

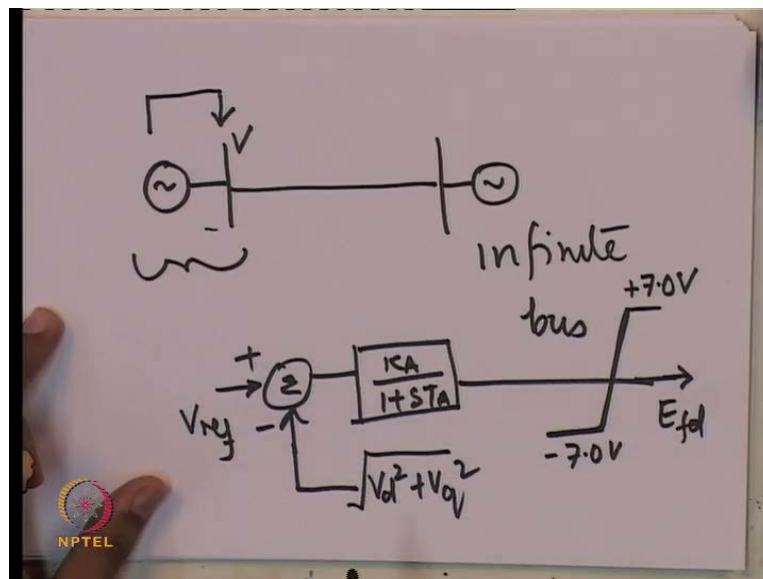
**Power System Dynamics and Control**  
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**Indian Institute of Technology, Bombay**

**Model No # 01**  
**Lecture No # 29**  
**Excitation System Modeling Automatic Voltage Regulator**

We had commenced the simulation of a single machine connected to a voltage source through a reactor. We had in fact just started that. The main point to be noted in that was the behavior of the AVR. Now, an AVR of course, is the automatic voltage regulator. The simulation studies of interest are the step response of the AVR to a step change in the reference voltage of the AVR. Also, we need to see in the simulation how with changing load conditions whether the AVR is able to regulate the voltage at the terminal of a synchronous machine connected to a voltage source or an infinite bus.

Of course, the important difference between the simulation we are carrying out now and what we carried out few lectures back was that, the synchronous machine is not directly connected to a stiff voltage source. But, it is connected via a very, a kind of toy model of a transmission line. And the transmission line in fact, is a model just like a reactor.

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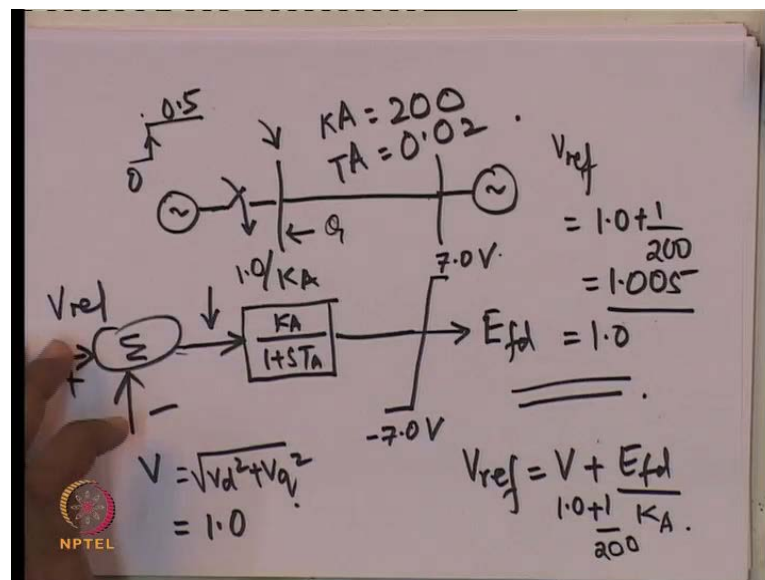


Now, if you look at what we are trying to do; the synchronous machine was connected via a transmission line to another stiff voltage source. This is the infinite bus, this is a

transmission line, this is a synchronous generator, this is the terminal voltage of this machine was being regulated. So, we took the feedback of the voltage at this point and fed it voltage regulator of this kind. Simple proportional kind of voltage regulator and the excitation system was assumed to be a static one.

So, **though** the model of it with normalized quantity this simple a plants with a gain of 1 and the limits of the static excitation system of course, were taken. This is fed to the field winding and of course, the feedback here is the magnitude of the voltage which is root of  $V_d$  square plus  $V_q$  square. We assume that there is no measurement delay here. We could if we wished model this delay by a first order transfer function. But, we will not do that right now. The simulation **was** is being carried out with the following steps.

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First, the synchronous machine is synchronized to this voltage source. It is in fact, it is kind of bumpless synchronization. We have talked about synchronization of a synchronous machine before. So, we will not talk of it again right now. Initially the voltage that the power of course, initially is 0 at the time of synchronization and the open circuit voltage is 1 per unit and the machine is running at practical at the rated speed. Now, in order to create 1 per unit at the terminals of the machine, you need to have  $E_{fd}$  is equal to 1. So, the output of the excitation system should be should give a field voltage which results in the terminal voltage being at its rated value at rated speed.

So,  $E_f$  is one. Now, to create this  $E_f$  of course,  $E_f$  is within the limits because the terminal voltage being 1 itself. So, 7 point 0 into 1 and minus 7 point it is within the limit so it is not getting clip. And to create this  $E_f$  you have got an automatic voltage regulator,  $V_{ref}$  and this is  $V$ .  $V$  which is nothing but, root of  $V_d$  square plus  $V_q$  square. Now, since this is equal to 1, this is one, the gain is  $K_A$ . So it follows that the value of this error here is 1.0 divided by  $K_A$ . So,  $V_{ref}$  in fact is nothing but,  $V$  plus  $e_f$  T by  $K_A$ . So one thing you should notice this is the proportional controller there is some steady small steady state error. In fact this steady state error is less if  $K$  is large.

So,  $V_{ref}$  and  $V$  are not exactly equal if you have got a proportional controller. You would need to have a small steady state error in order to get a nonzero  $E_f$ . So, this is something which you should keep in mind. So  $V_{ref}$  can be back calculated. So what is the  $V_{ref}$  which results in  $V$  being equal to 1.0 under open circuit conditions? Well, it is equal to this. So,  $V_{ref}$  is slightly greater than what is required to that is what **what**  $V$  we require. So, if this is 1 this will be  $E_f$  which is none 1 by say if  $K_A$  is 2; 100 which is a typical gain per unit by per unit then, your  $V_{ref}$  is equal to 1 plus 1 upon 200 so that is equal to 1.005.

So, you see that  $V_{ref}$  and  $V$  are almost the same provided  $K_A$  is large which is indeed the case.  $K$  is normally quite large. So, in our studies right now we will assume  $K$  is to 200 T A, the time constant here is we will take it as 0.02. It is small value and once we synchronize the machine there is no power flow. So, actually nothing **nothing** changes because you have just had a bump less synchronization. The conditions on the machine do not change. However if I increase the mechanical power out input for example, initially of course, before synchronization the machine is simple rolling. So, if I increase the mechanical power after synchronization, say I give a step change. It is not possible to give a step change to the mechanical power. But, let us just take it as a mathematical idealization. If I give a step change in the torque, electrical torque you will find at the power flow through this increase and presumably it remain in synchronism.

