Power System Dynamics and Control

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Model No. # 01
Lecture no. #04
Analysis of Dynamical Systems

In the previous class, we discussed the analysis of a simplified model of a power system. In fact, the model we chose was very simplified one of our single machine connected to constant voltage source and our focus was on the analysis of that system. We will continue that today. So, todays lecture is titled the analysis of dynamical systems. We continue with our analysis.

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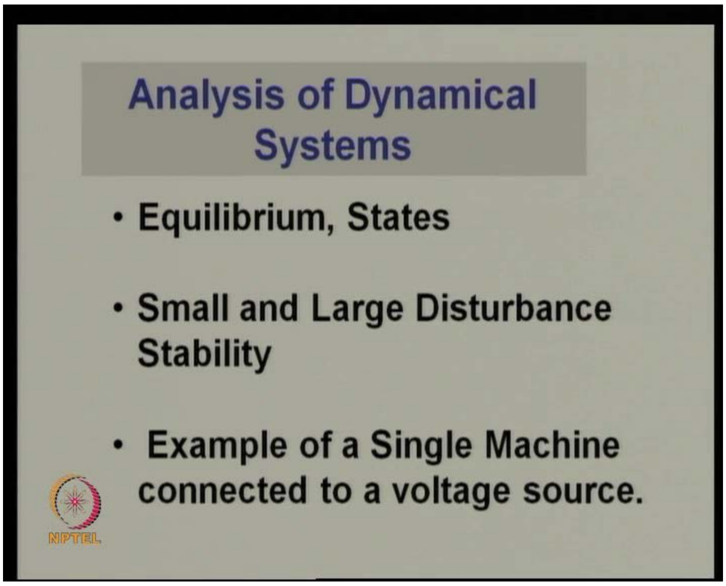

In the previous class if you recall, we discussed the concept of equilibrium or equilibria of dynamical systems, states, small and large disturbance stability and we began on this example which I just talked about, the single machine connected to a voltage source. In today's class, we will try to focus on large disturbance behavior of this particular system and then, we will turn our attention to the general analysis of linear system which is certain class of systems. We considered the toy model last time of a power system. So, let us just recap quickly on what we did last time.

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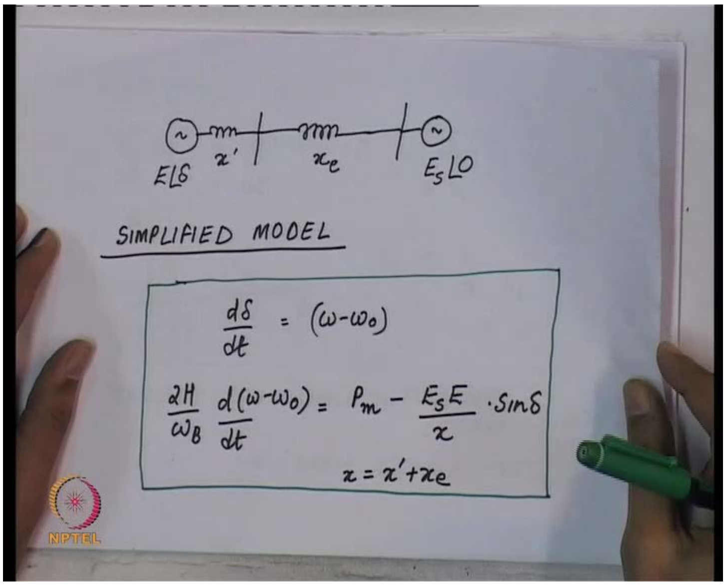

So, we had single machine connected to a voltage source. This was a stiff voltage source or we call it an infinite bus. The synchronous machine was modeled as a voltage source behind transient reactants. The phase angle of the voltage source was in fact delta which is related to the position of the rotating machine.

So, the simplified model, we did not drive this model. I just gave it to you. This is in fact a very simplified model as I mentioned in the previous lecture. It gives the motion of the rotor angle and the rotor speed deviations from omega naught which is the frequency of this infinite bus. This particular simplified model, what we did was first of all we tried to study the small disturbance stability of this particular system.

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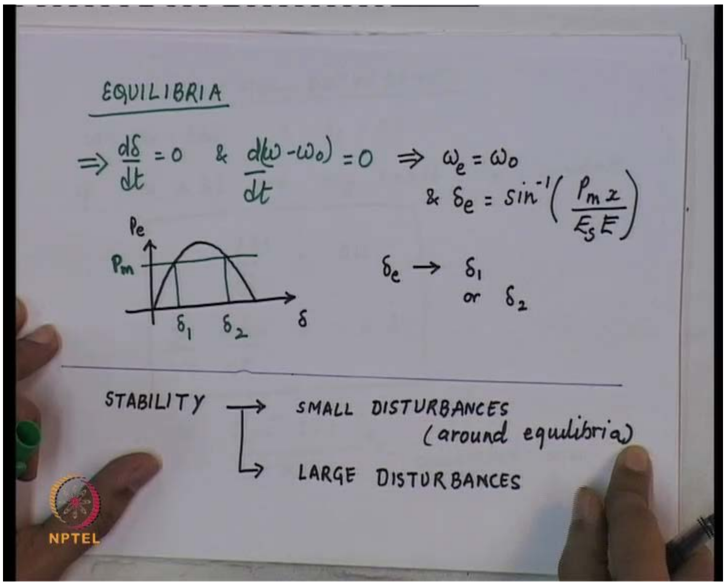

So, the first step which we took was finding out the equilibria of the system. In fact, equilibrium is defined as a point or the values of the states for which the derivative of the states become 0 and that yielded us two equilibrium points. Both have omega e equal to omega. This comes out of this particular equation and delta e was sine inverse of this quantity. This value of delta e results in this derivative become equal to 0.

Now, between 0 and 180 degree, they could be two equilibria. So, if this is for example, P m mechanical power which you assume to be constant, then there two equilibria delta 1 and delta 2. One is less than 90 degrees and the other one is greater than 90 degrees. Of course, you could have this continuing and you could actually have many more equilibria, but you noticed that they are spaced 200, that is 360 degrees away from these. So, actually they are indicative of the same position of the rotors.

So, we will not talk of equilibria beyond 180 degrees. We said upon ourselves to find out the stability of the system and we to some extent did an analysis for small disturbance around these two equilibrium. We need to really understand the large disturbance behavior. In fact, large disturbance behavior really is something which is important in the sense, that in the first lecture I told you of the phenomena of loss of synchronism. It is in fact a manifestation of the non-linear behavior of the system. So, small disturbance stability around equilibria is actually easier to analyze and easier to you know describe. So, that is what we did in the last class. So, just again looking at what we did.

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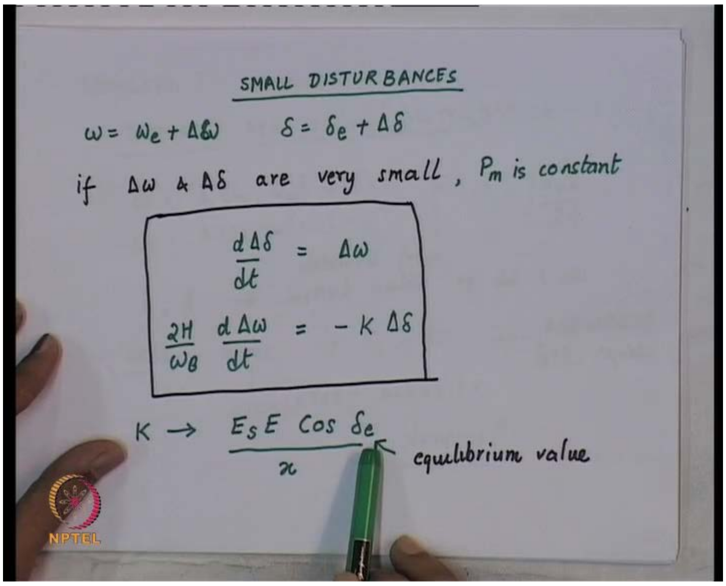

We assumed that we had small deviations around the equilibrium. So, we assume that delta omega and delta delta are very small, P m is a constant. Of course, I mentioned it some time back. Then, we got this particular model, where k was E s E cos delta e by x by delta e was an equilibrium value. So, if you are studying deviations around an equilibrium delta e omega e, then k would be this.

So, after that we obtained the response for small disturbance in fact whether a system is going to be stable or not after you given it a disturbance, that is, whether it is going to come back to the equilibrium is determined by the response of the system. So, response of this particular system was written down, in fact, it was guessed by analogy with a spring mass system. We did not actually derive it. We will do that of course for generally linear systems in the following lectures.

