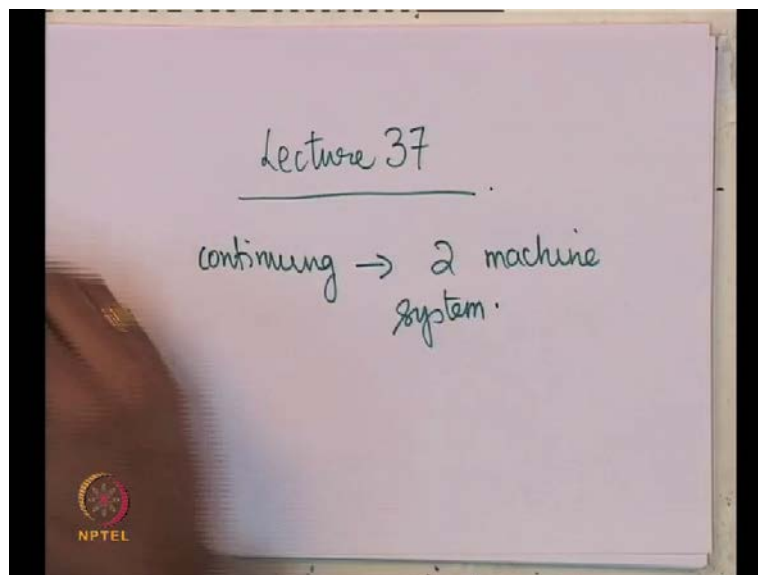


Power System Dynamics and Control
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Module No. # 01
Lecture No. # 37
Two Machine System (Contd.)

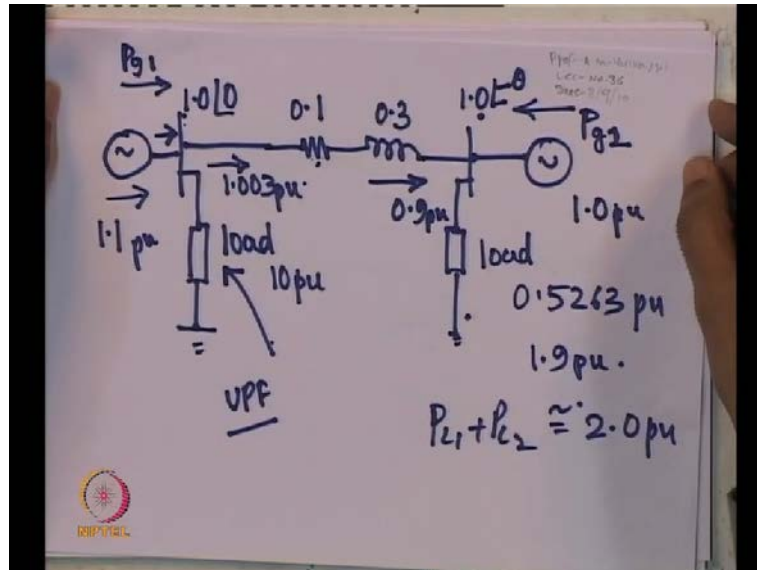
We proceed with our simulation of a two machine system. In the previous lecture we kind of, I gave you an idea how you can formulate the equations, we will actually go through **the** all the equations once. And then go ahead and actually simulate, I will show you the results of the simulation for a various disturbances like load change as well as a faults.

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Now, this is a lecture number 37 and what we will doing here is of course, continuing with our simulation of a two machine system. So, let us just have a relook at what the system we were trying to simulate.

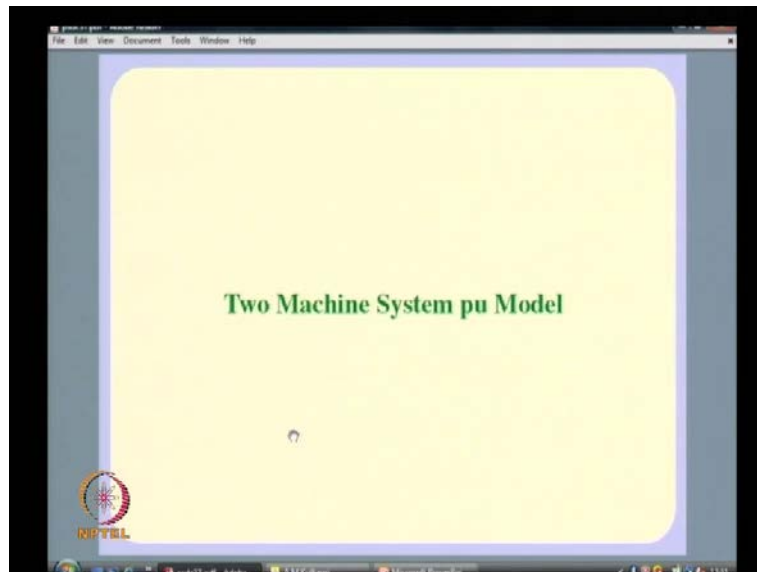
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This is the simulation, system **system** simulation and this is the initial steady state operating condition. Now, we will take this as the beginning or the starting point of the simulation; in fact, or you can say this is the load flow situation. And of course, the first step of any simulation is to back calculate initial condition, so if I know the voltages here, then I can get the values of for example; all the states and the field voltages of both generators based on the voltage at the terminals and the currents in steady state which are coming out.

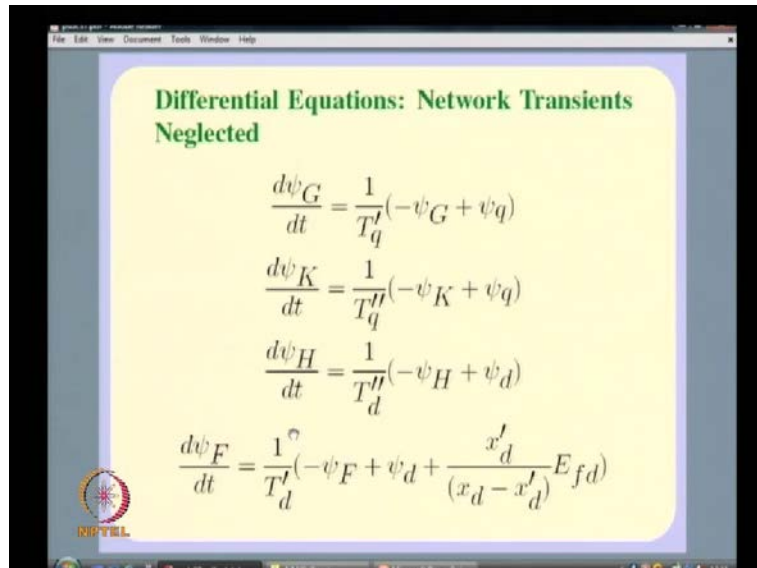
And of course, from the conditions from all this **you know** specifications which I have given for this scenario, you should be able to find out the voltage and the angles as well as the current output of the machine which will enable you, as I said to compute the initial conditions. You can refer to a simulation of a synchronous machine with an AVR connected to an infinite bus. We did that several lectures ago, where actually showed you how you can from the terminal conditions calculate all the initial conditions.

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So, let us look at the synchronous machine equations for the two machine system, I mean this includes both the machines and also how you will interface them.

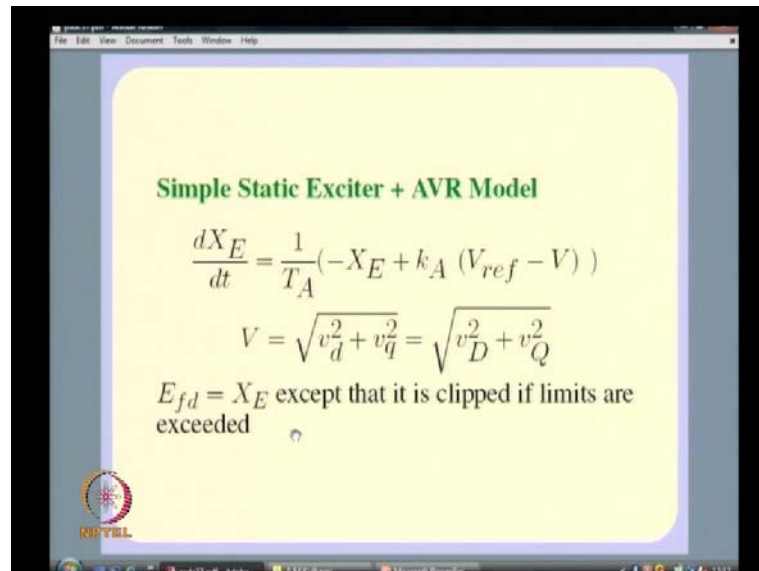
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So, of course, you know that the basic differential equations of a synchronous machine with stator transients neglected, we have just four differential equations per machine. So, when I say network transients neglected, what I mean also is that stator transients also are neglected. When I say stator transients are neglected, what I mean of course is that, $d\psi_d/dt$ and $d\psi_q/dt$ terms of the synchronous machine have been set to 0 and

the corresponding differential equation has been converted to an algebraic equation. So, what we have essentially are the differential equations of the rotor windings G, H, F and K.

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Simple Static Exciter + AVR Model

$$\frac{dX_E}{dt} = \frac{1}{T_A}(-X_E + k_A (V_{ref} - V))$$

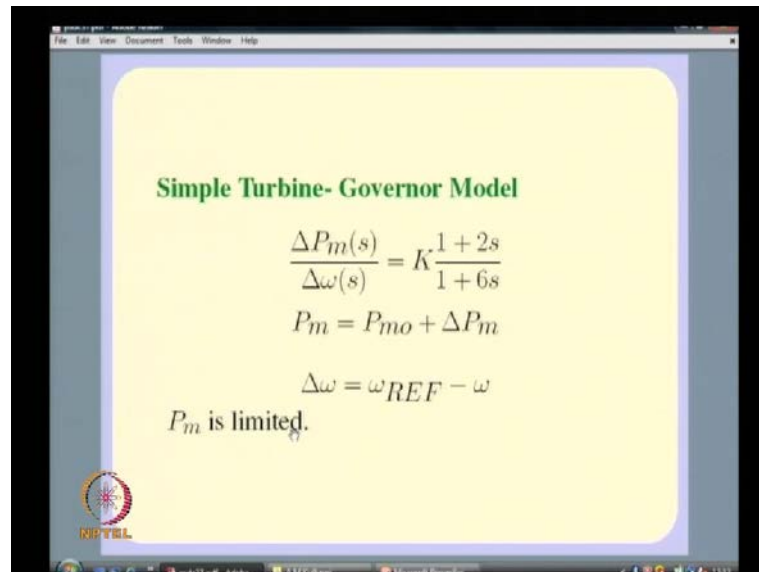
$$V = \sqrt{v_d^2 + v_q^2} = \sqrt{v_D^2 + v_Q^2}$$

$E_{fd} = X_E$ except that it is clipped if limits are exceeded

And we also have a differential equation corresponding to a simple static exciter plus AVR model. In the previous class I told you that, the AVR static exciter is modeled just by a transfer function $k_A / (1 + sT_A)$ that is the form of the transfer function. As a result of which it actually is embodying the differential equation in a state X_E which is given here. And of course, E_{fd} the field voltage is equal to X_E except when X_E hits limits; if X_E hits limits, we kind of clip E_{fd} to those limits.

So, this is basically the static excitation system model, this is the same for both the machines to keep things simple, we have considered a similar parameters for both the machines.

