Power System Dynamics and Control
Prof. A. M. Kulkarni
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture No. #41
Torsional Transients: Phenomena of Sub-Synchronous Resonance

Today's lecture will focus on studying torsional transients in a generator turbine system. The reason, why I have chosen this particular topic, as a kind of a case study is, because the modeling required for studying this these transients or these dynamics, are somewhat different from the modeling kind of modeling we did to study slow transient. Slow transients I mean power swings, and for example loss of synchronism, power swings frequency stability that is the common motion of the frequency in a multi machine system. These were relatively slow transients in which the way we model our network or even the turbine generator system was appropriate for the study of slow transients.

So, in today's lecture will focus on torsional transients, and the phenomena of a sub-synchronous resonance which I will try to describe to you, and we will also try to do an analysis of this particular phenomena.

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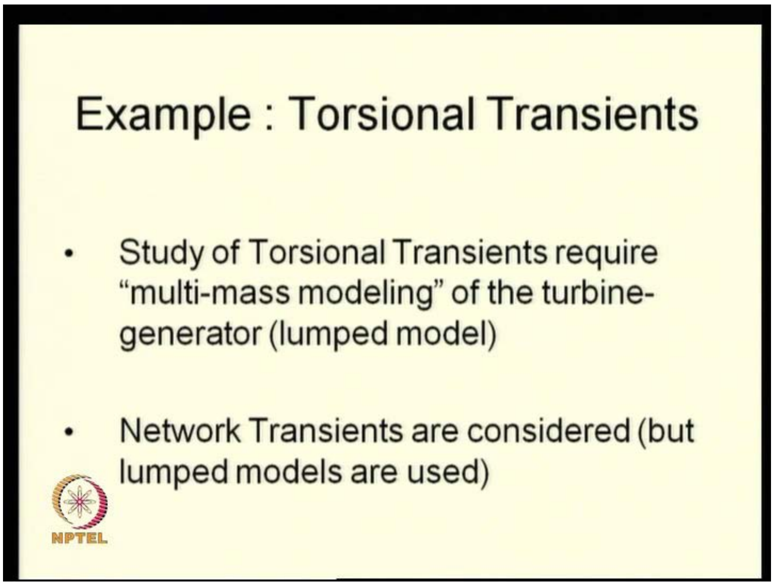

And what we really are going to study is the torsional transients. When I say torsional transients, it is really refers to the kind of torsion which is experienced by the shaft of a turbine generator system or there may be several shafts in case there are many turbines.

So, we need to model the shaft turbine generator system, the mechanical system as a multi mass by as a multi mass system which is connected by elastic shafts. So, what we normally do of course, what we have done so far. When we have modeled synchronous machine a generator the mechanical equations we have considered, it as one the turbine generator system is considered one lumped rotor mass whose inertia we call H.

Here, what we will be doing is that, instead of treating the generator and the turbine as one mass, we will be treating as the two or more separate masses connected by shafts. So, the phenomena in fact we are going to study that is the transients associated with the torsion in the shaft. Of course, requires you to model it in this way.

One more important point of course is that, the network in stator flux transients are not neglected in this study. As it will turn out the torsional transients of interests have a band width of a several hertz. I mean, from ten hertz onwards typically. And it would not be appropriate to neglect network transients.

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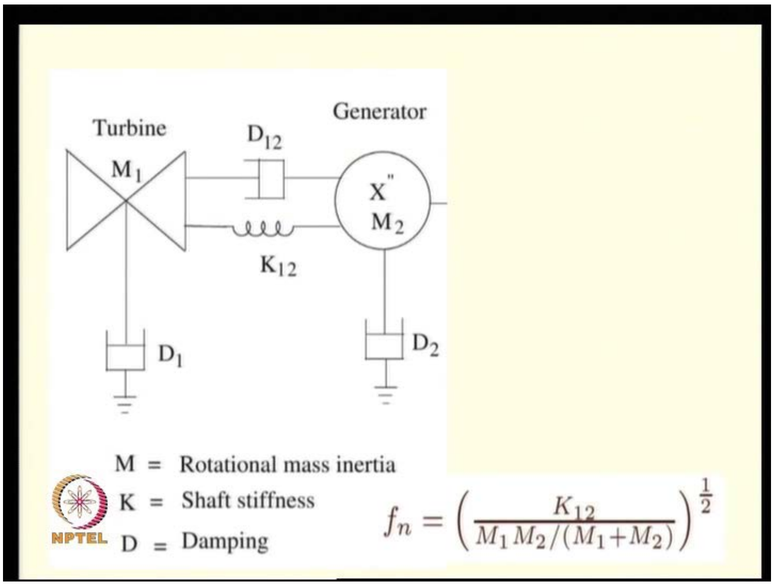

In fact, if you look at a mechanical system of a generator and a turbine just one turbine and a generator, then it can be modeled in the way, it is shown here. You have got two masses rotor masses, the turbine and the generator. The shaft mass also is there. So, in principle if you look at it in a very strict sense, even the turbine generator system along with the shaft is actually a distributed parameter system. because the shaft also has some mass.

But what we will do is, we will assume that the turbine generator system can be modeled well by two masses. In this case, in the figure which is shown we have got two masses connected by a shaft which has got a shaft stiffness which we call we do not escape and the rotational mass inertia is M.

And the damping is D of course, there can be damping, there can be viscous damping which is proportional to the speed of the turbine, the speed of the generator or the difference speed between then. So, that is why you have got D 1, D 2, and D 1 2

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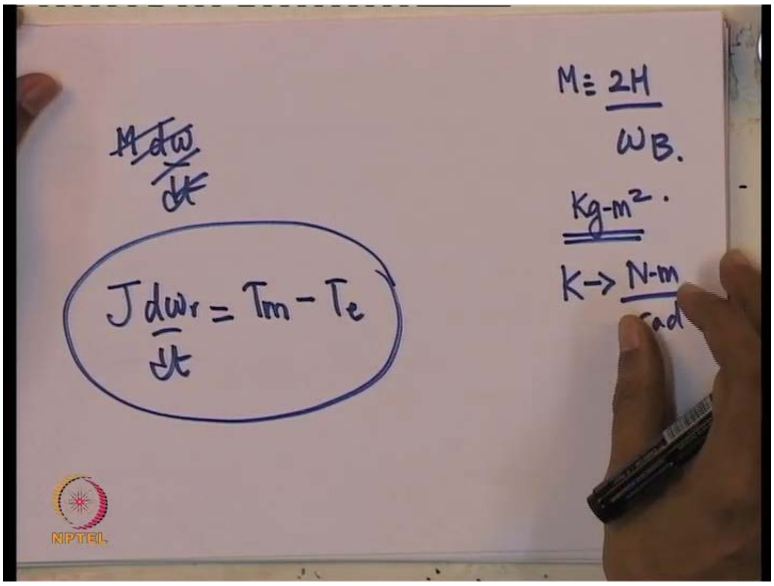

Now, if you look at this particular system, you will find that the equations of this system are effectively M d omega by d t. M is of course, if things are in per unit then M is 2 H by omega B. but   in case your your talking in actual M case units, you will have M is in kg meter square. And K will be in Newton meter per radian.

So, of course, if you write everything in m case units, then you will have J the same is, this J is nothing in case is in kg meter square. d omega rotational by d t is equal to mechanical torque minus yeah, the electrical torque. This was of course, the equation we used for single lumped mass. Right now, we have got two lumped masses connected by a shaft.

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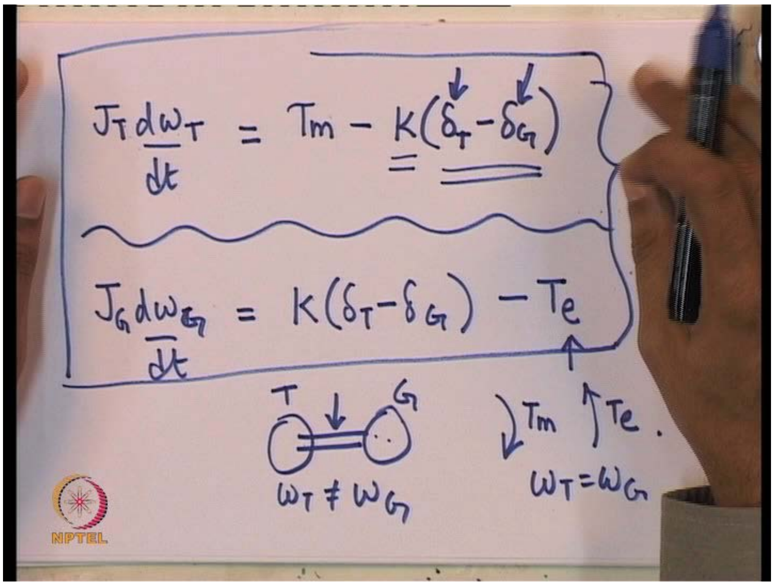

So, the equations are In fact, J 1 d omega by d t, omega 1 by d t is equal to for example, for the turbine I call this turbine in this particular figure which you see on the screen, there is one turbine mass.

So, if you look at the equations for that the mechanical torque is actually applied by the steam on this turbine mass. So, T m appears in this equation minus there is no electrical torque actually on a turbine. Actually, what you have essentially is K, the position of the shaft. So, you have got delta turbine minus delta generator. So, this is basically giving you the torque equation of the turbine.

If you look at the generator is equal to K delta T minus delta G minus the electrical torque. This is the electrical torque. So, actually if remember that, when we wrote the equation for a generator. If you recall the kind of equations, we were writing for the study of slow transients. We just had one J and T m minus T over applied on the same mass. So, what was the assumption there was that, if this is the turbine connected by a shaft to a generator and this shaft is got infinite stiffness, then you can treat this turbine generator system as one mass. On which, T m is applied in one direction and T is applied in another direction the speeds are equal.

