Power System Dynamics and Control
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Model No. # 01
Lecture No. # 03
Analysis of Dynamical Systems

In the previous two lectures, we have had a brief overview of power systems and stability problems associated with it. What I really told you in those lectures was a description of the phenomena, the basic phenomena. We now go on to the nitigrities of trying to analyze these phenomenon, try to understand how they occur and look at it in a more mathematical fashion.

Now, mathematicians will scoff at as in probably we horrified by the kind of the approximations we will be doing in order to get a basic understanding of the phenomena. You will understand of course that the nature of the models which we will use will be determined by that by which phenomenon, we want to understand.

So, today's lecture we will learn about some basic concepts of dynamical systems. We will also understand a few examples pertaining to a power system, but the main issue which we will try to tackle in this particular lecture is a general attitude, to get a general understanding of dynamical systems and how we can attack the problem scientifically. In the past two lectures, I have not really given any mathematical or rigorous explanation for the kind of phenomena we have seen, that is voltage instability or loss of synchronism. I just gave you a kind of a hint that the reason why the systems behave the way they do, they are due to the physical loss which govern there motion.

So, today what we will do is try to understand some kind of general ideas, understand general ideas of dynamical systems. The first thing about dynamical systems is what is equilibrium? Of course, we would like for example, a power system or any other system to be at an acceptable equilibrium at all times. This of course will not be true because power system is always subjected to some disturbance or the other. It could be a minor load change. For example, you switch off a light in your house. It is a load change or they could be really large faults involving tripping of components due to you know some short circuit and so on. Those are large disturbances. So, what we need to do is understand what you mean by equilibrium. Now, if you look at it using the very simple examples so what we will do is just look at a very simple example of a ball on a hill. So, you can just concentrate on what I am drawing here.

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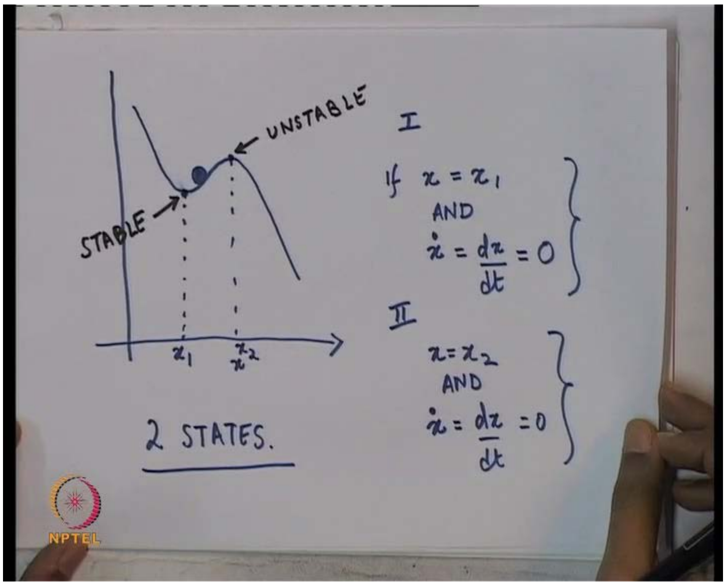

Suppose, you have got a hill of this kind, you got a valley here and a ball is somewhere on this hill. So, you got a ball on this hill. The ball will be at equilibrium if its x coordinate is here. So, this is what is known as equilibrium. So, I will call this x 1. Actually, what I have told you is not strictly speaking true. What I should tell you is that if this ball is at this point and its speed is equal to 0, then it is at equilibrium here.

We can always have ball which is rolling down and at this point, it has got some non zero speed. In that case, it will continue moving. So, when we say something is at equilibrium, we should really specify all the states corresponding to that equilibrium. So, what I should say is that if x is equal to x 1 and x dot or which is nothing, but a notational simplification of d x by d t. The rate of change of x is equal to 0. If these two things are satisfied, you are at equilibrium. The ball will be at equilibrium. Is there another equilibrium? Yeah, there is one more. For example, if I manage to place the ball here, I will call this x 2. X is equal to x 2 and x dot is nothing, but d x by d t is equal to 0. This also defines equilibrium.

So, if I place a ball right there at the peak of this hill very carefully, so that its velocity is 0 and and it is just exactly here, then the ball will stay there. So, that is also equilibrium. There are two equilibria here in this diagram. As far as I have shown you here, there are only two equilibria here and here and there are two states of the system. What you mean by state? We will not go into any rigorous definition, but in this context you see that these values of the states are the minimum information I need to give you to know whether you are at equilibrium or not. So, you should, you need to specify two things-the value of x and the value of x dot. So, this particular system has two states.

Now, you will notice that there is a qualitative difference between the equilibrium here and here. In fact, it could have come to your mind right away. This equilibrium 1 is different from this equilibrium 2. How is this equilibrium different? Well, in case I place this ball at this equilibrium and I give it a small disturbance, what would happen? It would kind of suppose, I gave it a kick while it was here. Suppose, somebody came and gave it a kick, so it would go up, then come down and then go up again and then, come down. There could be forces which you try to pull this thing back to equilibrium, but of course, if there is no damping, we will set off an oscillatory motion, but the fact remains that there are forces which you are pulling it back to this equilibrium. So, this particular ball would eventually stabilize at this equilibrium. The point is of course, there should be some friction here, so that it comes back to this equilibrium. Otherwise, it will just go on continuing to oscillate. Contrast this with a situation where the ball is here. If the ball is placed here at a 0 velocity, in case I give a small disturbance, it will just roll off.

So, this particular equilibrium is such that if the ball is hit a small push can really take you out of the equilibrium. So, in fact a kind of intuitive definition of stability can be got right here is that if you have got equilibrium and if I give a small push from that equilibrium, if the ball tends to come back to the equilibrium, eventually of course, it should settle back at the equilibrium. Then, that equilibrium point is stable. To put it more precisely, it is stable for small disturbances. So, that is one important thing which you should keep in mind is that there could be many equilibria, but if those states are subjected to a small disturbance like the ball being given a small push, are we going to come back to the equilibria. So, that is the basic concept of stability. In the example which I have shown you, this is unstable equilibria and this is stable equilibria.

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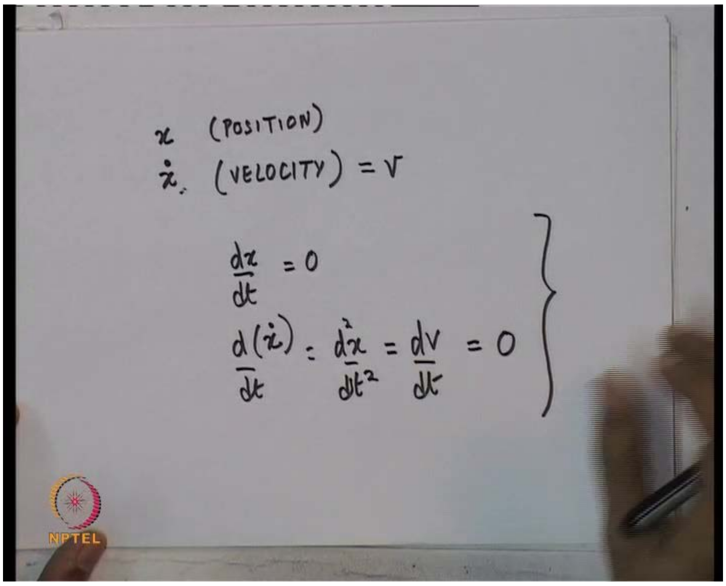

One of the ways we can really mathematically define in equilibrium is if we take the states for in this case x, which is the position and x dot, which is the velocity, if the rate of change of position and the rate of change of velocity which is nothing, but I will call velocity as dv in that case we will have.