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Simple Turbine- Governor Model

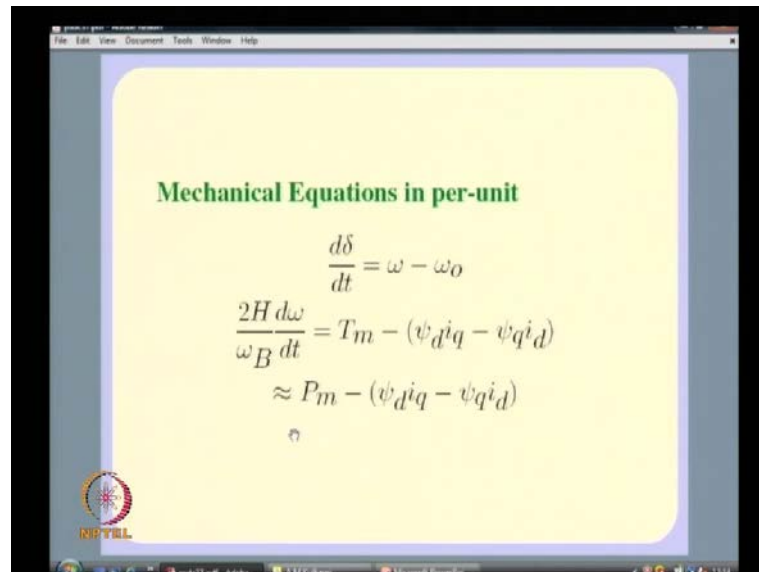
$$\frac{\Delta P_m(s)}{\Delta \omega(s)} = K \frac{1+2s}{1+6s}$$
$$P_m = P_{m0} + \Delta P_m$$
$$\Delta \omega = \omega_{REF} - \omega$$

P_m is limited.

The turbine governor model is also kind of a transfer function, of course you can write the state space equation corresponding to this transfer function, that **that** is something which I have not written here. But we will easily **you know** infer that this first order transfer function $K \frac{1+2s}{1+6s}$ is actually embodying a single differential equation. So, effectively you can write down this transfer function in state space form that the input is the error in the frequency or the deviation of the speed from the reference value, and the mechanical power is of course the set point, the load set point P_{m0} plus the output of this governor ΔP_m .

So, P_m of course is limited, and out here you see as in a static excitation plus AVR model, the turbine governor model, in fact the governor model effectively has a gain K , remember that the governor is a proportional type controller in this case. So, what we are considering is a proportional type controller in both synchronous machines. So, the turbine governor model, the steady state gain is not infinity like in integral controller, it has got a finite a steady state gain.

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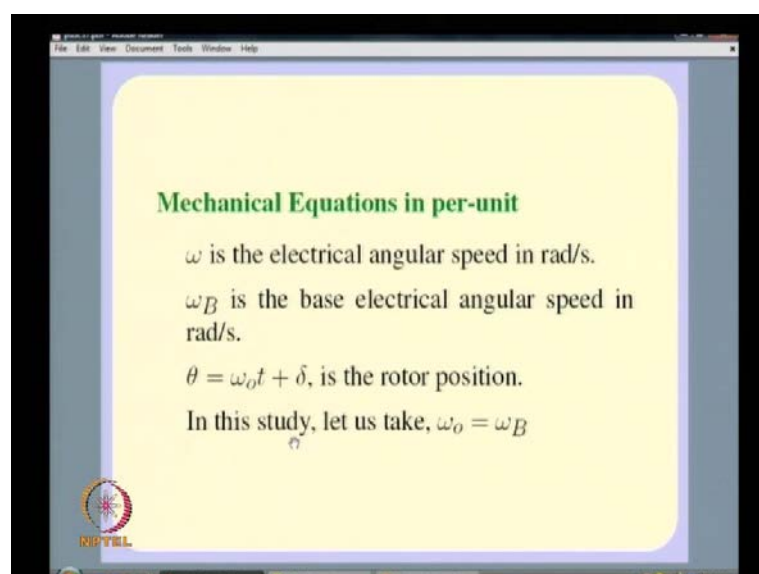
Mechanical Equations in per-unit

$$\frac{d\delta}{dt} = \omega - \omega_0$$
$$\frac{2H}{\omega_B} \frac{d\omega}{dt} = T_m - (\psi_d^i q - \psi_q^i d)$$
$$\approx P_m - (\psi_d^i q - \psi_q^i d)$$

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The mechanical equation, of course are the rate of change of a the rotor **the rotor** position, that is if theta is **theta is** of course the rotor position, so if theta is equal to omega t plus delta, then d delta by d t of course is given by omega minus omega not. And the rate of change of speed is of course the torque equation and in per unit, of course T m and P m are practically the same, the assumption of course is whatever transients you are going to study, the speed deviation from the nominal will not be too much. So, mechanical torque in per unit is the same as is practically the same as the mechanical power in per unit.

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Mechanical Equations in per-unit

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

$\theta = \omega_0 t + \delta$, is the rotor position.

In this study, let us take, $\omega_0 = \omega_B$

NPTEL

Now, if you look at the next equation rather a few more clarifications here. Remember that omega B in all what I have written is base angular speed; omega not is basically such that theta is equal to omega not t plus delta is a rotor position.

So, delta is equal to theta minus omega not t. In this study let us take omega not to be equal to omega B. So, what basically you are saying is that base frequency and omega not are taken to be the same. In fact, the thing we will assume this example to keep things simpler is basically assume that the steady state frequency is nothing but the base frequency. So, this is some simplifications which we are going to consider.

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Algebraic Equations (Both Machines)

$$\psi_d = x_d'' i_d + \frac{(x_d' - x_d'')}{x_d'} \psi_H + \frac{(x_d - x_d')}{x_d} \frac{x_d''}{x_d'} \psi_F$$

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'} \psi_K + \frac{(x_q - x_q')}{x_q} \frac{x_q''}{x_q'} \psi_G$$

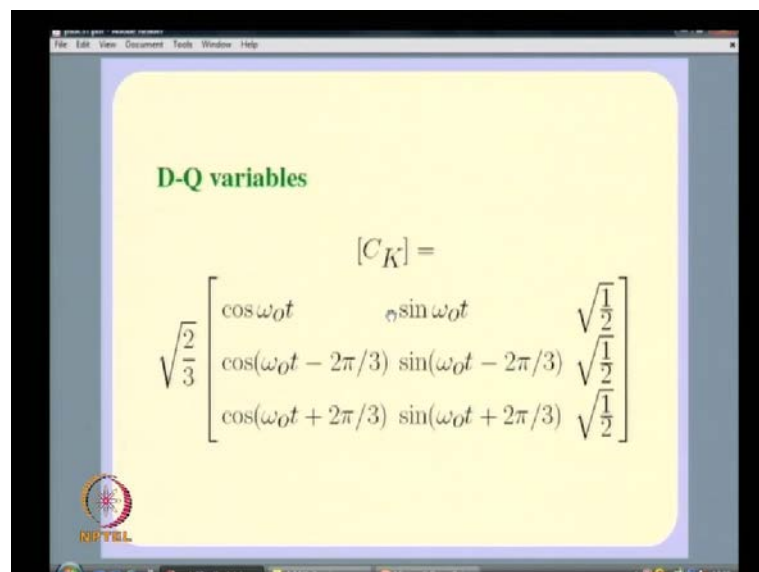
$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

We have the algebraic equation for both machines. In fact, along the d axis and the q axis the first algebraic equation is something of course **you know**, we derived the synchronous machine model. Second algebraic equation is actually got assuming the d psi d by d t is equal to 0 and in that, in this particular equation, this **this** term should have been omega, but we have approximated it to be omega B.

So, this particular algebraic equation has got constant coefficients into the, these variable psi q, i d and v d. Similarly on the q axis, you have got this algebraic equation; this is of course derived from the synchronous machine model. And this algebraic equation is in fact obtained by setting d psi d by d t and d rather d psi q by d t equal to 0, so the same approximations apply here as well as here.

So, we have got in fact four differential equations per, **you know** you have got four equations of the synchronous machine rotor windings, and one differential equation for the static excitation plus AVR, one differential equation for the simplified governor model, turbine governor model so that we will see basically six equations, algebraic differential equations per machine. So, they are two machines, there will be twelve states in fact, but the algebraic equations per machine are four. So, four algebraic equations per machine and they are listed down here (No audio from 8:30 to 8:45).

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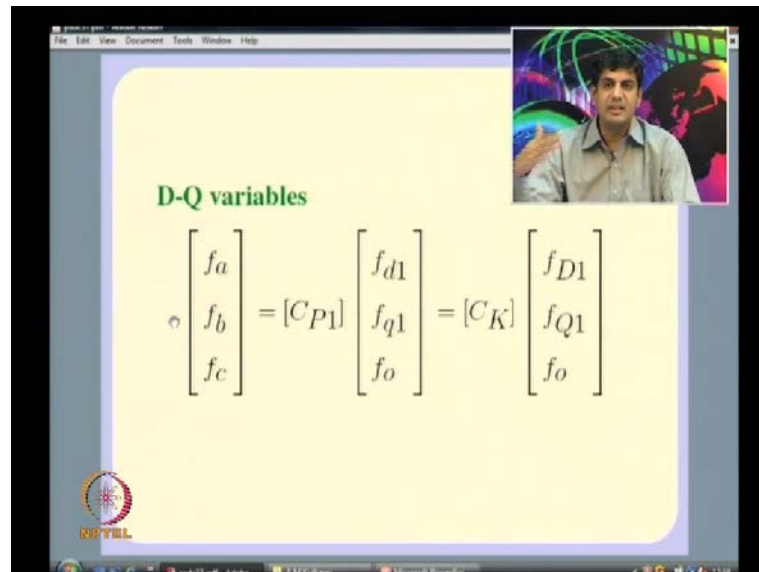
D-Q variables

$$[C_K] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \omega_0 t & \sin \omega_0 t & \sqrt{\frac{1}{2}} \\ \cos(\omega_0 t - 2\pi/3) & \sin(\omega_0 t - 2\pi/3) & \sqrt{\frac{1}{2}} \\ \cos(\omega_0 t + 2\pi/3) & \sin(\omega_0 t + 2\pi/3) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

Now, remember one thing, what I have shown you so far is the synchronous machines in the Park's reference frame. So, when you derive it for each machine, you will be using the Park's reference frame native to that synchronous machine.

But if I use a transformation C_K instead of C_P , where instead of using theta as the argument of the cosine and sin terms here, I will use omega not t, remember delta is missing here, **you know** if it was omega t plus delta, it would be theta corresponding to a synchronous machine. So, instead of having two different transformations for two different machines, what I will do is use a common transformation, whenever I am going to interface the variables corresponding to these two synchronous machines **ok**. So, what this will become clear in the movement.

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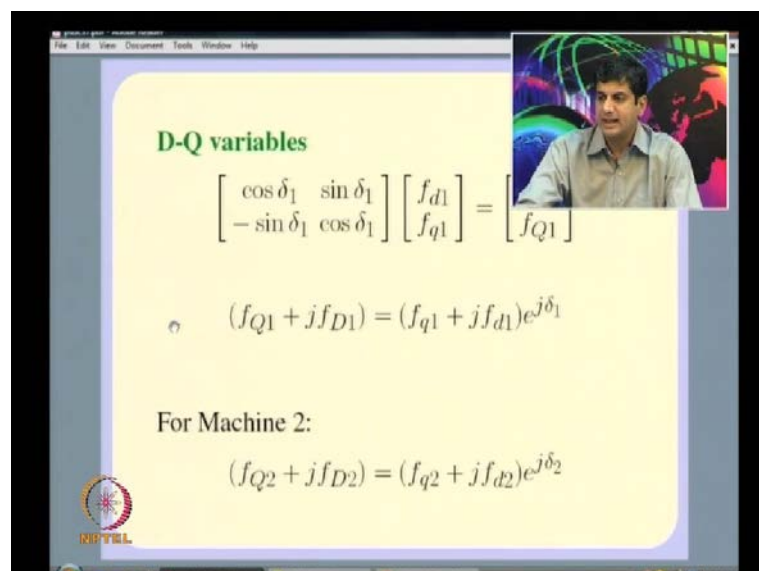
The slide displays the following equation:

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = [C_{P1}] \begin{bmatrix} f_{d1} \\ f_{q1} \\ f_o \end{bmatrix} = [C_K] \begin{bmatrix} f_{D1} \\ f_{Q1} \\ f_o \end{bmatrix}$$

The slide also features a small video inset of a presenter in the top right corner and the NPTEL logo in the bottom left corner.

