Power System Dynamics and Control
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Model No # 01
Lecture No # 33
Transmission Lines Prime Mover System

A transmission line is a power system component, which is a distributed parameter component, and spans, you know, the component essentially is, you know, spread over a large geographical distance. As a result of which, the first treatment of a transmission line typically involves the formulation of equations, in terms of partial differential equations. So, we did this last time. A transmission line is effectively modeled by these equations. They are also called telegrapher equations and these are partial, set of partial differential equations.

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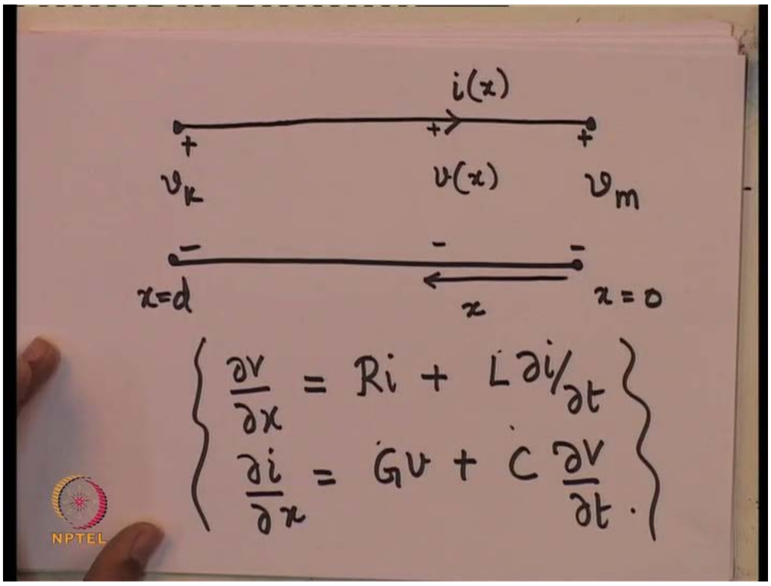

Now, if resistance and the conductance are neglected, I mean they assume to be 0, in such a case, they are normally quite small.

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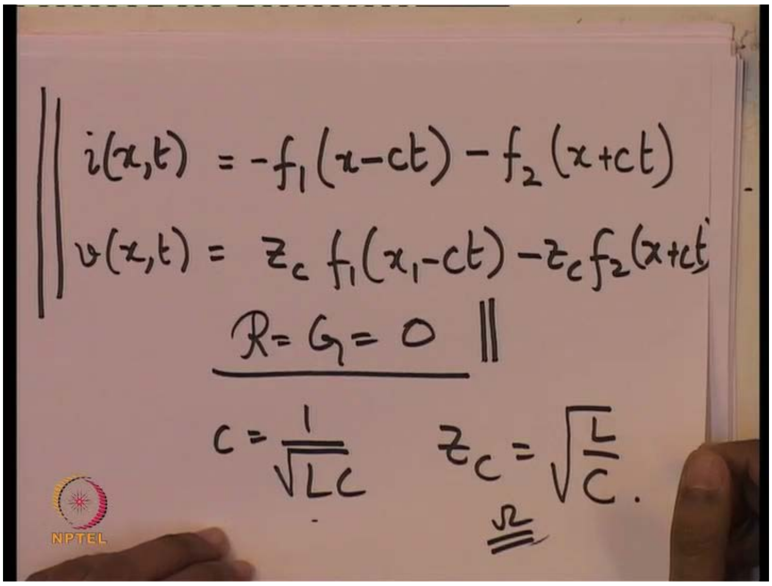

In that case, the response of the transmission line is given by the very well known traveling wave equation. This is of course, with the assumption that R and G are equal to 0. Since, small c here is the velocity of light, remember, the L and C here are the inductance and capacitance per unit length. Capital C, that is upper case C and upper case L are, in fact, the inductance and capacitance per unit length. So, this is one of the important equations, travelling wave equations.

Now, in this particular lecture, we will continue with our discussion of transmission lines. We were kind of poise at very interesting discussion in the previous lecture. We continue with that discussion and hopefully by the end of this lecture, we shall come to a fairly useable model of a transmission line, not only for high frequency studies but, also for lower frequency study, of lower frequency phenomena. Thereafter, we will go on to the study of prime movers. So, in this lecture, we will began with, so, will do the study of, we will continue our discussion of transmission lines. Sometime at the end of the course, of this lecture, we will also try to; I will try to introduce you to prime movers systems.

Now, one of the interesting discussions which we kind of left half way in the previous lecture was the model of a transmission line, the dynamical model of a transmission line. Of course, I have already given you a dynamical model but, it is in terms of partial differential equations. The solution, surprisingly for a last less case is very neat. The travelling wave equation looks very neat. Now, if you want to actually, you know, understand how a transmission lines behave, so you will have to use a partial differential equation solution.

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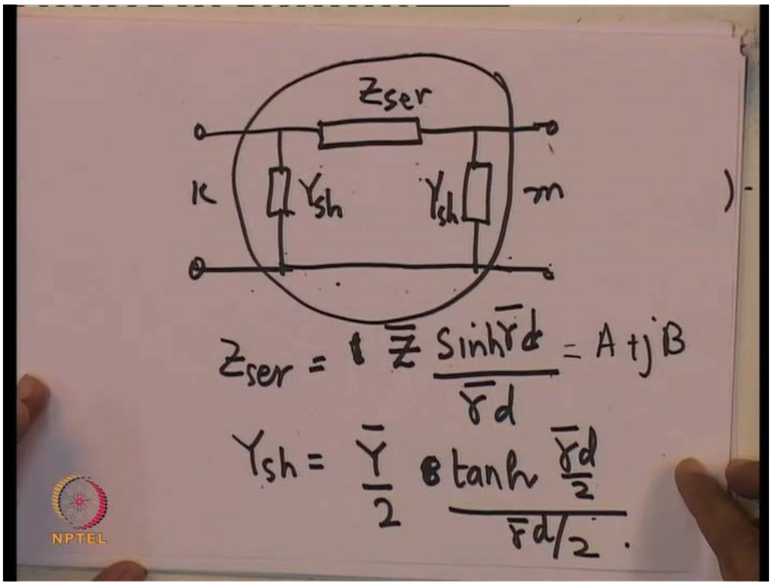

Now, we have in our under graduate years come across this model of a transmission line. This model is a pi equivalent of a transmission line, is a pi equivalent of a transmission line. But, it is very important to remember that, this equivalent using, in fact, lumped blocks of impedances susceptance or I should say impedances and admittance, are actually valid only for sinusoidal steady state conditions.

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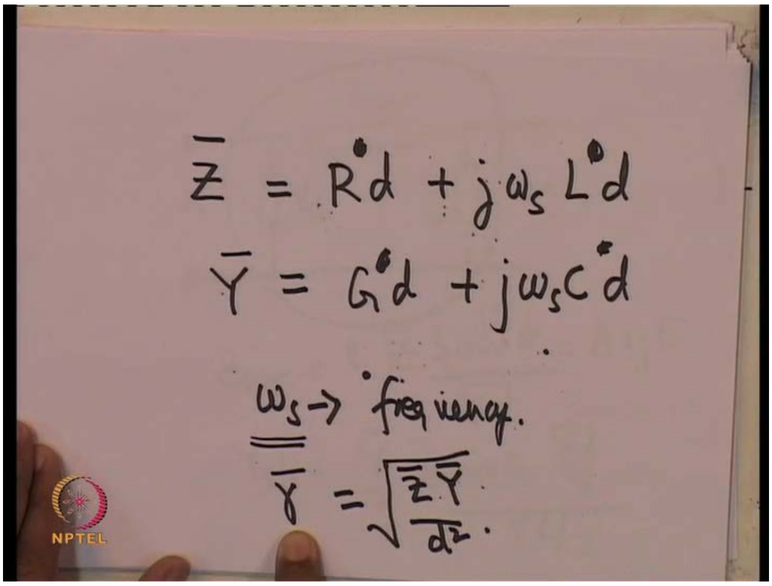

So, in fact, if you look at what z bar and Y bar mean in this equation, they are, in fact, impedance and admittance and what you have is gamma here. Gamma bar, in fact, is determined by this. So, remember that, frequency comes in this and distance also comes in this but, this is essentially a sinusoidal steady state model. As I mentioned in the previous class, we may be tempted to, you know, consider this model, as you know, correct for dynamical, for the study of dynamical phenomena as well. The answer is, strictly speaking, no. Remember that, this is a lumped parameter pi equivalent which tells you the correct terminal relationships in case of sinusoidal steady state. But, the correct solution for a loss less line, in fact, is given by these equations. Of course, in the case were R and G are not equal to 0, you would have to find ways of solving the partial differential equations. You will not get a neat solution like this, in case R and G are not equal to 0.