But, in case that is omega T is equal to omega G, but in case this shaft is elastic. It does twist a bit it has got some torsion. In that case, omega T is not equal to omega G. And of course, you the you cannot treat this as one lumped mass. So, for the slow study of slow transients we do consider that this is one lumped mass. And T m and T e applied to the same mass and omega T is equal to omega G.

But, In case of your studying torsional transients in the shaft, the shaft is treated as an elastic shaft. And remember that omega T and omega G are not the same the angular position delta T and delta G are also not the same. And the force on the shaft is K times, this is stiffness constant into the difference of the position of the torque turbine and the generator mass.

So, if if there is a twist or a torsion, then you have got a force. So, the equations of two mass connected by an elastic shaft system is actually given by this. Remember, the T is applied on the generator mass and T m is applied on the turbine mass.

Now, of course, you can easily show that the center of inertia movement of this is you can just, you know just rewrite this equations and add them up,

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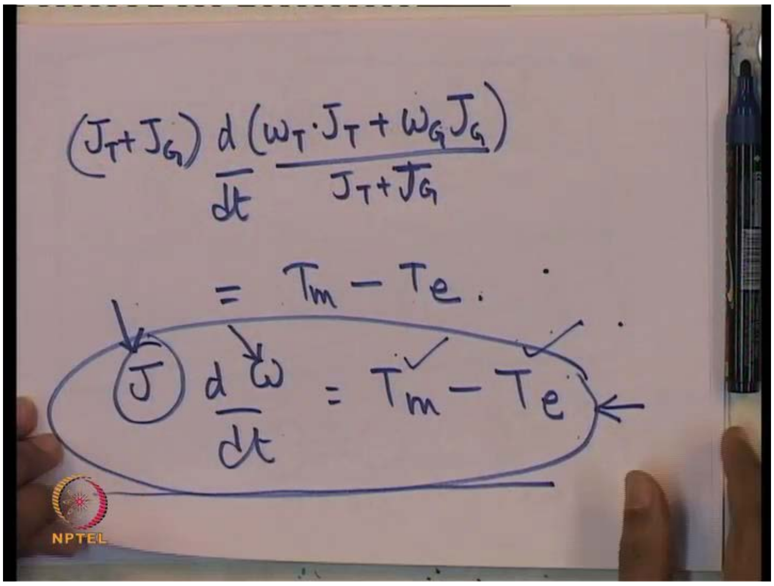

You will find that J T plus J G, d omega T into J T plus omega G into J G upon J T plus J G. This is center of inertia speed of the machine is equal to T m minus T e.

So, In fact, this can be written as J d omega by d t is equal to T m minus T e. So, please remember that whatever we were doing previously, I mean we were treating the machine torque the generator turbine is one mass on which T m and T e are acting is in fact, in some way valid. In the sense, that it is describing the motion of the center of inertia of these two masses.

So, if your the center of inertia of the two masses is affected by T m minus T e. But the individual speeds are determined by these equations. So, this is an important point which you should note whatever modeling we have done so far. Assume that the generator in the turbine shafts are one and the same, but remember that they are different when when connected by an elastic shaft, the speeds are different etcetera. But whatever we have done before is not invalid only remember that the speed mechanical speed of the of motion was in that case essentially, the center of inertia motion of this system.

So, whatever we have done before is not invalid in the sense, that we had considered T m T acting on the same mass, the rotor turbine generator as one lumped mass and omega T and omega G kind of equal. Instead, we should say now that when we modeled the turbine generator system for the study of slow transients we did not take into account the elastic shaft sections. but the center of inertia motion of that turbine mass rotor system. Equation was indeed correct I mean what we use essentially was that. So, we do not have to unlearn whatever we have done before, we do not have to unlearn the modeling which we have done before for the study of slow transients. Remember that the in the study of slow transients what we essentially to was J was the total inertia, J T plus J G omega was essentially the center of inertia speed. And T m and T e were acting on the center of inertia, you can say of the system.

So, we do not have to unlearn anything what we have done before as far as the slow transients are concerned. The only thing we should remember that the motion of the center of inertia was just one aspect of the motion which is for the study of slow transients. But remember that, actually there are two sets of equations and in addition to the motion of the center of inertia, you also have a an additional pattern. The additional pattern is essentially the oscillatory pattern or the staff shaft torsional oscillations which may result, because of this spring mass type system.

So, we have got two masses connected by elastic shaft. We have got the center of inertia motion which is essentially governed by the equation. This equation can be used for the study of slow transients, but in addition to that there are. In fact, faster transients associated with the rotor torsion. So, in fact, the two modes associated with this kind of system. One is an oscillatory mode and one is a common mode of motion.

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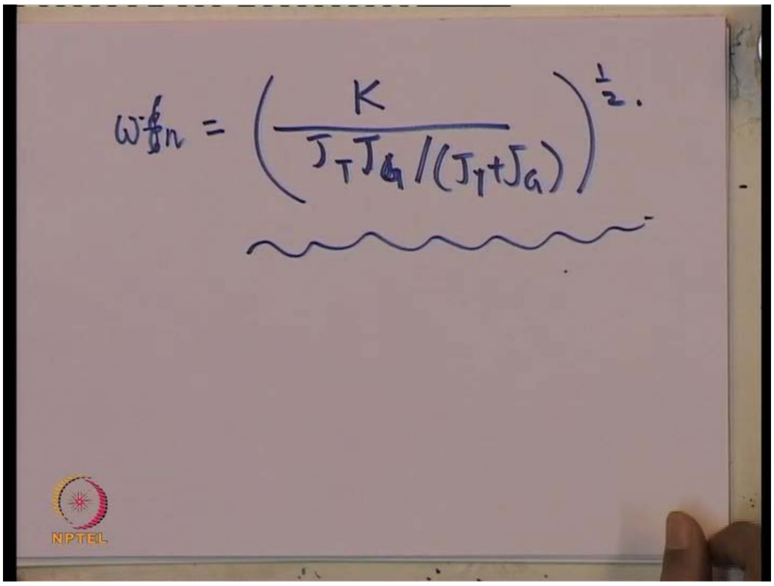

The oscillatory mode of motion has got an inertia natural frequency of oscillation which is given by the shaft constant divided by J T J G upon J T plus J G raise to half. So, this is effectively. In fact, I should say this is the kind of, this is In fact, omega n. So, if this is in this kind of equation results or this kind of frequency of oscillation results for this kind of system. So, we have got this additional pattern in the motion which I actually pertaining to the torsional oscillations in the shaft.

Now, the interesting thing why why do we really required to understand or study this, why did I select this particular phenomena is that the torsional transients. In fact, any any motion of this kind is exceed by changes in T m or T e. So, T m and T e In fact, are like inputs to this, you know kind of torsional system. And suppose for example, there is a transient in T e, it exceeds these torsional oscillations as well.

So, this additional pattern of motion is now we are trying to understand. Now, the reason why we as, I was just trying to tell you, the reason why we are trying to study these particular transients is that there was one interesting situation, wherein the electrical network caused torques, which actually caused these torsional oscillations to go unstable.

And under these circumstances, the fatigue on the shaft due to oscillations is you know, if you take a metallic any metallic shaft and you give it a very large displacement. Say, a torsion displacement, it tends to come back to its original position. but if the torsional displacement is too large or the displacement caused by the torsion is too large the shaft may actually get damaged, it may actually fail.

So, this kind of situation did occur because the electrical electrical torque which is an input to this mechanical system, cause an adverse effect in which the oscillations grew with time. And the system became unstable the torsional oscillations grew instead of dying out. Normally, one would expect that because of the frictional and wind age torques which are associated with a with a steam turbine and bearings. One would expect that eventually the torsional oscillations if excited would die down, there these damping damping torques, you know present in this system.

So, one would expect a torsional oscillations if they exist would actually die out. But because of adverse interactions with the electrical network, there has been an experience in the past. In which the shaft got damaged because the torsional oscillation did not die down because of damping. But because of the adverse interactions in the electrical network, they actually grew in time the oscillations grew and the shaft got damaged.

So, that is why I have chosen this particular topic, will now we we will try to understand the interaction of this shaft torsional system with the electrical network. How does the electrical network affect these oscillations.

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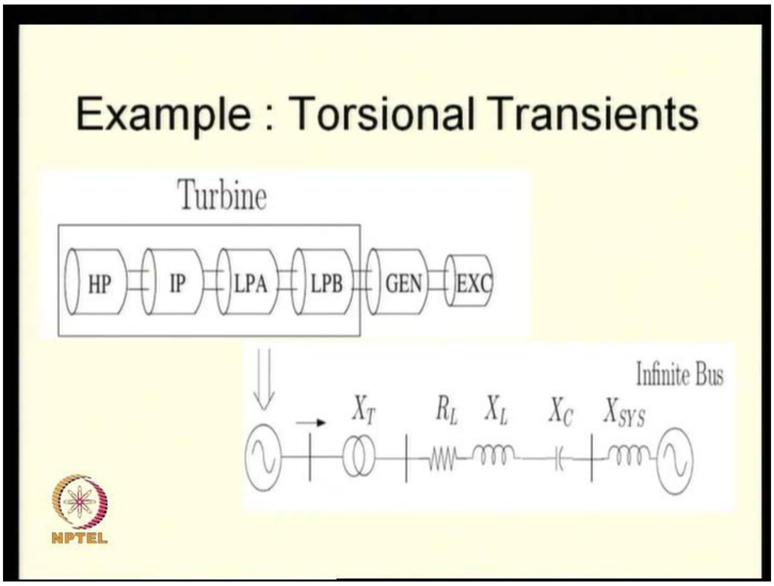

So, this is the topic of. In fact, today's lecture remember that all though I have given you an example of two mass connected by elastic shaft kind of turbine generator system. You can have several especially in steam turbines you may have several turbine stages. In fact, there may be a high pressure turbine, an intermediate pressure turbine, two low pressure turbines all interconnected to each other by couple to each other by a shaft. And then couple to a generator and maybe even a rotary exciters.