So, equilibrium is the point at which the rate of change of the states is all equal to 0. Now, as we saw in the previous slide in this particular example, there were 2 equilibria. Now, this is the general you know definition of equilibrium and states. How do you really analyze the small disturbance behavior of the states? So, that is something which we will do slowly and understand it using a power system example, the simplified power system example in this particular lecture, but before we move on to that, let us just look at some more interesting things about stability. You take this example again.

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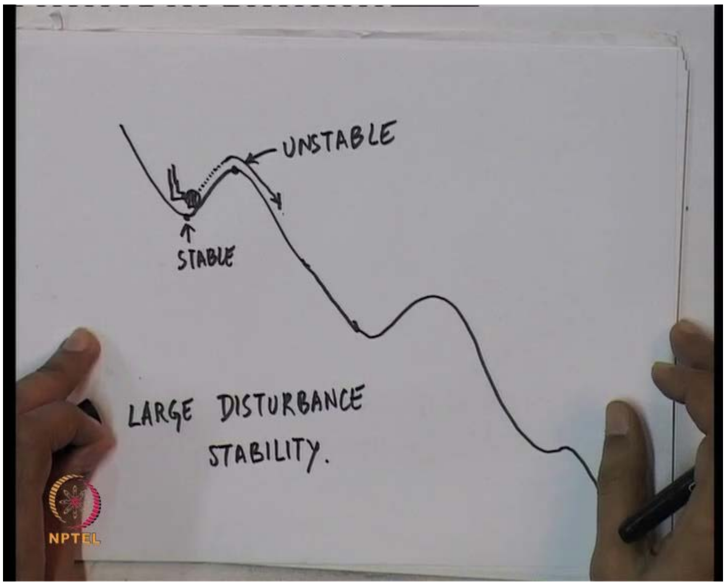

These are the two equilibria. This is unstable, this is stable for small disturbances and this is unstable for small disturbances. However, the small disturbance may be the point is if a ball is here and I give not a small disturbance, but a big disturbance, somebody really kicks it hard, then you may find that this ball if somebody kicks it, for example, this ball will tend to move upward. It is constantly being decelerated because of the force of gravity, but that force is not adequate to stop the balls motion before it reaches this point. So, what will happen is that if the ball rolls over this hill and starts rolling down here and if that happens, this hill just goes on like this or it goes on like this for example, you will find that the ball will never come back to this particular equilibrium.

So, although this equilibrium is stable for small disturbances, it is not for large disturbances. A big enough kick to this ball can make it roll over this hill and never come back to this equilibrium. So, this is a typical situation where a system is small disturbance stable at this equilibrium, but if a large enough disturbance is there, it will not return to its equilibrium. So, this is what is known as large disturbance stability. So, I hope you got a kind of a feel for these two concepts. Let us do one thing.

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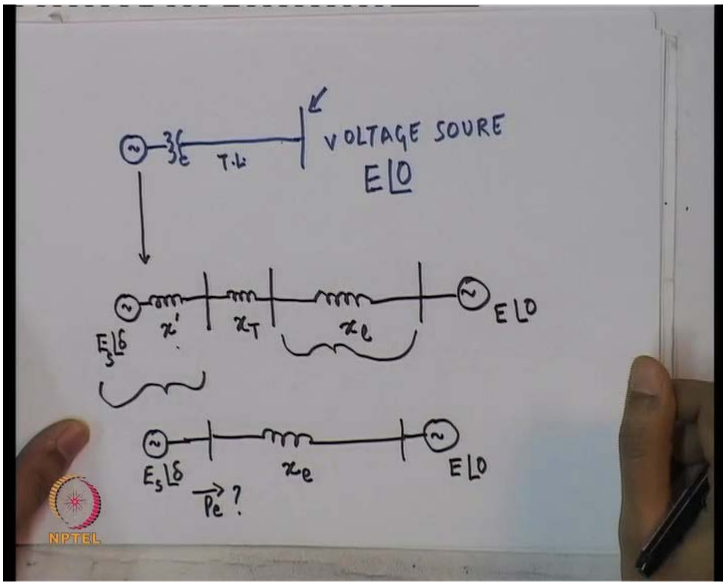

Let us take a simple power system example and go ahead in trying to analyze it. A very similar example will have two states. Consider a single machine, a synchronous machine connected say, via a transformer to a transmission line and a very large grid. The grid is very large and let say this practically a voltage source of constant magnitude E, constant frequency, so that its phase angle never changes. So, it is sinusoid, a three phase sinusoidal voltage source whose voltage magnitude frequency and phase angle simply do not change whatever you do. So, it is a very large system. This is a transmission line.

Now, when we are trying to analyze this system for small and large disturbances, what do we need to do? We need to really form a model. Now, the whole idea of this course in the later lectures is to get good models of all these elements like a transmission line, a transformer synchronous machine. One important point which you should notice is that we will never be getting exact models. We will never be using exact models, in the sense that every model will involve some approximation or the other. For example, look at this example. This synchronous machine let me model as a voltage source behind what is known as a transient reactants. How do we come to this model? We will do that in the course as the course moves on.

Let me just tell you that this is not a very respectable model of synchronous machine. I am just using it to highlight a certain phenomena. So, what I will do is a synchronous machine is model as a voltage source E s with a constant magnitude behind a transient reactants. So, although this E s is constant does not mean that the terminal voltage is a constant, E s is constant, delta is a rotor position. So, what we have is if the rotor moves, it directly reflects in the phase angle of the voltage which appears. So, this of course is not very difficult at least intuitively to understand that this could be a kind of a model of a synchronous machine, but there is absolutely no rigger. We are just I am just putting it forth to you. The transformer is modeled just by its leakage and a transmission line and this voltage source. A transmission line is just one lumped inductor x. This is x transformer and I will call this x line.

So, this is the electrical circuit model. We have really made a lot of approximations to bring a transmission line model from something very complicated. In fact, the transmission line is modeled by Maxwell equation, described Maxwell equations. So, it is a big long story of how we come to a lumped reactance equivalent of a transmission line. I hope I will get some time to tell you how it comes about, but right now, let us just take this model of a transmission line as a simple lumped reactance x l. So, I will further simplify this. I will just make it a bit. So, this circuit, as far this circuit is concerned, this is how it looks. So, x is nothing, but x dash plus x t plus x l.

