Power System Dynamics and Control
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Lecture No. # 42
Sub Synchronous Resonance Stability Improvement

I have hinted to you in the previous class, that the oscillations associated with torsion of the shaft of a turbine generator system, can be adversely affected by the behavior of the electrical network, that is the electrical under certain circumstances can destabilize the torsion oscillations, from a modeling perspective shaft torsion transient are faster than the transients associated with say relative angle swings, low frequency swings and frequency stability phenomena or the voltage stability phenomena which we discussed sometime back.

Therefore, the most important modeling difference is that for a study of a torsion transients and their interaction with the electrical network, one has to consider network transients and stator transients of a synchronous machine.

In today's lecture, we will continue our modeling, our effort in the previous class was directed towards trying to understand how a electrical network with series compensation in the form of a fixed series capacitor, (( )) and what kind of characteristics it has?

So, today's lecture we will try to complete that analysis and the icing on the cake, as far as the analysis is concerned is a starling phenomena, that is when we have a series compensated network with a series capacitor fixed series capacitor, we can actually cause shaft torsion oscillation to grow.

This phenomenon in fact is called sub synchronous resonance. It will become clear in this lecture, why it is called why the word resonance is used after all, we are talking of transient phenomena not kind of a forced response phenomena, but still you will understand why we can use the word resonance under these situations.

So, today's lecture we will complete our discussion on sub synchronous resonance, and we will also I will just give you a flavor of what we shall do in the next two or three lectures, that is the last part of the course on stability improvement.

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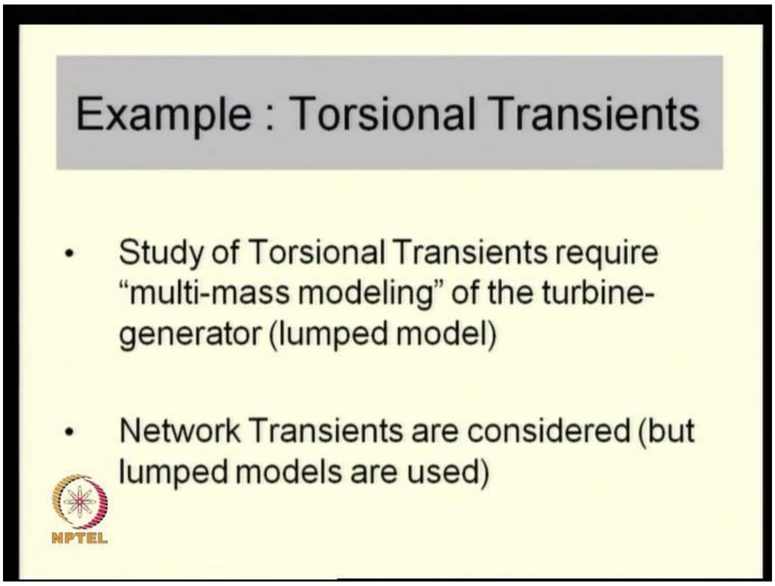

So, let us continue where we left off in the previous class, just to recap torsion transients require the modeling of multi mass modeling. Remember, that whenever we are studying torsion transients, the speed of all the turbines and the generator is not the same; though in the steady state they are same, the transient differences in speed are usually caused by these oscillations torsion. We consider network transients, because as we shall see the interaction with the electrical network under certain circumstances is very important.

But remember that we have to consider network in stator transients, but the frequencies involved are between say 10 hertz to around 50 hertz. So, often we can understand these phenomena using lumped models.

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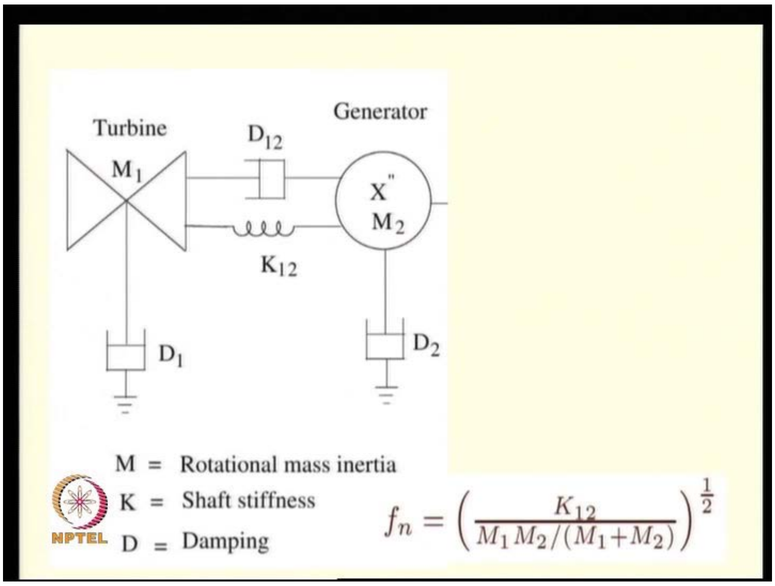

So, I just gave you a kind of motivational example, to show you that particular system. In fact, displace the phenomena of shaft torsion, oscillations is then the frequency; of course, depends on the shaft stiffness and the mass of the generator and the turbine

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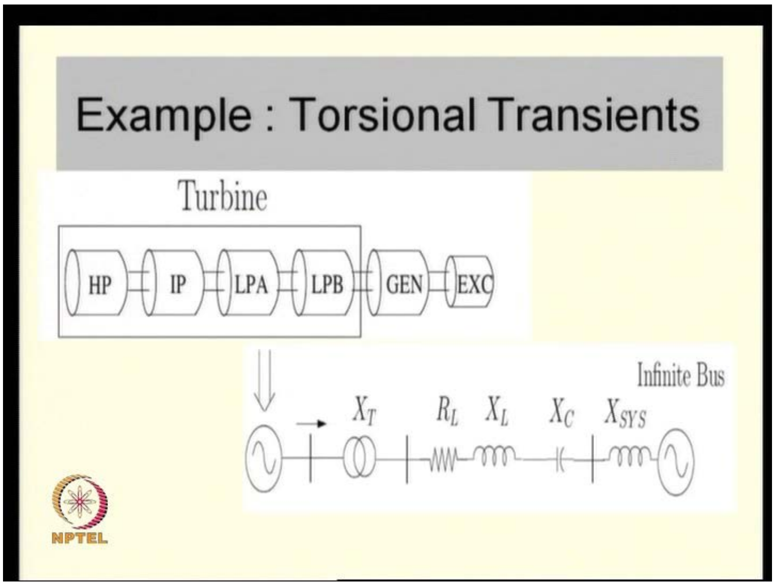

Now, we will try to understand the interaction with the electrical network. I hinted to you last time, that this problem arises often due to series compensation, that is whenever you get a fixed capacitor in the network; this electrical interaction becomes very prominent, in the sense, that they could be instability of this torsion oscillation. So, let us study this phenomenon, why it occurs?

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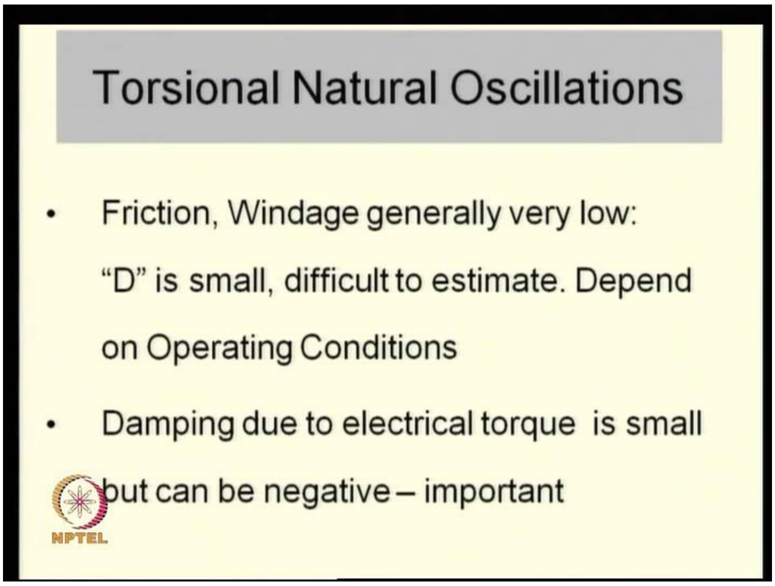

Now remember the natural oscillations of the shaft and mass and the shaft and turbine and generator system is effectively like a spring mass oscillation, but importantly the friction wind age is very low, is not much, the bearing friction, etcetera is very low. So, the damping, which we associate with this torsion oscillation, is actually quite small.

Now, electrical torque also affects the torsional oscillations. One can look at it, as an input, it is not strictly speaking an input. As we shall see the damping due to electrical torque can be even negative. So, that is very important, negative damping means that the oscillations can grow. So, this is how it can occur.

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Example : 6 mass

Mass	Inertia constant (H)	Shaft section	Spring constant pu torque/rad
HP	0.092897	HP-IP	19.303
IP	0.155589	IP-LPA	34.929
LPA	0.858670	LPA-LPB	52.038
LPB	0.884215	LPB-GEN	70.858
GEN	0.868495	GEN-EXC	2.822
EXC	0.0342165		

So, before we try to understand you know the properties of an electrical network in the previous class. In fact, the model network, we shall just go through that again, let us just take an example, of a 6 mass system there is a high pressure turbine, there is a intermediate pressure turbine, the low pressure the two stages LPA and LPB there is a generator, there is a rotating excitation system, a brushless excitation system.