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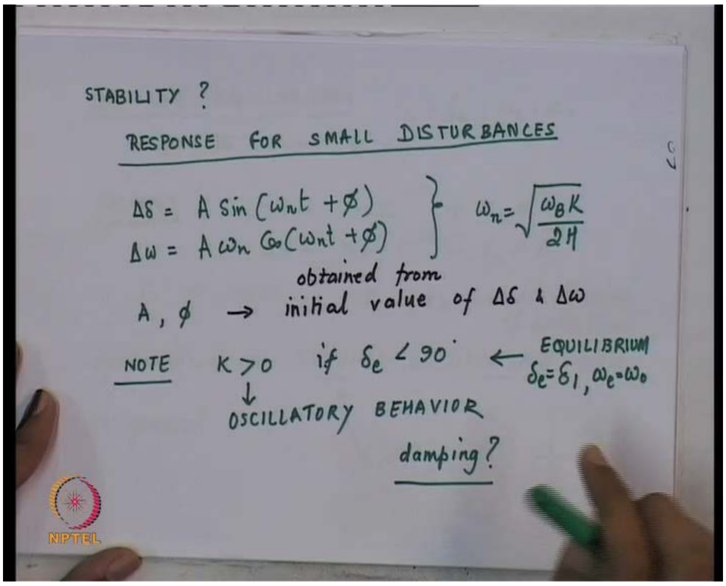

So, response for small disturbance is given by this. You notice it is an oscillatory response, where omega n is dependent on this value of k and h. It is a natural frequency of the oscillation in delta and omega. So, for small disturbance is around the equilibrium, we have got an oscillatory response, just one small point. This assumes that k is greater than 0. So, this is true actually. This response is true provided k is greater than 0. There is an oscillatory response if k is greater than 0, that is, delta e less than 90 degrees. The values of A and phi incidentally are obtained from the initially value of delta delta and delta omega.

Note that whenever we are talking of a stability of a system, we are really discussing the situation when we are away from the equilibrium. We are not at the equilibrium. If you are at the equilibrium, of course, you simple stay there. The point is that you are given a push from the equilibrium due to some disturbance. So, this disturbance could be things like step change in the voltage of the infinite bus or something small. So, we will not discuss what the disturbance is actually, but we will assume that it has been given a disturbance around equilibrium and of course, once you get disturbed, you will oscillate around the equilibrium provided your original equilibrium was less than 90 degrees, that is delta e was less than 90 degrees. This is true for the first equilibrium, that is, delta e is delta 1 and omega e is omega 0.

So, this is the first equilibrium. When I say first equilibrium, I am talking of this particular equilibrium point, delta 1 which is less than 90 degrees. Of course, point which may occur to you, would you call this oscillatory behavior is stable or unstable. The thing is that if you look at the response here, it just continues the system, just continues to oscillate around the equilibrium. So, would you call it as stable or unstable response? Actually, we would like the system to come back to the equilibrium. So, if you just oscillate around the equilibrium, you would call it marginally stable system. It is not really coming back to the equilibrium nor it is going away from the equilibrium with time.

So, of course, if there is some damping in the system, the oscillation will die on it. I am not proving this here, but you can really imagine it that if there is some kind of viscous damping or something in the system, it will be kind of oscillation will die down with time. In fact, in real system, in a real power system where in you model everything including the damp providing the field winding, there is usually some damping in the system. In fact, if you got control systems associated with the control of field voltage, you even can make damping negative. That is the oscillation which grows in time.

So, this is an issue which will handle later on. Right now, we have not considered any damping in our model. It is very simple model. We are not going to get any damping. It is just going to be an oscillation around the equilibrium. If you are near the equilibrium delta 1 delta is equal to delta 1, then that is what you will expect. Later on, we will come across more detail models of synchronous machines in which damping is present; some kind of damping will be there.

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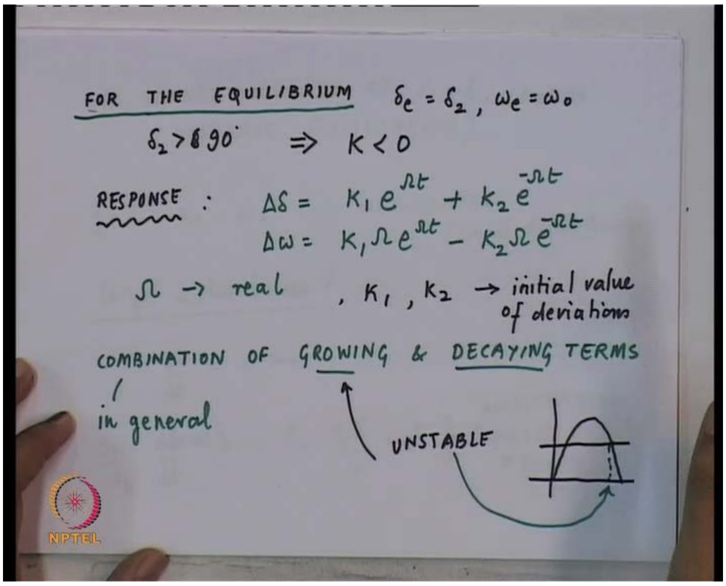

The other equilibrium delta 2 which is greater than 90 degrees, which is that unstable point k is greater less than 0. Why is it unstable? That is because the response turns out to be this and one component, aware this omega is real one of the component aware is going to be greater than 0, that is this or this can be greater than 0. So, what will happen is that you will have a combination of growing and decaying terms in general. Of course, there is an issue that will be k 1 or k 1 and k 2 are non zero. The answer is of course, k 1 and k 2 are dependent on the initial value of deviations and for more situation, most initial conditions, k 1 and k 2 will be non zero.

So, what you will have is you will have a combination of growing in decaying terms. The growing terms effectively mean that if I give disturbance from the equilibrium, you are going to move away from the equilibrium and this is what happens at this particular equilibrium point. So, this is what we discussed in the previous lectures.

One important point which you should note when we are talking of operating it unstable equilibrium, you cannot actually operate an unstable equilibrium part. If you give push, you are going to go away from the equilibrium points, any small portion. So, it is small disturbance unstable system and if you got small disturbance unstable system, you cannot operate at all. So, you have to be, you really cannot operate at this equilibrium delta 2 omega e is equal to omega naught. So, this is an important point which you should keep in mind. For example, the earlier equilibrium was to be unstable and that also would not be feasible to operate there either. So, this is an important thing. A system should be small disturbance stable if you want to operate it that point.

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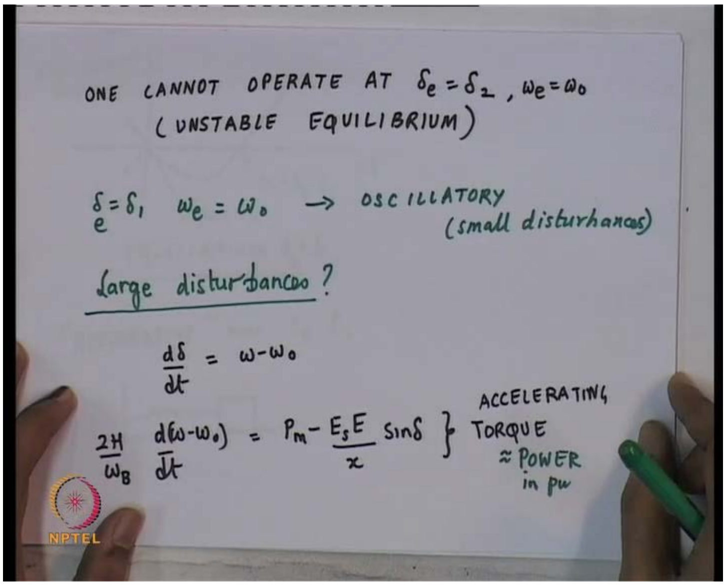

So, just to summarize, if delta e is delta 1, omega is omega naught, you get oscillatory response for a small disturbance. So, this is very important. The other equilibrium could have growing terms. So, it is an unstable system at the other equilibrium delta is equal to delta 2. For large disturbances, we cannot make any approximation based on omega or delta omega delta delta, delta being small. So, we cannot assume that the disturbance is small if the disturbance is not, rather if the deviation, initial deviation is small. If the deviation from the equilibrium is not small, you cannot assume that sine of the deviation is equal to the deviation. That is what basically we did in when we got the linear approximation.

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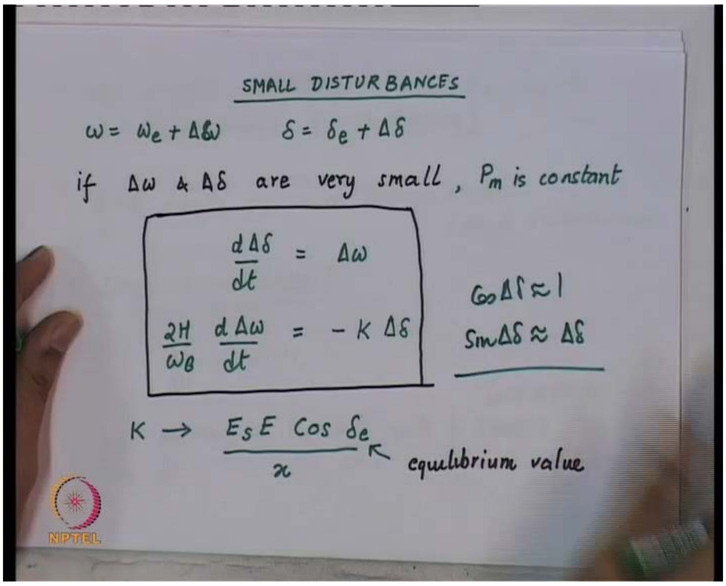

This particular approximation which we got was based on the fact that sine delta delta was approximately equal to delta delta. This was approximated as a key approximation and cos delta delta was approximately equal to 1. This is true only for small disturbance. For large disturbances, we cannot use this model.