So, the speed of this and this will be the same. These two machines or this **this** machine and this voltage source is going to be same. That is, in spite of the change in power the synchronous machine tends to remain in synchronism with this bus. But, it is important change. As soon as some current starts flowing that is some power starts flowing, the voltage here tends to drop. So, if you did not have a automatic voltage regulator what

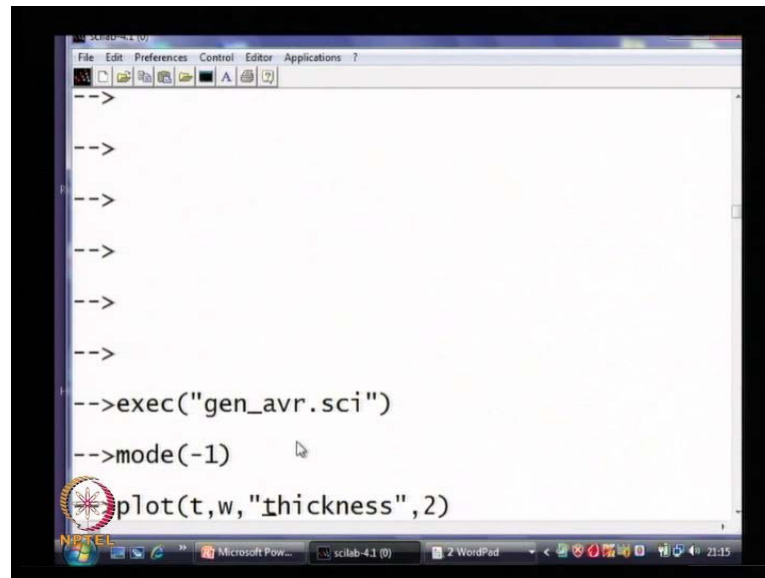
would happen is this voltage would it would drop and  $E_f$  would remain the same. But, with a voltage regulator, as soon  $V$  drops  $V_{ref}$  is of course, the same right now. If  $V_{ref}$  drops  $E_f$  increased. So, this voltage here is regulated and  $E_f$  is not a constant.

So, an automatic voltage regulator saves us the trouble of trying to you know, manually trying to change  $E_f$  every time we change the mechanical power. It does it automatically by satisfying this regulation function. So, that is for first test signal we will give to this machine. The second test signal will give **is give** a step change in  $V_{ref}$  keeping torque constant. So, what will do is increase the reference voltage of this a  $V_r$ . That is I am going to increase this voltage. Now, you know that if I increase this voltage will the mechanical power or the electrical power be affected? No. What instead **state** will get affected is, since this voltage is increased the reactive power output of the machine will change. So, this  $q$  here will change. So, this is one important thing which we will see now in our simulation. So, today's lecture we continue this simulation of automatic voltage regulator. In fact, it is system which contains an automatic voltage regulator.

So, what will do is first synchronize the machine it is a bump less, absolutely smooth transfer smoothed interconnection. Thereafter that is of course, than time  $T$  is equal to 0. Thereafter, we increase the mechanical torque of the machine. We increase the mechanical torque of the machine and thereafter we increase the reference voltage of the  $A V_r$  to 1.05. Now, I will, since is the absolutely smooth synchronization we have assume that the rated speed of the machine, the machine is running at exactly the speed which is equal to the infinite bus and the line to line  $r_m$  open circuited voltage is equal to the voltage which appears right at the point whether circuit breaker as to be closed.

So, you can see that there is going to be a bump less transfer of the magnitude of the voltage, is the phase angle also is 0 at a time of synchronization. So, what will have is the bump less synchronization first at time  $T$  is equal to 0. You will not see any transient because it is absolutely a smooth synchronization.

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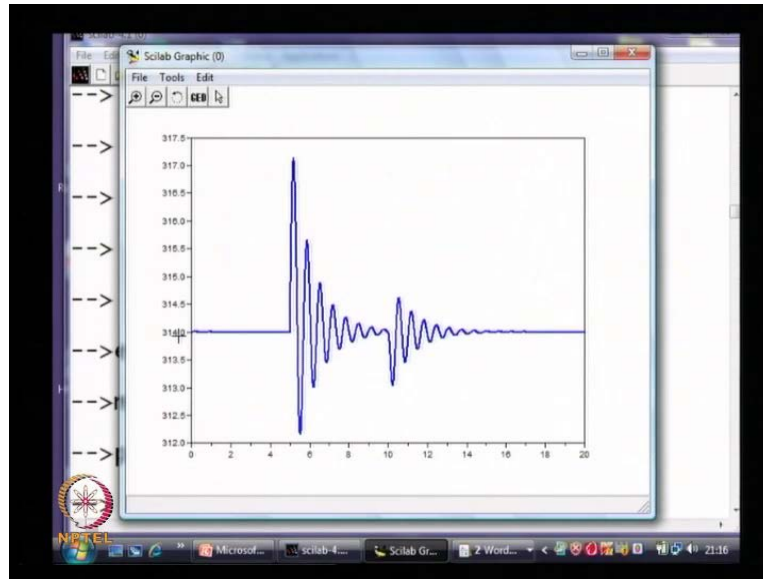