And the shaft sections which interconnects the HP and IP turbines, the IP and LPA turbine, the LPA and LPB and so on.

So, they are coupled while the shaft in this fashion the HP is coupled to the IP to the LPA and so on.

So, this is the typical data given in a first bench mark model, which is available in the literature. Now, for example, how do you take out the frequencies? Let us look at this, turbine generator system in isolation; we say it is not connected to the electrical network. Let us assume the damping is 0 it is not zero, but it is a bit difficult to estimate, it is quite small.

So, damping from purely mechanical sources is assumed to be 0. So, what are the model properties of this particular system? So, if I am going to try to analyze this system remember, what are the states? The states are the angular position of all the masses and the speeds of the masses.

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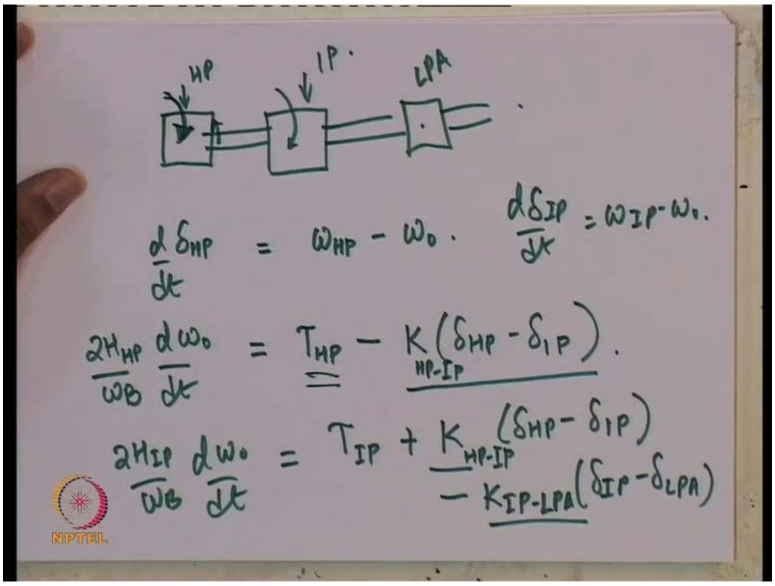

Remember, that if you have got two masses connected by a shaft, suppose this is the HP turbine and this is the intermediate pressure turbine of a steam, turbine generator system. So, you will have d HP by d t is equal to this is the change of the angular position and if you write this in per unit it will look like, this is equal to T HP. Remember, now we have got several turbine stages and the mechanical torque. In fact, is created in all these.

So, for the HP turbine and the mechanical torque may be T HP minus, we do not have any electrical torque, right at this mass, because this is the turbine. What instead, you will write here is K times, the K is the shaft stiffness delta HP minus delta IP.

So, similarly for the IP turbine you will have two h IP by omega B d omega naught by d t this is in per unit. So, T IP minus K times, so this is K HP I p. So, it will be plus delta HP minus delta IP minus K IP, LPA delta IP minus delta LPA. So, if the LPA turbine is connected here, the turbine equals the equations of the masses, the motion of the turbine masses. In fact, d delta IP by d t is equals to omega IP minus omega. So, this K of course, is the shaft stiffness. So, the forces are there is a driving force due to the mechanical torque created in the turbine and if the shaft is slightly twisted, you also have a torque, I mean there is a torsional force. So, you will have equations for all the turbine masses in this fashion and for

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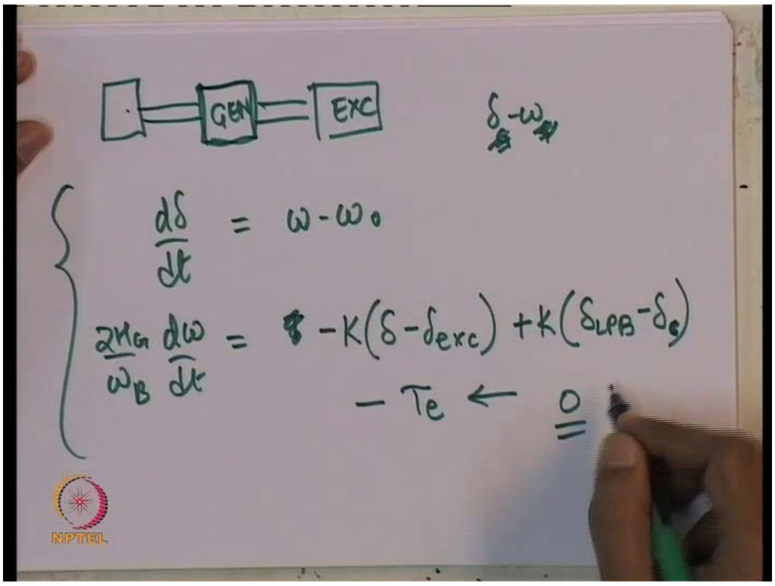

If you look at the generator itself, suppose this is the generator and this is the exciter the equations are d delta by d t del. I will not use and subscript for generator, you will call it delta itself and so this would be omega HP and this should be omega IP.

So, d omega by d t omega in delta is the speed of angular position of the generator. In the previous class, I had written a subscript G G, but for simplicity, we will just not put any subscript. So, this will be 2 h generator by omega B. So far a generator of course, there is no mechanism, there is the driving force actually comes from the shaft. In fact, you will have minus of K delta minus delta exciter, plus K time's delta LPB which is the mass connected here the turbine minus delta minus the electrical torque.

For the time being we will assume it to be 0 actually, because the electrical torque is a coupling with the electrical system. So, this is the expression for the generator similarly you can have an expression for the excitation system.

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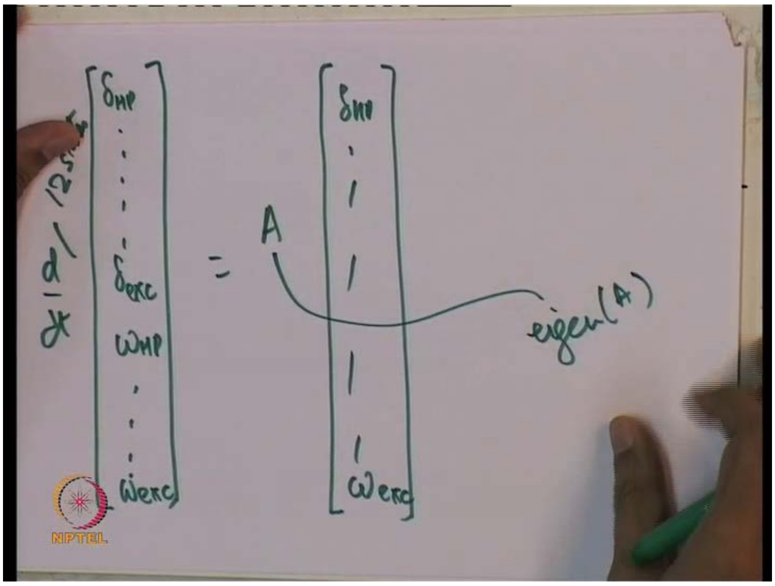

So, our states eventually will be for A S, you will have delta HP right up to delta x and omega HP right up to omega x. So, for 6 mass systems, you will have twelve states. So, d by d t of this will be a matrix a into the states again delta HP to omega x.

So, we can isolation, if you take the turbine generator system. In fact, we can obtain the Eigen values of A. So, find the Eigen values of A. So, what we will do is, first try to see that we get these torsional oscillations. We will just have a quick Skylab program to really get this. I have already run it before. So, I will just show it to you again, now you look at this program.

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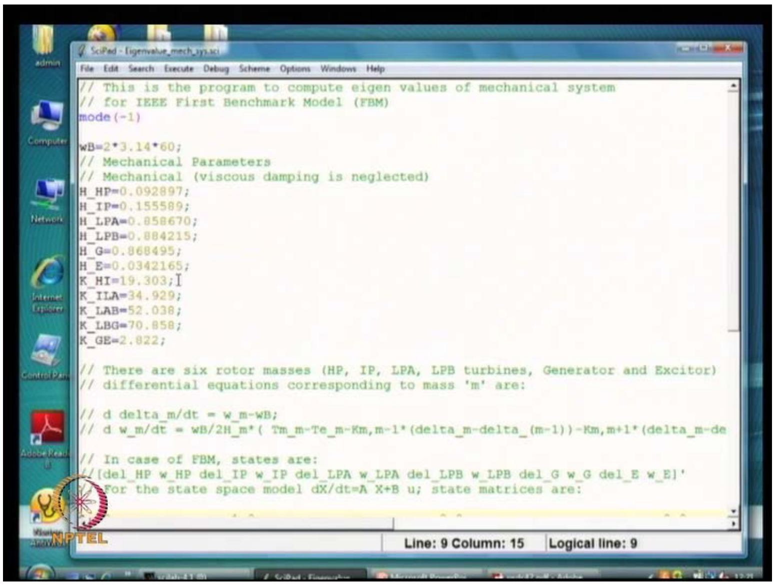

So, if you look at this program, what I have done it may not be absolutely visible on your screen. So, I will just quickly read through it, this is the data from the IEEE first bench mark model, you will assume omega is 2 point to 60 inches. So, 60 hertz system and the data corresponding to the turbine is given here, h the inertia constant value as well as the shaft constants.

