Power System Dynamics and Control
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Lecture No. # 36
Stability in Integrated Power System
Two Machine Example

We get to try to simulate a Two Machine System. And what we will try to do today is try to bring out the phenomena of the movement of the centre of mass or centre of inertia speed allow as well as the phenomena associated with the relative speed. Now, in the previous class, we had seen an analogy with the spring mass system; and from that we had infact inferred that, you know the there are two components of the motion there is the relative motion, the centre of inertia motion. The centre of inertia motion is affected by the external forces on the system.

Now, what we will do today is actually a simulated two machine system and at least some patterns, which we have identified in the simple spring mass system analogy, will be evident even here. So, that is what I wanted to show you. But we will do a a relatively detailed simulation, we will take a higher order machine model and we will also consider the effect of AVR and a governor.

Of course, the AVR and governor models which will be considering here, we fairly simplify, so but, they will hopefully be successful and try to bring out the essential or the important concept related to the phenomena in two machine system. Of course the aim of course, eventually will be to try to extrapolate what results we have got here to higher order or may really large multi machine system. Now, so in today's lecture will take this two machine example and try to study certain stability phenomena, which are evident in integrated power systems.

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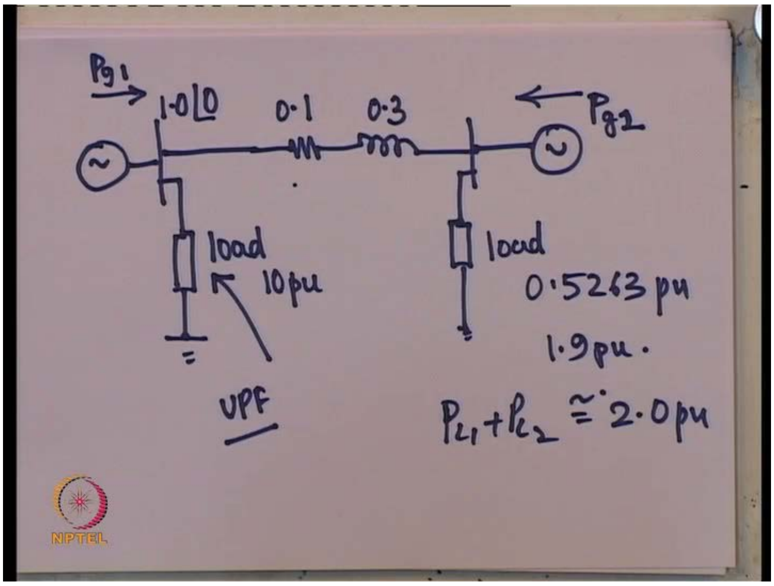

What we will do first will get get down to the actual work and the single line diagram of the system, which we are going to consider is is this. So, you have got this is the load, this is the transmission line, this is the generator, this is also load, this resistance here is 0.1 per unit, this is 0.3 per unit. This basically the important thing to be noticed is this interconnection, this is the transmission line of course, I remember you are just taking an r l kind of model of a transmission line. This is the lump model, remember that in our discussion of transmission line we it seen that lump representation.

You know even if you take a dynamical representation of a transmission line using the lump model, that is you know have an inductance here; which will of course, be associated with the dynamic equation lump model is likely to give you a reasonably correct results for low frequency phenomena.

But of course, if we are studying switching and lightening transients this is not an acceptable model. In fact, what you will see today is that, we may even neglect the dynamic associated with this, you know this reactance here. So, that of course, will be coming too shortly. So, you have got two loads the two generators are this is generator 2 and this is generator 1 the load in the at load bus, the load is a unity power factor load. So, if it is a unity power factor load let us say it is its resistance effectively let us represent it as a resistance, so that let us assume that the resistance is 10 per unit.

So, that would mean that the load is 0.1 per unit in case the voltage here is 1 angle, 1 per unit. So, if your voltage here is 1 per unit the load resistance is 10 per unit, then effectively the power is 0.1 per unit. Similarly, here we have got a resistance of 0.5263 per unit, which corresponds to a load of roughly 1.9 per unit. So, you have got a load of 1.9 per unit here and a load of 0.1 per unit here.

So, the total load in the system if you look at you know you can say P L 1 plus P L 2 is equal to 2 per unit, roughly there is also loss associated with this resistance here. So, if there is some power flow in this line there will be a loss. So, P g 1 plus P g 2 in steady state will be roughly 2 per unit plus the losses.

Now, one of the important points, which you I hope you noted in the previous lecture was that, when you considered two mass spring example, if you did not want the centre of mass to move you did not want the frequency equivalently. If you do not want the you know if you want to have any equilibrium in the centre of inertia frequency in this system, you should ensure that P g 1 plus P g 2 is equal to P L 1 plus P L 2 plus the losses; otherwise there will be of course, a transient in the centre of inertia speed.

Now, the thing here to be noted is that once you have got a integrated power system of this kind you cannot really say, that load one load two are separately being you know individually being met by generator 1 and 2 what we can say of course, is the total load is being met by the total generation.

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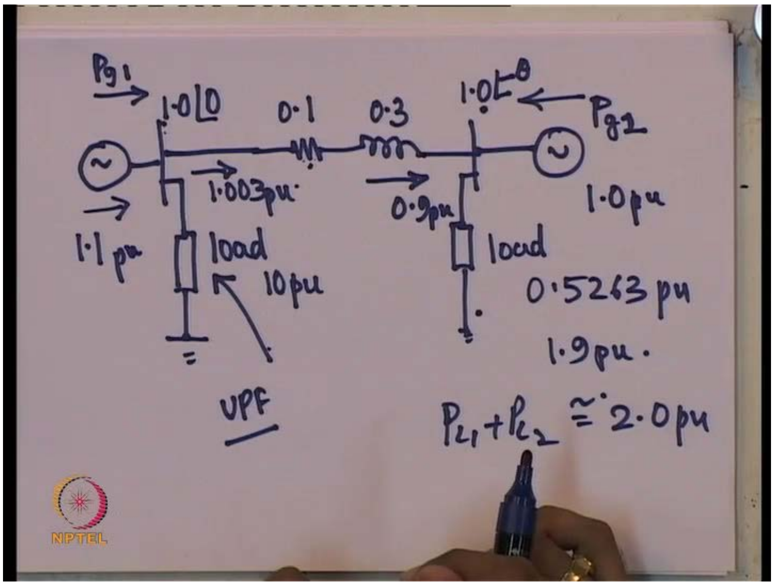

So, it is not necessarily true that P g 1 is meeting this load and P g 2 is meeting this load though of course, you can arrange it in such a fashion, you could for example, have all the load being met by the generation here and all the load here being met by this generation but, in this particular case will not be doing that. We will in fact, have roughly a power flow of 0.9 per unit of course, when I say power flow the received power here is roughly 0.9 per unit and so, most. So, if you look at how the generation is being done you will find it this P g 1 is not only serving this load but, also pushing some power to this line in serving apart of this load.

So, both the generators are roughly operating at 1 per unit power and P g 1 not only P g 1 and P g 2 are serving both loads but, it is not certainly true that P g 1 is you know is adequate only for this load. It is actually pushing some power along this transmission line. In fact, because they losses the power flow here, is roughly power at the sending end is not equal to the receiving end you have approximately 0.1 per unit losses.

So, actually if this is the situation you know equilibrium situation. In fact, you can if I tell you that voltage magnitude is being maintained at both buses at 1 per unit and this is 1 angle 0. And if you know the sending end power flow is roughly 1 per unit you can actually compute what this angle is. So, this is some 1 angle minus theta, so theta of course, will be a positive number in this convention, if I call it as minus theta. So, basically you can use the power flow expressions you you know they are the functions of the voltage magnitudes, the angular difference and the resistance reactance parameter of this line.

So, you will be able to compute this theta for this particular load flow situation. So, I am telling you that this is the situation, I will be giving you certain specifications. So, what all what are the things I have specified I specified the load powers, I specified the of course, the parameters of the system r and x here, I have specified the voltage magnitudes and I have specified the sending end power here, you should be able to solve and get this theta from that.

So, this is the initial equilibrium situation and as a result of this you know you will this you can infer that this is supplying 1 per unit power and this is supplying 1 per unit power, 1.1 per unit power here and 1 per unit power here, this is the equilibrium situation.

