

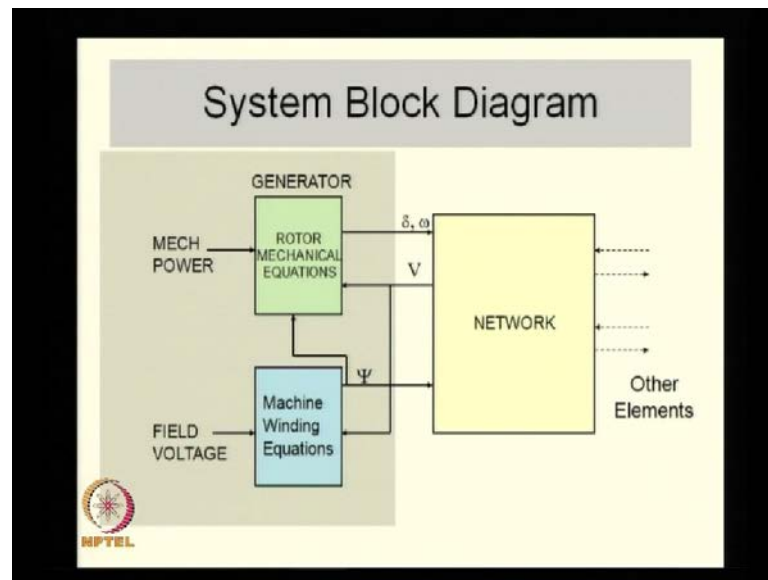
Power System Dynamics and Control
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Lecture No. # 35
Prime Mover Systems Stability in Integrated Power System

Our focuses in the past few lectures have been the modeling of some control systems and actuators in a power system; in fact, the previous lecture was **was** focusing on the modeling of a turbine governor system. Of course, our treatment was more of an electrical engineering kind of treatment, we did not really go into the details of how we got the transfer functions, or **you know** the small signal models of a turbine or even the control system and their actuators. We will not do this, of course in this course, but an interested reader is requested to look into books by Kundur; well, he is given a more detail treatment of this issue.

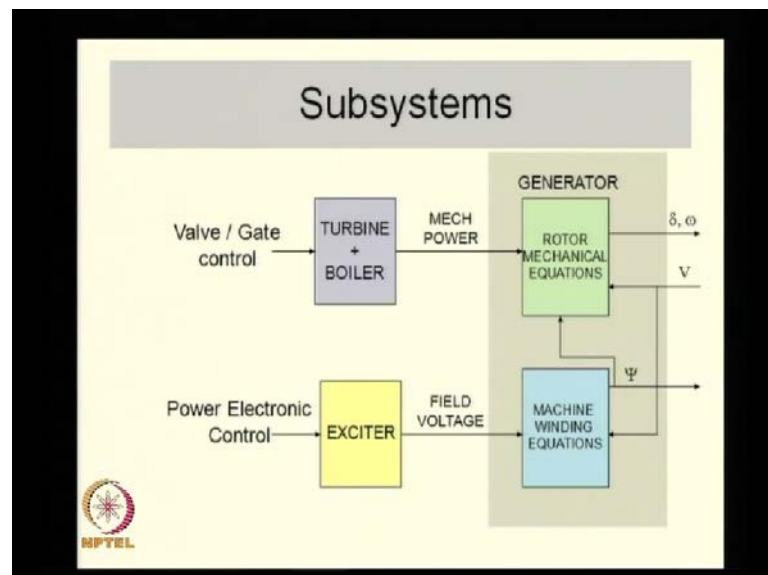
Now, so our lecture today will in fact, focus on some remnant issues in Prime Mover S Systems, and we will move on to the real intent of our study. In fact, the intent of this course itself is to study the integrated power system, what are the stability issues in an integrated power system? Now, we have in fact done of modeling of most sub systems, which are needed to study a power system, of course we have not, for example, studied you know some **some** important components like HVDC controllers etcetera. But whatever we have learnt so far should allow us to get a good idea of at least some of the major stability phenomena in power systems.

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Now, coming back to what our focus has been or rather the remnant issues in our study so far; we have been talking of the mechanical power sub system. If you look at the system block diagram of a generator connected to a network, you will find that the two major inputs to a power system, one is the mechanical power, and the field voltage.

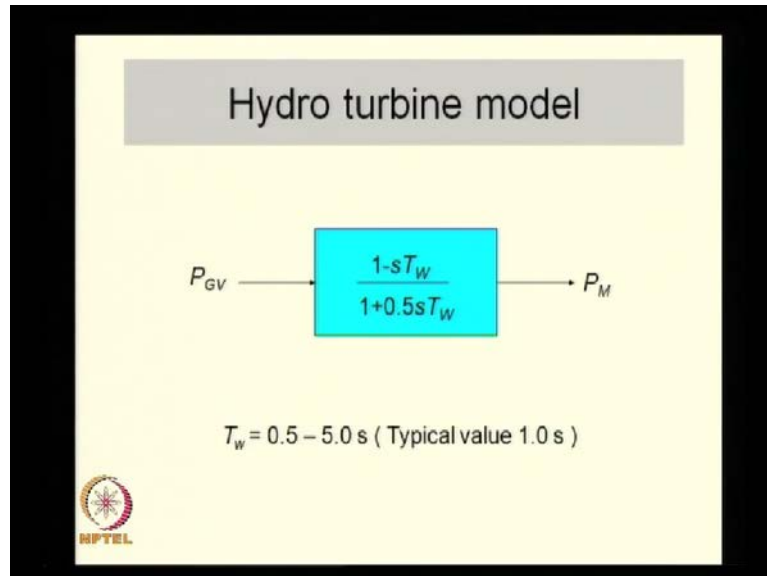
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We have of course seen that power electronic control is used to control the excitation system, which in turn **contains** controls the field voltage to a synchronous machine or a synchronous generator; valve and gate **control** controls the mechanical power output of

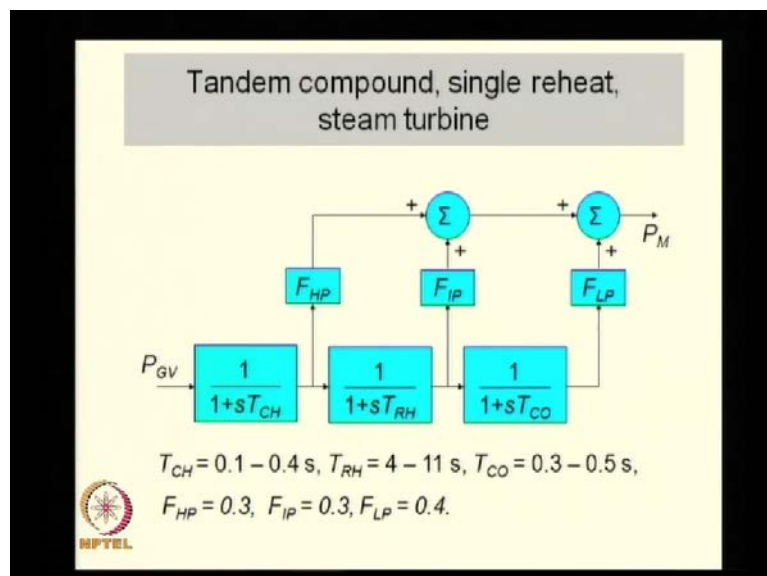
turbine and in case, it is a steam generating system, it is a boiler also. Now, in the previous class, we have seen the models of these **these** turbines systems.

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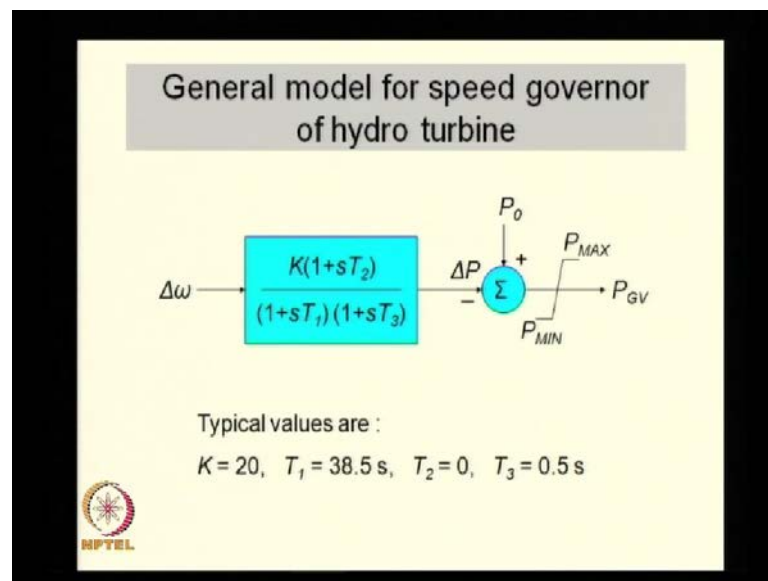
In fact, one of the models was of a hydro turbine. The hydro turbine model, of course is a this is a very, very simplified model of a hydro turbine, it is suitable only for some simple studies. In fact, one of the important points, which he brought out last time was that, the hydro turbine model is of non minimum phase type, it is got a 0 on the right hand side of the s plane.