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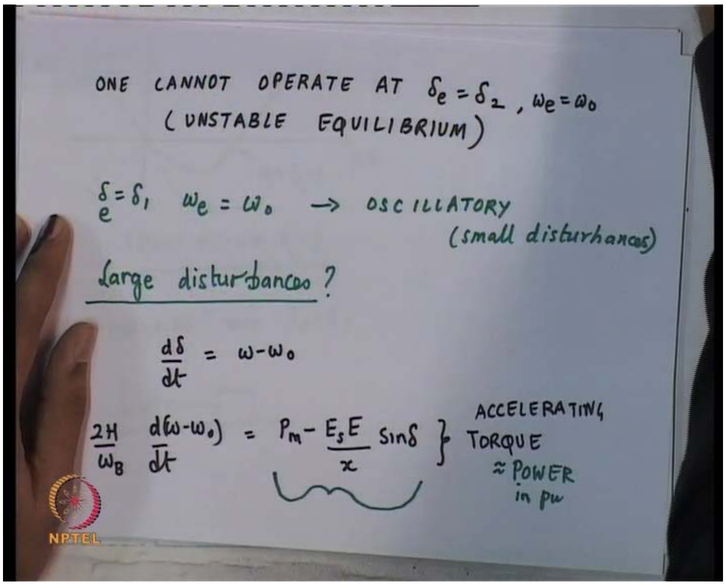

So, we have used the full grown model in which sine delta is retained. If you look at this particular term here, you can look at it as some kind of accelerating torque or accelerating power. We are talking in terms of per unit and will assume speed deviations from the equilibriums are not too large.

So, we can per unit torque and per unit power are in fact almost equivalent. So, we will just take this approximation. So, if you take this accelerating power here, it is accelerating power or approximately the accelerating torque. You will notice that as compared to this in this particular system, this is a linearise system; we are assuming that the accelerating torque is professional to the angular deviation here. If you do not assume small disturbance, this expression does not tell you that, it tells you that. It is depended on the sign of the angle.

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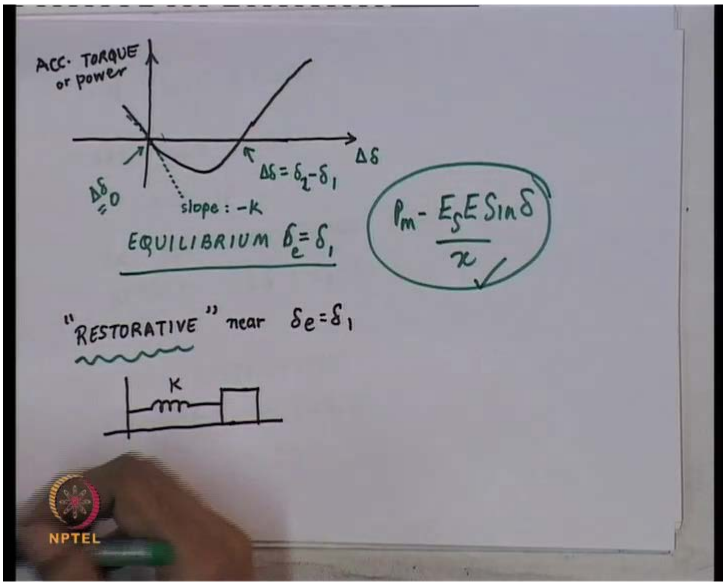

So, if you look at how this particular accelerating power or accelerating torque looks like. If you look at the accelerating torque or accelerating power, if you are at the equilibrium, that is delta delta is equal to 0, the act the equilibrium delta 1, the accelerating torque is 0, but it becomes negative, that is P m minus E S E sine delta by x becomes negative if delta delta goes away from the equilibrium and delta delta is positive. If it is negative, this is positive. So, it goes negative.

So, in some way, if delta delta is greater than 0, you will find if there is some tendency to get you back to the equilibrium because the torque is restorative in nature. That is very important point like spring mass system. You take a spring mass system and you give it push from the equilibrium. If it is a positive push, the spring is stretched and it has a tendency to pull it back. So, nature of the forces is restorative. Convulsively, if you push the spring, the spring is compressed and it has a tendency to push you back to the equilibrium. So, we can say that the torques or the forces are restorative in nature in this particular case, but look at this carefully. It is negative, all right, but after the point, it becomes positive.

So, if you look at this particular system, it becomes positive after some time. For this spring mass system, if you assume that the spring has got you know is ideal, then you can stretch it and force is always professional to the stretch. This is not true here. You will find it accelerating torque upon becoming negative. It is restorative till this point and thereafter, it becomes positive. So, for small disturbances, we can take it as restorative, but for large disturbance, it is not correct to assume it is restorative.

So, in fact, when we did the linearise analysis, what we effectively took was the slope of this particular curve at this point. So, this slope was the k which we talked about sometime back. So, for small disturbances, you can use this particular straight line with a slope of minus k, but for large disturbance, it is not correct to assume that the torque are the accelerating torque may not be restoring after some time. Now, the fact that the torque deviates from this linear or this line has got important consequences.

In fact, it goes; it kind of comes down after some. It gives the maximum and comes down after sometime becomes negative. You will find that if the disturbance is large enough, it is very interesting thing. You have got for the small, for small deviations the force is almost propositional to the stretch, the restorative force, but if you give the large enough disturbance, it will start to move, it will move and then, the restorative force suddenly start coming down and becoming negative. Now, the restoring force becomes negative, the mass will never come back to the equilibrium. So, what we should do? You know if the spring stretches up to a point, if it stretches up to a point at which this restoring force is become equal to 0, then you may not come back to this old equilibrium again. So, that is the important thing.

So, if there is a large disturbance, you may or may not come back to the equilibrium. You may actually just go away and you may just become unstable. So, that is an important point. Interesting point is that the restoring torque in this particular system becomes negative when the angular deviation reaches; angular deviation becomes delta 2 minus delta 1, that is, delta becomes delta 2. So, this particular point, we should ensure that we can kind of formulate kind of rule.

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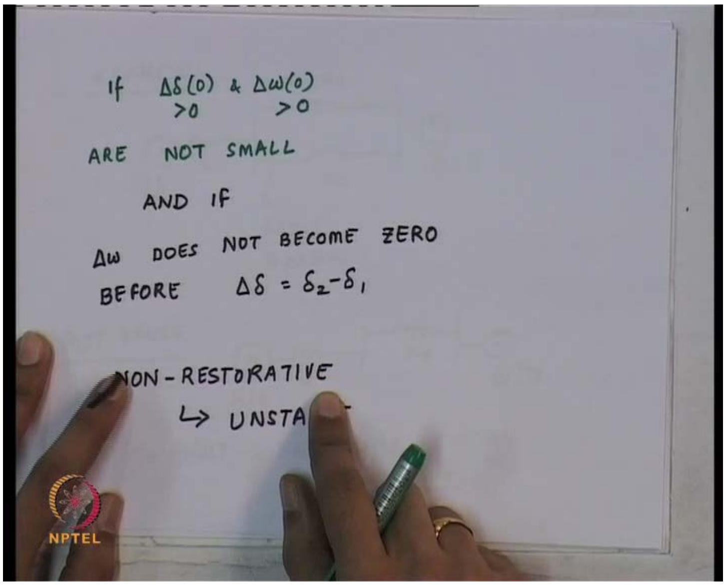

So, if I give disturbance because of which the angular deviation is greater than 0 and they are not small and if this speed deviation from omega naught does not become 0 before delta 1 becomes equal to, sorry delta become equal to delta 2 or delta delta equals delta 2 minus delta 1, then you have reached a place where torques are no longer restorative and you may become unstable. So, that is the basic reason why for large disturbance is a synchronous machine connected to a voltage source or later on, we will see connected to other synchronous machines could go unstable.

Now, the whole catch is after a disturbance are you going to be stable or not is something which depends on the response of the system because the rule says that is the speed going to be equal to 0 before the torques become non restorative. So, the thing is now you have to actually see what the response of this speed is, when it will become 0 and so on. So, that is something which is complicated. So, let us just take example of large disturbance.

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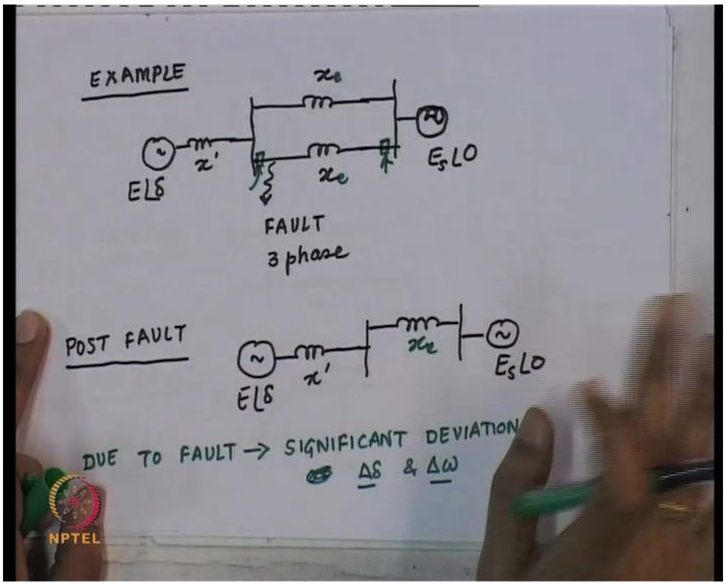

You have got synchronous machine connected to another voltage source. It is connected by two lines. On one of the lines, there is a three phase fault. So, you can have a three phase fault on a transmission line. If there is a three phase fault, there is a fault current in fact, the voltage here if it is a three phase bolted fault, then you will find at voltage at dips down to 0. There will be large fault currents and typically, there will be release in the system and circuit breakers.