Now, what are the equations which describe this system? Now, those who are purist would say, well this transmission line has to be modeled by a partial differential equation, this transform model probably again one has to use Maxwell equations. This synchronous machine well, I have just given you a model and not told you how you know it. It really comes about. It involves of course, Maxwell equations and Newton equations which described the motion of the rotor. So, just let me put forth particular model.

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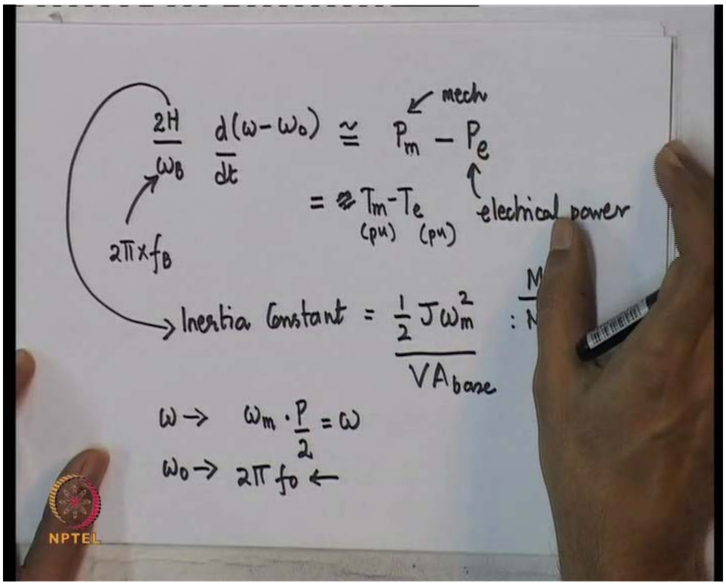

We will be doing the origin of this model later on in the course, but right now you just take it from me. I will just explain each element. This is omega B which is the base radian frequency, that is, 2 pi into the frequency base. H is nothing, but the inertia constant of the machine which is actually equal to half into J into the mechanical speed square divided by the volt ampere base. So, its units are actually mega joules per MVA. So, these are the units. In the expression for H, we should have omega m B square, that is, the mechanical speed base square. Out here I have written it as omega m square, but what really needs to be a, what is correct it is in fact omega m base square. So, please note that the minor error here.

Omega is the electrical frequency, electrical radian frequency of the generator. The electrical radian speed I should say. So, it is 2. If there are four poles, of course the mechanical motion into P by 2 is equal to the electrical speed. So, this is the mechanical speed, this is the electrical speed. Omega naught is 2 pi into f 0, where f 0 is the frequency of the infinite bus, that is the voltage source whose frequency and voltage do not change at all. So, this is f 0.

Now, this is basically P m is the mechanical power input. So, this is the mechanical power input, this is the electrical power output of the machine. Now, one important point is, remember that P m and P e are in per unit. So, that is a very important thing. You should note this P m and P e are in per unit, otherwise you will not find this dimensionally. So, P m and P e are in per unit. In fact, the mechanical and electrical powers, this in fact is in approximate equation. What should have been correctly written is T m minus T e in per unit.

So, sorry, in fact this is equal to T m minus T e in per unit or both in per unit, but I have written it as mechanical power minus electrical power. The approximation here is that mechanical power in per unit and mechanical torque in per unit, they are practically the same. So, this is one approximation which we have made right away. So, let me just rewrite these equations again.

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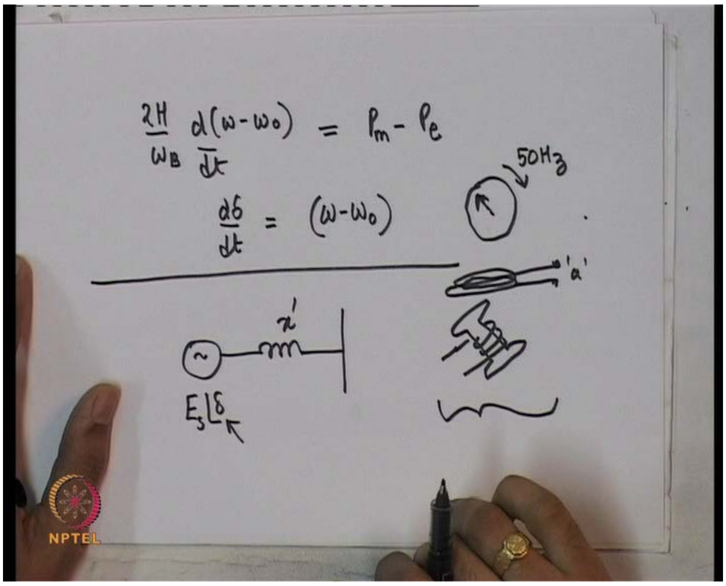

2H by omega B d omega minus omega naught is approximately equal to the mechanical power minus the electrical power in per unit. So, that is one important thing you should remember. In fact, the rotor position, the rate of change of the rotor position is given by omega minus omega naught. Now, the rate of change of rotor position as seen by a frame which is rotating at omega naught, so if the machine is rotating exactly at 50 hertz and your omega naught also corresponds to 50 hertz. Then, this angle delta will be a constant. However, if the frequency of this even transiently changes the position of say, this mark will appear to move, so let me just put it you know in a kind of physical way. If I am rotating at 50 hertz and I am looking at this mark which is also rotating at 50 hertz, this mark will appear stationery.

However, if there is a transient change in the speed, I will see this mark moving. So, that movement of this mark is captured by this equation. Remember, the position of the rotor, of the rotor of a synchronous machine also determines the phase angle of the voltage which appears at the terminals. That is why what you see here is this delta also has a bearing on the phase angle of the voltage of this voltage source in this model of this synchronous machine. This again is not difficult to understand intuitively because the position of the field which is on the rotor will really determine the kind of the phase angle of the voltage which appears in the stator winding say, the a stator winding.

So, you know the point here I wish to make here is that it is not difficult to see intuitively. We will do this in a bit more detail later that the actual position of this will determine the phase at a given time. The position of this will determine the phase of the voltages which I induced in the stator winding say of the a phase. So, we will do this a bit later. We will keep this for later to show this actually is true when we actually derive this synchronous machine model.

This is of course let me reiterate a very simplistic model of a synchronous machine. This in fact is not even a respectable model, but I will use it to highlight the certain phenomenon which is actually seen in practice. So, it is only used to show that a certain phenomenon actually exists. Now, if we look at the system here, what is P e?

