Power System Dynamics and Control

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Model No. # 01
Lecture No. # 32
Induction Machines Transmission Lines

One of the most important loads, category of loads are those formed by induction machines. In fact, we also discussed in the previous class that, induction machines are used in another context as generators in many wind farms. Now, so it was in the previous class I told you, how from a basic synchronous machine model for example, one point one model, the model in which you have got one rotor winding on the d axis and one of the q axis. You can modify it. So , as to get a induction machine model, for that you would had to set the field voltage to 0, remove all kinds of salient behavior for example, x t and x u, you would have to make to a value x and x 3 dash and x 2 dash also would be equal.

So, if you did that kind of thing and even made a time constants on both axis the same, you would come across induction machine model from the original synchronous machine model. So, let us just review what we are going to do in today's lecture. Today is the 32 nd lecture and will be continuing a bit of our discussion on induction machines and will try to cover a bit of transmission line modeling also in this lecture.

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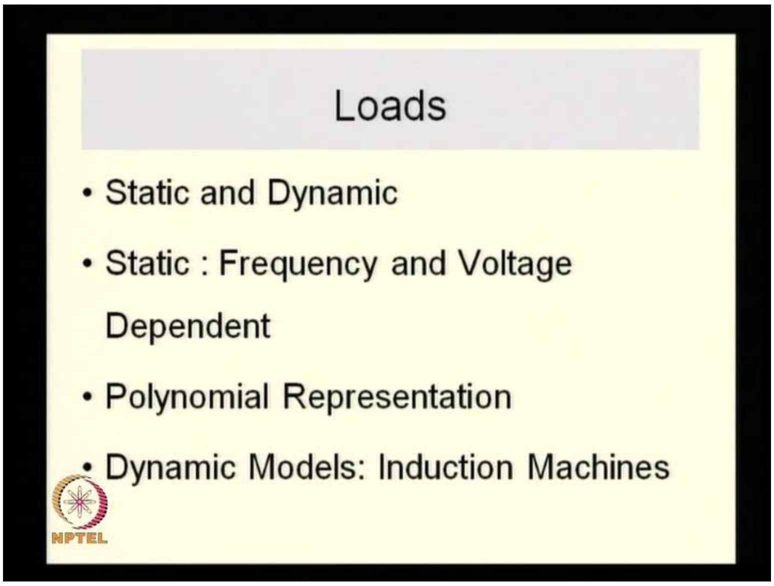

So, if you recall in the previous class, we had done a static and dynamic models or we had discussed the nature of static frequency and voltage dependent loads and we had begun on our induction machine model. Now, in the previous class, the point at which we left off was some of the issues when it came to induction machine models.

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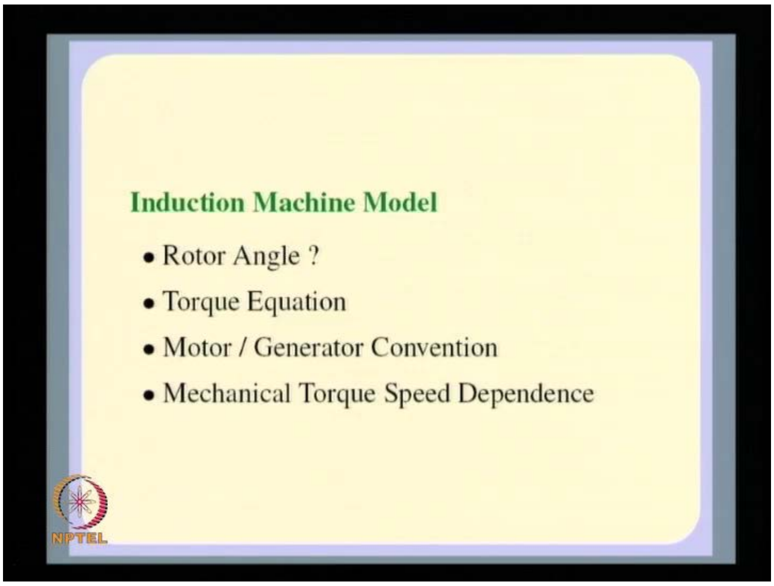

Although, I did mention that you can have, in fact, in induction machine model obtained from the synchronous machine model, there are few issues which naturally will come to your mind. For example, what do you mean by rotor angle in a situation where there is really no saliency in the machine. So, you cannot really align your axis to any particular, in any particular direction with respect to rotor because, the rotor is exactly symmetrical.

So, that is one issue which we need to tackle. So, if you look at the synchronous machine equations, they seem to be having a rotor angle dependence. So, how do we actually show that induction machine, in fact, are not dependent on the rotor angle, although you can get an induction machine model by, you know, just modifying a synchronous machine model.

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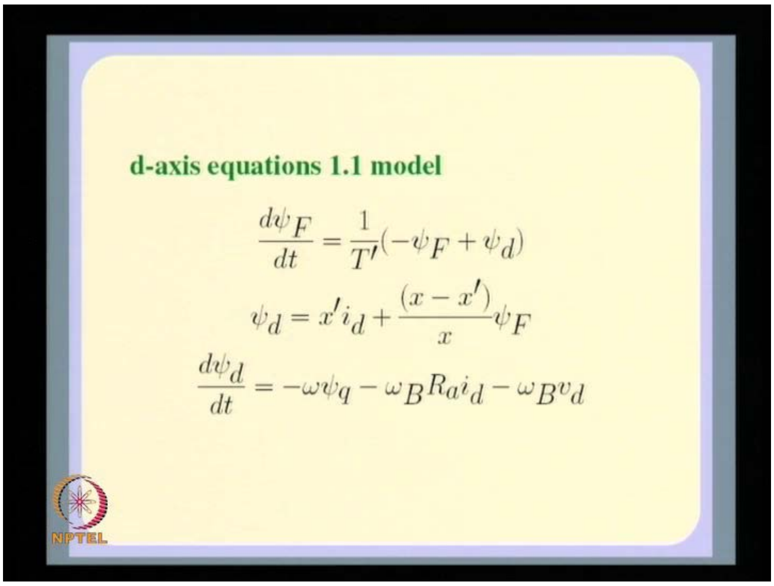

So, if you look at the basic equations which we had discussed last time, that if the induction machine model is obtained from a synchronous machine model by neglecting e F d and setting all the transient time constants and transient reactance's and of course, the steady state reactants equal on the d and q axis. So, this was the model for the 1.1model on the d axis and for the q axis this is what we have.

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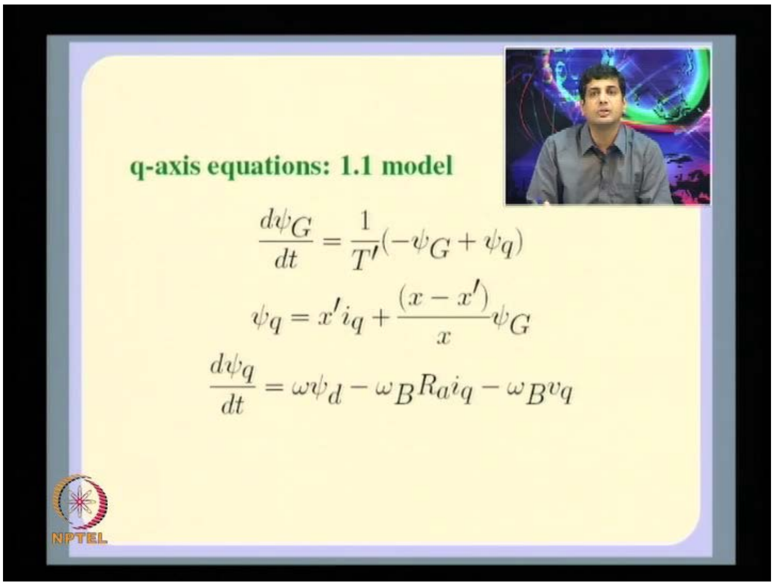

So, what you notice here, of course is, in both these equations, both on the d and q axis, the last equation for example, here is dependent on v d and next one, here the q axis equations, you have got v q here. Now, if you assume a three phase sinusoidal source, sinusoidal balance source of a constant frequency and you use parks transformation that is depending on the rotor position, then you will find it v d and v q become delta dependent. As I mentioned some time back, delta is only notional. It is kind of a abstraction in the context of a induction machine. It can be anywhere, but the point is, it does not affect eventually the observables or choice of the d, the d axis in the q axis.

So, in other words, does the rotor angle matter eventually? The answer is no and proving it is not; is not just you know, it is not, it cannot be shown, for example, in a couple of steps here. What we will do is, this something, it will come coming usefully even later. What we will do is reformulate the same equations using another transformation. Now, this seems to be a kind of complication. But, sooner or later, as it start working in power system dynamics, you will kind of get used to this change of reference frames.

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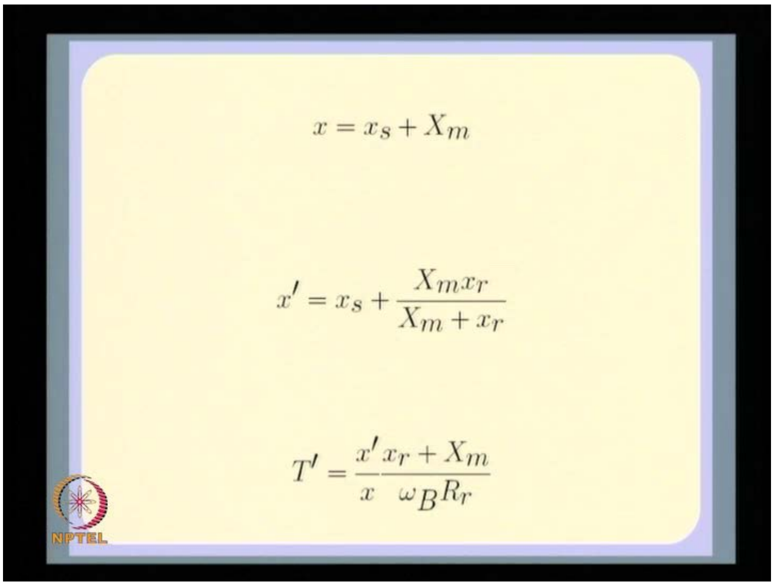

So, our normal reference frame, of course, before we go there, let us just repeat what we did last time. That is, the x to be used in these equations is nothing but, in terms of the typical parameters of a induction machine, x is equal to x s; the leakage reactants of the stator plus the mutual reactants of the stator. x dash, in fact, is obtained from the second equation. In this, x r is the rotor leakage referred to the stator and T dash similarly, is defined at the bottom with a R r, R r being the resistance of the rotor winding.

