetizing reactor reactance and also this capacitor really denotes the capacitance of the widening. So, this is a kind of a near realistic example; it is a model, a toy modal of a pulse transformer. I chosen the values in such a way that, it will aid us so in that sense I have not given exactly realistic values. If you just look at this particular example, without even having to solve it, you will notice that this is a very large inductance compared to the leakage which is just 10 mille Henry; this is 100 micro Farad and this is 0.1 ohm. This is a kind of situation in which, there are in fact components which are fairly differing in size; they are they are not of the same size; this leakage is much smaller than this and so on.

In fact, the main reason why I have given you this example, we shall actually pursue this example even further after this class is over is that this particular system have got some certain characteristics. It is in fact what is known as a stiff system. So, before I tell you what is a stiff system etcetera. Let us just look at this at steady state; in fact, if this particular system has is not excited at all in case it is not excited at all, it is easy to see that is if the voltage here is 0 or I keep this open, the equilibrium is when… Well what is the equilibrium of this?

The equilibrium point of this is obtained by setting the rate of change of the states of the system to be 0. So let us for example, choose the states. Suppose i L1 is a state; i L2 is a state and V C is a state. So, this is the initial conditions of these have to be given if I want to tell you what the response is going to be. But, if you for example, if I… When is for this particular circuit if the input u is 0 that is I connect this to voltage source which has got voltage 0; in that case, it is not very difficult to see that in steady state if di L2 by dt is equal to 0; it also means that the voltage here is equal to 0; so V C will have to be 0.

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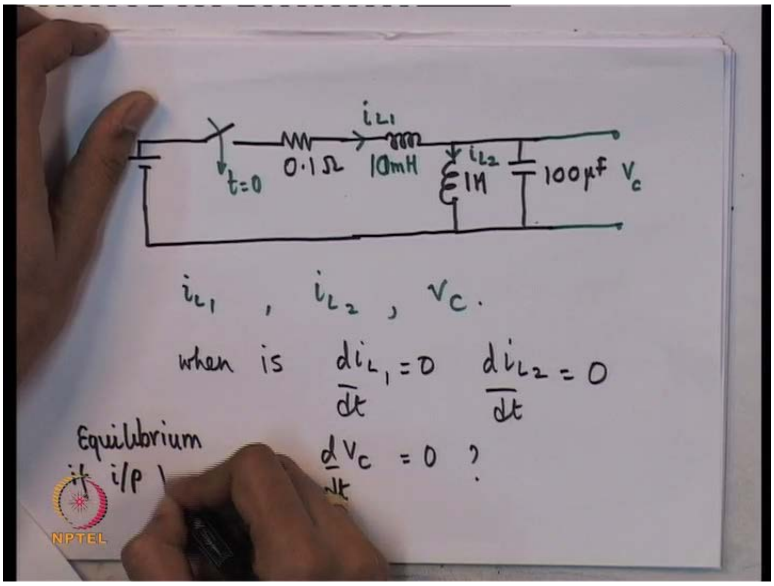

So the equilibrium, if input voltage is 0; input is 0 is V C is equal to 0; i L1 is equal to 0; and i L2 is equal to 0. This is the equilibrium for no input or for input u which is equal to v i is equal to 0. So, if the input is 0, in that case this is the equilibrium. It is not very difficult to find that out, we can do it formally that is i L1 by dt is equal to this is 10 is to minus 2 10 mille henrys; this 10 mille henrys this is 0.1 ohm. So you will have, if you apply this k v l in this loop, you will get is equal to or in other words, l di by dt plus 0.1 into i L1 plus V C is equal to v 1. So, I just got things some things on to this side when I applied k v l. So, this is the basic equation; this is the second equation and the third equation is of course, 100 micro Farad is equal to of course, if V i is equal to 0, that is if the input I make if the input were to be 0, then the steady state value is obtained by putting all these things equal to 0. So if V i is equal to 0, this is equal to 0; so V C also will be equal to 0; if V C is equal to 0 and V i is equal to 0; it means, i L1 is equal to 0; this is set to 0, then i L1 is equal to i L2 which means i L2 also is 0.