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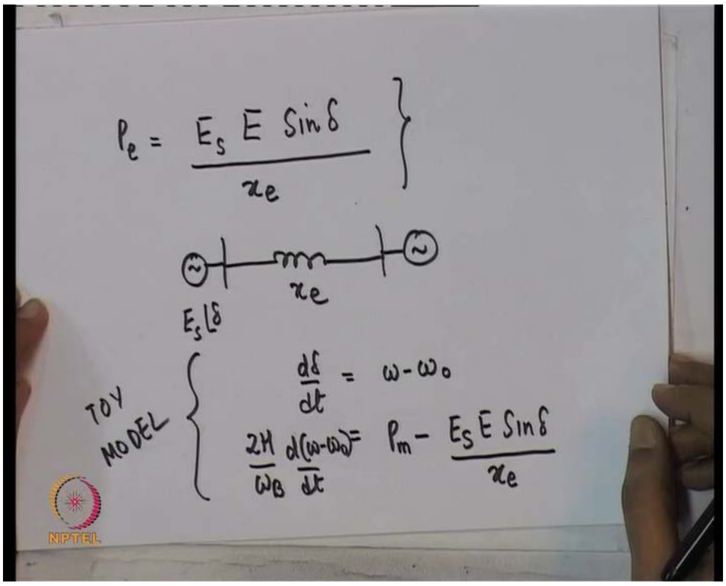

Now, P e is equal to E s. This is a power transfer between two voltage sources which you are quite familiar with x e. So, this is you can say the sinusoidal steady state power flow formula for a three phase system, a three phase balance system. Now, of course again somebody may raise an objection that well, you are studying system dynamics. Aren't you? So, why give a sinusoidal steady state expression for power? Actually, you should actually write the differential equations corresponding to this inductance here. After all this is inductance x e while, so this we cannot treat it as a reactance. Then, uses sinusoidal steady state formula and then, use it in our differential equations.

So, unless I actually make this justification, you will probably be unconvinced. So, I agree that I am not given you any such justification. Let me just tell you right now that if the phenomena we are trying to study using this model is much slower than the transients associated or the natural transients associated with this electrical network here. This is just a simple transmission line. We can use a kind of a quasi. This is a kind of a steady state formula for analyzing these equations. So, what appears actually a transmission line or transformer or a synchronous machine. It should be made out of many many many differential equations and it in fact does. We have reduced it to just two differential equations and this is you can call it a toy model. It is not a very respectable model. It is a toy model which will help us to understand certain phenomena. In fact, it highlights the phenomenon I wish to tell you quiet well.

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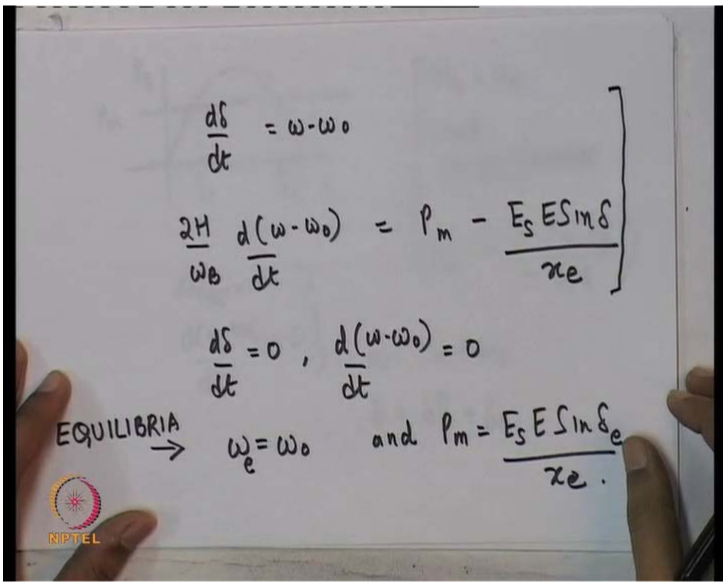

Now, how do I analyze this system? So, you have got d delta by d t is equal to omega minus omega naught and 2H by omega B d omega minus omega. When you first look at this equation and I tell you well, is this system stable or not stable. Well, the correct way to approach this problem is first of all take out what are the equilibrium conditions for this system. The equilibrium condition for this system is, remember how do we get the equilibrium conditions? By setting the rates of changes of the states which are in this case the speed deviation and the rotor angle delta equal to 0 in which case you will get the equilibria are omega is equal to omega naught and well, P m is equal to E s ES sine delta by x e. So, at equilibrium I will call this may be omega e. Omega e and delta e are the equilibrium values of this particular system now.

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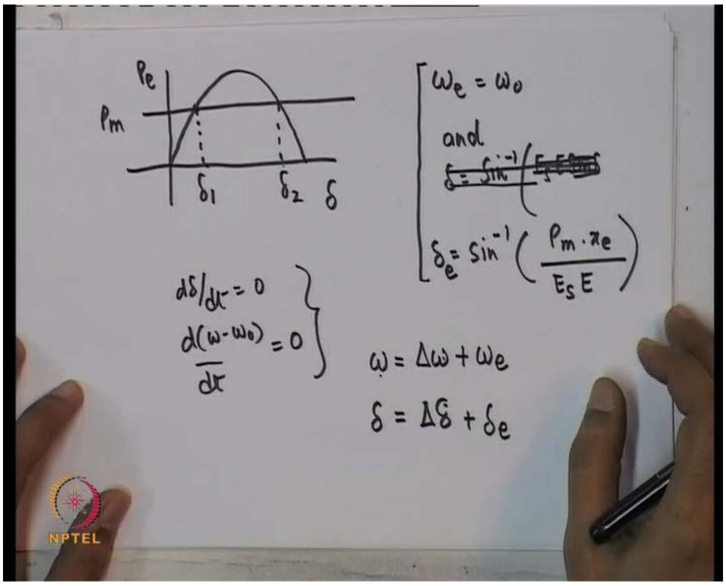

So, just look at it graphically. This is P e. P e is nothing, but e S e sine delta e by x e. So, this is electrical power verses delta and this is P m say, a certain value of P m. Then, the point at which they are equal defines the equilibrium, in fact from 0 to 180, the two possible equilibria of delta. Here this is one and this is another for this particular value of P m. So, let me just, so omega e is equal to omega 0 and delta is equal to sine inverse E s E sine delta, sorry rewrite this P m c.

Now, the point what we are trying to do here is analyze the stability of this system. Now, of course there are two equilibria. So, there is one here and other here. So, the first thing which I can try to understand let us not try to understand large disturbance stability right away. Let us talk about the small disturbance stability of this system. How does this system behave, in case you give small disturbances if it is initially at this equilibrium? So, the point here is suppose I am at this equilibrium. If I am at this equilibrium, I will simply not move. The reason is d delta by d t is equal to 0 and d. So, if these two things are satisfied, I will just stay where I am. So, the only way we can get a transient is to give it a small disturbance.

If as a result of a disturbance, the system slightly gets deviates from one of the equilibria, so it is at one of the equilibrium. Let us say delta 1 itself and it slightly deviates from there. So, let us call this deviation to be small. So, this is a small deviation. So, let us say omega is a small deviation from omega e, the equilibrium value and delta is a small deviation from its equilibrium value. So, these are small deviations. Why are we talking about small deviations? We shall see later that if we talk about small deviations, the analysis is practical. You look at this. The set of differential equations is described the motion of the system.