So, this kind of, these are the parameters which I used in model, 1.1model. But, for induction machine, they can be related to the cell; the leakage as well as the mutual reactances, as shown in this slide.

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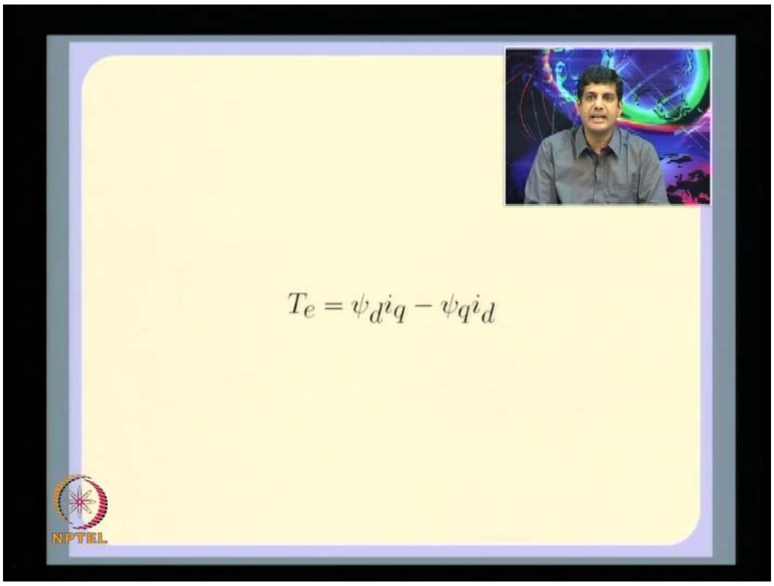

Now, remember that a torque equation is the same. It is i d i q minus psi q i d and we have used in this equation, the parks transformation.

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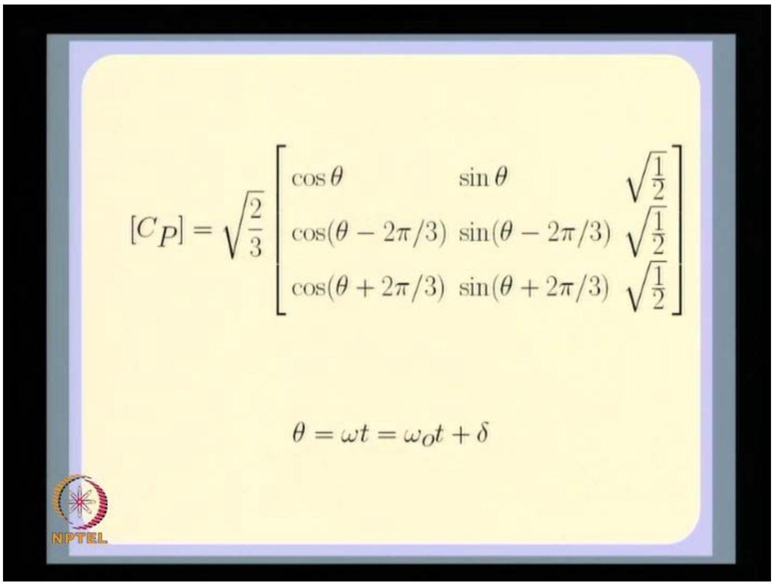

So, where theta is the position of the rotor, it is omega t plus delta. Now, let me introduce you to you another transformation, C K. In fact, it is also called crones transformation.

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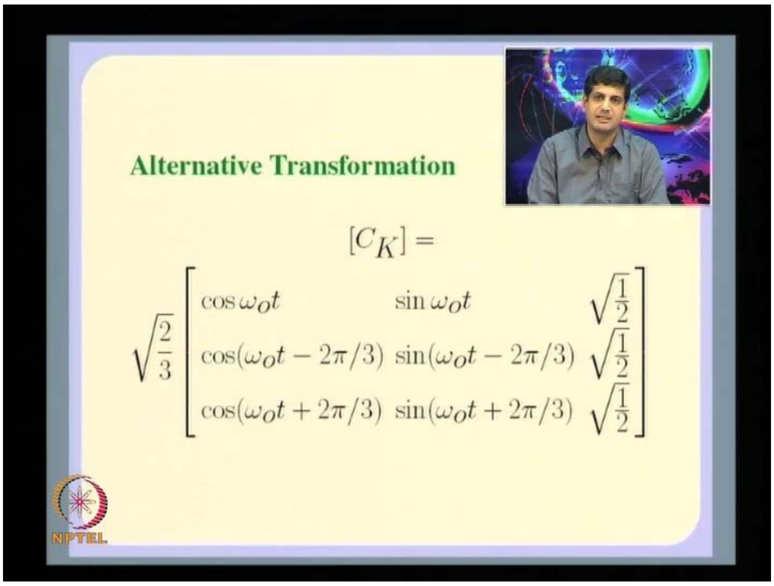

The difference between the transformation before and now, is that, instead of theta, which is omega naught t plus delta, you have got in this transformation, the argument of the cosines and sines contain just omega naught t. They do not have delta. So, this is not the same as the previous transformation.

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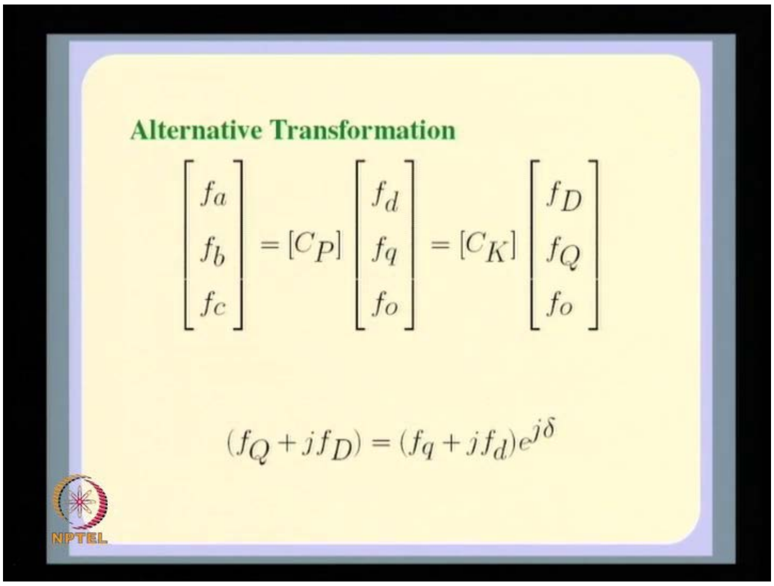

So, if I use this transformation to transform to the new set of variables from a b c, I had shown in this slide, that is instead of transforming to this small d q lower case D Q 0 variables, you transform a b c using c k into the variables f d f q and f naught. It is, you can really show that eventually, the lower case f D f Q and f 0 are related to the f upper case D Q and 0 by the relationship which is given below. That is, f Q plus j f D upper case is equal to f q plus j f d lower case into e raise to j delta.

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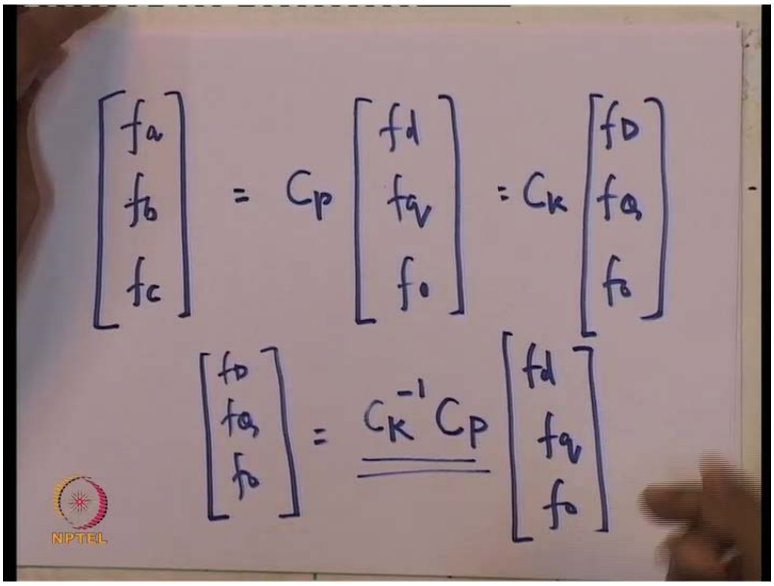

This is a compact way of writing this. In fact, you can really f a f b f c is equal to C P into f d f q and f 0 is equal to C K. This is another transformation which gets you somewhere else. It is another set of variables. So, it follows that f D f Q and f 0 is nothing but, C K inverse C P into f d f q and f 0.

So, from this, if you evaluate this, it can be compactly written down. In fact, f 0, of course in these both, in these transformations, f 0 are the same. But, f D and f Q upper case are related to f d and f q lower case by this transformation.

