Power System Dynamics and Control
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Lecture no. # 30
Excitation System Modeling
Automatic Voltage Regulator Linearized Analysis

In the previous class, we saw an interesting simulation in which, when we the change the operating point with an AVR in action, you found that there was a increase rather the system did not stabilize at a new equilibrium point you found that in fact, the oscillations were increasing the in fact, the oscillation or the swings you can say were increasing with time. Now, before we proceed forward with this may I just tell you that this kind of thing is actually observed in practice in fact, there have been situations in which the grid operators are noticed that seemingly spontaneously, when the operating point or rather when the operating point shifts, I mean this of course, is not you know tracked by a operator for every small change I mean for example, a load somewhere in the system could change and your operating point changes. In such a case, it has been observed in certain circumstances that the system does not seem to settle down to the new equilibrium point.

So, we in the previous class, we actually shifted over from doing the simulation to trying to trying to understand this using Eigen value analysis now of course, the system behavior essentially is non-linear as a result of which, if we want to analyze this using the linearized theory that is Eigen value analysis, we would need to linearise the differential equations and in order to do that we will have to actually first of all find an equilibrium point because when I say linearization I am basically, trying to characterize the system, in the system for small disturbances around an equilibrium point. So, your system, in fact the linearized system is actually dependent on the operating point.

Now, this also means that when I change the operating point the system Eigen values change. So, it is probably not very surprising that your system behavior does change with change in operating point, but it still would be nice to really analyze this particular system by linearise analysis and really try to predict using Eigen analysis, that indeed certain operating points are unstable.

Now, the starting point for the simulation was, we just synchronize the machine and then there after we gave certain disturbances like step changes in torque or the voltage regulator reference. So, we actually went to different operating points or we tried to go to different operating points by essentially, changing the inputs to the differential equations. So, the inputs of course, being the mechanical torque and the voltage reference.

But for Eigen analysis, what we will assume or what we need to do is we have to first find out the equilibrium conditions corresponding to an operating point itself, then linearise the system differential equations and once you that you carry out the Eigen value analysis on the resulting state matrix, the linear system of the linear system. So, the first step in fact is trying to linearise the system or to do that of course, we need to compute the operating point. The first step in operating point computation is. In fact, doing a load flow a load flow or a steady state analysis of the system. In fact, a load flow computes the steady state value of variables in the electrical network.

But what we need to also take out the initial or the equilibrium values of the states the states of the system. In fact, are the fluxes of the synchronous machine the currents well the currents are algebraically, related to the fluxes you also have a state corresponding to the excitation system. So, you have to effectively compute the equilibrium values of all these states, use those equilibrium values in your computation of the linearized system. Remember the linearization involves computing the you know rather computing, the partial derivatives of the non-linear terms and evaluating them at the equilibrium point.

So, equilibrium point computation is the first step. In fact, is the first step of even simulations of course, our simulation started off with synchronization. So, luckily we were the computation was very simple in the sense the currents were 0 and back calculating all the states was very simple, but when you have got a synchronous machine already connected to an say an infinite bus through a transmission line of reactance x.

We have to specify, what at least the broad operating conditions which exist of the load flow solution and then back calculate in a systematic manner. The actual equilibrium values of the states corresponding to this operating conditions, putting it in another way we have specified for example, what the terminal conditions of the machine are like this, it is giving out this much power the electrical power, it is also you know having a terminal voltage of say one per unit.

So, this is like a specification which you are giving, which describes the operating condition from that we actually need to compute. What this value of the rotor angle say delta is what is the value of the other fluxes are and so on. Which corresponding correspond to steady state corresponding to this operating point. So, today's lecture we will continue our linearized analysis.

In fact, in the previous class we to quite a bit of time to actually obtain the steps towards getting the equilibrium values of the various states. Now, it is a bit it was a bit tedious I admit. So, what we will do is do quick recap using slide. So, that I hope whatever you did not or found difficult to understand there will become immediately clear here. So, our next step is to just I will just outline the steps required to do the Eigen value analysis, the first step is get the equilibrium condition.

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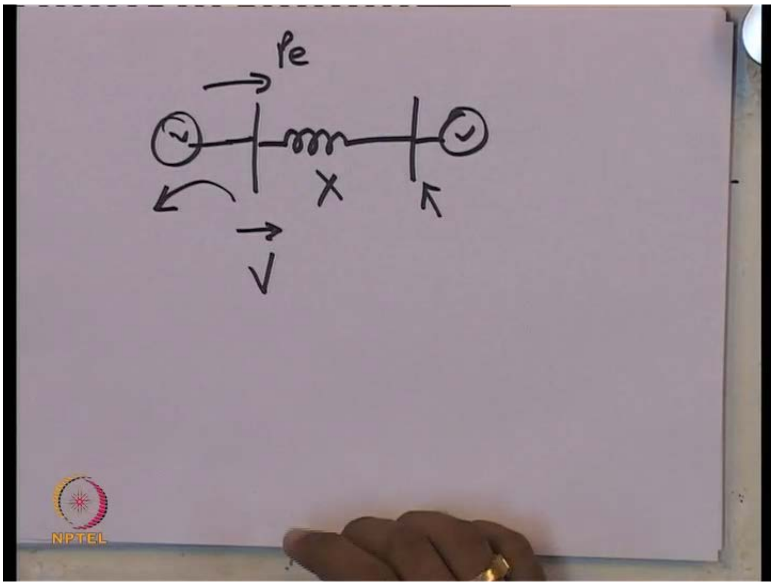

So, first thing I will specify. What are the things which I specify you have got a synchronous machine which is connected to an infinite bus. The infinite bus voltage magnitude and angle is specified the reactance of the lines is specified, we are assuming that the generator power output power P e specified as well. As we have also specified the voltage magnitude from these operating conditions and the parameters of the machine compute first the equilibrium conditions of the system remember that, the synchronous generator itself is has got a voltage regulator whose form we have discussed in the previous class.

So, let us quickly go through the steps involved in linearized analysis, we had initially planned to of course, model I will give you I thought I would give you the models of all the various components of a power systems and then move on to analysis, but I think this is the better idea. Let us do our analysis side by side with the modeling. So, today we will do the linearized analysis of the automatic voltage regulated simple power system. Now, let us pay attention to the, to the slides which I am showing to you now.

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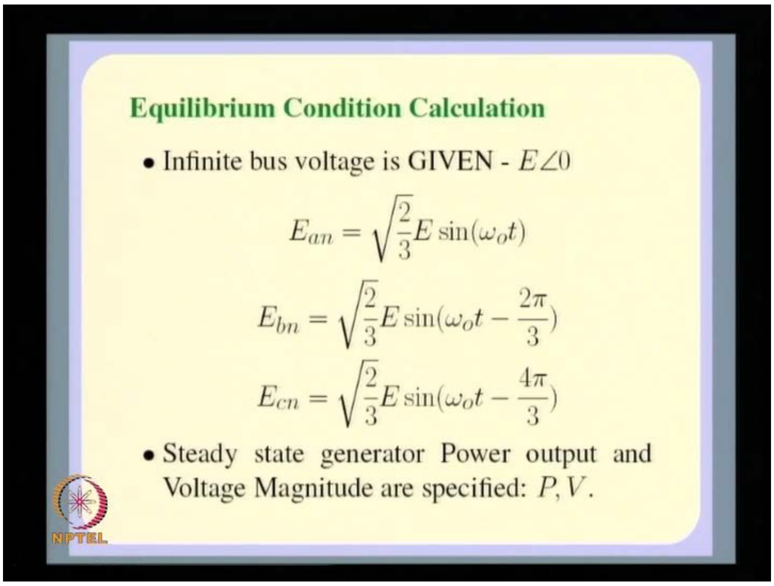

So, the first step in computing the linearized rather the equilibrium conditions is first point is the infinite bus voltage is given that is it is E angle 0 which effectively means that E a n, E B n and E c n are these this is what I mean, when I say the infinite bus voltage is E angle 0. What is specified as I mentioned some time back is that the real power output and a voltage magnitude is specified P and V.