So, release will detect that there is a fault in this system in this part of the transmission system and trip these circuit breakers. So, these lines get isolated. Suppose, fault you just have this particular line remaining in the system, this line is gone off. So, this is the typical stability question which you may come across for studying large disturbances, that is there is a fault like large disturbances like a fault and because of that fault, there is a significant deviation in these angles and the speeds from the equilibrium. Are we going to come back to an acceptable equilibrium after the fault is clear?

So, this is the post fault system. So, are we going to come to the post fault equilibrium? That is the question which we need to ask. So, just putting this let us try to formulate this as a kind of a problem.

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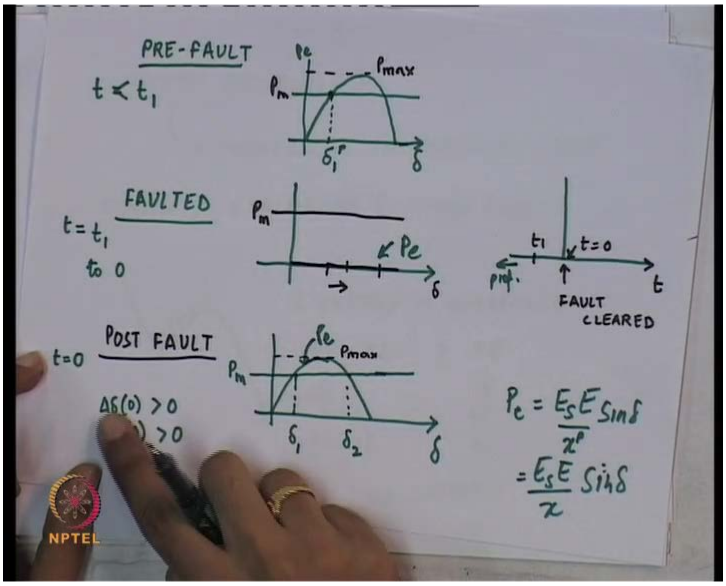

At time before t is equal to 0 and t 1, the system says in the pre-fault condition. So, this is in the pre-fault condition. In the pre-fault condition, we are operating at delta 1 with a superscript P. So, this is the equilibrium point when you have fault. Let us say, this fault occurs at t 1, the electrical power becomes 0. Why does it become 0? Because the terminal voltage is dropped down to 0. We will assume up three phase fault. So, the voltage drops down to 0. So, electrical power, this is in fact electrical power p e, it becomes 0. So, there is suddenly mechanical power become much greater than the electrical power and you will find the machines accelerate. Delta and omega will suddenly start increasing.

At time t is equal to 0, the fault is clear by the fact that release detect the fault and trip the faulted component that is a transmission line at t is equal to 0. So, at t is equal to 0, the electrical power will become this. This is the electrical power p e, the equilibrium. Now, r delta 1 and delta 2. Remember that P e, the electrical power after the fault is different from the pre-fault electrical power because you have tripped one line. Now, your electrical power initially was E S E by x pre-fault sine delta and now, it is E S E x sine delta. In the pre-fault system x p superscript P is given by x 1 x dash plus x e by 2. After the fault is clear, it is simply x dash plus x e.

Now, of course, we have these equations. These equations defined the movement of the rotor angle and we also see that due to a fault, your angles and speeds have become greater than 0. They have deviated from the equilibrium. So, there is a, there are substantial deviation, there is a substantial deviation from the equilibrium.

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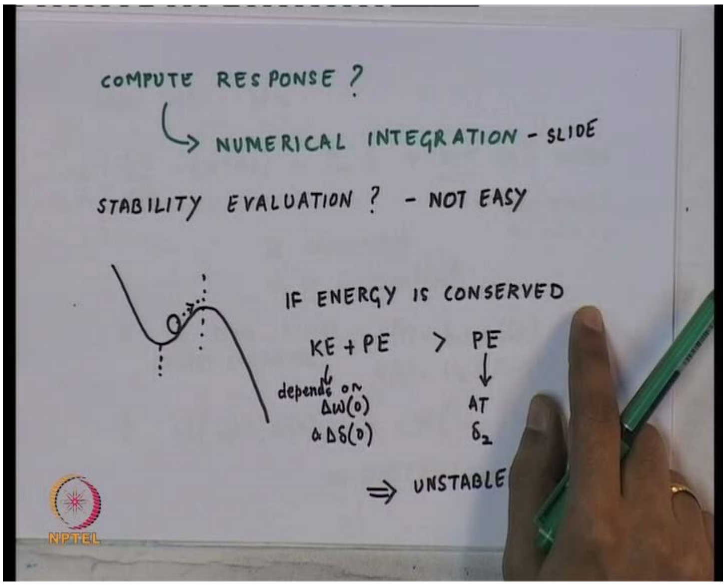

Now, how do you know whether the system is going to be stable or not? Well, you could numerical integrate the system. So, if you numerically integrate the system, you can actually find out. So, numerical integration means this particular integration is performed by discretizing this set of equations. How to do this is something we will discuss later in the course, but soon enough. If you do a numerical integration of the system, I will just show you know numerical integration.

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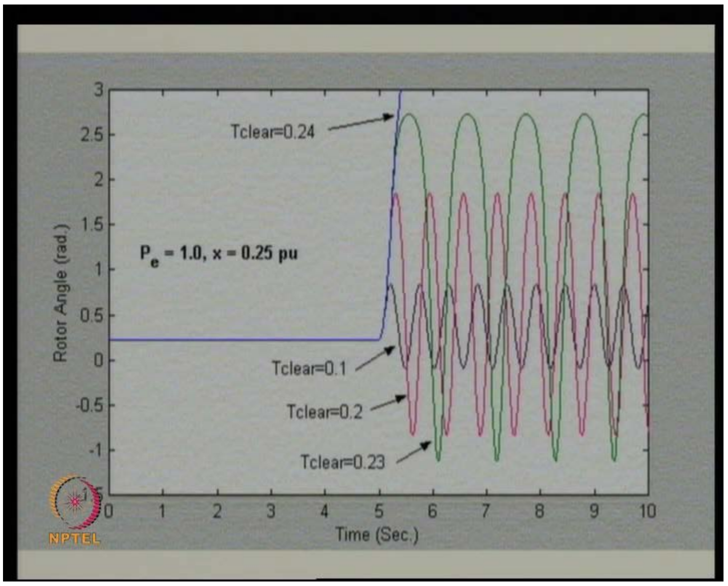

If the clearing time of the fault that is a fault duration, that is t 1 to t 0, that is the fault duration is small, the behavior may mix that after small disturbance behavior. It is oscillatory. If the fault deviation is increased, you will find that the oscillation magnitude increases and it still looks like a sign wave, that is, what small disturbance is like, but if now the fault clearing time and therefore, the deviation from the equilibrium increases beyond the point, you will find that the nature of the curve is like this. It is no longer looking like a sign wave. This is in fact numerical integrated solution of the system. So, actually use a numerical integration technique to find out this. How to divide it? Let me assure you we will learn it sometime soon in this course.

One important point is if the clearing time becomes greater than the certain value, you become, you loss synchronism. You become unstable; you are no longer going to come back. This angle just grows with time. So, this is a typical situation in which because of a large disturbance, we are not going to reach equilibrium. If the fault duration is large, then this is what will happen. Remember the equilibrium is stable. You know small disturbance stable, but rather we can say that it involves oscillatory behavior around the equilibrium, but for large disturbances, it never reaches the equilibrium again.

So, the question is of course, this is why do we need to do non-linear, why we need to do numerical integration in order to find out this particular response? The answer is this.

Whenever you have a non-linear system, unfortunately the response cannot be written down in terms of simple function which you know. So, this is not true for linear system.

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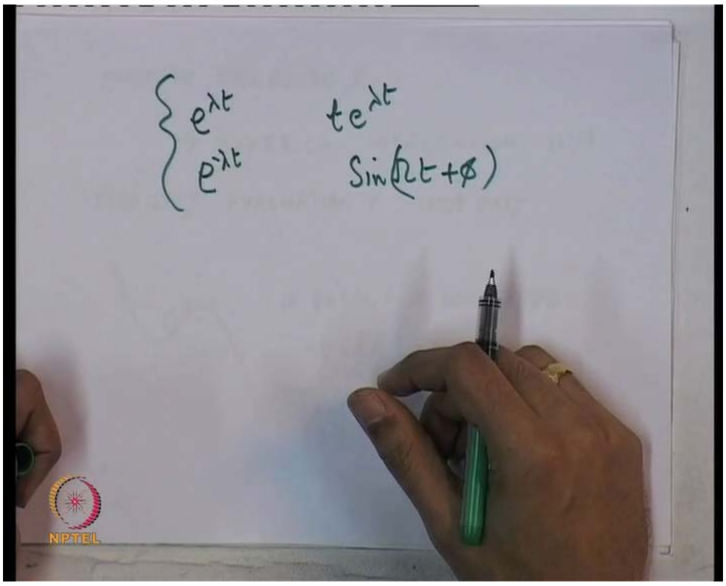

So, if you take linear system, you can write it in terms of it turns out that response is in fact in all linear systems, you will find that response contains terms like this, e raise to lambda t, e raise to minus lambda t, t e raise to lambda t sin. So, typically linear systems are super position of terms of this kind. You will find at this is how most linear systems. So, you take out any linear systems response and you find out this, but for non-linear systems, we cannot write down the response in terms of simple well known functions and as a result of which you may have in other than in very very special cases, you may actually have to, you can find out the answer only if you numerically integrate the systems.