So, the starting point of analysis is a particular equilibrium about which will be doing our analysis. So, one of the first steps you will have to do is back compute you know back calculate the phase angle or rather I should say all the states of the synchronous generator and the the states of the synchronous generator.

So, you we can calculate the equilibrium conditions or the equilibrium values of all the states of the system; the states of the system of course, are in case you are representing this by dynamical equation the currents, then all the fluxes in the machine delta and omega of the machine; so these two of course, of both machines, so this is how we will start our study. Now how do you proceed is from the load flow solution, I had already shown you in the simulations of in AVR how to come back compute the values of the states once you know the nature of the voltage at the terminal.

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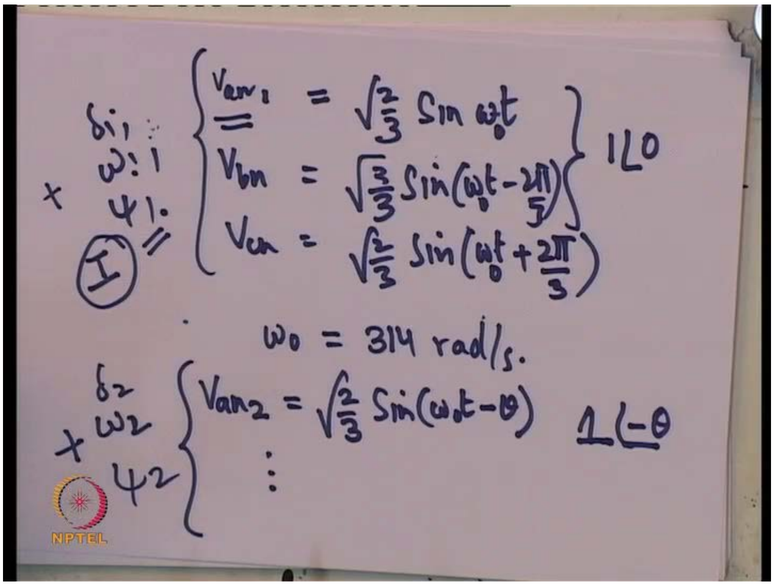

So, if I tell you that the terminal voltage is you know a certain you know has a certain wave shape for example, 1 angle 0, would mean according to our convention the voltage V c n the voltage across the phase V c n across the stator winding of the generator 1, would be root 2 by 3 sin omega t, this is what I mean by 1 angle zero. Of course, it is balanced and of course, V c n is equal to root 2 by 3 plus 2.0 pi by 3; of course, this omega is the equilibrium speed. So, the equilibrium speed let us call it omega naught without loss of any generality let us assume it is 50 hertz.

So, omega naught will be 2 pi into 50 that is 314 radiant per second approximately. Now 1 angel theta or 1 angle minus theta would mean that V a n at the terminals of that generator is this would be minus theta and so on. So, you will have V c n and V b n 2.

So, we can take out the wave forms of each of these generators now once you of course, know the terminal voltage is off the synchronous machine, you can compute rather we remember that the main question is how to compute the states of the system. So, delta 1 omega 1 and all the fluxes of that machine the equilibrium values you can do that if you know for example, the electrical power output of the generator and the reactive power output of the generator.

So, if you know the current under equilibrium conditions, if you know the current wave forms under equilibrium conditions just like you need a terminal voltage I can get the current wave forms. How do I get the current wave forms (Refer Slide Time: 12:21), by solving this circuit for a certain specifications you can get this theta; once you get this theta you will be able to get the current through this line, you will also be able to get the current Phaser through this load. So, the generator current can be obtained from the generator current Phaser you can compute the instantaneous values of i a, i b and i c from i a, i b and i c you can actually compute all the values of the states.

So, if you know i a, i b, I c and the terminal voltages as well then, you can actually back compute all the values of the states. So, this is something we have done during our simulation of an AVR, so i will not repeat it here you can refer to that lecture, the same thing can be done for the second machine. Once you know these terminal voltage is you can compute delta 1, delta 2 omega 2 and all the states of that of that machine.

Of course, if you are considering AVR and governor you will have additional states you just do not have delta omega and the fluxes, but you also have for example, the states associated with the AVR and excitation system and also the governor and the turbine turbine system. So, you have got these additional states, so there are additional states. So, I will just write this to denote this there are some other states, I have just put a sing plus here means that you have other states also.

So, first step is of course, compute the equilibrium values of all the states of the synchronous machine you can do that. Now remember this is one point which I am emphasized in the last class you are going to write down all the equations of your synchronous machine.

Now, one of the important points is that your equations of the synchronous machine or all in the (()) reference we have been derived it in the (()) reference frame. But, in case you are going to do any interfacing with other generators, it is important that before you apply K B L and K C L that is Kirchhoffs current law and kirchhoffs voltage law. All the voltages and current should be on a common reference frame.

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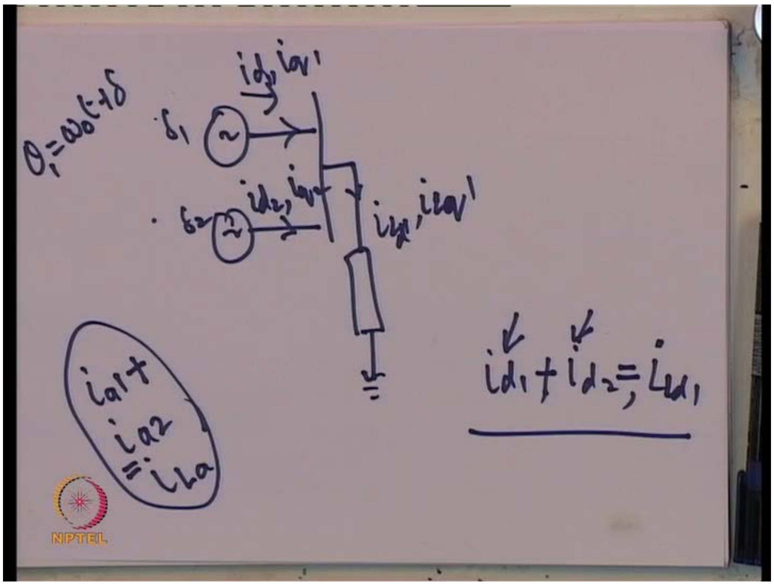

For example, if you have computed consider this situation, if you are considering system of this kind if you have computed for example, i d and i q using the (( )) reference frame attached to this machine, that is theta is equal to theta 1 is equal to omega t plus delta, these are the arguments used in your d q transformation. And if you have used another transformation and obtained i d and i q for this generator and suppose the current in the load is also computed using for example, or reference frame the d q transformation using this theta 1.

So, I will call this l one i L d 1 and i L q 1 in that case it is will not be possible to say that i d 1 plus i d 2 is equal to i L d 1 it is not possible because these currents although it is true that K V L; i a 1 the i a 1 i a 2 and i load the a phase they satisfy K V L Kirchhoffs sorry K C L Kirchhoffs current law.

But it is not true that i d 1, i d 2 and i l i L d 1 they will not satisfy K C L unless that is I will not be able to say that this plus, this is equal to this (Refer Slide Time: 16:27) that we cannot say this, we can say this yes, but we cannot say this. Because the transformation from these currents to these currents is not been done with the same transformation, the transformation used here and here is different.

So, what you instead do instead of using you know whenever you are doing any kind of interfacing with the network or with another generator it is important to write down your equations in the common frame of reference. So, even if you have for example, written down these equations here in the reference frame local to this generator that is you are using theta 1 in the arguments in the (( )) transformation and theta 2 in the arguments in the (( )) transformation here.

Whenever you are going to use K C L convert these currents to a common reference frame. So, one of the ways you can do it is of course, use what is known as (( )) reference frame or reference frame, which is not dependent on any generator.