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We also saw, how a steam turbine model looks like, this was a tandem compound, single reheat steam turbine; and you see in this particular model that, each turbine contributes a bit to the mechanical power. So, F H P, F I P and F L P are in fact components or rather than contributions of individual turbine stages, the high pressure turbine, low pressure turbine, and the intermediate pressure turbine to the final mechanical power.

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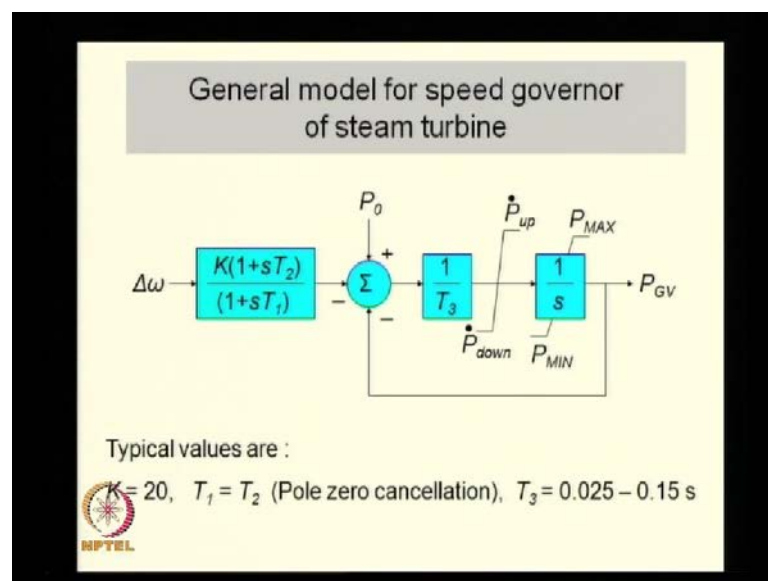
Now, as we have discussed last time, we actually control the gate or the valve position to achieve certain control objectives. For example, you could set your valve position or gate position based on load reference; by load reference I mean I want to have a certain output power of this synchronous machine. So, I set the turbine power equal to that load reference. The load reference itself is obtained from a energy management system, or system operator who tells plant operator to operate the system at a certain load reference, or the certain operating power level.

Now, in addition to that you can have, for example a speed control system remember that, if you got a system without an infinite bus; in such a case, the speed is not regulated, in the sense the speed is actually determined by the mechanical power inputs to the various generators. And in such a case, one may have a close loop control of this speed, and a general model for this speed governor of a hydro turbine is shown here, it is again a highly simplified model of a speed governor.

And one of the things which is modeled here is, of course the limiters, the maximum and minimum limits of the output of a hydro turbine are in fact model in this **in this**, the gate position directly maps on to the mechanical power output. So, what I have shown you here in some sense a per unit model.

For example, when I say per unit gate position is 1, it could mean that the machine will operate at in steady state and in this situation at 1 per unit, a mechanical power. 1 per unit of course, what I mean is the rated mechanical power of course. So, as I mentioned last time, in case you are using MVA base as in the swing equations, you will have to convert this valve position to the appropriate **you know** mechanical power in per unit on an MVA base. P_{GV} is equal to 1 means, rated real power output of the machine. So, if you **you** have to convert it to the MVA base, not the mechanical mega watt base. So, that is one thing which we have to take care, when one is using this for a system studies.

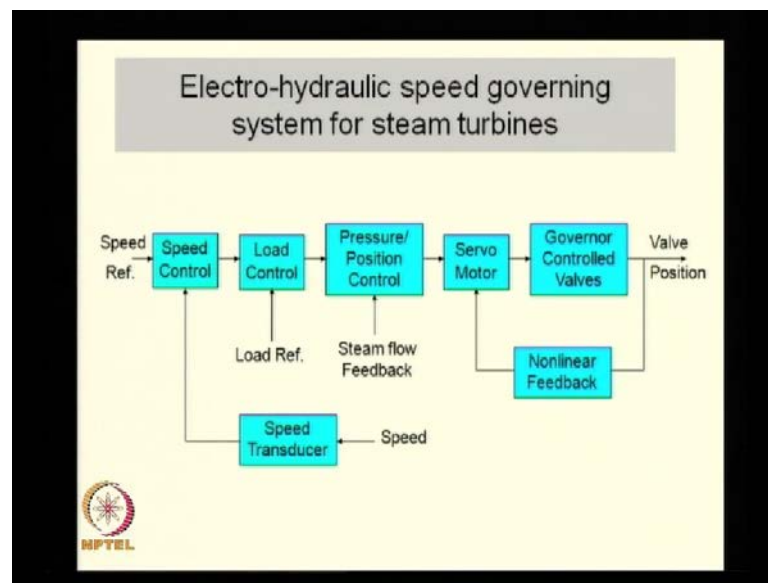
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Now, one more model which I will introduce here is of a steam speed governor of a steam turbine, the steam turbine speed governor, again we are modeling the actuator and as well as the speed governor, the speed control system. The gain of the speed governor is K and if you notice that a model of a speed governor includes the rate limiters **you know** P_{up} and P_{down} are in fact, rate limiters, there are limiters which come before the integrators.

So, they prevent a certain rate of change, rather they prevent the rate of change from increasing beyond the **limited** limiting values. So, this is our **our you know** model of a steam turbine, again let me emphasize that P G V is in fact, the gate position or the valve position in this case in per unit. So, **gate** valve position if it is equal to 1, it means that the valve is completely open and the machine will operate at rated mega watt, so that is something you should remember.

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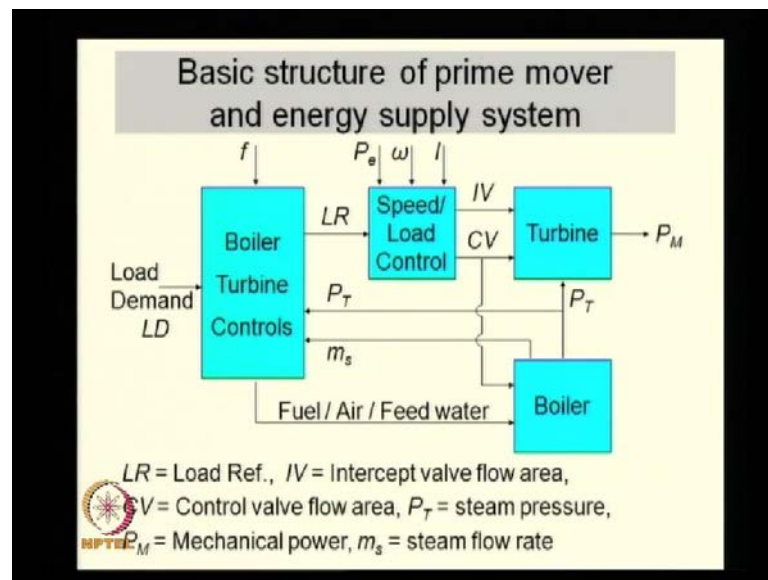
Now, if you look at an electro hydraulic speed governing system, if you look at the complete block diagram, of course I showed you a simple, very simple kind of transfer function block diagram. If you look at what goes into a electro hydraulic speed governing system, electro hydraulic speed governing system means, that all the control systems are implemented using usually digital electronics these days; also the sensing, etcetera is done using electrical sensors, in some sense **you know** you convert all the speed, etcetera to electrical values, then sample them and feed them to a digital control system.

So, this is an what **what** you mean by an electronic governing system, the word hydraulic comes because although the electronic governing system can tell you what the valve position should be, it has to be eventually implemented using the hydraulic system, because the force level, the force required to move the valve is quite high. So, you do not have any electrical system strictly speaking, you have got a hydraulic **you know** system

which actually moves the valves. So, the electrical governor just gives you the set point which then is implemented by the hydraulic actuators.

Now, if you look at a speed governing system, it actually looks quite complicated, in the sense that you have got speed transistor which meant measure the speed, compares it with the speed reference then you have got a gain, it is also called droop. The gain is in fact the **the** reciprocal of the droop, then you, this is combined with the load reference, that is the load set point which in fact, then tells you what to do regarding the position of the valve, and then it is implemented by hydraulic systems. And finally, it is implemented as a valve position which gives you a mechanical power output of the machine.