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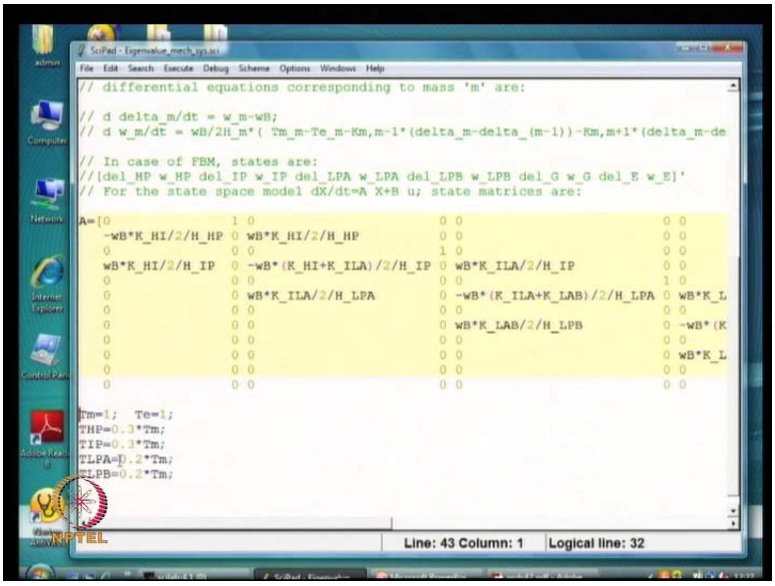

Now, this matrix is very simple. So, the states are arranged, of course, here as delta HP omega HP delta IP omega IP and so on. So, this is not required here, this part of the program is basically T m is equal to one T e is equal to one and T m actually is made out of components. You know in steam turbine, with many turbine stages the mechanical power actually is obtained as the sum of all the mechanical powers generated at the individual turbine. So, for example, the HP turbine could be 30 percent of the total mechanical power, may be generated at the high pressure turbine, 30 percent at the ii p turbine and 40 percent in the 2 LP turbine stages.

As far as, this particular analysis is concerned, you have got the matrix. Let us just take out the Eigen value. So, we will do a limited analysis here.

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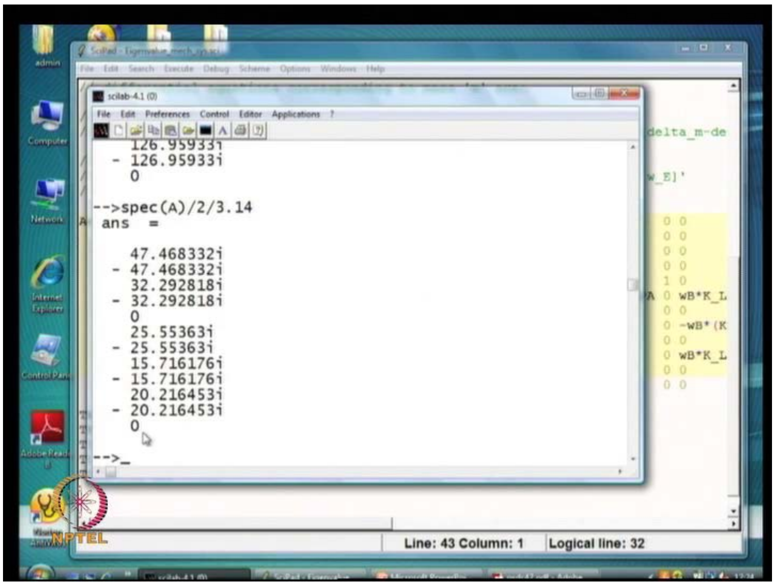

So, we will run this program, execute and form the a matrix. So, this is just forming the a matrix mechanical system. This is the mechanical system in isolation, assuming that electrical torque is not is 0 or it is a constant.

So, what you will find is, if you take out the Eigen values. The Eigen values are as follows, so this is in radian per second, so if you look at this, there is basically 6 pairs of Eigen values 5 of them are torsional oscillatory modes. So, do spec a divided by 2 divided by 3.14, this is going to give you the frequency of these oscillations as well.

So, this is the frequencies of the oscillations, which you see 2 0 Eigen values, because you have got absolutely an isolated system, and there is no friction.

So, In fact, if you have got a completely isolated system, that is a spring mass system of this or even if you take a isolated spring mass system, you will find 2 0 Eigen value, if you do not have friction, if you consider friction then one of these Eigen 0 values will disappear.

So, remember that the 0 Eigen values are generally related to the common motion, where all the masses move together. This is something we also did, when we considered the analysis of a two machine systems. So, similar things keep popping out in different situations.

So, to know friction you have 2 0 Eigen values and both these Eigen values are associated with the common motion. In fact, this can be even inferred from the Eigen vectors. So, for example, if you take out run this command, if you look at v colon one. So, what you will notice is, remember that, the states are arranged as delta HP omega HP delta IP omega IP and so on. What you see is delta and omega the terms which you get is of course j this is complex number, and then this is a real number. If this is a complex, this is a purely imaginary number, and then is a real number. That kind of behavior is not very surprising.

Whenever you have got an oscillation and delta is the derivative. The derivative of delta is omega. You know it is proportional to omega; in that case it is obvious an oscillation, if you have then delta and omega will be 90 degrees phase shifted. That is the reason why this delta Eigen vector component corresponding to delta is ninety degrees, so the Eigen vector corresponding to the speed. So, this is why we have it. You notice the different masses at different observability as far as this mode is concerned. For example, this mode is will be hardly visible or rather is not very well visible in the last 4 states. That is delta generator, omega generator, delta exciter and omega exciter.

So, this is the property of this mode, also you notice that the delta component here is negative while for the IP component it is positive. So, it means that when one of the masses or rather the displace of one of the masses is going ahead, the other is going behind. So, there is a kind of very common, the mode shape in any kind of oscillatory mode.

So if this is basically giving the nature or the relative movement of all the states given that the first mode alone is excited. So, if you lo at the second mode you will have some other property and so on. Now, one thing of our interest of course, is the 0th mode. So, if you at the 0th mode that is 1 2 3 4 for 5 th mode, one interesting and good thing you notice is that this particular mode appears to be equally observable in all the states all the deltas.

So, this is as far as our analysis of the mechanical system taken alone, without worrying about the electrical system. You just take the mechanical system in isolation these are the Eigen values.

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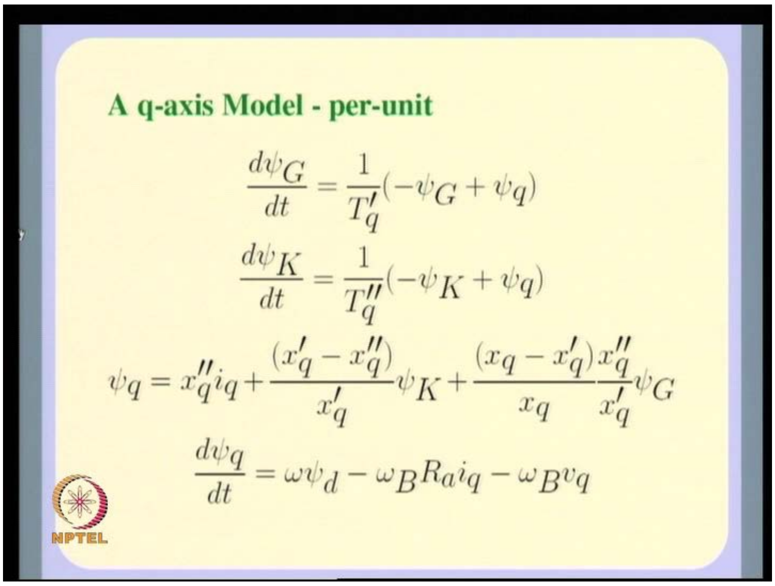

What about the electrical system, if you recall in the previous lecture we had gone through the modeling of the electrical system which consists of a synchronous machine connected to a transmission line connected to which is compensated by fixed series capacitor. So, you will just quickly go through this.

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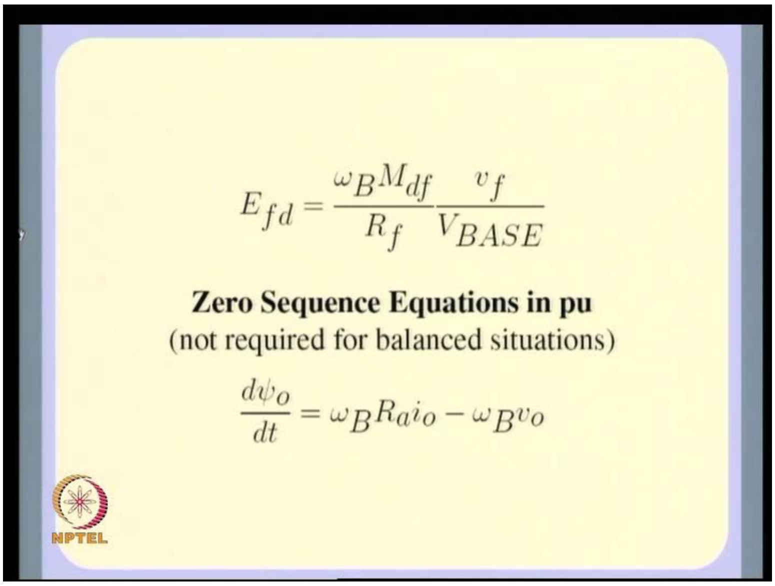

So, this is the synchronous generator model, but we did make some assumptions to make our analysis a bit simplified.