So, this is something which has to be made clear. Interestingly, the partial differential equation, the loss less case, the solution with the loss less case, in fact, tells us something about a transmission line which is very neat and nice. In fact, a transmission line using these, this solution, it can be shown that, if V k and V m are the instantaneous voltages at both end of the line, and I m and I k are in fact, the currents at both the ends of the line.

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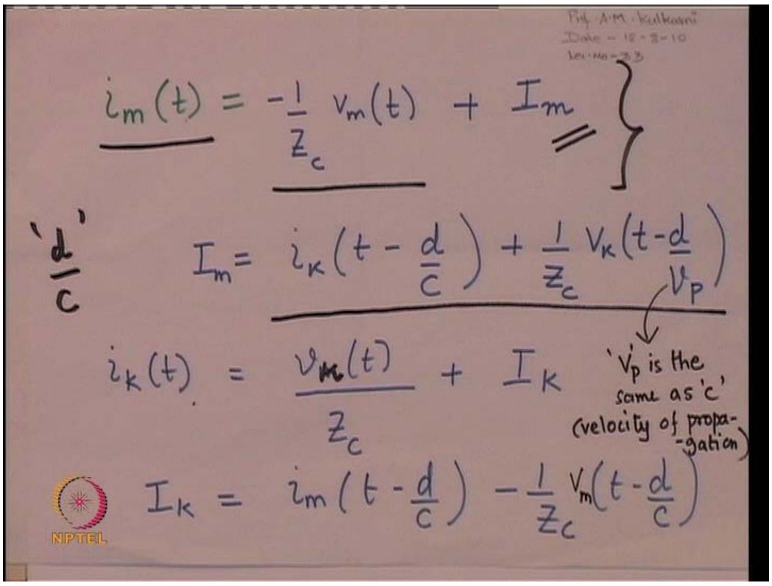

Then, you can represent i m as, i m is dependent on V m by this equation were z c is, in fact, the characteristics impedance. We have discussed what characteristic impedance is. z c is root of L by C, were L and C are, in fact, inductance and capacitance per unit length. Small c is the velocity of electromagnetic propagation which for, air is practically equal to the velocity of light.

So, that is, 1 upon square root of L C. Now, what is getting at is that, the current i m can be written down as a function of the current i m at a time peri[od] time instance t, can be written down as a component which is dependent on the voltage at that end of the line. A current source I m, which is, in fact, dependent on the currents and voltages, which exist at the other end of the line t by d by c seconds ago. t minus d by c, actually tells you that I m is equal to i k sometime before. What i k was sometime before and Similarly, it is also dependent on what V k was sometime before.

So, it depends on what was existing at the other end of the line sometime back. Similarly, I k can be represented in this way, where i k of t is equal to dependent on V k of t sorry V k of t and the current at the other end of the line. This I, capital I k is, in fact, a current source, which contains what are known as the history terms. So, if we look at, in fact, the first equation, what it tells is the equivalent at one end of the line.

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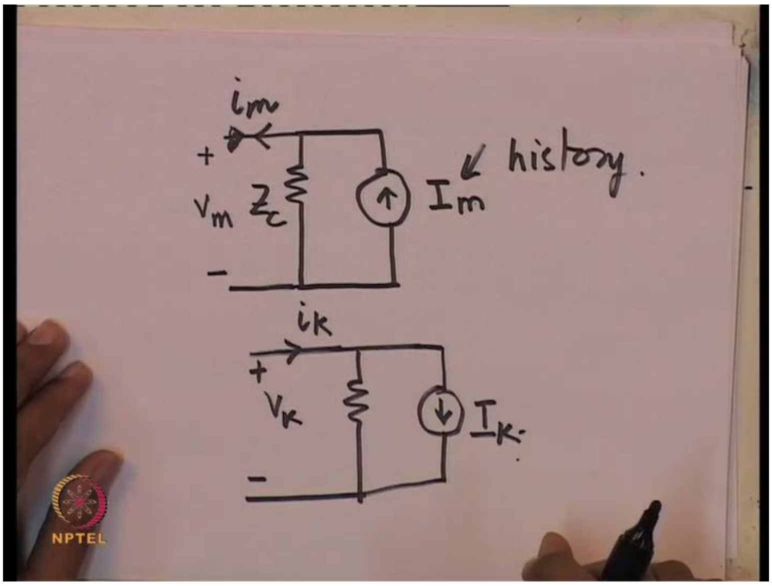

So, if you look at i m at one end of the line, you can see that this equation effectively tells you, I m can be obtained by a circuit of this kind.

So, all I have done here, of course, is represented this equation. I just represented this equation. I have just represented this equation by this circuit. So, if you manage to solve this circuit, you know, of course, you need to you know, define what else is connected to the system. But, the point is that, if I locate the equations there effectively, you know, representative of this circuit, it is a resistive circuit. This is a history term. Similarly, you can obtain i k from this circuit. So whatever, if you solve this circuit, you will get i k. This i m and i k, remember, capital I m and capital I k are, in fact, history terms which depend on the currents at the other end of the line. So, it is a very important thing to note this, that these are in fact history terms, which tell you about the current at the other end of the line and some time ago. That some time ago is of course, d by c, d is the length of the line divided by c seconds before.

So, in fact, it is an interesting point here that the solution of a loss less line, in fact, comes out to be simply algebraically related to the currents at the other end but, of course, with a time deal. It is pure transport dealing. So, in fact, this in fact is useful, if you want to for example, simulate a transmission line. So, if I tell you, you know, the conditions which exist at the boundary of a transmission line, the boundaries of a transmission line, you should be able to tell how the system behaves for other instance of time.

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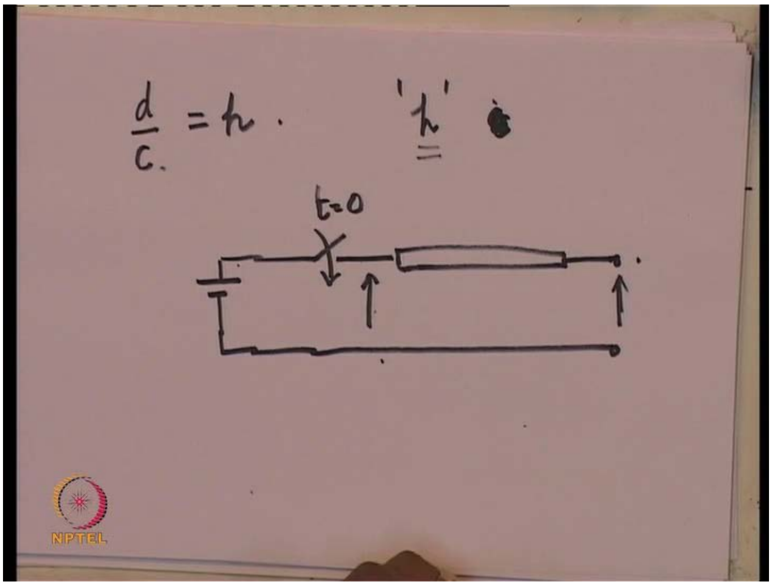

So, for example, if you look at this, in fact, equation, if you call d by c h, so if I call d by c h, where h is, you know, I can evaluate the values of i m t and i k t at discrete instances of time by using these algebraic equations quite easily. For example, I could try to understand the behavior of a transmission line for example, which is connected to a voltage source. So, this is our transmission line. I can use this. Suppose, I want to understand, how the voltages and currents behave, in case I switch on a voltage source at t is equal to 0 and I want to know, how this voltage at this end varies and how this voltage here varies under open circuit conditions. That is, the current at this end is 0 and the voltage here is defined at, t is equal to 0 onwards.

With this information, I could evaluate i m and i k at discrete instance of time. In fact, those discrete instance of time are the time required for the wave to travel from one end to the other. So, this is a very interesting kind of equation that we get which is of course, not true in case, R and G are nonzero. So, let me just again repeat what I what I said in case, I present to you a circuit, which needs to be simulated, a transmission line behavior which needs to be simulated, the dynamic behavior then, I can use these equations. These equations are, in fact, algebraic equations with a history term. The history term, in fact, involves voltages at the other end of the line, voltages and currents at the other end of the line.

So, if I tell you the boundary conditions, that is the voltage or current conditions at one end of the line and the other end of the line, you should be, for all other instance of time, be able to tell how the behavior is. In fact, it changes at every h interval of time. It changes after h, a period of h, where h is d by c, the amount of time it takes for the information to travel from one end to the other.