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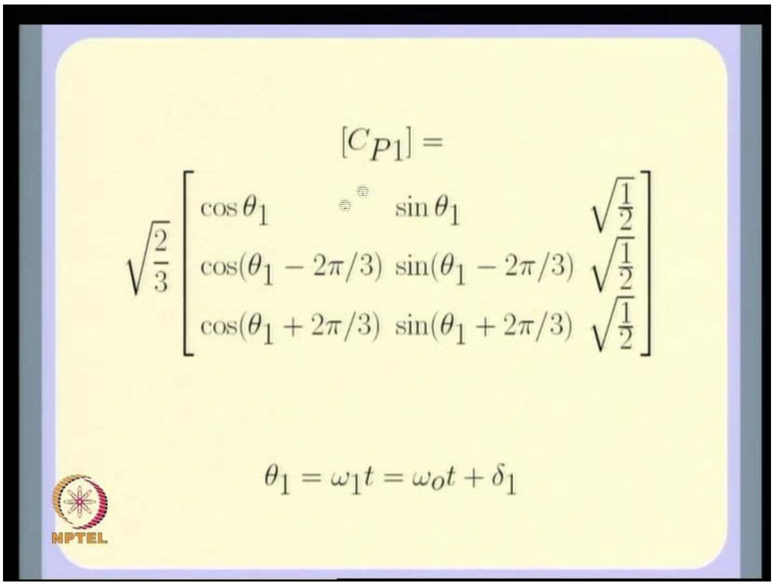

So, what you do is what I will try to show you here this is the (( )) transformation which you will use for the first machine, you will formulate all your equations in the d q reference frame but, you will be using this transformation of variables.

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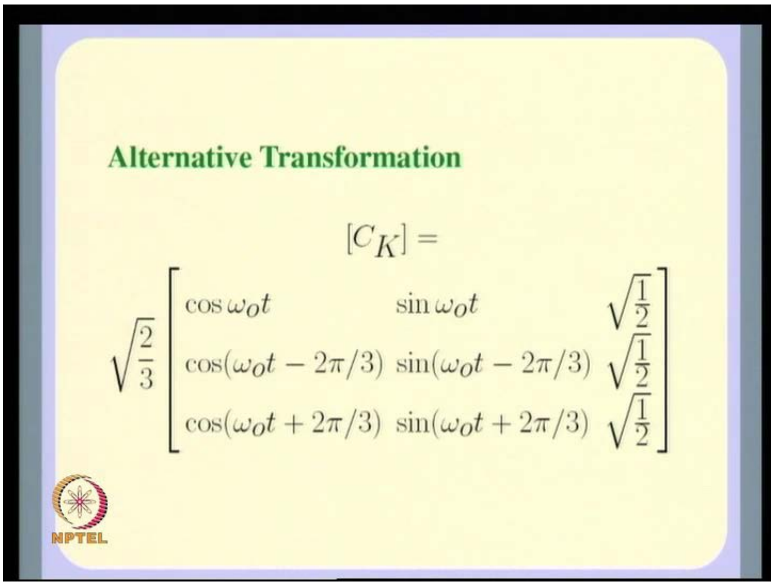

So, you will be using this transformation of variables instead of that can you use this transformation the answer is yes you can.

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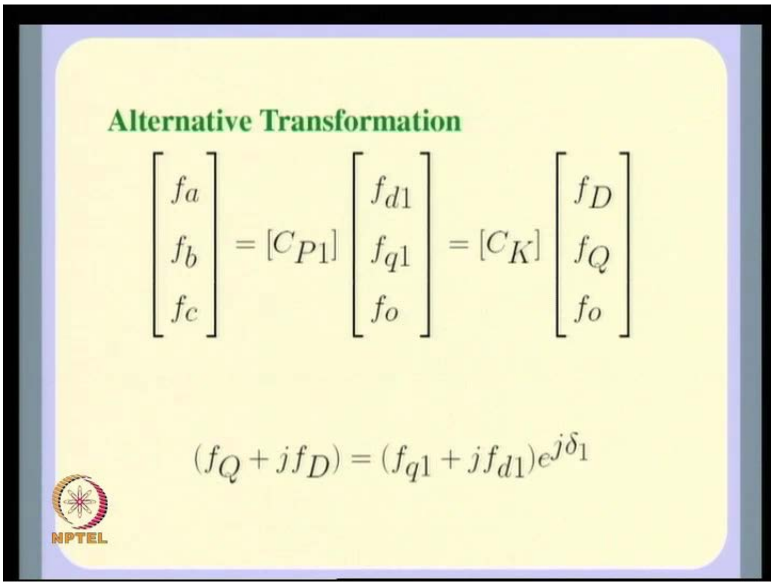

And the variables f q 1 f t 1 and f Q and f D are related by this relationship you can easily work this out remember that f a f b f c are the three phase variables they remain unchanged you are using a transformation C P 1 to convert to f d 1 f q 1 and f 0, if you use C K instead it is easy to see that if C K is given by this C P by this (Refer Slide

Time: 18:24). Then the variables in the upper case or capital D Q frame and the small d q frame are related by this relationship.

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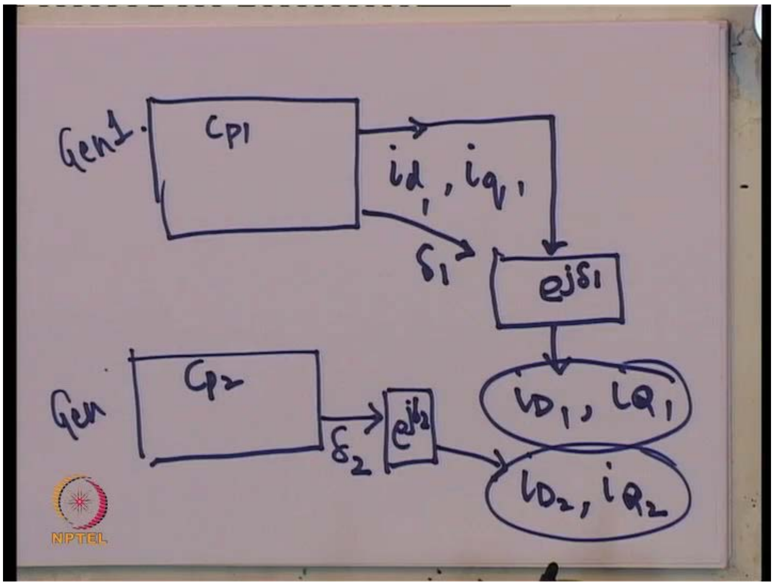

So, what really is the procedure you should apply in case you are trying to interface different generators what you need to do is you can formulate the generator equations in using C p the local frame, so you. So, from that local frame you will get i d and i q then convert this i d and i q using delta 1 to i d and i q. So, use this transformation e raised to j delta one as I mentioned here in the slide. So, can you have a look at the slide again (Refer Slide Time: 19:25).

So, you can use this slide to convert you know the small d q variables the you know capital D Q variables now what you do with this this is for generator 1 you do the same thing for generator 2. You can you can formulate all you equations in the local frame but, whenever you are going to use any kind of interfacing with the rest of the system.

You use these variables are on a common reference frame and you can use, they are using the same transformation from the basic a b c variable. So, once you have these variables on a common frame, you can apply K C L and K V L. So, this is an important point when you are modeling the system.

So, you can for example, write down the equations of a network in the d q frame of reference, remember we had formulated the you the equations of a transmission lines for example, in the a b c frame and then we can convert them in to the d q frame. So, the d q frame which you are going to use can use c k, the transformation c k.

So, the network the network equations as well as all the current injections from the various generators have to be transformed to a common reference before you apply K V L and K C L. So, what you need to do is of course, whenever you are interfacing with the network or with another generator directly you convert all the variables to a common reference frame, so that is very, very important.

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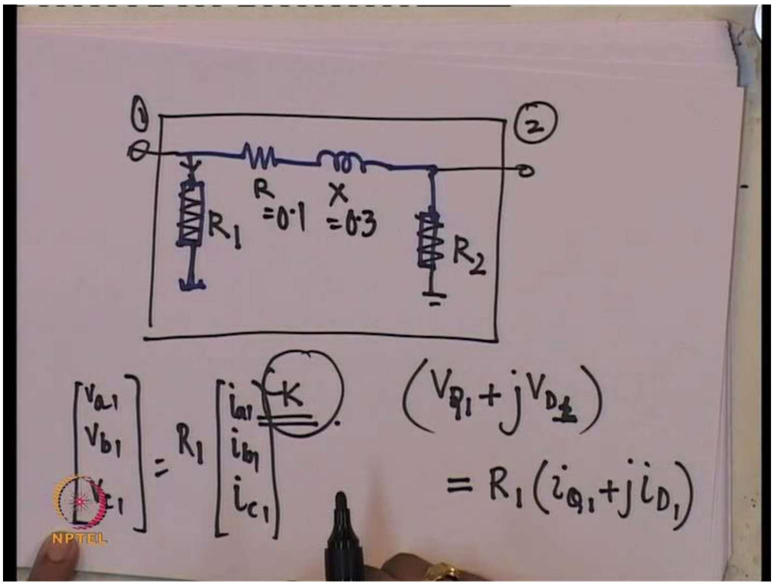

So, in this particular for example, system we have got a network like this is the load, which I mentioned sometime back was, in fact, a unity power factor load, which is of course, voltage dependent; because I have told that it is a resistive type load also. So, I assume that the load is voltage dependent it is dependent on the square of magnitude of the voltage. So, you have got a system like this, this is a network.