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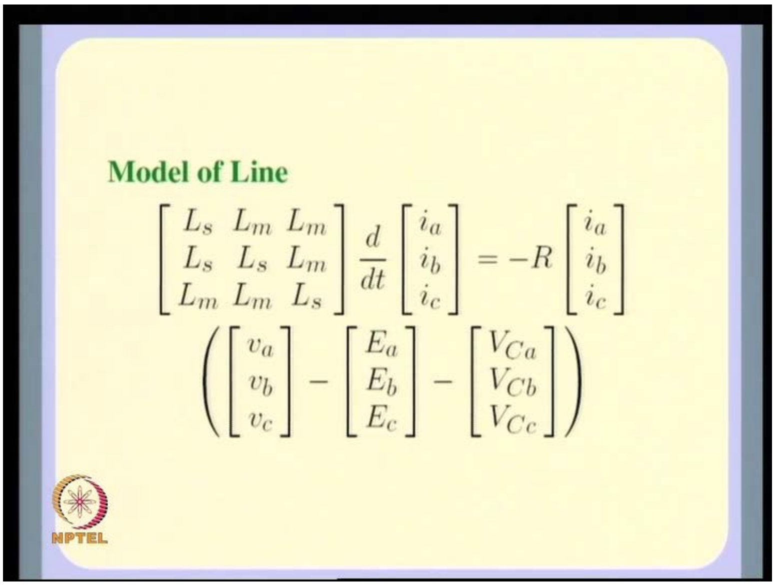

So, the assumptions were of course, that the line is a lumped is represented by a lumped inductor.

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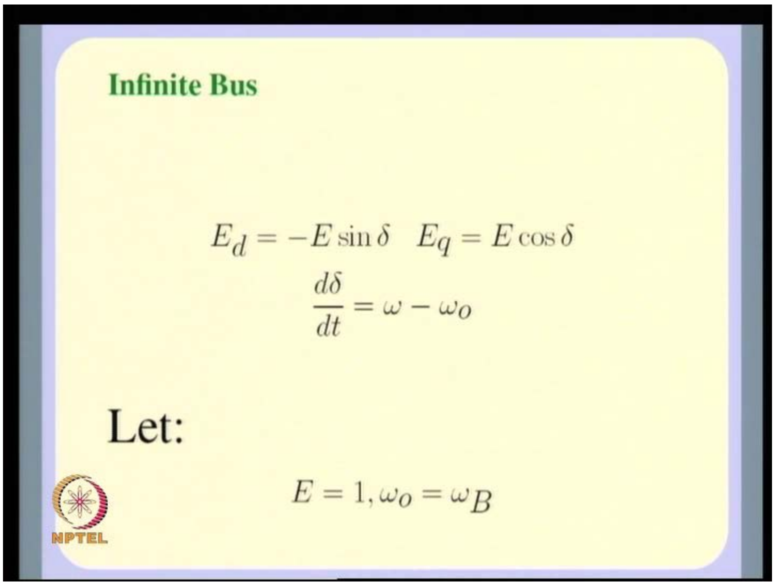

The second assumption, we made was we of course, we to E the infinite bus voltage as one and omega naught the frequency of the infinite bus as the rated speed. It was just a simplification it need not always be. So, that the infinite bus to which this synchronous machine is connected to, its speed should be equal to the rated speed or its frequency is equal to the rated frequency. This is just a simplification, we are doing for this analysis which likely to be very close to the rated frequency

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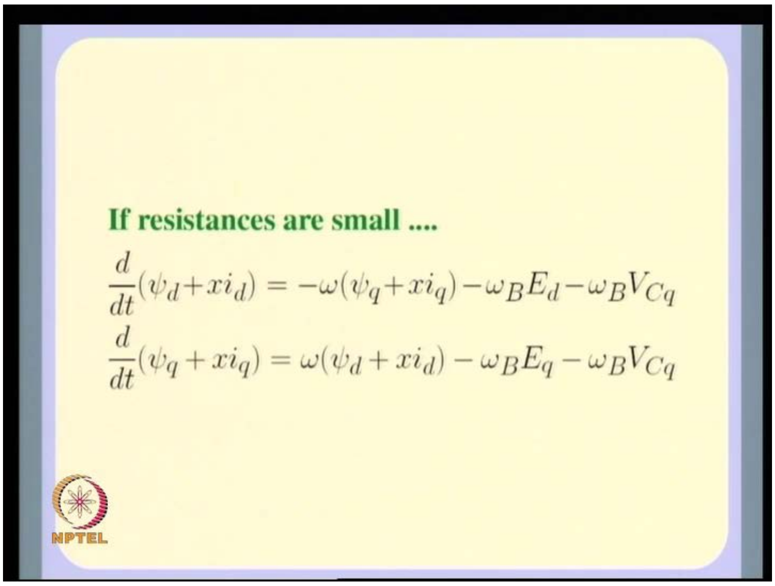

If resistance, are small we can combine the flux and the current equations. Remember that there was a redundancy of states, because flux and currents are related by an, algebraic equations. So, we do not have to separately write differential equations for the state of flux and the current. So, we combined those equations and made one equation and if resistances are small, this can be easily done.

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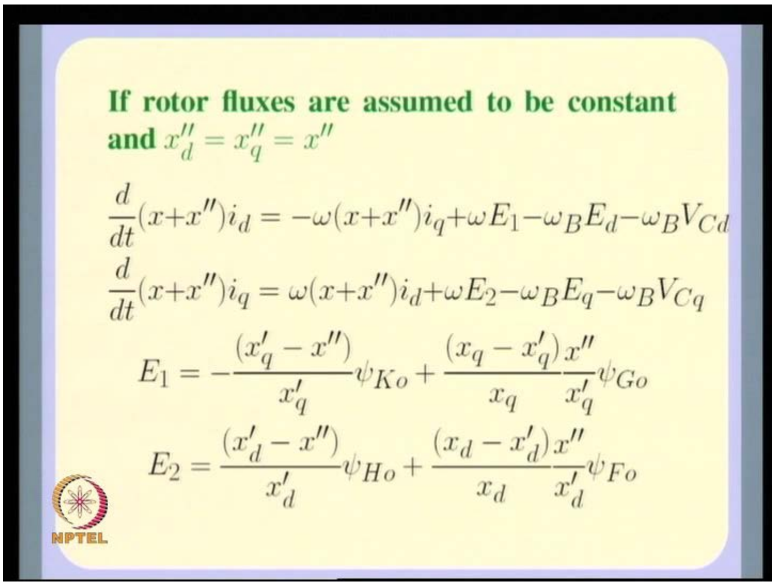

Thereafter, we made another assumption that the rotor fluxes are assumed to be constant. So, we can define E 1 and E 2 in this fashion and its possibility to substitute for psi in terms of I psi d and psi q in terms of i d and i q and get the first two equations. This is something we did in the previous class. So, I will not spend too much time on revising that again.

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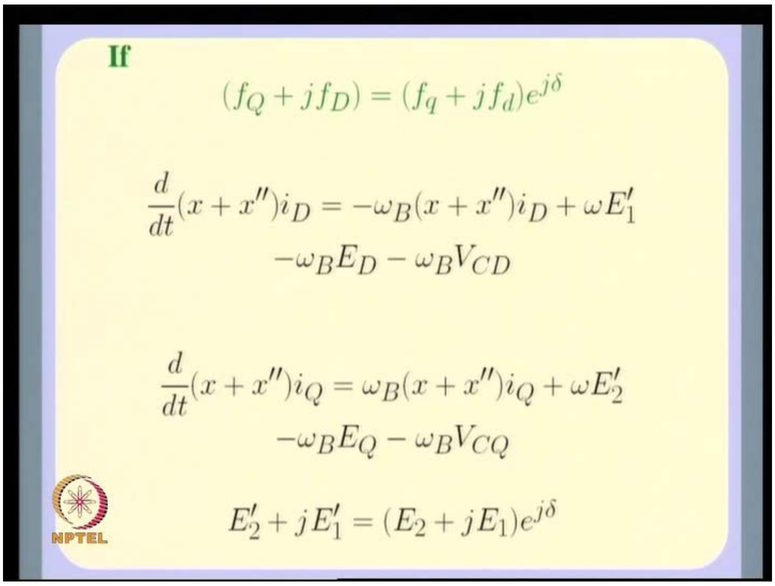

We then converted these differential equations into the upper case d and q variables. The advantage of doing that of course was the infinite bus voltages will become constant, if I use this particular transformation of variables.