So, the response in general for a non-linear system for example, the one we are talking of for large disturbance, we have to consider the non-linear systems. We will have to obtain from doing a numerical integrating. So, we need to rely on a computer, typically if the system is very large. So, that is one of the problem issue we will face in this particular course that even if you get the model to get to infer the responses is not always very easy. In fact, when you talk of a system which has got hundred of generators and tens of thousands of lines and buses and it is non-linear system to infer the behavior of the system becomes very tough because the response is complicated function. It can only be obtained by doing computer study of the system and that is this little bit worry some actually, philosophically worry some also. It basically says that unless you work out or really use very high level of complicated computation, you cannot actually infer how the system behaves and that is although you know the model.

So, this is very interesting because you know the physics of the system. The physics is very well known, but still you do not know how the system is going to behave unless you actually do study which probably requires a lot of computing resources. This is of course, not a system like this. A single machine infinite bus can be you know you can even do numerical integration without the lot of computer resources, but realistic system requires lot of computation. So, that is something which is philosophically interesting about the behavior of system. So, even if you know the physics of the problem, you know the equation has described each particular component. Once you integrate all of them, thousands of such components, how the behavior will require a lot of computation.

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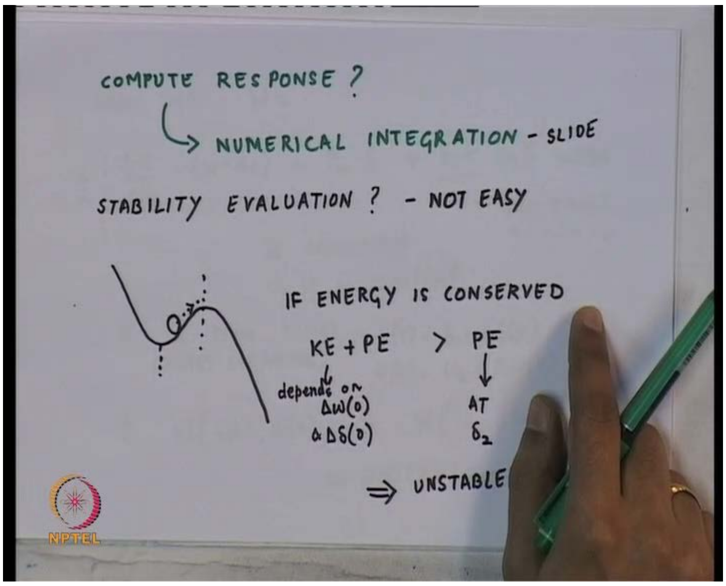

Interestingly enough, this particular system, this you know this particular system I mean only this system, in fact only this system model system are similar equations. Even though, computation of response requires numerical integration techniques stability evaluation, although it is not easy for this particular problem, it can be obtained. For example, look at this particular problem of a ball in a valley. We have talked about this problem in the previous section, previous class. I know because of certain disturbance, this particular ball has deviated from the equilibrium. It has got displaced by equilibrium by certain amount and it has also acquired some velocity because of the disturbance. Then, the question is, is this ball going to roll back or no?

So, one way of finding out you can make rule that if this ball speed becomes 0 before it reaches to this point, then it will roll back. So, at the limiting condition, the ball will just go right up to this, just reach here. It will just become, it will reach speed equal to 0 just at this point. This is a kind of limiting condition. Intuitively in this particular case, if I tell energy is conserved is another way of finding out whether this is going to happen or not.

So, I am not going to evaluate the response, but still from a certain criterion I can tell you whether system is going to be stable or not. If I compute kinetic in energy and potential energy of this ball at this point at this initial condition, if it is greater than just the potential energy at this point. This you know we assume kinetic energy is 0 at this point, then it will just be stable, that is the speed becomes 0. Just at this point it will be stable. So, the criterion is with a kinetic and potential energy here is equal to the potential energy at this point. I will call it delta 2. If it is greater, then it is unstable, but if it is equal to you will find the ball just goes and sits here. This is of course true only if energy is conserved. So, I am talking of very limited kind of situation, very specific situation. In fact, in our under graduate courses we learned about a criteria called equal area criteria. In fact, it is based on this particular feature.

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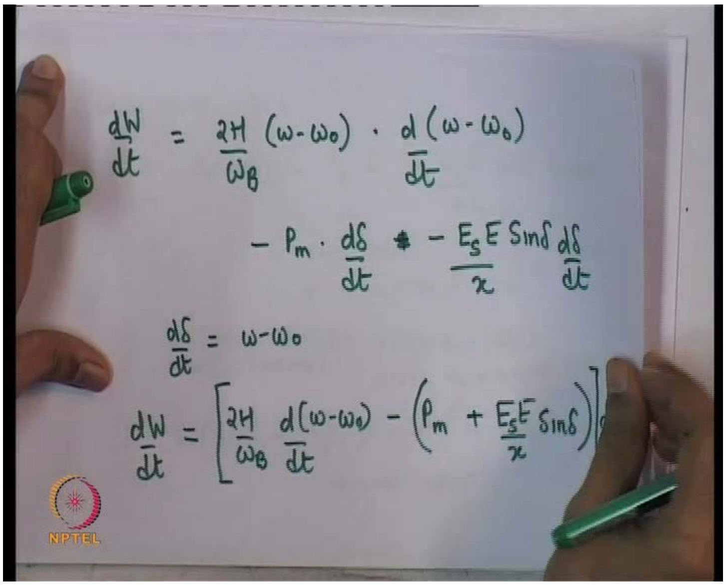

So, the only difference is that will have to define what is conserved actually. You can actually show that w which is equal to this particular quantity half of 2 H by omega B omega square minus P m into delta plus this into cos delta is the quantity which is conserved. How can you find it? Just try to evaluate d w by d t. So, shall we do that? Yeah. So, this is what it is. So, d w by d t is equal to 2 H by omega B into omega minus omega naught into d omega minus omega naught by d t minus P m into d delta by d t plus, well minus of E s E sine delta or x b delta by d t and d delta by d t of course, is equal to omega minus omega naught from our equations.

So, what we have is d omega by d w by d t is nothing, but 2 H by omega B d omega minus omega naught by d t minus P m minus this and this, there is some mistake in the signs. So, obviously I have written something wrong here. This should be minus and this should be plus, so minus. You look at this. So, if I define my w to be this, this is equal to 0. Why it is equal to 0? It is because of the second differential equation. This we know that this is equal to this.

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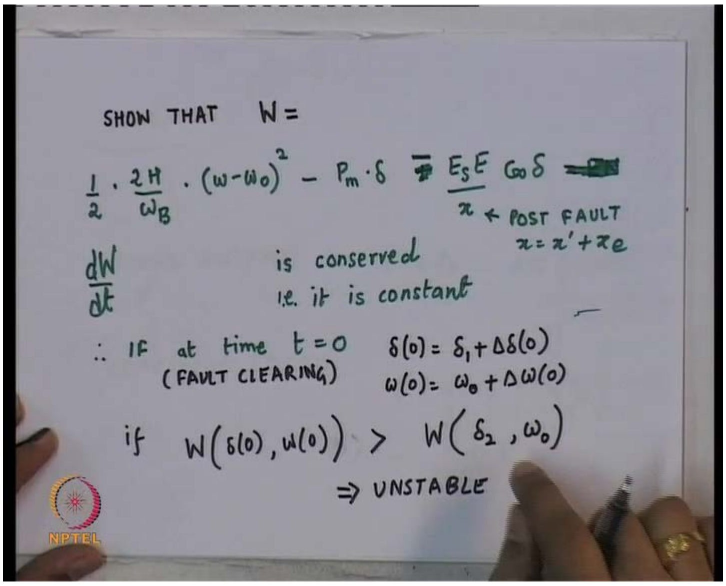

So, what we have is in this particular system? This single machine connected to an infinite bus. This is conserved. So, that is it is a constant. So, if time t is equal to 0, your deviations are delta in delta delta delta omega, that is, the values of delta 0 and omega 0 is these. Then, the condition for stability is that if the energy evaluated at the fault clearing time is greater than the energy evaluated at delta 2 omega 0 because omega, this is omega 0. First term will become 0. So, basically we are using the same criteria as we are using here. So, in fact, this is the basis for equal area criterion which we have learned in our under graduate years.

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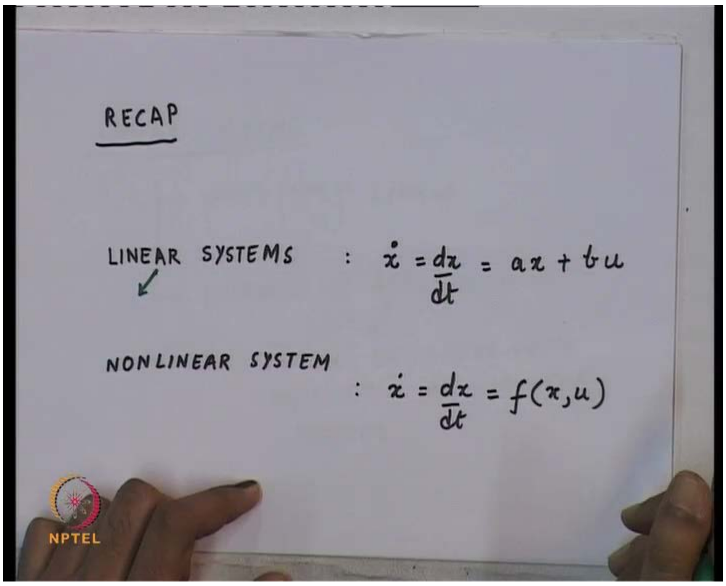

Now, we will have quick recap. The aim of this particular, this couple of lectures which we are having is actually you talk about general system. So, I have taken in example of single machine infinite bus system because we will get a kind of bird view of power system dynamics. We do not get lost in the detail of general dynamical systems, but of course, study of that is interesting and important. So, just a quick recap. A single machine connected to an infinite bus with two equilibria. One of the equilibria if you give small disturbance, you get oscillatory behavior. The other one is unstable for small disturbance. You cannot operate there at all.