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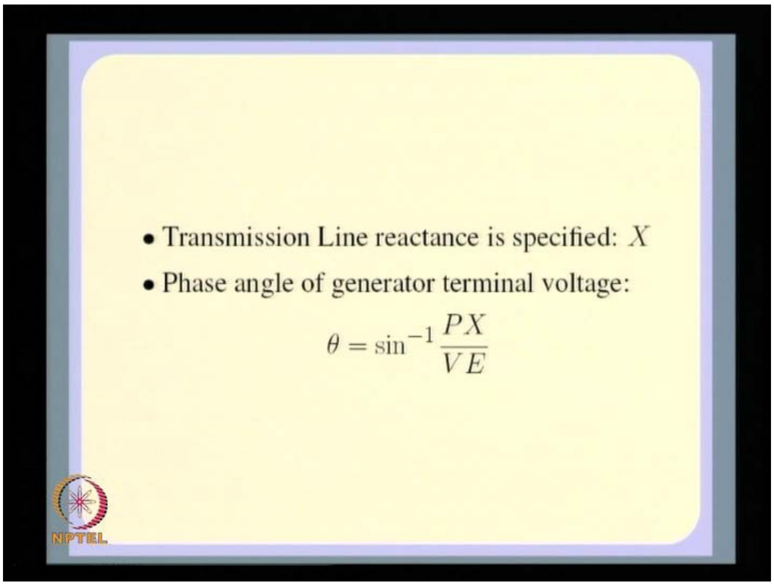

This transmission line reactance is also specified, the phase angle of the generator terminal voltage theta is computed by the formula sine inverse PX by VE. Note that in this context theta is the phase angle of the terminal voltage it is of course, a constant in steady state the reason why? I bring this to your notice is in another context we have used theta as the rotor position.

So, we do not define a new variable here we will just continue with what we have done, but you should keep in mind that theta is the terminal voltage phase angle and not the rotor position, which was used in another context. So, to avoid notational confusion I am clarifying this point, we assume that the transmission line has been modeled by a toy model it is just a reactor effectively of X, it does not have a new resistance.

So, the using the formula for power flow you know the power 3 phase power flow is V into E. The line to line voltage r m s magnitudes of both ends divided by X into sine of theta, theta minus 0 the angle of the infinite bus is of course 0. So, that is how we get the expression for theta this has to be calculated from the values of PX, V and E which are specified.

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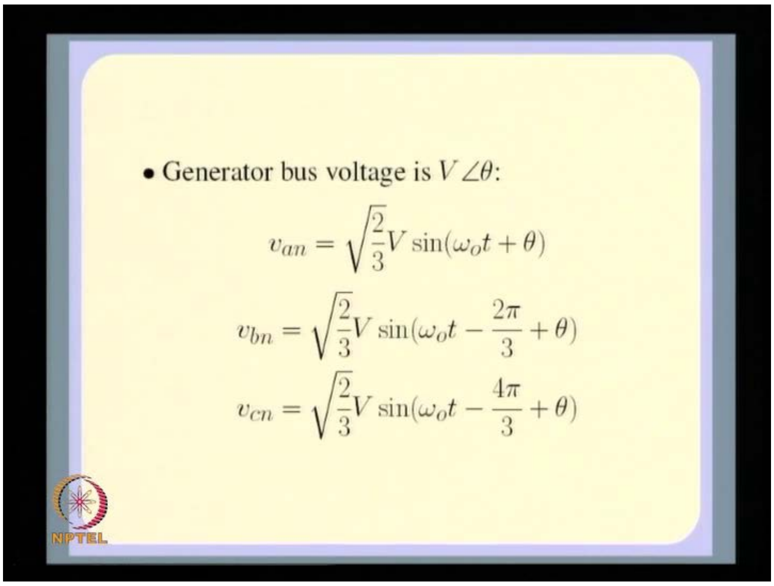

So, theta is obtained what does it mean? It means that if I get theta it means V a n, V b n and V c n are as shown. So, theta appears in this these sinusoidal terms corresponding to the voltage a to neutral b to neutral and c to neutral. So, we are assuming here of course, a star connected system, we are not considering any unbalance that is one point which you should recall. So, this is what I mean by V angle theta. So, theta is also obtained because we know the power specification.

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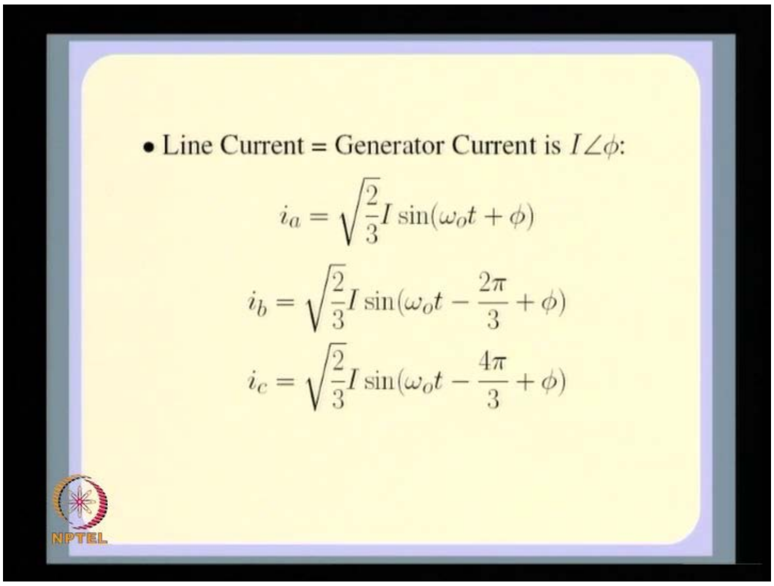

The next step is of course, getting current suppose current is I angle phi which also means i a is root 2 by 3, I sine omega is 0 t plus phi. So, if I say current is I angle phi this is what I mean are the line currents. So, this is what I mean. So, what is the value of I in such a case, if i a is related to I in this fashion then I and phi capital I and phi are given by simply this.

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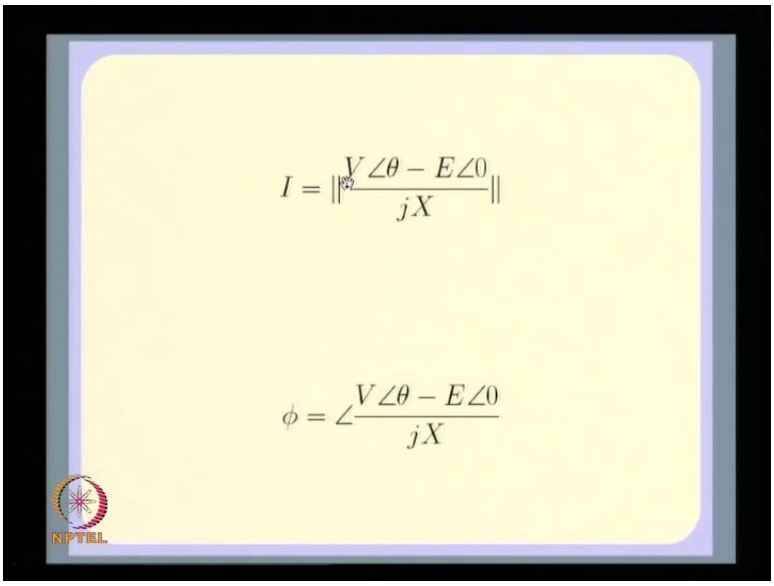

The magnitude of V angle theta minus E angle 0 divided by j X. So, I capital I is equal to V angle theta minus E angle 0 by j X and the phase angle is of course, the phase angle of this quantity itself. So, the phase angle of V angle theta minus E angle 0 by X is the phase angle of the current. So, once you have got theta you can also get I and phi and therefore, you can get i a, i b and i c the instantaneous values. So, if you start off with the instantaneous values of the infinite bus and if you are given PX and V you can get I and phi and therefore, get i a, i b, i c as per this formula.