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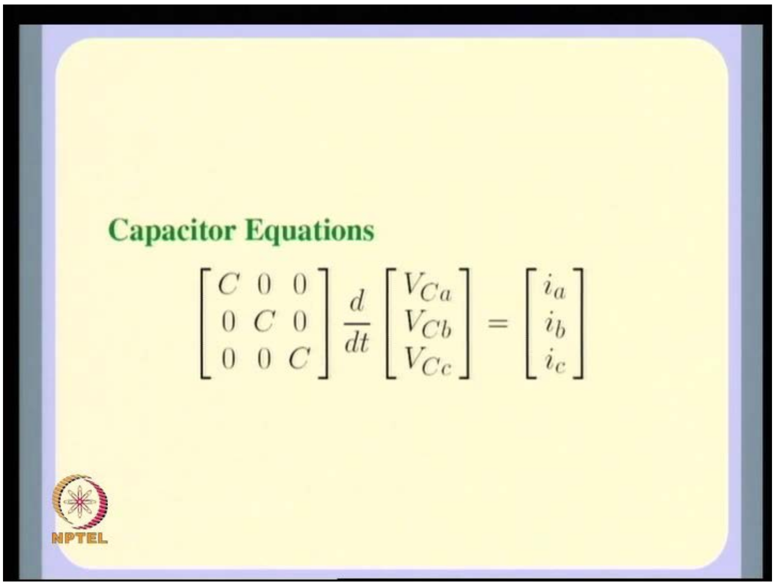

Now, the capacitor equations were these in the a b c frame of reference.

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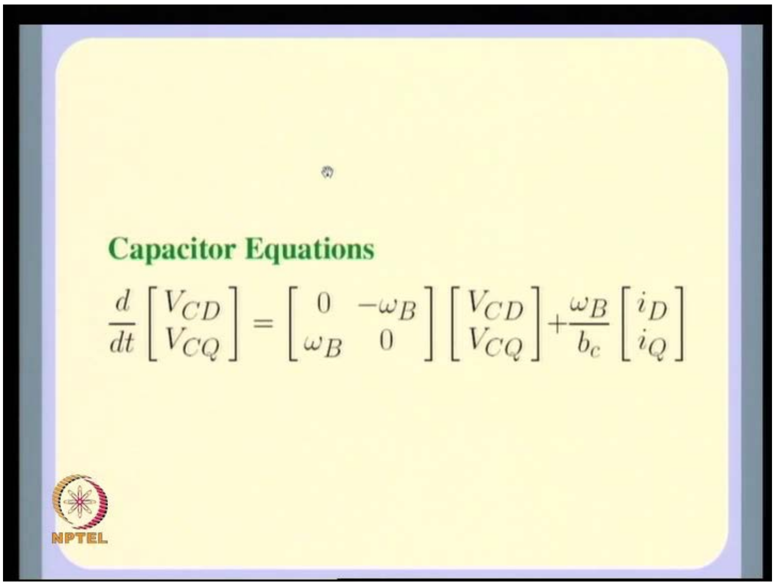

Then, we converted them to the upper case D and Q variables remember the upper case D and Q variables are obtained by using of transformation, like park transformation. But instead of theta we take a fixed omega naught T, the theta being the position, actual angular position of a synchronous machine.

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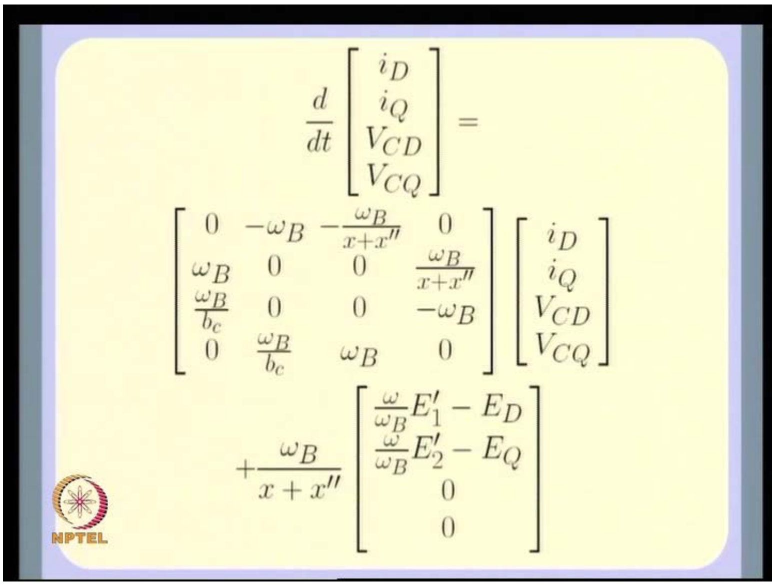

Now, if you combine all these equations you get, a state space equation of this form. So, you have got a linear state space equation. In fact, this a matrix is constant. But remember that, these are the two sources even in E 1 dash and E 2 dash are dependent on delta as well as the fluxes, the rotor fluxes which is the simplified analysis, we can assume to be constant.

But one thing you notice here is that the electrical system is going to be affected by omega, there is an omega here, which is the speed of the generator and E 1 dash and E 2 dash are also functions of delta which is the rotor position of the synchronous generator. So, what you notice here is the some kind of coupling.

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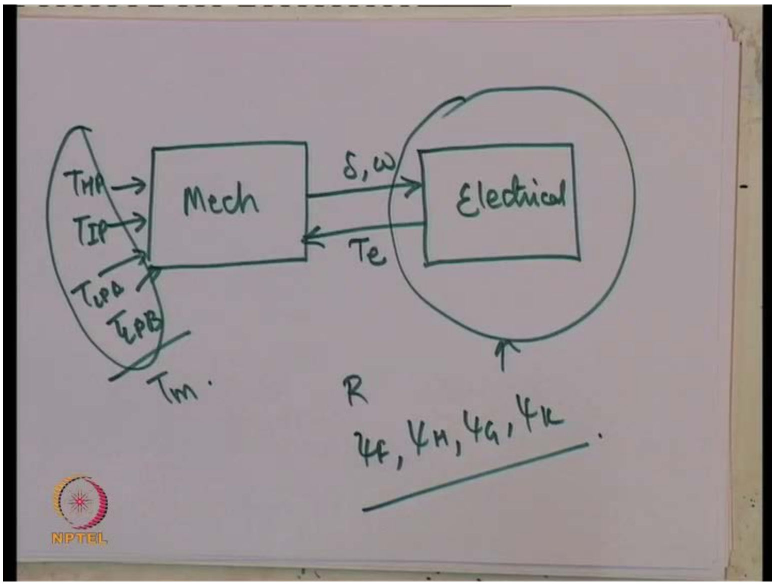

So, look at the mechanical system, you are giving, the mechanical torques are of kind of being generated at the various turbines, so LPA, LPB and electrical network. So, delta and omega of the generator, affect the electrical variables, as we are seeing in these state space equation and the electrical torque, affects the mechanical equation.

So, this is cumulatively gives you T m remember that. So, this is how really the interaction between the mechanical and the electrical system takes place. What we will do now, in an isolated fashion, just as we did for the mechanical system. We will assume that delta and omega are constants and just see what is the behavior of the simplified electrical network with all those assumptions, we have made.

What are the assumptions we have made, resistances are small of the stator as well as the transmission line, delta omega will assume to be constant, rotor fluxes are assumed to be constant. So, these are the assumptions we are making. So, we will just look at the properties of a electrical network which is connected to electrical network which connects the synchronous generator to an infinite bus.

So, we will just run write a simple program, to obtain the Eigen values of the same matrix simple. If you look at the same matrix, what you will notice. There is a kind of nice symmetry avail is not symmetry, really this is a kind of skew symmetry. So, what you notice here is minus omega B omega B here there is a minus omega B here is omega B this is omega by b c, there you will have minus omega by x plus x double dash, this should be minus here there is a small error please note it. There is a minus sign here for this element 2 4. So, there should be a negative sign here.

So, if you look at, this is a matrix or the state matrix corresponding to an electrical network lode at in isolation, actually there is a coupling with the mechanical system which occurs, because of these terms. So, if we just run a simple skylab program, to obtain the Eigen values of this system. So, if I have already programmed it. So, I will just directly run it. So, what we need to do is of course, execute a simple program, in which I have certain values of parameters, I have taken out. So, if you look at this, written out to be. So, if you take out the Eigen values of a compensated system, what you get is of course, it is a 4 by 4 matrix. So, you will get a set of Eigen values.

Of course, if you ought to really see the program, because I have not told you what the total x of the system and what the b c etcetera is. So, I will just open a file. In the electrical system, this X l is what I will be using is X l is the total x plus x dash x double dash plus any other reactance which may come in this series path. Its value is 0.7 and X C. So, X C is 0.3. The compensation level it is 0.3 divided by 0.7. So, this is the percentage of compensation which is used.

Now, the point is that the Eigen values of this system are these. Now what you notice is that, they are the two oscillatory modes. In fact, you will find that this is 6 23 this is the imaginary I is a purely imaginary number square root of minus 1. So, this is effectively a complex pair. There is no resistance considered. So, you get no real component now this these complex conjugate Eigen values or in fact super synchronous in the sense that the frequency of these is greater than 60 hertz.

So, one of the frequencies is coming out to be greater than 60 hertz and the other one is coming less than 60 hertz. So, you have got two frequencies, one is super synchronous, one is sub synchronous.