So, this is one interesting outcome of the travelling wave equation, that you can actually simulate a loss less transmission line quite easily. I mean, it looks very complicated but, if its loss less, you can simulate it quite easily. Now, one thing I should mention here is that, I am evaluating the currents and voltages at both ends of the line at discrete. I can easily obtain it at discrete instance of time, because of the nature of the algebraic equations which I am getting. So, this is somewhat different from the discretization, you know, kind of thing we were doing for other circuits.

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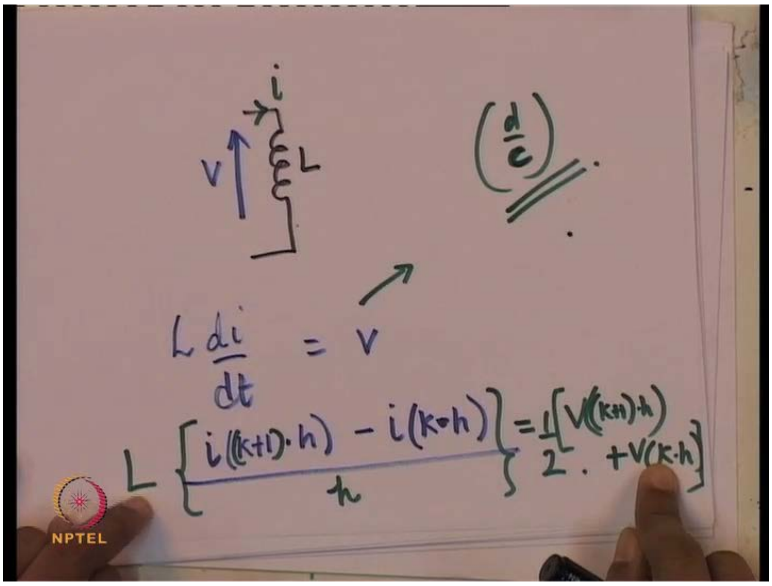

For example, if you have got a lumped inductance, if I discretize it using trapezoidal rule, if I discretize the equation L d i by d t is equal to the voltage across a lumped inductor, then I will get i at k plus 1 into h. This is the current at the discrete time instant.

So, if I have discretized using trapezoidal rule. This is k plus 1, of course, should be inside a bracket, into h divide by h is equal to V k plus 1 into h plus. So, this is of course, into l plus V of k into h into half. So, in fact, the discretized equation of, for a lumped inductor using trapezoidal rule is, in fact, this. Out here also, you will notice that the current at the k plus 1 instant, is dependent on the current at an earlier instant.

So, this is an interesting thing. So, if you discretize a continuous time lumped parameter equation again, you get basically the answer, you get a kind of when you discretize it, you get essentially an algebraic equations which relates new value of, in this case, the current, the new value of the current in terms of the history, the current and voltage history of this circuit. But remember that this, you know, dependents on a history term is local. In the sense, the current at the k plus 1, the instant depends on the local current and voltage at the kth instant. But, in a case of a transmission line, remember that our current, you know, at the one end of the line at the kth instant, depends on the current and voltages at the other end of the line, d by c seconds before

So, this is something which is fixed by the nature, that is, the length of the circuit as well as the physical parameter. Let see, which is the velocity of light for an overhead line? Whereas, when you discretize a lumped parameter, a lumped parameter device like an inductor, lumped inductor, you again get an algebraic relationship. But, remember that the dependence on the history term, rather the history term is still local. In case of a transmission line, the current at one end of the line is dependent on the current at the other end of the line at a previous time instant and the time instant is really dependent on the distance.

So, whereas here, it depends on the time step you have chosen for discretizing this continuous time differential equations. So, there is slight difference between what you are getting there and what you are getting here. But, it turns out that since this kind of algebraic relationship is obtained for a transformation line directly, you can interface the transmission line equations and the equations obtained by discretization of a continuous time lumped parameters element like inductance and do a simulation.

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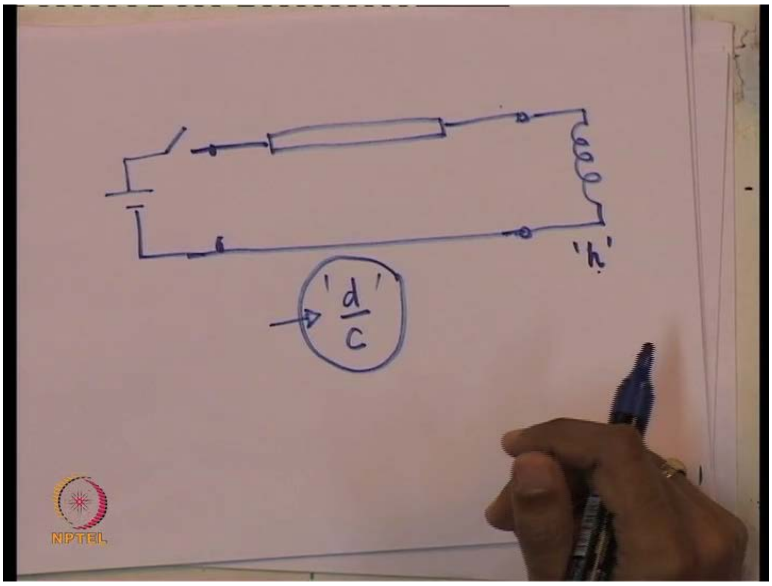

For example, it is it is not very difficult to do a simulation now because of these facts, of a system like this. You have got a transmission line, and at the other end, you have got a lumped parameter inductor.

So, when you discretize this, you will get basically algebraic equations with history terms and of course, the discretization interval is h. Out here, you again get voltage and current at both ends, dependent on one another but, of course, there is a delay element. They depend on the history terms with, you know, the history being the relevant to what what was the situation d by c seconds before.

So, if actually, if I chose h to be a multiple of d by c or h equal to d by c, it should easily, one should be easily able to interface the discreet time equations which you get here with the algebraic equations which are given by these, which are given here. So, this is an interesting thing. In fact, this is the way, you know, transmission lines are represented in programs called electromagnetic transient program. So, this is how things are done here. You got a transmission line; you assume it, suppose you assume its loss less, then the the currents and voltages at discretize instance of time can be obtained by these algebraic relationships. Continuous time system lumped parameter systems are connected to the transmissions lines, in fact, can be brought to a similar form because of discretization by some numerical integration method. In, fact if the h is either a multiple of d by c or equal to d by c, this this becomes even more easier to interface all these equations.

So, just chew upon this and you can actually do a simulation of a transmission line connected to lumped capacitor or lumped inductor and so on. It is not very difficult to do that. So, only thing is of course, that there is an important assumption here that the line is loss less. So, can you think some means of, kind of taking into consideration, you know, the losses in the line that is nonzero R or G. The answer is, well the the equation which we get, travelling wave equation or this algebraic relationship which we got, is true only for R is equals to G is equal to 0. We just cannot use this and we cannot use this. We cannot use this simple algebraic relationship between the currents and voltages at either the end of the line which we have just discussed sometime back.

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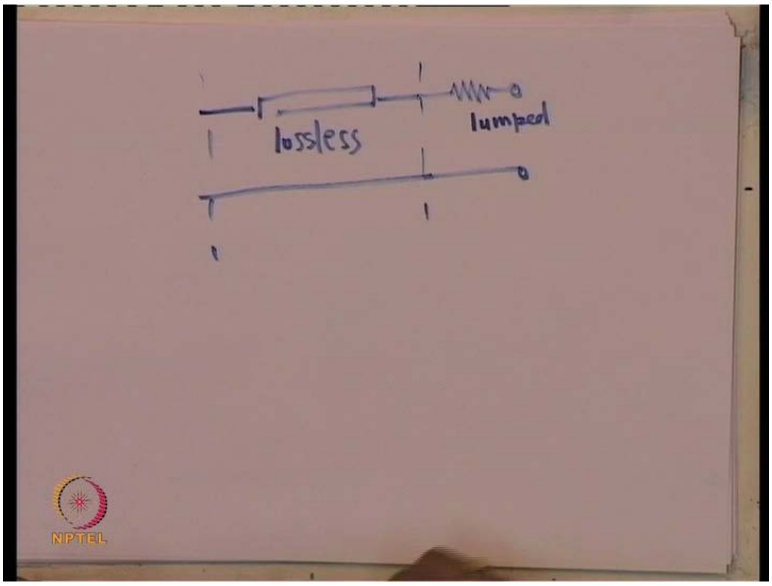

So, if R and G are to be brought into the picture, the best idea would be to consider loss less line as consisting of loss c line, as consisting of loss less line plus the effect of the resistance is considered as a separate series resistance which is connected separately outside. So, this is a lumped resistances plus a loss less line. So, this is what seems to be reasonable reasonable thing to do. I mean, of course, the validity of this approximation needs to be checked out. I mean this is of course, something which you occurred to as a nice trick to, you know, use this algebraic relationship even when the system is a lose system.