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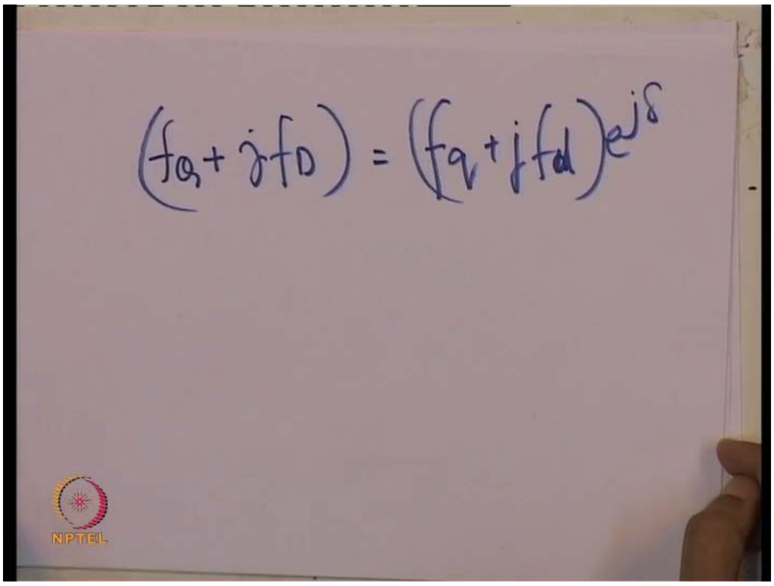

Now, this is, I will not derive it here but, you can compactly write it as shown in this slide. That is, f Q plus j f D is equal to f q plus j f d into e raise to j delta. What I am trying to say here is, since the rotor angle, in the case of a induction machine or rotor position in case of an induction machine is a bit of an abstraction. The rotor angle is a bit of an abstraction because, there is no saliency in the rotor. It makes sense to reformulate our differential equations, not in terms of the parks transformation but, in terms of the crones transformation, which is a, which contains omega naught t as the arguments of the sines and the cosines.

And omega naught, of course is a constant in this case. There is no delta coming into the picture. So, we use such a transformation instead of parks transformation. So, this is what is very important. So, if I actually reformulate the equations, you know, in the capital d q frame of reference, I will call it the capital d q frame of reference or the crones frame of reference.

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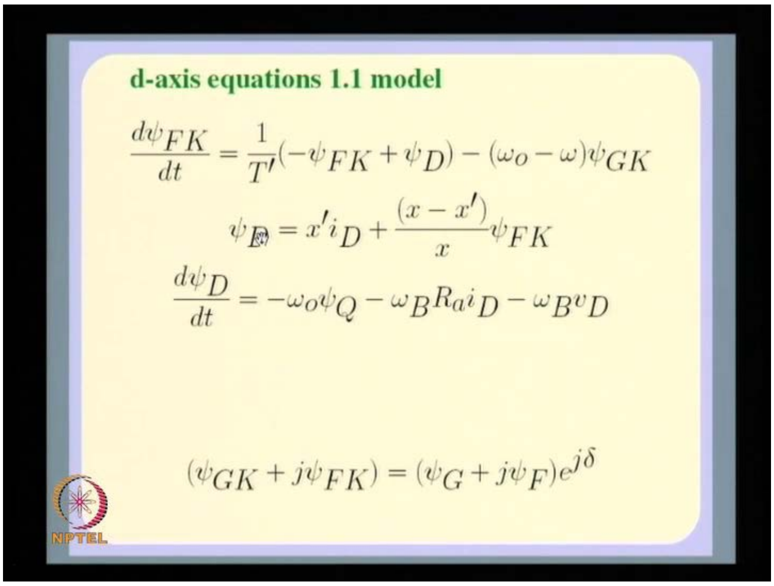

The equations look similar with this additional term. So, if I write down, you know, for example, d psi capital D by d t, the equations for it look the same except here, you get omega naught. So, this is one change you will see.

Another important thing is, I have also changed the variables psi capital G and psi capital F to the new frame. So, actually if you look at the new variables psi G K and psi F K, in fact, they are related to these old variables. So, if I do that, you know, I just substitute for the old equations with the new variables. Your differential equations look like this in the new variables. So, you have got now, new variables psi F K psi capital D on the d axis and similarly, on the q axis, psi G K and psi Q.

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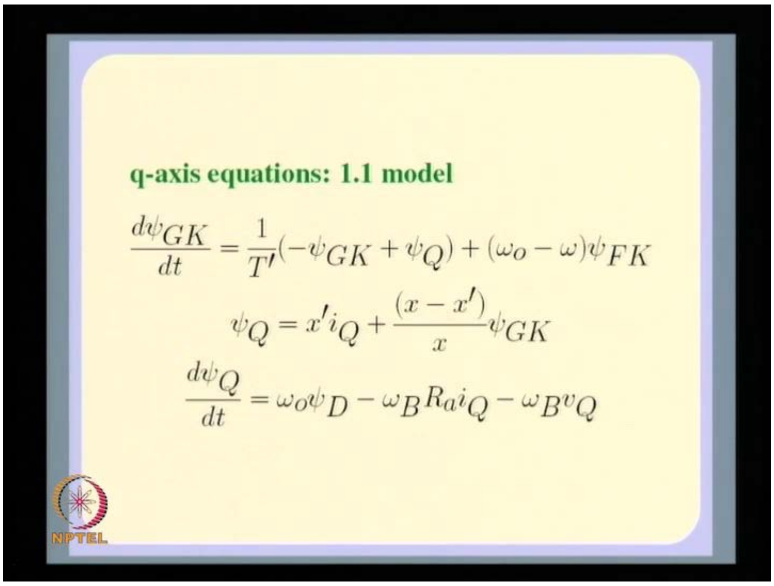

Now, what is the advantage of doing this or why are we writing down the equations in terms of different variables. Now, the important thing here is, if your source is a three phase balance source of frequency omega naught, you will find that v q v capital Q or upper case q and v capital D are in fact, independent of delta. In fact, you will find that they are constants which are independent of delta.

So, although the earlier differential equations of the induction machine was certainly valid, what we have done is, come to a form in which the inputs, v d and v q, are in fact constant in the new variables, in the new new transformed variables. They are not dependent on delta. So, it is better to get rid of the concept of delta or the abstraction of delta in case, you do not have salient pole machine like in induction machine.

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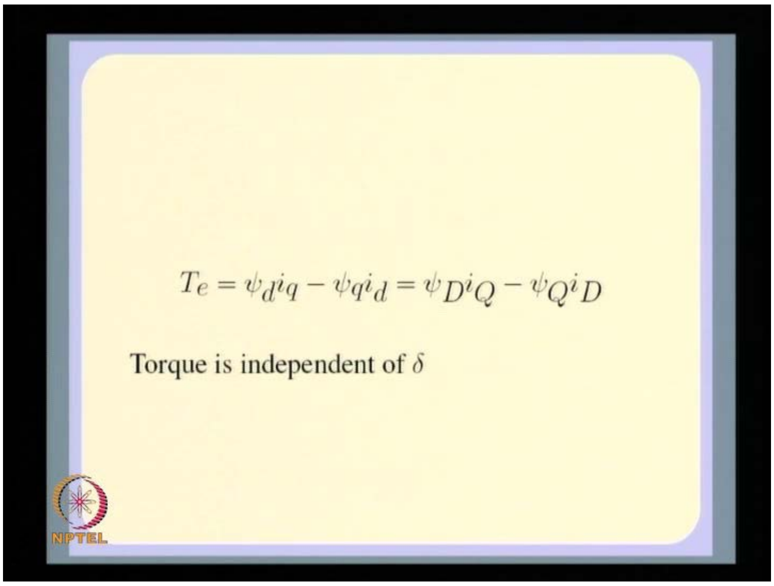

So, the important thing, of course is something which you can prove using this basic relationship which I have shown you here, on the written slide. You can show that, of course and i d plus j i q ,sorry is equal to i Q, sorry it should be Q and this should be D and this should Q and should be D.

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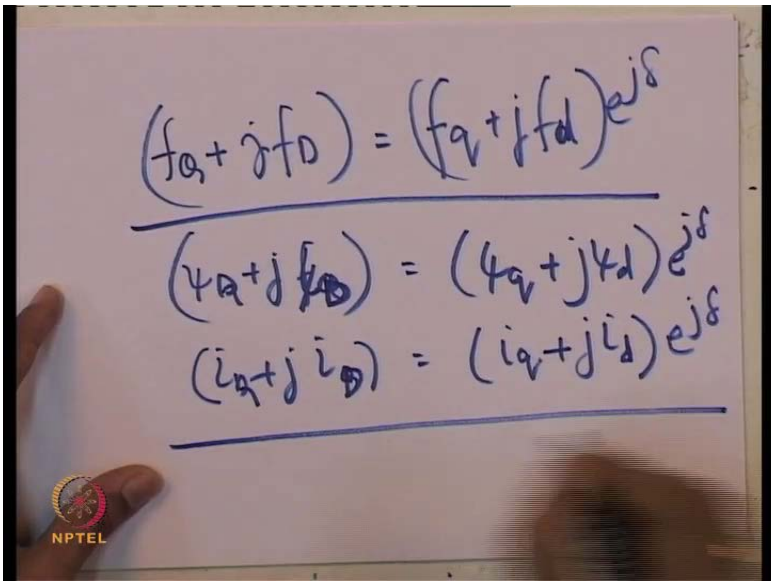

So, if I actually compute psi D i Q minus psi Q i D, it will turn out, that it is equal to psi d i q minus psi q i d.

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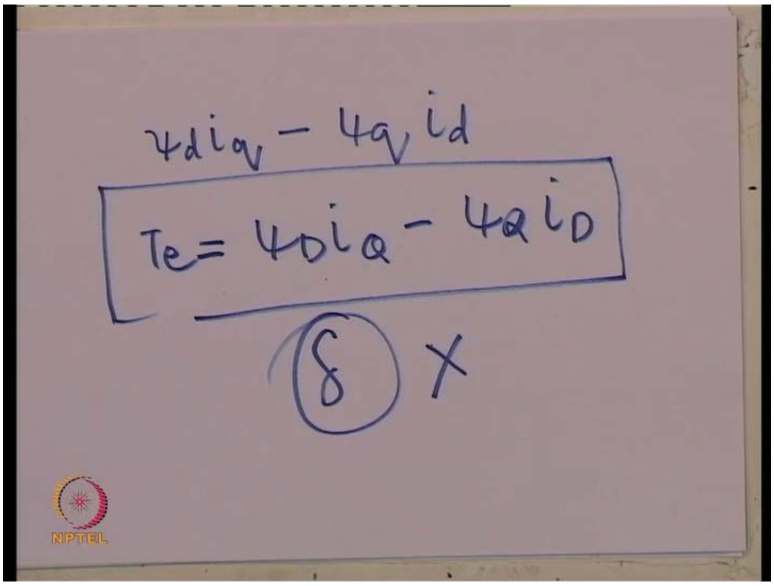

So, what I want to say here is, of course, that the torque equation also, the electrical torque is independent of delta; is not dependent on delta. So, I have formulated my equations in the crones variables or using a transformation, which is, you can say rotating at a constant frequency, instead of using parks transformation and I am able to formulate my equations of an induction machine, so that, it is independent of delta. Of course, that also means that the rotor angle is not important in some sense. So, we know for example, that an induction machine under steady state conditions, under load or no load, in fact under loaded conditions, its speed is not equal to the frequency or the electrical speed of the machine is not equal to frequency of the source to which it is connected to. In fact, if you load an induction machine you get a slip.