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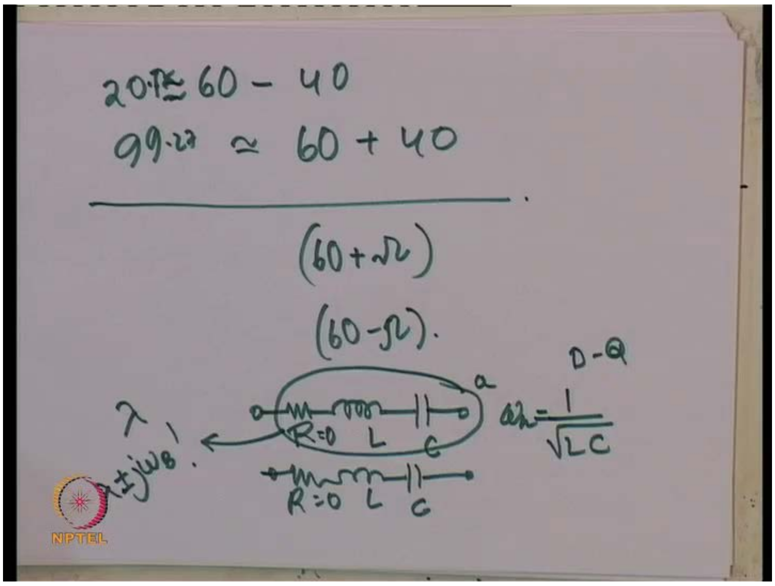

Now, if you look at these modes carefully one of the modes is 60 if you look at 20, 20 is nothing but 60 minus 40 and 99 is roughly 99.27 is roughly 60 this is roughly 20.72 this is 60 plus 40. So, it appears, that the Eigen values are appearing in a certain form. So, you have got 60 plus some Eigen value some frequency and 60 minus some frequency this is very typical. Why is it typical? If you take system like this, suppose this is a 1 phase of a 3 phase line if the Eigen values of this system lets say R is equal to 0. So, you have just L and you have got C.

If you just take this single phase in isolation, then you write down the equations in the a b c, a variables you will find that; obviously, there is going to be an oscillatory Eigen value which has got a frequency 1 by root L C .

Now, this is in the phase domain, if I convert a balance 3 phase network of this kind with R is equal to 0, of course, this R is equal to 0 is not important in this it is true, if R is not equal to 0. The point is that, if I take this and write the differential equations for a linear network. You get an Eigen value lambda for the a phase and similarly for the B and c phases, an interesting thing is that, whenever reformulate these equations in the D Q variables, then your Eigen values will be plus minus G omega B.

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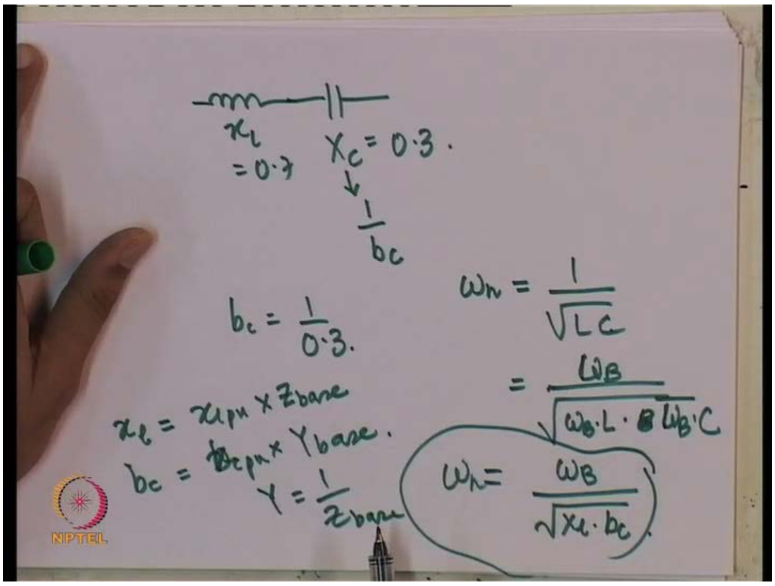

So, remember for example, in this network, you have got X l is equal to 0.7 in each phase and x c is equal to 0.3. So, if you look at x c is nothing but 1 by b c. So, b c is equal to 1 by 0.3.

Now, if you look at this phase, in isolation if you have just written the equations of this phase, the differential equations of one phase in isolation, you would have the natural frequency of this is 1 by root L c. So, this should be equivalent to having omega B into L into b c, you can say c is nothing but what we will do is, just multiply omega B in the denominator. So, you will get this. So, this is nothing but equal to omega B divided by root of X l into b c. So, this is what we get as a natural frequency.

Now, of course, X l is given in per unit. So, X l actual is nothing but X l in per unit into z base and b c in per unit is equal to b c, actual value is b c per unit into y base. So, y and z base are related like this.

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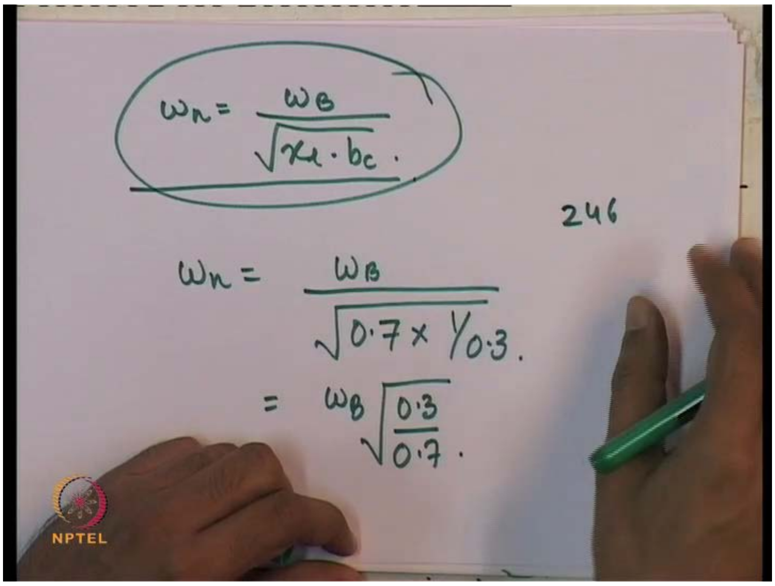

So, it is obvious that the natural frequency will be the same, whether you express X l and b c in per unit in actual values.

So, this is the expression of the natural frequency of an electrical network considered, in isolation. So, if you look at this particular equation, for these the values which we discussed here. What we will get if omega n is equal to omega B divided by root of 0.7 into one upon point. So, that becomes omega B into 0.3 by 0.7.

So, if you look at this omega, B is nothing but around 377 for 60 hertz. So, omega B into square root of 0.3 divided by 0.7. So, the radian frequency comes out to be 246. So, as I mentioned sometime back, the D Q in the D Q reference frame, formulate your equations and compute the Eigen values.

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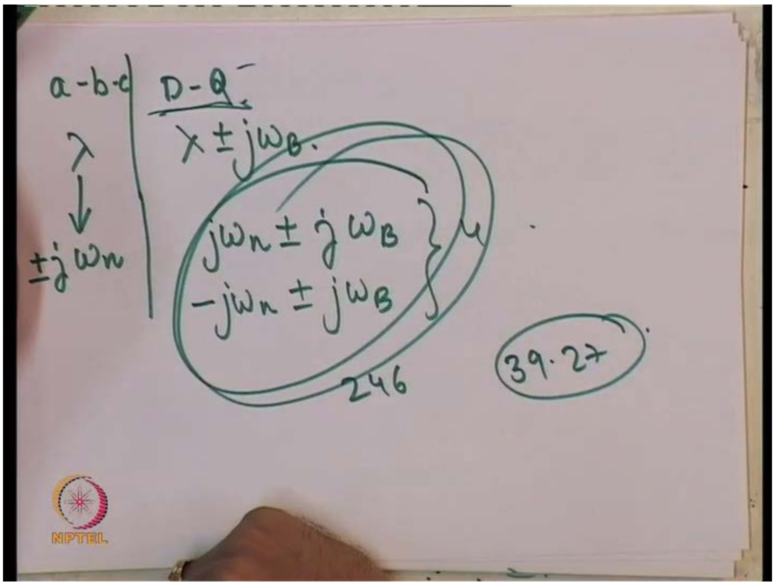

Then your Eigen values will be suppose, in the a b c or each phase taken individually you got an Eigen value of lambda in the D Q you will get omega b. So, if your Eigen values in the a b c, turn out to be plus or minus omega n. What you will have is j omega n plus or minus j omega B. So, if for minus omega and you will have omega n plus or minus j omega B. So, you will get 4 Eigen values like this, in D Q reference frame. The 0 sequence of course, is neglected with the assumption, that everything we are considering here is balanced.

So, if you look at 246 radian per second, which is appearing on the screen, and actually take out its frequency. So, this is divided by 2 divided by 3.14 this is the hertz frequency its 39.27. So, it is not 40 exactly, its 39.27. So, the 4 Eigen values, which we got by doing this analysis, in the D Q reference frame is these Eigen values.

So, remember that you get these kinds of complex pairs of Eigen values. So, if you got a series compensated network you do get these oscillatory Eigen values and interestingly some of them are sub synchronous.