So, the a b c variables, of course are the same all what we are doing is changing using the transformation C K to what is known as the D Q variables. So, if you look at how these transformations and the variables are related, you get the equation as shown here.

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The slide displays the following equations:

$$\begin{bmatrix} \cos \delta_1 & \sin \delta_1 \\ -\sin \delta_1 & \cos \delta_1 \end{bmatrix} \begin{bmatrix} f_{d1} \\ f_{q1} \end{bmatrix} = \begin{bmatrix} f_{D1} \\ f_{Q1} \end{bmatrix}$$
$$(f_{Q1} + jf_{D1}) = (f_{q1} + jf_{d1})e^{j\delta_1}$$

For Machine 2:

$$(f_{Q2} + jf_{D2}) = (f_{q2} + jf_{d2})e^{j\delta_2}$$

The slide also features a small video inset of a presenter in the top right corner and the NPTEL logo in the bottom left corner.

Now, if you look at **you know** you just do C K inverse C P 1, or do C P 1 inverse C K, you can get a relationship between f D 1, f Q 1 and F D 2, f upper case D 1 and upper case Q 1.

So, this relationship is essentially of a matrix relationship, but you can easily see that it **it** is it essentially gives you can be compactly written by this complex relationship. Similarly, for machine 2, if I use **you know** C P 2 which uses the argument for the cosine and sines as θ_2 is equal to $\omega t + \delta_2$, we will get basically the relationship between the capital or rather the uppercase Q and D variables as related to the local variables which use a local Park's transformation.

So, whenever am going to, what I will do is will get the equations to the uppercase Q and D variables, the lowercase q and d variables are using a transformation which is native to that machine. The uppercase D and Q transformations for machine 1 as well as machine 2, both use C K which is independent of **theta** delta. So, whenever we are going to, so f Q 1 and f D 1 uppercase are in fact variables which are obtained for a common transformation. So, f Q 1, f Q 2, f D 1 and f D 2 uppercase are in fact obtained from a the same transformation.

So, the point is that if **if** all the a, b, c variables are transformed using a common transformation C K, then we can use KVL and KCL, that is Kirchhoff's Voltage Law and Kirchhoff's Current Law while writing down, when interfacing both machines.

Now, the important thing is we do not have to transform all the differential equations to the uppercase D Q variables at all. What we need to do is whenever we are interfacing these variables, whenever you are writing the algebraic constraints relating to these variables, that time you should insure that the all the **the** variables involved in that interface are obtained using a common transformation C K. Otherwise, you will not be able to use the algebraic relationships which are obtained from Kirchhoff's voltage and current laws, so that will become quite clear now.

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Algebraic Equations (Both Machines): Assume $x_d'' = x_q''$

$$\psi_D = x_d'' i_D + \mathcal{F}_1(\psi_H, \psi_G, \psi_K, \psi_F, \delta)$$

$$0 = -\omega_B \psi_Q - \omega_B Ra i_D - \omega_B v_D$$

$$\psi_Q = x_d'' i_Q + \mathcal{F}_2(\psi_H, \psi_G, \psi_K, \psi_F, \delta)$$

$$0 = \omega_B \psi_D - \omega_B Ra i_Q - \omega_B v_Q$$

If you look at the algebraic equations corresponding to both machines, what I have done is the algebraic, the variables i_D lowercase i_Q lowercase i_D lowercase and i_Q lowercase have all been transformed, the equations have been transformed, the same relationship is there, but now I am using different variables, I am writing the same relationship in terms of different variables.

Now, the interesting thing is that, if I write down these algebraic equations in terms of the uppercase variables, I will have to combine those **you know** equations in fact, and you get this kind of form of the equations.

Now, the interesting thing is that if you choose x_d'' and x_q'' to be equal, what you find is that this capital D variables ψ_D is related to i_D by simply x_d'' , ψ_Q is related to i_Q also by x_d'' . So, if you assume that the sub transient's saliency is not there, that is x_d'' and x_q'' are equal, then an interesting thing which follows is that the algebraic equation corresponding to ψ uppercase D is related to i uppercase D by a simple constant coefficient which is not dependent on delta.

The other terms are all gathered out here and they are all states, F_1 is a function of the states, so these are non-linear function of the states. So, the algebraic equations effectively have been re written in terms of the capital D Q variables. Now, this of course can be done for both the machines. So, when you are talking of machine 1, you will have

of course an additional subscript of 1 every where. So, of course, I have not done that, but remember that this has to be done for both the machines.

So, what you will have effectively is that these four algebraic equations for each machines, for both machines there will be four algebraic equations, but remember that the variables in these four algebraic equations will be having a common transformation from the a, b, c variables. So, you can actually use these uppercase D and Q variables directly whenever you are using any algebraic constraints like KVL or KCL.

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**Algebraic Equations: Network + Load:
Transients Neglected**

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{r\omega_B}{x} & -\omega_B \\ \omega_B & -\frac{r\omega_B}{x} \end{bmatrix} \begin{bmatrix} i_{1D} \\ i_{1Q} \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} g_{D1} \\ v_{Q1} \end{bmatrix} - \begin{bmatrix} v_{D2} \\ v_{Q2} \end{bmatrix} \right)$$

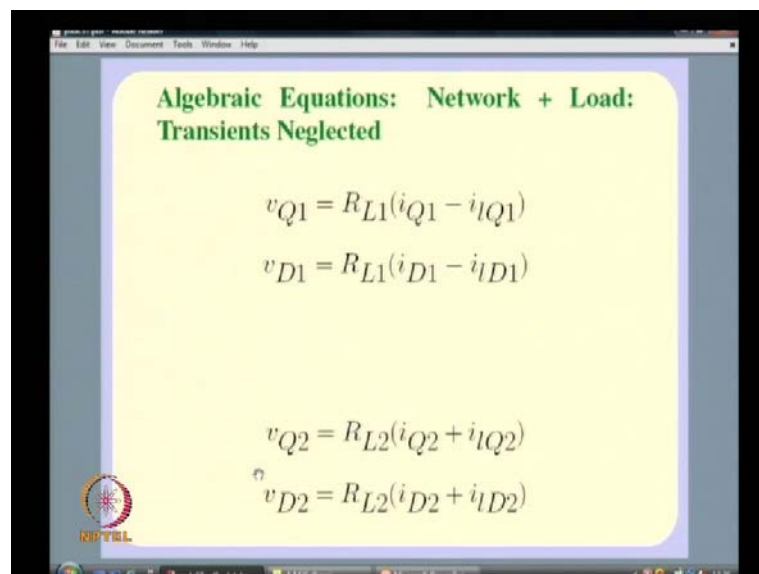
If I neglect, of course if I neglected the stator transient, that is $\frac{d\psi_d}{dt}$ as set equal to 0 and $\frac{d\psi_q}{dt}$ as set equal to 0, then there is no real logic in retaining the fast transients associated with the network, remember that the network was already simplified **you know** we may get in lumped RL model from what was essentially a partial differential equation model. So, that is of course, a big jump, big modeling simplification, now we go ahead and do another modeling simplification that is we neglect the network transients.

So, if you neglect the network **network** transients, then the equations are essentially like this v_{D1} and v_{Q1} , and v_{D2} and v_{Q2} are the terminal voltages of the synchronous, two synchronous machines and both are of course obtained using a common D Q transformation. So, you can get them and subtract them, they are kind of compactable.

So, we can use the voltage on either side of course, is obtained from a common transformation. So, this equation is actually valid.

We could not have for example, written down here, small a lower case D 1 and Q 1 and lower case D 2 and Q 2 that would be in correct. So, remember that this change is there. So, what we have effectively done is obtain these algebraic equations of the network, from the essentially from the differential equation of the lumped RL model of the inter connection.

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Algebraic Equations: Network + Load:
Transients Neglected

$$v_{Q1} = R_{L1}(i_{Q1} - i_{lQ1})$$
$$v_{D1} = R_{L1}(i_{D1} - i_{lD1})$$
$$v_{Q2} = R_{L2}(i_{Q2} + i_{lQ2})$$
$$v_{D2} = R_{L2}(i_{D2} + i_{lD2})$$

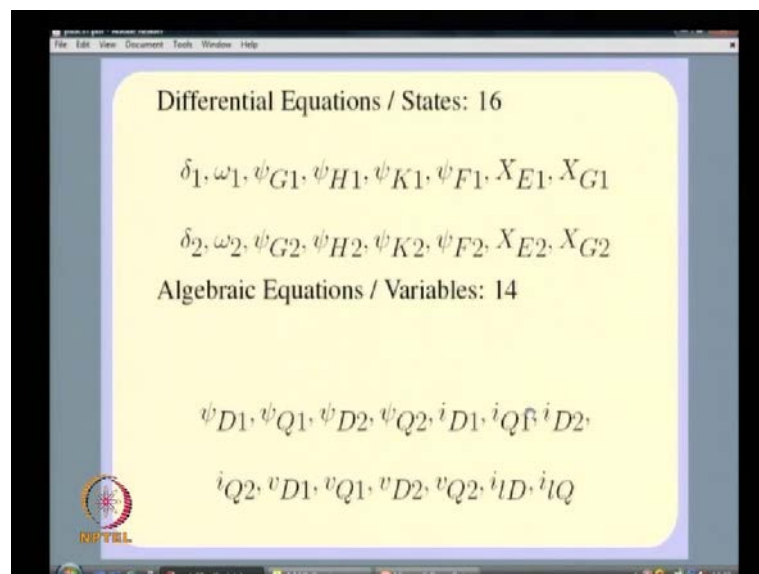
There are some more algebraic equations, what we have done essentially is what is v_{D1} and v_{Q1} , since there is a resistive load, if you focus on this diagram which is here, if this is a resistive load, it is a unity power factor load, but I also additionally say that it is a **you know** kind of a resistive load. So, the voltage here is equal to the current into the resistance, if the resistance is R_{L1} , then **you know** the voltage and the current are simply related by an algebraic relationship.

Now, so the, what is the current flowing through this, it is the current of the generator minus the current through the line. If I call this current through the line is I_L , in that case you will have v_{Q1} is equal to R_{L1} into the difference of the generator current minus the line current.

Now, this 1 and 2 actually are super (0), actually this should be just I L Q this 1 is not necessary. So, this is a small error which you should correct v D 1 is equal to R L 1 into i D 1 minus I L D, please remember to remove this 1 here, it is not required there is only one line current. So, there is no need of this additional subscript 1 out here.

Similarly, v Q 2 is equal to R L 2 into the current through the resistance, that is the generator current plus the inter connection line current, again please neglect this super script 2 here which appears in this term I L Q. So, just remove this super subscript I am sorry, so you can remove this subscript. So, you have got this these algebraic equations, please remember to make these corrections.

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So, eventually you have got 16 states, what are the 16 states? In fact, these are delta 1 omega 1, then the four rotor windings of the first machine, and then the state corresponding to the simplified AVR and exciter model and X G 1 is a state corresponding to a simplified turbine governor model.