So, this is the whole rotary system of turbine generator system. And in such a case for example, here you have got 1 2 3 4 5 6 masses. You have got, In fact, 5 modes of oscillation and one one pattern of motion which corresponds to the center of inertial motion wherein all the masses move together rotor masses move together.

So, if you look at this kind of a turbine generator system, please remember that, if you are got 6 masses, you have 5 modes of torsional oscillations. And one mode of common motion all the masses move together. The common motion of the all the masses moving together has already been studied in some to some extent. When we studied the, you know low frequency swings, we treat it the whole all the rotor masses of the system of the turbine generator system to be one. And the T m and the T E is actually applied to just this one equivalent mass that was essentially you know representative of the common motion of the turbine generator system in which all the masses move together this is, but remember that, this is just one of the patterns of motion of this turbine generator system. They are 5 other, in this case torsional modes, these are oscillatory modes, the frequency of oscillation of course, depends on the masses and the shaft data or the shaft parameters of this particular system.

Now, the point is of course, that the torsional oscillations are affected by the electrical torque whether electrical torque appears like a kind of an input to this system. So, that is why in this particular diagram I have shown you basically the electrical network as well. Remember that, the generator is connected to a transformer and then to a transmission line and then it is connected to a system the say the say an infinite bus or voltage source.

So, in this particular system, it turns out that under certain circumstances you can have adverse interactions between the turbine and the generator turbine generator system and the electrical network. And what you see very importantly is that this particular system in a in which I say there is a possibility of adverse interaction has got a series capacitor connected to it.

Now, a series capacitor is connected to a transmission line. This transmission line is represented is a lumped R L circuit series compensation. This capacitor or capacitive series compensation of a transmission line is done to reduce the effective X of the line. Now, why is that done reducing the effective reactance of a transmission line. The thing is, which we have discussed some time ago is that a reduced reactance in a transmission system allows for more power transfer, secure power transfer. In fact, if you have got a system in which the reactance between generator or the reactance between a generator and a voltage source is lesser, you can show that the amount of that it is more secure under large disturbances. So, if your got lesser reactance, you can show that for a given large disturbance a system with lesser reactance, system reactance.

Is more secure, in the sense that it can sus, it can sustain a large disturbance without losing synchronism. So, in fact, that is the reason why you have series compensation of transmission lines.

Now, if you look at basic problem which I was trying to address or which I gave you an idea about was that. Under certain circumstances, a series compensated electrical network can cause shaft damage. This is something which I was trying to really tell you about or this is the whole lecture is about this.

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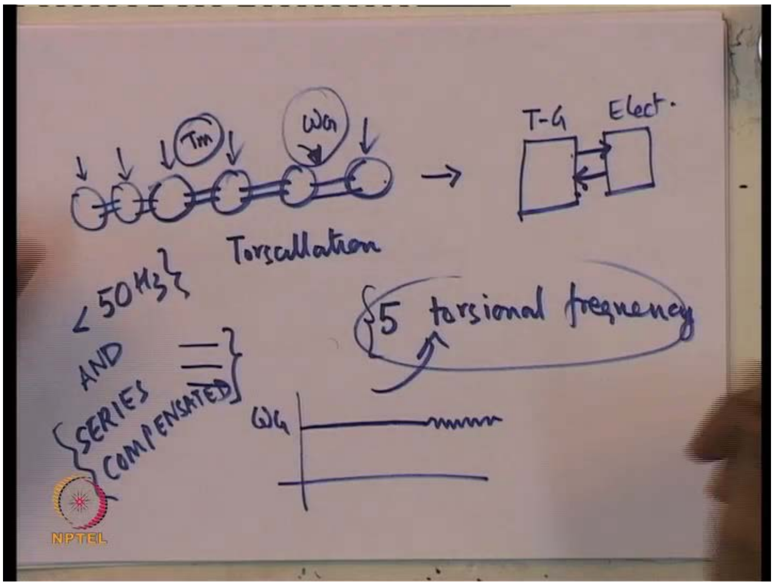

Now, how can you may wonder that how does effectively the turbine shaft come into the picture. Remember that, suppose, this is your turbine generator system these are several masses and suppose let us just try to take a kind of intuitive way of looking at things. Suppose, I because of some random disturbance or some random change in the mechanical torque in the turbine may be it may be a very very small change in that that would cause torsional oscillation to be setup after all you give any system which has got certain torsional frequencies. In this case, there are 5 if there are 5 or 6 masses they have got 5 torsional frequencies. If you have got two masses you have got just one torsional frequency like I showed you here, but if you have got 6 masses you will have 5 torsional frequencies.

I give a kind of a disturbance to the say to the mechanical torque. In that case, you will find that. You know, in general, if I give a kind of a disturbance say to 2 or 3 shafts at the same time find that all these torsional oscillations will be excited.

So, if I give a push here here here here and here suppose. Some disturbance, because of which I you know displace these masses from equilibrium. You will find that these torsional oscillation frequencies are excited remember one small point which you should not ever lose track of is all these masses are rotating at you know the rated near about the rated frequency under the normal circumstances.

So, if it is a three thousand rpm turbine you will find all these masses are moving at three thousand rpm, there is no relative motion between any of them under equilibrium. but if I give a push to one or more turbines, it is likely that one or more torsional oscillations would be excited. This is what I actually wanted to say probably, would not have been very clear the way I put it earlier.

So, if I give a push or any disturbance or displace one of the masses let us say somehow, I give an impulsive torque to one of the turbines just you know, just think of it a some kind of thought experiment give a impulsive push to one or more of these turbines turbine stages you will find that these various torsional modes are excited.

Now, if these torsional modes are also observable in the speed the generator speed. In that case, you will find that the voltage is induced on the stator of the turbine will also contain signatures of this torsional frequencies.

So, instead of having a three phase 50 hertz supply, if I give a small push to any of these turbines and that triggers of torsional oscillations which are observable in omega omega G which the speed of generator you will find that it will cause.

So, instead of the speed of a turbine generator system or the generator speed being a constant, if it has oscillations. These will manifest in the voltages which appear at a terminal of the synchronous generator and if that happens it will obviously, cause some currents in the transmission system. So, if your got a voltage source which has got you know torsional frequency components or the complement of torsional frequency components. You will find that, it will cause currents also of that frequency because your network will respond to that.

Now, once you have got currents as a result of this disturbance, it will cause torques. Now, so let me put it this way you have got a torsional turbine generator mechanical system. This is the electrical network that any torsional, if you excite torsional oscillations here. They will have effects on the electrical network in the sense, if the electrical network also will see some currents because of this torsional oscillation.

And if this currents interacts with the torques which are present in the generator to cause electrical torques, which enhance the oscillation which in effect created it. One can kind of see that these oscillations may actually grow with time. Are you getting what I am trying to say. So, so you have got a turbine generator system there torsional oscillations which are excited torsional oscillations cause variations in the generator speed. The variation in the generator speed cause variations in the electrical voltage which appears at the terminal of a generator that causes currents in the electrical network, disturbance currents in the network. The electrical currents which are caused cause torques disturbance torques. Now, if these electrical disturbance torque is such that, it enhances the existing torsional oscillation which is exact which is excited you will find that the torsional oscillation may go unstable.

So, this is a kind of a cause and effect reasoning of how under certain circumstances, it is possible that the electrical torque causes a torsional mode to get become unstable. Of course, it all depends on whether the electrical toques caused enhance the existing oscillation or not. And this enhancement can actually occur under very special circumstances and that those special circumstances are when the electrical network is compensated by series capacitors.

And the torsional oscillation is sub synchronous in nature. That is, if the torsional oscillation is less than 50 hertz torsional frequency is less than 50 hertz and you have got an electrical network which is series compensated. Under these circumstances, it turns out that the electrical torques are generally destabilizing in nature that is, they tend to enhance the oscillations which cause them.

So, the tors you gave a disturbance cause a torsional oscillation, it cause a changes in electrical voltage that cause currents disturbances in the currents. The currents cause electrical torques, electrical torques enhance the existing oscillation. This can occur, if the torsional frequency is less than 50 hertz and the system is also series compensated by the use of capacitors.

So, this kind of phenomena is there for called sub synchronous resonance. because it is like a situation in which the electrical system acts like an input to the torsional turbine generator torsional system and the input is such that, it enhances the existing oscillation. So, it is like almost like a situation which we call as resonance. In fact, I will show you that a series compensated system has got a resonant frequency which you know comes close to the torsional frequency. It could come close to a torsional frequency and then you would have a classical resonance kind of system, where the electrical system is kind of you know giving a input to the torsional system, torque turbine generator system which is near about one of its natural frequencies. So, you know the oscillations can grow.

So, this kind of rough kind of cause and effect description of this phenomena is something which I have given you now. It is obvious that you would have found that I am not explained everything in a very rigorous fashion. For example, you may say well actually this turbine and generator in electrical system are one system, it is not right to treat the electrical system as a kind of a input to the turbine generator system. It is a one combined dynamical system that is indeed correct. What I have done is given you a cause and effect analysis in which the oscillation is caused by the turbine generator system and a electrical network reacts to it and then what you get out of it is an input again to the turbine generator system that is why use the word resonance you know, it is a kind of a forced rather the forced response to an input.

But, this is not you know those who are who have done this course. Of course, would not like the way I have described this phenomena, it is not a very rigorous way of describing how this phenomena occurs. So, let us actually try to you know model this system. And try to see what are the steps in the modeling of course, I will not you know, it is not possible to explain in a cause and effect fashion when your when your system size is become greater than 10 or 20 or 30 differential equations .

So, that is the reason why actually started up the discussion in a very non rigorous fashion. but now what I will do is, I will just try to describe to you the models which are used why the electrical network under series compensated conditions can have a sub synchronous resonance frequency that is something which I will also try to explain to you. And therefore, show you a case study where in the turbine the one of the torsional oscillations is in fact, unstable.