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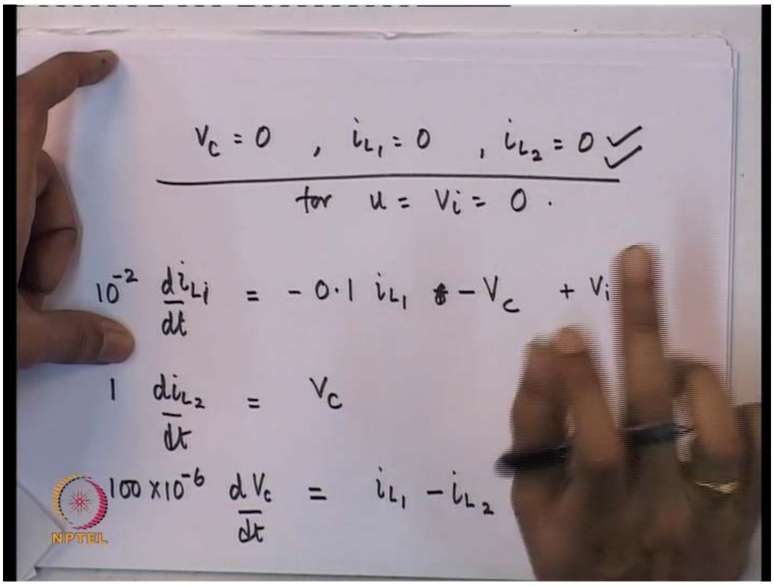

So if V i is equal to 0, then this is the equilibrium condition. This is obtained by setting all the derivatives equal to 0 that is it. So this is, if you do not excite it at all, let all you excite it with 0 voltage source, this is the equilibrium condition for the system. However, if as if I shown here I excite it with a 1 volt source; in that case, I can write this compactly in this fashion. To save sometime, I will omit all the algebra and write this equation so for t is equal to 0, this is A and this is b into u this is b into u.

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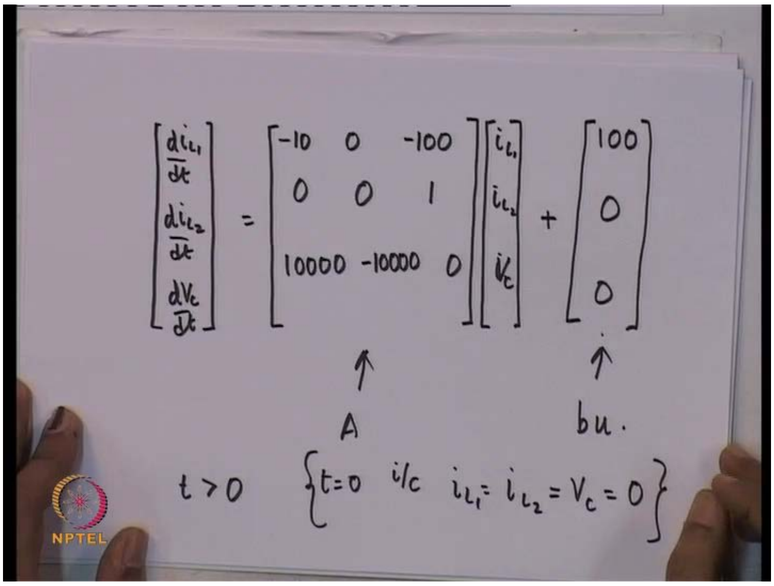

So, this is basically our system and this is the system after t greater than 0 and let us assume that at t is equal to 0, the initial conditions are all 0. That is all the currents and the voltage are actually 0; so let us assume at t is equal to 0, the initial conditions remember in all linear dynamical systems or all dynamical systems, we need to be given some conditions like the initial conditions in order to tell what the future response is going to be. So, let us assume the initial conditions are this is the initial conditions only at time t is equal to 0. So, the response is if this is A and this is b u, you will get sorry e raise to At 0, 0, 0; this is the initial condition plus 0 to t e raise to A t minus tau into b u tau; so this is after time equal to 0 after time is equal to, 0 this is how it looks like.

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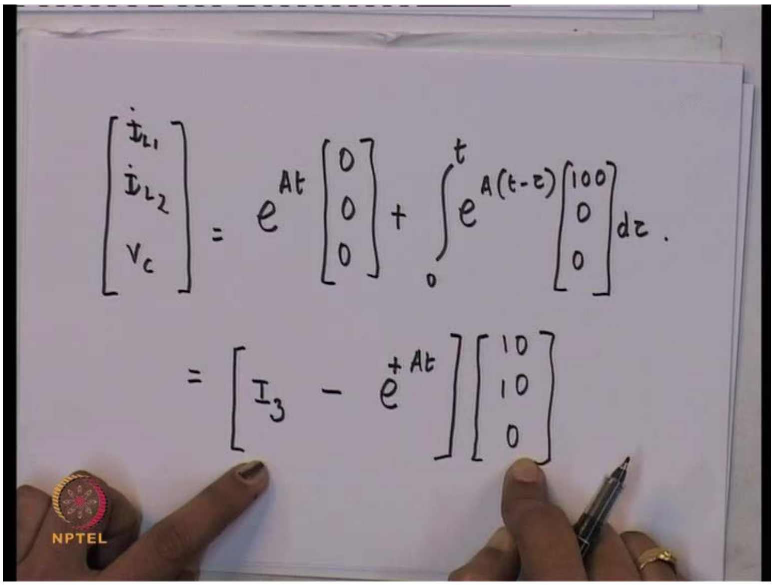