So, this is your network and you can write down your equations of this directly in using the transformation C K. So, for example, the current here in this resistance, let us call it R 1 and R 2 is for example, you can directly write the relationship. If this is bus 1 and this is bus 2 and you can write V d 1 plus j V d 2 sorry I am sorry sorry. So, V q 1 plus j V d 1 is equal to R 1 into to the current through this. So, I will call this i Q 1 plus j i D 1; actually what effect I have got the this is a complex relationship, it is a rather these are complex numbers.

But, what you need really to see here is V q 1 is equal to R 1 in to i Q 1. So, V d 1 is equal to V d 1 into r 1 into i D 1. So, these are actually two separate equations here instead of writing it in matrix form I have written it compactly in this form. So, you can easily get these equations from and applying the transformation C K in order to convert to the capital D Q variable. So, once you use this C K you can convert this to D Q you can convert this to D Q and this is what you eventually get the algebraic equation here will be like this.

Similarly, if you look at this you know the equation this X is for example, 0.3 and R is 0.1; in this example you will see that this, in this system the equations for the transmission lines of course, are differential equations. So, of course, if assume that you can lumped model with just series lumped reactance, then you have got this differential equations. Now we saw in the our discussion in the transmission lines, if you convert this using D Q transformation if you convert this using C K what you will get in steady state, I am not talking of that you know the differential equation.

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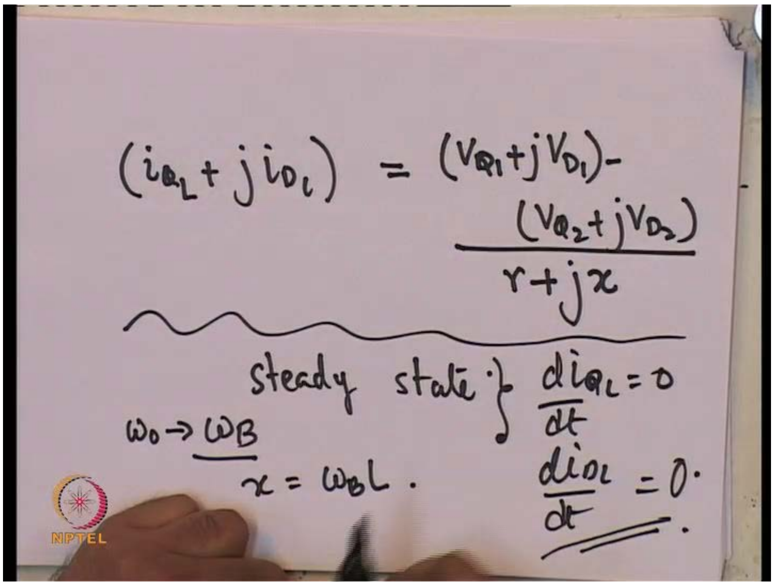

In steady state you will get very surprisingly i Q line j i D line is equal to v Q 1 plus j v D 1 minus v Q 2 plus j v D 2 divided by r plus j x. So, this is the relationship you get in steady state this of course, assumes this assumes that omega naught is same as the omega B the base frequency. So, x is equal to omega B into L, so this is the assumption but, this only true in steady state, if you want to write the differential equation. In fact, took into account the rate of change of current that is I have assumed here that d i Q L is equal to d t and d i D l by d t is equal to 0.

So, this is an assumption which I made, so just remember this, so this only valid in steady state. Otherwise of course, I will get a differential equation in d i D l and i Q l and this is something which you have discussed sometime back in our discussion of transmission lines.

So, for every element of this network or this part of the system (Refer Slide Time: 25:44), you can get differential equations or algebraic equations. If you assume that the network is always in steady state and is not, what we call as neglect network transients, then actually all these equations become you know if all the d by d t's are neglected then all these become algebraic equations.

And surprisingly algebraic equation look very neat, they almost look like Phasor equation actually these algebraic equations for example, this is representative of two algebraic equation which we get when we set this equal to 0. So, this complex notation is a compact notation as well. Now, so, if I neglect the network transients if I do not wont to neglect them of course, I should write down the differential equations, you can neglect the differential equations provided the transients of interest are slow. So, do not make this assumption for example, while studying lightening or switching transients or fast transients.

Suppose you are trying to understand you know, how the network interacts with a fast controller like an HVDC controller power electronics in HVDC controller, then of course, please do not make this kind of assumption. In fact, you may even want to model a transmission line by a more detail equivalent (Refer Slide Time: 26:59). You may try to use a travelling wave model of a transmission line. So, this neglecting of d by d t's are very big assumption to make provided but, it is provided you are interested in understanding only the slow transients.

One interesting thing is that if your load is just a resistance load and your transmission line is also represented by algebraic equation instead of differential equation with the understanding that you have of course, you are going to study slow transients, then you can represent the network completely by algebraic equations.

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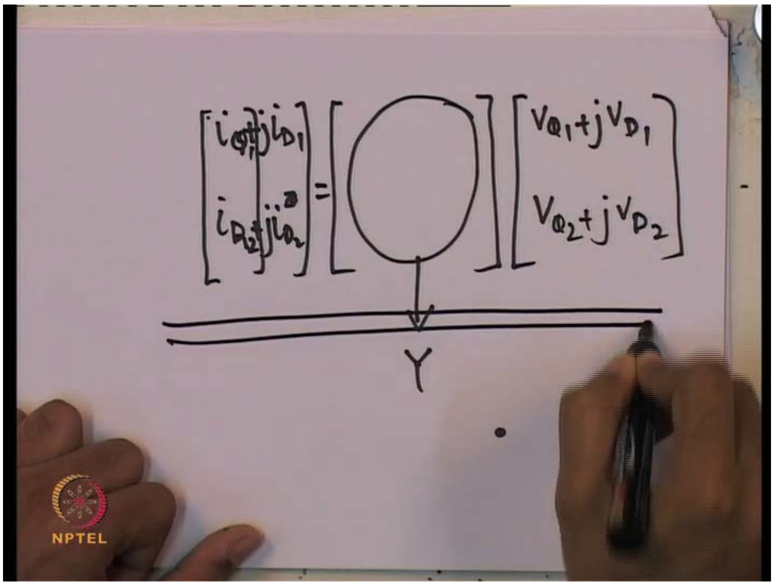

And. In fact, you will notice that you should be able to write down for all nodes for example, in this system (No audio from 27:45 to 28:56) I am sorry. So, this will be i Q 1 plus j i D 1 is i Q 2 plus j i D 2, so this work also will not look very nice. So, this will be like your admittance matrix. In fact, it is the admittance matrix, you just try to work it out you will just find that what you will get the admittance two port admittance matrix for this system. Of course, this representation of your system network is assuming network transients are neglected, so that is something which you should notice.

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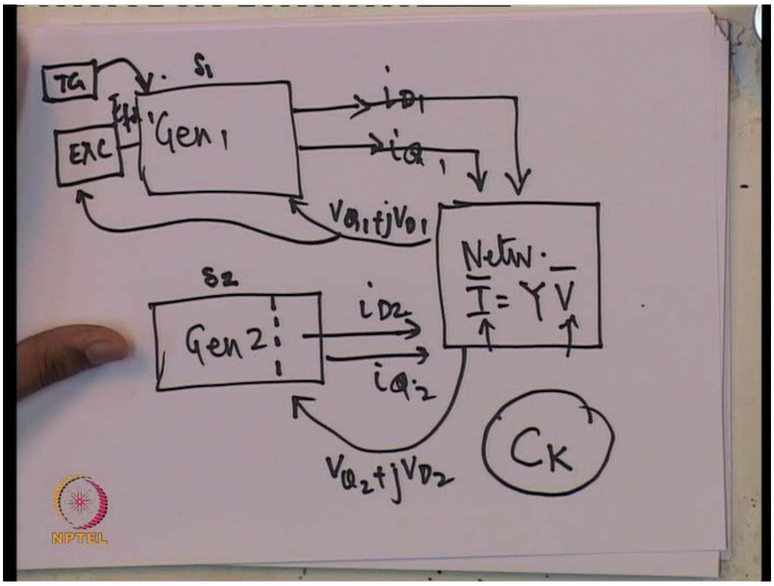

So, what you have here is of course, the generator is represented by this differential equations from which you can get the currents i D 1 i Q 1. So, generator 1 currents i D 1 and i Q 1 these currents are injections to your network static network and static load. So, load also is actually absorbed as a part of the network, because it is purely resistive of course, this will not be true in case you have got rotating loads in which you are representing for example, large induction machine by differential equation. In that case you cannot represent it as a part of the static network.