So, this is something I will try to explain to you by a modeling this electrical network in little bit more detail. So, why does series why do series compensated network networks connected to turbine generator systems especially tees steam turbine generator system cause rather cause torsional oscillation to grow under certain circumstances is what you will try to understand.

Of course, you may wonder I used just just know, I used the word steam turbines. In fact, you can have you know you you have a turbine as well as a generator even in a hydro turbine, but it turns out that in a hydro turbine generator system the generator mass is much much larger compared to the mass of the turbines. So, in some ways you can say that the generator in some say screens the electrical network from the torsional system. So, this is this is the kind of a thing which we will try to understand for a steam generator the SSR phenomena the sub synchronous resonance phenomena for a steam generator.

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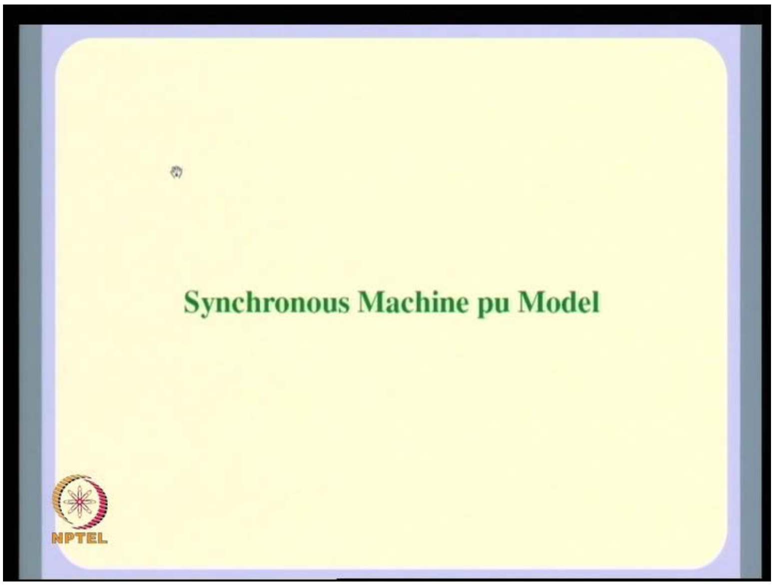

So, let us first look at how would you model a synchronous machine and electrical network which is compensated by a transmission by a series capacitors.

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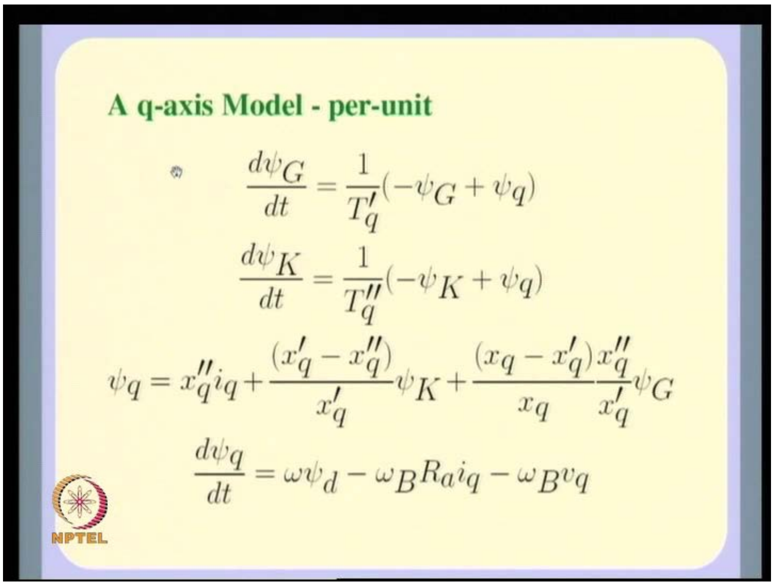

So, let us just look at this modeling. So, we will just we have already done this modeling of a synchronous generator this is the q axis equations in per unit psi G psi K psi q are the fluxes on the d on the q axis .

The three differential equations and one algebraic equation which relates psi q, psi K and psi G, to I q. So, the third equation is an algebraic equation. The d axis model is similar only remember that the d in the d axis the field flux or one of the fluxes of or rather the field voltage appears in the equations. So, this E f d in fact, is proportional to the field voltage. So, otherwise the equations look quite similar.

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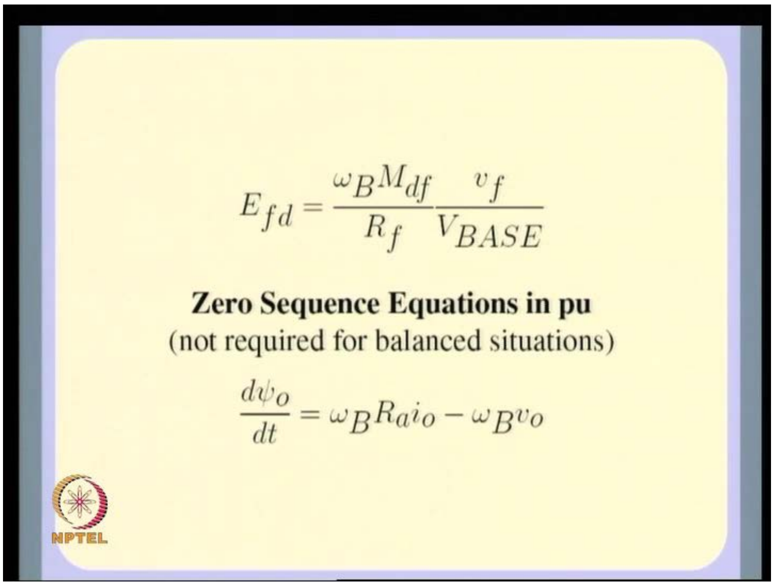

So, in fact, E f d is proportional to the field voltage. We do not of course, consider the zero sequence equations. In fact, this phenomena can be understood. In fact, it is a balanced kind of phenomena. So, we will not can be understood even under balance situation these things occur. So, we will not need to consider unbalance for understanding this phenomena. So, this zero sequence equation in some sense is redundant in this analysis.

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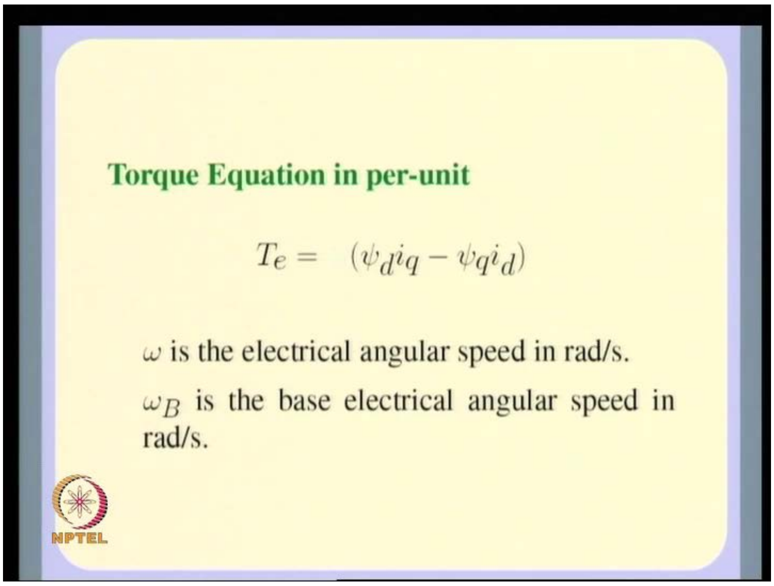

The torque equation is something we already know in the above equation or the equations which I have shown you so far. Omega is, in fact, the electrical angular speed in radiant per second and omega B is the base frequency.

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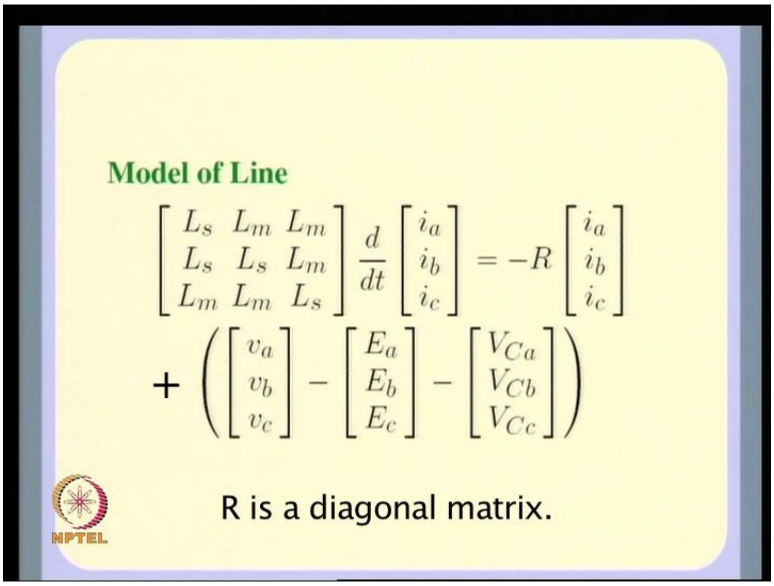

Now, if you have got a transmission line which connects a synchronous generator to a a voltage source, a three phase voltage source whose voltages are E a E b and E c. V a V b V c of course, in this are the generator terminal voltages and V C a V C b and V C c are, in fact, the voltages of the capacitors.