So this turns out to be, we will cut a long story short; this will turn out to be 0 and if you evaluate this I will skip the steps, I will just write down the final answer I 3 minus e raise to minus At sorry into 10, 10, 0. So I just evaluate this particular matrix, this particular this is of course 0; so if and I actually evaluate this from 0 to t, I will get I 3 minus e raise to At into 10, 10 and 0. So which of course can be written down; we know what e raise to At of course is a short hand for P e raise to lambda 1, lambda 2 of course, this is 3 by 2, 3 matrix; so this is what we get.

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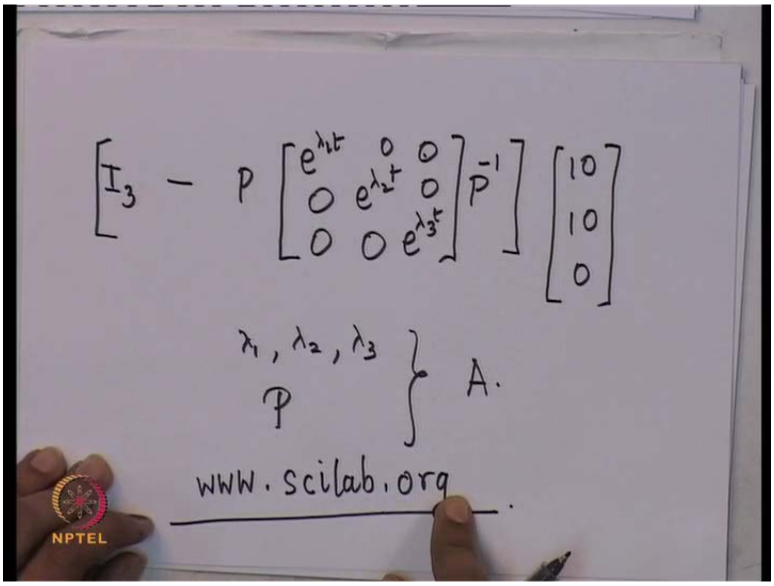

So this is our response. Now of course the point is what is in fact, we are assuming here of course here, it is diagonalizable; this needs to be actually checked. So, we need what is lambda 1, lambda 2, lambda 3 and also we require this matrix P. So this is what we really require in this particular system. So this of course, has to be obtained from A. Now, for very simple second order systems, I showed you how you can take out the Eigen values and Eigen vectors we did a very simple example in the previous class, but if you really want to do an example in which is a third order or fourth order matrix; if you use useful to use some software and here I will show you this computation using the software sci lab, note down sci lab is freely downloadable. It is available at this point of time at this particular web address www.Scilab.org.

So, I will show you this computation on sci lab; so just keep this in mind what we are doing. If we start up sci lab, this is what you will get. Now, I will not really tell you all about the syntax of using and so on. What I will try to you know just show you the commands and the syntax is fairly intuitive so you should be able to pick it up on your own. So, what I will do here in order to take out the Eigen values and Eigen vectors, I first enter the matrix A this is nothing, but 10, this is the first row, then I enter the second row; this is the syntax of entering the matrix, then you have got the third row.

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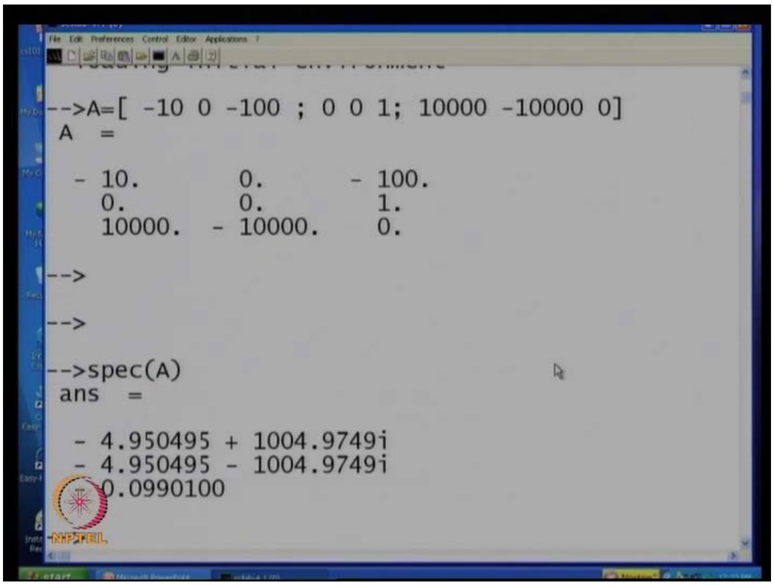