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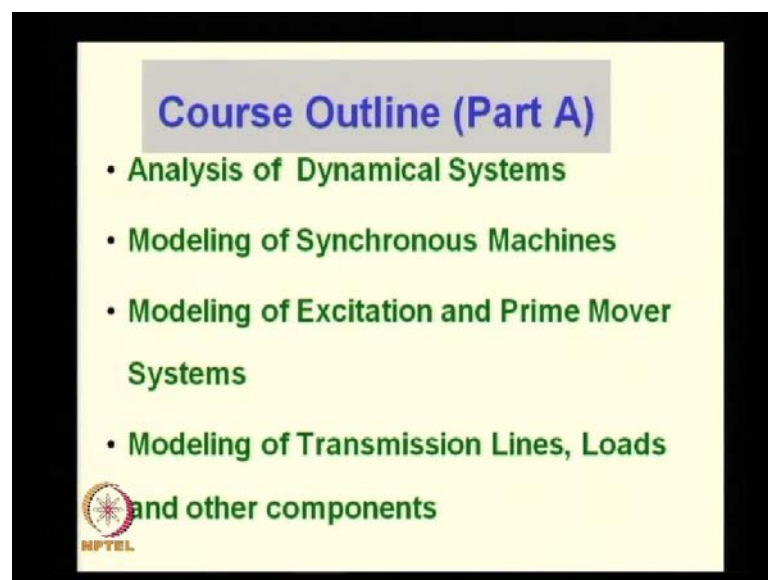
Now, one thing which you should remember, of course in the steam turbine system is that, just by changing by valve position in fact, one cannot in a consistent way, continuous way change the mechanical power output.

In case you want to change the mechanical power output, you should also change the fuel input and feed air and feed water into the boiler, you for example, in a hydro turbine by changing the gate position, in fact you directly change the mechanical power. But in the steam turbine, by changing the valve position, you actually change the flow, **the flow will** change in flow will result in change in pressures; the pressure has to be maintained by changing the fuel and air and feed water into the boiler.

So, in fact just changing the valve position, cannot on a **on a** sustained basis change the mechanical power output. So, what you need to do is, of course the turbine controls or rather the valves controls have to be coordinated with the boiler controls, so this is what this diagram is trying to show you.

So, the boiler control, the boiler in fact controls the pressure and the flow, and turbine valve controls eventually the mechanical power. So, the point is rather I should put it this way, the both the pressure, steam flow rate and the valve position will determine eventually the mechanical power output of the turbine, so the if you look at the block diagram, it will look like this.

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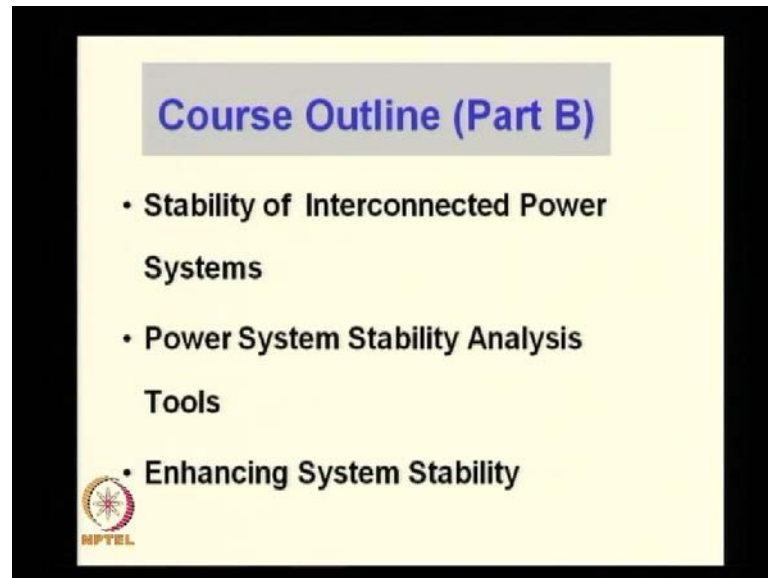
We now move on to **you know** try to understand the behaviour of an integrated power system, so far let us just look at what we have done. We have actually in the beginning of this course, in the **in the** first few, first ten lectures, analyzed dynamical systems.

We tried to analyze dynamic systems in general, we learned tools of Eigen analysis as well as numerical integration, then we took off an rather large component of a course, in fact, it has taken almost 25 lectures, that is relating to the modeling of synchronous machines, excitations, and primo systems, and also primo transmission lines, and loads.

Now, the other components we will not really do in much detailed in this course, when I talk about improving power system stability, I will introduced to **to** you few other

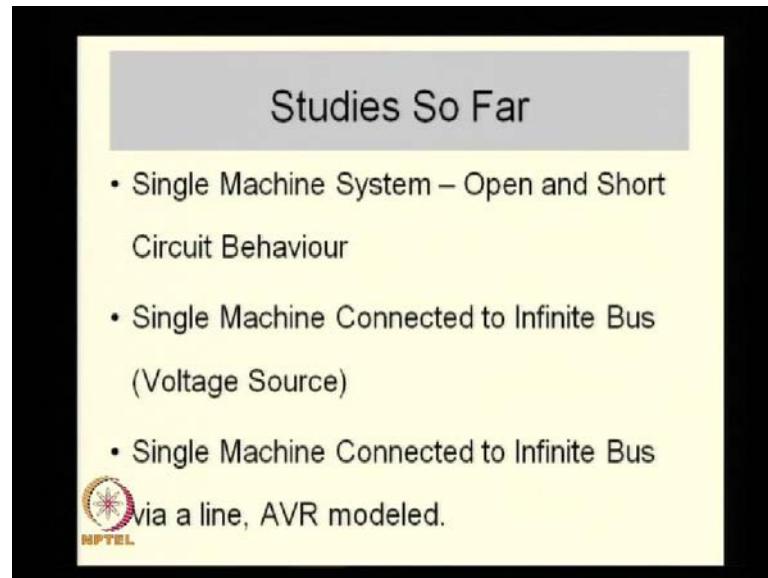
components. Now, what we have got right now is the system model, in fact we have seen them, we have even done certain studies using the models at various stages in our modelling itself.

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Now, what remains to be done in the last part of this course is, understanding the stability of the interconnected power systems. We will just have a quick look at some of the common power systems stability tools which are available to you, and some of the methods of enhancing system stability. So, this is what really remains; now luckily in our course, while we were doing modeling, in fact we have done a bit of analysis as well, what have we actually analyzed?

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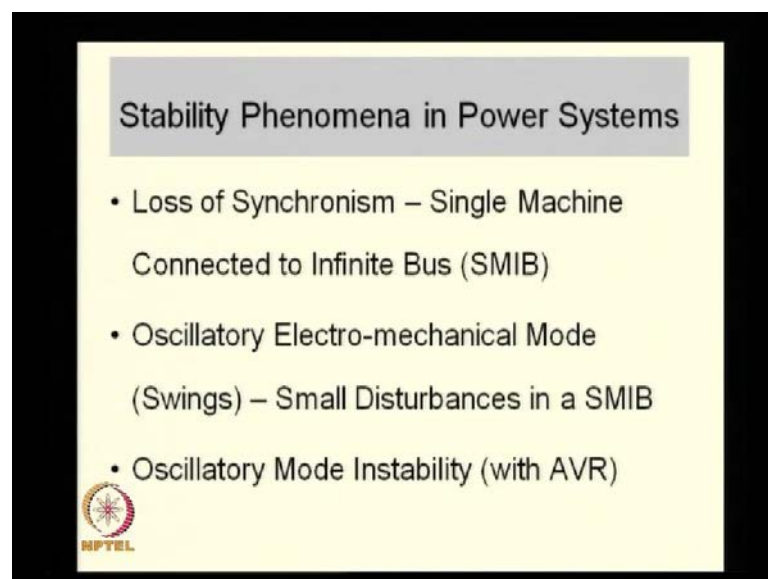
Studies So Far

- Single Machine System – Open and Short Circuit Behaviour
- Single Machine Connected to Infinite Bus (Voltage Source)
- Single Machine Connected to Infinite Bus via a line, AVR modeled.

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The studies which we have done so far are the single machine in system, we have done an open circuit and short circuit behaviour study, and we have also done a study of a single machine connected to an infinite bus which is nothing. Infinite bus is of course,, is a stiff voltage source whose frequency, phase angle, and voltage magnitude is not changing. We then went on to actually do a single machine connected to an infinite bus with the AVR model, the excitation system model, the automatic voltage regulation of a AVR is also modeled in this kind of study.