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-->  
-->exec("gen_avr.sci")  
-->mode(-1)  
plot(t,w,"thickness",2)
```

We will do the simulation right away. **yeah** So, I have already written the program. So, this time I will not show you the program. It is a simulating the system using Euler method. Euler method can be used only if I remove the stiffness out of the system **remove the stiffness in the system.**

So, I have neglected  $d \psi$  stator transient  $d \psi$   $d$  by  $d t$   $d i$   $d$  by  $d t$  and so on. So, this is something we have explained before in our earlier class. So, what I will do is concentrate more on the results this time. So, let us just see what happens to this speed. So, I will plot. **Yeah**

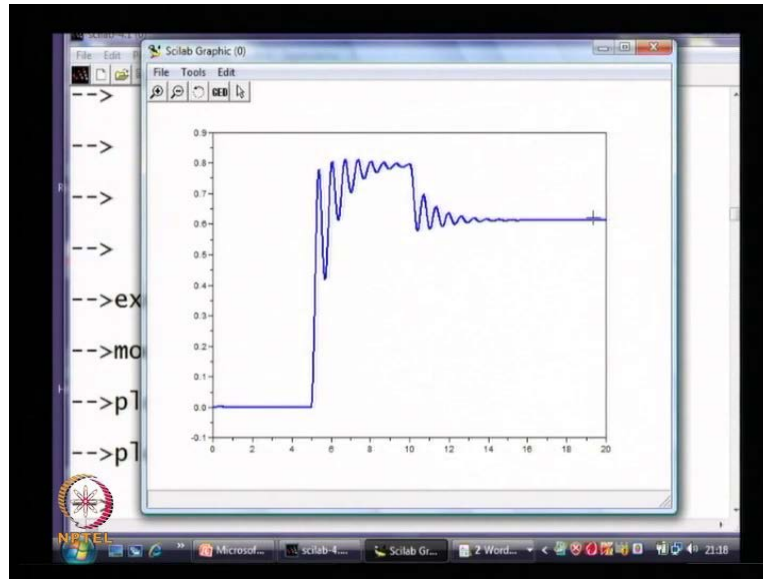
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So, what you see here is, a almost a bump less transfer here, a synchronization. Here at this point I give a step change in the mechanical power. If you increase the mechanical power in this system, the infinite, the machine does not loss synchronism in fact the step change in torque is 0.5 per unit from 0 to 0.5 per unit. A machine remains in synchronism in the sense that, it the speed of the synchronous machine although it deviates in transients; there is this swing which you see, a low frequency swing it tends to come back to synchronism. The speed tends to come back to synchronism at ten seconds. I have given again another step in the voltage reference of the synchronous machine.

So, you see that in a both cases the machine tends to remain in synchronism. That is, the mechanical the speed of the machine equals to the frequency of the infinite bus. Now, you can also see various other parameters for example, delta.

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If you look at delta of the machine this is of course, in radiant. The machine is synchronize when delta is equal to 0 and speed is equal to 0. So, you have got this bump less synchronization right in the beginning. And as soon as this give step change in torque, the phase angle of the synchronous machine, the delta of the synchronous machine rather it changes. You see this swing which is also observable in a speed, it is also observable here.

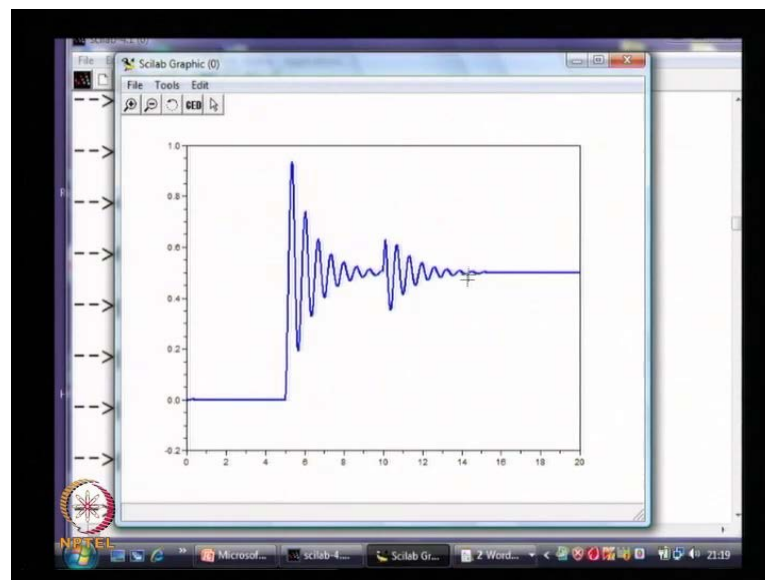
So, you see this swing this is one particular mode which is associated with a electro mechanical variables; delta and omega especially. Also, when you change  $V_{ref}$ , you find at delta actually slightly decreases. Now why is that so? You **are** have not changed the mechanical torque at this point. You have change delta to 1.0 5 from 1.0 0 5. So, if you make this step change in  $V_{ref}$ ; delta reduces because  $E_f$  d as changed. Once you change  $V_{ref}$ , we are effectively changing  $E_f$  d also. That is why delta changes. You again see a swing and which settles down to a steady state value. So the delta settles down to a steady state value.

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-->
-->exec("gen_avr.sci")
-->mode(-1)
-->plot(t,w,"thickness",2)
-->plot(t,delta,"thickness",2)
-->plot(t,P,"thickness",2)
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Microsoft Pow... Scilab 4.1 (0) 2 WordPad 21:18
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So, if you actually plot the mechanical power or rather the electrical power output of the generator which is  $e V i d$  plus  $E q i q$  we will find that d drive. yeah

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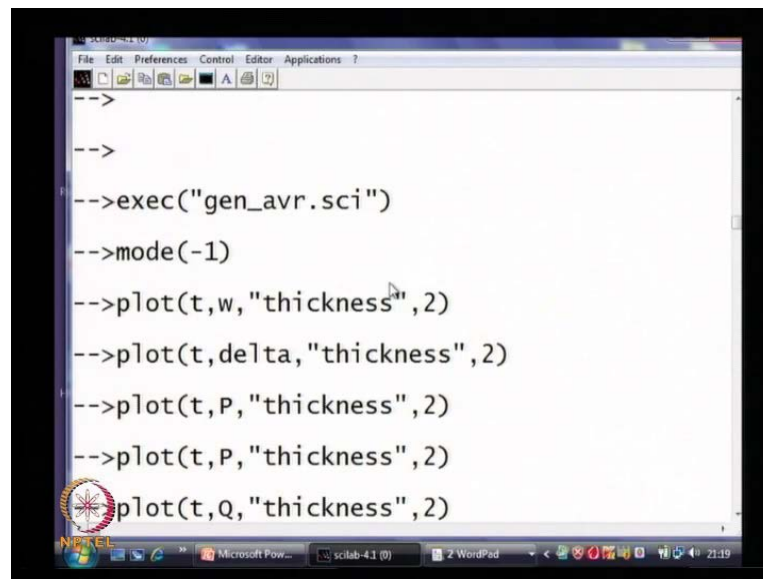


So, what you see here is a of course, a mechanical power changes to 0.5. The mechanical **sorry** the electrical power follows the mechanical power. You should remember, you have got an infinite bus. The synchronous machine is connected to an infinite bus. So, if the speed remains finally, the machine remains in synchronism and mechanical power become equals to the electrical power. So you see that the mechanical, electrical power



output  $P$  is equal to the mechanical power. It eventually becomes 0.5. If I change the  $V_{ref}$  of the machine, **if I change the  $V_{ref}$  of the machine** the reference voltage of the AVR of the machine; you find that the output electrical power  $P$  does not change. **did not suppressing** This is not surprising. After all, by changing the reference voltage of the machine you are not changing the mechanical power. The mechanical power becomes equal to the electrical power in steady state if the mechanical power does not change, the electrical power does not do so.

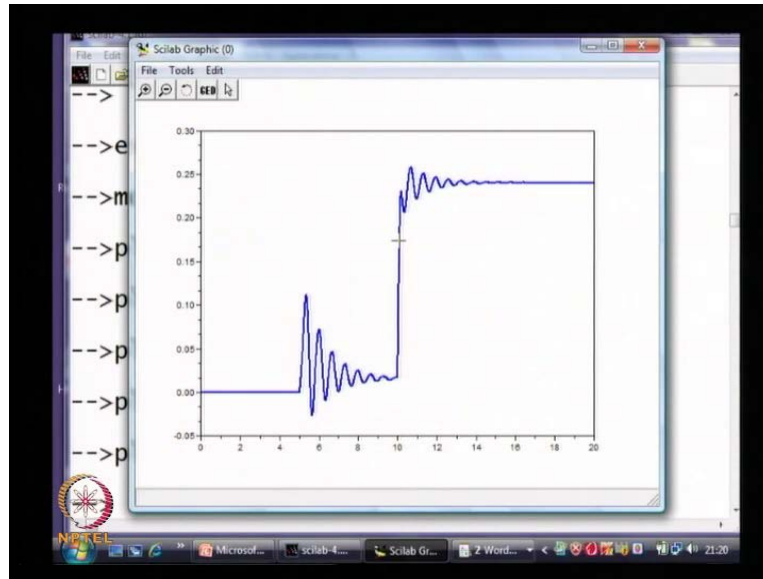
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-->
-->
-->exec("gen_avr.sci")
-->mode(-1)
-->plot(t,w,"thickness",2)
-->plot(t,delta,"thickness",2)
-->plot(t,P,"thickness",2)
-->plot(t,P,"thickness",2)
-->plot(t,Q,"thickness",2)
```

So, this is an interesting point here. What if I plot  $Q$ ?  $Q$  is the reactive power output of the machine **okay**.