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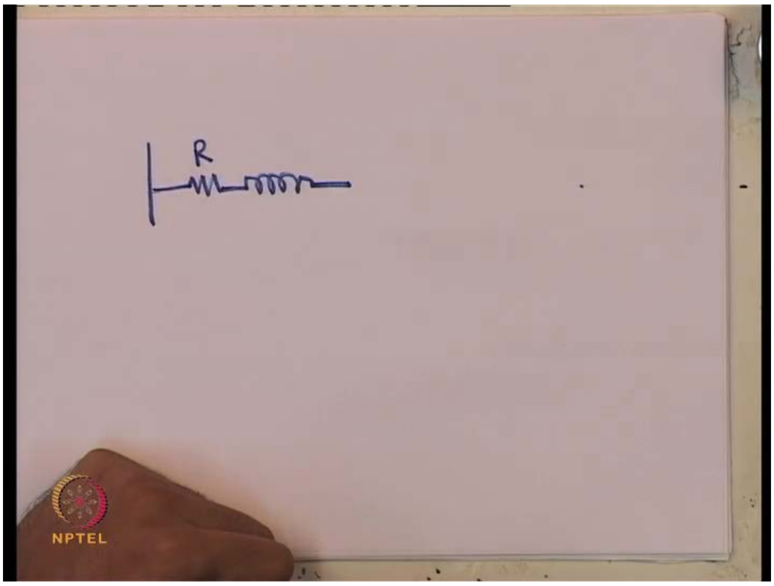

So, if you are looking of a interconnection like this a lumped electrical network are. This is a three phase network so, in fact, although the resistances can take as equal in all the three phases. The three phases, the inductances are all coupled.

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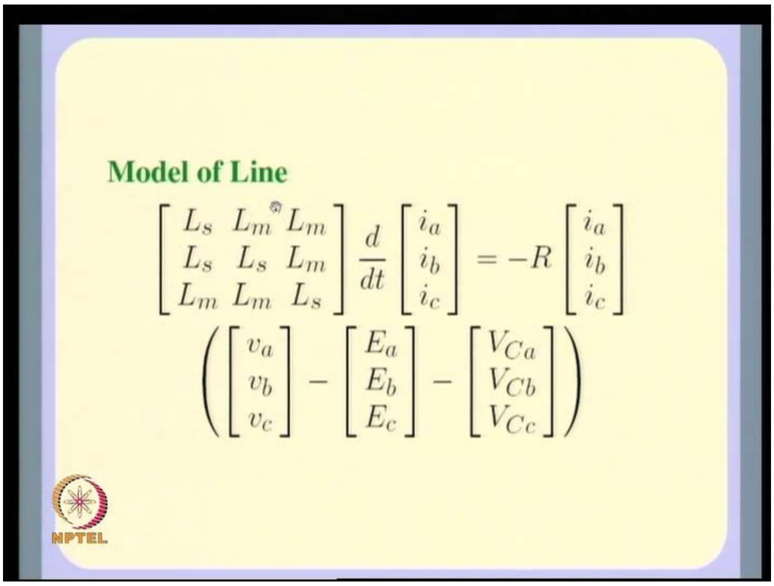

$$\begin{bmatrix} L_s & L_m & L_m \\ L_s & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = -R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \left(\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} V_{Ca} \\ V_{Cb} \\ V_{Cc} \end{bmatrix} \right)$$

So, you have got if you look at this slide, you will find that L s L m L m are the mutual inductances. L m are the mutual coupling between the a b and c phases. R is the resistance of the line. V a V b V c is the generator terminal voltage.

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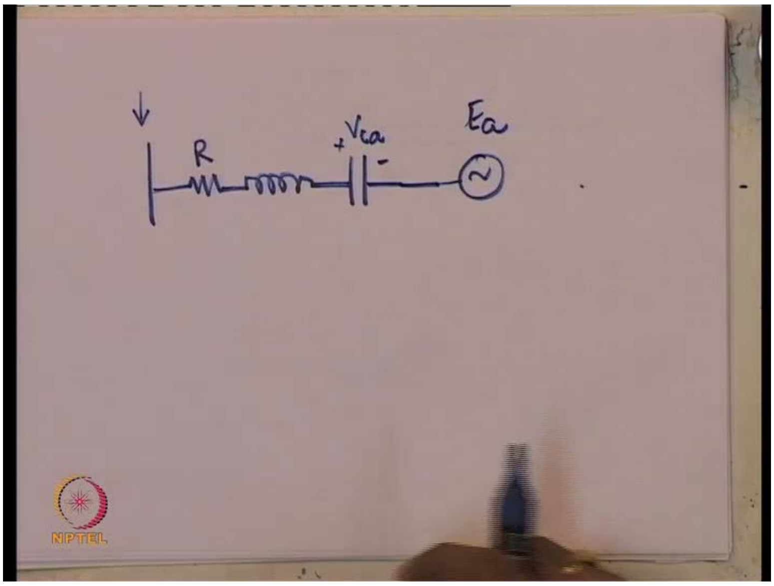

Then V c a V c b V c c are the voltages of the capacitor for the series compensated network. And of course, the infinite bus or the voltage source voltages are E a E b E c. So, I have just shown it for a phase. So, this kind of network is there, this is what I will be studying you and showing you. This phenomena of adverse torsional interactions of the electrical network with the turbine generator system.

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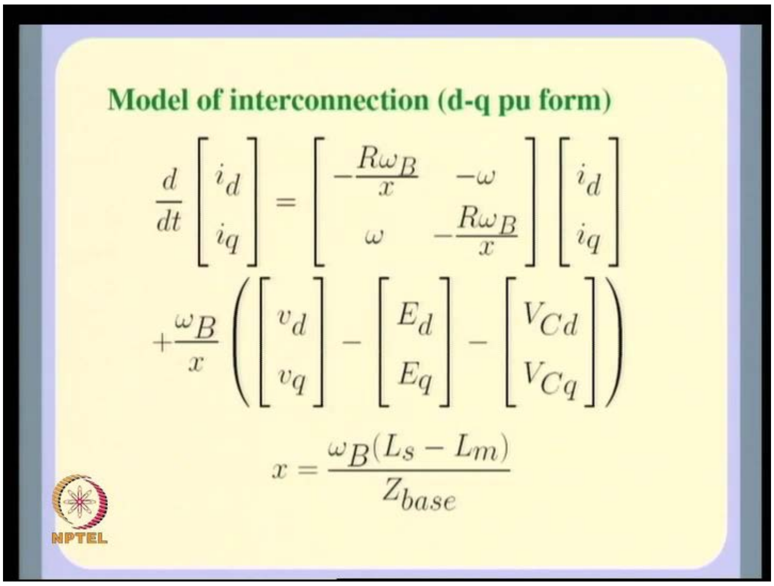

Now, if you convert these equations to the d q per unit form. This is what you will get, I am not the zero sequence equations. So, this is something we had done before when we studied the simulation of a generator and AVR. A generator with an AVR for a single machine infinite bus simulation which we did several lectures ago the only difference here of course, is that you have got these capacitor voltages also coming into the picture. So, this V C d and V C q are the capacitor voltages.

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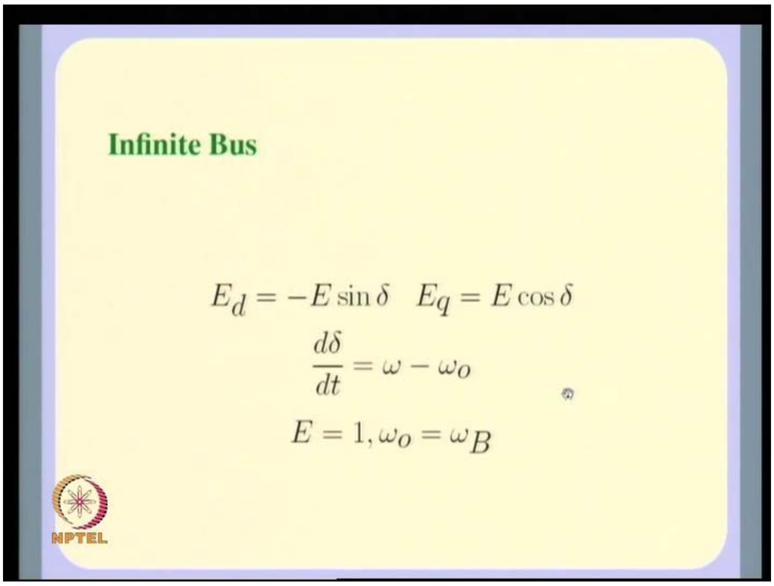

Of course, the infinite bus d q components, if you have got a voltage source E a E b E c then the infinite volt bus voltages are minus E sin delta and E q sin delta.

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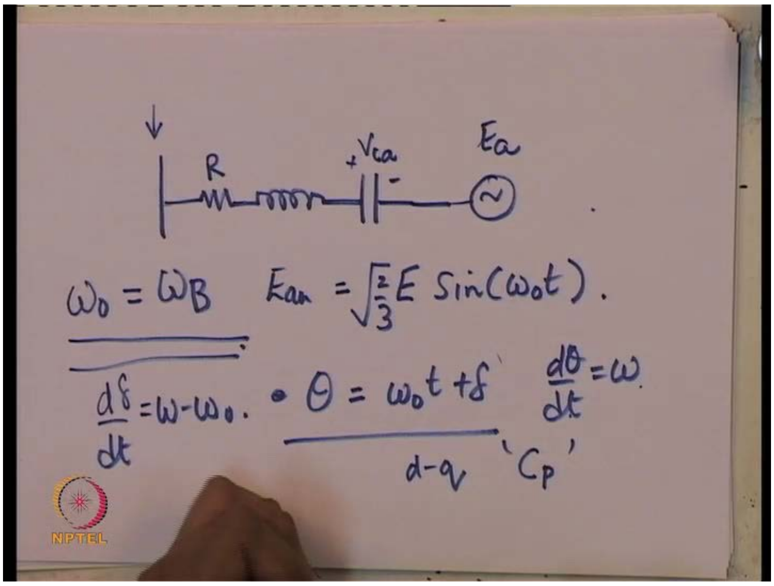

This of course, assumes that E a to neutral for a star connected infinite bus would be root 2 by 3 sin omega not T. And theta the position of the synchronous machine is omega not t plus delta. So, with this assumption E d and E q come out to be this. Remember that the d q transformation uses parks transformation we have learnt this c p transformation which is a function of theta.

So, this is what we get one of the important which you should note is d delta by d t as a result of this equation d delta by d t is nothing, but omega. So, d delta by d t is d theta by d t is omega. So, d delta by d t from this will be equal to omega minus omega not.