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Stability Phenomena in Power Systems

- Loss of Synchronism – Single Machine Connected to Infinite Bus (SMIB)
- Oscillatory Electro-mechanical Mode (Swings) – Small Disturbances in a SMIB
- Oscillatory Mode Instability (with AVR)

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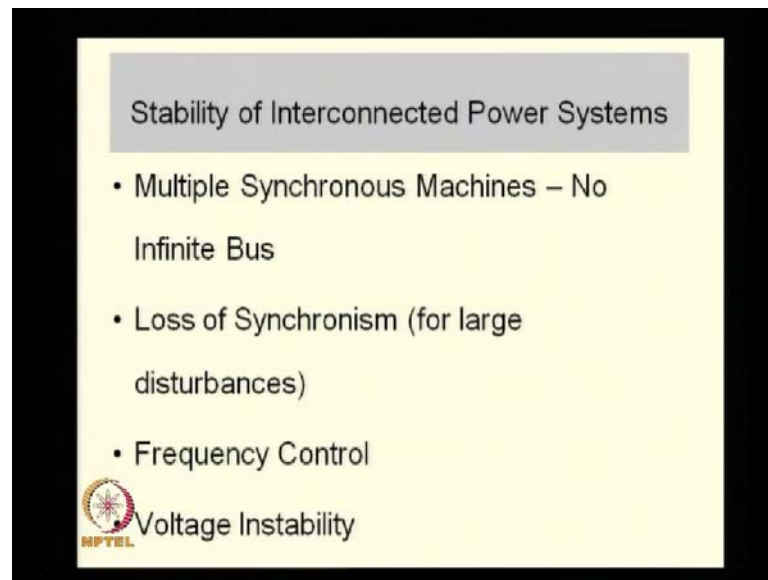
So, this in fact, brought out certain phenomena, first thing was of course, when we did a machine connected to a voltage source, we see phenomena called loss of synchronism. So, if you give a disturbance to a system, you may if the angular deviation is very large, you may find that the system losses synchronism. So, this is something which is seen in a single machine infinite bus system, this is something which we saw. We also saw that, when we connected synchronous machine to an infinite bus, we call it as single machine infinite bus system, one of the modes or one of the patterns which we see, especially in the rotor angle in this speeds are one of the patterns is oscillatory and usually that oscillation damps out.

So, if we give a small disturbance, we see an oscillation in the response, especially of delta and omega. And we also see it in power; it is observable in many states. Now, this oscillation is roughly, typically around 1 hertz, it is between 0.5, 1, 2, hertz **you know** in that range in a real power system; in fact, some of these phenomenon actually seen in larger power systems as well. So, the fact that we did single machine, the kind of an academic study, it does not detract from the fact that, in fact some of these phenomena are seen in the real power system. Now, one more interesting fact which we saw was oscillatory mode can become unstable with certain control systems like an AVR.

So, we did see in the last part of the study of an AVR, when we did the simulation study that for certain operating conditions, if we give a small disturbances you get an oscillatory instability, the system does not settle down back to the equilibrium point. This was the small disturbance phenomena in the sense that, it could be predicted from Eigen analysis of a linearised system around an equilibrium point. So, now what we are going to do is a big grand mixture of things, we are going to now **you know** remaining part of the course study, some important stability phenomenon.


In fact, the major hassle of modeling a synchronous machine, etcetera is been kind of done with, now we will use these models and make an integrated power system. So, what will effectively do when I say integrated power system, in fact single machine infinite bus is just integrated power system. But what we will do is, we will try to do a more realistic study where we do not have a infinite bus or voltage source, but you got many synchronous machines connected to other.

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Stability of Interconnected Power Systems

- Multiple Synchronous Machines – No Infinite Bus
- Loss of Synchronism (for large disturbances)
- Frequency Control

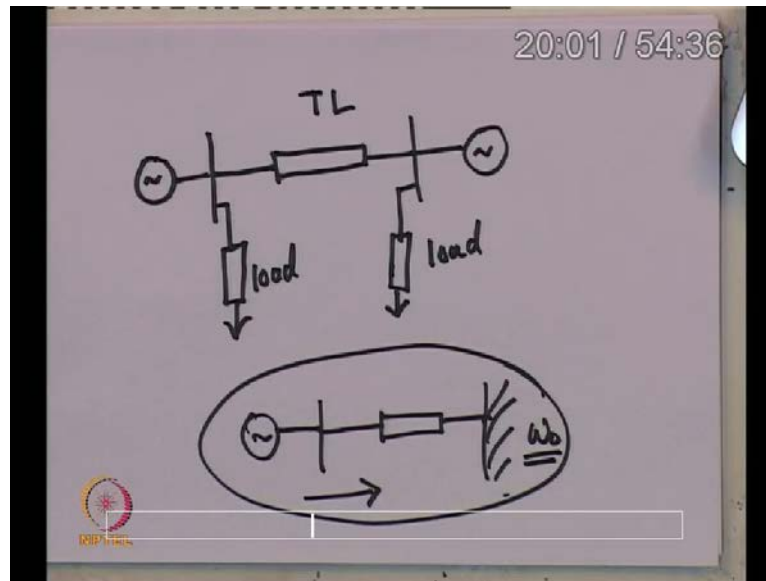
 Voltage Instability

So, we are going to do a multiple synchronous machines kind of study. And what we are going to do is just check out, whether the loss of synchronism is something we can see or whether we can simulate this.

Remember, loss of synchronism, of course is happens when there are large deviations due to large disturbances. So, what we are going to see is whether we can have, we can show loss of synchronism for multiple synchronous machines as well. One of the things which of course, we have to do just after our study of speed governor and turbines systems is speed frequency control, how do we control the frequency in our power system. And the last important phenomena which we have not really spent a lot of time or we have not really talked about it much is relating to voltage in stability.

So, these are phenomenon which could like to discuss in our next part of the course which is on stability of interconnected power systems. So, a part of, of course even this has been covered in our modeling, when we considered single machine infinite bus system with some components like AVR, etcetera model. So, we have to just go on and graduate to a somewhat larger system. Now, the system which can really tell you a bit about frequency control as well as **you know** relative angle phenomena like loss of synchronism is a two machine system. So, our next job, so to speak could be to study a system of this kind.

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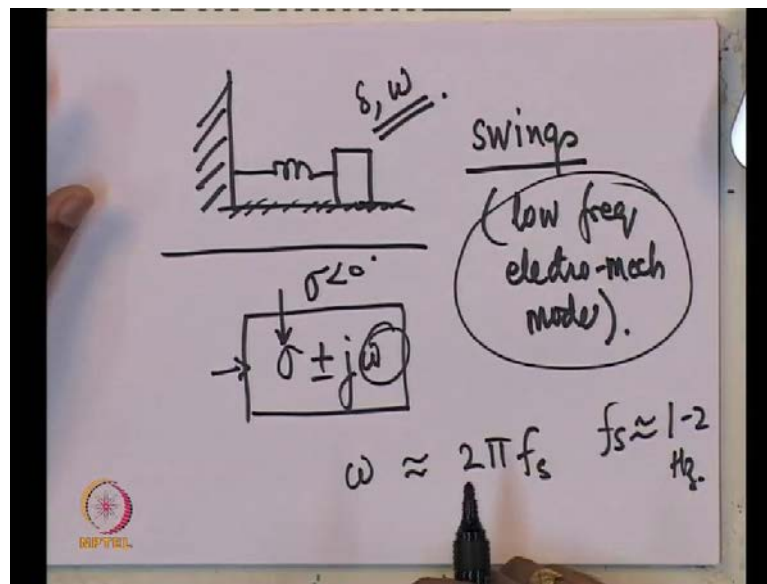
So, let us we will do the simulation in the next class, two machines, these are two synchronous machines, these are not voltage sources, connected to each other by a transmission line each having its local load. So, this is a kind of, we graduate from a single machine connected to an infinite bus to a two machine system with its individual loads at two buses is a transmission line which allows exchange of power between the two systems. Now, this system is actually different from having a single **single single** machine connected to an infinite bus, what is the difference?

The first and foremost difference is that, the frequency of this system will settle down to what's value is something which we have to investigate, it really depends on the control systems employed to control the frequency. In fact, in a single machine infinite bus, we did not have to worry about controlling the frequency, we really the synchronous, or rather the infinite bus, the synchronous machine; if it were in synchronism with the infinite bus, the infinite bus frequency itself was the steady state frequency. We could in fact, push in power into this infinite bus which the infinite bus happily was able to absorb. In a single two machine system with a load, a natural question would be that, if for example, mechanical power were more than the total load in this system, what could happen?

Now, the frequency obviously will change of the synchronous machine, the speed of the synchronous machines will change, but how do we really in our minds in some way

distinguish between relative motion, and the overall motion of the both the machines. In fact, it turns out that everything comes out quiet neatly. Now, before we go on and try to **you know** I will tell you the steps to analyze this kind of system in a **single machine** two machine system with its load with an AVR, with governor, turbine governor model. Let us just think of a kind of an equivalent a very crewed analogy, a crewed analogy for a two **two** machine system are first of all single machine infinite bus system.

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The analogy of a single machine infinite bus system as far as the just the electromechanical mode is concerned, when you are talking only of what are known as swings, which are nothing but the low frequency electromechanical mode. If you are studying swings in a single machine infinite bus system, you can use this kind of very very crewed analogy of a single machine infinite bus system to study the low frequency electro mechanical modes. Note that, this analogy which I am giving you is not going to give you correct numerical results or anything of that kind.