So, this is a network which is represented by I injection is equal to Y in to V you know (Refer Slide Time: 29:15) I injection of course, means this vector. Similarly, the other generator is injecting i D 2 and i Q 2; remember i D 2 and i Q 2 are functions of the flux by an algebraic relationship.

So, of course, i D 1 and i Q 1 i D 2 and i Q 2 and all the current injection and voltage vectors here, are obtained from a b c using the C K transformation or from remember that if your generator has been formulated generator equations have been formulated using the local paths. Reference frame using theta 1 and theta 2 in that case you have to use delta 1 and delta 2 to transform those currents to those compatible with this transformation. Now remember that the network, once you give the current injection from the network effectively you get by solving the network you get the information which will be required to compute the next value of the states.

So, v Q 2 v D 2 and v Q 1 and v D 1 are, in fact, voltages at the terminals of a synchronous machine in this reference frame. So, again you have to use delta 1 and delta 2 in order to get the same voltages in the small d q local reference frames. And those can be used by the differential equations; in fact they are inputs to the differential equations. So, this is how your systems look like.

In fact, if you look at the exciter it will also be taking this information, because it requires the feedback of the voltage magnitude at the terminal of the generator compare it with the local reference voltage and give the field voltage to this generator is that. And of course, it goes without saying that the turbine governor system is something which affects the mechanical power input to the generator this present here also. Now, one of the interesting things which it is kind of a diversion but, you can try to prove that if I use the variables C K the capital D Q variables.

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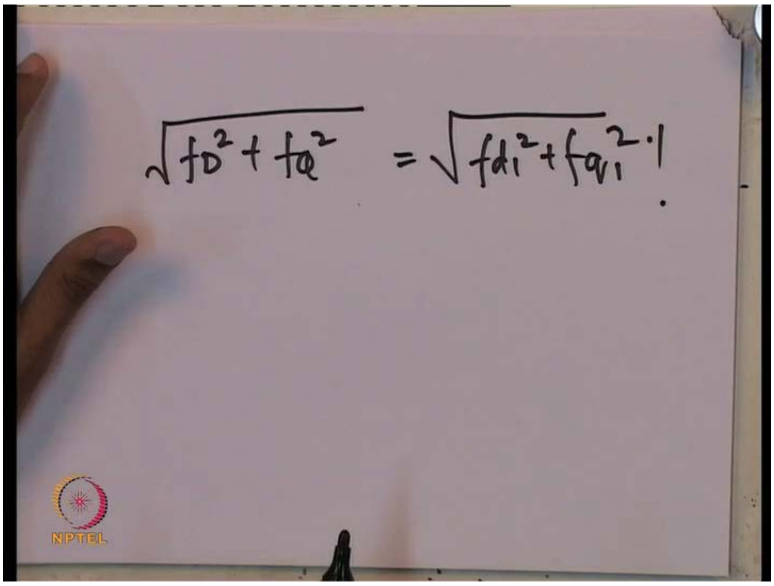

Then you can show that f D square plus f Q square any for any a b c variable if you transform it to the capital D Q frame it is also equal to. So, this is an interesting thing where can you use it for example, if you are trying to compute the magnitude (Refer Slide Time: 32:07) instantaneous magnitude of the voltage at the terminals of a generator; remember we are had a discussion about what we meaning we can assign to instantaneous magnitude.

So, if I want to use the instantaneous magnitude one way we can define it is v D square plus v Q square which is also equal to v which is small capital D 1 square plus v small capital Q 1 square. So, that is an interesting point which is of course, which can be used in actually program. So, what I am trying to do here is of course, my main aim here is to actually tell you about phenomena but, all the same I have tried to tell you a bit about how you will formulate your equations and actually solve them.

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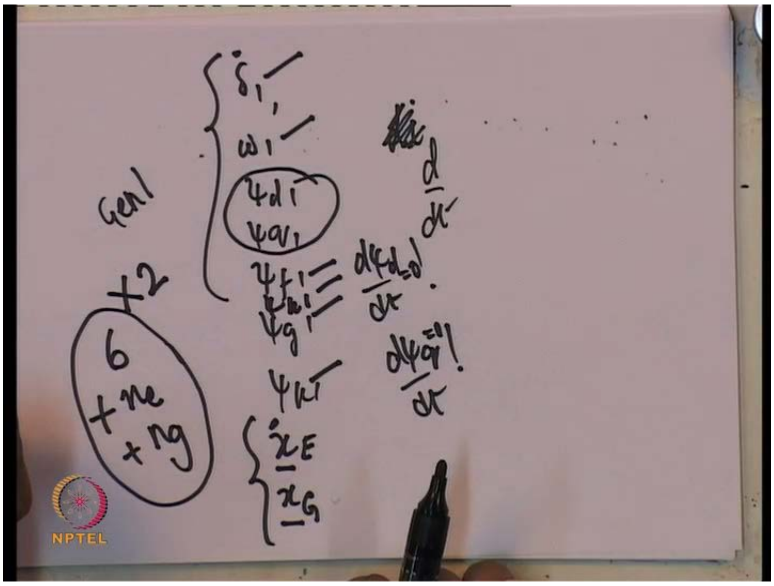

So, what you really have at differential equations delta 1 omega 1 corresponding these are corresponding to the generators delta 1 omega 1 the fluxes psi d, psi q, psi f, psi g, psi k, for the first generator. The states they may be 1, 2, 3 or 4 depending on how you represent your exciter. So, I will just call this x c these are states of the exciter I will call them by x under bar, because there maybe more than one. Similarly, the turbine and the governor may have several states associated with the turbine, the actuator and the controller, this is the governor.

Similarly, the exciter may involve some states corresponding to the excitation power apparatus as well as the AVR the Automatic Voltage Regulator and other controllers. So, these are the states of the system, the generator 1; similarly you have the states of the generator 2. And then you have got the states of the network, in case you are neglecting the states of the network that is neglecting the d by d t for example, in this example you could neglect d i by d t and d i q by d t and would set them to 0, which means of course, that the model is suitable only for slow transients.

Then, in that case you will not get differential equations for the network the network will be represented only by static equations of course, if your neglecting the network equations, it make sense to neglect the transients associated with d psi s it is just the consistency.

Neglect these transients also or set this equal to zero also. So, what you will have instead of if you are neglecting network and stator transients, then your states are 1, 2, 3, 4, 5 they are five have I missed out one there is one psi h 1 also, so 6, so in that 6 states plus the states associated with the exciter and turbine governor system. So, you will have six plus you know let us say n e plus n g this is for each generator. So, you will have you multiply by this by 2 is the total number of differential equations which you will have.

Remember that the differential equation corresponding to psi d and psi q and the network differential equations can be set to 0 or they can be converted to algebraic equations provided you are studying slow transients. But, do not make the this assumption in case you are studying faster transients.

So, this is are the total number of states and one way of solving this whole system is to discretize of course, the differential equations using some numerical integration method the algebraic equations. In fact, you you will you can just directly solve them. Now, in fact, if you look at the algebraic equations, which are there the algebraic equations actually can be solved are they linear algebraic equations just think over that are you going to get linear algebraic equations.