One of the assumptions we will be making is that, the infinite bus frequency omega not is equal to the base frequency or the rated frequency. So, I will not make a distinguishing distinguish, I will not distinguish between omega not and omega b. So, please remember that in all our analysis.

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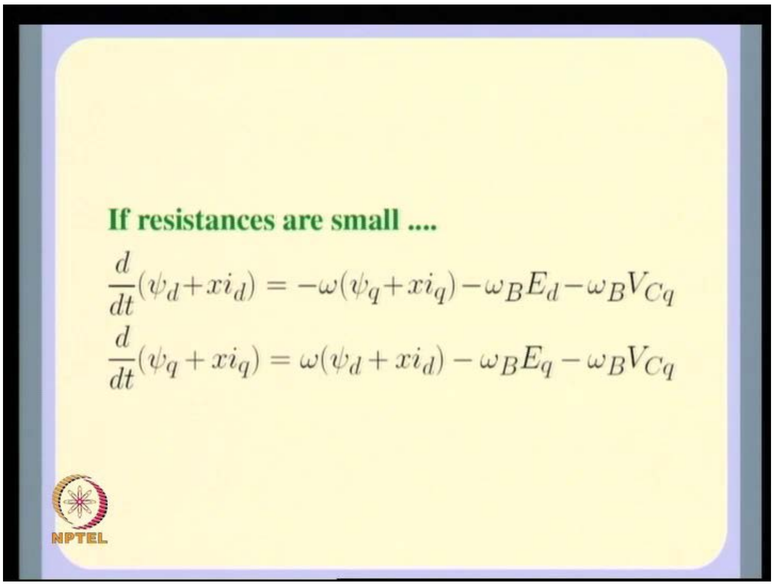

Now, it turns out that we have written our down a differential equation for the currents i d and i q.

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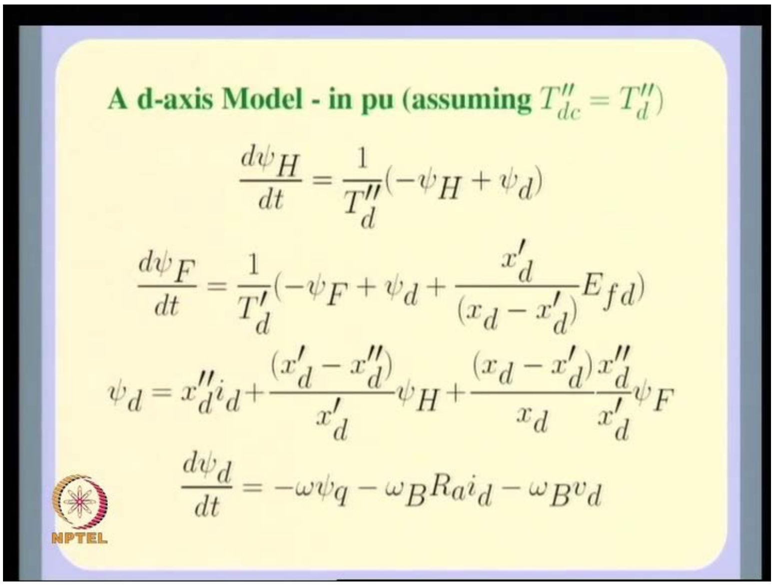

There is also a differential equation for psi d. And psi d and i d are also related algebraically.

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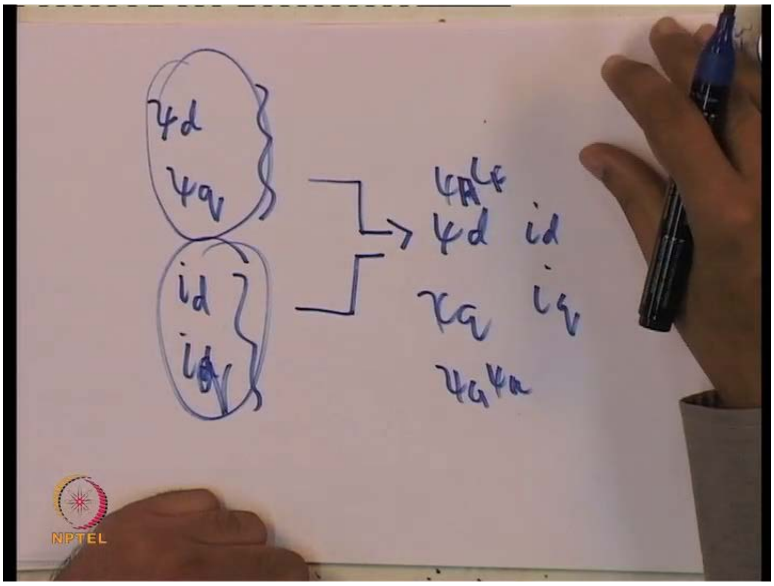

So, let me just put this matter to you psi d psi q are states. So, there differential equations will describe them. I d I q also I have written it, in terms of differential equations they are independent equations here.

But, independent I mean, separate equations. Of course, they are related. We also have an algebraic relationship between psi d i d and psi q i q. And, in fact, psi d psi g psi psi

psi h here psi f and psi g and psi k. So, we have got an algebraic relationship as well. For example, the third equation here gives you the algebraic relationship between i d and psi d. So, the point is that do we really need since there is a relationship between psi and i. There is no need to write these equations separately, I mean you do not have to write two sets of differential equations once for psi d psi d psi q and i d i q. There is no need to do that because they algebraically related.

So, what we will do is, I will combine these equations and make them into one equation. So, there is no need let me repeat to have separate equations for psi d psi q i d and i q. No need for separate differential equation because they are algebraically related. In fact, if I do represent them separately and I give the give initial conditions psi d psi q and i d i q which are not compatible with the algebraic algebraic relationship between them, I will end up with the problem. I will be giving inconsistent kind of initial conditions.

So, it is better idea not to write separate equations for psi d psi q i d and i d i q because they are algebraically related. Of course, this problem will not arise or rather this issue will not arise.

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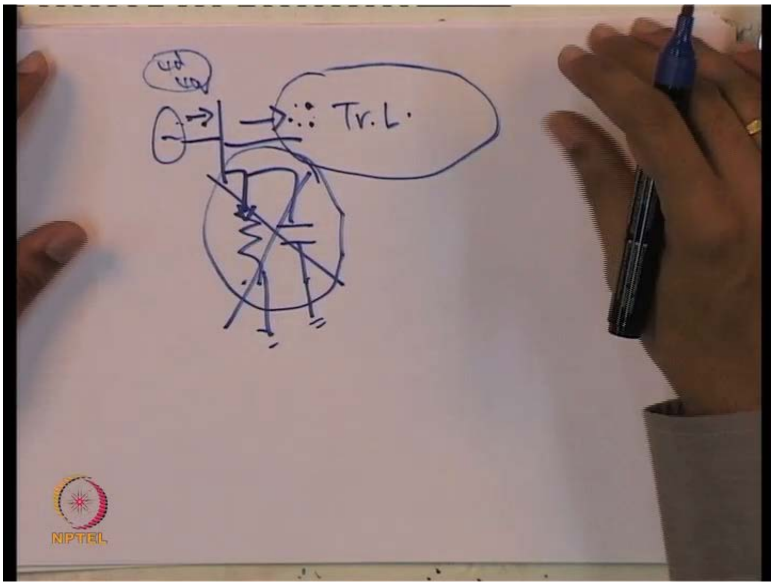

If you have got something in shunt between for example resistive load or a capacitor in shunt with the generator and the transmission line. In that case, the generator current and the transmission line current are different. So, you can you will have to you can write separate equations for this and separate equations for psi d and psi q because the currents here and the currents here are not not the same.

And. So, this is one thing you should remember that if there is no shunt connection between the generator and the transmission line which is modeled by a lumped inductance. In that case, you do not have to write separate differential equations for psi d psi q and i d i q. Of course, somebody may say that look you do actually have a shunt connection in the form of parasitic and all that, but in this particular model we shall not consider parasitic capacitances etcetera, which exist between any electrical structure and ground say.

So, we will assume that nothing is connected in shunt and as a result which we will just try to have one combined equation for psi d psi q and i d i q, instead of two sets of equations for that.

So, how can we do this simple way of doing it, the many ways you can do it, is just to add up the differential equations of psi d and x i d. And if you do that, it is interesting that this is what you will get, if resistances are neglected. So, if you assume that a transmission line resistance is small which is not really true. but just for the simplified analysis, let us assume assume it is true and the stator resistance of a synchronous machine is also small which is, in fact, true its very very small. In that case, we get this particular model by simply adding up the two equations what effectively we have done is got rid of V d and V q out of this. And we have got just once a set of two differential equations, instead of four differential equation separate equation and psi d psi q and i d i q.

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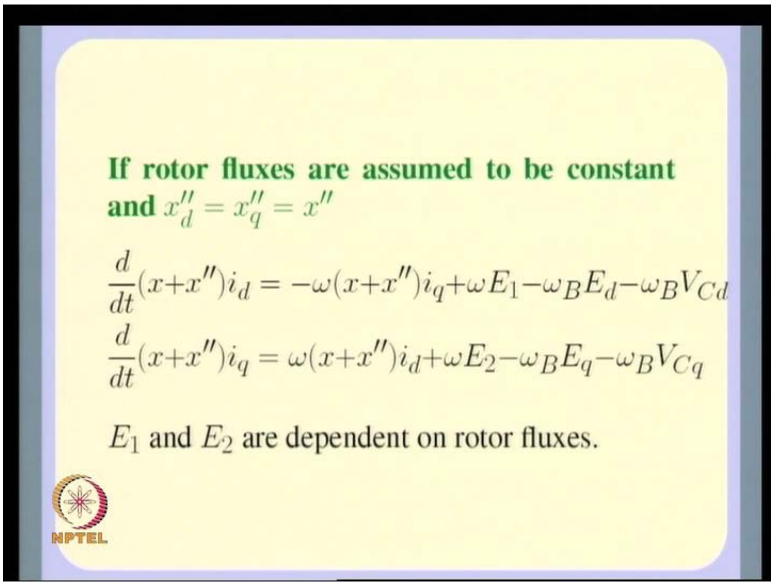

Now, if we assume in addition that x d double dash and x q double dash are also equal and equal to x double dash.