It is just telling you roughly, how the electro mechanical mode behaves, or what really this is just an analogy which gives you behaviour similar to the electro mechanical mode in a single machine infinite bus system. If you recall, what do I mean by electromechanical mode, **well** if you give a single machine infinite bus system a disturbance, you are going to get several modes, many of these modes are going to be associated with the damper winding and field winding flux.

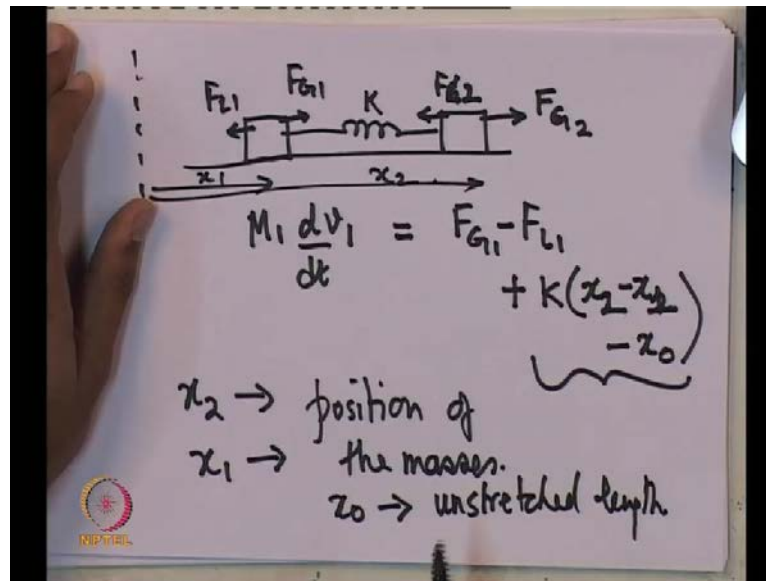
It is a coupled system, remember you cannot associate a mode with a certain state variable directly, but in addition to the modes associated with the changes in the fluxes of the machine, there is an electro mechanical mode. If you recall we had got a low frequency electro mechanical mode which is manifest as Eigen value in this small signal linearised model of the system. The Eigen value had an omega of roughly, if you recall the studies which we did of roughly 2π into f swing where f swing was roughly 1 to 2 hertz.

So, we have actually come across this mode in our study of the AVR, if you are you can go back to those lecturers and just have a look at the kind of Eigen values we got. Now, this particular Eigen value which you call the swing mode λ is actually seen or observed mainly in the electro mechanical variables delta and omega. Now, delta and omega, in fact are relating to the rotor position and a rotor speed.

Now, if you look at this analogy, here also you have got an oscillatory mode in this system. In fact, if you give this masses small push, you are going to get an oscillation and of course, if there is a some friction in this flour here, you going to σ get a damped oscillatory response, that is sigma will come out to be less than 0. So, this is basically how you can expect the electro mechanical mode to behave in a single machine infinite bus system. So, it is a crewed analogy, do not read too much into it.

For example, this kind of model of or analogy of this system will not tell you about the λ of AVR behaviour of the field flux behaviour. So, please remember that, this is a very crewed analogy, now if you go on and look at two machine system and you just talk about the electro mechanical behaviour of two machine system.

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In fact, you can understand it quite well by a system of two masses connected by a spring. Now, of course, the spring has got say a K , kind of a stiffness constant of K in Newton per meter, and you may have external forces like F_{G2} and F_{L2} and F_{G1} and F_{L1} on this mass. And if you write down the equations of this system, for the first mass you have got $\frac{d}{dt}$, $\frac{d}{dt}$ velocity of the first mass is equal to F_{G1} minus F_{L1} minus K times x_1 minus x_2 , rather maybe it is a better way to write this is x_1 minus x_2 minus x_0 , so you will have plus here minus x_0 .

So, this is nothing but in fact, it is K times the stretch of the spring. So, x_1 and x_2 of course, are the position of the masses. So, x_1 and x_2 are the positions of the masses, so you take some reference position and see x_1 and x_2 , position x_1 and x_2 . So, if you look at the equations, they look like this, x_0 is of course is the un stretched length of the spring of the spring. So, actually this is one of the equations, so I will just write down all the equations one by one.

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The image shows a whiteboard with handwritten equations for a two-mass spring system. The equations are:

$$\frac{dx_1}{dt} = v_1$$
$$\frac{dx_2}{dt} = v_2$$
$$M_1 \frac{dv_1}{dt} = (F_{G1} - F_{L1}) + K(x_2 - x_1 - x_0)$$
$$M_2 \frac{dv_2}{dt} = (F_{G2} - F_{L2}) - K(x_2 - x_1 - x_0)$$

A small logo is visible in the bottom left corner of the whiteboard.

So, the correct way of writing the equations would be $\frac{dx_1}{dt}$ is equal to v_1 , $\frac{dx_2}{dt}$ is equal to v_2 , $M_1 \frac{dv_1}{dt}$ is equal to $F_{G1} - F_{L1} + K(x_2 - x_1 - x_0)$ which is the pull of the spring, and $M_2 \frac{dv_2}{dt}$ is equal to $F_{G2} - F_{L2} - K(x_2 - x_1 - x_0)$, that is if the string spring stretched by the second mass, you will find out the force tends to pull it back.

So, this is the kind of equations you will get for a two mass spring system. So, this is what I am trying to sell as an analogy of a two machine system, only to highlight how the electro mechanical modes look like. Now, we can do Eigen value, this is these are inputs and we can do an Eigen value analysis of this system. Now, the interesting thing about this system is that, if you do an Eigen analysis, you will find that in the absence of any friction, a very interesting thing you can see is maybe I, what I will do is write down the a matrix for this system.

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$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{M_1} & +\frac{K}{M_1} & 0 & 0 \\ \frac{K}{M_2} & -\frac{K}{M_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ x \\ x \end{bmatrix}$$

L.A.

So, $\frac{dx_1}{dt}$, $\frac{dx_2}{dt}$, $\frac{dv_1}{dt}$ and $\frac{dv_2}{dt}$ will be a times x_1 , x_2 , v_1 , v_2 plus some inputs. So, I will just write this is inputs, inputs in fact for, x_1 , x_2 are 0 and here you have got something which is related to $F G 1$, $F G 1$ $F L 1$ divided by M_1 and x . And also $F G 1$, $F G 2$, $F L 2$ and x_0 , so these are the inputs in this system, a matrix will look like this 0 0, in case **in case** there is no damping, you will find that.

And you will find here that it is **minus K** **K oops** minus K plus K K K minus K divided by M_1 , divided by M_1 , divided by M_2 , now this matrix is singular (Refer Slide Time :29:00). In fact, if a matrix is singular, we can show that it is got at least 1 0 Eigen value, so you are taking out the modes of the system.

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$$\lambda_{1,2} = \pm j \sqrt{\frac{K}{M_{eq}}}$$
$$M_{eq} = \frac{M_1 M_2}{M_1 + M_2}$$
$$\lambda_{3,4} = 0, 0$$

e^{0t} $t e^{0t}$

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You can just try this out as an exercise, you try to take out the modes of the systems, you will find that the modes of the system are the first Eigen value. In fact, the first two Eigen values you can get as a complex pair **you know**. So, your response is going to be having in oscillation, and in case there is no friction of course, it will be a un damped oscillation, this is j into root K by M equivalent, this is something you can work out, this M equivalent is nothing but M 1 M 2 upon M 1 plus M 2. So, you have to actually take out the Eigen values of this particular matrix, it is not very difficult to do, you just try it out as an exercise.

And the third and the fourth Eigen values are in fact repeated, that two 0 Eigen values in this system. Now, what we expect is of course an oscillation and two 0 Eigen values will of course mean, that there will terms like e raise to 0 t in the response. Now, one more thing which you can try to verify is that, the fact that they are too non distinct Eigen values, 0 and 0 implies in this case that in this particular case remember, it implies that you do not have linearly independent Eigen vectors, you do not have four linearly independent Eigen vectors corresponding to these four Eigen values.

And as we have discussed in our first few lectures, **you know** first ten lectures of this course, in such a case you are going to get terms like t e raise to 0 t also in the response. So, you have going to get oscillatory response plus terms of this kind, e raise to 0 t, of course is nothing but 1. So, if you look at it from purely mathematical perspective, you

can actually this particular two mass spring system, and you will find that it is got an oscillatory mode.

And you have got these two 0 Eigen values and which in fact, result in the motion of the motion of the center of inertia of this system. If you look at the Eigen vector, the right Eigen vector corresponding to the complex pair of Eigen values, you will notice that this something you need to really take out and leave that as an exercise to you.