So, this is your A matrix the Eigen values and Eigen vectors of A can be obtained from a command called spec of A. That will give you actually the Eigen values associated with this system. So, the Eigen values associated with this system are roughly minus 5 plus a complex number, this is a complex number roughly this one; this is approximate I have just rounded off. Now, the point is these are the Eigen values. One important point you are seeing is that you are getting Eigen values which are not real numbers, they are complex numbers. Even if they are complex numbers, I told you the stability criterion is that the real of lambda should be less than 0.

So this is in fact, a stable system because the real parts are all less than 0. One important point which you notice here again is that although, these are complex number they seem to appear in complex conjugate pair. In fact, a general result which can be stated that if A is a real matrix, then it is Eigen values are real and or whenever they are complex, they will appear in complex conjugate pairs. So you can have real Eigen values and or complex conjugate pairs.

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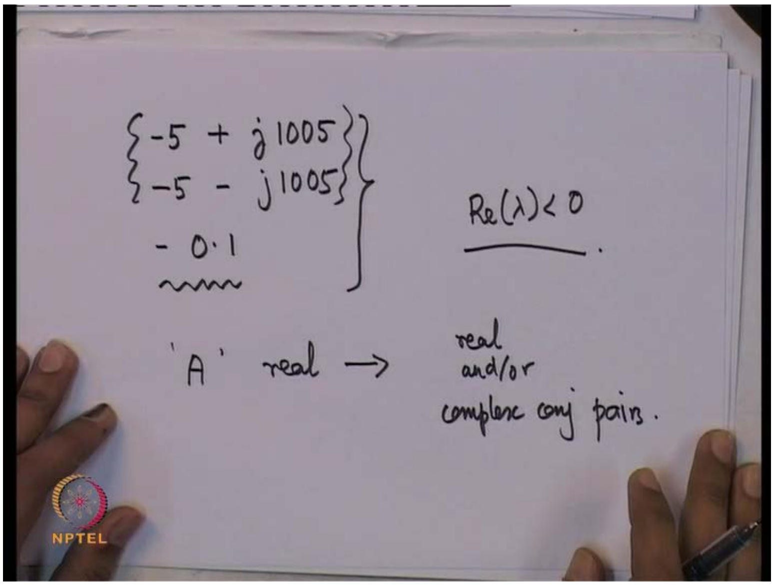

Your final response as I is shall show you in some time is in fact, quite is going to be real it is not going to contain imaginary terms at all. So, please remember that you are your Eigen values whenever for real matrix you will get Eigen values as a real and or complex conjugate pairs as you have seen here. One more thing which you should notice here remember, we are trying to do a study of interpreting the results as well. This Eigen value and the magnitude of these Eigen values are very very much different. In fact, you will see that this particular Eigen value is small. In fact, the small Eigen magnitude Eigen value is associated with slow rates of change.

So, you will find that your response will contain one pattern which is relatively slow and one pattern which is corresponding to a high magnitude Eigen value, the rate of change of that pattern will be fairly fast. So in fact, a larger Eigen value is associated with faster rates of change. So, that is one important point which you should note. These of course Eigen values are three distinct Eigen values, so our response will be a superposition of modes. In fact, so we do not have to worry about Jordan form or what happens how do you actually solve with repeated Eigen values. This is a luckily we have got a system with three distinct Eigen values.