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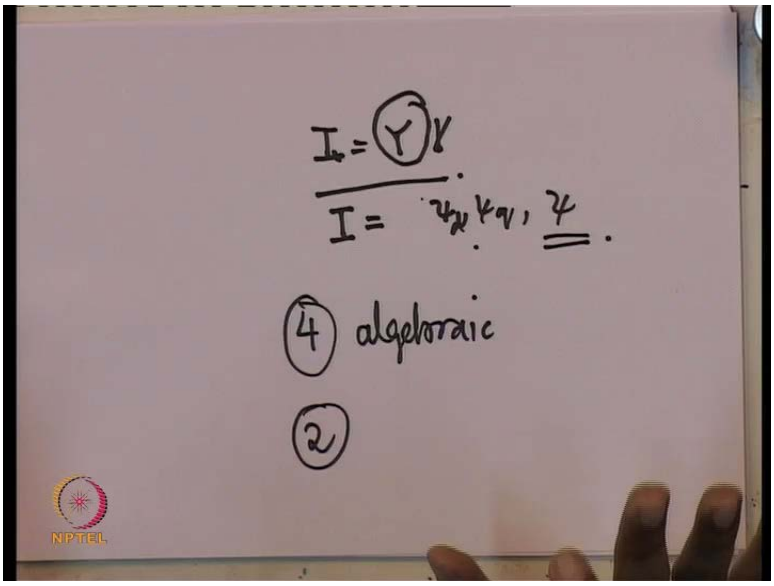

The answer is kind of the point is that the network to a means for example, the way we have written it is, in fact, a linear network. So, to get if I give you I to get V simply involves solving this linear system of equations. But remember that I itself is a function of psi d psi q. So, you have got is a function of psi d psi q of each generator, as well as the other fluxes remember that equation which we had.

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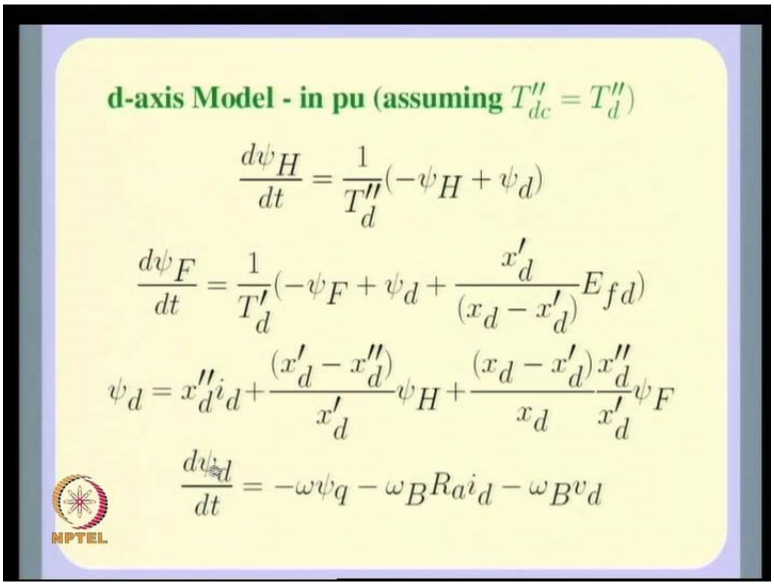

This for example, psi d is equal to x t double dash into i d plus these states. In addition if you set this to 0 d psi by d t is equal to 0 when your studying slow transients, they may got you know an algebraic equation here and another algebraic equation here.

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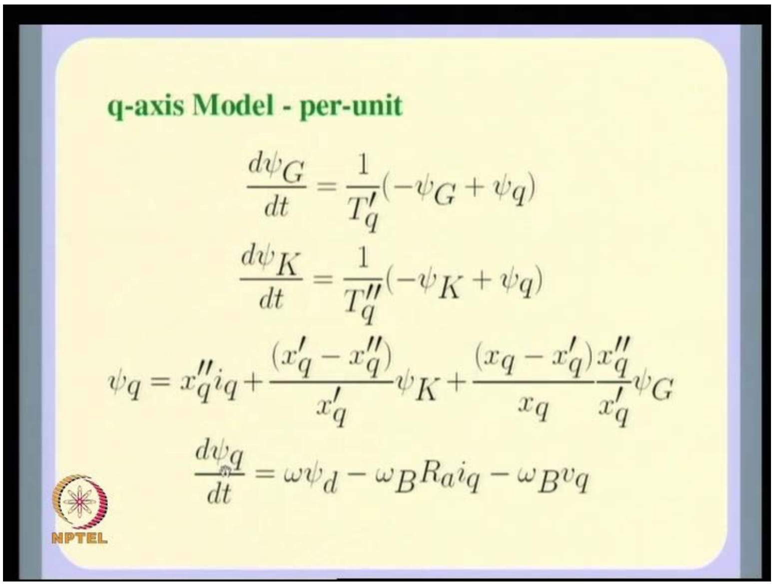

Similarly, you have got for the q axis an algebraic equation here and in case your neglecting stator transients in our algebraic equation here. So, in fact, if you neglect stator transient the generator itself has got 4 algebraic equations.

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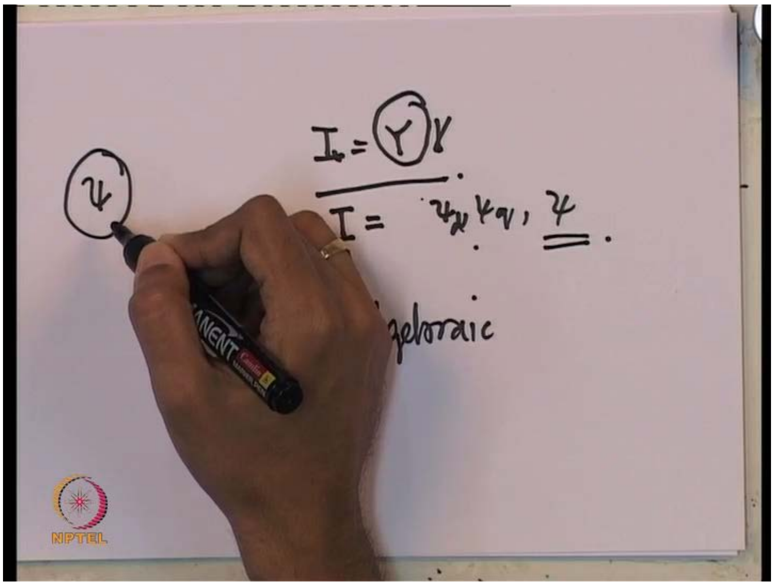

4 algebraic equations the network is all algebraic equations. So, in case you neglect stator transients of 4 algebraic equations, otherwise you have got two algebraic equations which relate i and psi. So, what you need to do is gather up all the algebraic equations and then solve them but, remember the algebraic equations can be solved but, remember that they have got certain inputs. The state variables that is psi f, psi h, psi g and psi k for each generator as well as delta and omega, delta come into the equations how do they come into the equations remember that the algebraic equations you see here (Refer Slide Time: 38:19) has psi K and psi G.

So, that is how these states come into the equations moreover v d and this particular equation v d v q v d sorry v q and i q has to be converted to capital D Q reference frame. So, you need to have delta remember that when you you know what you call interfacing all the algebraic equations together all the current should be got to one reference frame and that requires you to actually use delta. So, the algebraic equations are, in fact, functions of delta.

So, at very times step, when you solve algebraic equations there will be a functions of delta at that point. So, In fact, solving these algebraic equations will require you to do you know in principle a matrix inversion. Actually matrix inversion is not a very nice way to do things, when the matrix is very large we will be actually using when you come to larger systems it would not be a good idea to use a you know explicitly compute inverses.

Because not only to they require a lot of storage usually an inverse even if your original matrices, when your solving algebraic equations are (()) when you compute an explicitly an inverse the matrix becomes full. So, that is one of the reasons why we will not you know explicitly take out inverses and of course, if your algebraic equations or functions of delta and delta is changing at very time step one problem which you should grapple with is that for every step you will have to solve a set of algebraic equations.

So, at every step you will have to redo this kind of solving of algebraic equations, you can avoid all this if you can do some tricks. So, these tricks of course, ill not spend right now time on, we are talking about very small system. So, solving algebraic equations at every time step itself is not a very difficult or very heavy computational burden.

But, remember when you are trying to solve very large systems you will be faced with a problem of how to you know represent the system or what what kind of trick to use. So, that you will not have to solve very large number of algebraic equations and you do not have to actually do inversion or even you know what you call what the the technique which is used of course, l u factorization and you do not have to do this factorization at every time step.