You will find that the Eigen vector components are such that, when one of the mass, if you look at a oscillatory Eigen value, if one of the mass is moving this way, the other mass is moving in the opposite way. So, if this something which you can look into the at, you can infer from the Eigen vector components. So, you will find that when you look at the oscillatory mode, it effectively involves the two masses swinging against each other which is not a surprise, if you give a push to the spring, how do you get this kind of response.

Now, if you give a push to the spring, another kind of responses is also possible which involves the motion of the mass, both the masses together. In fact, that motion is not oscillatory; it depends on how much force you have given it. So, if you kind of given asymmetric disturbance, it is going to cause this; if you move give, if both these masses have got an initial speed which is equal, you will find that will keep on moving. So, displacement of both masses will just increase with time if there is no friction.

So, you will find that the system has two modes in fact, one is in motion, relative motion and the other is the center of inertia motion that is the center of mass motion, where both the masses move together. And that motion in fact, if is not damped, both the motions are not damped; the relative motion is not damped if you do not have any friction.

And also in case both the masses have equal initial velocity, you will find that this mode is excited and it will just go on moving, since there is no friction. So, that is that explains the presence of this t term in the response. So, if you do an Eigen analysis of this, this is what the kind of response, this is the kind of the response you will infer and in fact, kind of is consisted with our intuition.

Shall we expect that the system is going to behave like this, now we done one analysis even before, but I will just repeat it; because it is relevant at this point of time.

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$$\frac{d(x_1 - x_2)}{dt} = v_1 - v_2$$
$$\frac{d(v_1 - v_2)}{dt} = -K(x_1 - x_2) + \text{(other terms)}$$

$F_{g1}, F_{L1}, F_{a2}, F_{f2}$

That if you look at $d(x_1 - x_2)/dt$, it is equal to $v_1 - v_2$, and $d(v_1 - v_2)/dt$ is equal to what you will get is which are in terms of $F_{g1}, F_{L1}, F_{g2}, F_{L2}$. So, this is obtained by simply adding these two equations (Refer Slide Time: 34:10). So, just remember that if I just look at the difference of x_1 and x_2 and v_1 and v_2 , you get what look like simply like our **you know** single mass connected to infinite wall kind of equation, these also equations of a simple harmonic oscillator, a force simple harmonic oscillator. So, the difference **you know** the relative motion is oscillatory, because it follows the motion, simple harmonic motion.

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$$\frac{d(x_{com})}{dt} = v_{com}$$
$$\frac{d(v_{com})}{dt} = \frac{F_{g1} - F_{L1} + F_{g2} - F_{L2}}{(M_1 + M_2)}$$
$$x_{com} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} \quad v_{com} = \frac{M_1 v_1 + M_2 v_2}{M_1 + M_2}$$

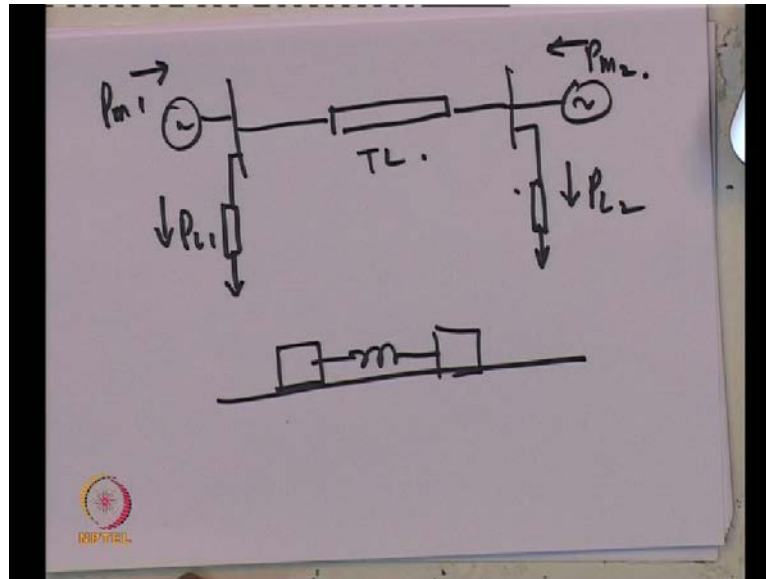
And interestingly, If you look at $\frac{dx_{cm}}{dt}$, I will tell you what x_{cm} means, this is obtained by simply adding of this and this equation and this and this equation (Refer Slide Time :35:31). So, you add up these two equations, you will get, so these are in fact the center of mass variables (Refer Slide Time: 35:40) (No audio from 35:40 to 36:16). So, if you look at just the center of mass motion, the center of mass velocity, the rate of change of the center of mass velocity is dependent just on these external forces F_{G1} , F_{L1} , F_{G2} and F_{L2} . In fact, it is equal to the sum of the forces in one direction that is F_{G1} plus F_{G2} minus the forces in the opposite direction F_{L1} minus F_{L2} .

And so, the center of mass, in fact motion in this particular case does not seem to be dependent on, it just depends on what is the balance of the external force is. Of course, some of these forces, if they are functions of velocity or **you know** individual velocities or individual displacements. In such a case, **you know** when we coupling between the center of inertia motion and the difference motion. But in case, F_{G1} , F_{L1} , F_{G2} , F_{L2} is are constants, you will find that the center of inertia motion is completely decoupled from the relative motion.

But of course, it need not be true that F_{L1} , F_{L2} are actually or F_{G1} and **FL** F_{G2} are absolutely decoupled or not dependent on the velocities, individual masses, and velocities, and displacements. So, this decoupling in practice may not be true, but roughly speaking, **you know** you will find that the center of inertia motion, the center of mass motion in this two mass spring system can be at least mentally we can decouple, **you know** roughly decouple rather I should say, it is a roughly decoupled from the relative motion.

So, you can look at these two modes in some ways independently, **you know** you can look at the relative motion and the center of mass motion. Now, what really do I want really to how this all related to power systems, the point is that one of the basic understand the electromechanical modes in a power system is by looking in this analogy. If you have got say two mass, two generators is connected to the individual load, so this something I do some time back.

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In a very very crude fashion, we can these are the loads, this is the transmission line, in a very very crude fashion we can understand one pattern of this behaviour, the electromechanical mode of this system, there are many **many** more modes, because your machines remember our model by many mode states, corresponding to fluxes and other control systems as well.

It is equivalent to having the electromechanical motion can be intuitively understood by this analogy. So, what I really wanted to say is that, the electromechanical motion of these two synchronous generators is not very different from you will expect out of a two mass spring system. What you will find is that if the disturbances, you will find that the synchronous machines tend to oscillate against one another, in the single machine infinite bus case, the machines are oscillating, the angle delta was with respect to the infinite bus voltages.

Here, you are what you are seeing is that, the machines oscillate, there is a relative motion which is oscillatory in this two machine system, this something I have not proved, I will be actually showing you a simulation. And later on, we will also do Eigen value analysis of multi machine system; where I will actually prove that it is true that you are actually getting this kind of behaviour as far as a electromechanical motion is concerned, you also have other modes, remember. But the electromechanical modes you will find that they are oscillatory modes, there is a relative motion between the machines,

you also have a center of inertia, **you know** motion in which the machine, both the machines may accelerate together or decelerate together.

And that really is determined by the load generation imbalance. So, instead of what we talked of $F_G 1$ and $F_L 1$ in this context, they are the mechanical power input to this machine and the loads. So, what we have here is two kinds of motion, one is the motion of the center of inertia of this system, and one is the relative motion of the system. The moment of the center of inertia if it depends on the total load generation balance, this is what really comes out of this analogy that is why I actually mentioned it, depends on the load generation balance. In fact, if there is a load generation imbalance, the center of mass of the system and the center of inertia of this system will keep on changing.

Now, the important thing of course is, if I make $F_G 1$, $F_L 1$, $F_G 2$, $F_L 2$ functions of the center of mass speed, then one could in fact get some kind of control over this center of mass motion. So, the center of mass motion could be changed by changing $P_m 1$ or $P_m 2$ that is using governing systems. By using governing system in fact, we can make $P_m 1$ and $P_m 2$ functions of the speed and therefore, control the center of mass motion or the center of inertia motion. Similarly, if $P_L 1$ and $P_L 2$ are loads which have functions of frequency, there in also we find some **some** kind of leverage over the center of mass, center of inertia motion.

So, the important thing to be noted is we do have relative motion, but when you come to two machine system without a voltage source, the overall frequency or the center of inertia frequency will be dependent on the load generation balance, this is not true in a single machine infinite bus system, because the single machine infinite bus system has an infinite bus which maintains the frequency constant, in fact it is kept constant. So, what we require for good operation or normal operation of the system is that, relative motion should be stable that is, there is no relative motion in steady state in synchronous machine is connected to each other.

If the relative motion is unstable, then you will find that the power flow will keep on jumping around, a phenomena which we discussed right in the beginning of this course. Load generation imbalance is required to be maintained **load generation balance is required to be maintained**, otherwise the generators will not run at an acceptable speed, you may have the speed continuously changing or in case we have got governors or load

frequency dependence, your frequency will settle down some value, but it will become unacceptable at times.