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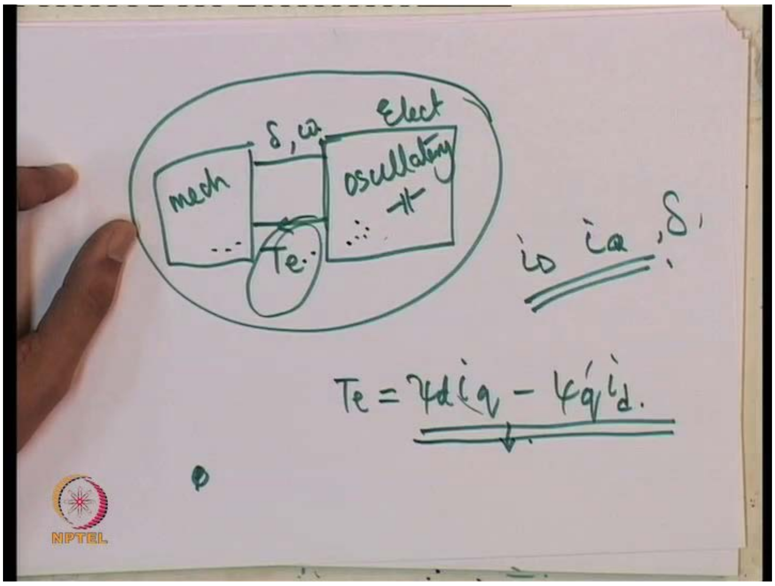

So, the point now is, you have got an electrical network, which is series compensated it has got oscillatory Eigen values or oscillatory response and the mechanical system also is in oscillatory response delta and omega T e.

Now, you can put the electrical and the mechanical system in isolation and say that the earlier oscillatory response. This in case, you have got a series capacitor, then you have got an oscillatory response, with sub synchronous and super synchronous components.

So, the thing is, when you put these together, the Eigen values of the system, becomes the coupled system. Remember, that delta and omega affect the electrical system, if you look at our final equations, here the electrical system is affected by omega and the mechanical system is affected by the electrical torque, electrical torque of course, is equal to psi d i q minus psi q i d.

So, we can actually compute this, from the electrical states i d and i q and of course, there will be also dependent on delta because here it is, i d and i q are lower case. So, you will have a dependence on delta as well.

So, in fact, the point is, the equation is non-linear the product term. Remember, psi d is related to I psi d is related to i d and psi q is related to i q. So, this is an non-linear relationship. This relationship is not linear. T is nonlinearly dependent on the terms of the electrical system, the synchronous generator in the electrical network and the electrical network also is in a non-linear way related because this comes out to be a product term. You know E 1 dash will be a function of delta and omega is also a mechanical variable.

So, this kind of non-linear relationship exists. What you need to do is, which not compute will or rather compute the linearised system of Eigen values.

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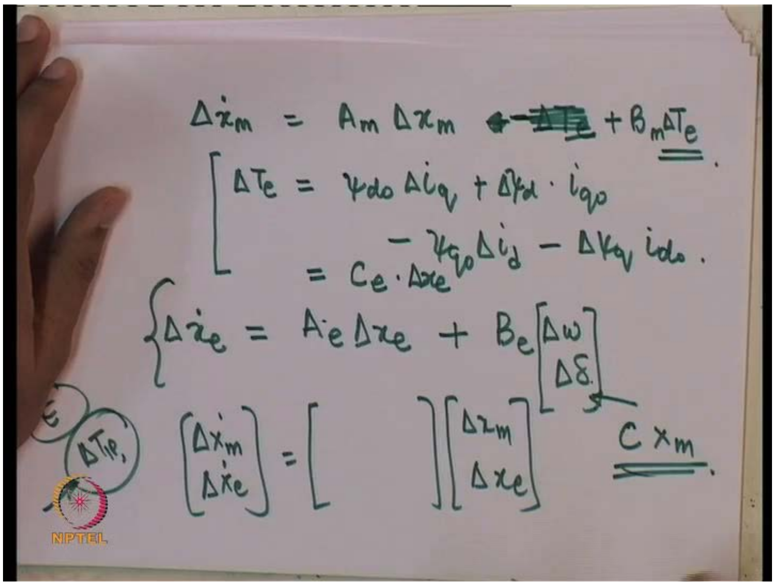

So, what you need to do, the mechanical system you will have the delta and omega variables. So, I will call them x m is equal to A m into delta x m minus delta T e. that is not going to affect all the states directly, it will affect the generator states directly. So, you actually have B matrix into delta T e and delta T e itself is equal to psi d 0 delta i q minus delta psi d into i q 0 minus psi q 0 delta i d minus delta psi q i d 0.

So, this is the coupling thing, similarly you have got the electrical network equations, electrical delta x c, this you have to do a linearization. This a matrix is of course linear, you do not have to bother the nonlinearity comes here, because this is a non-linear this is how you have to linearize this delta T e plus B e into delta omega and delta, delta. The delta, delta and delta omega are in fact, related to x c. So, they are some matrix c into x m. So, I will call this also as, some kind of C e into delta x e and this is C into x m.

So, you have to couple these two equations and get one grand state matrix. So, you will get one grand state matrix of this kind you will assume all the inputs delta IP HP delta. The mechanical inputs are constants. So, mechanical power in torque inputs to all the turbines will assume to be are constant and also the infinite bus voltage will assume to be a constant. So, this is how you will formulate your equations. What I have written here is absolutely general, you can relax all the assumptions which we have made so far, relating to the resistance of transmission line, the rotor fluxes x d double dash not being equal to x q double dash is not being equal to x T double dash.

So, all these assumptions can be relaxed and this can be an absolute detailed model. You need not take the simplified model, which I had discussed sometime back, the 4th order model, that assume the rotor fluxes are constant, resistance is were small and so on. You can relax all those assumptions. Now, one small thing which I probably bit out of place, but I missed telling you is that, there is nothing in this particular rule which I gave you relating the Eigen values, when equations are formulated in a b c reference frame and the D Q reference frame, remember there is nothing special about this being purely complex.

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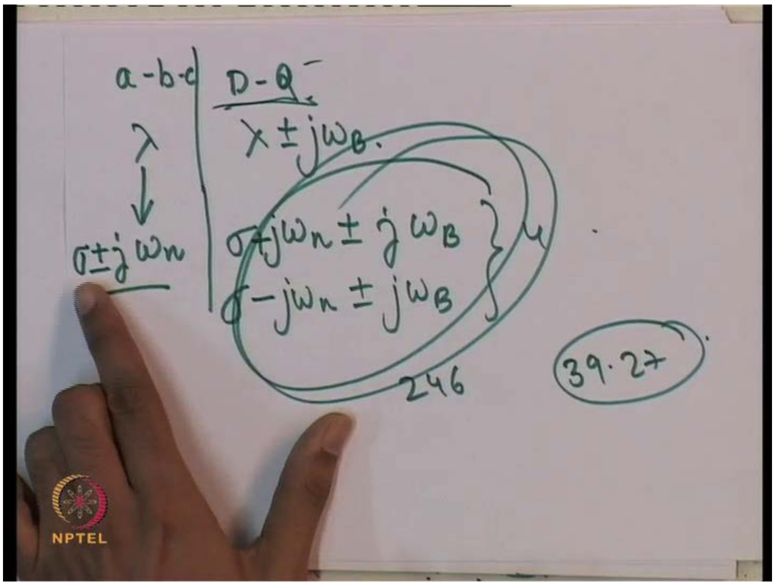

So, if this is sigma plus j omega n, this will be sigma plus. So, this sigma will appear here also. So, there is nothing special about this being purely imaginary. This kind of relationship holds to even this is a complex number with a real part. It will also appear here. So, this is something which I forgot to tell you.

So, we can actually using more detailed models, put all these things together. Now, the point which is very important is that, when I said that the electrical and mechanical system interact with each other what happens, well you can actually do the Eigen value analysis of the mechanical and electrical system put together is something what we did here, put them together and just do the Eigen value analysis.

But, even before we do that lets have try to have some kind of insight, into what will happen in case, the mechanical oscillatory frequency becomes close to or equal to the oscillatory frequency of the electrical network. Let us look at it in a cause and effect manner. So, this is also called at damping torque kind of analysis.

Assume that the mechanical system is oscillating at a fixed frequency, an undammed oscillation is there, suppose of the mechanical system.

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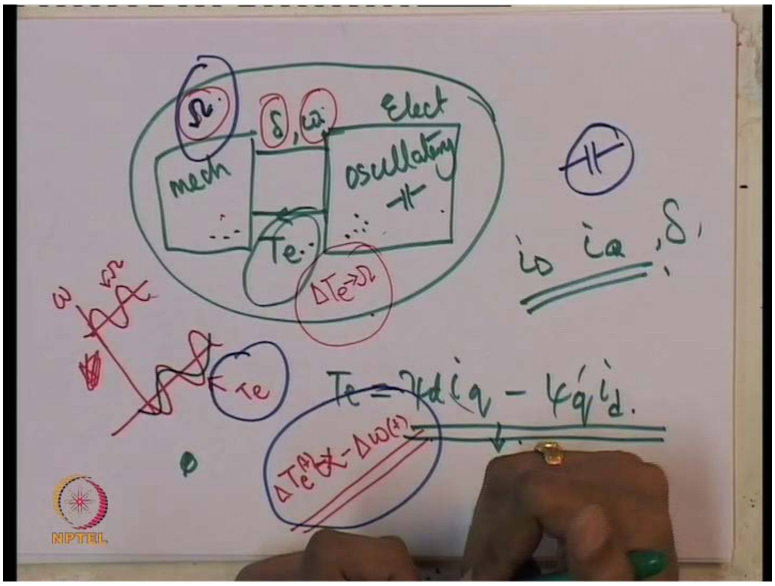

Let us say, its oscillating at a frequency omega. So, if this oscillation omega is observable in delta and omega, then the electrical network is like a input to the electrical system, let us assume that the systems are kind of decoupled.