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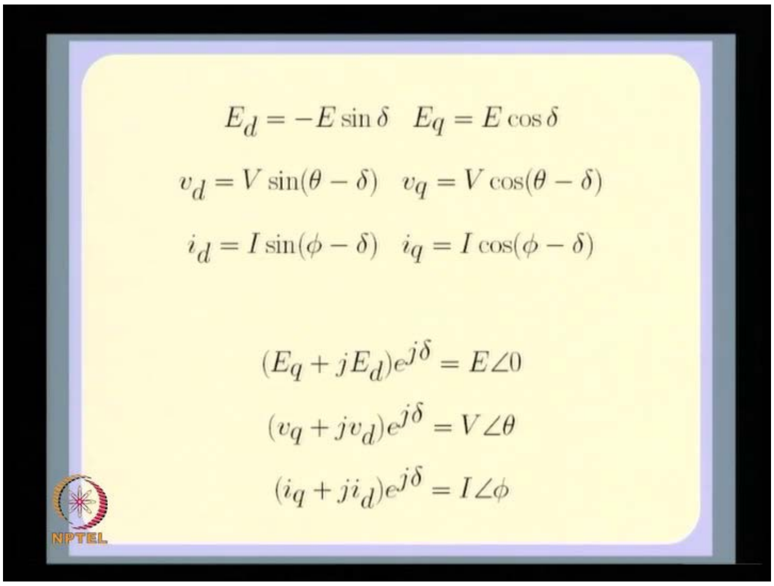

So, once you do that remember that the infinite bus voltage E a n, E b n and E c n. If we transform to parks reference frame, we have done this before for this 3 phase voltage source, you will get E d and E q the d q components of the infinite bus voltages as E sine delta minus of E sine delta and E cos delta similarly, you can show that V d and V q given that V a n, V b n and V c n are the form, which I had shown you about a couple of slides or 3 or 4 slides back similarly i d and i q are given by these formulae.

So, this is what you get in case you do the d q transformation of the voltages. Now, what do you have you have got theta you have got V, you have got E, you have got i. So, you have got these values by just back calculating as I had mentioned some time back, but of course, you do not have delta we still do not have delta this is something you need to compute so. In fact, I cannot get E d, E q, V d, V q, i d and i q. The equilibrium values yet till I tell you what delta is, but remember if you just by observing E d and E q the form of E d and form of E q, you can show that you can compactly write this in this fashion right.

So, for example, E q plus j E d into e rise to j delta is equal to E angle 0 that is E plus j 0 similarly V q plus j V d into e rise to j delta is V angle theta which is nothing, but V cos omega cos theta plus j V sine theta similarly for I and phi and i q and i d. So, what we know is not E d and E q, but we do know what E q plus j E d into e rise to j delta is know, we know what i q plus j i d into e rise to j delta is. So, the next step of course, is trying to find delta itself because from that we can get all the d q components of the voltages.

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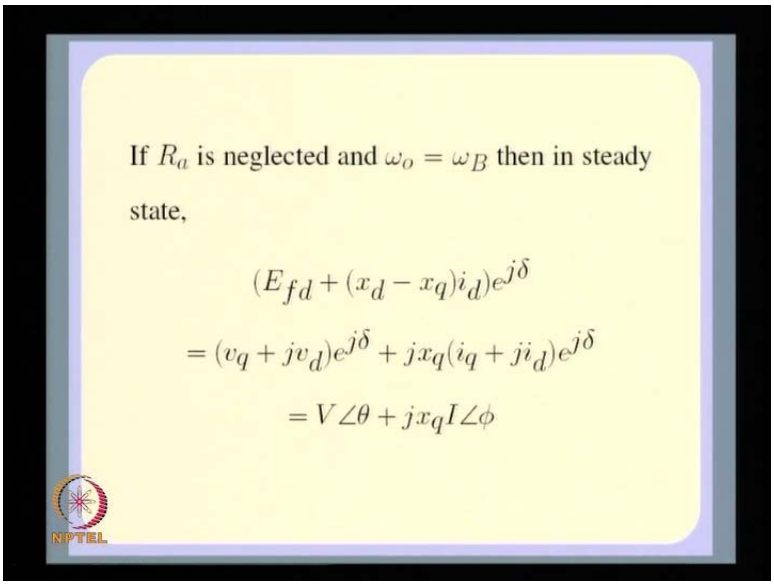

Now, one of the things which we derived in the previous class was that if, R a is neglected and if we assume that the infinite bus frequency is omega 0, which is equal to the base frequency then in steady state, we have E f d plus x d minus x q into i d e rise to j delta is equal to V q plus j V d into e rise to j delta plus j x q into i q plus j i d into e rise to j delta. So, this is something we did in the previous class I have just read it out, but if you would probably some of you would care to look at what, we did some time around the end of the previous lecture.

So, what the interesting point here is of course, the E f d, E f d plus x d minus x q into i d into e rise to j delta is equal to finally, something what we know. So, that is V angle theta plus j x q into I angle phi. So, that is something we know so. In fact, we know the right hand side of this equation, we know what V angle theta plus j x q into I angle phi is because we know x q, we know I we know V and we know theta and phi.

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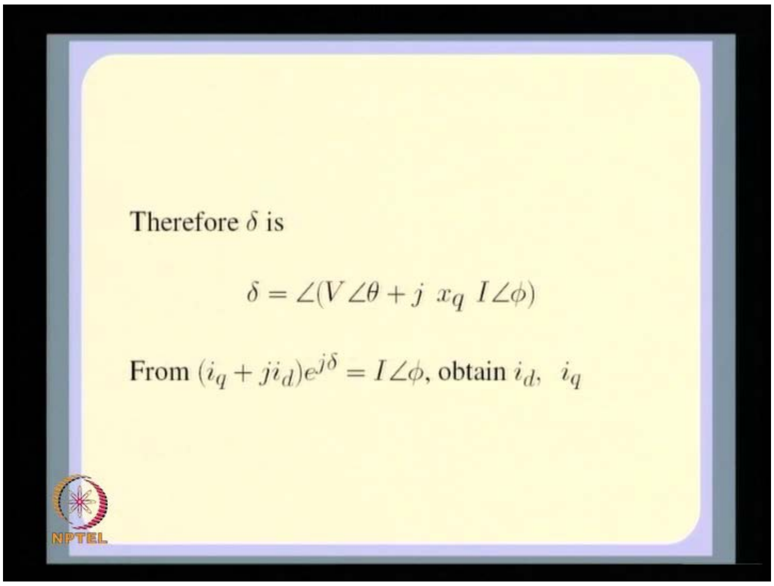

So, the point here is that because of this we can compute what the value of delta is going to be. So, once effectively what I mean to say is if you lo at the previous slide we know V angle theta plus j x q into I angle phi. So, because of that we know E f d plus x d minus x q into i d into e rise to j phi. What do I mean may be I will make it clear here.

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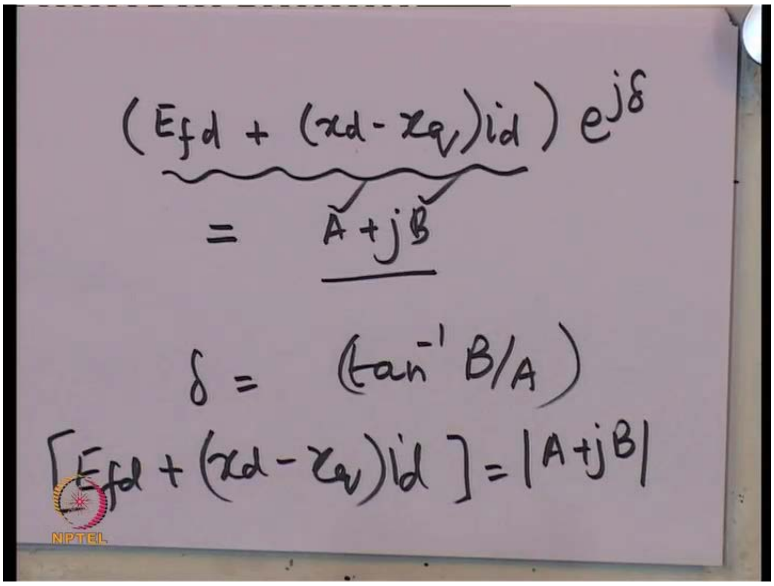

So, if E f d plus x d minus x q into i d into e rise to j delta is a known complex number. So, suppose it is A plus j B this is just you know suppose, it is this what it means is since this is a real number. Since, this is a real number what it means is delta is nothing, but the angle of A plus j B or you can say the tan inverse B by A. So, that is what I mean by angle of the complex number and because of that it also follows that the magnitude of this is the real number is nothing, but the magnitude A plus j B this we know.