So, you consider a lose transmission line is a loss less transmission plus the effect of the resistance is considered separately as a lumped element. So, this is how you would try to simulate or understand the effect dynamically behavior of a transmission line. There is one more small issue which you need to tackle. This is something I will not spend a lot of time, but, you can just think of, in case, I want to do a simulation of a transmission line like like it is shown here and the discretization of the lumped element like an inductor which is connected, say at one end of the transmission line is done at a time step of h. And, d by c, that is, the length of the line divided by the speed or the velocity of propagation is not an integral multiple of h. In that case, it turns out to be somewhat difficult to interface the algebraic relationships which you get of the transmission line with the algebraic relationship, which you get by discretizing the lumped parameter continuous time differential equation with a discretization time interval of h.

So, what one would normally do under that such a circumstance is try to get h to be rather d by c to be an integral multiple of h, as close to it as possible, I mean not be exactly equal to, d by c may not be exactly equal to k by h but, you can make it approximately so.

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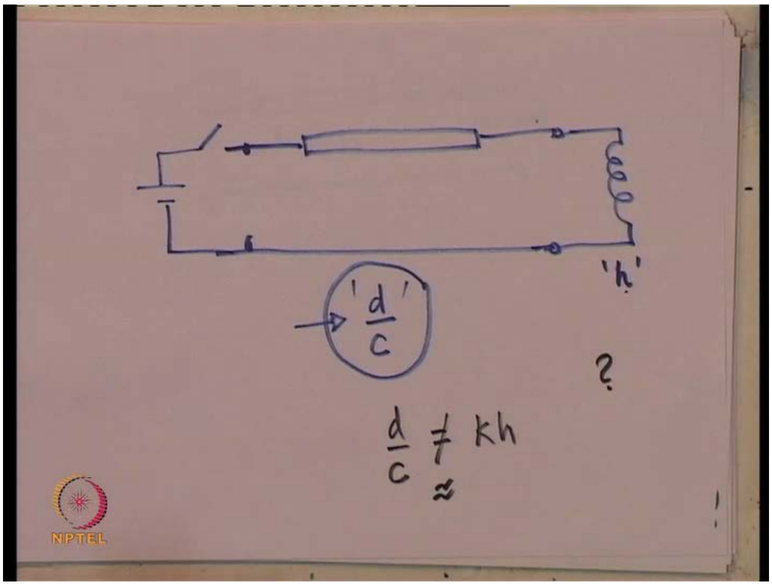

Alternatively, you would need, in case you do not, you cannot really achieve this. What can you do? So, this something you can think over.

This is left to you to think over what would be a reasonable or, you know, a satisfying way, let me say of handling a situation where d by c is not an integral multiple of h. As a result of which, it becomes difficult to interface the algebraic relationships obtained from the travelling wave equations of a transmission line, with the discreet time equations arising due to the use of some numerical integration technique for a lumped element.

So, this is essentially how you would, this is of course, particular issue which you may need to tackle. But, there are reasonable ways to really solve this problem and I leave this to you to think about. The next issue is something which I actually left you in the last lecture was you have got this lumped pi equivalent of a transmission line from sinusoidal steady state analysis. The question is, would that lumped parameter, let me call it model of a transmission line, a lumped parameter model of a transmission line obtained from sinusoidal steady state analysis safes to really simulate or mimic the behavior of a real transmission line. That is a question which you would like to answer next.

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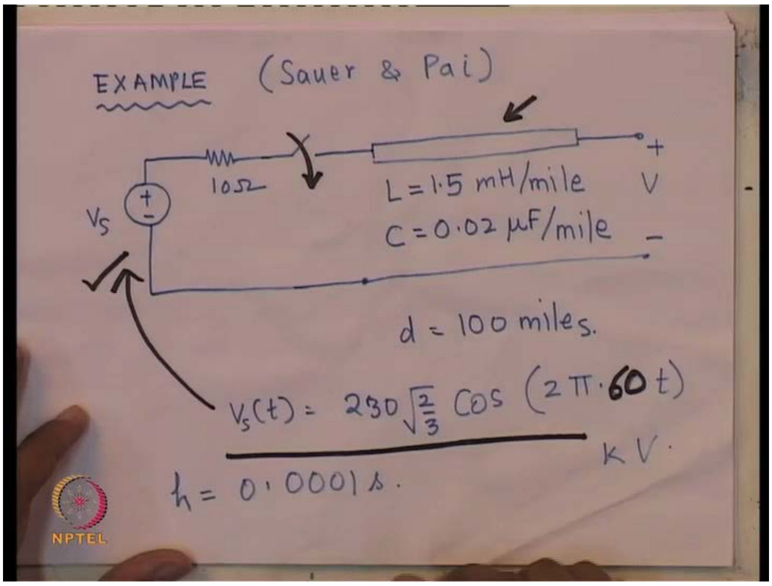

So, as I, you know, discussed in the previous class, I mean I left you with a problem. If you recall what we did last time, the problem was this. This is an example from the book by Sauer and Pai where you got a transmission line which is 100 miles. It has got these parameters L is 1.5 mille hendry per mile, C is 0.02 micro ferret per mile and you switch on a voltage source which has a sources of 10 ohms onto the transmission line which is open circuited at the receiving end. Now the question which I post to you was that, if I try to simulate or understand the behavior of this transmission line by using the algebraic equations given here the algebraic equations given here and see how this system behaves.

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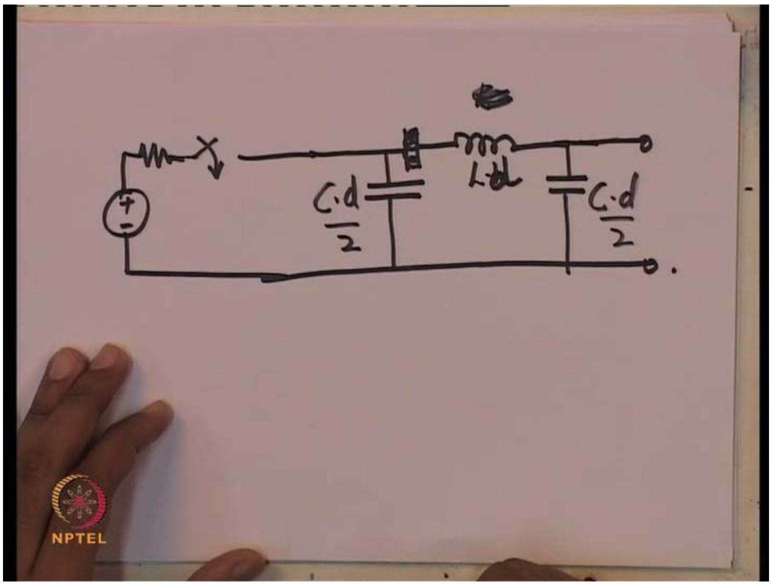

In that case, how the response differ in case I took instead a lumped parameter model of a transmission line, where the induct, this inductance I assume to be L into h. Remember, L is inductance per unit length L into d and this is C into d by 2 and C into d by 2. C is of course, the capacitance per unit length.

So, the question is that, if I use this lumped parameter model of a transmission line and try to find out how this system behaves, in fact, you can analytically get the response or even numerically integrate and obtain the response. How does this system behave as compared to a proper simulation of a loss less line using the travelling wave equation? So, you use the algebraic equations with history terms which are actually obtained from the travelling wave solution of a transmission line and see how both of them compare. So, if you do that, you get somewhat surprising result. So, what is that result.