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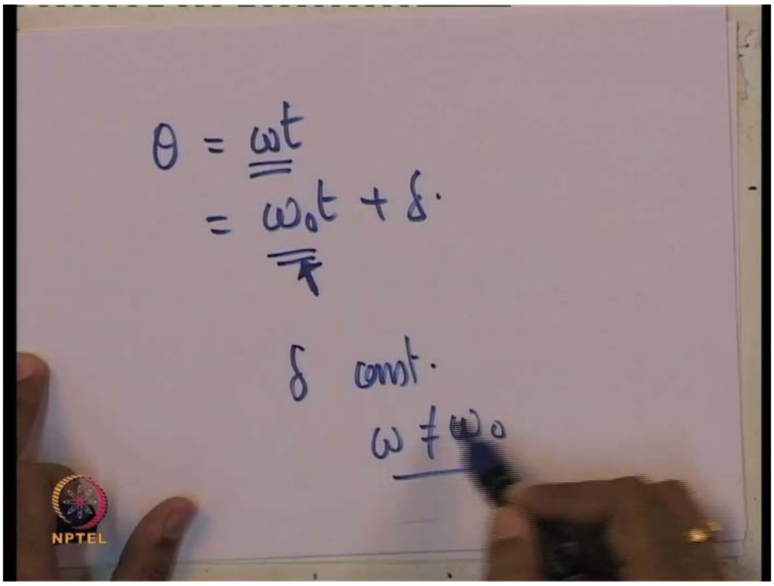

Now, if you took the classical definition of the delta, the delta is nothing but, theta is equal to omega t is equal to omega naught t plus delta. You see that, in case the speed of the induction machine is different from the speed of the transformation and therefore, also of as I told you, it is also equal to the frequency of the voltage it is connected to, voltage source which the induction machine is connected to. We will see that delta is constantly varying if omega naught equal to omega naught. And, you know that a induction machine can operate stably even this is, even if this is true. In fact, there is always a steady state slip when you load the machine.

So, what I have done really here is, come to a formulation using the crones transformation, which is completely independent of theta. The induction machine, in some sense, can happily operate at the speed, which is different from the the the omega naught. Now, this is not true of a synchronous machine. In a synchronous machine, in fact, you will find q, you will find that the torque, in fact is a function of delta. It is related in some way to delta. So, in case delta if not a constant, in such a case, you will find that the torque is also not a constant. But, this is not true as far as a induction machine is concerned.

So, it may be a good idea in induction machine to formulate your equations in the capital D Q frame but, you could formulate your equations in the, you know, the parks reference frame as well. But, the the changing delta, whenever there is a slip is of no consequence eventually. So, that is what really I wish to tell you here. So, you can get the induction machines as equations as I mentioned to you.

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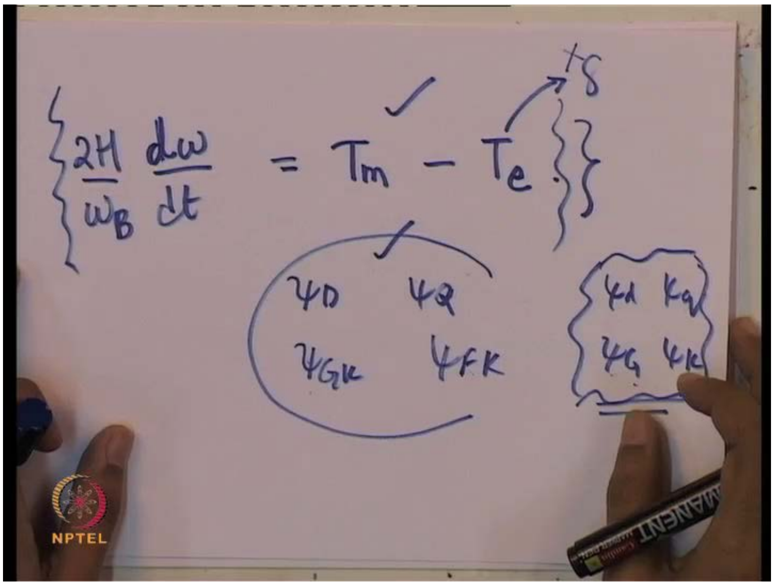

A few more points. First thing is, what is the torque equation of a, of the machine. I told you, we are working with the, the torque equation, in fact, is exactly the same. This is the per unit torque of a machine. In fact, this is correct, where T is nothing but, what I had written some time before; it is this.

So, this is the differential equation which you have to use along with the differential equations of the various fluxes, that is psi D, psi Q, psi G K and psi F K. Alternatively, you can also use psi d, psi q, psi G and psi K but, remember delta will keep on varying in these equations. So, in steady state, you will not find psi d, psi q, psi j and psi k as constants. Since, this is not a function of delta, t is not a function of delta, you do not really actually have to write the separate equation for delta itself. Nothing, if you formulate your equation this way, you do not really have to write the delta equation at all. The differential equation corresponding to delta at all, because, delta never appears in any of these equations.

Now, one of the important things of course is, this is the equation of an induction machine. In fact, it is derived from the equation of the synchronous machine. But, if you want to operate, you want to really study the operation of a motor instead of a generator.

It is important to remember that we, when we first formulated this swing equation or this, in case of the synchronous machine, the direction of T m and T e were as follows.

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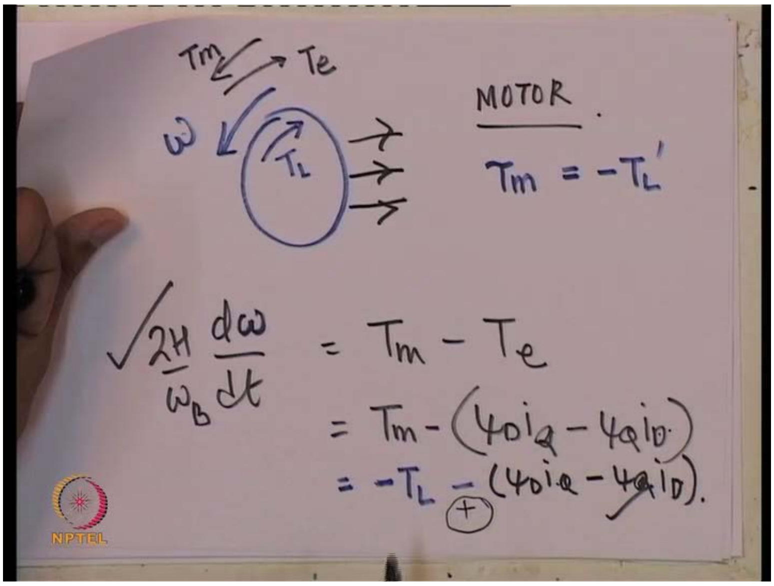

We assume that your machine is moving in this direction and T e is like this and T m is like this. So, it is correct to say that the speed of the induction machine is equal to T m minus T e. You know motor, you know motor remember that T m, is in fact in the opposite direction of the speed of rotation. So, if you are in a motoring mode, T m is in fact minus T L dash, where T L dash is the mechanical load on the machine. So, you do not have T m in a synchronous machine. A synchronous generator T m would have been the prime mover torque. But now, the load torque on a motor would be minus T L. So, if I am writing down the equation, this is the speed of the machine, rate of change of the speed of the machine. This is not slip, remember this is speed of the machine is equal to minus of T L. In case, it is a motor with T L in this direction. T, then you will have minus T L minus.

Now, just remember, of course that, whenever we are writing, this is a correct equation. There is nothing wrong in this equation. It is minus T L, minus of this psi D i Q minus this. So, this is correct. There is nothing wrong in this equation. You see, the direction of speed T is this expression, for T e in this in direction T e in this direction is given by this expression, T L is in this direction. So, this is the correct equation. But, one more small, rather, somewhat minor point is, in case you are studying a motor, another change you would probably like to do is, assume that the currents i D and i Q are going into the motor, rather than coming out of the motor. In our generator convention, we had assumed that the currents are going out. So, these currents are in fact, the currents coming out of the machine. So, all the in all the equations, wherever current appears, it is referring to the current going out of the machine. So, of course, if you change the direction of the current, this will have a positive sign here. Similarly, in other places, you will have to change the sign, in case you change the direction of the current.

So, this something which you should remember that, in case, you are taking the direction of current inward, then you have to change the sign of this. Then, our equations, absolutely self consistence; there is no problem with them. So, this is regarding the motor convention. The second point which I would like to talk is about T L itself. T L itself is the mechanical load on the machine. A particular load, has a, you know, normally we would like to characterize each mechanical load also by some torque speed characteristics.

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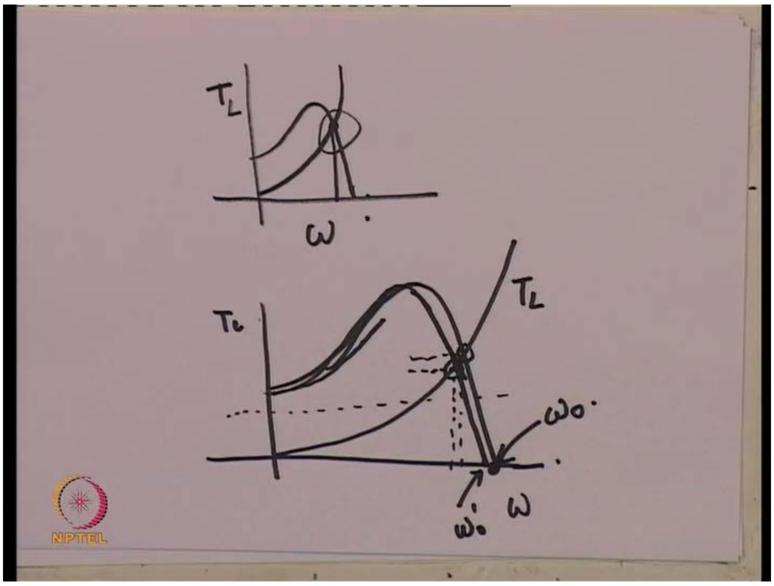

Now, for example, a fan. You know, you can have a, if you have a fan, you will find that if torque speed characteristic is something like this, the torque verses the speed. So, this is the torque speed. This load, this is the load torque verses speed characteristic and of course, if you got an induction machine, which is driving this fan, then the operating speed is given by a point at which both these things intersect.