So, this is the kind of input to the electrical system. The electrical system will respond and you will find that in delta T e also you will find this oscillation of sigma. Now, if this delta T e the oscillation, which is created, is such that it actually enhances the oscillation which is already there. For example, omega of the generator is oscillating at a frequency omega is the frequency of oscillation of the generator speed, around the equilibrium value. So, turns out that the electrical network has an electrical torque which is created like this.

So, electrical torque is like this. So, electrical torque is in some way, if you look at these two way forms is some way proportional to minus of omega. So, suppose this is true. So, you know what this looks like this is a kind of a situation, where the mechanical system is getting an electrical torque, which is proportional to negative of the speed of the generator. The things are exactly in outer phase, in such a situation, what will happen if the electrical torque will enhance the oscillation, it is almost like having negative viscous friction and the oscillation will increase.

So, this is the kind of a cause and effect non rigorous reasoning while, sometimes it is possible that the electrical network can cause electrical torque which enhance the oscillation with initially caused this electrical torque variation in the first place .

So, this kind of cause and effect analysis can reveal that, we have a lot of potential problem. Now, one of the important things which you should note is that, this need not be true, but it turns out that for a series compensated network, which is for a network compensated by a fix series capacitor. If the frequency of mechanical oscillation is sub synchronous, then this kind of situation is likely to occur, and if that occurs one can expect that there can be some problem.

Now, I hope you get the reason, why we can have a potential problem. This problem will be very much enhanced, in case this frequency is equal to the frequency of this in the D Q frame of reference. So, if the frequency of delta and omega the torsional oscillation frequency is very nearly equal to the oscillatory frequency of the network. Whether the network is kind of model in the D Q reference frame, then this T e can be very large or this problem can really become very significant.

So, the electrical torque may really cause a very adverse reaction and the oscillation may start growing with time, that is why this thing is called sub synchronous resonance because the electrical torque is found to have this kind of phase relationship or rather it has got a significant component of this kind. So, it has got a significant component of this kind in case, the frequency is sub synchronous and the network frequency comes close to this, then you can show that the electrical torque has got a fairly large component, which is proportional to a kind of phase with the electrical speed.

So this is the kind of cause and effect reasoning. If you feel uncomfortable about this kind of reasoning, no harm you can actually do the Eigen value analysis and see.

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Example : 6 mass

Mass	Inertia constant (H)	Shaft section	Spring constant pu torque/rad
HP	0.092897	HP-IP	19.303
IP	0.155589	IP-LPA	34.929
LPA	0.858670	LPA-LPB	52.038
LPB	0.884215	LPB-GEN	70.858
GEN	0.868495	GEN-EXC	2.822
EXC	0.0342165		

So, let us take a case study and end this lecture. So, if you look at this particular, will take this example of a 6 mass system.

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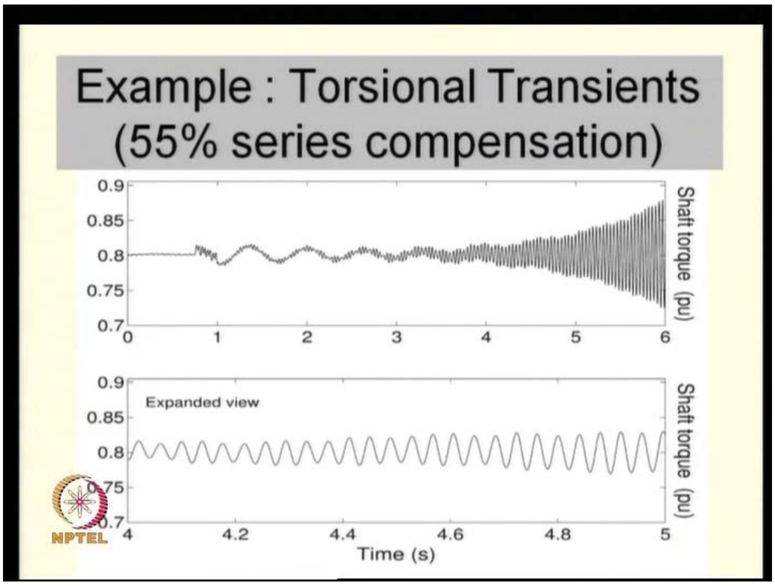

If I compensate this network by 55 percent series compensation, there is one oscillatory mode which goes unstable; this is what we find by simulating the system. Remember, if you want to simulate this phenomenon, you will have to use E m T p like programs in which stator transients and network transients are not neglected.

So, if you look at the expanded view, you can see that this oscillation has got 1 2 3 4 5 around 5 complete cycles in 0.2 seconds, would mean around 25 hertz. 25 hertz would correspond to a frequency of roughly 160 radian per second.

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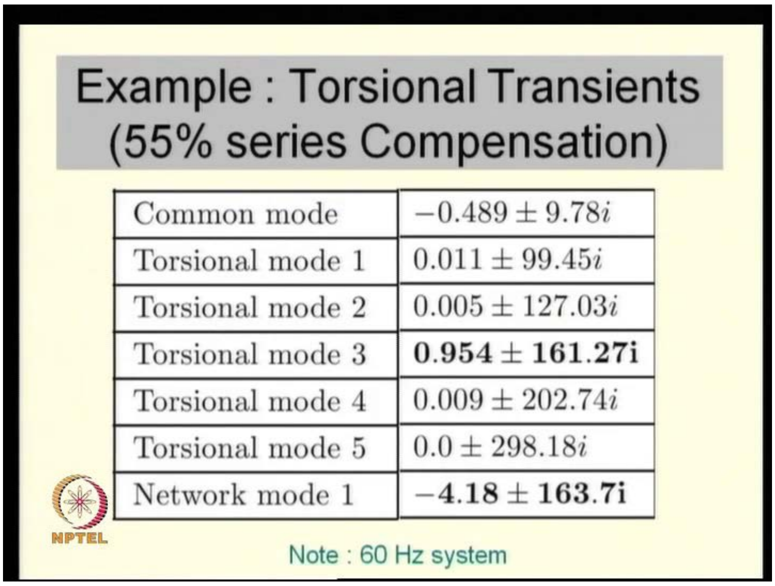

So, look at this Eigen value analysis is also predicted by small signal analysis, while doing a small signal analysis, we can actually correlate what happens in simulation.

The sub synchronous network mode, if its frequency becomes close to one of the torsional modes, we have evaluated the torsional modes sometime back, remember the torsional mode frequencies have hardly changed.

But, the important thing is that the torsional mode damping has become negative that is the real part of the Eigen value has become positive and this happens because the coupled electrical network has got an Eigen value or has got a natural frequency in the D Q reference frame formulation, which is absolutely close to this. So, whenever network frequency comes close to this the torsional mode becomes unstable if it is sub synchronous.

Now, one important thing of course, which I missed telling you is that how much effect the electrical torque has on each individual mode depends on you know the controllability of a mechanical system mode by the electrical torque. In fact, in this particular system the torsional mode 5 is hardly affected by the connection or the coupling to the electrical network, it shows that this mode is practically uncontrollable by anything you do in the electrical network.

This is something you can actually analyze by looking at the Eigen vectors of the mechanical system. In fact, if you look at the Eigen vectors of this corresponding this to this particular mode the both right and left Eigen vectors, they will have very, very small components corresponding to the generator mass.

So, delta and omega components, that is delta of the synchronous generator and omega of the synchronous generator, the Eigen vector components corresponding to these two states are generally very small, both the right Eigen vector as well as the left Eigen vector. That indicates that this 5^{th} mode is not very well controllable by anything you do on the electrical network and this is what is actually seen in this Eigen value analysis as well.

So this is what happens in case of a sub synchronous torsional mode. There is a matched kind of frequency in the electrical network, as well this electrical network is sub synchronous because of the fact, there is a oscillatory natural frequency of the electrical network because of the series capacitor.

I have not shown the sub synchronous network mode or the other modes of the system I have just shown you, only the torsional modes, the sub synchronous network mode and very importantly the common mode of this system is given here. It is the low frequency mode, which you have already studied before.

If you recall what I said last time you do not have to unlearn what we did as far as the low frequency swing oscillations of one hertz were concerned. Remember that when we had done a study of a two machine system, we had considered the whole turbine mass system as one and therein we had seen that there is a low frequency oscillation of around 1 to 2 hertz.

Now, what we have done is we have represented the turbine generator system is a multi mass model; therefore, you are getting all these torsional frequencies, but this particular mode, in which all the masses of the mechanical system move together still persist and basically is manifested, when you couple it to the electrical system as this common mode or swing mode.

So, the 0 Eigen values, which we saw in the mechanical system, which is isolated from the electrical network, now transforms itself into this low frequency swing mode. Now this mode is of course, seen in this simulation, you see this low frequency mode as well. This simulation incidentally has been done for a step change in torque a very small step given in the mechanical torque this is the disturbance which is given of another turbines.

So, please remember just to summarize, this particular phenomena you can have adverse torsional interactions at sub synchronous frequencies due to a series compensated network. This is basically the crux of the important thing which you should take back with you after this lecture. I thought I would be able to start on improving stability methods in this particular lecture, at least I would give you an introduction to that but we will do it in the next class.