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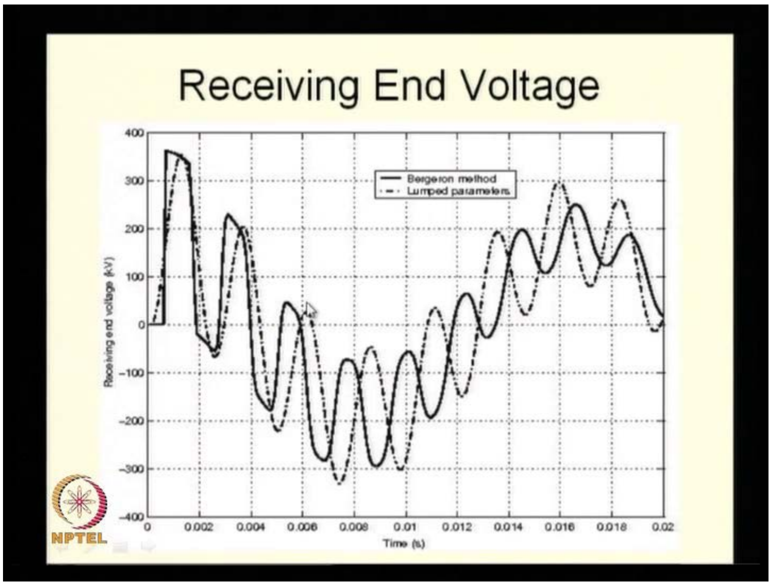

So, if you look at, this is actually the response of the transmission line. The the bold, what you see as the bold line here, it looks a bit like square wave initially, is actually the response obtained from the travelling wave model. That is, using the travelling wave kind of response of a transmission line, that is using the model, the algebraic model of a transmission line with history terms as I sometime mentioned sometime back. Remember that for this particular circuit, the receiving end reacts or you will see something happening only after the wave reaches the other end.

So, this is what you will get using the, you know, the kind of the algebraic equations with history terms which are valid for a loss less transmission line. This is also called Bergeron method. Instead, if you use the Pai model using lumped parameter that is lumped inductance is in capacitances, you get what is seen here is a dashed line. One of the important differences between the travelling wave response or the detail response of a using the travelling wave model of a transmission line and the lumped parameter response is that the lumped parameter response starts immediately after the disturbances is initiated at the sending end.

So, you see the effects right away, whereas, there is a clear time delay when you consider the travelling wave model. The travelling wave model of course, is more accurate. So, what you really, although we are talking of a comparison, remember that a travelling wave model is actually they more accurate one. An interesting thing of course, which you see here is that although there is of course, a difference between the travelling wave model and the lumped parameter simulation of this system, there is some similarity too. In fact, if we look at the response, it seems to be some kind of a filtered response, you know, the lumped parameter response is a kind of a most smoother and you can call it a kind of low frequency part of the response.

So, if you look at, it is similar. It is not the same, of course, but it is similar. And, in steady state, if you, of course, if you allow this particular system to settle down, remember what is the system we have considered. Please have a look at it again. If you look at this again, it is a sinusoidal source switched on to an open circuit at in line.

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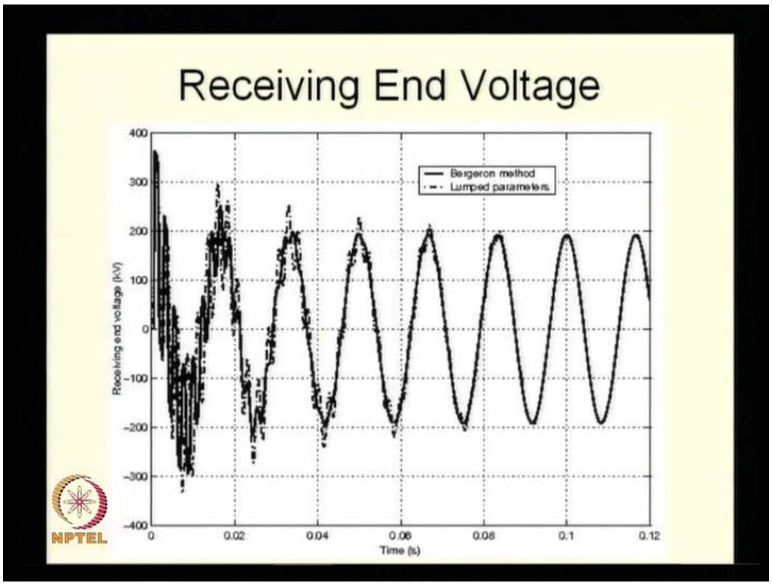

So, if you wait for a long enough time, one would expect that the system would reach a sinusoidal steady state. The interesting thing is that, if one looks at, you know, the receiving end voltages after the system is allowed to settle down, you will find at, in fact both the lumped parameter model as well as the travelling wave model also called a Bergeron method, settle down to the same value. Now, why this is so? In fact, this is not surprising at all. Remember that the pi equivalent is valid. It is valid, in fact, for the sinusoidal steady state.

So, the travelling wave model is essentially settling down to the same steady state. So, as far as the sinusoidal steady state is considered, the Pai model of a transmission line is in fact correct. In fact, there is nothing wrong with it. Only thing is that, the initial part of this transient which we saw in expanded portion in the previous slide, there is some difference. There is some difference in the initial part of the response, using a more detail partial differential equation model as compared to the lumped parameter model.

So, this is what essentially you will be losing out, in case, you know, you choose a lumped parameter kind of a model. Now, it is obvious that if you are not interested in the high frequency transient, which are seen right at beginning of this plot, then it appears that you can just as well use the lumped parameter model.

So, if you are not interested in high frequency transients, in fact it is a good enough approximation to even do dynamical analysis with the lumped parameter model. But remember, the origin of the lumped of the parameter model. The origin one was actually from two port equivalent and the sinusoidal steady state condition. But, you can actually use it, if you want to obtain the low frequency behavior of transmission line. So, it is ok to use Pai equivalent model.

So, this is what we get from this particular study. What you cannot, of course, get from this model, the lumped parameter model is the behavior just after the disturbance. Then, there is a substantial difference. So, this is something which we saw some time back that right at the beginning, there is a difference between the responses. Although, the responses are similar, they are still different. Now, going on further, in fact our discussion so far has been restricted to, a kind of a single phase line, a distributed parameter model of a single phase line and we actually got the travelling equation wave equation for it. The question which we can ask our self is, if you got more than one line.

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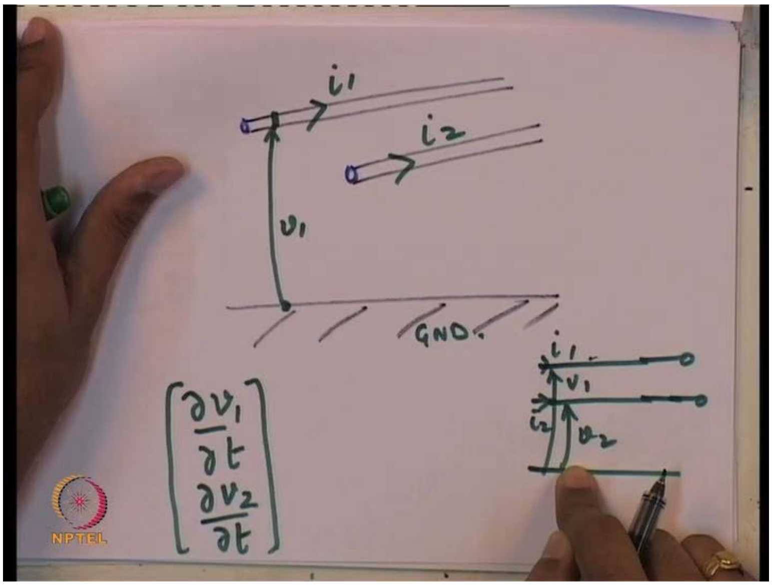

For example, suppose you got a bipolar line. That is, the electromagnetic environment consists not only of two lines but, something like this, two transmission lines and the ground. So, the ground also could be a part of the electromagnetic environment. There could be current, for example, in the ground. In such a case, our equations, the equations of a transmission line are in fact, suppose the current through this is i 1 and this is i 2, then we can show that the at any, just like in the previous case, you can write down the equations, like these are partial differential equations.

Remember that, what we are doing different from the previous case, is that we have introduced another conducting kind of system into the analysis. This is a ground. So, the electromagnetic environment is slightly different from the previous case where you just had two lines in this universe and nothing else. Now, we have got v 1, say at a distance x as before. This is v 1. This is the loc[al] the voltage of this point with respect to ground at this point. So, one should be very clear about what we mean by this. Similarly, the voltage of the other wire, at any point of the other wire with respect to ground locally is v 2.

So, you have got v 1 and v 2. Of course, I have shown this slightly tilted, so you can look at it this way. This is ground, so this is v 2, this is v 1 and this is i 1 and this is i 2 and like before, you have got the sending end and the receiving end and so on. So, the equations for v 1 v 2 i 1 i 2, v 1 and v 2 are the voltages with respect to the local ground here are in fact given by, I am sorry this should be, yeah, so, we will have a matrix here is equal to, just a moment we will just yeah.