So, therefore, we can get the value of this. So, this is what we the this is where we were last time. In fact, once you get delta a lot of thing suddenly you know become known for example, once you get delta one can get since you know I angle phi you can get i d and i q. Similarly, you can get V d and V q and E d and E q.

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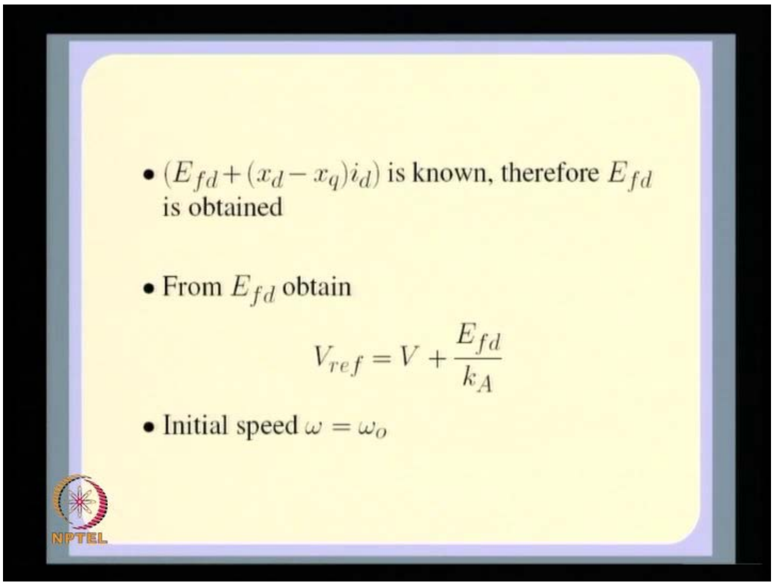

More over since, you know now i d and you know of course, x d and x q you can actually get E f d. So, you can get the equilibrium value of E f d which results which results in the operating condition. You are trying to describe from E f d of course, you can obtain V ref.

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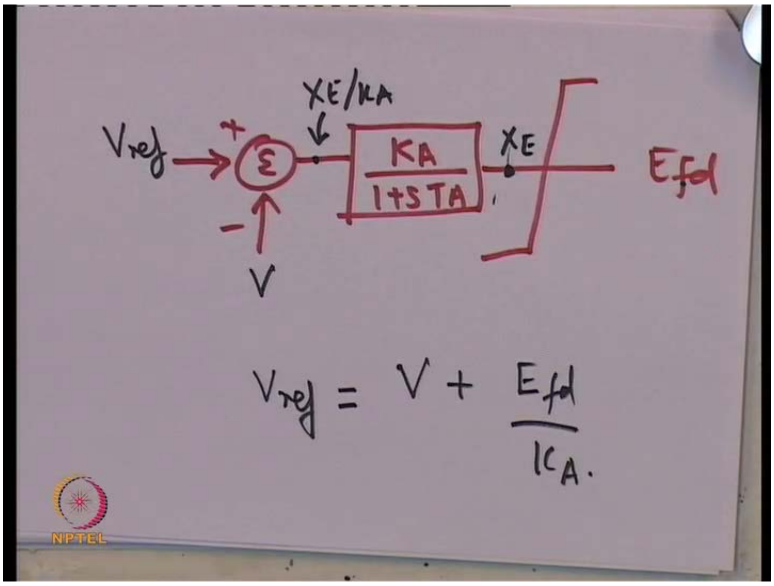

Now, one interesting point which you should note which we mentioned last time too is that since we have got a proportional controller in our we have modeled a AVR with a proportional controller this is E f d and of course, they are limits. Now, we will of course, assume that the operating point is such that we are not exceeded any limit in that case E f d is equal to X E and in steady state this will be X E by K A.

So, this will be X E by K A and if this is V ref and this is V we will come to know the value of V, the value need to give to V ref in order to get V at a terminals of the generator in steady state is that. So, this is an important point remember that the steady state gain of this transfer function is K A, K A by 1 plus T A has a has a steady state gain of K A. Now, of course, there is something which was implicit in what we did the we have got delta, but what is the initials, what is the initial angular speed of the rotor omega. In fact, it is equal to omega 0. Why is that?

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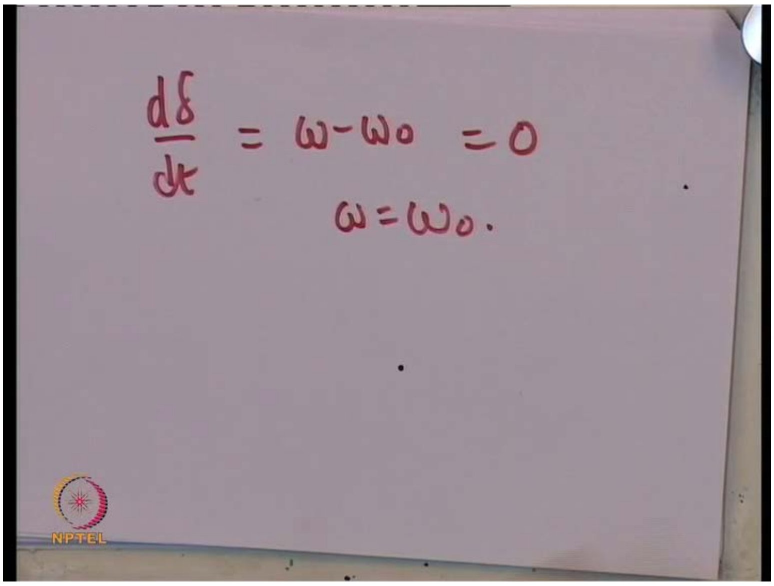

So, because if we said d delta by dt equal to 0 is equal to omega minus omega 0. Suppose, I set this to 0 after all we are computing the equilibrium values you will get omega is equal to omega 0. So, that is why we get it as I mentioned. Now, what we have done so, far is actually compute I have given you a procedure a step by step procedure to compute the initial values of delta E f d, V ref and T m as well T m in per unit is equal to electrical power in per unit provided, the speed is equal to the base speed the equilibrium speed is equal to the base speed.

So, mechanical power and electrical power are equal and mechanical torque in per unit at if the machine is operating at the base speed is equal to the, that both the torque and the power are equal in per unit. Now, we of course, I have made one assumption that the resistance of the generator is very small otherwise of course, there is a bit of a loss and the electrical power output of the generator is not equal to the mechanical power, there is a bit of a loss.