So, this is the operating speed of the machine, so, or rather the steady state speed of the machine. So, this is the steady state speed of the machine. Now, one of the points which you should appreciate at this junction is that, in case, so, you if you look at a torque speed characteristic of an induction machine, this is the electrical torque verses speed and this is the load characteristic. This is the load verses speed for a fan type load. For a, may be, you may, can also think of a constant tower torque load. For example, if you are lifting up something, you know, like a through a lift, then the torque is a constant. It is not function of the speed. Torque is nothing but, the mass into gravity. This is propositional to the mass in the gravity, which is being lifted, the mass which being lifted against the gravitational force.

Now, the operating speed, as I mentioned was this. Now suppose, an interesting point here is that, suppose this, of course, frequency here is omega naught. At when the slip is 0 or the speed of the mechanical speed of machine becomes equal to omega naught, the electrical torque become 0. Now, an interesting thing is, if frequency changes, what happens to the torque. If frequency changes, you will find the torque speed characters. In fact, the torque speed characteristic in fact changes and it becomes 0 at some other mechanical speed. Because of this, you will find, so if there is a small change in the torque, you will find this point slightly shifts and operating speed also changes. In fact, the amount of the, even the torque changes. So, what you find is, if you are driving a fan type load, you will find that, if the electrical frequency changes from omega naught to omega naught dash, so the source frequency changes from omega naught to omega naught dash, the power output of such a motor would reduce. Therefore, the input power also would reduce to a to a certain extent.

So, in fact, if you got fan type mechanical load, it also implies that the steady state load of your machine, the electrical load of your machine is frequency dependent. This is something which, in fact, this is one of the mechanisms by which load becomes frequency dependent. Will conclude this, our discussion of induction machine with simple interesting dynamical example. This is not something to do with our classical power system analysis but, nonetheless, we are at a point where we can actually analyze this system. That is, the behavior of our induction machine which is connected to just a set of capacitors.

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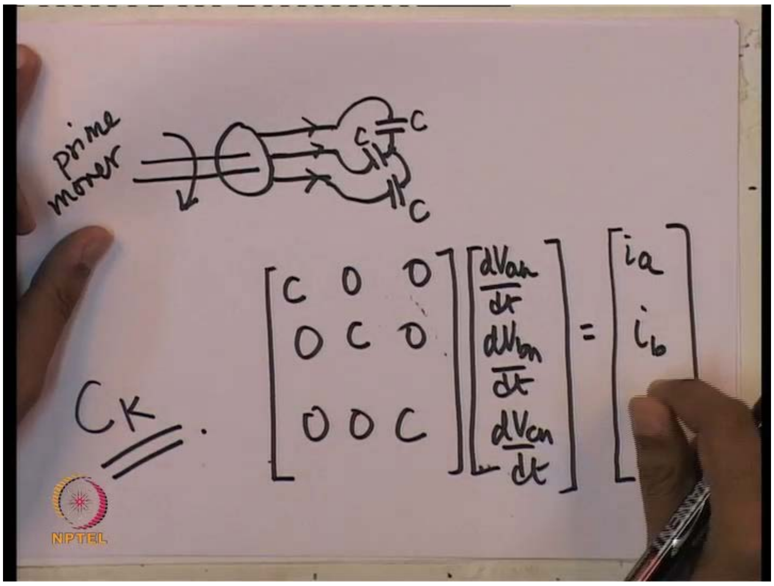

So, if you got an induction machine and you connect to, say a star connected bank of capacitors, like this. The stator winding is connected to a star connected bank of capacitors and suppose, the induction machine is being driven; is not the motor. Suppose, it is a, it is basically like a generator. It is being driven by some prime mover at a constant speed. Let us assume this. So, there is a prime mover, which is rotating the induction machine at a constant speed. The induction machine stator is not connected to a voltage source but, it is connected to a bank of capacitors. In such a case, a very interesting thing is, which is observed in practice is that, the induction machine self excites. That is, you will find that some voltage appears here.

So, if there is some residual magnetism in the machine or there is some residual charge on the capacitor, you will find that automatically if you, of course, connect an appropriate value of this C, is the balanced star connected C, you will find that the machine suddenly self excites. This is the, by the way, very, from physical perspective, very very interesting phenomenon. It practically says that, if you got a bank of capacitors, which is basically just, you know, metal and dielectric and you have got a machine, which is just a ferromagnetic material and copper and you rotate the machine low and behold, you have got some voltage appearing at the terminals of the machine, you know, without having any other magnate or battery available with you.

So, this is something very interesting about self excitation as a phenomenon. What I wanted to tell you here today is that, with the tools which you have right now at your disposal, you should be able to show the self excitation occurs. What exactly self excitation? You rotate a machine with, you know, practically you know, with no excitation source. Except the fact that the machine is being rotated by the prime mover at a certain speed, say the rate at speed. What you want to show is that, if there is some small, you know small, however small in initial condition, you will find that voltage grows and the machine self excites. Eventually, of course, what you normally find in practice is that, the voltage grows up to a point and settles down, because of saturation of the ferromagnetic material which consists the which constitutes the induction machine.

So, if you look at it from a mathematical perspective and analytical perspective, what you have to do is write down the equation of the induction machine. So, we have for example, written down the equations of the induction machine. You do not worry about the torque. We will assume that the prime mover is somehow maintaining the speed of the machine a constant. So, the equations of the induction machine are as shown in this slides. So, the first three equations, two differential equations and one algebraic equation and q axis equations. So, these are the equations is the induction machine. You will have to just interface them with the equations of the capacitor bank in the d q frame of reference.

Now, remember, we have not written down the 0 sequence equation. We will assume absolutely a balanced set up and therefore, the 0 sequence equations are completely decoupled. So, we do not have to include them in this analysis. They are decoupled completely from this, our set of d q equations. So, what do you need to do, well what do you need to do is right down the equation, the capacitance, capacitor. So, if you look at the equations as they given here, you will find that the equations of the capacitor are C, C and C, d v and by d t is equal to the current i a, i b and i c.

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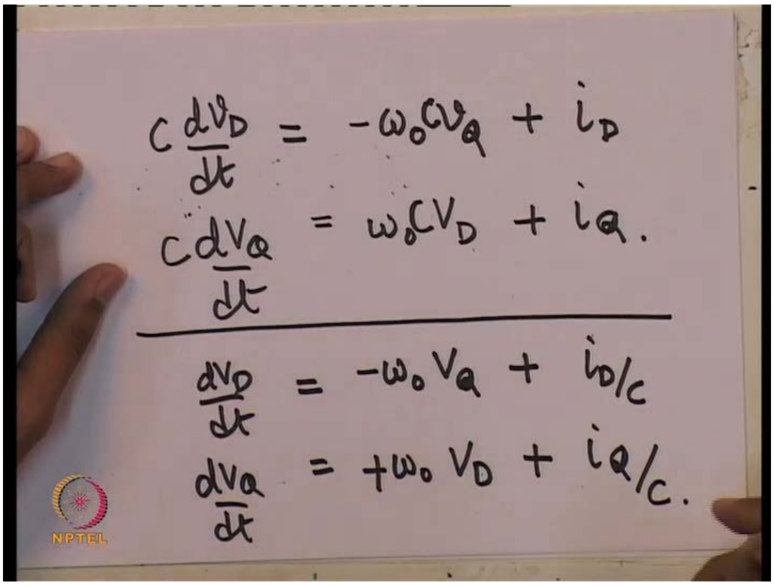

So, what you get here is, if you transform these using the transformation C K, what you will find is this. Something I will leave in the exercise to you, d v D by d t is equal to, so, in fact, this will be omega omega naught C. So, if I, actually if I write this equation again, so you will have d v D by d t is equal to minus omega naught v Q plus i D by C.

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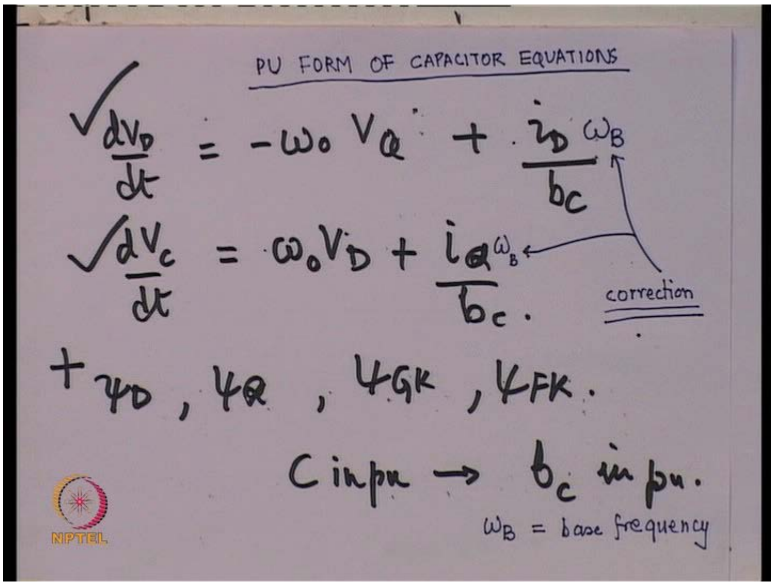

You will have, of course, now we can write this in per unit form, in which case you will have d v D by d t is equal to minus omega naught v Q. Now, these are in per unit, plus i D per unit divided by c per unit. But, in case you are in per unit, you can also write it, c in per unit is the same as the susceptance in per unit. So, there is one, so normally, you will be given the susceptance in per unit. So, that is why I am writing it in this form.