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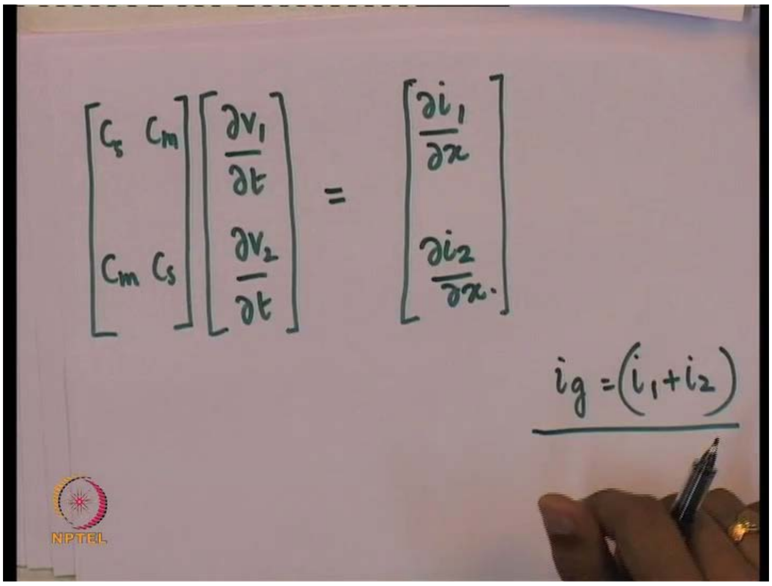

So, what is different in this case as compared to the next? Remember that v 1 is the voltage of the first, at a point on first wire with respect to the local ground. v 2 is the voltage with respect to ground, the local ground for the second wire. i 1 is the current in the first wire; i 2 is the current in the second wire. You could have in fact, current in the ground. In fact, the ground current, in case i 1, if i 1 is not equal to minus i 2, in such a case, you can have currents to the ground. So, in some sense, what I am doing here is including the effect of ground or the electromagnetic environment around these two wires as well. So, this is basically equation which you get and a similar equation exists for the current as well.

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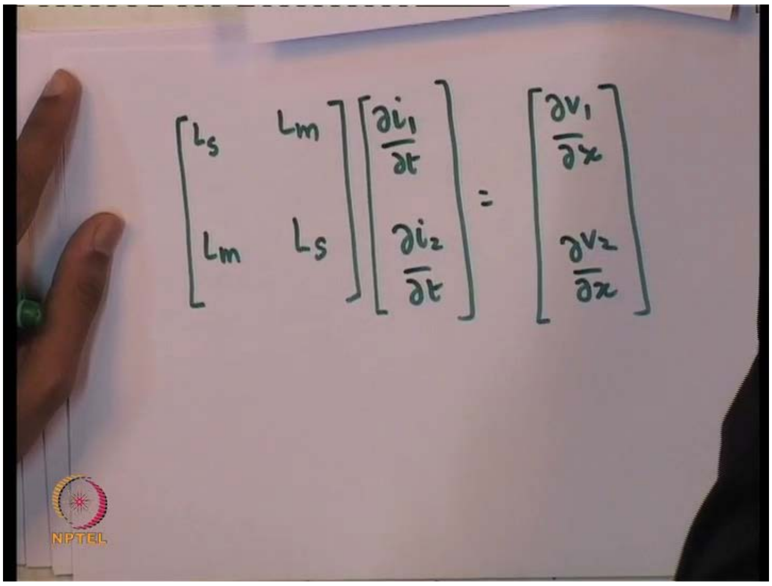

So, you have got L s L m. These are inductance. You have got an inductance matrix is equal to this, for loss less case, of course. Now the question is, we had considered the equations of a transmission line earlier, this of course, with losses. But there, we had just the voltage across the line and the current. There is only the current. Whatever the current flowed here was equal to the current, which, so this is whatever the current, suppose you have 1 amp flowing in this direction, 1 minus 1 amp would be flowing in this direction.

So, this was the situation before. Now, we have got the ground with possible current flow through the ground and you have voltages with respect to the ground v 1 and v 2. So, this is what is different. So, we have got more variables and you also got a bit of coupling. These variables are coupled to each other. So, how does one solve in such case. So, this is the actually as a simple case of how you can use mathematical tools to solve this kind of situation. Now, in fact with r and g equal to 0, we know the solution of this equation is actually the travelling wave solution. Can we directly apply this to this situation? The answer is no. We cannot directly apply because now, you have got a coupled set of, you have got two sets of equations and each of them is in some sense, a vector equation.

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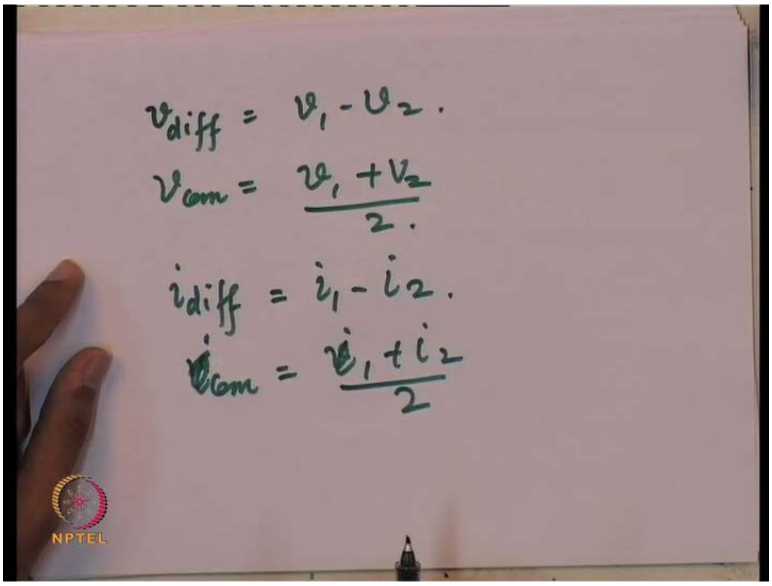

So, how do you solve this problem? In such a case, if you have defined the difference voltages v 1 minus v 2 and the common voltage as v 1 plus v 2, say divided by 2. Similarly, i difference and v common is equal to i 1 plus i 2 divided by 2. In fact, the common current is in fact, proportional to the ground current. So, if I transform v 1 and v 2 to v diff v com i diff i com, I can reformulate these equations. This is something I leave for you to do. You can reformulate the equations in i diff i com v diff and v com and one very interesting thing occurs, is something you can, is very very easy to prove.

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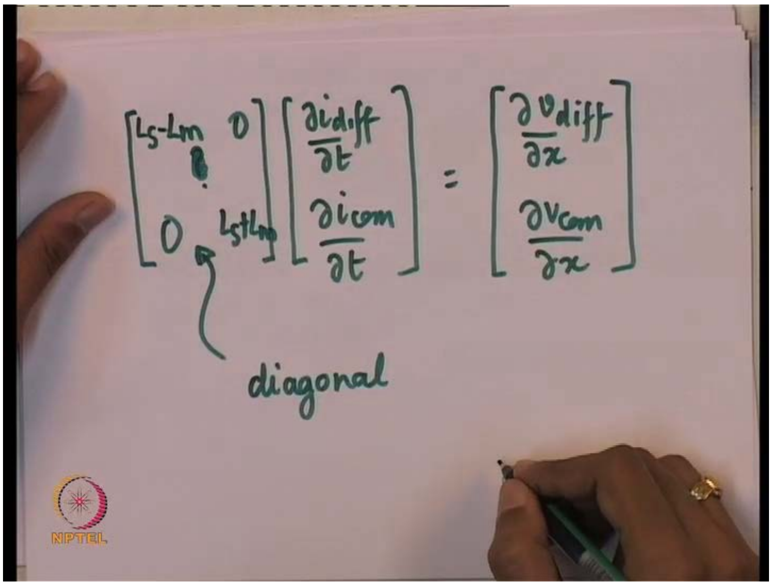

So, you have got for example, d i diff by doe t, d i com by is equal to, you can reformulate the equations in terms of these new variables. The surprising thing is that, this is a diagonal matrix, in case you use this transformation of variables. In fact, the terms are in fact, L s minus L m and here you get L s plus L m 0 0.