Similarly, you have got the same thing for the other machine. So, there will be a total of 16 states and the algebraic equations are 14, you could have counted all the algebraic equations, 4 for each generator that will become 8, plus 6 equations which I just described some time back, 2 of course for the line, and the 4 equations which I just mentioned some time back, remember that the number of variables, algebraic variables

also are 14. So, it should be possible whenever you are writing a program to actually use **you know**, you can solve the algebraic equations and write them in terms of the states.

So, this is something you can do, remember that at every point, at every instant of your simulation on numerical integration, remember that ψ_{D1} and i_{D1} , ψ_{Q2} , ψ_{Q1} , i_{Q1} and so on are dependent on the states. So, if you recall that if you look at these equations here, you see that eventually ψ_{D1} , this equation is dependent on the states. So, there is an overall dependence on the states, just remember that, there is an overall dependence on the states.

Now, we begin our simulation, remember all the assumptions we have been, which we have made, we have made lots and lots of them, but some of the important ones are that we are considering, we are going to consider relatively slower phenomena, we are not going to consider very fast one, they are basically we are going to study electro mechanical transients and what **what** we of course done is also assumed in some cases replaced ω by ω_B . So, what the assumption is that you were not deviating too much from ω_B . So, all our studies are for frequencies which are near the nominal frequency, **is it ok**.

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**Simulation case study :
Two machine system**

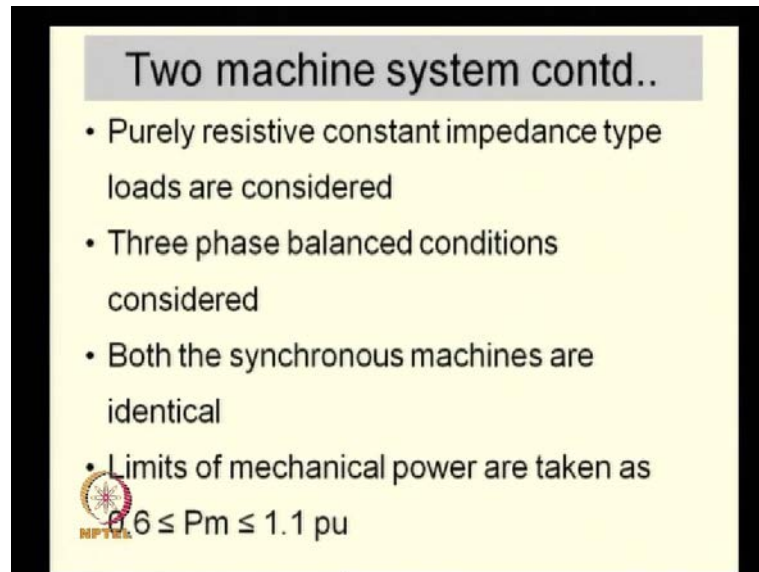
Following are the simplifications.

- AVR's are modeled by simple transfer function $\frac{K_E}{1+sT_E}$ ($K_E=200, T_E=0.02$)
- Turbine-governor systems are modeled by simple transfer function $\frac{K(1+2s)}{1+6s}$

$K = 20$) This gain is in pu power/ pu speed.


Now, let us get back to our case studies. So, I will just get **get** you to that slide, remember we have made the simple models of the AVR and turbine governor. So, this is something you can watch on your screen.

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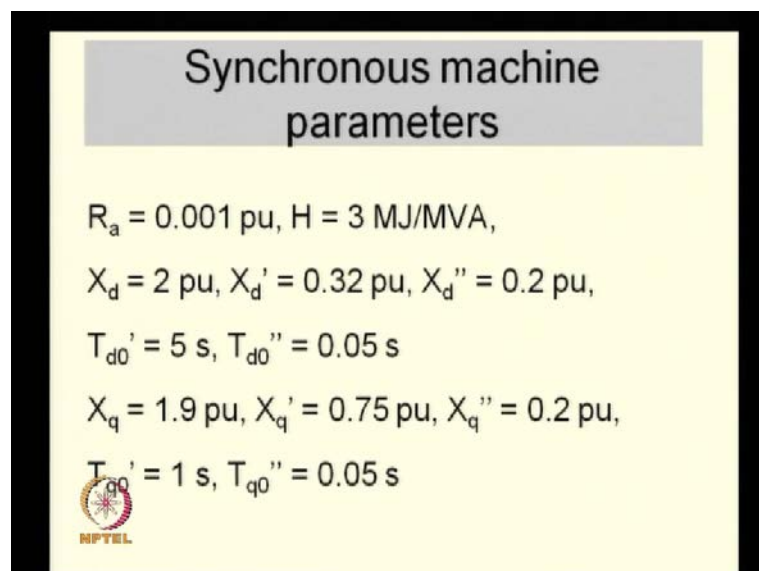
Two machine system contd..

- Purely resistive constant impedance type loads are considered
- Three phase balanced conditions considered
- Both the synchronous machines are identical
- Limits of mechanical power are taken as $0.6 \leq P_m \leq 1.1$ pu




So, once you have made these simplified models and obtain the differential equations, and of course made a few more assumptions, we will of course considering three phase balanced disturbances, there would not be any unbalanced disturbances.

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Synchronous machine parameters

$R_a = 0.001$ pu, $H = 3$ MJ/MVA,
 $X_d = 2$ pu, $X_d' = 0.32$ pu, $X_d'' = 0.2$ pu,
 $T_{d0}' = 5$ s, $T_{d0}'' = 0.05$ s
 $X_q = 1.9$ pu, $X_q' = 0.75$ pu, $X_q'' = 0.2$ pu,
 $T_{q0}' = 1$ s, $T_{q0}'' = 0.05$ s



Synchronous machine data.

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Eigen values of the system			
Network transients considered			
With governor		Without governor	
0.00	-28.71	0.00	-28.71
-0.16667	-37.33	0	-37.31
-0.392 + 0.398i	-47.995		-47.997
-0.392 - 0.398i	-52.261		-52.264
-1.8758	-45.152 + 311.43i	-1.892	-45.152 + 311.43i
-2.2142	-45.152 - 311.43i	-2.214	-45.152 - 311.43i
-1.544 + 9.749i	-1191.1 + 308.33i	-1.265 + 9.824i	-1191.1 + 308.33i
-1.5439 - 9.749i	-1191.1 - 308.33i	-1.265 - 9.824i	-1191.1 - 308.33i
-14.673 + 9.066i	-26473 + 314.06i	-14.704 + 9.034i	-26473 + 314.06i
-14.673 - 9.066i	-26473 - 314.06i	-14.704 - 9.034i	-26473 - 314.06i
-12.467 + 22.299i		-12.464 + 22.289i	
-12.467 - 22.299i		-12.464 - 22.289i	

And I have done the programming for analyzing this system, the first result which I have want to put forth to you are the Eigen values of the system for obtained after a small signal analysis around the equilibrium point corresponding to this situation. So, if you look at the situation again, this is the equilibrium situation, I have back calculated the condition, linearized the differential equations, we have done this before, so I am not doing it again.

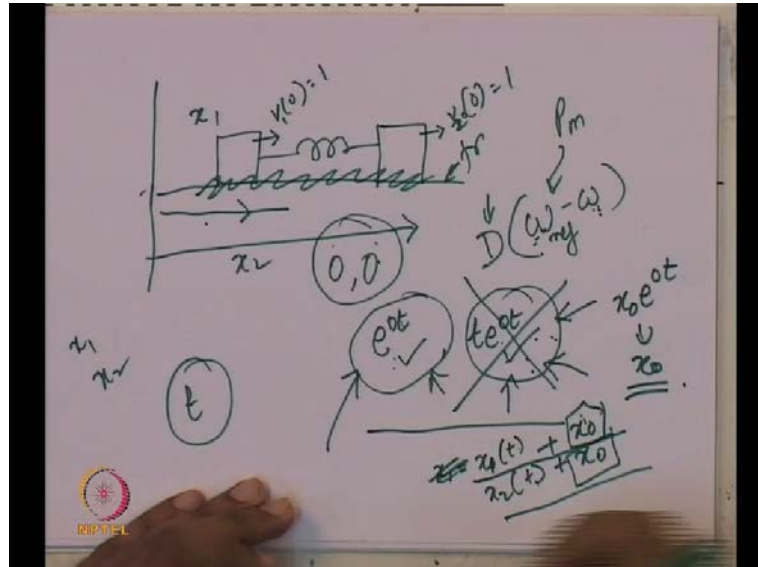
And I have actually tried to obtain the Eigen values of the system for this equilibrium point. So, if I give small disturbances, the behavior will be captured by the linearized kind of response of the system for small disturbances. So, if you look at the Eigen values of the system, there is some interesting things you will see. If you have no governor, if you have got no governor, there are two 0 Eigen values.

Then you have got, you will have two 0 Eigen values and then you have got a low frequency oscillatory mode which is not so well damped. So, this is the low frequency oscillatory model which is not so well damped **I am sorry**, you also have other Eigen values, some with the relatively large real part and some very large Eigen values on the right hand side.

So, without governor, if I consider network and stator transients, that is I do not neglect d ψ d by d t d ψ q by d t as well as d i d and d i q by d t of the network, in that case you

will have some very high frequency or high magnitude Eigen values corresponding to fast transients **ok**. So, this is what you get, what are these 2 0 Eigen values.

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Remember I had mentioned to you that whenever you have got a system like this, this is just an analogy, a two mass spring system, you the thing is that if you have got essentially a swing mode which is an oscillatory mode as well as two 0 Eigen values. Why do you get these two 0 Eigen values, you have got essentially.

If there is no friction of this surface then your response has two components corresponding to these 0 Eigen values, they are repeated Eigen values and in this case in fact, you can show that you are going to get for the spring mass system a response terms e^{0t} and $t e^{0t}$; we have discussed this long time back when we are considering linearized analysis, when you have got repeated Eigen values, you can in some cases get this kind of response.

In fact, if you do not have friction, we do get two 0 Eigen values for a two mass spring system, there is something you can verify by writing down the equations. So, what does this mean is that, if I give a, if for example these two masses have equal initial velocities say V_0 is equal to 1 and V_2 is equal to 1. Both these masses will move, if there is no friction, they will keep on moving, getting what I am trying to say. So, they will keep on moving in case there is no friction.

So, what you will find is that, the displacement from an arbitrary reference will have a variation of t , because if you give an initial condition in these states which are equal, you will find that it just keeps on moving. So, it is not surprising that you have got these two 0 Eigen values and therefore, you have got this kind of response.

So, remember that when you do not have a governor, what does it mean? It means that you do not have any control over the load generation balance; you do not have any control. So, it is equivalent to having the situation where you have got constant external forces acting on this mass spring system, but no friction, there is no **no** frequency dependence of these external forces.

So, if there was friction of course, things would be different. In fact, by putting a governor, what you are doing is essentially making some external force on this system which is proportional to the change in velocity. So, in fact the governor changes the mechanical power with in response to $\omega - \omega_{ref}$, $\omega - \omega_{ref}$. What it means essentially is that, you are introducing a kind of a viscous damping coefficient into your **you know** torque equation and as a result of which when you have a governor, you do not have these two 0 Eigen values, so that is an interesting point.

So, just remember this, remember that in case you have got a load which is frequency dependent or **(0)** you have got a turbine governor enabled, you will get only one 0 Eigen value, because in that case you do not expect this $t e^{-\lambda t}$ term to be there. In fact, if you in case you have friction and you have got an initial velocity, you will find it eventually this mass will grind to some halt, grind to a halt after some time. So, you cannot have this kind of term, in case there is this viscous damping introduced by either load dependence on frequency, or mechanical power dependence on frequency which is introduced by a governor.