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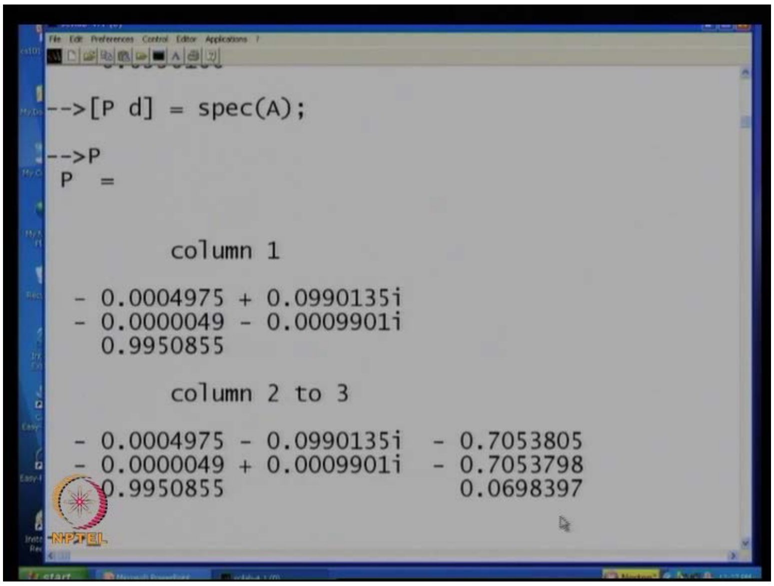

Now, I mentioned some time ago that your response is this. Now, we also require in order to get the response to find this value P; P is the Eigen vector matrix. In fact, there is a command in Sci lab which allows you to take out the Eigen vector matrix that is P. So if you do this, P in fact gives you the right Eigen vector matrix. In fact, you get the columns 1, 2 and 3. Remember, column 1 itself is complex; column 1 is complex; column 2 is also complex, because they correspond to complex Eigen values and the column 3 is real. In fact, the column 3 corresponding to the Eigen vector associated with the real Eigen value. In fact, we should not just let this go as it is. One important point you will notice is that the Eigen vectors associated with the complex conjugate pairs turn out to be complex conjugate themselves.

So you say this particular Eigen value Eigen vector is minus practically 0 and this is almost 0.1 into J or I; I is actually square root of 1 that is J. I mean I have been using J the software here uses I. You will find that the Eigen vectored component here is the complex conjugate of this. So, this is an important property of real matrices. So even you are Eigen vectors turn out to has certain properties. One more interesting point is, I will just write this Eigen vector matrix; it is the Eigen vectors and roughly this is roughly, very roughly this is J into 0.1 this is practically 0, this term here. The second term practically is 0 and the third term is practically 1 roughly. This is minus J 0.1 0 1 and this is minus 0.7 minus 0.7 and 0.06.

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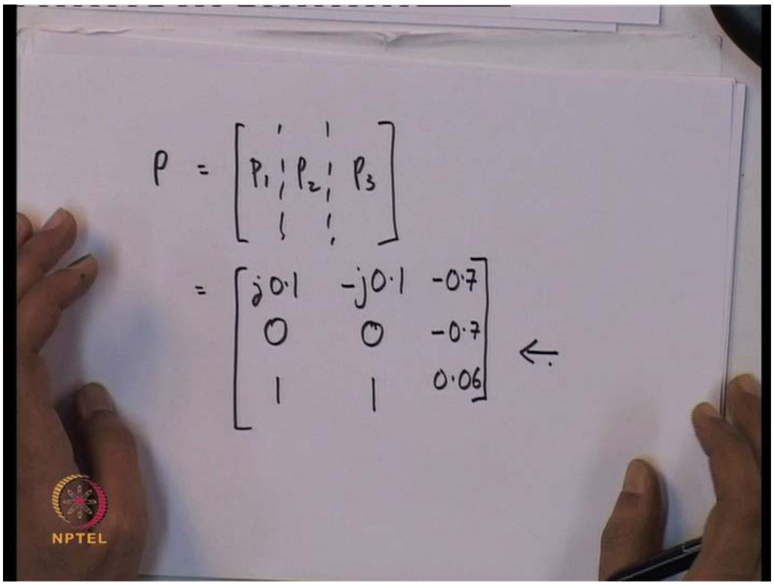

So, please have a look at the screen again, the computer screen. I just approximated these columns just rounded it off. I mean rather I should say I approximated it rather than round off; it is practically 1 and is a very small number here. So, this is how I get P. You can concentrate on the sheet which I am writing on you will get a P matrix of this kind. So, if I numerically evaluate now I have got lambda 1, lambda 2 and lambda 3 lambda 1 lambda 2 lambda 3 and I also have got P1, P2, P3 and the P matrix. So, I can actually take out P inverse, so P inverse is take out the inverse of P matrix. So I just print this out; so this is the inverse of the P matrix. The columns of this column 1, column 2, column 3, but if you look at the row that will give you q i, the you know q i transpose. So, I have also computed P inverse in this particular case. But, let us now evaluate the time response of x; this is what I eventually wanted to take out P this and P inverse.