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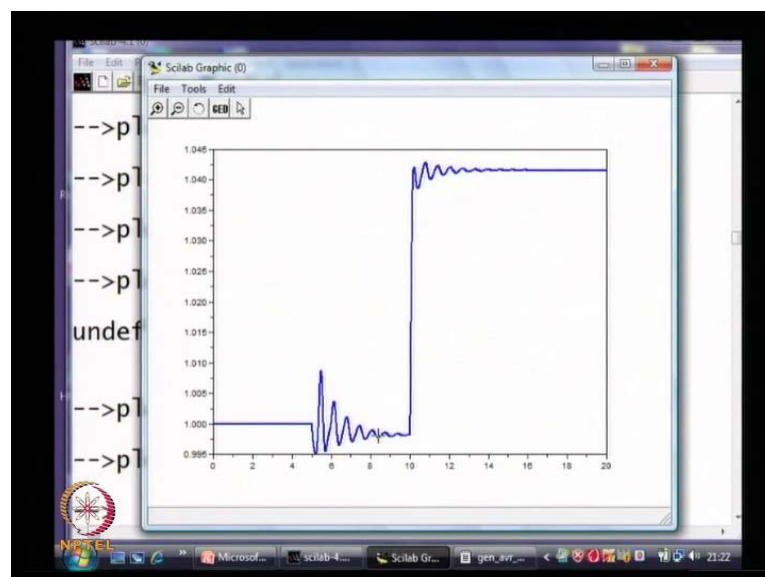
If you look at the reactive power output of the machine; if I, **if** of course, the machine is initially synchronized. There is no real or reactive power output at the point of synchronization. But, if I am change the mechanical power; the electrical power also changes,  $E_f$  also changes and you will find at the generator in fact supplies the bit of reactive power. There is some reactive power output of the generator. If I change  $V_{ref}$  on the other hand, if I change the  $V_{ref}$ , what you are doing is not changing much the real power does not change. But, the reactive power changes quite substantially. So, by changing the reference voltage of the AVR; we infect in fact changing the reactive power output of the generator. and of course, the terminal voltage also will change under these circumstances.

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File Edit Preferences Control Editor Applications ?
-->plot(t,delta,"thickness",2)
-->plot(t,P,"thickness",2)
-->plot(t,P,"thickness",2)
-->plot(t,Q,"thickness",2)
-->plot(t,V,"thickness",2)
!--error 4
undefined variable : V
-->plot(t,Vg,"thickness",2)
plot(t,Vgen,"thickness",2)
```

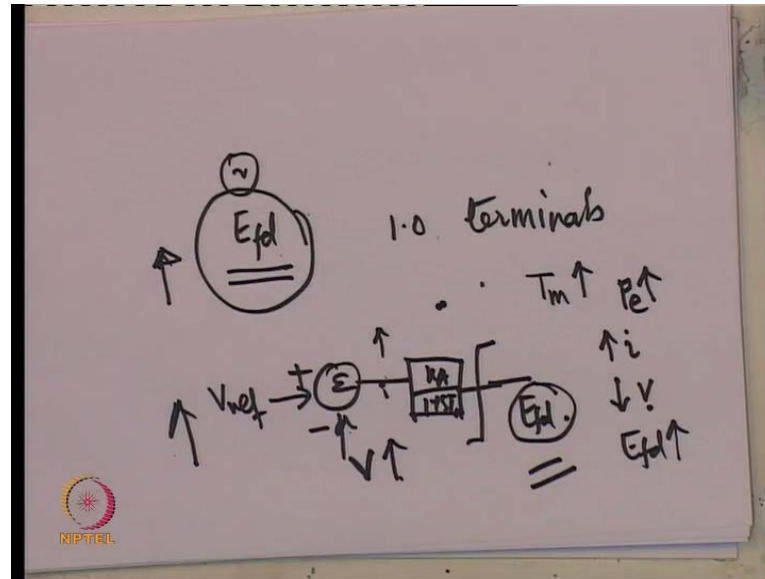
So, let us just look at the terminal voltage of the machine. Remember that the first disturbance or first step change is that of the synchronous mechanical torque at ten seconds you have given step change in the reference voltage of the a  $V_r$  from 1.0 0 5 to 1.0 5. So, let us just plot the terminal voltage. I think it is called  $V_g$ . yeah So, if you look at the terminal voltage, well I plotted something wrong. I will just just movement a moment. We will just look at the variable which is corresponding to it. It just split slipped of my mind. It is  $V_{gen}$ . So you just see what  $v_{gen}$  is. It is a terminal voltage of the synchronous machine.

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You see that the terminal voltage of the synchronous machine is initially one. When I increased, if I increased the real power output of the machine by changing the mechanical power at 0.5 seconds; we notice that the voltage slightly drops. Now, why does the voltage drop? That is the important point which you should **pointed** ponder upon. We will try to address it here.

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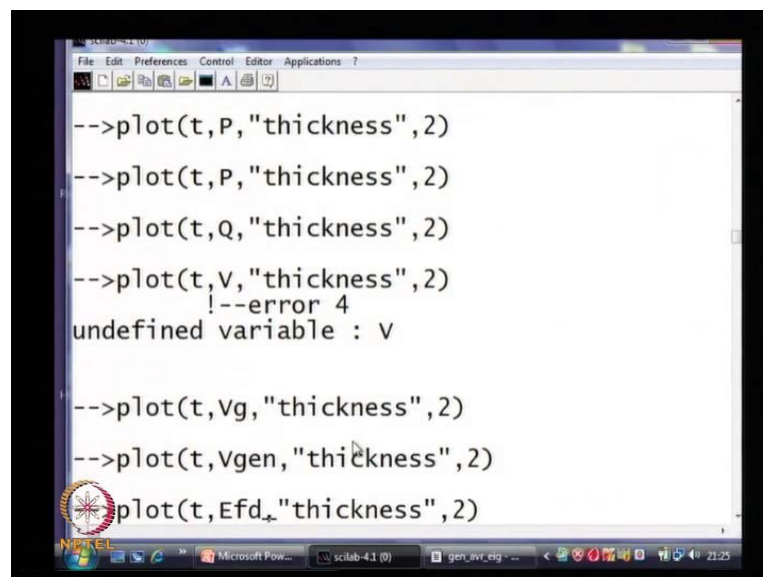
What happens actually is that when you have got synchronous machine under open circuit conditions, there is a certain field voltage which will produce 1 per unit at the terminals.

Once you load the machine, on the other hand that is you increase the mechanical power in and therefore, the electrical power. What happens is that, the terminal voltage tends to drop. Now, because the terminal voltage tends to drop, E<sub>fd</sub> is increased. So, if E<sub>fd</sub> is increased, V<sub>ref</sub> is the same, so let us just do the, you know kind of **Cos** Cause effect kind of analysis. Mechanical power increases, electrical power also tends to increase because of that. If electrical power increases, there will be from current 0, there is some current. So, current increases. Because current increases, you will find at the terminal voltage tends to drop. The terminal voltage tending to drop will force this automatic voltage regulator to increase E<sub>fd</sub>. So, if E<sub>fd</sub> is increased, it also means error has changed. V<sub>ref</sub> we have not touched right now. We are just giving a step change in mechanical power. So, what happens is effectively if the error as in steady state if E<sub>fd</sub>

required is more, we will have to drop more. This because you are using a propositional controller and steady state error is nonzero. So, to get more  $E f d$  you will have the steady state error will have to be more. So,  $V$  slightly decreases.

So, once you load the machine, since you got proportional type automatic voltage regulator, you find that the voltage slightly decreases. Of course, if I give step change in  $V_{ref}$  itself; so if you look at what I am done here now. Now,  $E f d$  is certain value. Now,  $V_{ref}$  is increased. If  $V_{ref}$  is increased of course, you will find that the error would have increased. If error increased,  $E f d$  will increase. As a result of that,  $V$  will increase. But, remember that  $V_{ref}$  and  $V$  are not going to be exactly equal. That is why even though if I made  $V_{ref}$  is equal to point for 1.0 5 at ten seconds, you find at actually the voltage settles down to around 1.0 4 2 5. So, this is the kind of interesting **load** look at the simulation. It should correlate well with our steady state analysis. If you look at of course, this something we have discussed even before. You have got to you should keep **in I** an eye on this swing. Now, if you look at this swing which is there, which is seen in almost all the quantities, it is a very prominent mode. You will find at it is oscillation of roughly 2 **yards** hertz, slightly less than 2 **yards** hertz.

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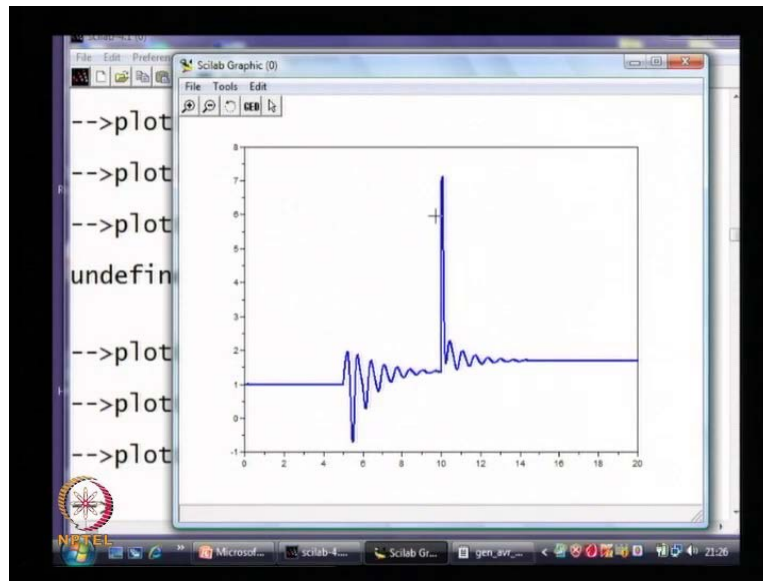