So, in this case of course, I will load you the constant resistance. So, of course, you do not expect that the load to be frequency dependent. So, if you do not have a governor, you will have 2 0 Eigen values and $t e^{-\lambda t}$, t kind of response, remember of course, that even if you put a governor there is still be a 0 Eigen value or in effect a response term which is equal to $e^{-\lambda t}$. Why is that so, remember that if in this mass spring system, if my response is $x_1(t)$, if your response is $x_1(t)$ and $x_2(t)$, you can add an

arbitrary constant term to both of them and still your differential equations will be satisfied.

So, an interesting point is that if your solution to your δ_1 and δ_2 or in this case for a spring mass system, the displacements x_1 , x_2 is $x_1(t)$, then $x_1(t) + x_0$ also will satisfy your differential equation, this is because whenever x_1 and x_2 appear in your differential equations, or even in this synchronous machine example, you will find that everything depends on the difference of the angles, the absolute value of δ_1 and δ_2 does not determine the electrical powers, etcetera. None of the components in this system are dependent solely on the absolute or the actual angle at phase angle at a given point.

For example, the power flow, the steady state power flow here depends on the difference of the angles at the two ends; it does not depend, for example on this alone. So, what it follows is that if your δ_1 and δ_2 are solutions are the responses following a disturbance or even the steady state responses. If I add a constant term, a constant term is $x_0 e^{0t}$ which is nothing but x_0 itself, and then also your differential equations are satisfied. Therefore, do not be surprised when you do an Eigen value analysis and find out that you are getting a 0 Eigen value wherever you are studying this kind of system with the kind of differential equation and differential algebraic formulation which we have written.

So, of course whenever you do an Eigen analysis, also another thing you try to remember is that you may not get this as exactly 0 whenever you do a numerical analysis, you may get this slightly non 0, but that will be because of numerical errors, precision errors. So, of course when you have a governor, now coming back to this problem, having a governor introduces changes this 0 Eigen value to a negative real Eigen value which implies that they cannot be any $t e^{0t}$ kinds of terms in your response.

Another thing is that two additional Eigen values come about, because they are two extra states which you have considered with when you model a turbine governor. If you of course, assume that the mechanical power is constant that is what this without governor means, you will not have these additional two Eigen values, but when you have a governor, you will have these additional Eigen values. Again, remember that when we consider network transients, we have got these high frequency terms.

(Refer Slide Time: 31:16)

Eigen values of the system			
Network transients Neglected			
With governor		Without governor	
0.00	-28.76	0.00	-28.78
-0.166	-37.26	0	-37.24
-0.639 + 0.384i	-46.054		-46.053
-0.639 - 0.384i	-51.539		-51.548
-1.885		-1.8944	
-2.208		-2.2096	
-2.026 + 9.939i		-1.386 + 9.957i	
-2.026 - 9.939i		-1.386 - 9.957i	
-14.376 + 9.433i		-14.466 + 9.381i	
-14.376 - 9.433i		-14.466 - 9.381i	
12.286 + 22.977i		-12.273 + 22.999i	
-12.286 - 22.977i		-12.273 - 22.999i	

If you neglect network transients, in that case you will find of course, that the high frequency terms are missing, but other than that, you have got again these 0 Eigen values, you also have the low frequency oscillation terms. These are also called as swing modes and you have also got additional modes **cause**, because of there are many **many** more states. But the important modes which you need to remember are these 0 Eigen values and these swing modes, these are associated with electro mechanical transients. Remember of course, that how do I know that, I cannot I have just stated this factor; I have not proved that these Eigen values are associated with the electro mechanical variables, deltas and omegas.

But in fact, you can actually prove this using participation analysis which we discussed some time back, when associate the modes with electro mechanical transients, primarily with the electro mechanical transients. But of course, this does not mean that these **these** modes are not dependent on the other states, but when I say electro mechanical modes, these are essentially, primarily associated with deltas and omegas. The other states also influence these modes, but they are primarily associated with delta and omega, that is what I want to say.


So, with governor and without governor, you have similar behavior with and without network transients considered, you still have a similar behavior. So, we can to some extent justify this approximation of neglecting stator and network transients. So, we go to case 1, now I will quickly go through a few simple disturbances. Suppose, I have got this

I give a step change in this load, I change this resistance from 0.52 per unit to 0.7 per unit. So, actually the load reduces, because the **resistance is reduced sorry** resistance is increased. So, if there is a load through off that some part of this load has been removed.

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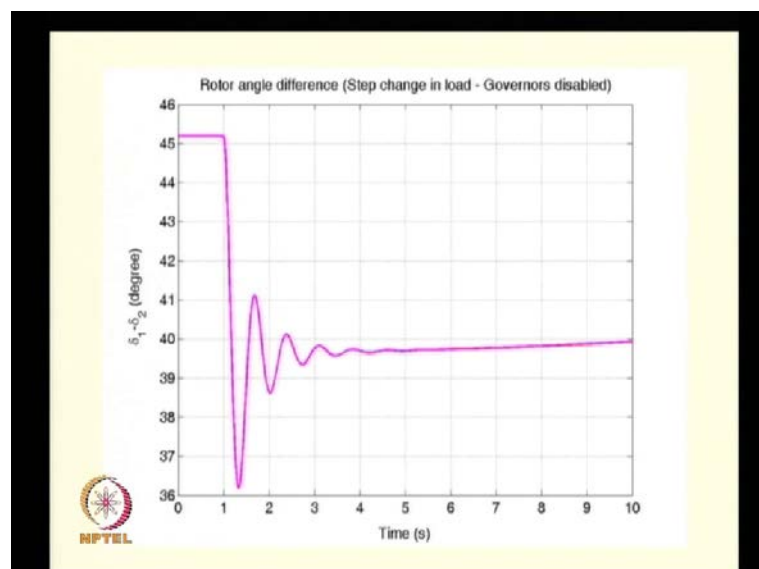
Case - 1

- Step change in load at bus - 2.
- Load resistance at bus -2 is increased from 0.53 pu to 0.7 pu (Load thrown off).
- Governors are disabled.



In that case, if I do not have governors.

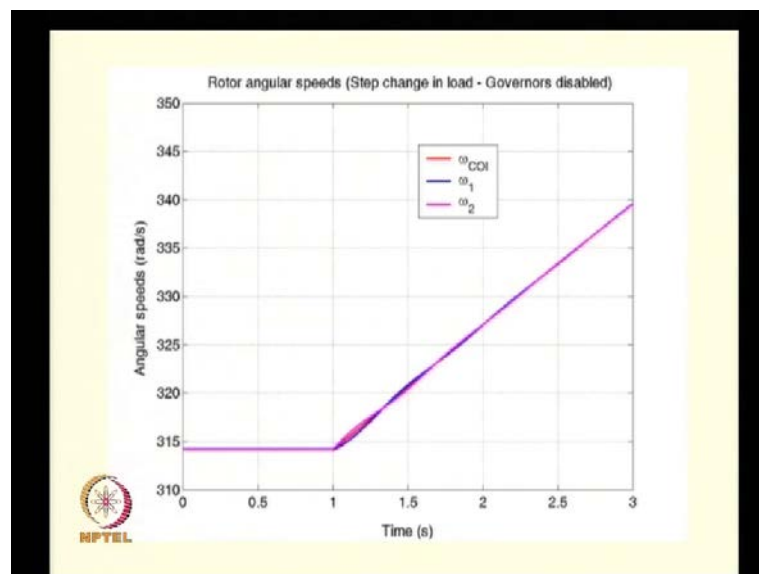
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If I look at various quantities, first thing you will find is the rotor angular difference increases or rather decreases, why does it decrease, from 45 degrees it comes to roughly 40 degrees.

So, the angular difference δ_1 minus δ_2 , both are **you know you know** rotating and the angular difference between them is around 40 degrees after the disturbance, while it was 45 degrees before the disturbance. Why does that happen, while when we are looking at it is that when you through of some load here, the load here reduces, on this line reduces. So, the angular difference between these two busses reduces, and the corresponding rotor angle also reduces.

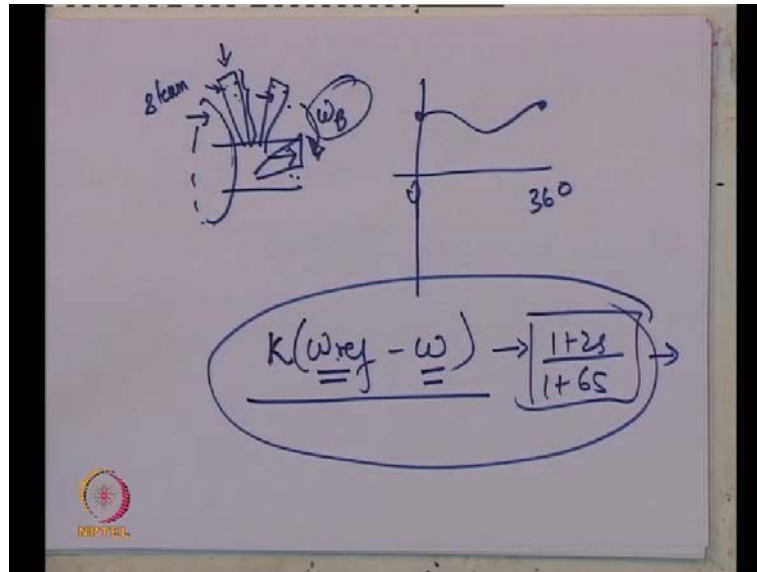
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Now, remember when you say no governor, you are not changing the mechanical power; if I do not change the mechanical power, remember this is like a frictionless system. In fact, there is no dependence of the external powers, mechanical or load powers on the frequency. And if there is a load generation mismatch, what you of course find is ω_1 ω_2 as well as the center of inertia frequency which is defined as $\frac{h_1 \omega_1 + h_2 \omega_2}{h_1 + h_2}$, both linearly increase.

So, you see this linear increase and of course, if you allow this to continue, then after a point your generators will be tripped out. The reason is that if your frequency goes greatly away from the nominal frequency, there is a chance that you would damage the turbine and generator system, why is that so?

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Actually turns out that the turbine is made out of blades, they are made out of blades and you have got a steam flow through this, through these blades. Along this periphery, the steam flow is not uniform, it is not perfectly uniform, and there are variations, natural variations in the steam flow. So, a steam flow may be 0 to 360, may be like this, slight may be of course, this is an exaggerated electrical engineering or electrical engineer's description of what happens.

Now, what happens is that, in case you are rotational frequency deviates from nominal, if it deviates from nominal, you will find that these turbine blades start getting **start getting** periodic forces across, I mean are applied across the blades. So, if your steam flow is changing, this is the variation of steam flow and you are running at the base speed, you will of course see that the force on the blades is actually changing.

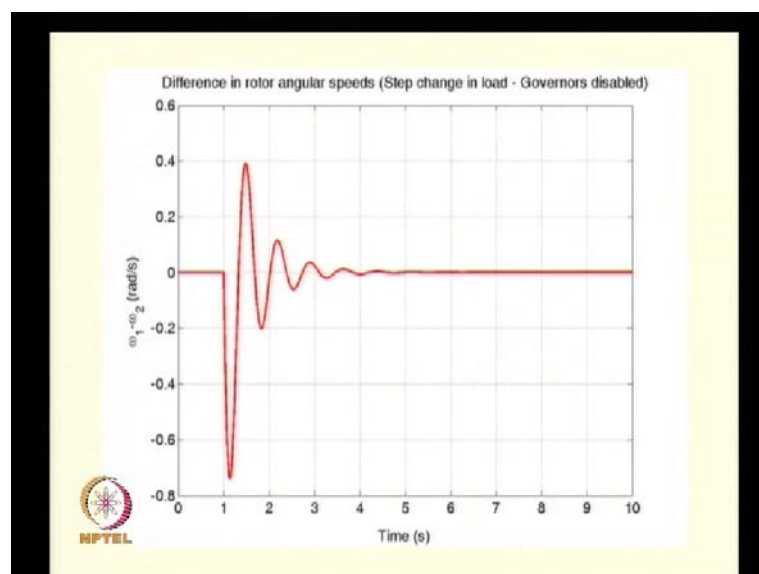
Now, the thing is if you deviate from this ω_B , the frequency of the force which these blades see will change. Now, what happens is in case the frequency with of the forces which these blades see, starts coinciding with the natural vibration of these blades, then there will be huge displacements. See, you are giving a periodic force which is equal to the natural vibration frequency of the blade.