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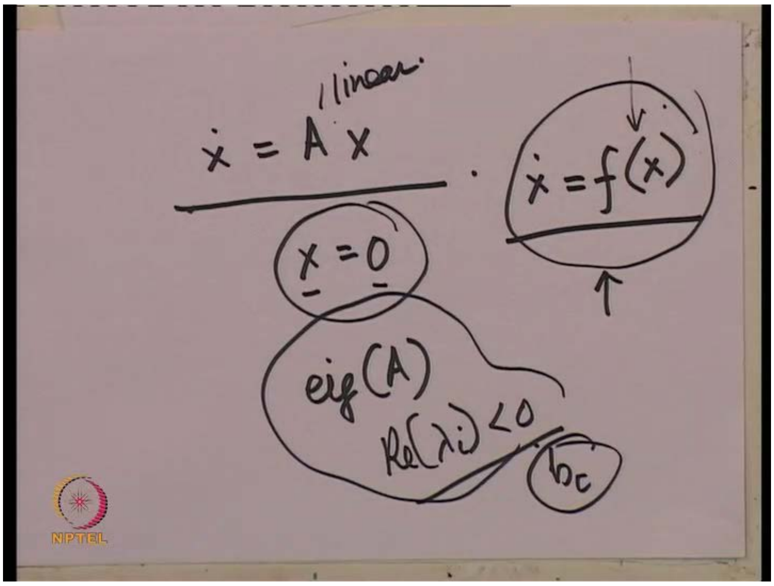

Now, the differential equations corresponding to this, plus the differential equations corresponding to psi D, psi Q, psi G K and psi F K can be combined together. I mean you can write the whole thing down as in the form X dot is equal to A X. There is no other source of excitation. There is no other source of excitation. Will assume speed is constant. So, we do not write down the prime mover mechanical equation. We just assume that speed is constant. Now, what you need to do is of course, what are the equilibrium conditions? X is equal to 0, I mean all the states are equal to 0, is an equilibrium condition. Then, what you need to do is, find the Eigen values of A, for different values of b c and what you will find very surprisingly is, that is, such a system will have Eigen values with real part less than 0, if b c is greater than a certain value. In fact, b c is greater than a certain value, which is related to the reactants. You can actually get the condition explicitly.

So, very very interesting thing can be analyzed mathematically, that self excitation will occur in a induction machine, if connected to a bank of capacitors and driven by constant speed prime mover. You will find that at a certain value of this capacitors, the system is unstable at the equilibrium point. As a result of which, you will find at, you know, you will find at some voltage appears, you know, for any nonzero initial condition which exists. So, it may be some residual flux in the machine or some residual charge on the capacitor, then the machine will just simply self excites. So, it is a very, you know, interesting and exciting phenomenon. Of course, one important point, which in this case is very critical, is that the equation which I have derived for the synchronous machine, as well as the induction machine, is shown to be linear.

Now, remember that, if you have got a linear system and it is unstable, you will find that for any nonzero initial condition of the state, it will simply, the system will simply blow up it; will just go on increasing to infinity. This does not happen in practice. The reason is that, the ferromagnetic parts of the machine tend to saturate and if that happens, what it also means is that, our model here, linear model here is inadequate to capture that phenomena because it is a linear model. So, in fact, some time before, I had mentioned to you that, there are, you know, ways which are not very theoretically, you know, trueable kind of ways to account for saturation of a synchronous or in induction machine. If you actually did manage to account for saturation and make this model non-linear, in that case, you you ought to have been able to show that, there would be another equilibrium point and the system would go to that new equilibrium point. That equilibrium point is not a 0 equilibrium point.

So, what I mean to say is that, with saturation, the system becomes x dot is equal to f of x. This is the linear system which we handling right now. What we are, we probably what, the thing which you can prove is that, if b c is greater than a certain value, you will find that at the equilibrium point, x is equal to 0, this system is unstable. Therefore, the system tends to self excite. If you take a non-linear system, first of all, you may be having more than one equilibrium point, if you take into account saturation. One of the equilibria is stable and the other is unstable. So, if you look at self excitation, we will find that the system, kind of, you know, settles down to a new, you know, let say an equilibrium, periodical equilibrium.

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So, that is one interesting thing which you can further chew upon. I will now show you an video graph experiment, which really shows the self excitation phenomena. What we will be seeing in this demonstration is, in fact self excited induction generator, which is driven by a dc generator which maintains the speed practically a constant, rather induction self excited. We will just do this again once more. What I will be showing you now is a experimental demonstration of self excitation phenomena. In this experiment, we have an induction generator, which is driven by a dc motor, which keeps the speed of a keeps the speed of rotation constant and we will connect a bank of capacitors across the induction machine. What you will see is, of course, that if the amount of capacitance is adequate, you can actually have a voltage build up at the induction generator, at the output of the induction machine terminals.

So, that really shows you, what is known as self excited induction machine phenomena. Now, let us see the demonstration.

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So, what you are seeing here is a dc machine, dc motor which is cup and this of course, is the bank, three bank, three banks of delta connected capacitors, each with a switch, which we shall connect across the induction machine terminals.

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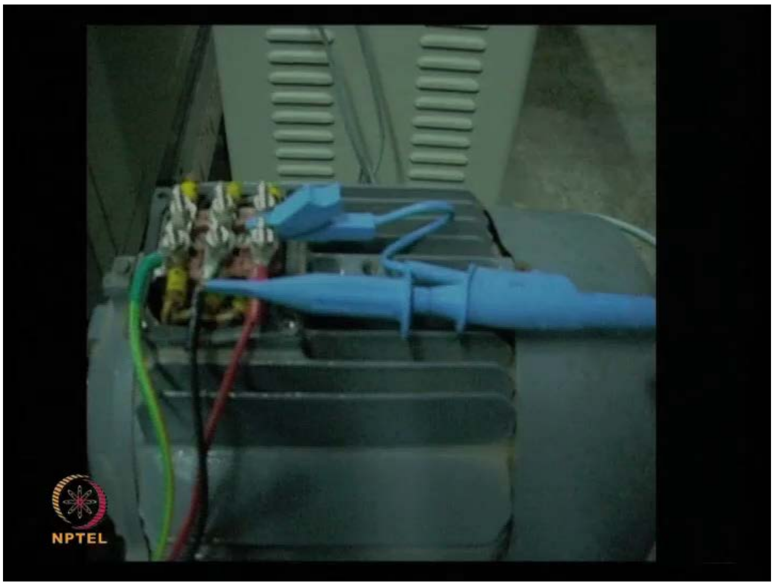

So, the dc motor will drive this induction machine which you are seeing and we shall, of course, monitor the output voltage through one of these probes, which is fed to this oscilloscope.

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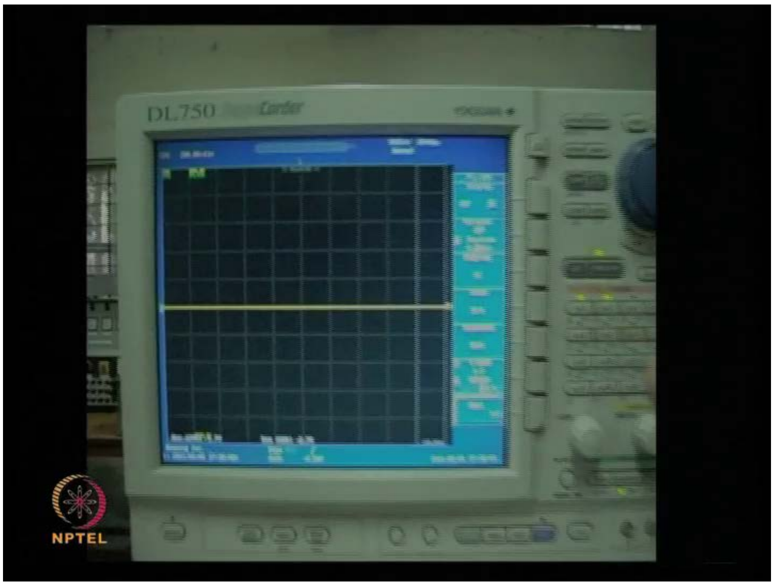

So, what will do now is, start this machine, dc motor. There is one more motor, one more machine right in the middle also but, that is playing no role as it is not energized. There is one in the middle also, here.

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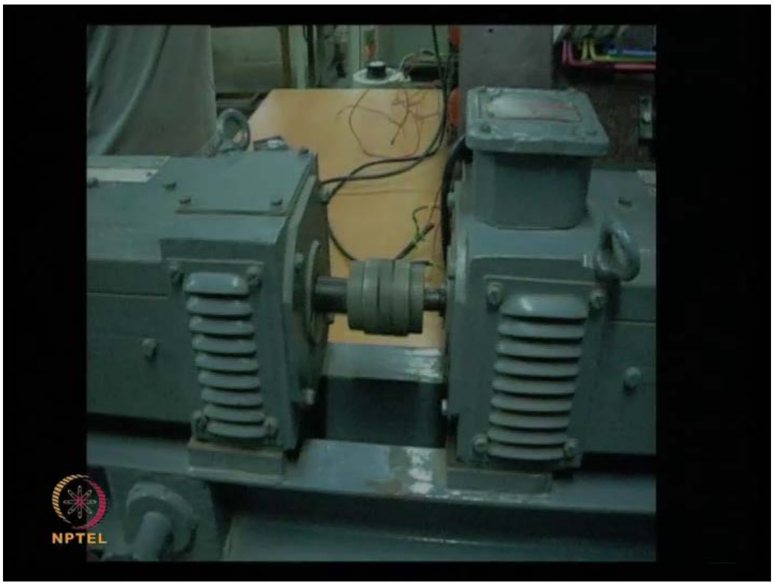

So, the dc motor is being started, the machine on the left. So, we build up the speed and you see, that the output voltage of the induction machine, there is probably some residual magnetism because which you get little bit of voltage. But, it is not really much. So, what is happening? Now, I will switch on one of the banks of the capacitors but, you see that really there is no voltage really building up at the induction machine terminals. That is, just a little bit, probably due to the residual magnetism. One of my students is removing one addition probe, which is not required for this experiment. It has no bearing on the outcome of the experiment, of course.