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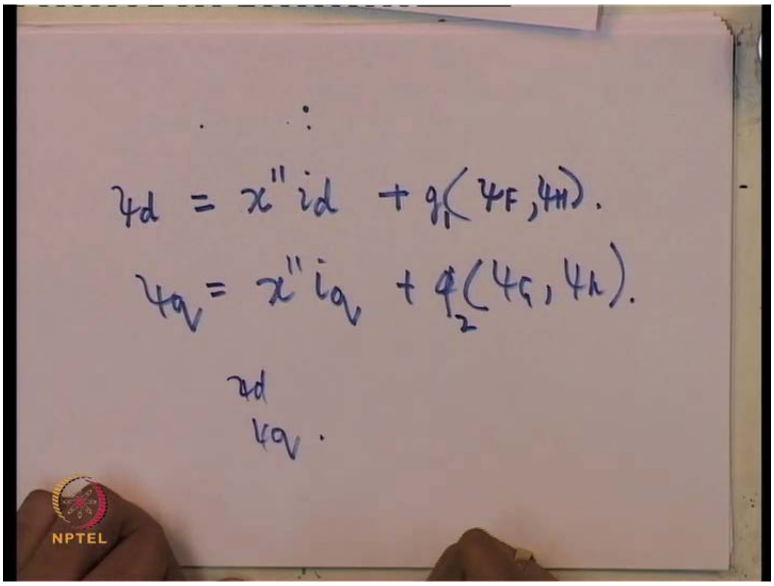

Then, it is very easy to see that since psi d is equal to x double dash i d plus something which is dependent on fluxes psi f and psi h. It is a linear relationship, I will just call it G. And psi q is equal to x double dash i q plus f, I will call this G 1 and G 2 psi G and psi K.

I can substitute this substitute for psi d and psi q in this particular equation and what I will get is essentially what you see on the screen. So, you will get I have replaced psi d and psi q, in terms of i d and i q. And in that case, you will get this particular relationship E 1 and E 2 of course, we will be dependent on psi f psi h psi g and psi k. So, what I have done is substitute it for psi d and psi q in the previous equation.

So, the previous equation was this. This itself was obtained by combining the stator flux and transmission line current equations. And this is what we get. Now, let us do another step remember that E 1 E 2 are, in fact, dependent on psi f psi g psi f psi g psi h and psi k. So, E 1 and E 2 are dependent on the rotor fluxes and what we get here next is this. Of course, in the, this particular equation there is one important assumption, which I have made that the rotor fluxes are assumed to be constant. So, d psi f by d t, d psi h by d t, d psi g by d t and d psi k by d t are assumed to be zero. So, they are, in fact, constants.

So, remember that in the modeling which I am doing this is a very simplified model of the synchronous generator and transmission line. I have assumed resistances are negligibly small, x double dash, x d double dash is equal to x q double dash then the rotor fluxes are assumed to be constant. Why they assume to be constant. The presumption here of course, is that the study which I am going to do is relating to fast transients. And the fluxes, the rotor fluxes are relatively slow. So, that is the kind of assumption I am made when I said that the rotor fluxes are assumed to be constant.

So, this particular model which we get assumes these three things resistance is a small, the transient sub transient reactance’s are equal on the d and q axis and the rotor fluxes are constant.

So, this E 1 E 2 are, in fact, dependent on the rotor fluxes. Now, if I if you look at this particular equation E d and E q are dependent on delta. So, I would like to rewrite these equations by converting the d q variables, these are lower case d q variables obtained by parks transformation which uses the rotor position of the generator.

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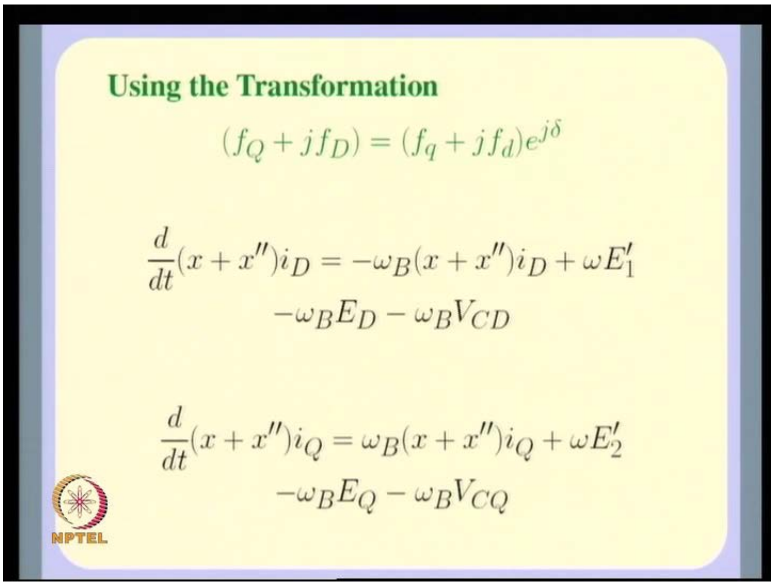

I transform this to the capital or upper case D and Q using this transformation. We have done this before in our study of a induction machine and so on. We convert it into a capital D Q or upper case D Q reference. In fact, if you recall in two machine system also we had converted all the equations of the inter phase variables the voltages and currents to this upper case D and Q variables, so that we could use KVL and KCL on them.

Here again we apply this transformation. So, what I have done is the previous equation, I have just transformed to the new variables using this transformation. And, if you do that what you obtain is what is shown here.

Now, the important thing which you should note here is that, E capital D and E capital Q are, in fact, constants. They are not dependent on delta. So, this is one important thing which you see here. Now, we the remaining question here, in the model of the electrical network is what is V CD and V CQ these are the capacitor voltages in the D Q reference frame.

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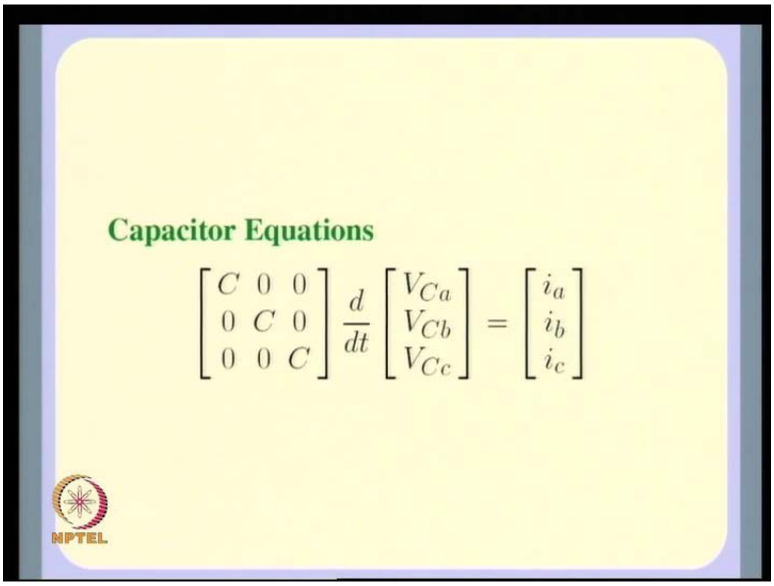

So, if you look at the capacitor equations for the three phases. These are suppose three lumped capacitors connected in series with the transmission line then the differential equations look, the way I have written them down here.

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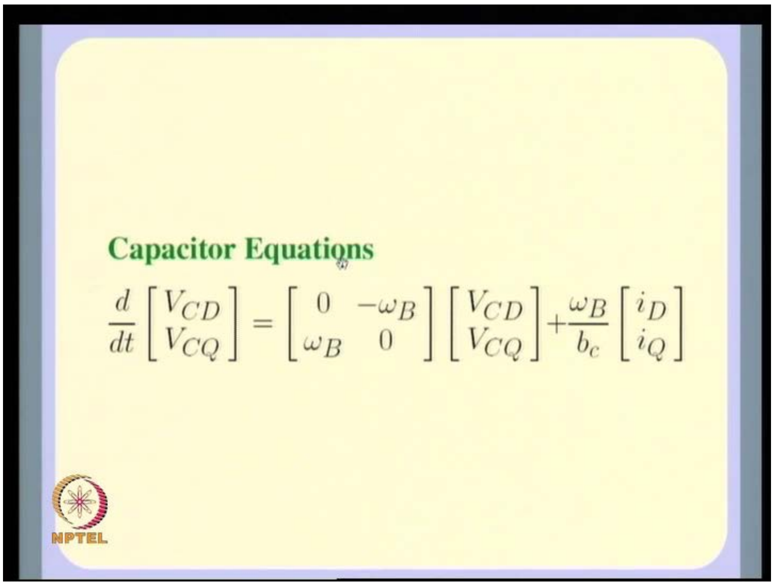

Now, if I transform them into the D Q reference frame what you get is this. Remember zero sequence has been neglected or rather disregarded here. because we are going to talk only of balance situations and the zero sequence equations are, in fact, decoupled completely from the D Q equations. What you notice here of course, in this equation is that you have got this additional term, in addition to the currents. This is of course, coming because we have used a time varying transformation to get from the A B C to the capital D Q variables or the upper case D Q variables. So, please remember that whenever we apply a transformation to the A B C equations. We effect effectively get this additional terms.

Now, once you have got this. This is so what we have done is effectively model electrical network. If you look at the equation, these are equations of a capacitor and these are the equations of the transmission line and stator fluxes assuming that of course, that the rotor fluxes are constants.