If that happens at a certain frequency, if the rotational frequency deviates from ω_B , then you will find at the frequency of the forces on the blades also changes, and if it coincides with the natural frequency of vibration, the blades will get damaged.

So, nobody allows, especially a steam turbine, in a steam turbine generator system nobody allows you to deviate, the frequency to deviate much away from the nominal speed. Because typically you will find that, after around a 1 hertz deviation from the nominal, 1 or 1 and a half hertz from the nominal, you may start coinciding with the resonant frequency. The forces which are incident on these blades start coincide, the frequency of that starts coinciding with the natural frequency of the blades, so that may damage the blade.

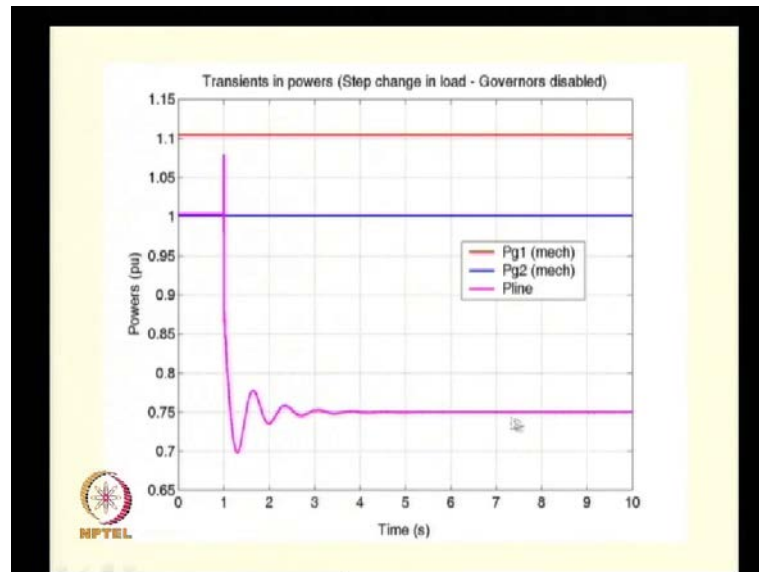
So, **no** after a certain speed deviation, either on the positive direction or the negative direction, you may have to trip out the generator, you either have an over speeding control which suddenly changes the mechanical power like a governor, or you have to trip out the generator. So, this is not acceptable, of course remember that because the angular speeds are changing, does not mean that the machines are losing synchronism.

(Refer Slide Time: 38:26)



So, if you look at the difference frequencies, it is constant in fact, $\omega_1 - \omega_2$ is constant. Only thing is that center of inertia speed is increasing, because the unbalanced external force, because you have changed the load generation balance.

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Also the line power which is line power comes to a constant. So, actually from a point of view of synchronism, synchronism if you look at it, we have not gone out of **we have not gone out of** step, the mechanical powers are constant, the center of inertia frequency is changing, but the relative speeds are in fact constant.

So, they have not loss synchronism, we have what we are saying is that the overall frequency has changed. So, that is something you have to keep in mind, we are talking of relative motion instability, we are talking of a continuous change in the center of inertia speed. So, both machines are almost together decelerating, or in this case they are accelerating together, they are not moving together, but they are both accelerating.

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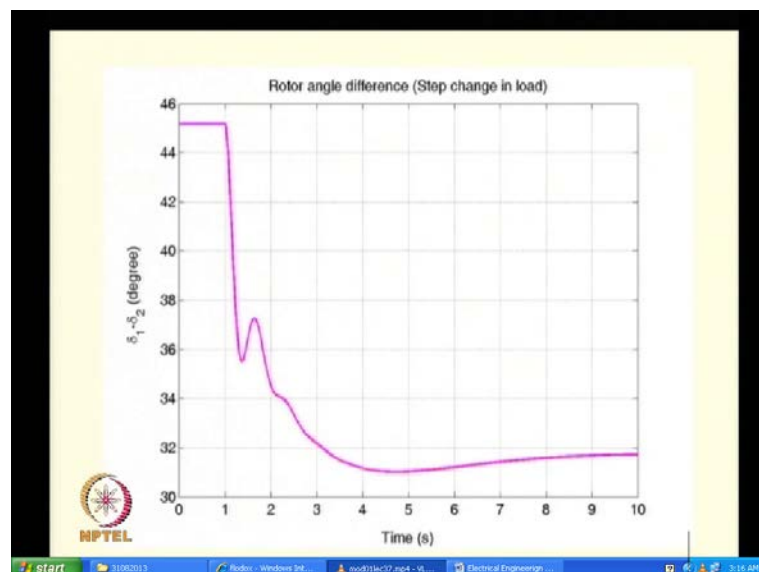
Case - 2

- Step change in load at bus - 2.
- Load resistance at bus -2 is increased from 0.53 pu to 0.7 pu (Load thrown off).
- Governors are enabled.



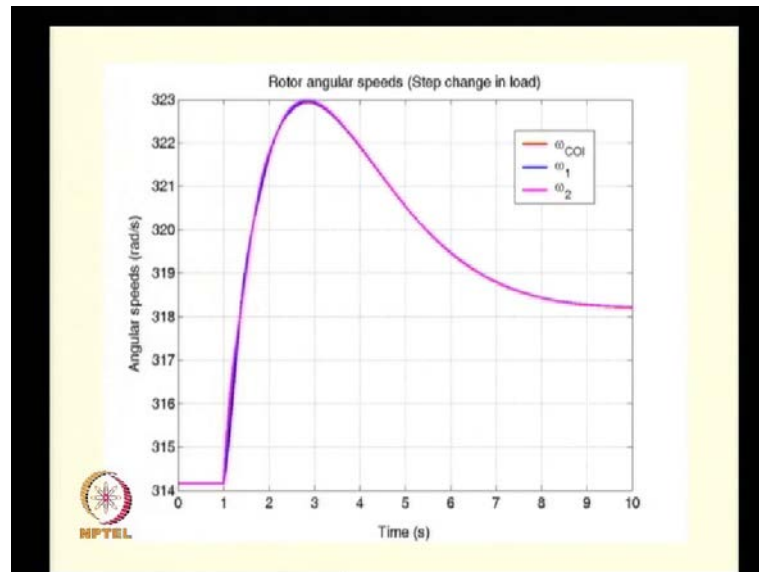
If you look at a case 2 with the governors are enabled.

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What you notice is, of course the mechanical the transient looks a bit different, and in fact it settles down to a different value, you look at the angular difference, it settles down to 32 degrees instead of 40 degrees in the previous case.

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The angular speeds reach equilibrium, there is a decrease in this, there is an increase in the center of inertia speed as well as the speed of each individual mass, rotor mass, but eventually the speed reaches equilibrium.

The equilibrium speed is not equal to the previous speed. In fact, if you note something that the governors are proportional type governors, in the sense that the mechanical power is changed using an algorithm like this, the governor has got this kind of **you know** transfer function and of course, the turbine has got a transfer function. So, the turbine governor transfer function is $k \frac{1}{1 + 2s} \frac{1}{1 + 6s}$, now this k is a proportional controller effectively, it is also the inverse of the droop characteristic of the machine.

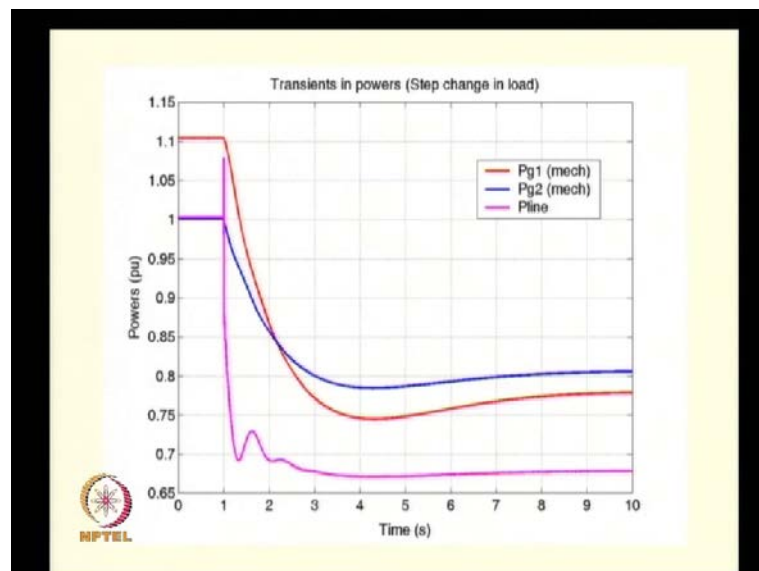
Now, the point is that you can change the mechanical output of the machine only if ω_{ref} and ω are not equal. So, in case this proportional **proportional** kind of governor will always give you a steady state error. So, your frequency does not come back to the original frequency, there is a frequency change that is something which you should remember.

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The mechanical speeds, the difference speed, the relative motion is stable, it is not unstable, the speeds, the difference speeds become equal to 0, but the overall speed of both the machines settles down to a higher value.

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Here the mechanical powers are changing, why they are changing, because we are having a governor.

So, if you have got a governor, your mechanical power will change and of course, the line powers also will change, because governor is present here as well as here. So, the

point is that if governors are present on both machines, each of them will change their mechanical power; if they change the mechanical power, the overall power flow situation changes. So, load has changed, mechanical power here is changed, and the mechanical power here is changed. So, if all the three things change, you will find the frequency goes to a new equilibrium.

And **you know** your line power also settles to a different value, the angular difference between the two buses also will settle down to different values. Now, of course, the point which you should note here is that governors on both machines are present. Now, what if we had just one governor or a governor present only on one machine, basically your mechanical power of only one machine would change, only one machine would take on the load change.

And of course, if you just had governor on one machine, only one person would contribute to the **you know** to trying to get the frequency to equilibrium. So, if you keep the same gain and just disable one governor, your frequency deviation will be more, only one generator will respond to the load changes.

It can also be inferred that since the load, the amount of power change of each turbine is proportional to k into $\omega_{ref} - \omega$, by changing k or ω_{ref} ; you can in fact change the amount of contribution of each generator. So, why is that so, because remember that speed, eventually if you are remaining synchronism speed of both machines is going to be the same.

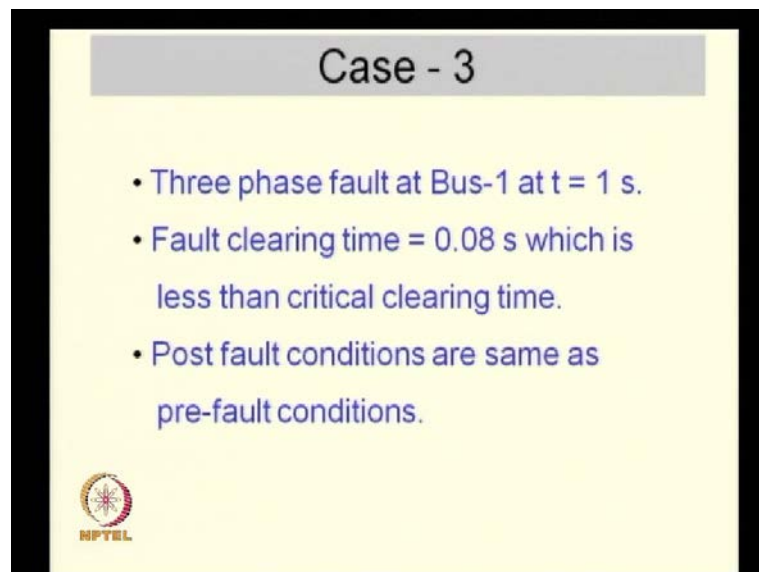
Now, if speed is the same and ω_{ref} of both machines is the same, then the amount of power change each generator takes up is proportional to $\omega_{ref} - \omega$ into k . So, if k_1 and k_2 are different, I mean you have got two different gains or two different droops **on the governor** on the generators of each machine. In that case you will find that, the each generator takes on different load change, a kind of extreme condition is where the gain of one generator of one governor is much **much much** higher, like one of the governors, for example has a steady state gain of **you know** a 300 and the other has got a gain of 10.