```
-->plot(t,P,"thickness",2)
-->plot(t,P,"thickness",2)
-->plot(t,Q,"thickness",2)
-->plot(t,v,"thickness",2)
! --error 4
undefined variable : v

-->plot(t,Vg,"thickness",2)
-->plot(t,Vgen,"thickness",2)
plot(t,Efd,"thickness",2)
```

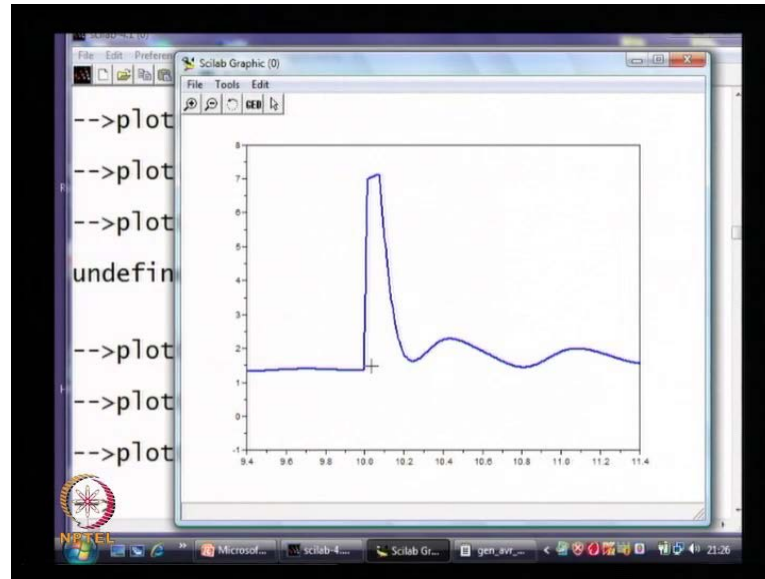
Now, will also look at  $E f d$ . We have not looked at  $E f d$ . Let see how  $E f d$  behaves.

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So, look at  $E_f$  vs  $t$ . Once you give step change in  $V_{ref}$  sorry mechanical power,  $E_f$  increases from one which is the value required to get 1 per unit at the terminals of the generator under open circuit conditions or no load conditions. If I increase the mechanical power, if I want to get a 1 per unit at the terminals, I will have to increase  $E_f$  and that is what exactly the voltage regulator as done. It has increased  $E_f$  from around 1 to approximately 1.3 or 1.2. Now, if I give another step, if I give the step change in  $V_{ref}$ , you find at  $E_f$  again increases. Of course, that is true. Because if I want to get higher voltage, I will have to increase my  $E_f$ . Now, remember that once I, the moment I give step you see that the change in  $E_f$  quite large. Why is that so?

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That is because this is extremely high gain system. Remember I told you, in order to get a very good response of the voltage regulator we do use very high gain in our automatic voltage regulator. This is because in order to **opposite** offset the relatively slow response of the field winding, what we do is we give a big push initially to the field voltage so that to get the field winding moving in the sense that we change the field current. So, that is why you provide for a very large, when you are designing excitation system and a  $V_r$  which provide for large gains and a fairly large range in which  $E_f$  can vary.

Normally  $E_f$  did not vary more than, under steady state condition more than 3 per unit 1.2 3 per unit is all what is required. But, normally the sealing is captivated more than 7 or 7 or around 7 time the terminal voltage. So, the sealing voltage is around 7 per unit plus or minus 7 per unit. So of course, if I give sudden change like a step change in  $V_{ref}$  the field gets forced. But, of course, it cannot exceed 7. So, it is kind of gets clipped at near about 7. So, what you see here in the simulation is that the  $E_f$  is try to increase to a very high value in order to get a very good response from the sluggish field winding but, it is clipped at 7.

So you see this almost flat top here. You see this and of course, once the field winding gets going there is no need to have such a large  $E_f$ . And  $E_f$  finally, settles down to a more comfortable value which is roughly around 1.7. So, this is an interesting point. So, this is the, showing the effective of the limit at the clipper which is there. You can also

have a situation if I give a step change in the reverse direction of course, E f d will decrease. So, I will not do that simulation but, you can well imagine what will happen and in fact you can try out this doing this particular simulation. It is very, it is an interesting simulation. Now, let me tell you something which is very, very interesting and important.

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```

gen_ovr - WordPad
File Edit View Insert Format Help
vd(k) = -Ra*[1 0]*A3*x_old -x_old(2)// -Raid-psiq
vq(k) = -Ra*[0 1]*A3*x_old +x_old(1) // -
Raiq+psid

id(k)=[1 0]*A3*x_old;
iq(k)=[0 1]*A3*x_old;
if(t>5)
    Tm=0.5; //
end
if(t>10)
    Tm=1.0;
    Vref=Vref;
end

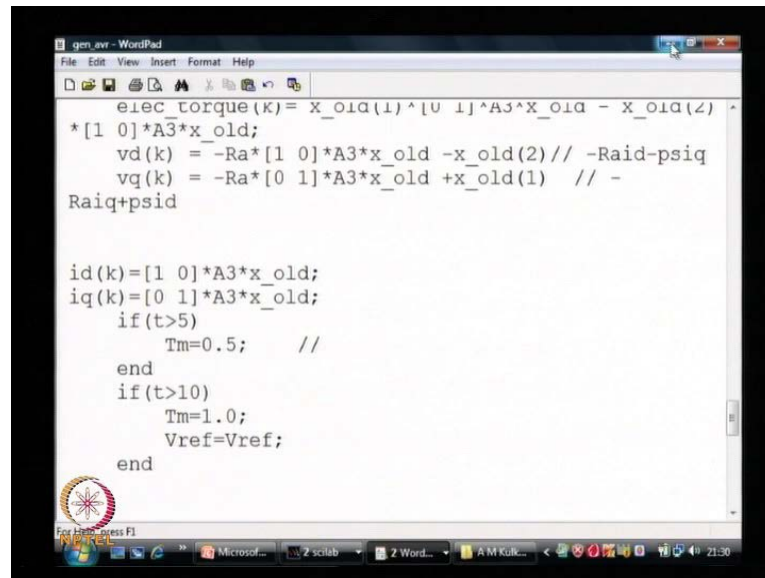
delta(k)=delta_old+(w_old-w0)*tstep;
w(k)=w_old+tstep*(P/2/H*(Tm_old-torque(k)));