So if you finally compute the response, I will omit the algebra what you will get is i L1 of t; i L2 of t and V C of t is equal to… This is the final response 10 minus 10 e raise to minus 0.1t plus 0.1 e raise to minus 5 of t sine of 1005t, and this is 10 minus 10 e raise to minus 0.1 of t. Please try to make it work it out on own with Sci lab, I am just writing down the response directly. This is of course, making certain approximations since I rounded off certain terms. So, this is your final response so you i L2 t is 10 minus 10 and so on.

Now, one important point is that lambda 1 and lambda 2 which were in fact, a complex conjugate pair here seem to be after lot of algebra seem to be giving you a sinusoidal term here. Remember, that things get so arranged because of the fact that Eigen values appear in complex conjugate pairs. The complex Eigen values are in complex conjugate pairs, you will find that you should be able to make these simplifications and your response is in fact, will turn out to be in terms of sine's and cosine's there is no complex number in the response eventually, because you will be able to combine all the complex terms appropriately in order to get these terms.

Now, one of the important points after a bit of you know approximating the Eigen values and Eigen vectors, these are the these are the Eigen vectors and these are the Eigen values. One thing you will notice here is that the there is exponentially decaying component of the response, there is the forced component, because of the input which you have given and there is also a component here, which is oscillatory in nature; it is a decaying oscillation; similarly here.

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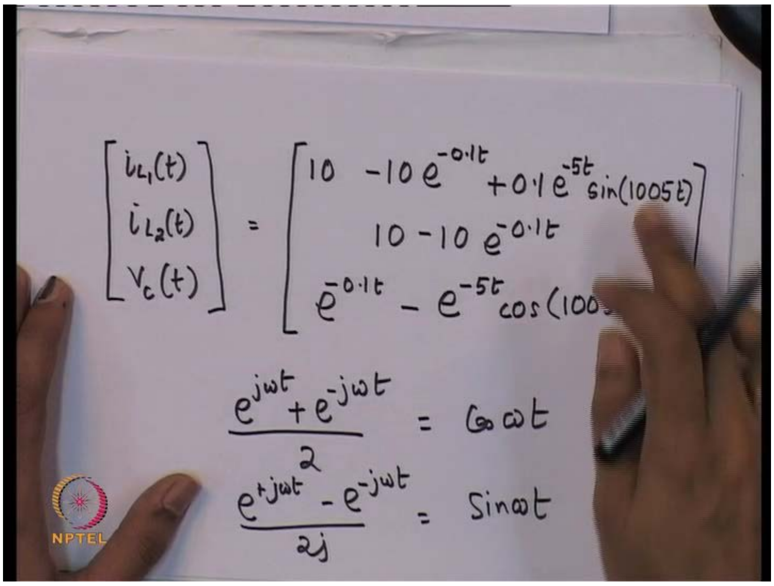

So because of the fact, that you have got a complex conjugate pair of Eigen values, the mode which you get will be oscillatory. There is a oscillatory mode and is also an exponentially decaying mode. Another important point which you notice is that the oscillatory mode is practically not observable in i L2; that is a very important point. So, what you are finding is that, in this particular system for all practical purposes of course, we did make a few rounding of kind of approximations. But, for all practical purposes there is no, there is no observability of this oscillatory mode in this i L2. In steady state, i L2 turn out to be 0; i L2 is 0; V C also turns out to be 0. If I put t tending to infinity V L and i L, V L, V C and i L2 becomes 0 and i L1 becomes 10. The set if you look at this if V i is 1 volt, what are the steady state conditions here? In steady state remember, the inductor will become a short; a capacitor becomes open circuited. So, what you have really is, if this a short, this is open circuit; this is a short. Your current through here will be simply 1 divided by 0.1 which is 10 amperes; this current here, this particular capacitor here in steady state becomes open circuited; this inductor here becomes a short circuit.

So what you will find is eventually, you will have 10 amperes flowing here and going into this, but the voltage here is equal to 0. So the steady state, in case you excite the circuit with the non 0 input is going to be that i L1 and i L2 are going to be 10 amperes and V C is going to be 0. This is 10 and 10; this is going to be 0 as t tends to infinity. So, this is an important thing which you should be able to do. By setting t tending to infinity, you should be able to take out a steady state final that is the final values of these terms. Now, what you really are seeing here in fact this particular example, we do not we would like to interpret the results appropriately, because the Eigen vectors are in fact giving you some idea about the observability of modes. If you look at P which is right Eigen vector, you see that the second state that is i L2, the component is practically 0. That is why it is not observable, you will not see the oscillatory mode observable in the second state which is i L2.