In that case, the governor assuming the ω_{ref} s are the same, the governor with a larger gain takes on more **more** of the load, it takes, it increases its mechanical power more, where as the other generator does not increase its mechanical power as much.

Now, the extreme condition is where you have got an integral controller on one of the governors, an integral controller essentially has got infinite gain in steady state **you know** 1 upon s , remember. So, you will get infinite gain in steady state. So, if you have got one governor which has got a integral gain, it will take on the complete load change, and the other governor we left high and dry, in the sense it will not contribute anything to trying to maintain frequency, of course this is assuming ω_{ref} on both machines is the same.


Now, just think of, think about it, can you have integral controllers on both governors? This is something you need to think about, these are question which not obvious, you think over it a bit, you cannot have in fact, both governors having integral gain, why it is so is something you can think about.

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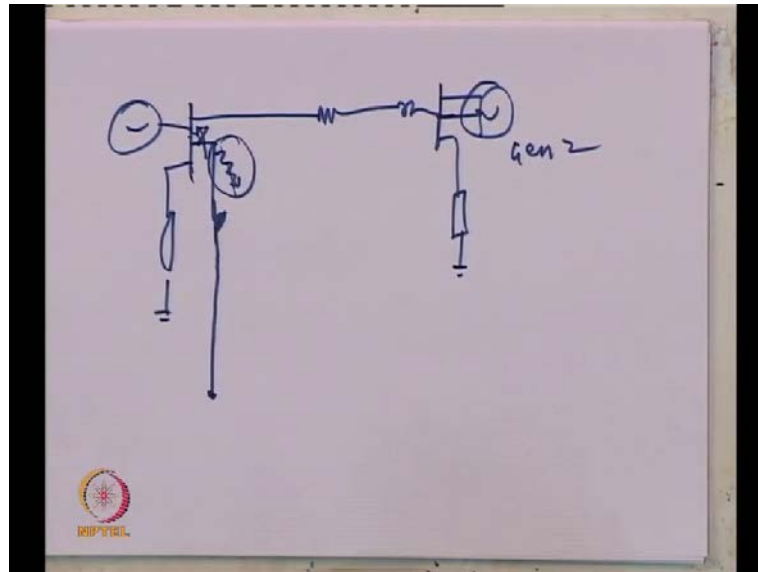
Case - 3

- Three phase fault at Bus-1 at $t = 1$ s.
- Fault clearing time = 0.08 s which is less than critical clearing time.
- Post fault conditions are same as pre-fault conditions.



Now, the 3rd case is not a load change, what we will do is we will have a three phase fault; it is a balance fault at bus 1 at 1 second. And we will clear the fault and after 80 milliseconds, that is four cycles. Now, of course remember that force fault conditions are assumed to be same as pre fault conditions. So, it is a kind of temporary fault, it is we just come back, so when the fault is cleared.

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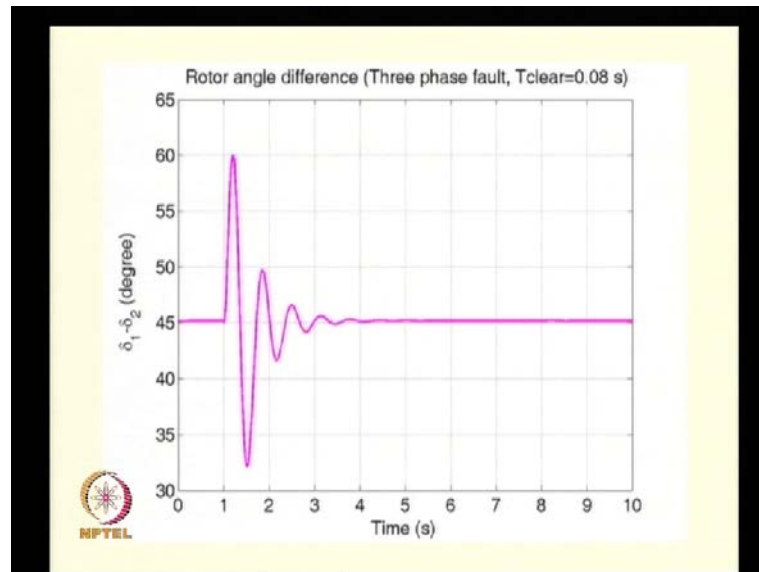


So, you can consider this as a situation where in bus 1, suppose you have got an open line, this is your **your** bus, this is a generator 2, this is a generator 1, the same system, this is a load and this is a load. So, this is your same system and you have got a transmission line out here which is open circuited here at this end, it is a short transmission line.

And you have got three phase fault on this line near about this bus; in that case, you will have a short circuit at this bus, three phase short circuit at this bus which has to be cleared by opening the line, the post fault and the pre fault situation is going to be the same. When before the fault, you had connected on open line here; it is a short open line.

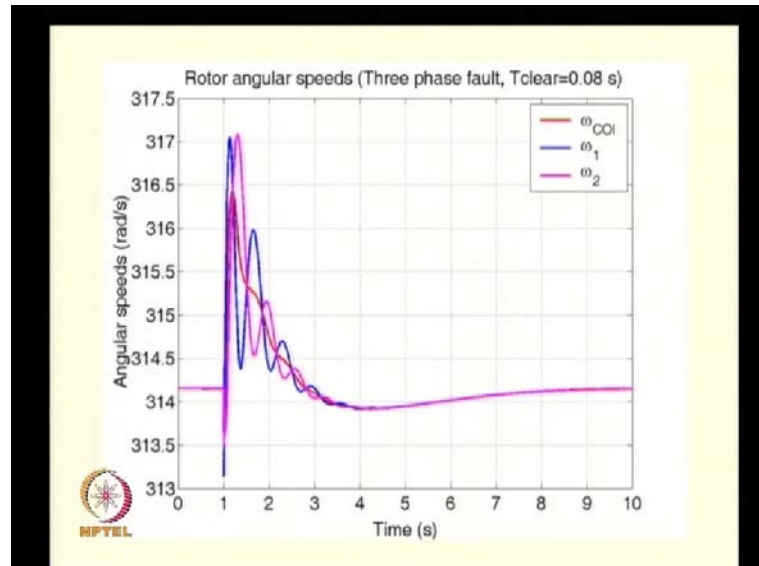
So, it is like having nothing here, you had a fault for some time which is cleared by opening the line. So, what you have is pre fault and post fault situation is the same, assuming of course, this is the short line, and open circuited short line is effectively doing nothing to the system initially.

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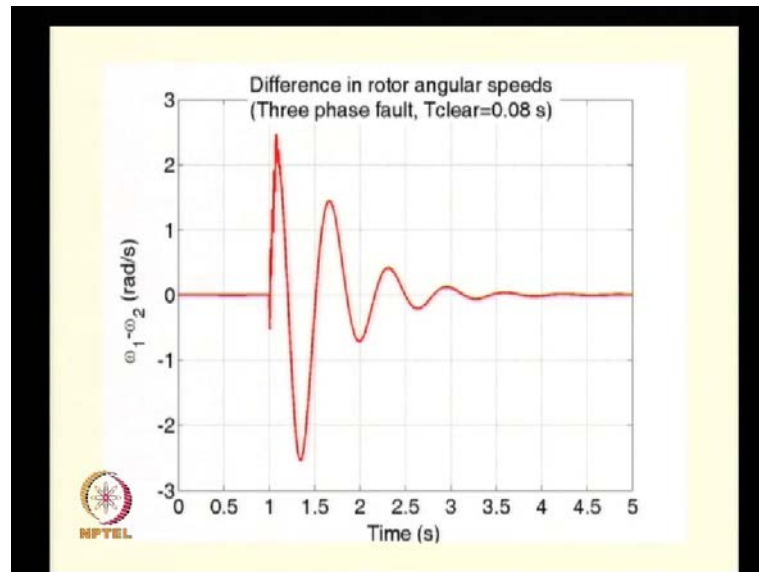
So, under these circumstances, what you find is if your clearing time is this much, you will find that $\delta_1 - \delta_2$ in fact is stable.

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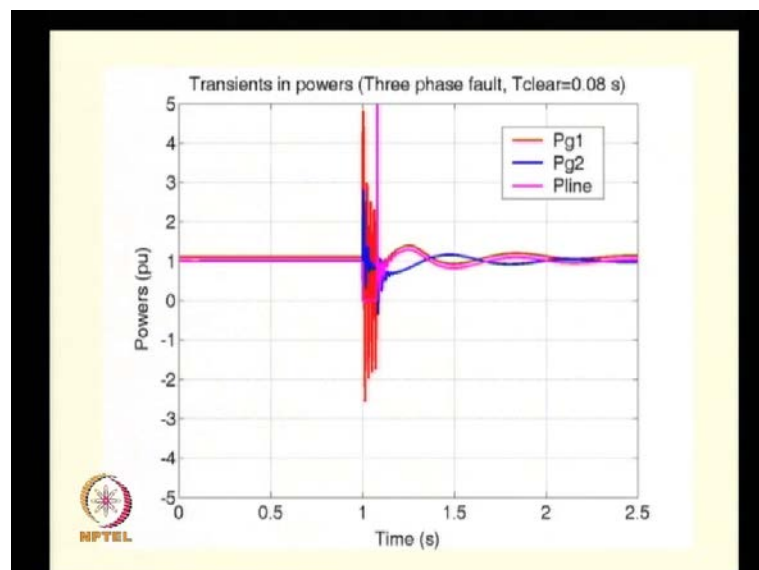
Since, pre fault and post fault systems are equal; you will find that there is no change in the load generation balance situation, no rather flows change, so losses also do not change. So, your angular speed simply comes back to the original value, they are transients, but overall angular speed settles down. So, it is a stable large disturbance, it is stable for this large disturbance.

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Omega 1 minus omega 2 is a relative speed, it is also stable.

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The power is again are stable, actually this simulation has been done considering network transient, we have model network transients. So, what you see is essentially in the beginning of this transient here, for example are some high frequency components. So, if you consider network transients, you will get these high frequency components, this is a high frequency component here as well, so this is how your delta 1 minus delta 2.