How do you solve this? Unfortunately, it is not equally easy to solve this. You will have to actually knew kind of integrate these set of equations, but without actually doing the integrations say, numerically or by some other technique, can we try to understand the behavior of this system. The answer is yes. For small disturbances, yes quite easily. So, let us talk about only very small disturbances around the equilibrium. Why are we talking of small disturbances? Because once we have small disturbances, we can make certain approximations. So, you take our original equations.

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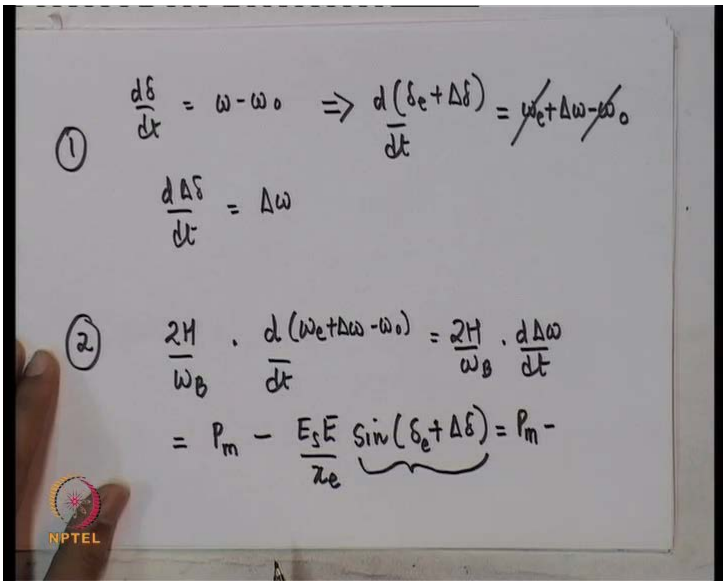

So, what I do is d delta by d t is equal to omega minus omega naught which implies d delta e plus delta delta is equal to omega e plus delta omega minus omega naught. So, actually omega e and omega naught are the same. Delta e is a particular value. So, the derivative of that is equal to 0. So, you have got finally for small disturbances, fine, for the small disturbances, the other equation. This is the first equation; this is the second state equation. You have 2H by omega B into d is nothing, but 2H by omega B into is equal to P m minus P e. Now, P e is E s E. We will assume these to be constant. All these things are to be constant sine delta plus delta e plus delta delta. So, that becomes equal to P m minus, well we can apply the formula for sine delta plus delta delta e plus delta delta.

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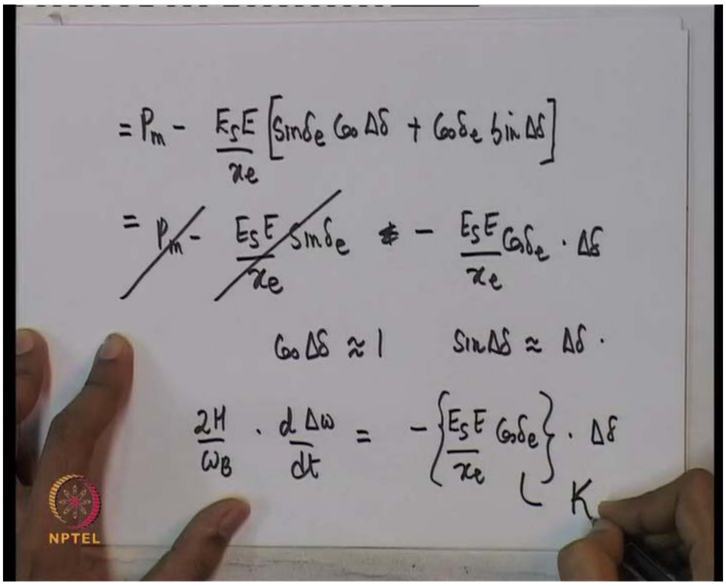

So, we will go to P m minus E s E by x e sine delta e cos delta delta minus, sorry plus cos delta e sine delta delta. So, what we have is P m minus E S E by x e sine delta e plus, sorry minus E S E by x e cos delta e. Is this a variable cos delta e and sine delta e? No, these are cos delta e. E is a particular value, so it is evaluated. It is not a variable into delta delta.

So, this is using the formula that cos delta delta is approximately 1. If delta delta is very small and sine delta delta approximately delta, so what we have here is the second differential equation becomes if P m is equal to con is a constant. Then, these since at equilibrium E S E sine delta e by x e should be equal to P m. These two can get cancelled. So, what we have here is basically E S E by x e cos delta e delta delta. This is a constant value evaluated at the equilibrium point. So, what we have? I will call this capital K.

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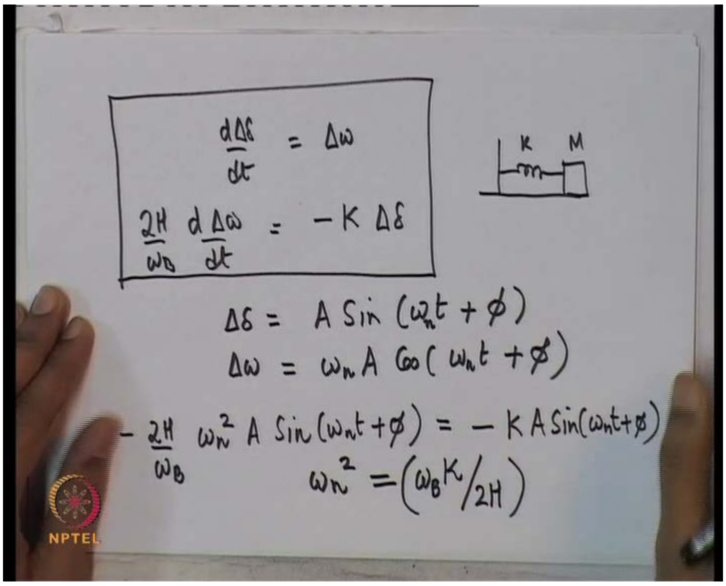

So, what we have here is the differential equations for small disturbances around an equilibrium are d delta by d t is equal to delta omega and d 2H by omega b b delta omega by d t is equal to minus K delta delta.

Now, the reason why I got you up to this point is this particular differential equation is actually easy to solve. For small disturbances, one can get a kind of an exact solution for this differential equation. So, what is the solution of this? Now, how to get the solution of this in this particular case or in a general case is something I will try to tell you later on in this course. Right now, let me just suggest a solution. This looks very much like the equation of a spring mass system M and K, where M is 2H by omega B and K is the spring constant of the spring. The differential equations which you get for this are very similar to this differential equations which are update.