Similarly, actually if you see here in V C, you will not see the exponentially dying mode to the same extent as you will see it here. In fact, it is almost 10 times more seen in these two states then it is seen here. So, that is what we really got in the final response if you focus on the sheet which I have written down here. You will find that, where as the coefficient of this e raise to minus 0.1t is 10 here it is only 1 here. So you will see that relatively, you can observe this particular mode more in these currents than in the voltage of course, voltage and current are in compatible, because the units are slightly different. So, one should take care in interpreting the results. You cannot compare 10 amperes with 1 volt so that is one issue which you should remember. So, what I said was only you know kind of giving a some relative feel. Now although, we have written down the response, it is a good idea to simply plot the response.

We will just plot the response using Sci lab and you can of course, see how the response looks like, because it sometimes difficult to picture if you are new to studying a dynamical system, how the response is going to look like. So, if you for example see, what is the response of i L1 or what is the response of V C? So I will just try to you know write down the response. So let us assume that, I will evaluate the response at some time steps.

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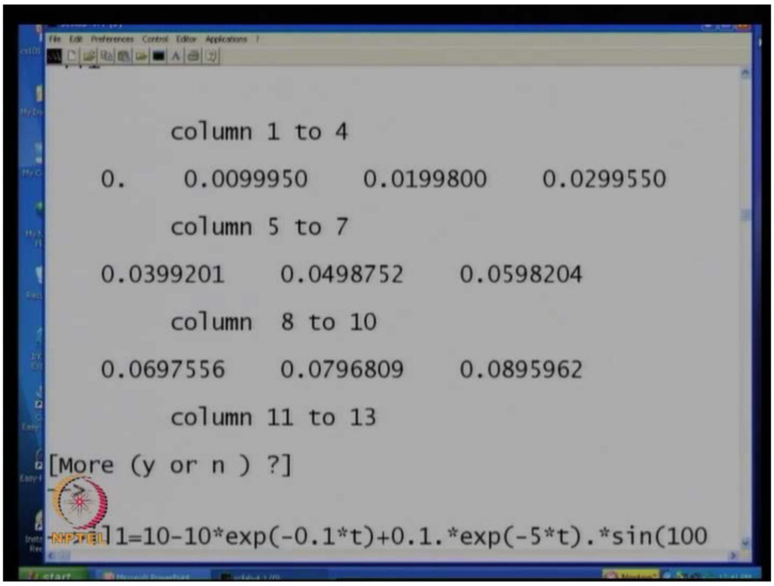

So, I will I will try to evaluate the response at the time steps t. So at discrete time steps, I will evaluate the time response and plot it. So the time response of course, of i L1 is given by 10 minus 10 star is the product to time plus 0.1 into exponent of minus 5t sine of 1000 and 5 times t.

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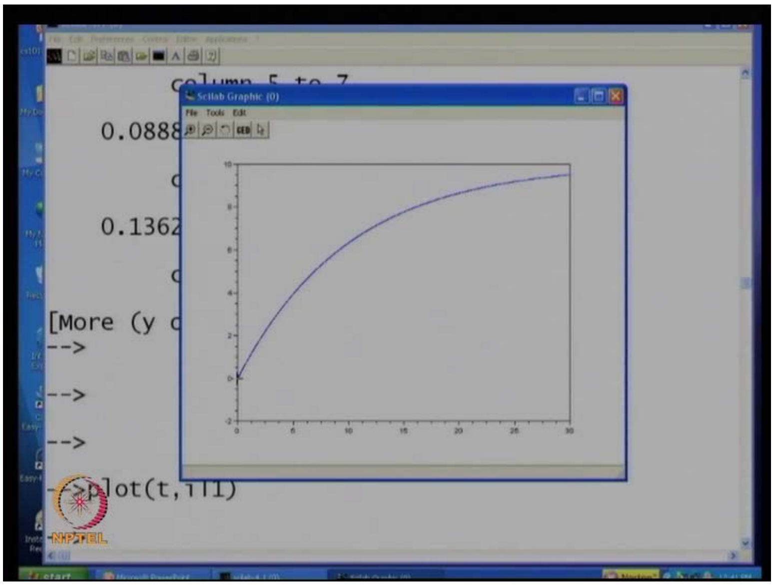

So, this is so i L1 I evaluated i L1; i L1 is simply this formula which is evaluated out here. I simply evaluated the formula which I had mentioned before. So what I will do is plot time versus i L1. So what you see is, this is the waveform of i L1; it practically, if you look at what it contains; it is basically containing mainly this term e raise to minus 0.1t it is very easy to see this particular term. Remember that, e raise to minus 0.1t has got a time constant of 10 seconds. So, around four times that time constant you will find that the system here is settling to its final value. An important point is that, you are not able to see much of the oscillatory response.