```

Suppose, I if you recall I have written down this program and disturbance is which I given are at t greater than 5 seconds. I give a step change in mechanical power at t greater than ten seconds, V ref is given a step. So what I will do is, I will not give step change in V ref. I will keep it as it is. But, at ten seconds I will give an additional step to T m. So, I will put mechanical power is 1 per unit which is at rated speed equal to the rated power output of the machine. So, what I am done is I am trying to load the machine completely.



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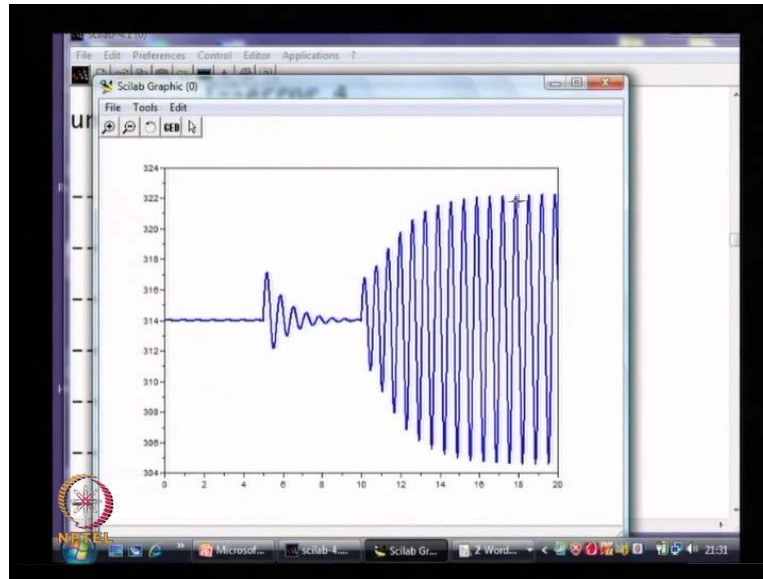


```
elec torque(k) = x_old(1) * [0 1] * A3 * x_old - x_old(2) *  
* [1 0] * A3 * x_old;  
vd(k) = -Ra * [1 0] * A3 * x_old - x_old(2) // -Ra * i - psi q  
vq(k) = -Ra * [0 1] * A3 * x_old + x_old(1) // -  
Ra * i + psi d  
  
id(k) = [1 0] * A3 * x_old;  
iq(k) = [0 1] * A3 * x_old;  
if(t > 5)  
    Tm = 0.5; //  
end  
if(t > 10)  
    Tm = 1.0;  
    Vref = Vref;  
end
```

Just one small point which I need to **rewrite** reiterate here is that, normally one cannot give such step changes in mechanical torque. This is an, just an idealization. It just is a kind of toy simulation in that sense. In later classes we shall see about modeling of the prime mover system themselves and you will note that you cannot give such step changes normally.

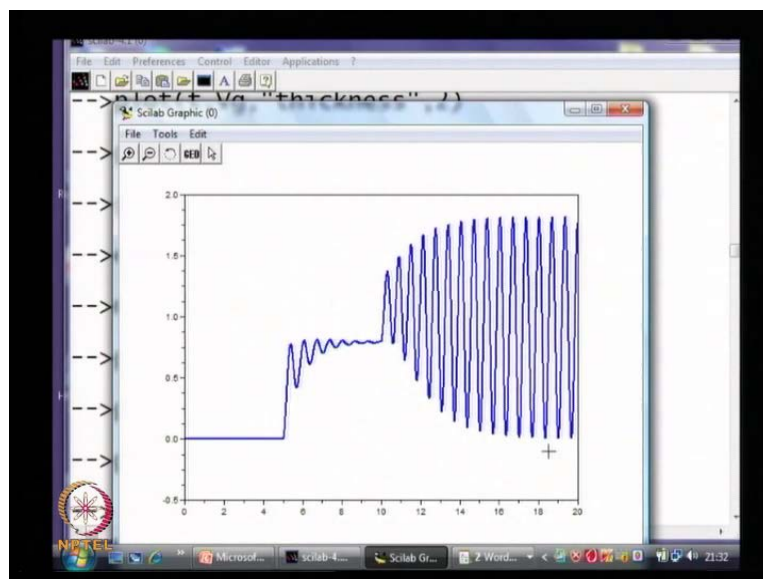
So, let us just try to redo this simulation. **yeah** In the mean time I will just close this graphics graphical window which is of the previous simulation. So, if you look at the simulation now.

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Now if you look at for example, just look at speed. There is an interesting problem which seems to have **reason** risen now is that, the speed does not seem to settle down. In fact the machine to off for all practical purposes seems to have **loss** lost synchronism. You see that speed is increasing with time and if you plot for example, delta also, we will of course, close this and redo this.