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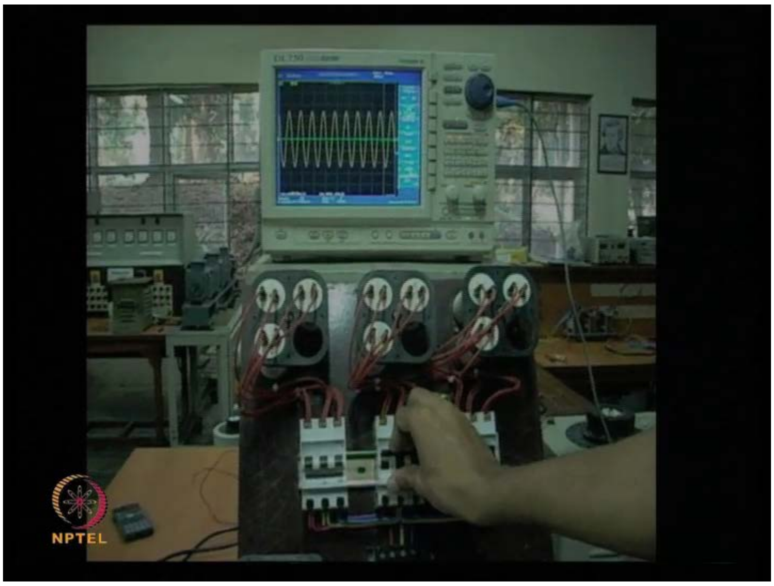

Now, will switch on the second bank and what you see is the voltage builds up spontaneously at the terminals of the induction machine. So, this is an example of self excitation. If I switch off the capacitor bank, one of the, one of this capacitor banks then, you see that the machine gets the excited and will the voltage will no longer be sustained.

So, we will do this experiment again. We will switch on this capacitor bank again, but remember, that the capacitor bank may have some residual charge on it. So, what we need to do is discharge these capacitors, the d agent energized capacitors. So, we will just discharge through a resistance. So, we see what, that is what we are doing right now. We will discharge it here. yeah There is no need of doing that because, it is not energized. It was not energized anyway. Now, we will just redo that, yeah you see that the machine self excites again. So, you can have an induction machine, self excitation by connecting capacitors across it.

So, this is so much about induction machine model. Remember that, when you have got large induction machine, you may have to model them in power system studies. So, for example, large power plant auxiliaries or induction machines in large industries, you may have to model it by dynamical model. Alternatively, you can try to even model smaller induction machines or in some cases, even a larger machine depending on the nature of the study you are interested in, by a static polynomial kind of model. I told you that, induction machine, which we say with a fan load, if you look at static characteristics, it shows the frequency dependents.

So, let us now move on to another, you know, equipment or another component of a power system, which is absolutely important, is a transmission line. You know, transmission line is an element. It is a distributed parameter element. In the sense, that it is defined by, you know, if you look at the, if you look at from first principle, in fact, all electrical equipment from first principles, would satisfy Maxwell's equations. But, if you look at some of the low frequency behavior of transmission lines, you can model them by distributed parameter equivalent, where you have got shunt capacitors in series inductance. But, these are distributed parameter devices.

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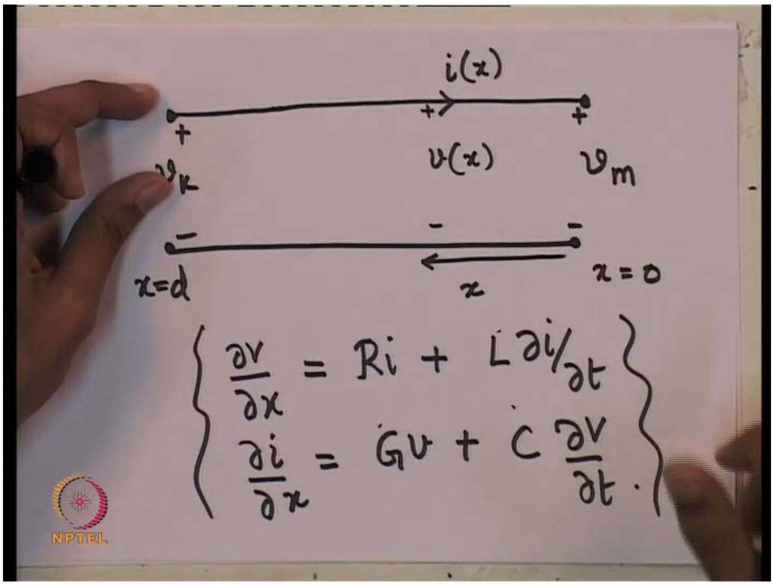

So, the basic mathematical equations which which describe transmission lines are given as follows. So, if I just right down this, the equations, so, if you have got a point k, this analysis practically follows what is given in Sauer and Pai's book. So, if this is, I call x is equal to 0, point of the transmission line. This is x is equal to d, it is a transmission line element with of length d. In that case, an x is measured in this fashion. So, you have got, this is, v of x and we will assume current in this direction, i of x. Then, the equations of transmission line are, this is something, of course, I would not derive, it is a big exercise in itself to, from the basic Maxwell equations to show that these are the roughly the equations will describe the transmission line for 50 hertz kind of behavior. So, these are the equations of the transmission line. They are partial differential equation and the current direction and voltages at a distance x are described by these equations.

Now, R L G and C are per unit length parameters. So, just one small note of caution. It is, you may think L is in landry, no it is landry per unit length and C is capacitates per unit length. So just, this is an important thing. These are the partial differential equations will describe transmission line, you know, especially for power, you know, power frequency behavior. So, most of our, these equations are valid for that kind of thing.

Now, the solution of, the general solution of these equations is something which you have done in your under graduate years, in a course on electromagnetic or perhaps even in maths or power system itself, is a, the current at any time t and distance x from from one end, is given by, just a moment, just have this in view.

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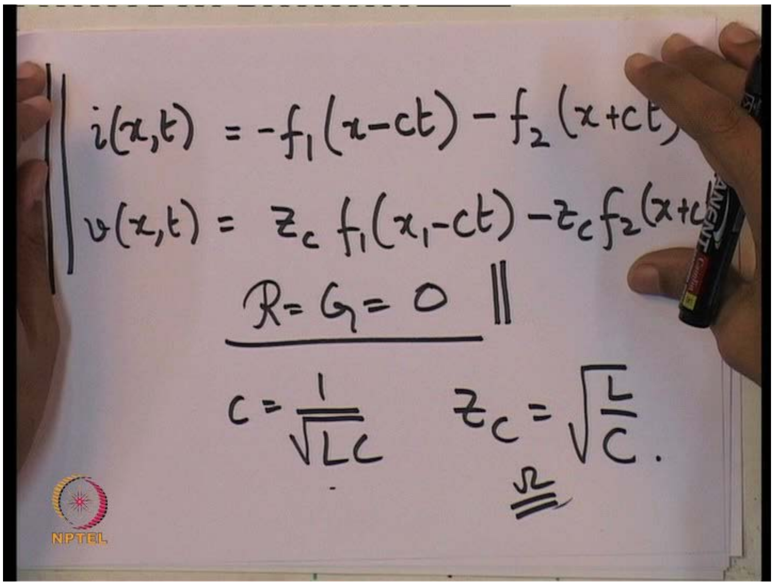

So, v of x at a distance x from this side, is given by, we are just following the notation of Sauer and Pai, where in this, when you are, this of course is true only for last less line, that is R is equal to G is equal to 0. The resistance and the conductance, shunt conductance per unit length is actually 0. So, in that case, we get what is known as a wave solution of these partial differential equations and where, C is equal to 1 by root L C and Z C is equal to root of L of C. This is the the dimensions of homes of a resistance. This is, in fact, the speed, propagation of velocity. So, it has got the dimensional distance per unit time.

So of course, it is important to note that this equation, this solution is valid only for loss less line. e h v lines, in fact, you will find that they come to this, close to this loss less behavior. The resistance is low compared to the reactive components. That is, that resistance for unit length as compared to the x per unit length is much smaller. Remember that, these functions f 1 and f 2 really dependent or dependent on what what are the conditions on the boundary. So, if I tell you about the condition at the boundary, for example, what is connected one end and what is connected at the other end and I tell you its behavior with respect to time, in that case, I should be able to tell you, what the behavior of I, any other i or v at any point on the line.

So, this is a solution which valid for loss less line. Now, the point of course, which I have I want to make here is that, this is the solution of the transmission line under transient condition of a loss less line, under transient condition. If you are talking of the sinusoidal steady state, if you are talking of sinusoidal steady state behavior, in such a case, you you must be recalling, that if I take a transmission line, this is k and this is m.

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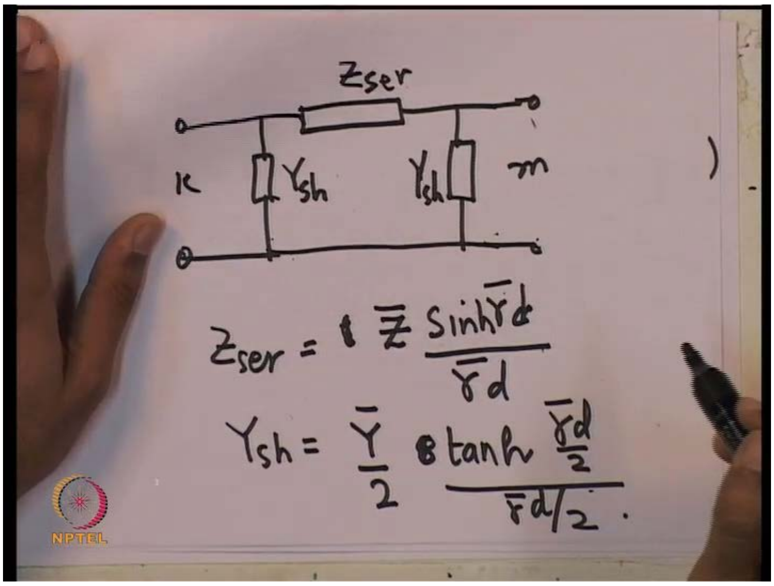

I will be able to mimic or at least capture the behavior under sinusoidal steady state condition. What do we mean by sinusoidal steady state conditions? For example, I applied voltage source here and a way for all transient to settle down and then, I measure for example, the terminal behavior here. You know, I find out, for example, what is the voltage which appears here; say this line is open circuit.