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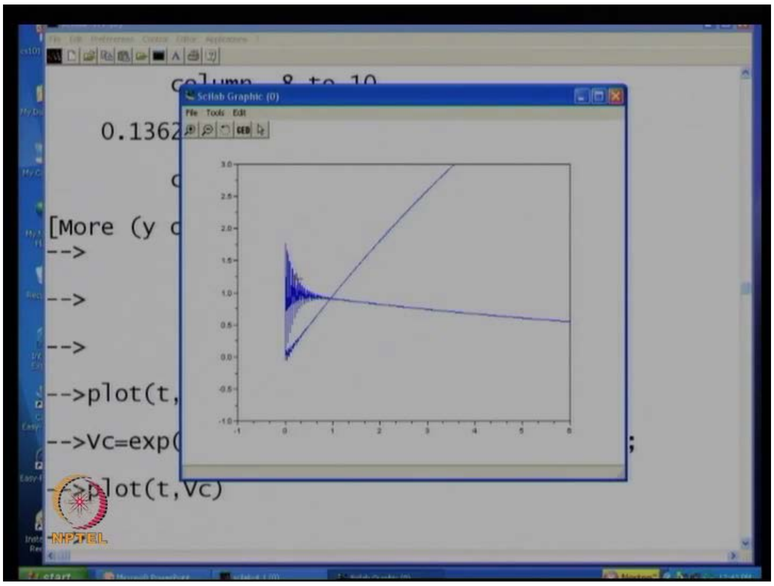

Let us just look at, in contrast V C; V C is nothing, but so I will plot V C, what you see is if this is i L1, this is what V C is. So V C of course, has got us this oscillation here this initial oscillation in fact is that 1000t Cos of 1000t, which you have seen here and there is a also a component which is decaying. So, you find that the response is of course, a superposition of the two modes and both modes in this case are fairly visible. So you see this oscillatory mode in the beginning of course, the rate of decay of the oscillatory mode is more e raise to minus 5t decays faster than e raise to minus 0.1t. So, this is basically the response which you get. Now if of course, I plot i L2; i L2 is simply 10 minus 10 star x 0.1 star t; so if I plot i L2, this is what I get. It is almost actually kind of its plotted over i L1.

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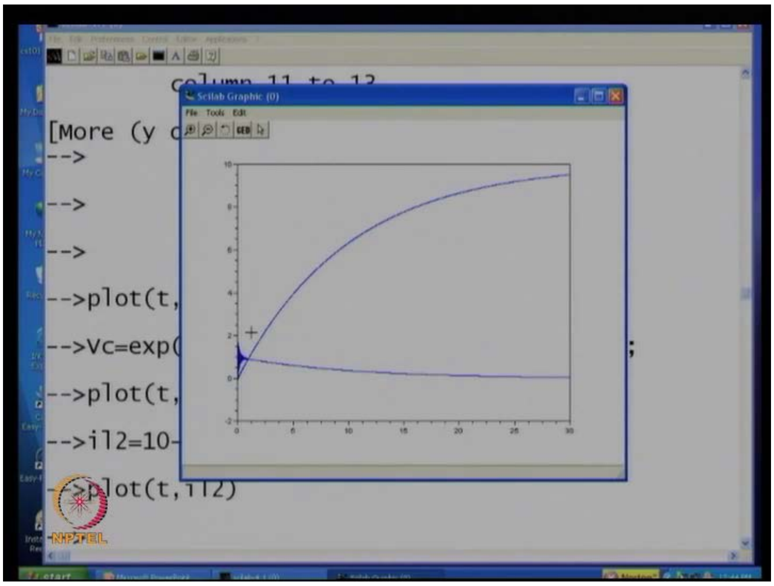

So, i L1 and i L2 practically have the same response. So, you see the different modes are seen to in to in different extents, in the different states. In this particular example, we could really interpret the presence of the complex Eigen value pair, the faster and slower responses, and also in some ways interpret the Eigen vector. In fact, in an this particular example I have chosen specifically to illustrate a very very important modeling approximation, which we will see in the next class. This particular system is in fact, what you call a stiff system which is a combination of a slowly moving transient e raise to minus 0.1 t, and a faster - relatively faster oscillation which also decays faster. So, these kind of systems are called stiff systems, and whenever you have stiff systems, we have may have problems solving them numerically, we shall see later. Numerical integration may be a problem using simple methods, but we also would be able to make some good, and modeling simplifications. So, with that we stop todays lecture. In the next lecture, I will introduce you to some very important modeling principle.