So, this something of course, you will not probably understand at this point, you can of course, just remember this issue whenever you are going to study large you know large systems when you are actually trying to make a programmed when which which is trying to you know trying to simulate a very large system. But, for small system you will have to solve algebraic equations you can even take the inverse explicitly at every time step it is not a very big computational burden for a small system.

But, remember that this issue when you are talking of very large systems you will have to use some tricks in order to make your computation burden a bit lower (Refer Slide Time: 41:43).

So, this is how you will simulate the system, so what you need to do of course, is get the equilibrium conditions you have got the algebraic equations discritize the differential equations. Incidentally when you discritize a differential equation it becomes an algebraic equation, so eventually all of your simulation becomes the solution of algebraic equations.

Now, once we start simulating the system, in fact, if your if your all your states are at equilibrium, your system will just stay where it is but, for example, if I change something in this system. For example, I change this resistance this load resistance I change the load effectively, then you will find that the algebraic equations have changed and as a result of which the equilibrium itself has changed. Now when you change an equilibrium your at present you know at a certain equilibrium, now the equilibrium itself has changed. So, your away from the new equilibrium your your at present at the old equilibrium and you want to go to the new equilibrium, so there will be some transient.

So, what you need to do is of course, whenever your simulating the system you need to tell your program at this point of time change the algebraic equations. So, once you change the algebraic equations you will start seeing a transient, because you have not at a new equilibrium your not your your initially at some other equilibrium.

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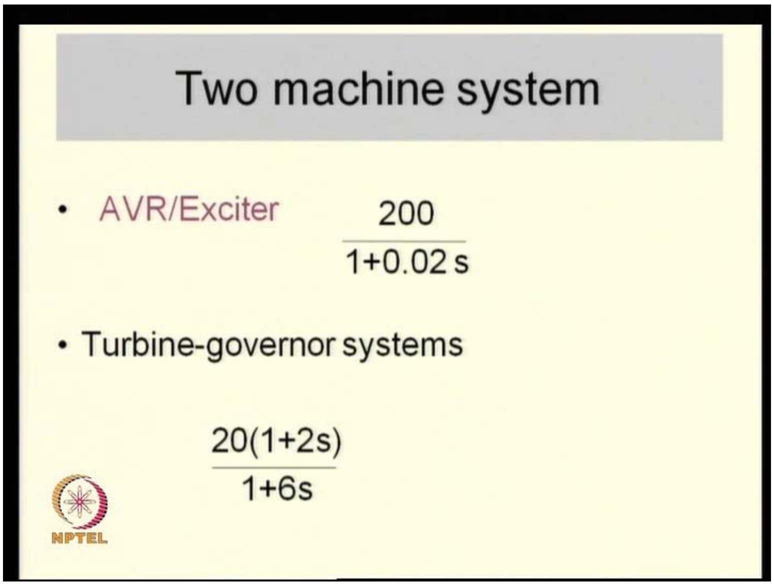

So, let us directly start understanding, so where we are we were here at this point let us represent both the generators are identical, which have got identical control systems we have got a automatic voltage regulator, which is practically maintaining the terminal voltage of this system at 1 per unit.

So, even, so your AVR control strategy is to maintain the terminal voltage at 1 per unit. So, ill just get this diagram again, so that we can have a better idea of what is happening. So, this is our system diagram (Refer Slide Time: 43:54) your maintaining this terminal voltage here at 1 per unit incidentally you have considered a very simple system with no transforms and so, on. You can actually increase the complexity of this system but, that would not be necessary to try to tell you about the basic phenomena.

So, right now let us consider only this simple system where I am maintaining this terminal voltage here at 1 per unit using an AVR. The turbine governor system of both machines has got this transfer function; so 20 into 1 upon 1 plus 2 s upon one upon divided by 1 plus 6 s.

So, this is a very, very simplified model of a governing system and a turbine. So, a governor is assumed to be a simple gain of 20 and the turbine has we assume has got a transfer function 1 upon 1 plus 2 s upon 1 plus 6 s. So, this is a very simplified model of steam turbine, in fact, it is basically neglecting the dynamics associated with the cross over piping as well as the steam chest. So, it is it is a kind of model which is very simplified but, this is enough of course, to tell us about the phenomena like load sharing etcetera but, please do not use this model in case you want to get realistic results especially for large disturbances.

Both the AVR as well as the excitation system as well as the output of the governor are limited in sense that we do not of course, change the mechanical power beyond certain limits and also the AVR the field voltage is not allowed to exceed certain limits. The limits in case of the AVR are plus or minus 6 per unit. So, you know you have got fairly high you know high sealing voltages for the AVR.

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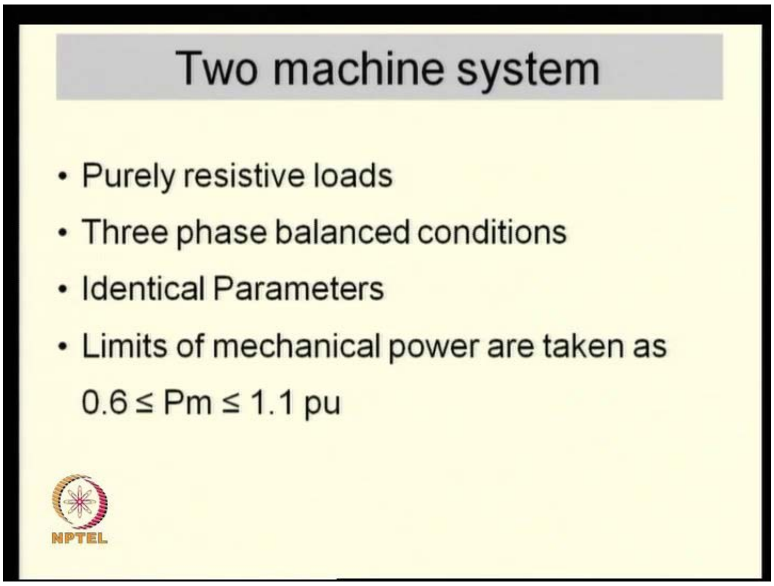

Now, what we need to of course, understand next is some of the important issues is purely resistive loads or loads are resistive, we are considering three phase balance conditions all the the transmission line the loads all are balanced even our you know disturbances, which you will be considering are balance. In fact, an interesting thing to chew upon is in case of what unbalance does it complicate our analysis the answer is yes

In fact, if you try to formulate the d q equations of a unbalanced network you will find that it is dependent on omega naught t. So, that makes you know the algebraic equations also time dependent you know. So, this is something very interesting, which you can think about.

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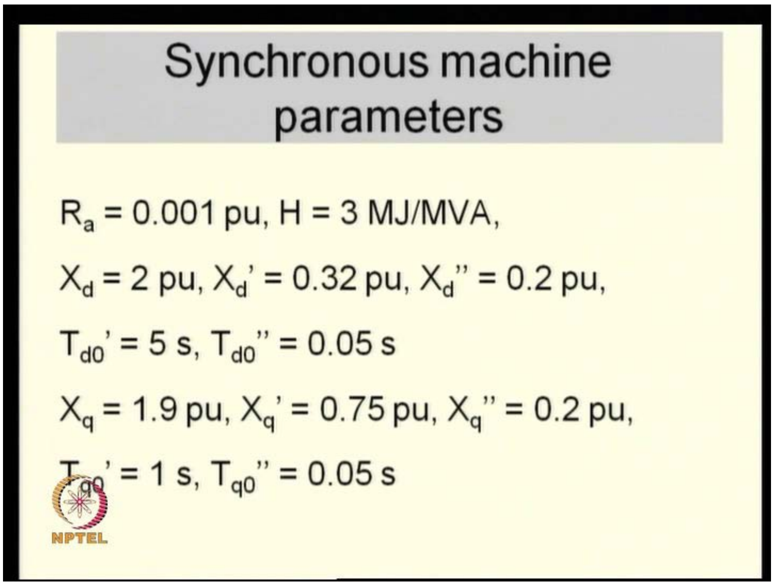

We have considered identical parameters in both machines, this is just an interest just just for keeping things absolutely simple. The identical synchronous machine parameters of both machines are the parameters of the synchronous machine are given here. So, one of the interesting points, which you should notice here (Refer Slide Time: 47:03), well it is not really something which we have imposed in this is that this generators is generating 1.1 per unit power. So, actually it is generating slightly more than the rated m v a of the machines. So, this is some something which, we have is that is why I would say that this is a very academic example, why would anybody load a synchronous machine more than its MVA it cannot be done really.