For the first equilibria of large disturbance, you can become unstable, but large disturbance behavior requires you to consider the non-linear behavior of the system, that is the restoring torques is non linear functions of the angular deviation. So, what you have is basically large disturbance behavior is much more interesting in the sense that it shows that if you got a large enough disturbance you know no longer, it is an oscillatory response, but something like an unstable response and actually, this unstable response in this particular situation is in fact the loss of synchronism phenomena which you have learned before. So, if you recall in the first lecture, I said that machines which are connected to each other, synchronous machine connected to another synchronous machine or synchronous machine connected to a good voltage source, it tends to remain in synchronism with that with the rest of the system. So, if you give a small push disturbance, it tends to oscillate and come back to the equilibrium, but if you give the large disturbance, it may lose synchronism.

So, one important point is the inherent characteristic of the system is like this. The physics of the system is such that it behaves in such a way. So, if you give push for small disturbance and it will oscillate and come back to the equilibrium. If there was a large push and it will lose synchronism. So, this is as much true for this toy example as it is for large system. Of course, before we get lost into a lot of mathematical detail involving you know variables like x and y, let us look at you know what happen in the large system.

If you got very large system, you got many machines connected to each other. So, what happens for large disturbance in large system? The thing is that you have this loss of synchronism phenomena, but as I mentioned in the first lecture, you have got groups of machines, their angles, they move together. Their angles move or increased relative to the angles of another group of machines or many other. They can be three groups of machines also. You know this particular phenomenon occur in realistic large power systems and you know it is not easy to analyze. It is ok to analyze this small system by equal area criterion, but it turns out to be a big challenge to analyze large non-linear system.

So, coming back again, linear systems we are not actually defined linear systems in a very regressive fashion. In this course, in fact a lot of things are invited, you have not defined. So, for example, linear systems are the equation of this kind. Wherever constant coefficient, the rate of change of a state is equal to a constant coefficient into a state, of course, this is single order system. This first order system you can have higher order system as well which is of two sets of states. Non-linear systems, this function can be a bit more complicated. It is not simply a constant coefficient into the state. It can be a complicated function like sine x as we have shown in single machine infinite bus system.

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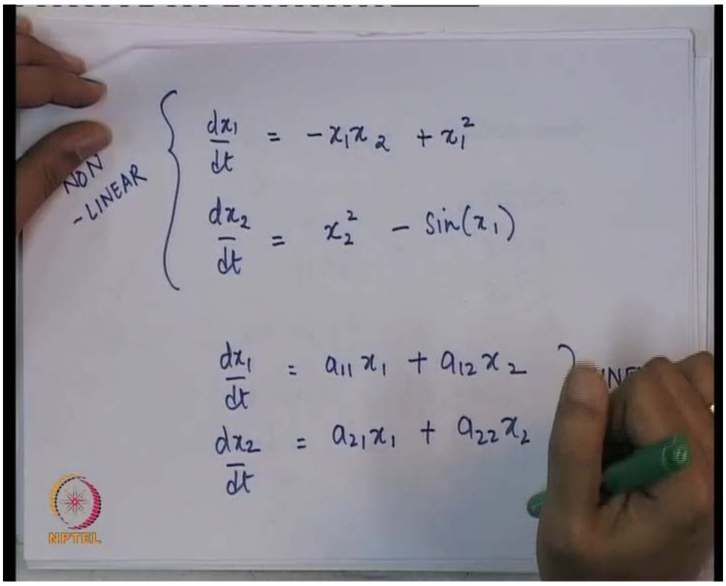

So, for example, you can take higher order system as well. For example, this is the higher order non-linear system rate of change of x 1 is the coupled system. Rate of change of x 1 is minus x 1 into x 2 plus x 1 square x 2 itself. The rate of change is again dependent on x and so, this is the example of non-linear system. You can have similar coupled equations of order 3, 4, 5000, one million and so on. A linear system, a coupled linear system can be of this form. In fact, when you are talking of a linear system, this is how it is. In fact, in these two systems are not given an input. You can have of course another input quantity plus something into u and this also could be a function of u as well which is an input like P m is an input in our single infinite system, infinite bus system. So, you could have input as well, but we do not have an input, it is called an autonomous system. So, these two are autonomous system without any input u.

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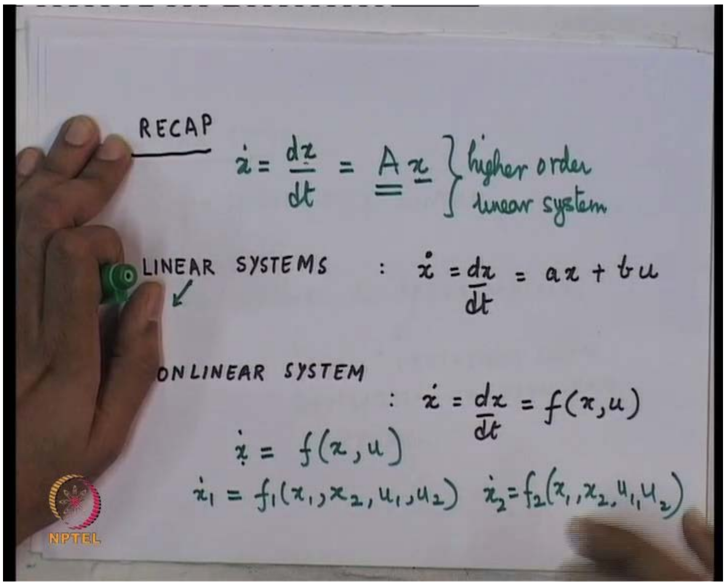

In general, linear system can also you know if you are having a higher order linear system, good way of writing this is A into x. So, A is a matrix, x is a vector, x is this, d x by d t is also a vector of the individual derivative of the states. So, this is the general way of writing a higher order linear system. A higher order non-linear system would be d x by d t or we call it x dot because x dot is easier, saves less, saves of bit of space. A non-linear system could be f x u, where x is a vector, f is also vector. So, you have got in fact like x 1 dot is equal to f 1 x 1, x 2, u 1, u 2 and x 2. This is the possible system f 2, x 1, x 2, u 1, u 2. So, this is a coupled non-linear system of two states, having two states.

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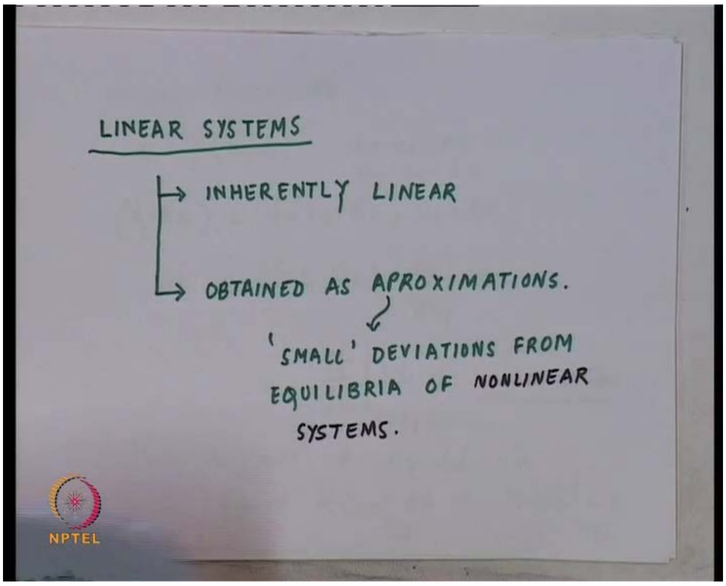

Now, linear system when we are talking of maybe, you know may come across physical system which is inherently linear. That can happen. Most physical system is silently turned out to have some non-linearity or other, but we can actually have some systems which are inherently linear. For example, I can design a system which is for all particle purposes linear. Some systems are inherently linear, other linear systems are obtained. So, when you really come across a linear system, they may as I said it may inherently linear or it may arise to an approximation.

For example, original non-linear system when we consider small deviation from equilibria of non-linear systems, we end up with linear system. So, this is what in fact we did for our single machine infinite bus system. We started off with a non-linear system. This is the non-linear system because you are analyzing small disturbances; we could get a small disturbance model around an equilibrium point. So, if this is the linear system and that was the non-linear system, so linear system are obtained as approximation, that is, small deviation from the equilibria of non-linear systems.