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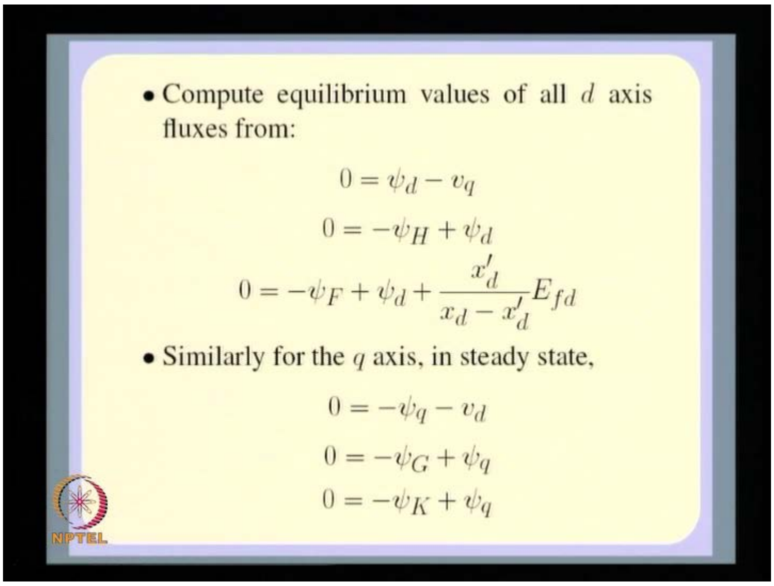

Now of course, once you get all the, you get E f d and you of course, can obtain sci d and sci q as well. In fact, once you have got E f d and you have got i q, i d and so on. You can compute what sci d is going be from sci d, you can get the values of psi f and psi H as well. So, this is how you actually compute the equilibrium values of all the states one by one of course, in the q axis psi. If you are operating at omega 0 which is equal to the base speed in that case psi q will be equal to minus of V d. So, as a result you will get the value of psi q and from there you can get the values of psi G and psi k, which are the other states in our state space description of the synchronous machine.

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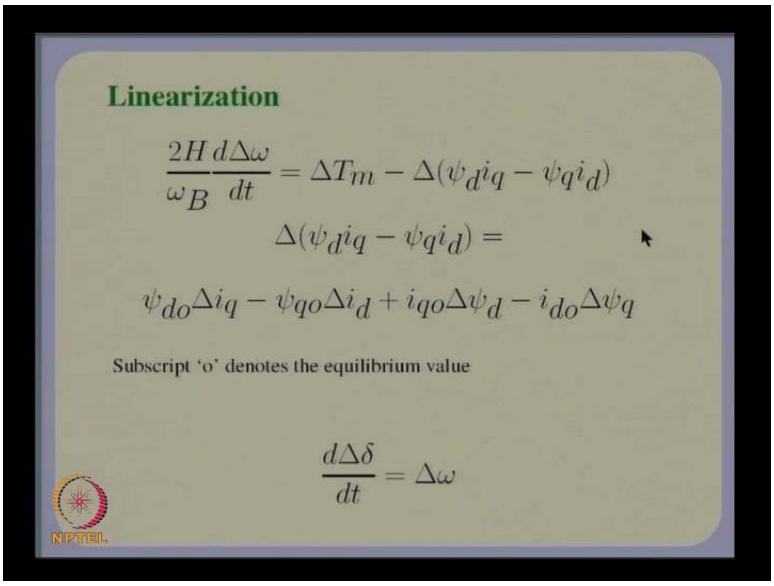

We will quickly go through the linearization of the differential equations of a synchronous machine connected to an infinite bus via a line and with an AVR. So, the let us just talk of the first you can say one of the differential equations is relating this, which really relates the rate of change of speed to the torque's acting on the system. So, for example, d omega by dt d delta omega by dt is proportional is proportional to delta T m minus delta t E and of course, you can linearise this using this equation.

So, this is a linearized equation, remember that the subscript 'o' here the subscript 'o' which appears just after this here or here really denotes, the equilibrium value of the states the other differential equation is linear to begin with. So, it just becomes the linearization is very straight forward here.

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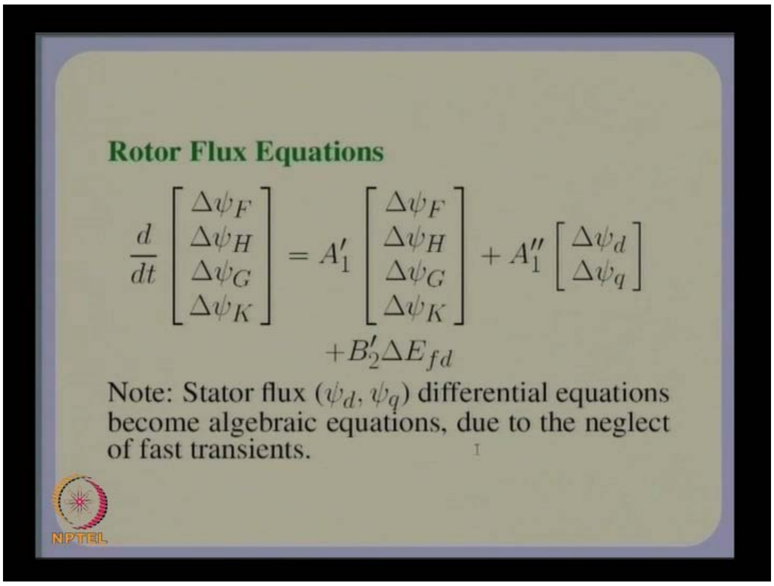

The rotor flux equations are. In fact, if you just look at these equations by themselves they are linear again, remember that stator flux is no longer the psi d and psi q are no longer states we shall see that they. In fact, obtain from the algebraic equations obtained by neglecting or setting d sci d by dt equal to 0 and d psi q by dt equal to 0 of course, A 1 dash, A 1 double dash, B 2 dash are given by these equations.

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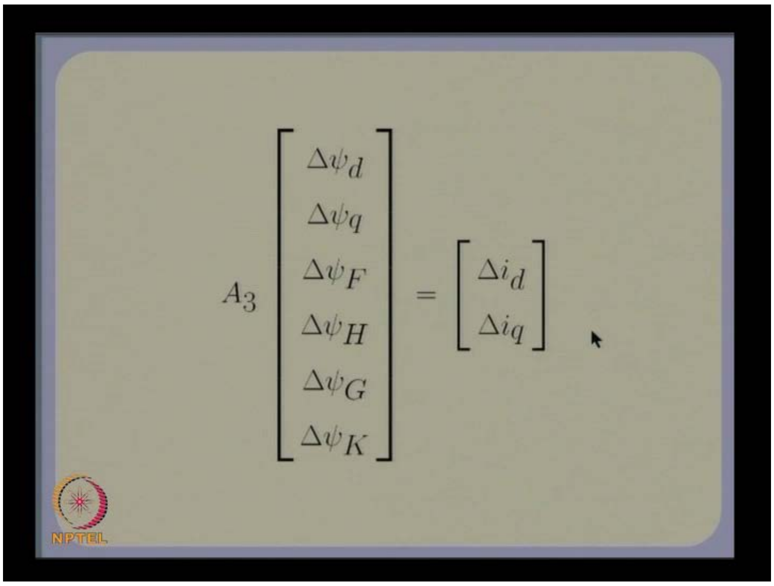

And this is the algebraic equation which relates i d and i q to the stator and rotor fluxes, where A 3 is given by this.

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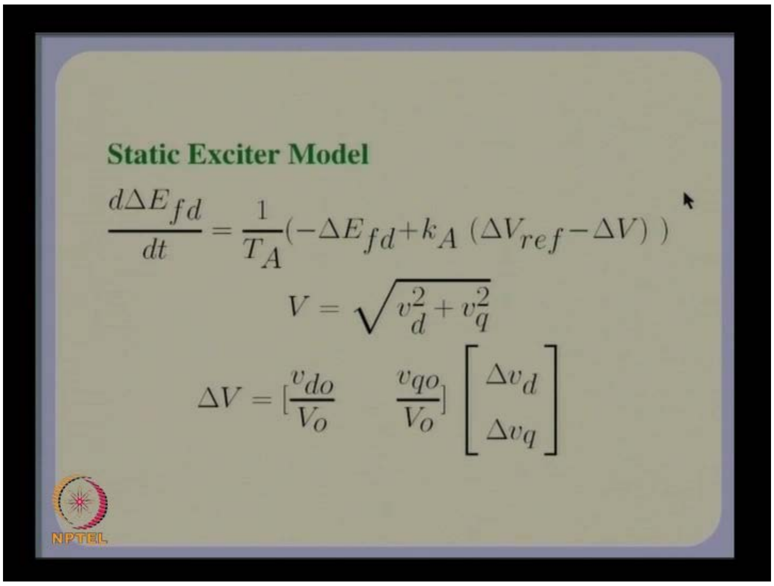

The static excitation system model we assume of course that the exciter is not at its limit in that case x c and E f d are identical and the different linearised differential equation is given by this; remember that V is equal root of V d square plus V q square, which is a non-linear function.