So, center of mass frequency also has to be controlled, now let me give you an example. If there is a disturbance, you will find that the relative motion between generators, say in India you have got a synchronous grade and say the machines near Mumbai **you know**; let like Trombay or running in steady state in synchronous in synchronism with machine, say in Arunachal Pradesh which is more than 2000 kilometers away, they are running in synchronism.

Now, if there is a sudden load generation imbalance, that is some load gets switched off, you will find that the center or you may excite some relative motion, and you will also find that **you know** the center of inertia is changing, now the center of inertia frequency will change, keep changing if there is a load generation imbalance unless steps are taken to bring back a load generation balance.

The two mechanisms for doing that, one is changing the turbine mechanical power, or hoping and praying that the load frequency dependence will get into a nice equilibrium. For example, if it is a small load change, a very small load change takes place in this system, and then what happens the frequency the center of inertia frequency, the common frequency of the system changes.

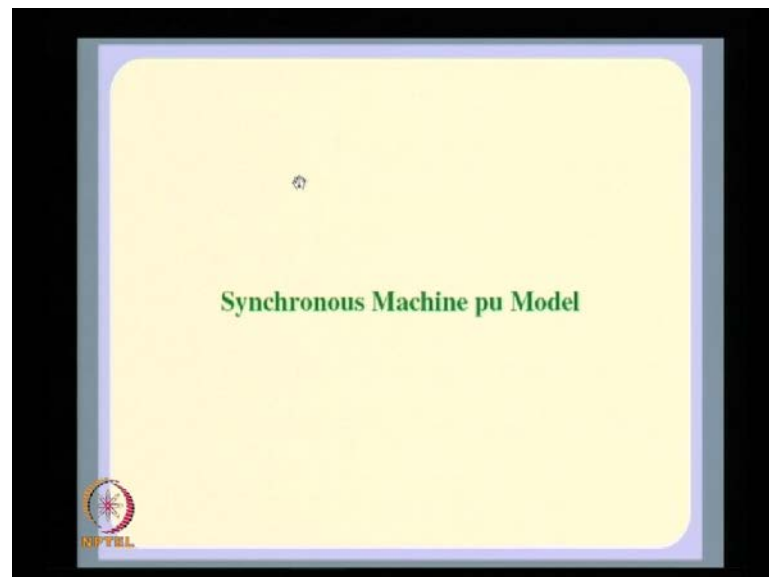
Now, when the frequency changes the load it themselves change, for example rotating load if the frequency falls, they will rotate slower and draw less of power. So, you may reach an equilibrium, you may also reach an equilibrium because of governor control, you are changing the mechanical power itself. So, remember that the center of inertia motion is controlled by the load generation imbalance, relative motion is caused due to asymmetric disturbances in this system like faults on some bus, and you may find that relative motion is excited.

So, this is what really we want to show you. So, this is a kind of curtain raiser to the simulation I am going to try to show you tomorrow. Now, there are one or two important issues when I am trying to go, when I am going to simulate this kind of system, after roll I am going to show you a simulation of two machine system. So, tomorrow if you want to really simulate such a system, a two machine system, what are the issues.

Now, we have done a single machine infinite bus system, you may ask what **what is what is** more is to be told, I mean you have already done the model of an AVR, you have also done the model of a synchronous machine, we have just done the models of a turbine generator which tell you what the mechanical power is.

So, you can write down all the dynamical equations and simulate them, now one of the important thing is how do you interface the equations of various synchronous machines. Now, what is the issue here, the point is that all the synchronous machine models have been **you know** formulated in what is known as Park's reference frame.

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So, let us just look at the synchronous machine model, we have already done this **(O)**, but it is a good idea to keep reminding our self of the equation.


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q-axis Model - per-unit

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'} \psi_K + \frac{(x_q - x_q') x_q''}{x_q x_q'} \psi_G$$

$$\frac{d\psi_q}{dt} = \omega \psi_d - \omega_B R_a i_q - \omega_B v_q$$


So, we will just see them, we will see them again. So, this is the q axis model, there are flux differential equations, remember that the two mass spring analogy which I showed you of a power system subsumes, rather it just neglects any other dynamic. So, you are getting an analogy only of a particular pattern in the motion, remember that whenever you are studying a detailed synchronous machine model, you will get many **many** more mode. So, I **I** cannot, I just cannot over emphasize this point.


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d-axis Model - in pu (assuming $T_{dc}'' = T_d''$)

$$\frac{d\psi_H}{dt} = \frac{1}{T_d''}(-\psi_H + \psi_d)$$

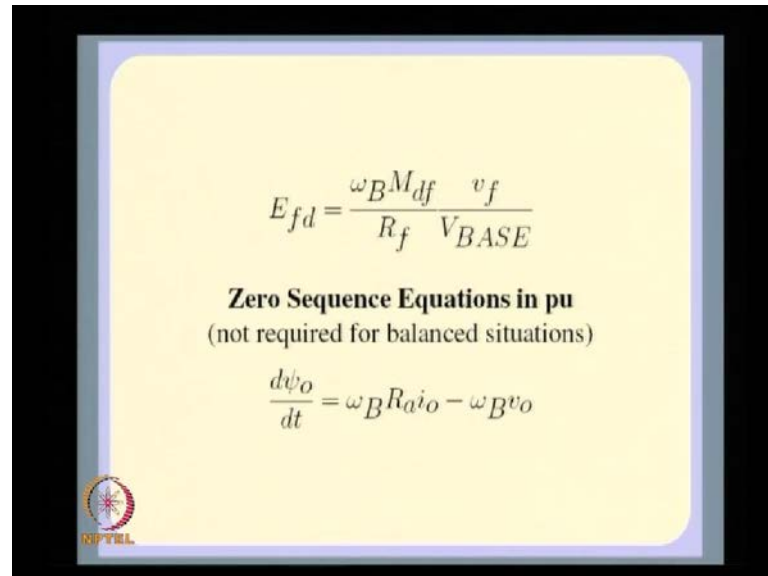
$$\frac{d\psi_F}{dt} = \frac{1}{T_d'}(-\psi_F + \psi_d + \frac{x_d'}{(x_d - x_d')} E_{fd})$$

$$\psi_d = x_d'' i_d + \frac{(x_d' - x_d'')}{x_d'} \psi_H + \frac{(x_d - x_d') x_d''}{x_d x_d'} \psi_F$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - \omega_B R_a i_d - \omega_B v_d$$


So, this d axis model, remember that ψ_d ψ_q i_d i_q which appear in this equations and v_d v_q are obtained using.


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$$E_{fd} = \frac{\omega_B M_{df} v_f}{R_f V_{BASE}}$$

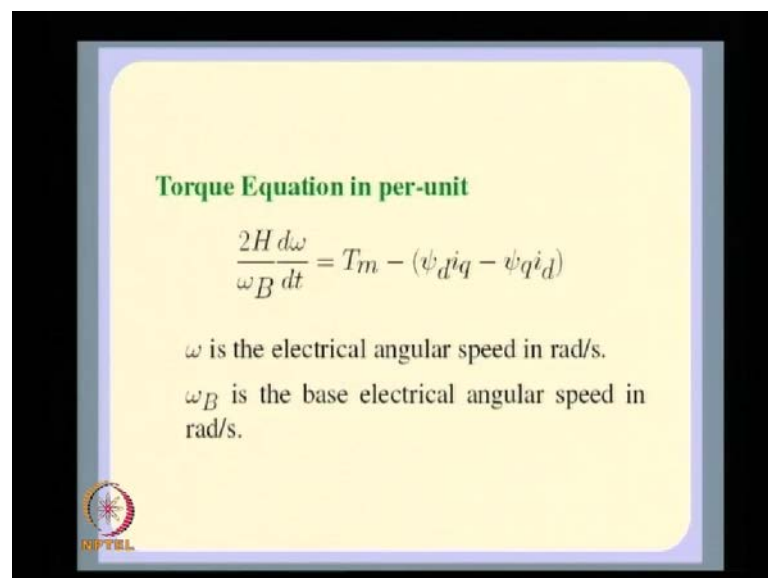
Zero Sequence Equations in pu
 (not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$



Of course, before we go to that, **yes** E_{fd} the equation for E_{fd} .