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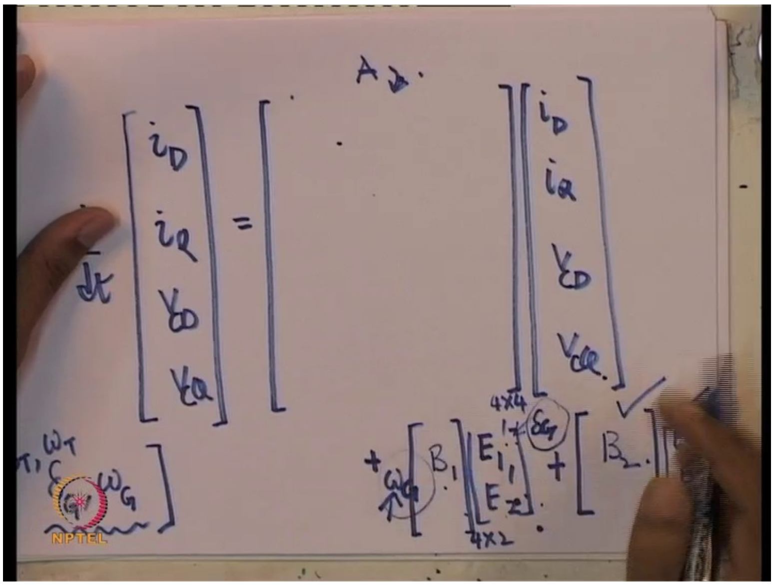

So, in fact, if you look at the d q equations, if you look at the d q equations you have got i d i q V D V Q. If you look at the electrical network d by d t is equal to for the electrical network and generator stator, rotor fluxes are assumed to be constant.

These are only the electrical equations remember that the there are equations pertaining to delta and omega. In fact, d delta by d t is equal to omega minus omega not. And of course, I should whenever I am going to talk now shaft torsional oscillations, I should make it clear that the delta here. You know the, we have to make a distinguish we have to make a distinction between the generator speed and delta and the speed and delta corresponding to the other turbine masses. So, this is very important.

So, you do have of course, the mechanical equations for the turbine masses they may not be one there may be 5 or 6 and your generator mass. So, these there will be differential equations corresponding to that we will look at them a bit later.

Now, the equations here are, if you look at the equations they will be a matrix a matrix here the state mat plus something corresponding to E 1 dash and E 2 dash into omega plus another matrix E d and E q. Now, one of the assumptions remember, what I have made. So, you will have a matrix which is 4 into 4 with the assumptions which I have made. This will be a 4 into 2 matrix, this will be 4 into 2 matrix E 1 E 2 E d E q. E d E q are going to be constants E 1 dash and E 2 dash are going to be dependent on delta. I just said E 1 and E 2 are constants. Then why are E 1 dash and E 2 dash dependent on delta. Remember, these were the original equations, if you look at the screen E 1 and E 2 are dependent on the rotor fluxes they are dependent just on the rotor fluxes.

When I do the transformation to the upper case d and q variables E 1 dash and E 2 dash are going to be dependent on delta. So, just remember that E 1 and E 1 dash and E 2 dash are dependent on delta where as E 1 and E 2 are not. So, E 1 dash and E 2 dash are obtained after you transform the lower case d q variables to the upper case d q variables.

So, E 1 dash and E 2 dash, if you look at what I am writing E 1 dash and E 2 dash are dependent on delta. These are dependent on delta, there is also omega dependence of this entire term this matrix is 4 into 2, this is also 4 into 2 matrix.

This A matrix is in fact, constant we will we will write this matrix down in the next class. This A matrix here is constant. This, I will call this the B matrix here, is B one matrix here is constant. And there is B two matrix here also is a constant.

So, what we have really is a system of fourth order system. We get a fourth order system for the electrical network including the generator electrical equations. The reason why we are just 4 equations is because we have assumed the rotor flux is to be constants. And therefore, the differential equations relating to that are no longer into in the picture. V d and V q, in fact, I should call this V CD and V C Q, I am sorry. So, these are, in fact, the states corresponding to the series capacitor of the electrical network.

And this particular electrical network has got E 1 dash and E 2 dash which are functions of delta and the rotor fluxes. The rotor fluxes are constant, but delta need not be is of course, can vary during a transient condition. This delta, in fact, it is delta of the generator, it is not the delta of remember, now I should make a distinction between the generator delta and the delta of all the turbines because the turbines in principle during transients could have a position of the rotor position of the turbine mass could be different from that of the generator mass and so so the speeds.

So, omega should really correspond to the here to the electrical speed of the generator mass. And delta of course, here is a delta corresponding to the generator mass. So, this is something which we should keep in mind.

So, this is one term which is like an input to the electrical system which is a function of the mechanical variables and the constant rotor fluxes, constant. because you have assumed them to be constant, this is a constant term and a constant matrix. So, this electrical network, in fact, has two inputs one is the infinite bus voltages, the second is the voltages. In fact, they are like voltages itself. The variation of these voltages of a synchronous generator are; in fact, dependent on the mechanical variables.

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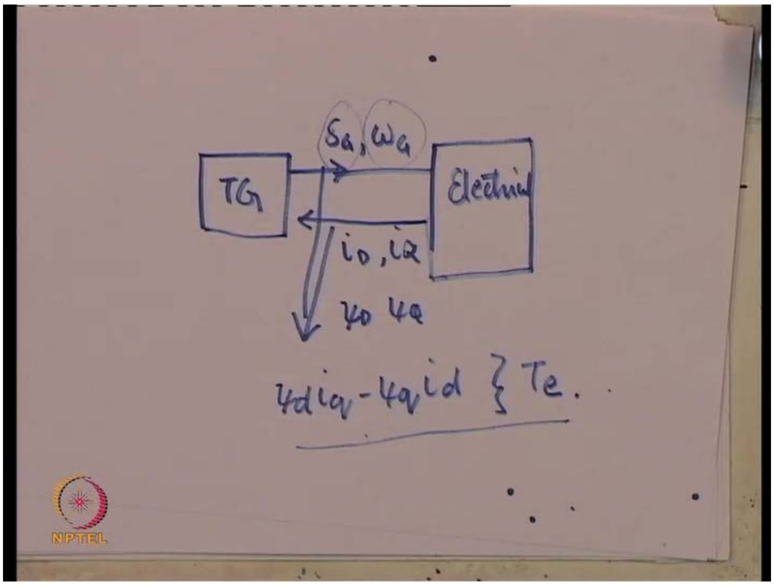

So, if you look at the electrical and if you look at the turbine generator system in some sense, the differential equations or the turbine generator system give you delta and omega G the speed and the rotor position of the generator. And the electrical network will give you i D, i Q. And of course, from i D and i Q you can also get psi D and psi Q and also from both of these things you can get psi d i q minus psi q i d, this is the electrical torque.

So, turbine generator interacts with the electrical network through these variables and this in turn affects the turbine generator through the electrical torques. This is what I meant when I said that this is a kind of a interaction between the turbine generator system and the electrical network.

And the next lecture, I shall show you that this particular interaction can be adverse for the torsional oscillations, if the torsional oscillations are sub synchronous that is there frequency is less then omega b the radiant frequency is less than omega b and the electrical network is also compensated using a series capacitors using series capacitors.

So, under both these circumstances the torques actually may be such that the enhance the existing oscillation. So, this is something which we will complete in the next lecture. And what I will try to show you is, you know using a case study, numerical case study is that these oscillation torsional oscillations can be destabilized by a series compensated electrical network and thereby you can damage the shaft under certain circumstances .

So, with this let us end today's lecture after doing the torsional oscillation, study the case study we will move on to understanding some some specific few cases on how you can actually improve system stability. This is something which you have not actually studied we have spent a lot of time on modeling and trying to show you, some phenomena like low frequency oscillations, the effect of AVR governor and now of course, the adverse interaction between an electrical network and the turbine generator is something which I will complete in the next lecture .

But we will spend a little bit of time the end of this course in some sense, the end ending part of this course on some simple case studies to illustrate how you can improve stability. This is something we have not concentrated. So, far as far as today's lecture is concerned what you need to remember before we start of the next lecture is that we have to model a turbine generator, if you want to study torsional oscillations by a more differential equations for each lumped mass you will have a differential equation .

And one thing which is a something which I did not prove, but you can easily infer that if you have got a multi mass multi mass for representation of a rotor of a turbine generator rotor. You will have if you have n masses rotor masses, you will have n minus one torsional frequencies this is not proved. but you can actually prove it. This is an interesting exercise. You also have the common motion where in all the masses move together this is, in fact, equivalent to what we used for low frequency oscillation studies in which assume that the whole system was whole turbine generator system was one rotor mass. We do not have to unlearn anything what we have done before because you were looking at only one aspect of the motion. In addition, now you have got torsional oscillations.

Now, these torsional oscillations can be relatively higher frequency more than ten hertz. In that case, you have to if you what to understand interaction of the electrical network through the electrical torques which are generated with this torque turbine generator system. In that case, you may have to model the turbine generator the electrical network without neglecting the stator flux transients and the network transients.

That is what you have done today, you have not neglected the d by d t is the d i d by d t is these are the d psi d by d t or d psi q by d t. This is a major departure of a departure of from what we did when we studied low frequency transient where very routinely we use to neglect the rates of change of flux the stator fluxes as well as a network current rates of change in the d q variables.

So, this is the major departure from what we were doing before and by, in fact, it is it is important to remember in this course although that our focus has been to study slower transients in which we could make certain assumptions there exist very important and interesting phenomena, in fast transient phenomena also in a power system.

And sub synchronous resonance or adverse interaction between the electrical network and torsional turbine generator system is one of them. In fact, I did not actually show you an adverse interaction, I just kind of put forth this tantalizing possibility that the electrical network may adversely affect the shaft torsional system. Now, this is something which I will show to you numerically in the next class, that this actual possibility exists for an electrical network which is series compensated. So, this is something we will do in the next time.