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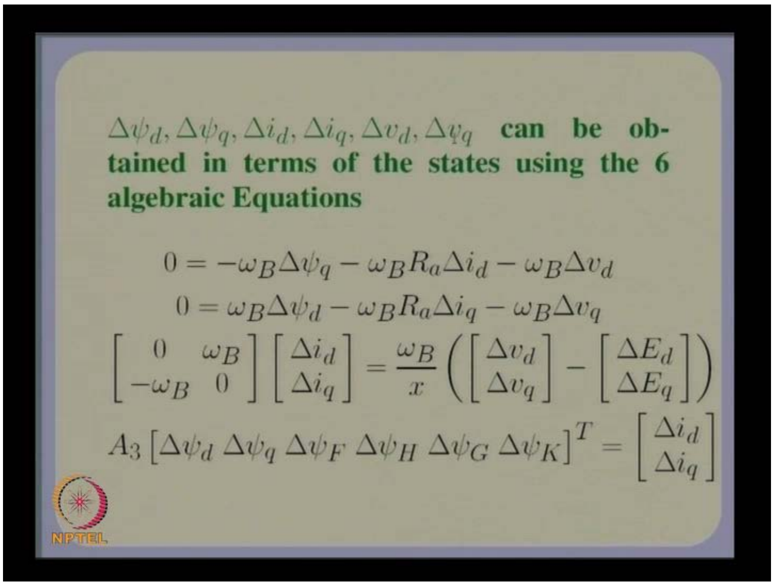

So, when you will have to when you, when you linearise it you will get delta V is equal to this. As before as discussed in the previous lecture psi d, psi q, i d, i q and V d y, V q, I can be obtained in terms these psi d, psi q, i d, i q, V d, V q appear in the differential equations, but we can actually eliminate them and write them in terms of the states using the six algebraic equations, which are linear equations these are the six algebraic equations.

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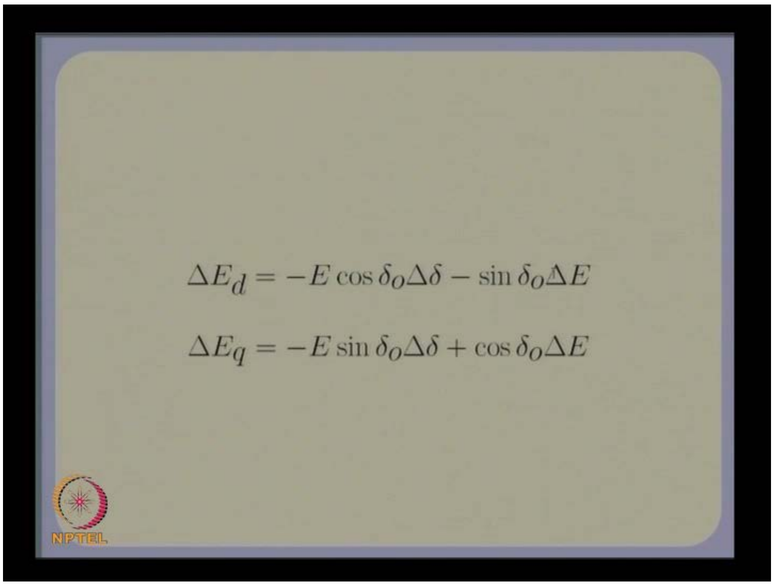

The linearized form of delta E d, E d and E q is given by these two equations remember that delta E, we assume that E is a constant. So, actually delta E is 0. So, we will just have delta E d is equal to minus E cos delta 0 into delta delta and minus delta E q is equal to minus E into sine delta 0 into delta delta.

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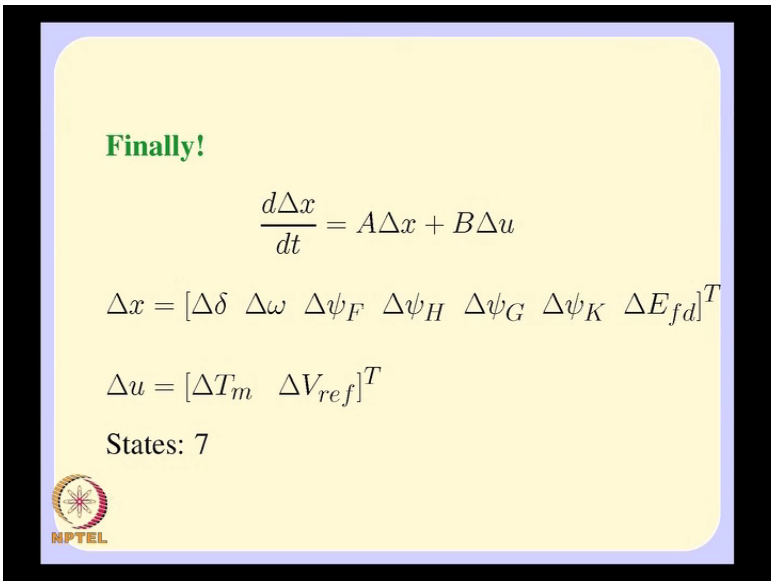

So finally, we obtained this set of differential equations remember that delta psi d, delta psi q, delta i d, delta i q and delta V d and delta V q do not appear here, because they have been written down in terms of state variables, which are delta delta, delta omega ,delta psi, f delta, psi H, delta psi G and delta psi k and substitute it in the differential equations, so that we get it in pure states space form that is d delta x by dt is equal to A into delta x plus B into delta u.

We can now use the A matrix which we obtain finally, to do the Eigen value analysis. The system small signal behavior is also a function of the equilibrium point. So, once you get the equilibrium point plug it into all your linearized equations and you get a final states space form like this, once you linearised it you get your Eigen values. So, let me put what we are trying to do, we are trying to see how to linearise the synchronous machine connected to an infinite bus through a reactance, which is also having the generator also having automatic voltage regulator and the excitation system which is modeled.

So, what we are going to do is see if we can replicate or get a good validation of the result simulation which result, which we got in the previous class that is for a certain operating condition. It was found that the system does not settle at an equilibrium point is it actually seen by Eigen analysis as well.

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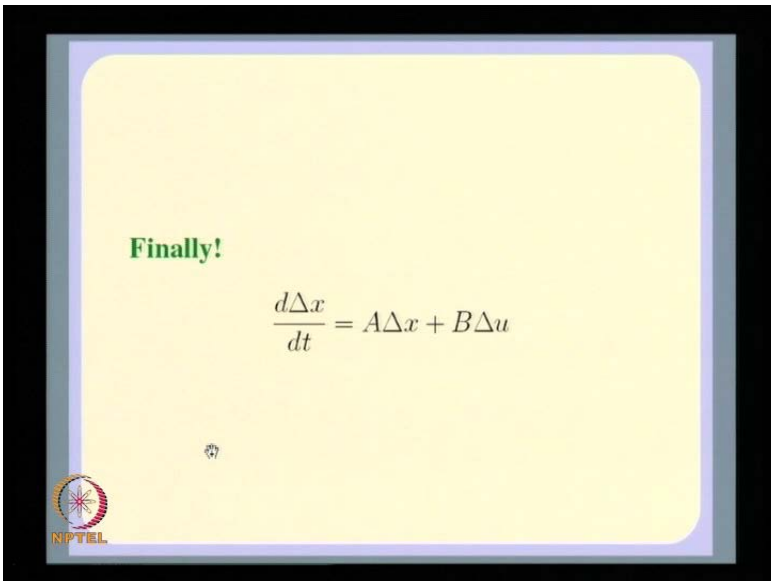

So, let us see whether that is true. So, what I need to do is of course, write a program to run this.