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Torque Equation in per-unit

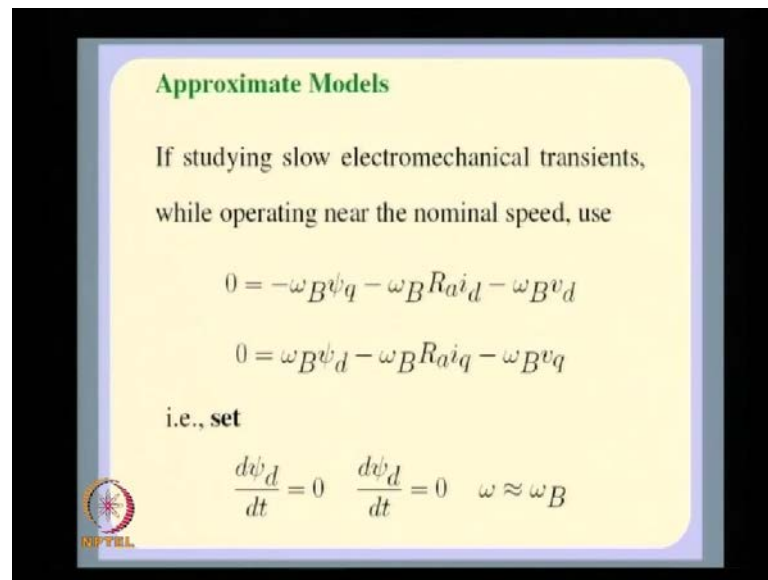
$$\frac{2H d\omega}{\omega_B dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.
 ω_B is the base electrical angular speed in rad/s.



And the torque equation which I in terms of again $\psi_d i_q$ minus $\psi_q i_d$.

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Approximate Models

If studying slow electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$
$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., set


$$\frac{d\psi_d}{dt} = 0 \quad \frac{d\psi_q}{dt} = 0 \quad \omega \approx \omega_B$$



Remember, this is an approximate model, in case you are studying slow electromechanical transients, you can set $\frac{d\psi_d}{dt}$ and $\frac{d\psi_q}{dt}$ in those equations, just set them equal to 0 and use the resulting algebraic equations.

So, if you recall, this is an approximation we have discussed before, the last equation here and the last equation here can be made into algebraic equations. So, this is an approximation we can make, because right now we are going to study the electromechanical transients, the phenomena we are going to try to study in this are the electromechanical transients, our focus is electromechanical transients and not the fast transients.

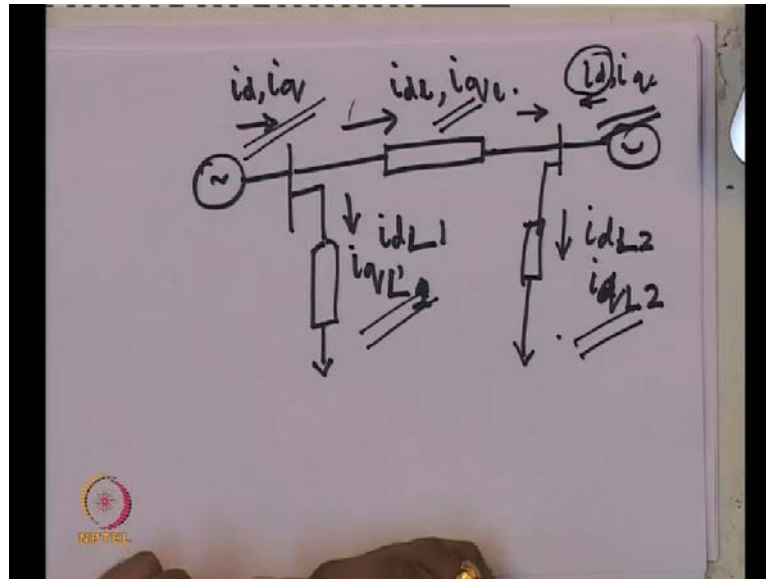
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$$[C_{P1}] = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \theta_1 & \sin \theta_1 & \sqrt{\frac{1}{2}} \\ \cos(\theta_1 - 2\pi/3) & \sin(\theta_1 - 2\pi/3) & \sqrt{\frac{1}{2}} \\ \cos(\theta_1 + 2\pi/3) & \sin(\theta_1 + 2\pi/3) & \sqrt{\frac{1}{2}} \end{bmatrix}$$
$$\theta_1 = \omega_1 t = \omega_o t + \delta_1$$

Remember, that whenever I say that ψ_d ψ_q i_d i_q and v_d v_q , we are using Park's reference frame and one of the things here which has to be noted is that, whenever you are using Park's reference frame, we are using the rotor position of that generator. So, θ_1 here refers to the position of one generator and the equations which I showed you for example, **you know** are in Park's reference frame. So, **for** the generator for which you have written down these equations, we have used a transformation where θ is the rotor position of that particular machine. So, θ_1 is $\omega_1 t$ plus δ_1 .

So, in fact in those equations, we should write ω_1 δ_1 wherever they appear, they refer to that specific machine. So, if you got two machines, in fact you will have C_{P2} and you will be using the transformation C_{P2} for that machine in order to get the equations in the d q frame of that machine, so the equations as they are formulated or using some kind of local frame of reference or local transformation.

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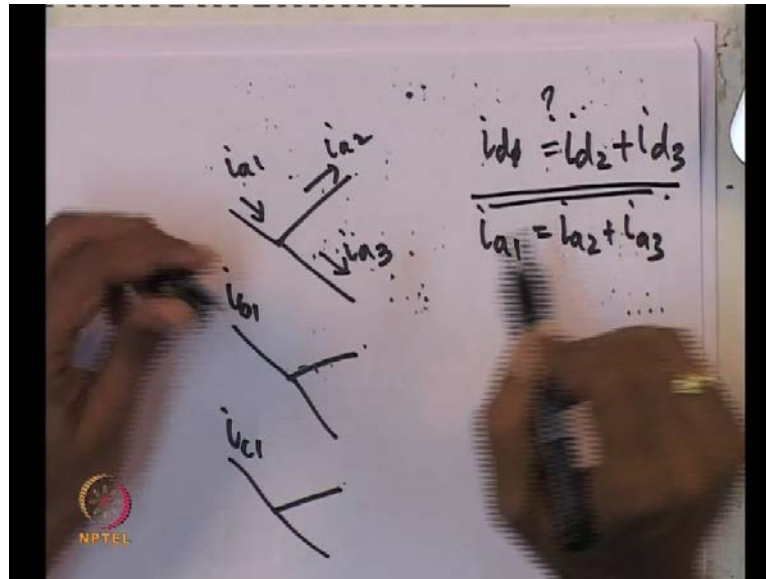


Now, in case you want to interface the two machines. What do I mean by interface is something I have to talk to you about. If I want to interface the two machines, for example the two synchronous machines are connected to a load. So, can we write, for example the equations like the current i_d i_q when you are applying **K** KCL, that is Kirchhoff's Current Law, this is we know that d q components of the current through the load, these are the d q components of the current through a transmission line.

And here you have got the d q components of the current through this load, I will call this the load 2, I will call this capital L 1, and capital L 1, this is capital L 2, this capital L 2. Now, can I say for example, can I use **KVL** and say that i_d the generator current here is equal to the generator current here plus the generator current here that is the question I want to ask. Similarly, can you say i_d here is equal to the transmission line current. So, rather it is equal to $i_d L 2$ minus i_d lower case 1, is it **is that true.**

The point is that you cannot apply KCL unless all these currents here are on a common reference frame, why is that so? That is something which we will chew upon and discuss in the next lecture, you have to get all the currents, the transformed currents to common reference frame before applying KCL, KCL that Kirchhoff's current law, for example on a bus at a bus says for an example $i_a 1$ plus $i_a 2$ plus $i_a 3$ is equal to 0. Suppose, you have three currents $i_a 1$, $i_a 2$, $i_a 3$, $i_a 3$ incident for the first phase a phase at a bus, then it is true that $i_a 1$ plus $i_a 2$ plus $i_a 3$ equal to 0.

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So, what I need to say is, so if you have got a three phase system, so you will have i_{a1} is equal to i_{a2} , hope so you will have i_{a1} is equal to if these are the directions, you will get according to KCL it is like this. Similarly, for b phase and c phase, so you will get similar equations for this, but does that mean that i_{d1} is i_{d2} in this case i_{d1} , this the transformed current using i_{a1} i_{d1} and i_{c1} , is it equal to i_{d2} plus i_{d3} , suppose where i_{d2} of course is the d component of the current through this branch.

So, this is the question I wish to ask you; the answer is **yes**, this is true provided the same transformation has been applied for these currents, the branch 3 current and branch 2 currents. If that is true, then of course, you can write this too. So, it is important what I wish to say, of course is important that a transformation which we use to transform the a b c to the d q frame, if it is common **if it is common** in that case, I can apply Kirchhoff's current law for the, for all the currents which are incident on a bus. But if the transformation of the a b c variables are using different transformations for the different branches, in that case you are not going to be able to use KCL directly.

So, this is something which you should keep in mind for the next lecture. So, in the next lecture, we will do a simulation of this two machine system with loads for changes in loads and so on. And hopefully, whatever I have said about the change in center of inertia, speed, and the **relative** relative motion will become very clear after **after** you look at that; so, on to the next lecture.