Then, when I say sinusoidal steady state means that after all the transient are died down, what is the behavior? So, you have already done this sometime previously in your under graduate years. So, if I call this as, you know, z s and Y s s h Z series and Y shunt, will find that z series is equal to Z bar into sin of gamma bar d by gamma bar d sin h, hyperbolic snitch function and this Y s h is equal to Y bar by 2 hyperbolic tan function. So, in fact, so if you take sinusoidal steady state response, you can use this pi circuit.

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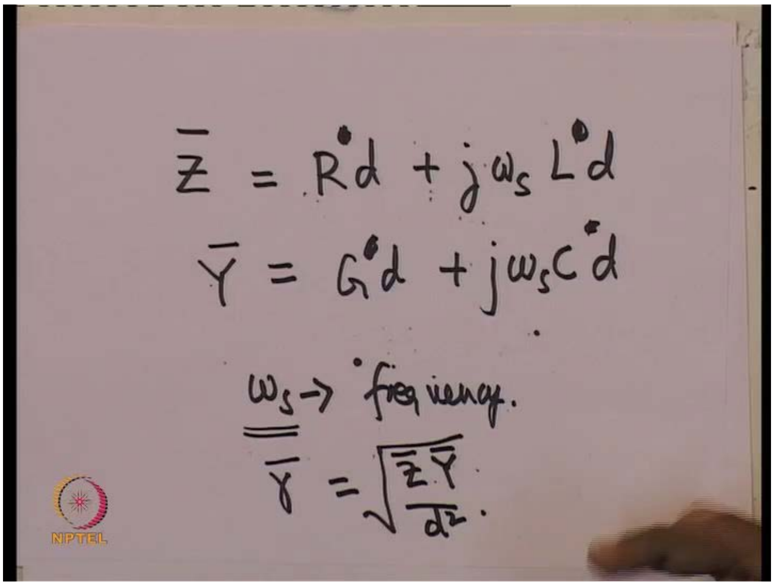

You can use this pi circuit with an impedance. z c is impedance and shunts, shunt admittance given by these values, where Z bar is equal to R dash d. d is of course, distance and Y bar is equal to G dash d plus j omega s C d. Omega s is of course, the frequency of the sinusoidal sources.

So, when we are taking of sinusoidal steady state, the frequency which corresponds to the sinusoidal sinusoidal are omega s and of course, root of Z Y Z bar y bar divided by d square. So, this is the sinusoidal steady state representation of transmission line as a two pot network. Remember that, this is, this solution is valid only for sinusoidal steady state conditions. So, as is mentioned in a, the book by Sauer and Pai, we should resist the temptation of trying to use this lumped pi equivalent to pot equivalent of a transmission line under sin, which is valid under sinusoidal steady state conditions. We should resist the temptation of using it for all kinds of transient analysis.

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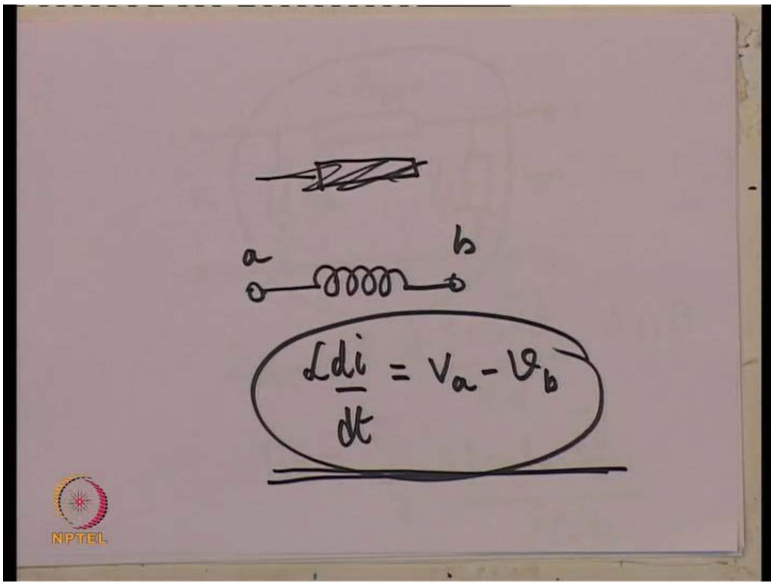

So, often you will find, you know, for example, very implicitly, people, for example, represent the transmission line, even under transient conditions by a series inductance, a short transmission line by series inductance and then, write down the transient equation L d i by d t lumped transient equations like this. Now, is this correct or no? That is the basic doubt you may have. So, let me just tell you what situation is this. You have got the differential, partial differential equation corresponding to the behavior of the transmission line which to some extent can be set to be physically valid for the studies normally encountered for power system, in power system analysis. You can use distributed parameter differential, partial differential equation model.

It has got wave like solution in transients. It has got traveling way kind of solution. Under sinusoidal steady state conditions, you will find that the behavior of a transmission line can be represented by two pot network consisting of lumped, a lumped pi network. Now, can we, the question is, can we utilize this lumped pi equivalent which comes, the r n l corresponding to the lumped pi, you know, you have got this impedance z series and y shunt, which is you just wrote down; can we use them? For example, if you look at this y series and shunt, if you look at the equations which come about, you will get basically, a plus j b. Then, you divided, divide this b divided by omega s that will give you the equivalent inductance to be used in this sinusoidal steady state model. Then, use the lumped parameter differential equation once we get the L from this and then, use it in our analysis for transient behavior.

We should avoid this temptation because, rather we can, and in fact we do it sometimes. The question is, is it valid? The answer is, strictly speaking, no. It is not valid but, under certain circumstances, you will get close behavior to what you observe, rather you can use this kind of approximation of representing a transmission line by lumped parameters obtained from sinusoidal steady state model. But, of course, you will be committing some errors because, this is not really hard, how the transmission line behaves during transient. It is a traveling wave behavior.

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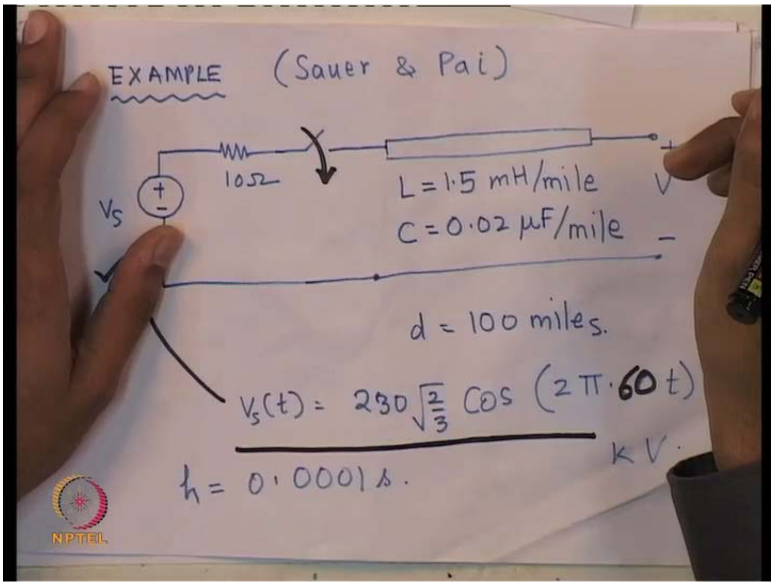

So, there is a example given in Sauer and Pai books, which is very apt and in which, he has asked us to find out, yeah it is basically an example here from Sauer and Pai. You have got a voltage source V s and it is connected to a resistance of 10 ohms. This voltage source is switched on, this voltage source is a sinusoidal voltage source; it is 230 kilowatts line to line. So, we are just talking of single phase model of transmission line. So, 230 kilowatt line to line system. So, the phase to neutral of this will be 230 divided by root 3 and the peak value is 230 into root 2 by 3.

So, this is the voltage source here. It is a sinusoidal voltage source of 60 hertz. So, you have 2 pi into 60 into t. This is switched on to a resistance of 10 ohms to 100 mile transmission line, distributed parameter transmission line with parameters L and C given as 1.5 mile hendry per mile and capacitors per unit length is 0.2 micro ferret per mile. Now, the thing we need to check is, when we do the simulation of this. How to do the simulation of this is something will discuss in the next class. When you do simulation of this and compare it with what we get if you simulate a lumped equivalent.

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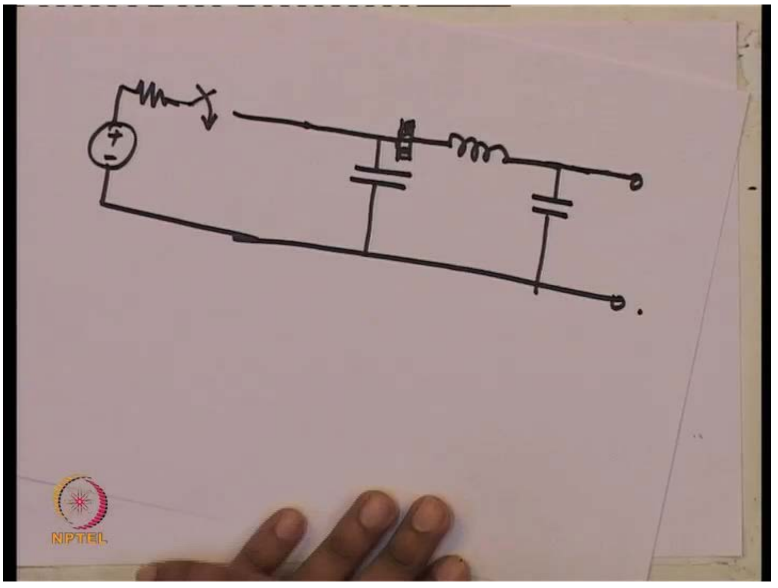

Same system except that now, we are going to use lumped equivalent. I am sorry, you will have an inductances series, inductance lumped equivalent of the loss less line.

So, we have to compare the behavior of this model of transmission, of distributed parameter model of transmission line with what happens, if you take lumped equivalent of a transmission line based on the sinusoidal steady state representation. So, that is an interesting thing and will compare the results and discuss this problem in more detail in the next class.