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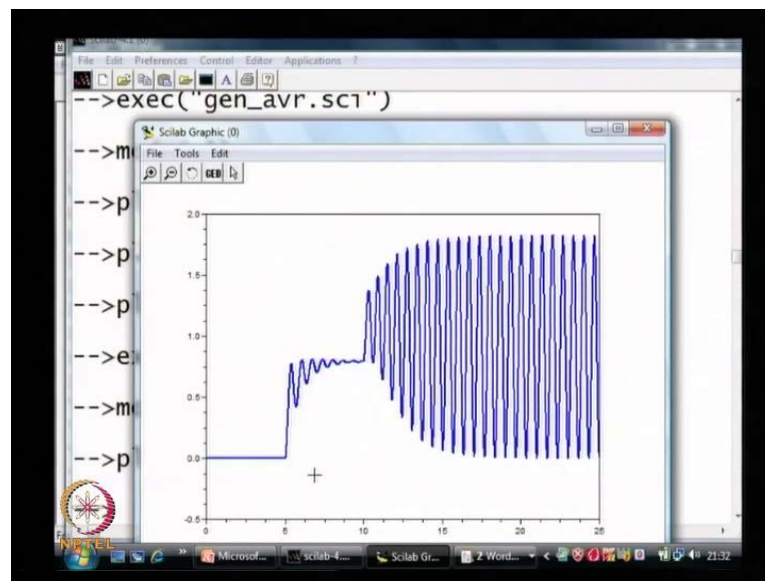


**Yeah** So you see delta, it does not seem to be reaching a steady state. It seems to be growing with time. So, these oscillations are growing with time. So, why is that

happening? And in that is an interesting point which you should ponder on. I mean it is, it is going on increasing with time.

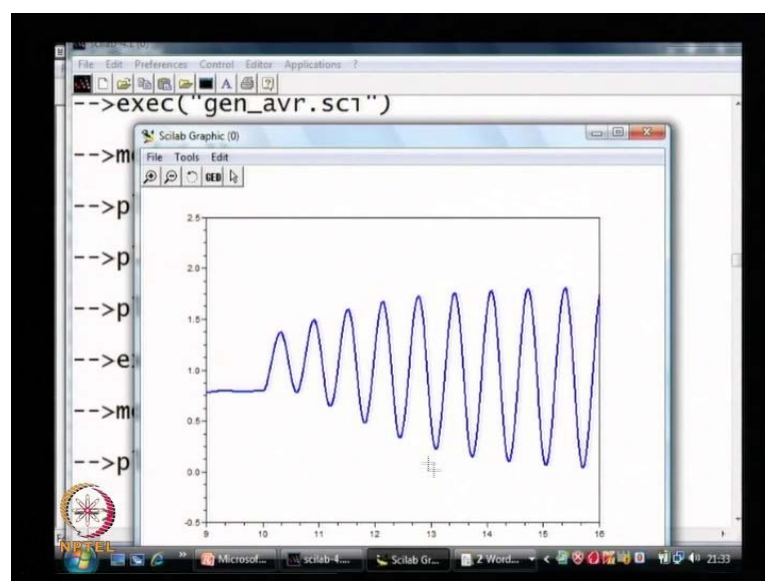
In fact, if I simulate for a longer time, this is a simulation for 20 seconds. If I make it as 25 seconds simulation, let us see what happens. Simulate for a slightly longer time and redo this. Close this.

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The delta seems to have kind of, is not settling down. It in fact if you look at what is happening is that it is increasing.

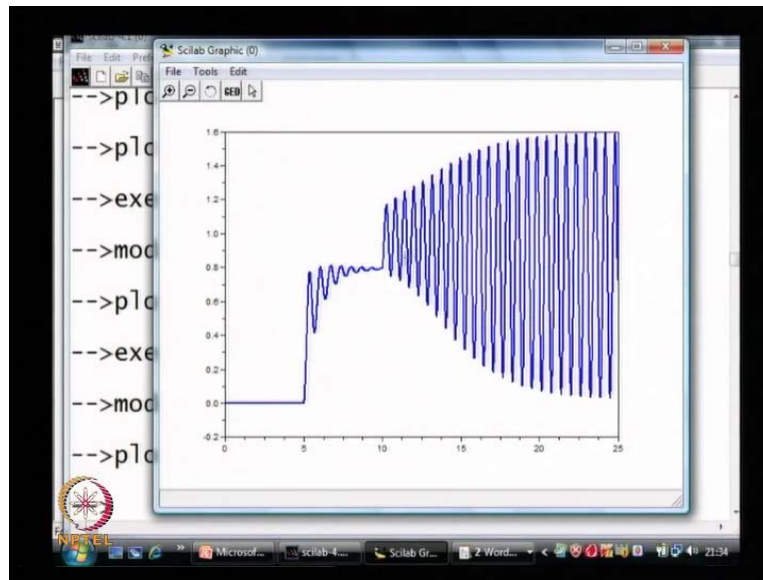
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So, this looks as if this particular operating point or equilibrium point which it should settle down to does not seem to be stable. It seems to be small signal unstable. So that is an interesting thing. So, the feedback system which we've got seems to, seems to be suggesting seems to be creating a situation for certain operating points which leads to a oscillation which does not seem to be dyeing down.

So, if you, in fact, may be if I give a make vref also increase at the same time, let us see what happens. Let us give you another simulation. So, what we saw in the previous simulation was that, there is an oscillation in delta. But, it is kind of increasing with time. That is not you know, usually we expected the delta decrease and you reach steady state. So, by giving step changes extra we were expecting the thing to reach steady state. But, it appears that there are situations in which steady state is not reached.

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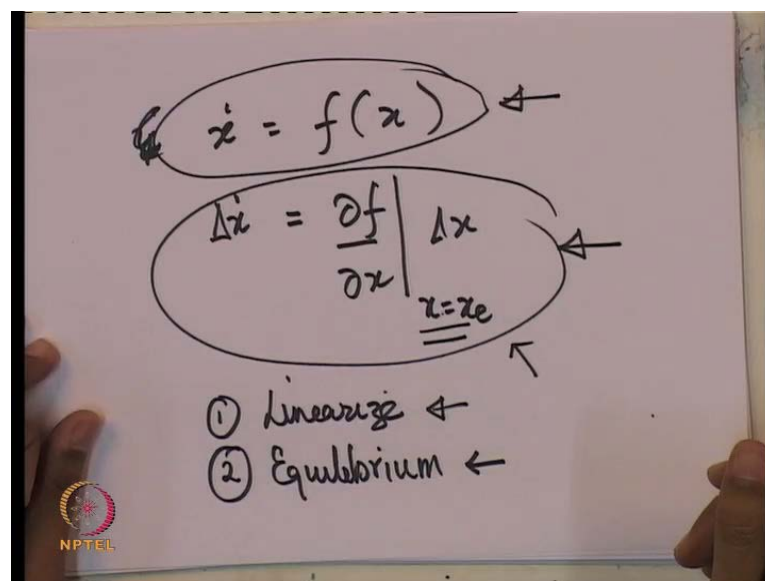
In fact, this seems to be even worst. Your angle seems to have increased even more. So, or perhaps its **moral** more or less is the same but, you see that this is also unstable. So, actually what we are effectively coming to a situation where the equilibrium point under our study appears to be unstable. So, what we should expect is that, at this operating point when do a linearize analysis of this system we should get Eigen values with positive real parts. So, that is what we need to correlate in our analysis now. So, let me just again point out what I want to say. We are having a situation, when we try to change the power from 0.5 to 1; we see that the system is not settling down to the new

equilibrium point. Now, what it what this means is that, why is it so actually why should the behavior depend on the equilibrium point? Well, one of the important point you should note is that, this is a non-linear system. So, when we do a linearise analysis we will be linearlizing the system around each operating point.