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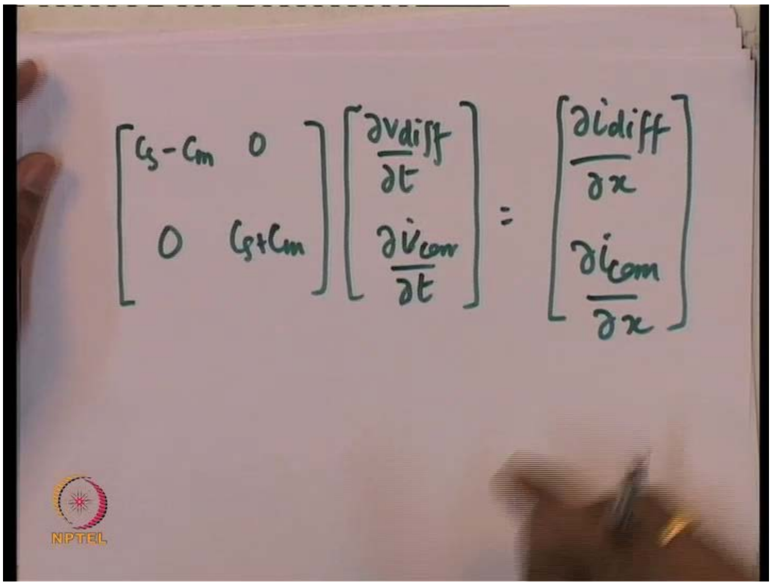

Similarly, you will have C s minus C m 0 0 C s minus plus C m doe v diff. So, what you have essentially is, this kind of decoupling takes place between the i diff and i com variables.

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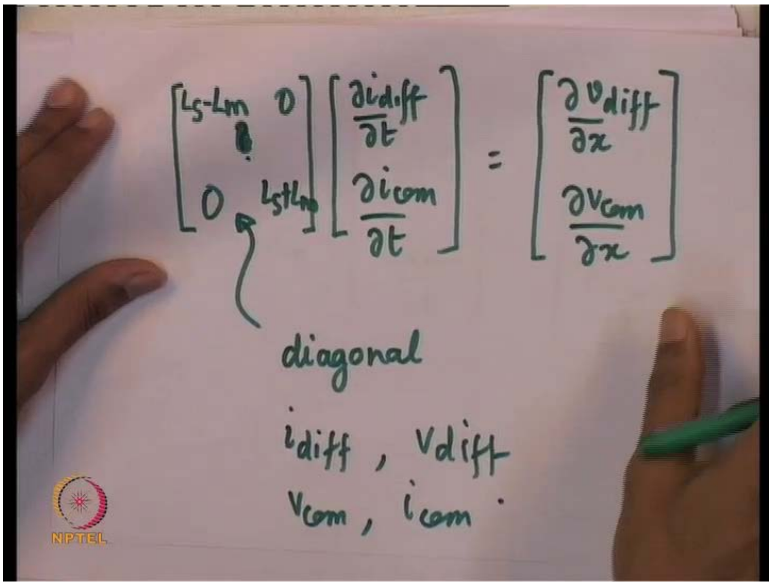

As a result of which, you can take the pair i diff and v diff and v com and i com and essentially, since v diff, the equations between i diff and v diff are completely decoupled from the equations of v com and i com and the equations in fact, are very similar to these ones. Of course, with r and g equal to 0, you can actually get a travelling wave solution for v diff i diff v com and i com separately because, there is complete decoupling between these and get the solution for this system as well.

So, it is very interesting thing, which I have tried to tell is that, you can use transformations. This is basically, linear transformation of variables in order to get decoupling. Once you get decoupling, you can get the travelling waves solution for the diff variables and the com variables separately and if you want to get v 1 and v 2 eventually in the end, you just super impose the solution. You just add up the solutions, you know, by using the reverse transformation. I should say add up and use the reverse transformation to get the original variables.

So, it turns out that, even in cases where you have got a system, you know, you can imagine that, if you got three phase system.

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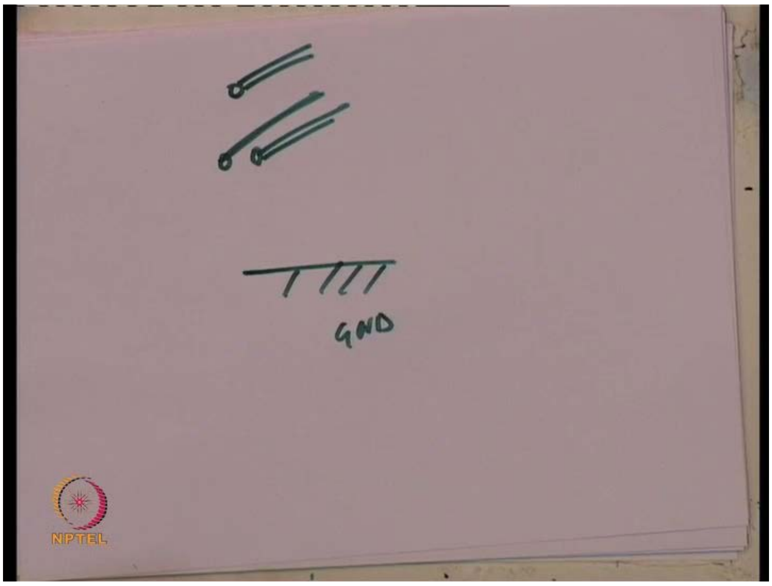

So, if you got three phase system and with ground as well, this is a typical electromagnetic environment which you will see. In fact, you can have clients in close proximity to each other et cetera. It turns out that, although they are coupled partial differential equation in the original variables like v a g, that the voltage of a phase with respect to the local ground, if you if you formulate the equations in that way using the the phase variables with respect to ground, the voltage is of the phase variables with respect to ground or the individual phase currents, it turns out that, in fact you will get some kind of coupling.

But, if it is symmetrical system, as in the, you know, the case which I showed you sometime, you had a symmetry C s C m C m C s. The matrix which related all the original variables was symmetric. Now, if the symmetric, if the system is symmetric, it turns out that some of the transformation we have studied before can be applied to the three phase case and we could actually do, kind of, we can get a neat model in terms of the new variables.

For example, you may ask, can I apply the d q transformation which I have used for studying synchronous machine to a three phase transmission line? The answer is yes, you can, provided it is a symmetric transmission line.

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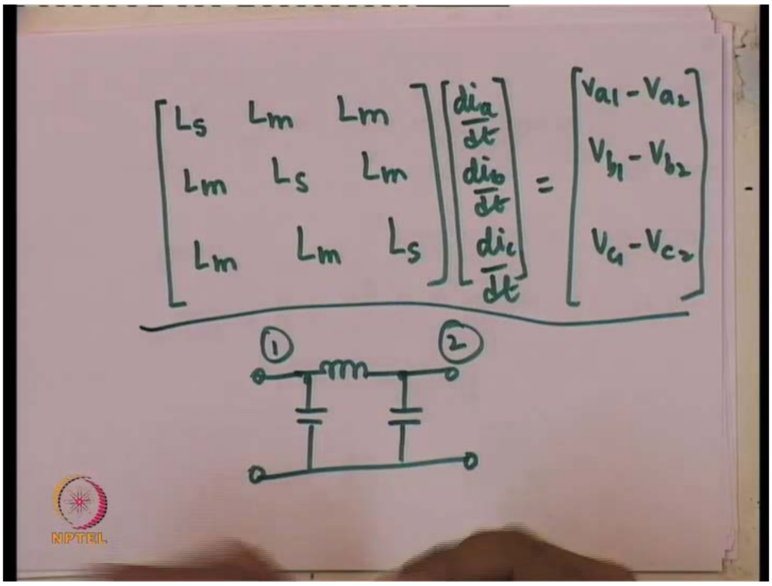

So, if you got for example, a transmission line which is, which has got parameter like this, it could be a lumped parameter representation. In which case, of course, a transmission line becomes for example, if you are studying transmission line using the lumped parameter approximation, we have already discussed this in the beginning of, rather sometime ago, so you will have d i a by d t for a three phase system is equal to V a 1 minus V a 2 V b 1 minus V b 2 and V c 1 minus V c 2 and of course, so this is 1 and 2.

So, actually V a 1 means the voltage a with respect to the local ground at this end of the transmission line. V a 2 is the voltage with respect the local ground at the other end of the transmission line. So, see this is the lumped lumped parameter model of a transmission line which you have, suppose, I wish to use. Now the question, which is the limited question we have asking here is not about the lumped parameter or distributed parameter lines is whether, but, can we get a transformation to make this decoupled set of equations or rather I can ask you a very more specific question. If I transform i a i b i c to i d i q and i 0 and V a V b V c also to, you know, the d q variables, what will happen of this, you know, equations.