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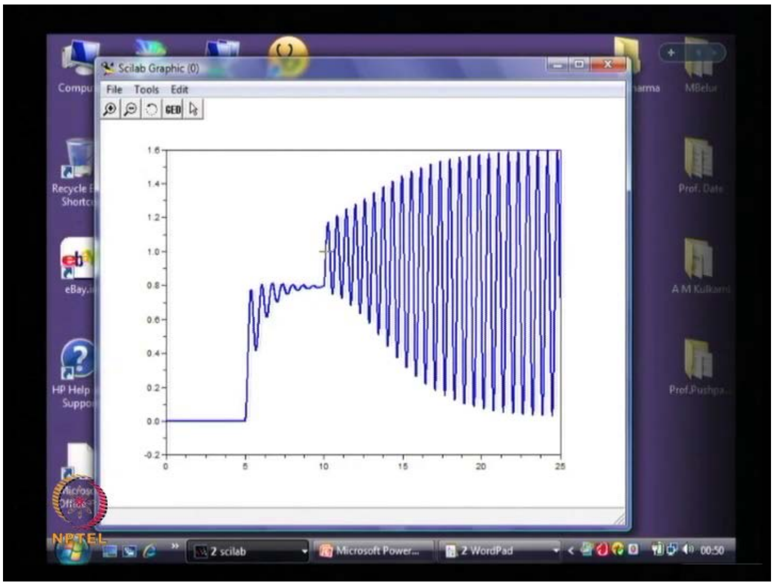

So, I have done that remember this is what we were trying to analyze. We synchronize the synchronous machine this is of course, the simulation result not the result of the Eigen analysis of course, this we synchronize the machine gave a step change in the electrical the mechanical torque then, we gave another step change to make it approximately one per unit.

And what we saw was this is the plot of course, of delta we saw that for the second disturbance the system does not seem to be settling down, but seems to be increasing with time, its seems to be increasing with time and what we conjecture of course, is probably this equilibrium point is not small disturbance stable. So, we will just confirm this shortly.

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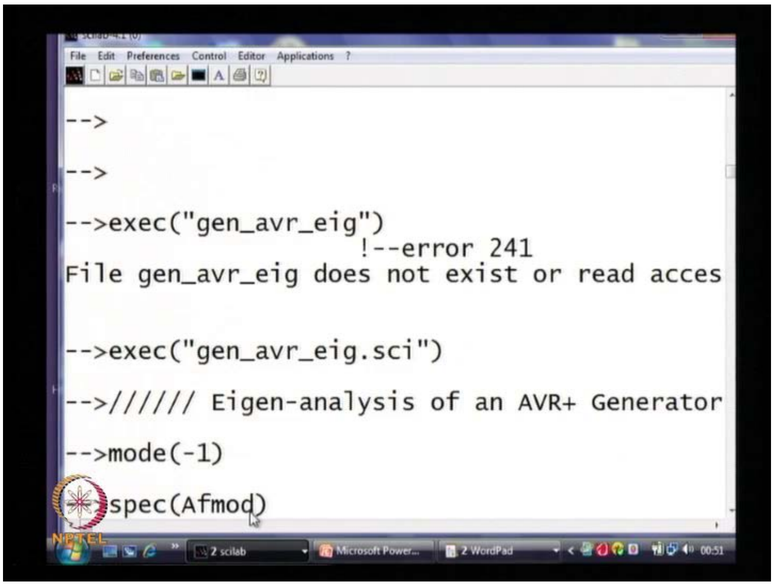

So, what we will do is I have written down a program of course, this time I will not shown you the program. What I will do is I will just run this program, what we have got is gen AVR eig. So, what we will do is we will just run this program first and once you do that knot sci, what we will do is take out the Eigen values, the Eigen values are corresponding to which operating point well I have I am trying to take out the Eigen values in this particular, you know in this program for the operating point corresponding to the electrical power equal to 0.5.

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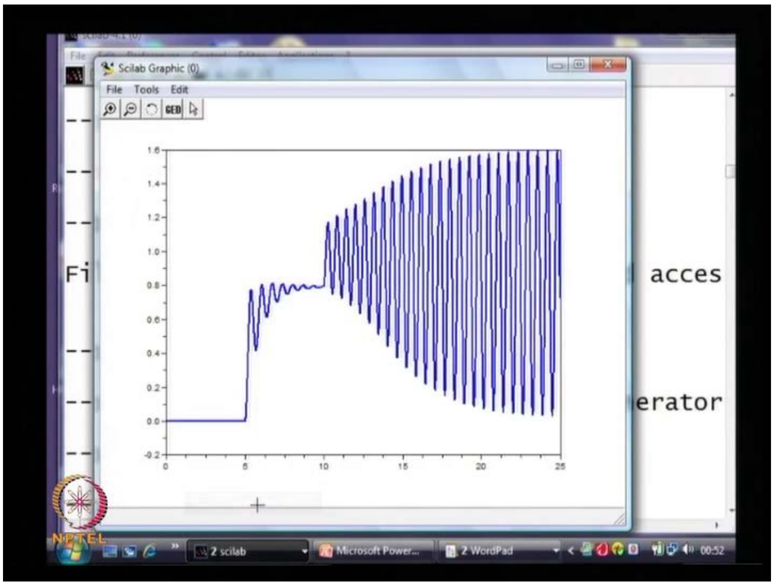

Remember when I give a step change to the electrical power or the mechanical power to 0.5. The electrical power also will go to 0.5 this particular equilibrium point, which corresponds to the first disturbance in our previous simulation is seems to be stable because you see that the oscillation seem to be settling down. It is only the subsequent disturbance which gives a step change in the torque to the tune of 0.5 which such that, total mechanical torque because one per unit it is then that the operating point is seem to be unstable.

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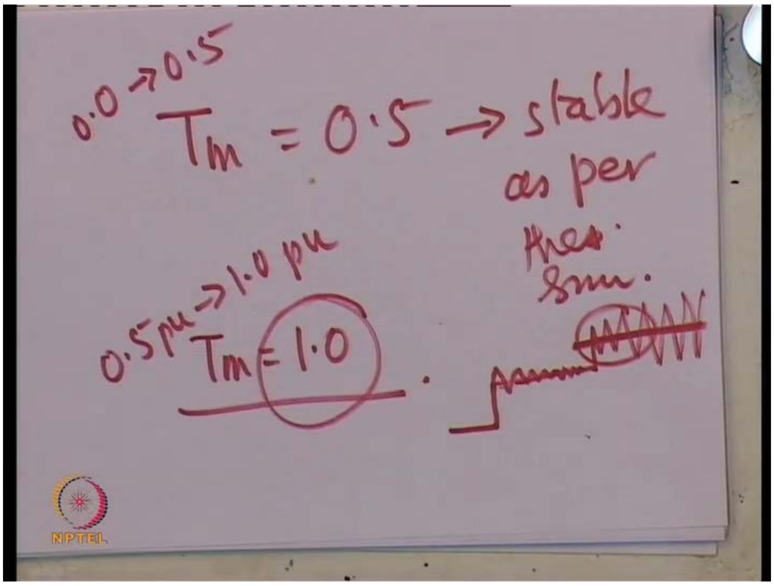

So, what we will do first is try to take out the Eigen values of the linearized system around the operating point corresponding to T m is equal to 0.5, this is stable as per the simulation. So, we just take out the Eigen values and what we see here is all the real parts are negative this confirms that this is. In fact, a good there is a one to one kind of correlation between what we see in the simulation and in the Eigen value analysis. So, what we are really seeing is that, the system even the Eigen value analysis predicts that the equilibrium point corresponding to T m is equal to 0.5 is indeed stable. Now, if I make T m is equal to 1 per unit.