So, this is a small drawback of our example but, it is a minor issue in the sense that again as I mentioned this is just a conceptual example to tell you about some stability phenomena. So, this is a just a reality check.

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Eigen values of the system

Network transients considered			
With governor		Without governor	
0.00	-28.71	0.00	-28.71
-0.16667	-37.33	0	-37.31
-0.392 + 0.398i	-47.995		-47.997
-0.392 - 0.398i	-52.261		-52.264
-1.8758	-45.152 + 311.43i	-1.892	-45.152 + 311.43i
-2.2142	-45.152 - 311.43i	-2.214	-45.152 - 311.43i
-1.5439 + 9.749i	-1191.1 + 308.33i	**-1.265 + 9.824i**	-1191.1 + 308.33i
-1.5439 - 9.749i	-1191.1 - 308.33i	**-1.265 - 9.824i**	-1191.1 - 308.33i
-14.673 + 9.066i	-26473 + 314.06i	-14.704 + 9.034i	-26473 + 314.06i
-14.673 - 9.066i	-26473 - 314.06i	-14.704 - 9.034i	-26473 - 314.06i
-12.467 + 22.299i		-12.464 + 22.289i	
-12.467 - 22.299i		-12.464 - 22.289i	

Now one other things you can actually do is for get a Eigen values of this system. In fact, if you you know you have got a algebraic and differential equations you can linearize around the differential equations around an equilibrium point; note that these are non-linear equations and you cannot directly get Eigen values corresponding to a state matrix, because the equations are non-linear.

So, what we can do instead in in if you want to do small signal stability analysis is consider an equilibrium point. Get the equilibrium values of the states form the linearized matrices for this system and then compute the Eigen values of the state matrices. And very interestingly if you consider network transients, that is if I consider the d i by d t is of the network d i d by d t and d i q by d t of the network I do not set it equal to 0 or I write in differential equation.

Similarly, if I write in differential equations of psi d and psi q as well for both generators in that case what you notice. In fact, these are two tables one on your left here, the first two columns on the left the first two, the last two columns on your right are the equations with governor and without governor.

Now, what you notice here, is some if network transients are considered some very large Eigen values are seen with real part the imaginary parts are near about 314 radiant's per second that is near about omega naught; these are very large Eigen values. Similarly, without governor also you have got this large Eigen values but, network and stator transients are considered stator d d by d t or d psi d by d t d psi q by d t are considered you get very large Eigen values.

With and without governor the difference is that without governor you have got two 0 Eigen values you have two 0 Eigen values with the governor you will get one 0 Eigen value one negative Eigen value. And you also get two additional Eigen values that is because the number of states increases with the governor you know but, the turbine governing system has got one state. So, if your got two generators you will have two extra states. And therefore, you have got two extra Eigen values here as compared to here without governor.

So, of course, one other thing which is very striking is you know is this particular mode this complex pair of Eigen values which are representative of the electro mechanical swings, we have see this swings before in a single machine infinite bus system they appear here to but, the important thing is without governor in addition to this swings you also have these two 0 Eigen values.

So, in fact, in the previous lecture I mentioned that if you have got a two spring mass, two mass spring system you have got one oscillatory mode and one mode corresponding to the motion of the center of inertia. In fact, the motion of the center of inertia is associated with two 0 Eigen values in case there is no friction in surface.

So, if you do not actually provide for any friction and you have got a two machine or two mass spring system you have got a complex pair of Eigen values and two 0 Eigen values which really talk about the motion of the center of mass of the system.

In fact, if you give an unbalanced or rather if you give a disturbance which causes this motion of center of inertia to be excited, you will find that. In fact, if you do not have friction this will just keep on moving and as a result of which displacement of individual masses will keep on changing with time.

In fact, you give a suppose this spring mass system you give a disturbance to both masses and they start moving together they will keep on moving because there is no friction.

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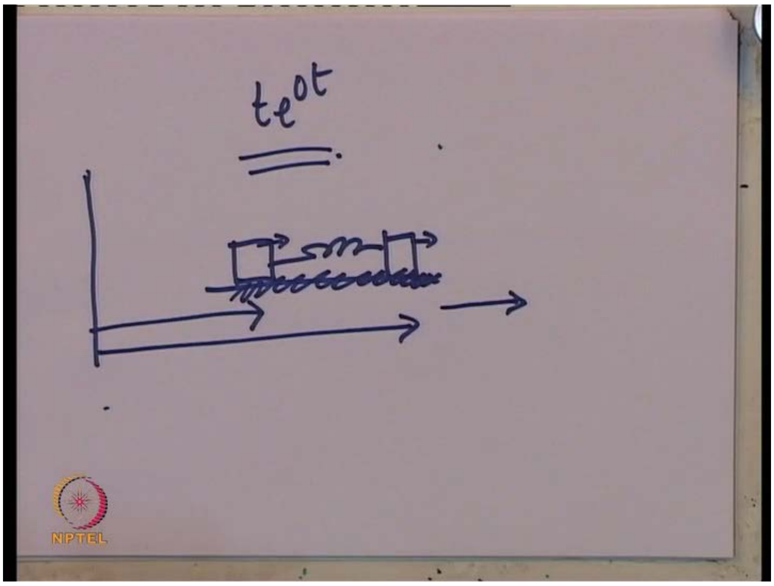

So, in fact, the two 0 Eigen values result in a motion result in as you mentioned last time, a component of motion which has got which looks like this. So, if you have got a spring mass system and if I give a push to both masses, you will find that the whole system starts moving, the center of mass of the system starts moving this side, and if there is no friction it will continue moving. So, if you look at the displacement with reference to some reference you will find that the displacement keeps on changing the states will keep on changing.

Of course if you put some friction, you will find that even if I give an initial displacement this will eventually settle down. So, if there is some initial displacement of the center of inertia you will find that this will kind of settle down. So, if you have certain, so what you will find in a Eigen value analysis is in fact, a reflection of this. A governor in fact, is something which kind of plays the role of viscous friction. In fact, if your loads of frequency dependent also you would have another mechanism in which you are had friction.

So, what happens is that in case there is certain load generation balance, if you have friction or if you have got generation and load of function of frequency that is what viscous friction really means. If you have got something which is a function of the frequency if you have make your mechanical power or the load power or function of frequency, you will find that the system kind of settles to an equilibrium you know you will settle to an equilibrium speed, even if there is a imbalance in the you know external forces on the system.

So, that is one important point, so if you look at the Eigen values of the system if you have no governor you have got two 0 Eigen values. And therefore, in case there is any load generation imbalance there will be a continuous increase in the velocity as well as the displacement or the angular displacement of the machines; this is something we will see in the simulation in the next class.

With the governor of course, you are bringing some frequency dependence in the generated power. So, in some sense, there is a mechanism by which frequency can reach an equilibrium in case there is an imbalance. So, if there is a load generation imbalance frequency will change, if frequency changes you will find that the mechanical power changes. So, equilibriums speed is eventually reached.

So, in fact, with the governor one of the Eigen values becomes negative (Refer Slide time: 54:35). So, these two Eigen values, in fact, with governor and with governor are, in fact, associated with the motion of the center of inertia of the system. And this swing modes or these oscillatory or these Eigen values corresponding to low frequency oscillations, here as well as here are nothing but, the swings modes associated with the relative motion of the machines.

So, although you have got many other Eigen values of very interesting thing is that the pattern associated with the electro mechanical states is, in fact, very close to that of the pattern observed in just a two mass spring system. In a two mass spring system you just have four states; in a two generator system with loads you have many, many more than four states. But, this phenomenon which you see which corresponds to the electrical electro mechanical modes can be captured by this, what seem to be a crude analogy of a two mass spring system.

Now, in the next lecture we will actually do a simulation of the system this is an Eigen analysis. In fact, you know they are many, many, many patterns which you see many, many Eigen values which you are seeing here (Refer Slide Time: 55:48) but, we shall also see a simulation in which you will try to I you know try to see or look for these kind of patterns in the behavior.

And we will in fact, see that for small disturbances the Eigen analysis and the simulation; in fact, match, but for large disturbances we do see instability in relative motion what you have all always called in this course as loss of synchronism. So, with this kind of curtain raiser for next times lecture, which will be actually showing you the simulation results, let us conclude here.