So, let me not derive any solution for this. Let us assume that the solution for this particular system of equations let us guess it. So, let us say delta delta is equal to A sine omega t. I will call omega n t. This is the different omega n from the omegas we have been talking of plus 5. So, what will be delta omega if this is delta? You will have since this is the derivative 5, so these are well, we do not know. What we do not even know whether this is the correct solution or not, but if you plug it in it actually satisfies this equation. The first one is obviously satisfied if this is true, but you just plug it in this equation. So, what you have is 2H by omega B. You will have omega n square A sine. This will be minus is equal to minus K A sine omega n plus 5.

So, obviously this is satisfied if omega n square is equal to yeah. So, omega n square is equal to K k by 2H. This is omega B. So, this is true. Of course, these equations get satisfied and indeed our guess solution is correct if omega n is taken to be this. So, let us just move on. So, have we got the solution yet? No, we have just said that.

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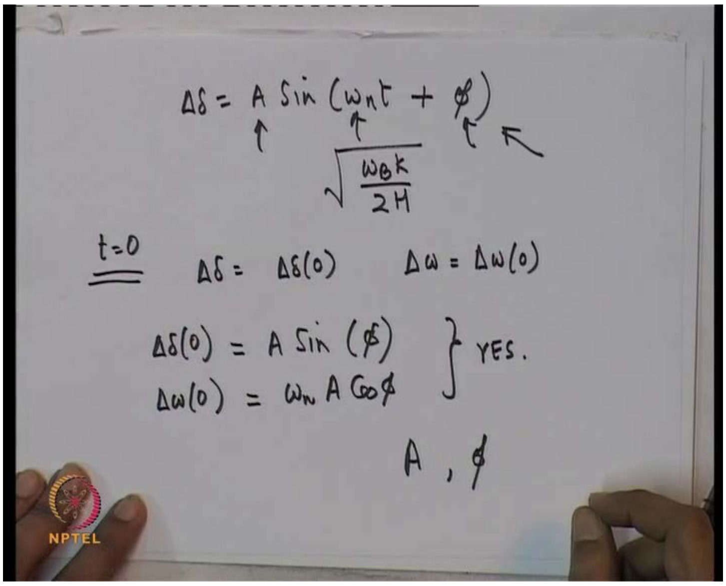

Out of this, we have got what omega n should be, that is it should be square root of omega B K by 2H can be positive or the negative square root. The things we do not have yet are A and 5. We only know that this particular solution, this solution is in fact correct. So, how do we get this a and 5? Well, let us say that at time t is equal to 0, we are not at equilibrium. We have been slightly displaced from the equilibrium. So, at time t is equal to 0 suppose, remember that the whole idea of doing this analysis is to see how we displaced from the equilibrium. If we are at a equilibrium, of course there are no transients at all because the rates of change of all the states are equal to 0. So, suppose t is equal to 0, we have and in that case, just substituting it in this particular equation delta is equal to A sine 5, right. So, actually if you look at the equations of this, this is delta delta, this is delta omega. So, what we have here is time is equal to 0. This is how things will look like.

So, if I am given, if I give you this delta and omega 0 that is time t is 0 to 0, can you take figure out what is 5? Yes, you can find out what A is and what is 5. Therefore, we get the complete solution because if I know what a is in that case and if I know what 5 is, you know what omega n is. So, I have got a complete solution for how this system behaves. So, this is what we get as the final solution of this particular system. Now, what I have shown you is that this particular system is stable. Well, not really stable, it has got an oscillatory motion. Is it really surprising? The answer is no.

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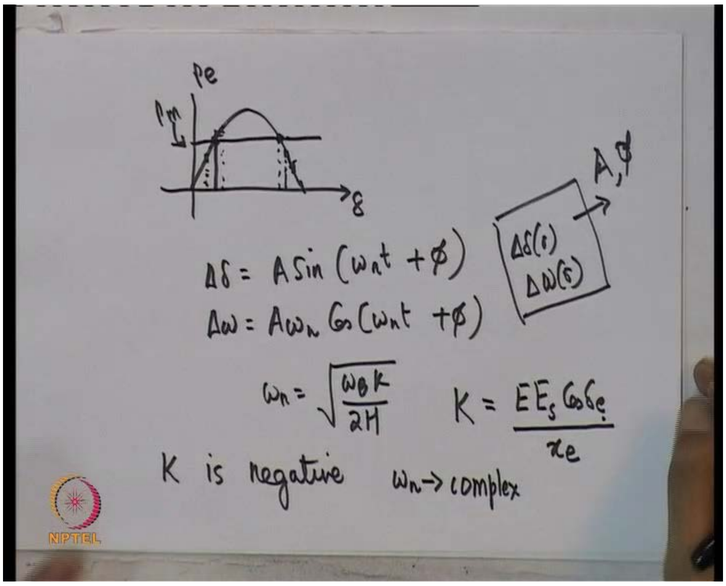

Suppose, I am at this, I am at this equilibrium. This is P m, this is P e, this is delta and I am at this equilibrium. If I give a small push, if I give a small push to this system, that is del. If this gets slightly away from this equilibrium because of a disturbance and I am suppose, at this point let us say delta is slightly away from the equilibrium and say, speed deviation is 0. In that case, P e becomes greater than P m, the machine will decelerate and try to come back to this equilibrium. Similar thing happens here if you are here, P e is less than P m. So, the machine will accelerate and come back to this equilibrium.

So, however the motion is kind of oscillatory, the restoring torques which are there are in fact proportionally. For the small disturbances, they are practically proportional to the angular deviations and therefore, you get a kind of spring mass behavior of this system. Of course, what about this equilibrium? In this equilibrium, it is not oscillatory. For example, my delta is here, speed deviation let us say is 0, but my delta deviation is there and slightly deviated from the equilibrium. In that case, P e is less than P m. So, the machine will accelerate. If it accelerated, it actually goes away from the equilibrium point. So, one would expect that this particular equilibrium point is not stable, it is unstable. So, how come I got this solution? It seemed all fine, right. This is my solution where A and 5 are obtained from the initial conditions of that is from this, you get A and 5. So, everything is fine.

So, this seems to say it is oscillatory, where omega n is. So, where am I going wrong? I mean this particular equilibrium point is unstable. Remember that this K if we recall our derivation, this K is nothing, but E E s cos delta equilibrium upon x e. This delta e here is dependent on the equilibrium point. In fact, this particular equilibrium point is greater than 90. So, this k is negative. If K is negative, omega n is a complex number. So, what we have is omega n is some complex number.