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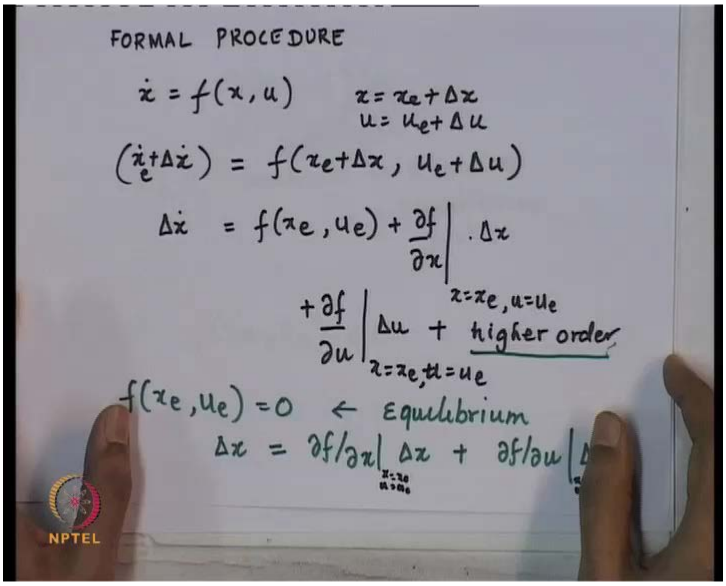

Of course, there is actually a formal procedure to get a linear system, a linearised system of originally non-linear system. So, if you have got x dot is equal to this is a non-linear system; you assumed as small deviation from the equilibrium. You substitute this and this into this, get a Taylor series approximation. This particular term is equal to 0 because at equilibrium by the definition of equilibrium x dot should be equal to 0. So, at equilibria x dot should be equal to 0. Therefore, f of x e u e should be equal to 0.

So, what you get eventually is delta x is the partial derivative of f evaluated at the equilibrium point. It is a very important evaluating at this at equilibrium point plus d f by d u partial derivative of f, sorry doe f by doe u evaluated at the equilibria. So, of course, you know this is an approximation because in neglecting higher order terms in this Taylor series expansion. So, the higher order terms like delta x square delta u square extra those kinds of terms we are actually neglecting. So, what we end up is linearised system with the understanding that it will probably give you good results or give you correct results near about the equilibrium point.

So, if disturbance is a small, this should give you good idea. Actually, this should be delta x dot. This should give you a good idea how the system is going to behave. In fact, there are you know kind of exceptions to this particular rule.

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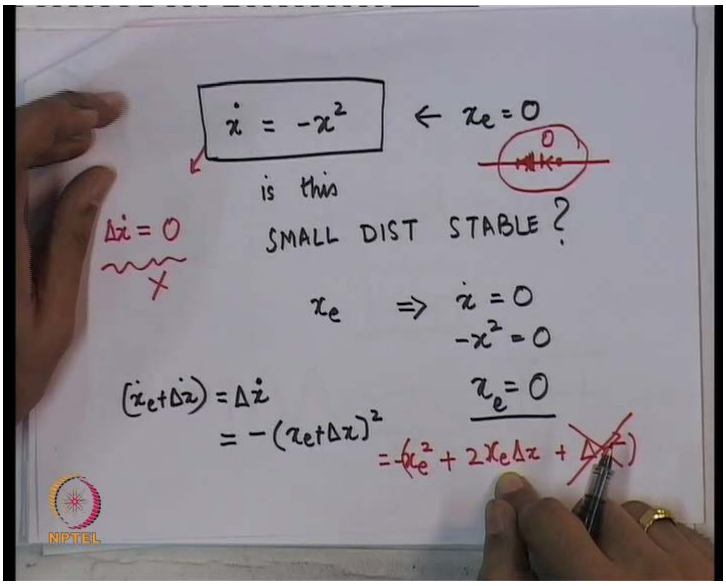

For example, if you try to apply this kind of procedure to evaluate the small disturbance stability of the system x dot is equal to minus x square around the equilibrium x is equal to 0. See actually this as an equilibrium x e is equal to 0. That is obtained simple by putting the derivative here equal to 0. So, how do you get the equilibrium z is equal to 0, x e becomes equal to 0. So, the equilibrium of this is this. So, if you linearise the system, you will have x e plus delta x dot which is nothing, but delta x dot because x e is value. So, derivative of it does not make any sense. In fact, it is 0. This is equal to minus of x e plus delta x square. So, that becomes equal to this is equal to x e minus of x e square plus 2x e delta x plus delta x square. Now, we will neglect this term. I told you we will neglect higher order terms of small disturbances when we evaluate small disturbance stability. X e because, actually it should be like this x e is equal to 0. So, the small disturbance model of this system is by the procedure I have told is in fact, since x is equal to 0, it is 0.

So, it says that if you have, if you got small deviation from the equilibrium, there is no movement. This is clearly not true. So, this is an exception to what I said, the formal procedure which I told you before. So, there are certain systems in which you cannot apply this formal procedure in order to tell how the behaviour for small disturbance is going to be. For small disturbances in fact, this particular system if x is negative, then you will get this x dot is positive. So, if you got this is 0, the equilibrium if you are here, this system tends to move to this equilibrium. So, in fact, if you start from here, you may actually come back to this equilibrium. If you are here on the other hand, if x is positive, x square is positive minus x square is positive is negative and you will come back to this, I am sorry I made a mistake here.

If x is negative, you tend to move away from the equilibrium. So, if x is negative since minus x square is negative, you move further away from the equilibrium. So, actually the non-linear behavior, the small disturbance behavior of this system does not seem to be obtained from the simplified system which we obtain by using the procedure which are described in the previous slide. So, obviously there are some exceptions to try to understand the small disturbance behavior by neglecting higher order terms, but these exceptions are very very rare and for all practical purpose, for almost of the system, you are going to encounter in fact, this simplification will tell you everything about the small disturbance behavior of this system.

So, please do not get roughly too much by the example which I gave, that is the pathological example. For most systems by this formal procedure, you can get the small disturbance behavior of the system.

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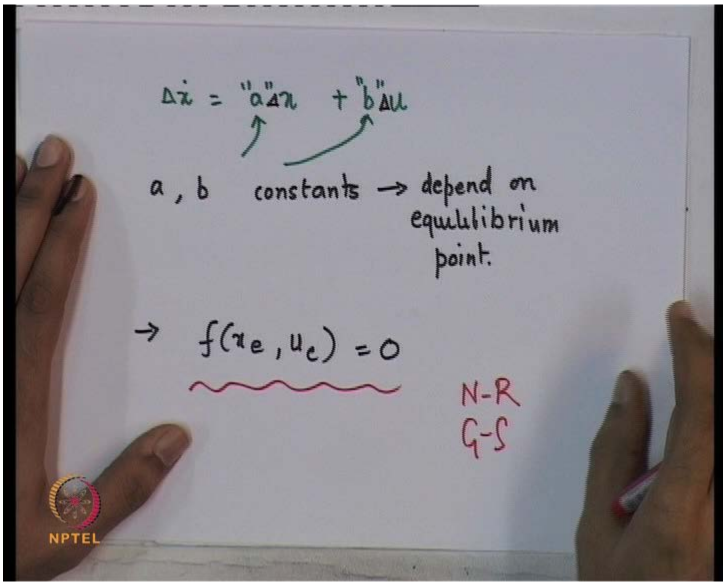

One important point to get the equilibrium, we need to solve this particular equation. If this is the non-linear equation, in that case, you will have to use numerical technique like Newton-Raiffeisen or Gauss-Seidel method to obtain x e and u v. So, even this is an interesting step when you are going to linearise the system in order to obtain the small disturbance model.

So, just to recap, a linear system may be obtained from a non-linear system and that linear system is obtained by neglecting the higher order terms. In most situations, the system so obtained will give you the correct small disturbance behavior for most system which is going to be that way.

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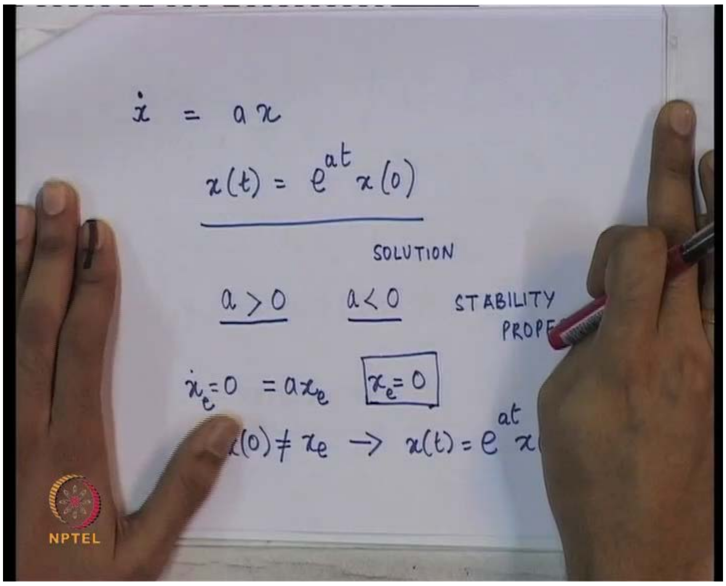

Now, coming to a very important topic The analysis of general linear systems. So, we are going to talk about general linear systems. Now, if you look at this particular example x dot is equal to a x, the solution of it is very simple. You can verify that x of t is equal to e raise to a t x 0. You take the derivative of this.