So, remember that when I gave a step change of in the simulation from 0.5 per unit to one per unit in the mechanical power we first saw of course, the first step change which was from point 0.0 to 0.5 resulted in the new equilibrium point being stable, you saw this equilibrium point was stable in second step, we saw that it was not stabilizing it was growing with time.

So, what we are now going to investigate is the nature of the Eigen values around the operating point corresponding to T m is equal to one per unit. So, now we will do that. So, in my program I need to just change this operating point to say one and see what are the small signal characteristics of the system around this equilibrium point.

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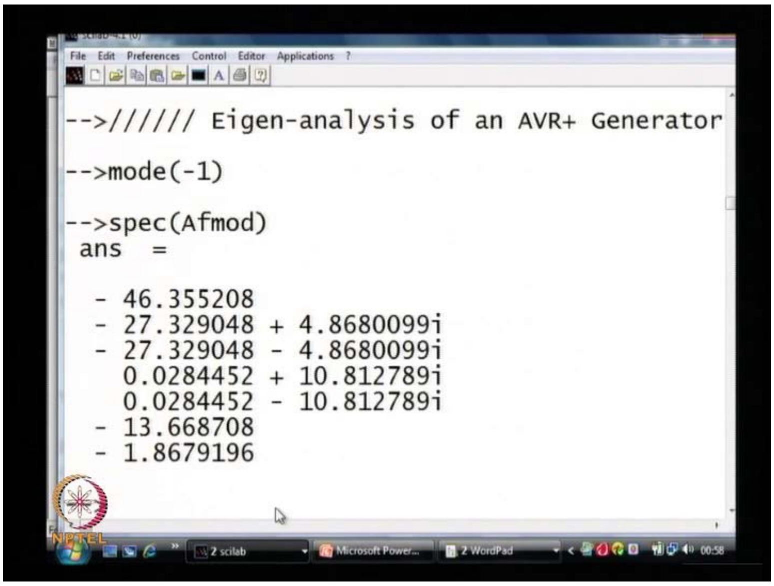

So, I will just run the program again well. Now, what you see is that the this swing this low frequency oscillation here which is has got a imaginary part of 10 radiance per second plus or minus 10 radiance per second, it is real part is positive. So, this is actually an unstable system of course, one point we will notice is that the real part is 0.02 plus 0.02.

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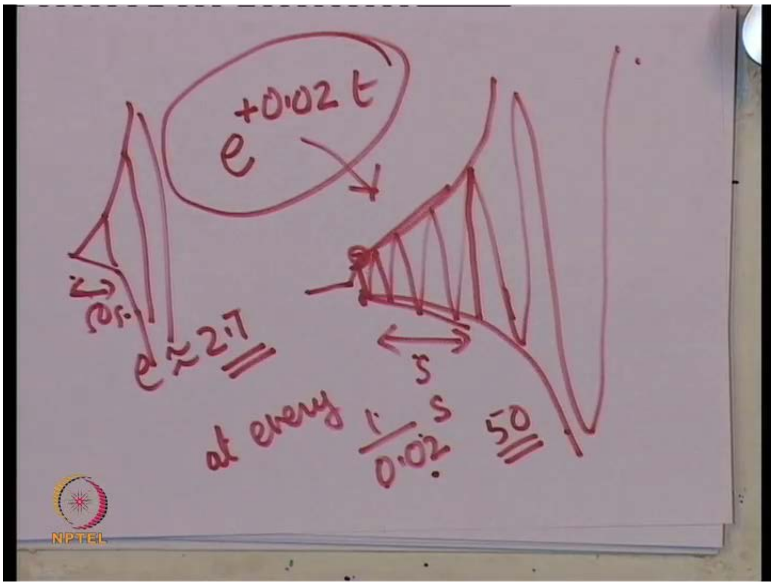

So, actually the growth of these oscillations for small near about the equilibrium point will be e into 0.02 times t. What I mean is that your oscillations, if you give a if your near about the equilibrium point your oscillations will be growing at this rate, you will find this is exponentially growing like this. Now, the two things, which you should notice about the simulation, which do not seem to be consistent with this although the fact that this is unstable equilibrium point seems to be validated using, the Eigen value analysis.

There a couple of things we have not considered that is if you look at the rate of rise of this oscillation, it seems to be much more than what is predicted by Eigen analysis. Eigen analysis predicts that e rise to 0.02 t, this is the rate of at which this growth should take place. Now, e is around 2.17 or. So, at every one upon 0.02 seconds you will see that there is a there is a amplification of 2.7, over the value which was there 2.7 seconds back. So, if this is 2.7 rather if this is one upon 0.02 seconds back you will see an amplification factor of 2.7.

So, if t is equal to 1 upon 0.02 you will find that the amplification, which has taken place in this interval one upon 0.02 seconds will be 2.7 times. So, what it says is that one upon

0.02 is nothing, but 50. So, every 50 seconds there is an amplification you know it kind of there is an amplification, which will occur just 2.7 times. So, there is a amplification which is occurring which is 2.7 times of the value which is 50 seconds back.

But what you see here in the simulation, if you look at the simulation result the rate of rise is. In fact, much faster the doubling for example, has taken place in less than 10 seconds this is probably due to the fact that, I have used Euler method in simulating this system. So, I think from my side this is the last time, I will try to use the Euler method it is giving an instability much more than what is predicted by Eigen analysis. The rate of growth is much, much faster Eigen analysis predicts, that this almost tripling of the response or the disturbance every 50 seconds, but here it is growing much, much faster it is doubling almost every 10 seconds.

So, that is probably a result of Eigen analysis there is one more point which, with which we will conclude this lecture. You notice that the oscillation is not just growing with time, what I expected was linearise analysis predicts that the system is unstable. What does it means that the envelop of this oscillation should go on growing as I am showing you on this sheet it should just go on blowing up, but what happens is actually the oscillation is instead settling down.

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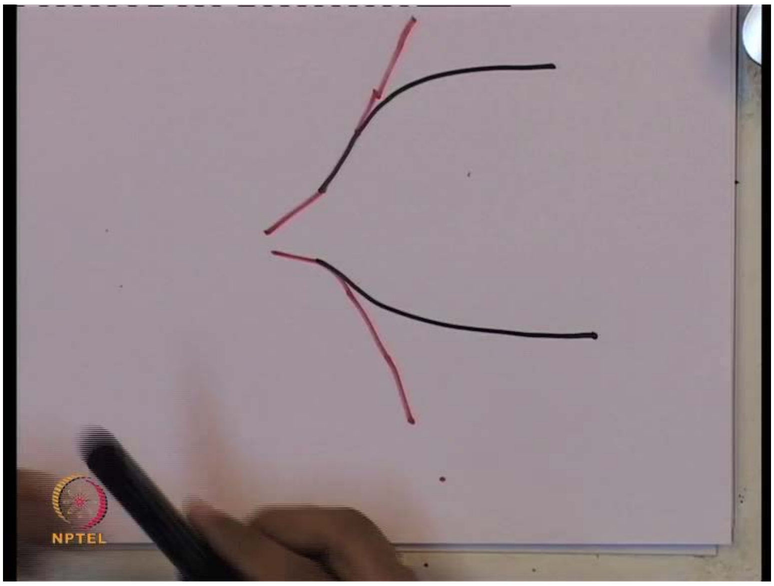

Now, is this due to the numerical method used or is there some other issue well remember that, once the oscillation increases in magnitude our linearise analysis is no longer valid. So, what is likely what is happening probably is that the non-linear behavior is no longer what the linearise analysis would predict. So, rather the linearise analysis is not valid once the system blows up. So, that is one important point.

So, what you see in the simulation is not exactly what you see in Eigen analysis except for the fact that, the Eigen analysis correctly predicts that this equilibrium point is not stable, but the continuous blowing up is not actually occurring. Now, why is this? So, is some this is are the interesting and small remanent point, which we will discuss in the next lecture. So, from there onwards of course, we will now go on to the modeling of other components in the power system. So, we will do that in the next class.