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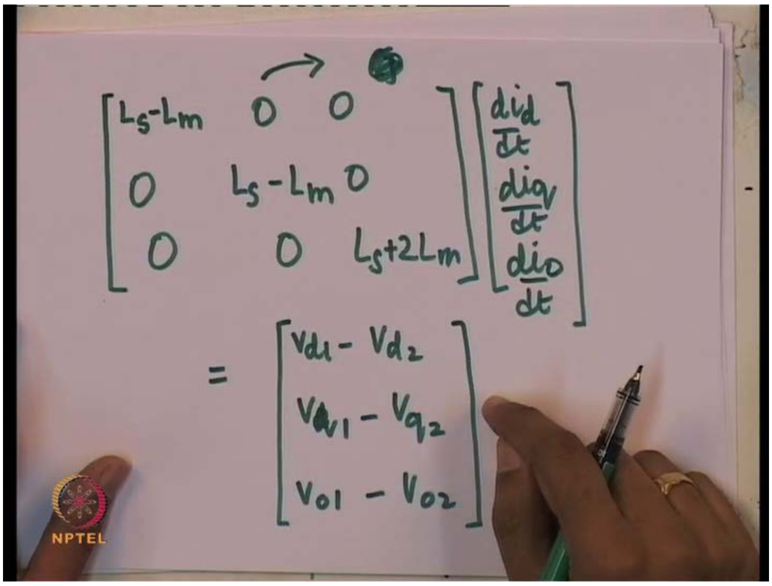

So, what the question is, the differential equation in fact, will look like this eventually. Where this is nothing but, what obtained by applying the transformation of variables. So, what I have done is applied at time variant transformations c p. Do you recall that the time invariant time variant transformation C p, which is a function of theta. Theta is the angular position of a machine. Of course, now the question arise is, which machine or there is no machine here. But, suppose I take any machine and use its theta and transform this this set of equations, in that case, you will get basically, and this matrix will get transform to this matrix. It in fact turns out to be diagonal. How you will get it?

Basically, I have replaced i a i b i c by i d i q and i 0. So, what what you of do is, suppose I call this the L matrix.

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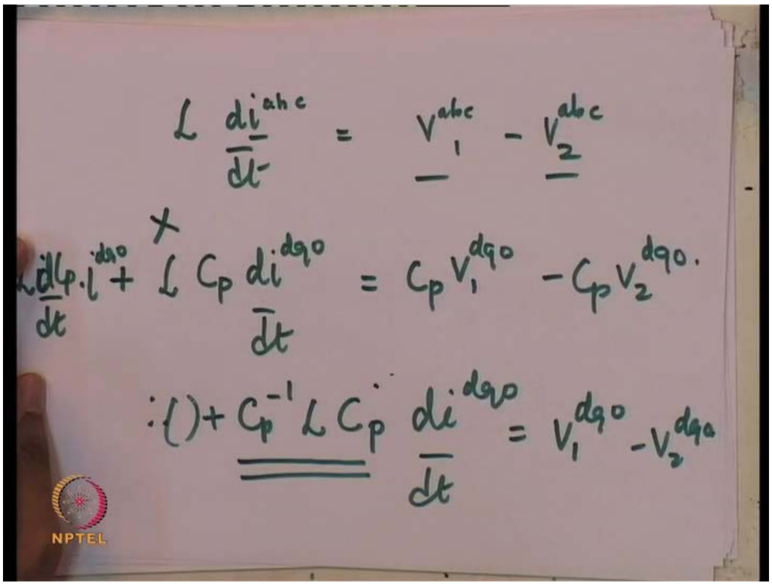

So, what you have is L d i. This is just written in compact fashion. So, I am just writing these equations in some kind of, it is just a compact form of way of writing this equation. These are all vectors. This is the, L is the matrix. So, when you transform, you will get L. This is C p. So, this is what I get. Is this correct? Is it fine? So, in fact, if you evaluate C p inverse L C p and L matrix is symmetric, it turns out that this new L, that is C p inverse L C p, it turns out to be a diagonal matrix.

So, it is an interesting and nice thing, for nice thing to happen, in case, if did use the d q transformation using spark transformation of some machine. Now, is this equation correct? Well, the answer is no. There is an error which we have actually done. So, this is in fact not true. Remember that, d of C p into i d q 0 is, will have another component d C p by d t into i d q 0. C p is also function of time. So, actually this equation is not correct.

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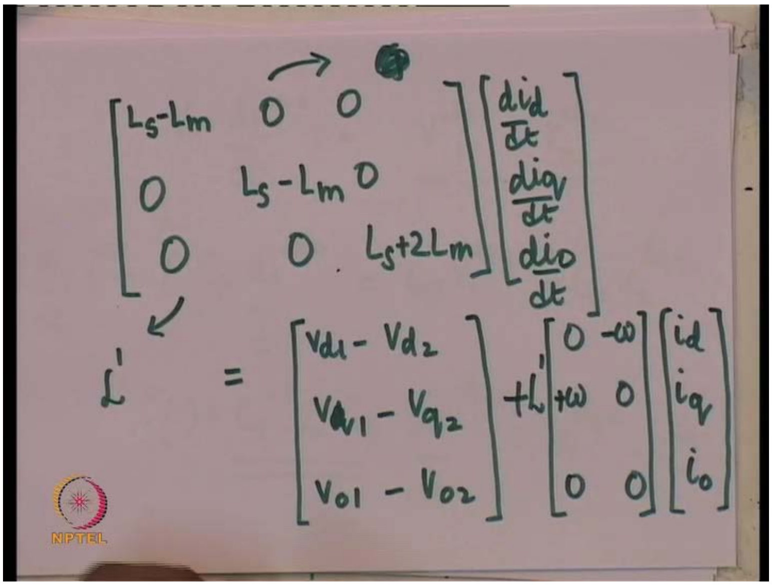

So, there is going to be an additional term here, which has to come, which is, in fact this is something you can try out yourself. It is going to be, so it is going to be this into this matrix. So, it is, suppose I call this matrix L dash. This will become L dash. So, basically what you will have is this equation. So, remember that this is because you have to take the derivative of the time varying transformation, whenever you going to rewrite the equations in terms of the d q variables. So, you are going to this get this extra term.

So, actually you can in fact, write down the equations of a para transmission line element. For example, the series reactance of a transmission line, in case you are talking in terms of the lumped equivalent, you can even write the partial differential equations in terms of the d q variables. There is nothing, you know, special about the lumped equivalent here. So, you can actually get the equations in the d q variables. So, you can actually interface the transmission line equations with eventually machine equations which are there in the d q variables. This can be done.

So, this is an important point. So, the final equation which you get for the transmission line or in fact can be written down in the d q variables in this form. In fact, one important point which I must emphasize here is that, in case this matrix L is not symmetric, that is there is some unbalanced or asymmetric in all the phases. In such a case, applying the d q transformation may not be, will not be of much use because you will not get this kind of nice time invariant and diagonal form of the matrix L dash. So, in case your L is not symmetric, it is not, it really arises from a unbalanced kind of configuration of the transmission line conductors.

In such a case, d q transformation will not yield you a useful set of equations. But, in case there is symmetry, you find that d q transformation is in fact, gets you nice decoupled equations in d q and nice diagonal form of the inductance matrix. So, this is an interesting point here about d q transformation, the equation of transmission line in the d q frame of reference. Now, there are few more things I need to tell you about transmission line. So, my earlier promise of introducing you to the prime mover systems will have to wait a little bit.

So, in the next class, we will just continue our discussion of trans. There is some remnant discussion about transmission line modeling, which we will continue and then go on to prime mover systems. Incidentally, in one of the examples, we did to show you the behavior of an a v r, the automatic voltage regulator. I did model the interconnection of a generator to an infinite bus by a reactor. In fact, if a transmission line is quite short, it turns out that, you know, you can model a transmission line for slow frequency transients by a lumped element. This is what in fact, this lecture told you that you could be used lumped pi equivalent of a transmission line to even get the dynamically response of the system to a very good approximation, if you are not interested in the very fast transient which occurs just after the transient is initiated.

So, in fact we did, we just did show the kind of behavior of the model using detailed travelling wave kind of model, as well as a lumped parameter simulation. So, we did actually do that and show you that a lumped parameter model does give you response which is reasonably for slow transients. So, it gives the reasonable dynamical response. So, you can under certain circumstances, depending on what you really interested in looking at represent a transmission line by a lumped parameter model. So, that is what you can take back from this lecture. In addition, the d q model of a transmission line can also be derived and it yields a neat model of transmission line. In fact, which, this owned whole through, in case you got an unbalanced parameter kind of transmission line, the d q model will not be very useful under such circumstance. But, of course, transmission line which is reasonably long is also transposed.

So, again a good approximation the system is balanced and you can apply the d q transformation and get d q model of the transmission line. So, these are the few things which I would like to take back from this lecture. We will continue a bit about the transmission lines in the next lecture and then, move on to prime mover systems.