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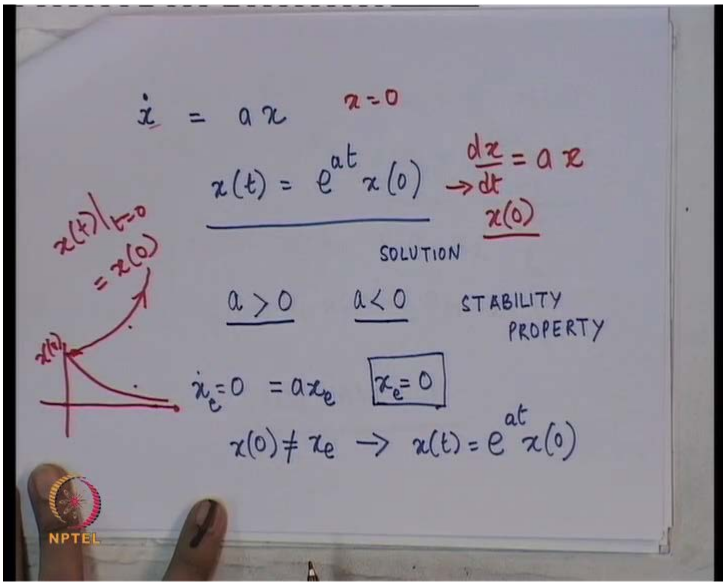

It turns out to be a into x. So, obviously this is a solution of this system. Now, another consistency check issue is that add x is t is equal to 0 is x is equal to x. So, if I told you that at time t is equal to 0, x is equal x 0, this also should be satisfied as far as the response is concerned. So, your plugging t is equal to 0. You will find at x of t at t is equal to 0 is equal to x of 0. So, this is also consistent.

So, this is the solution of this system for this initial condition. The interesting thing about the linear system which I have shown you here is that first of all, response is written nicely in terms of function we fairly well, we understand fairly well that is if e is greater than 0, you will have growing response is less than 0, your oscillation died out with time. So, if I start off from system from an initial condition x of 0, if A is less than 0, you will find it is going down to 0 which incidentally is the equilibrium value. The equilibrium value of the system is x is equal to 0.

So, if A is less than 0, this is how the response is going to be. If A is greater than 0, this is how the response is going to be. Just from the value of A, you can tell whether this system is stable or not. So, if I start from initial condition which is not equal to the equilibrium and A is less than 0, you are stable if system comes back to the equilibrium. If it is not true, of course A is greater than 0, you are going to go away from the equilibrium. So, this is unstable, this is stable. Now, so this is what I meant when I said that linear systems are a bit easy to understand. The nature response is or in terms of well known functions. You look at this system now.

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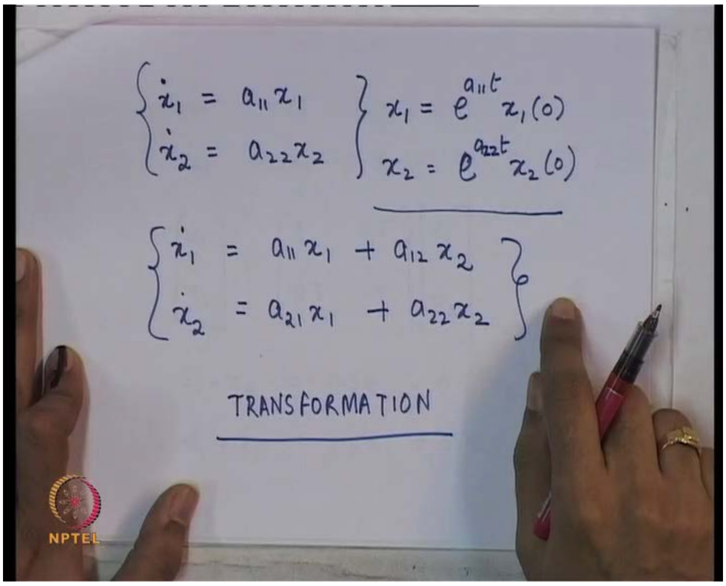

This is the second order system. The response is very easy to evaluate because x 1 is dependent on x 1 and x 2 dependent just on x 2. So, the solution of this system is very easy. You just have to apply this individually whatever I did in the previous slide to the individual states, but what if I have system which is coupled. This is second order coupled system. In that case, the solution no longer seems easy in the sense that it does not, you know I cannot infer right away. This is certainly not a solution of this. You can just plug in the value of x 1 and x 2 in this system and you can find out that this is not going to be solution of this.

So, the solution of this requires you to do bit of algebra and actually, by trying to understand this particular solution, we introduce ourselves to very very important tool of engineering which is the idea of transformation. So, I will give you the kind of simple system in which a transformation can be used in order to make it simplified. So, the idea of a transformation is something which I will discuss more in detail in the next class more formally, but just look at this simple system.

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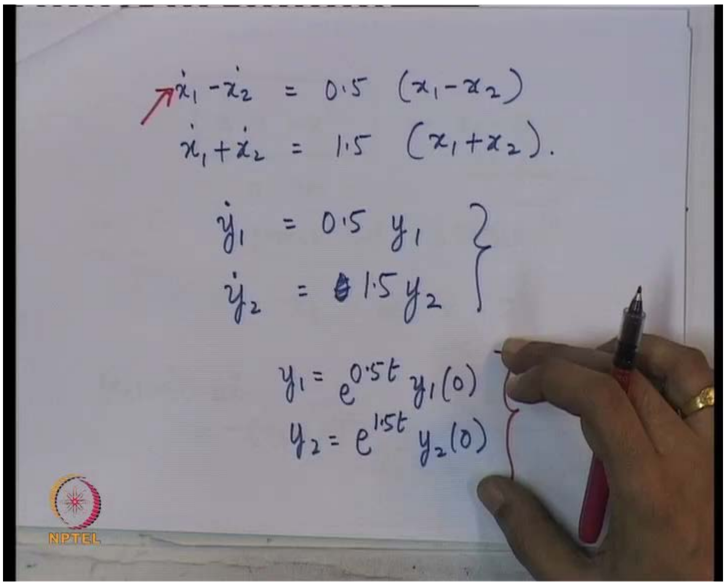

X 1 dot is equal to this and x 2 is this some system I want to analyze. This is a coupled system. Again the solution is not obvious. Just by in fraction is not easy to find out. I can write this in this fashion is a matrix, x 1 dot is equal to x 1 plus 0.5 x 2, x 2 dot is equal to 0.5 x 1 plus x 2. So, this is what I have written in this matrix form. Now, to analyze this system, let us just do one thing. We do not look at how x 1 and x 2 individually behave. We rather look at how the difference and sum of these states behave.

So, I just subtract these two equations in a linear system and subtract this, simply subtract in this. What I get and when I add them up, this what I get? So, for this particular system, I am not saying it looks neat, right but this is only for this particular system. I have given this simplified system so is to introduce to you the idea of a transformation. So, instead of looking at variables x 1 and x 2, look at these variables. So, what does it lead you to? It leads you to x 1 minus x 2 is 0.5 into this, x 1 plus x 2 dot, sorry x 1 minus x 2 dot is this, x 1 plus x 2 dot is this. So, if I call this is y 1, this variable x 1 minus x 2 as y 1, this is what you get and the solution in y 1 is very very simple, right. It is it has become decoupled system like the one we have considered some time ago.

So, this leads you to a simple solution. Of course, I have not got the solution in terms still of x 1 and x 2. We will do this formally in the next class. So, this is something we will do. It will lead us to the general analysis of linear systems.

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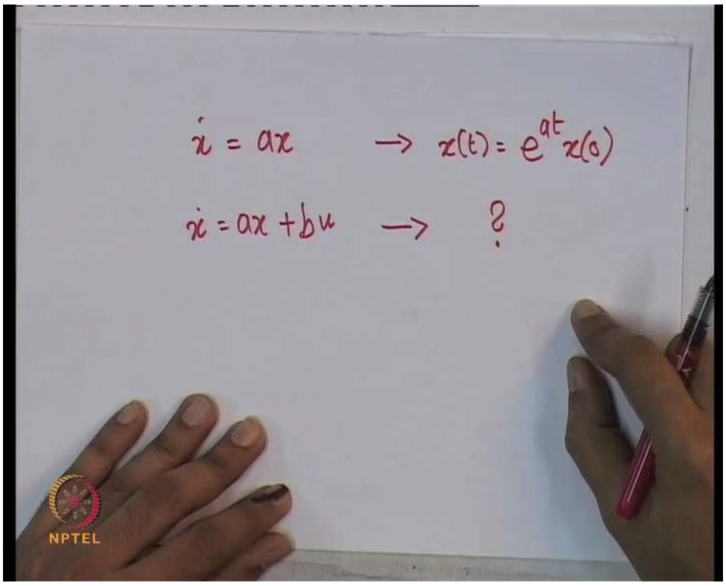

One small point I wish to make here is that if you got a system x ex dot is equal to ax, your solution is x of t is equal to e raise to a t x of 0. If your system is x dot is equal to ax by bu, what is the general solution? The answer is that you can write down the solution again very neatly in the sense that you can easily infer the properties of the response, but we will do this in the next class. So, we will leave two things for the next class. One is a more formal way of understanding the linear transformations to analyze these linear differential equations and also, what happens when you got you know some input, some forcing function like u. So, these two things we will do in the next class.

In the next few classes, we will kind of review a lot of analysis and we will be doing more of analysis of general systems, but all these things which you are going to do are going to be useful later when we apply them to realistic power systems and that is something let us keep it at the back of our mind. Sometime, we will also find out that some of the modeling principles also require you to do a bit of analysis. That is why I have decided to tell you a bit about analysis first before going into modeling. That is a reason why we have gone into the analysis of general systems. With that we, this is as far as this particular lecture goes. We will meet next time. Thank you.