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Case - 4

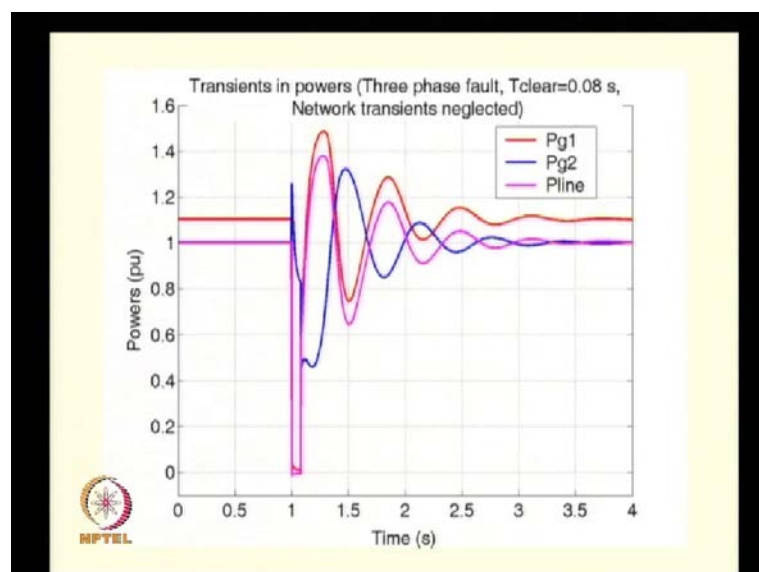
- Three phase fault at Bus-1 at $t = 1$ s.
- Fault clearing time = 0.08 s which is less than critical clearing time.
- Post fault conditions are same as pre-fault conditions.
- Network transients are neglected.



If you neglect network transients, you get almost the similar response, but remember that there is some, there are some differences. For example, the maximum deviation here is around 70 degrees, where as when network transients were considered, it was almost 60 degree, so there was a 10 degree difference.

So, when network transients are neglected, you do have some difference in the result. In fact, network transients neglected gives you a somewhat larger angular deviation.

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And but otherwise, the response looks almost the same, you just see that there is no high frequency **you know** transient **you know**, because you have neglected network transients.

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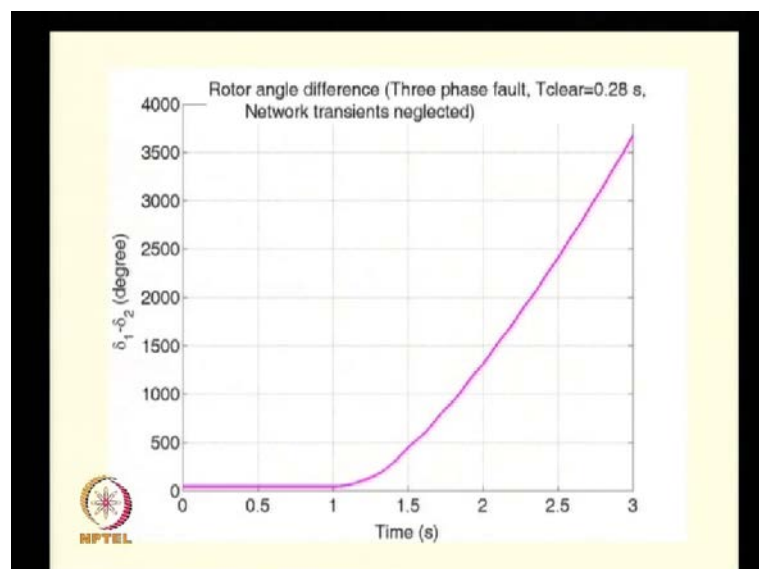
Case - 5

- Three phase fault at Bus-1 at $t = 1$ s.
- Fault clearing time = 0.28 s which is more than critical clearing time.
- Post fault conditions are same as pre-fault conditions.
- Network transients are neglected.



Now, we will consider when we increase the fault clearing time to 0.28 seconds, but the network transients of course are neglected.

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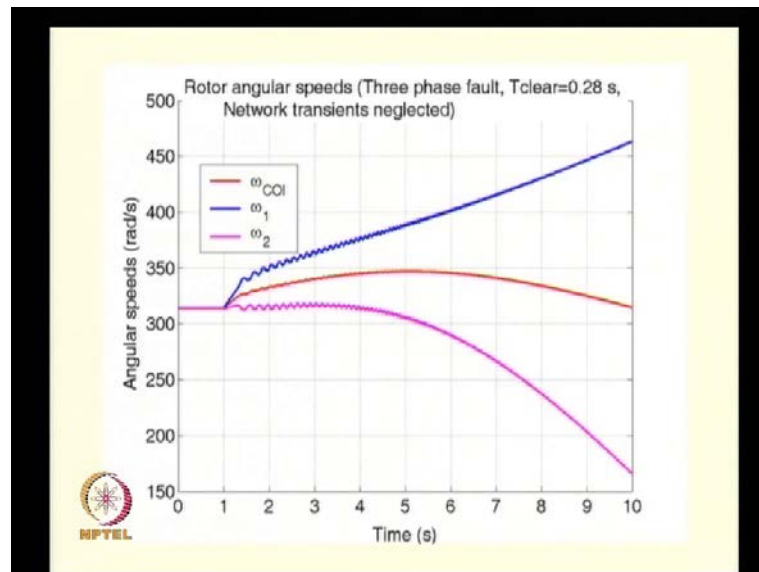


The interesting thing is that the angular deviation under this circumstance grows in an unbounded fashion. So, if you have a larger clearing time that fault is cleared, but after a long clearing time, so longer clearing time means that, because of this sudden

disturbance, the deviation of the states from the post fault equilibrium are going to be more.

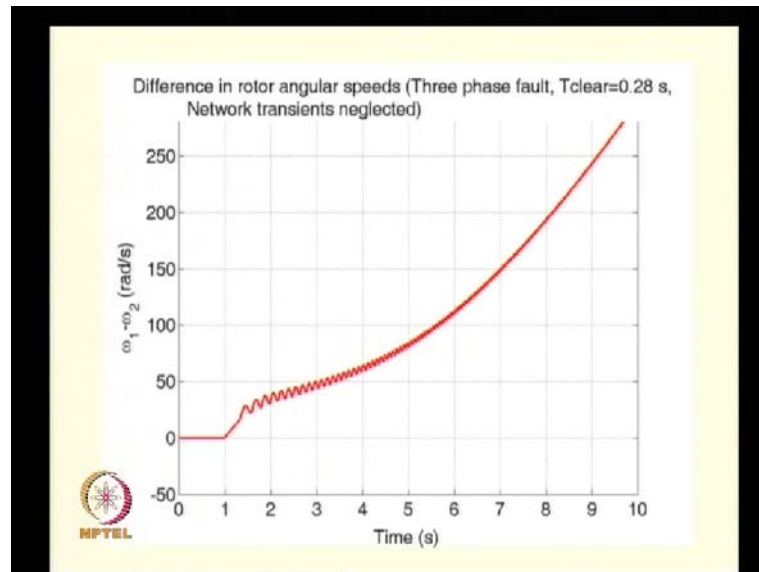
So, if your deviation is too much, you may not **you know** pull back into synchronism, as a result of which the relative angular motion is becomes unstable. So, please remember, this is relative angular motion becoming unstable.

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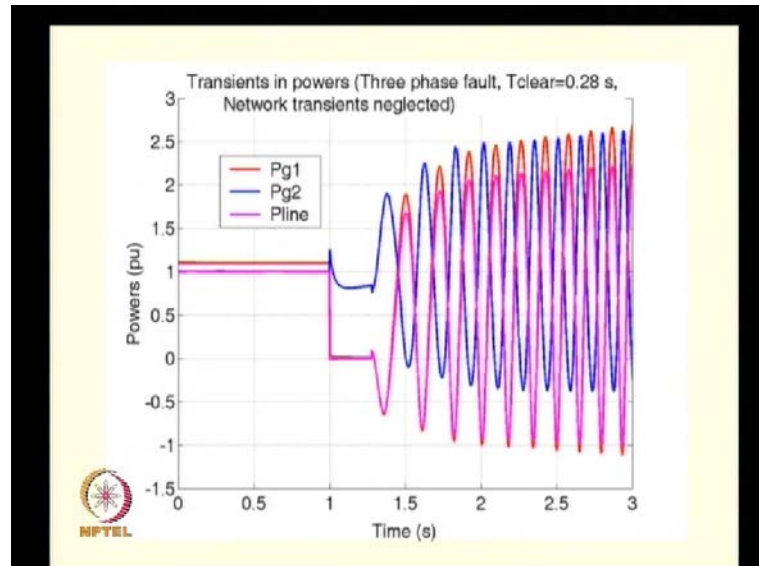
If you look at angular speeds, they too **you know** deviate from each other, although ω_{COI} is relatively **you know** within bounds ω_1 and ω_2 the relative angles increase, this is contrast with the earlier situation. In the earlier situation when we had a load through off without governors, ω_{COI} went on increasing, but the relative motion between them was stable, this is exactly the converse situation. So, what we have seen is that if we give a disturbance like a fault which is not cleared for a long time, you may go out of synchronism.

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And you will see the relative motion is unstable. So, relative out of synchronism refers to relative motion.

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If you look at the **mechanical** electrical power output of each machine, you will find that it is oscillating that is a kind of electrical a wild oscillation. In fact, it even reaches negative values, positive and negative values. So, if you lose synchronism, this is what **what** your mechanical power is going to look like. So, if your mechanical power is of course going to look like this, you cannot continue operating in this fashion. So, all

though I have kind of shown you a simulation **which** in which the loss of synchronism, kind of remains for several seconds, no action is taken, you are just allowing the angles to **you know** increase; another word for it is the poles of both machines are slipping with respect to each other.

So, you are just allowing that to continue, if you allow it to continue, there will be wild oscillations in power, and you cannot operate in this fashion, you will damage the shaft, for example of the machine. So, what usually you do is that in case you detect an out of synchronism condition, you will have to separate out the two machines.

Now, if you separate out the two machines, you will have to essentially trip the interconnection between them. But remember, if I trip the interconnection between them, I will have basically another problem to contend with, what is that, the local load generation balance in this two islands which I have got, which I have kind of disconnected is not **you know**, there is a fairly large generation imbalance in both these islands. And **you know** then, you have to really exert the governors as well as any emergency load shedding schemes, may be required to quickly get an equilibrium in the center of inertias of each individual system, or **you know** now they are disconnected systems. So, I have to ensure that the frequency of each system reaches an equilibrium.

So, you have to really control the prime over and load power on both systems once you disconnect. So, let us whatever was I telling you, you are interconnected, you gave a fault, the fault got cleared, you lost synchronism, there are wild variations in power. So, you cannot operate that way, you have to trip out the interconnection. Once you trip out the interconnection, you are left with another problem that is of load generation imbalance in the two disconnected islands which has to be really sorted out.

So, loss of synchronism is not a nice scenario and even if you manage to island or trip out, or disconnect the two systems when they are lost synchronism. Why do you need to disconnect, because there **(O)** be wild oscillations in power, voltage, and so on, so you have to disconnect. Once you disconnect, you have got the problem of maintaining the frequency in the two individual systems which boils down to trying to maintain the load generation **imbalance** balance in these two systems.

Now, of course, an interesting point is that, if your power system experiences a loss of synchronism situation, usually there are out of step relays which detect this and trip, also

an interesting point is that the wild deviations in voltage and current cause distance realized in a transmission system to mistake it for a fault and it causes transmission line tripping.

So, whenever there is a loss of synchronism scenario, you may have **you know** what you call an uncontrolled splitting of the system due to distance relay operations. So, you may what I said that, you need to disconnect the two machines, they may disconnect on their own in the sense that relays may actually trigger and trip out in case such a situation occurs.

So, there we will kind of summarize this very interesting study which we did on electro mechanical transients in a two machine system. Just remember, of course that we have considered the electro mechanical, both the relative frequency, relative motion as well as the center of inertia motion, there are other modes and patterns in this motion, they are not very well observable in this response.

Those patterns also can go unstable for example; if you do not design your control systems properly, for some inappropriate gain of AVR and governor, you may find some other modes which are not necessarily electro mechanical modes also go unstable.

So, these kinds of things also can occur, we will summarize at least this electro mechanical transient's part in the beginning of the next lecture and then move on to some understanding. What happens when you are got more than two machines, like our grid which is more than **more than** 500 or so, large machines, synchronous machines, all interconnected to each other. So, this is something we will discuss in the next lecture.