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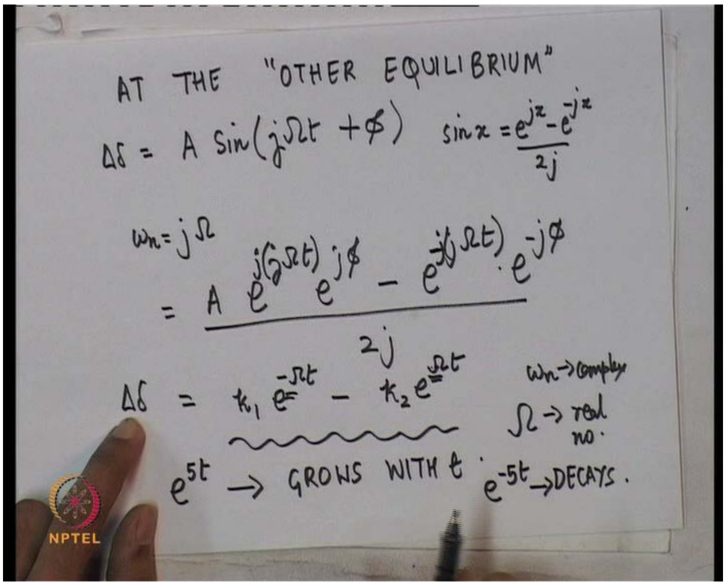

So, in that case, things get a bit complicated because now our solution is delta delta is equal to A sine, a complex number omega n. I will call it let omega n be, so at the other equilibrium. At the other equilibrium, delta delta is A sine J. Remember omega n is complex plus 5 which is nothing, but A. We can expand this. This is nothing, but E, (Refer slide time: 45:13-45:29) right.

So, this is nothing, but small values, sorry it should be j into j omega t here and it should be again, j into j omega t. Remember that this sine x is nothing, but equal to E raise to j x minus e raise to minus j x upon 2 j. So, this is what I view. So, it is nothing, but (Refer slide time: 46:03-46:14).

So, now your solution of delta delta if you are at the other equilibrium in which omega n turns out to be complex, the solution comes out to be like this, where omega of course is a real number, but omega n of course is a complex number. In fact is the purely imaginary number. Not only a complex number, but it is purely imaginary too. You look at this solution. If this is a real number, either this or this is going to be positive. So, minus omega will be positive or this or this will be positive. Now, remember that e raise to 5 t for example, grows with t. So, of course, e raise to minus 5 t decays. So, your solution for delta delta at the other equilibrium for a small disturbance is going to be a super imposition of these two terms. If the initial conditions are such that k 1 and k 2 are non zero, remember k 1 and k 2 are obtained from A and 5 and if this k 1 and k 2 are non zero, you will find that there will be one component of this solution which grows with time. It increases with time.

So, if you are at the other equilibrium point, what we have really shown is that you will be unstable; you will really grow with time. So, to summarize at least the small signal part, remember what we have done really is analyze the small disturbance stability of a single machine connected to an infinite bus. Using a very simplified model, we have seen that for a particular small disturbance, the system not for a particular small disturbance, for small disturbances, the system gives an oscillatory behavior for the equilibrium which is lower than 90 degrees and if it is of course the equilibrium point which is greater than 90 degrees which we have shown between 90 degrees and 180 degrees, the system is unstable, that is if I give a small push, the deviation grows with time. If the deviation grows with the time means, it will not come back to that particular equilibrium. So, after two equilibriums which we have got between 0 and 180 for delta for a given mechanical power, you see that one of this system is one of the equilibria is actually unstable.

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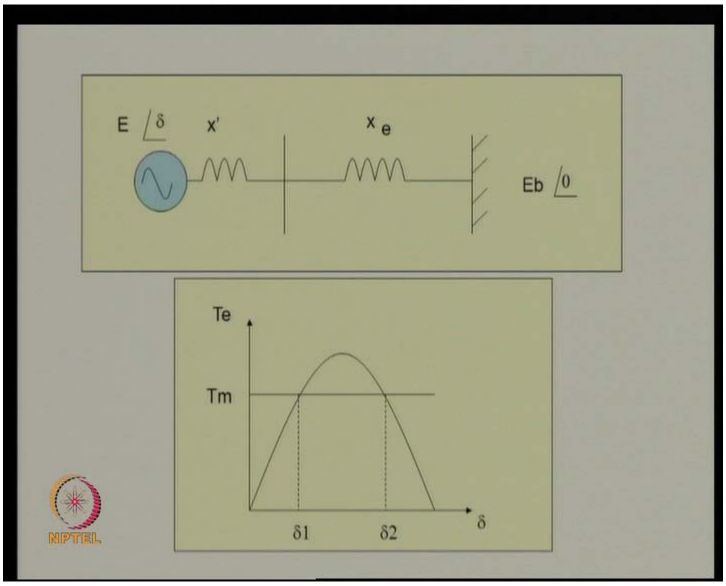

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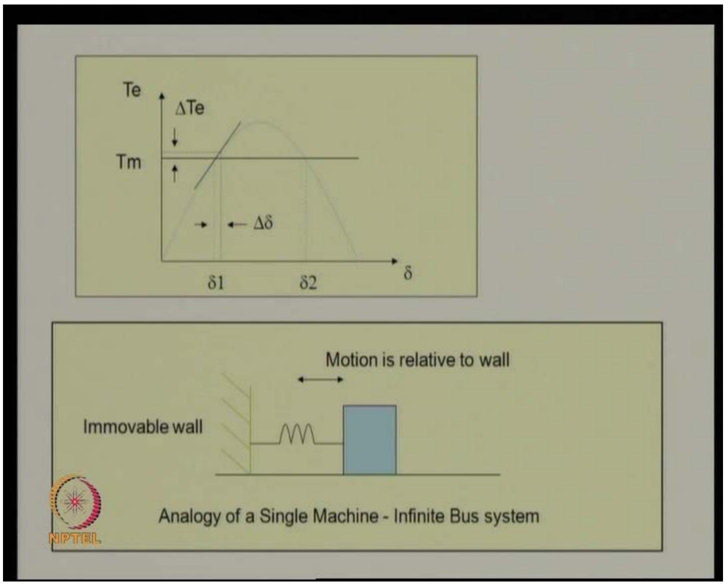

So, to summarize a single machine infinite bus behaves like more or less like a spring mass system at the equilibrium point delta 1.

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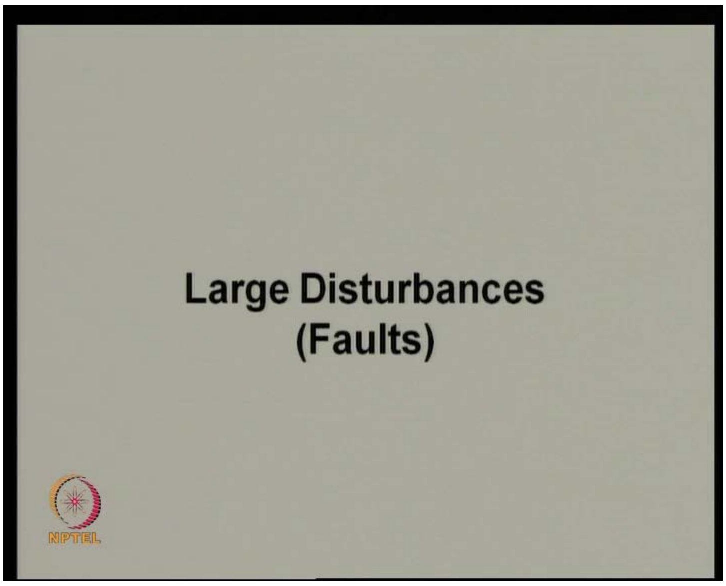

Now, last disturbances something we will learn in the next lecture, but let us do one practical. In this particular lecture, we have been looking at more of the mathematical treatment of a particular set of a differential equations. I said at the beginning that although, we are going to use a very toy model of the synchronous machine, it is actually going to replicate what we actually see in practice, a particular phenomena which we see, that is the oscillatory behavior etcetera is actually seen in practice.

So, if you recall in my previous lecture, I had shown you a small particular graphic of a disturbance did occurred in Tata power system and you could see that there were oscillations. So, these oscillation actually occur in practice. One good check we can do right away you know with this toy model is what is the frequency of oscillation, what are the frequency of oscillation if you are likely to face.

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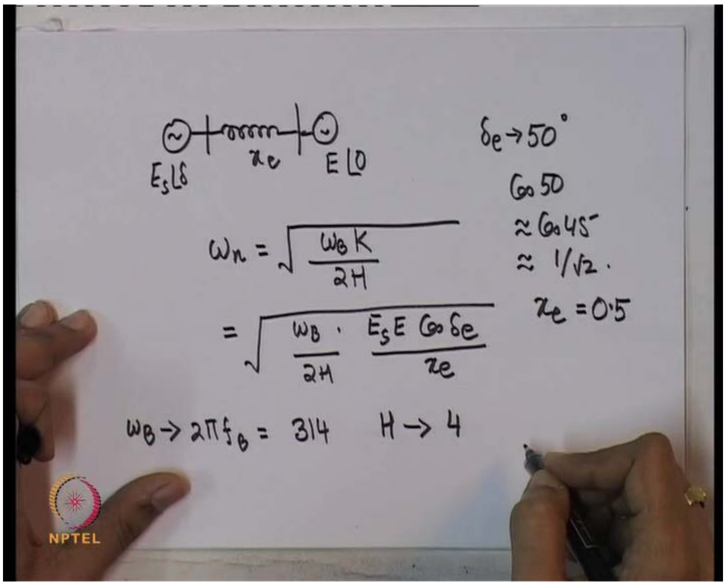

So, for example, you take this particular system, you call this E s angle delta. This is e e, e angle 0 and this is the infinite bus or constant voltage source. We saw that omega n is equal to root of yeah omega n is equal to root of omega b capital K upon 2H. This is nothing, but omega B. This K is nothing, but (Refer slide time: 51:02-51:14)

So, if I take omega B for 50 hertz system, the base frequency is nothing, but roughly 2 pi into 50 which is nothing, but roughly 314, roughly. H the typical value of the inertia constant of a machine let say, it is 4 mega joules per MVA. Let us say E s and e b are 1, cos delta e, let us say delta e is 50 degrees. So, cos delta e would be cos 50 is approximately equal to cos 45 is approximately equal to yeah 1 by root 2, yeah. X e let say, it is a cumulative impedance of the transient reactance of the generator, the transformer as well as the transmission line. Let us say, it is 0.5 per unit.

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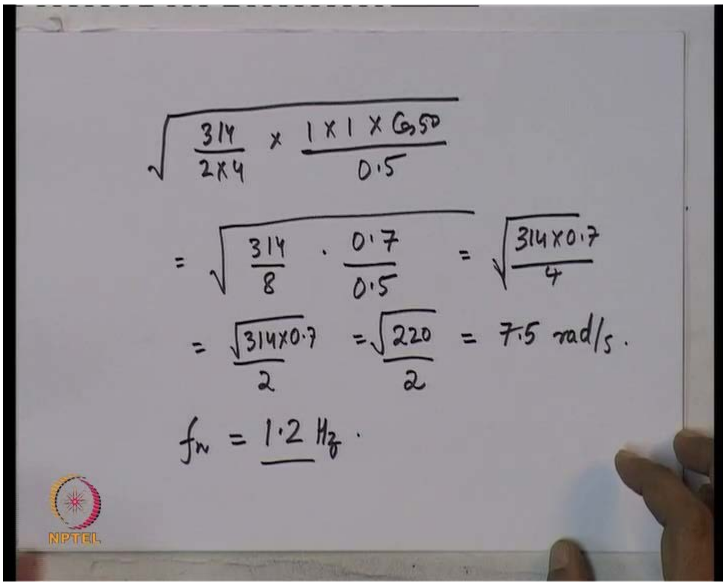

So, in that case, what we have is 314 by 2 into 4 1 into 1 into cos 50 by 0.5 is nothing, but 314 by 8. This is 1 by root 2 that is 0.7. Very roughly this is nothing, but 314 into 0.7 divided by 4 nothing, but half of root of 4 into 0.7 which is nothing, but roughly very very very very roughly. So, this is nothing, but let us say 15 divided by 2 around 7.5 radians per second.

So, the frequency of these oscillations is going to be roughly around 1, say 1.2 hertz. So, this is basically the natural frequency of oscillation which you will see if the system is subjected to small disturbances. So, if you are at the stable equilibrium point and if I give a push, in that case you will get a oscillation. So, the machines are rotating at 3000 rpm for a two pole machine or 1500 rpm for a four pole machine, but over and above that you will find that the speed of the machines over and above the 3000 rpm or 1500 rpm depending on the pole number, you will find that there is an oscillation taking place of 1, around 1 to 2 hertz. So, actually these oscillations have been observed in practice.

So, give a disturbance to a machine, it slightly oscillates. According to the solution which we have got for this particular system, if the system oscillates, it keeps on oscillating because you have got A sine omega t omega n t plus 5. So, it just keeps on oscillating, but actually there are reasons because of which usually these oscillation dies unusually. So, these oscillations die down. How do they die down? Unfortunately to the equations which I have given you in the toy model do not tell you how they will die down because that particular component of the model has not been included.

So, the point which I wanted to say here is we have taken a very simple system and shown that we give a small disturbance to it and it oscillates. Now, the real intention of doing this particular example was not so much to trying to tell you about the behavior of the electro mechanical behavior of a single machine connected to a voltage source, but the real intention was to tell you about how we can actually systematically analyze this system. In fact, we wrote down the differential equation and got the equilibrium linearised system or what we say is we saw how we can analyze the system for small disturbances. We guessed the solution. Now, this is one thing which we will try to do later again where we will not guess the solution, but actually derive it. We just guess the solution is A sine or omega t n or omega n t plus 5.

In the next several lectures, we will actually see how we can derive the solution for such system. The interesting thing which came out of this without having to go into very detailed models, we could infer that if such a phenomena does occur for typical values of system parameters, you will get oscillations of around 1 hertz and these are actually seen in practice.

Now, in the next lecture, we will just look at the equations for last disturbances. I told you last disturbance behavior can be quite different from the small disturbance behavior and we will try to see whether we can infer certain phenomena for last disturbances. So, to sum up in this particular lecture, we have really tried to see the concepts of equilibrium and small disturbances stability using an example.