So, when we do the linearise analysis around different operating points our Eigen value is going to b,e are going to be different. So, while step changes to 0.5, T m is equal to 0.5 or v ref is equal to 1.0 5 which you did in the previous simulation were all stable. You see that in trying to come close to T m is equal to 1.0, there seems to be a problem. So, the operating point to which we are trying to get to, appears to be not stable not small signal stable itself. So, what we need to do here is try to understand this instability of the swings. You see this low frequency swing if you except to be stable and to die out is growing with time because of the feedback **feedback** control system of the a V r.

So, if we kind of try to a linearise analysis, how do we go ahead? Now, linearise analysis is something we've done before. I will just quickly retrace out the steps. So, what we have in our equations of a synchronous machine. If you look at the equations of a synchronous machine, these are in fact non-linear. So, we've done the linearization of such systems before. So, what we need to do is instead of having a set of system  $\dot{x}$  is equal to  $f$  of  $x$ ; what we need to do really is do a linearise analysis around an equilibrium point of this system.

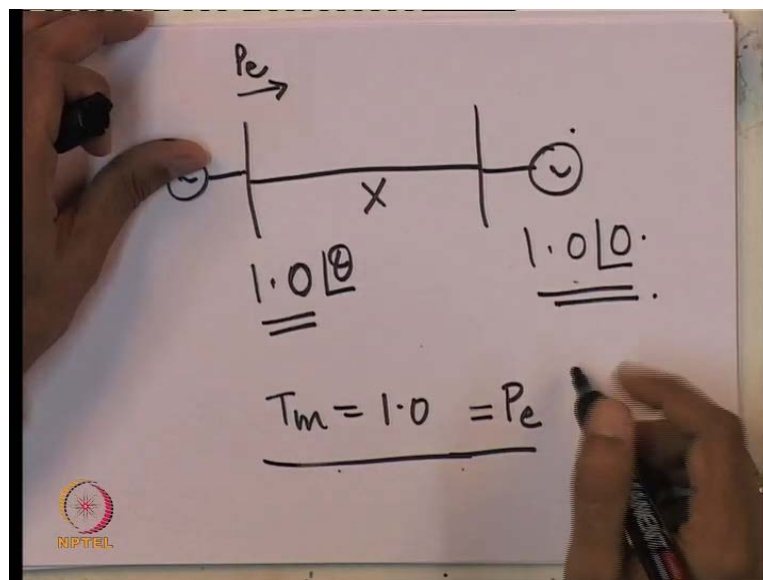
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So, if we've written down our equations in this form, we should now see we can do linearise, we have done a simulation of this system. This is a full non-linear system. We can try to understand the stability of the equilibrium point by doing a linearise analysis in this form. Remember, that when doing a linearise analysis you have to take we have to linearise it around an equilibrium point.

So, there is one important thing is, first thing is we have to linearise. Linearise in the sense use tailor series expansion of the non-linear functions which appear here. That is called linearization. And evaluate those you know after truncation of the higher order terms. You evaluate this at the equilibrium point. So, second thing is you've to take out the equilibrium points. So, linearise analysis would require first of all you to take out the equilibrium point and the second part is you linearise around an equilibrium point. Is that okay? So, this is what you need to do. How do you compute the equilibrium point? So, in fact long ago, I mean, I am sure it is not too long ago, you must have done the load flow analysis. A load flow analysis is in fact is a kind of an equilibrium analysis of the system. But of course, that is a in some sense the starting point of computing any equilibrium. In our case we've got a very simple system in fact computing the equilibrium points is very, very easy.

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So let us just go ahead and see how you can do it. Suppose, I want to find out this, the equilibrium values of the states corresponding to this scenario. You have got this

transmission line which has got a plain reactants  $X$ . This is your infinite bus which we take as 1 angle 0. That is I have already defined what the infinite bus voltage is  $r V_n V_b n$  and  $V_c n$  in the previous class. Let us say the terminal voltage of the machine is 1 per unit. it is near about 1 per unit and  $T_m$  is equal to 1. If  $T_m$  is equal to 1 and this is at rated speed then it also means that electrical power output of the machine, we are of course, neglecting the losses in the resistance of the synchronous generator they are really very small so compared to this speed. So, we will assume that electrical power output at the terminals of the machine is same as in per unit as  $T_m$  because, the speed is equal to the rated speed. We will assume that the frequency of the infinite bus is also equal to the rated frequency. So, if  $P$  is the output of the machine, this is 1 per unit what is the phase angle of the terminal voltage of the synchronous machine? That can be easily found out. You know that for a system with reactants  $x$  voltage magnitude 1 on both sides, the phase angle of the voltage here  $\theta$  is such that  $P_e$  is equal to 1 per unit is nothing but, voltage magnitudes at both ends divided by  $x$ .

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The image shows a whiteboard with the following handwritten equations:

$$P_e = 1.0 = \frac{1.0 \times 1.0 \times \sin \theta}{X}$$

$$\theta = \sin^{-1} \left( \frac{1.0 \times X}{1.0 \times 1.0} \right)$$

$$I = \frac{V \angle \theta - E \angle 0}{jX}$$

In the bottom left corner of the whiteboard, there is a logo for NIPTEIL.

This is of course, a multiplication sign and this is, it looks the same. So, we will just make this larger here into  $\sin$  of  $\theta$ . So  $\theta$ , you can calculate  $\theta$  from this. So, what, let me just remind you what we are doing. We are computing the equilibrium point corresponding to this operating condition and thereafter doing a linearise analysis. So,  $\theta$  is  $\sin^{-1} P_e$ . This is at equilibrium divided by 1 into 1 into  $x$ . So, this is  $\theta$ . We also can compute the current flow through this. Once you've got  $\theta$  you can get




the current. So, the current phasor can also be obtained. So once you've got the current phasor. So, current is equal to  $V$  angle  $\theta$  minus  $E$ .  $E$  angle  $0$   $E$  is 1 per unit  $e$  is the infinite bus voltage divided by  $j$  into  $x$ . So, this is effectively the current  $I$ . Once you get the current  $I$ , we are effectively can get the, what the current looks like.

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**Model of interconnection (d-q pu form)**

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left( \begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$


$$x = \frac{\omega_B L}{Z_{base}}$$


So, what we've effectively done is, if you actually look at, if you look at what we have in another way what we've done effectively is, set the left hand side  $d$  by  $d t$  is equal to 0.

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$$\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = - \left( \begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$


---


$$(i_q + j i_d) = \frac{(v_q + j v_d) - (E_q + j E_d)}{j x}$$




And thereof, therefore, we've got  $i d i q$ . If you look at the equations, which we saw on the screen you can have a look at them again.  $0 \text{ minus } X \text{ and } 0 \text{ is equal to minus of } V d$   
 $V q \text{ minus } E d E q$ . Now, this is effectively the same as saying  $i q$ . You can just write this in very compact form  $i q \text{ plus } i d \text{ is equal to } V q \text{ plus } j V d \text{ minus } E q \text{ plus } j E d$  divided by  $j \text{ of } x$ . You can just expand this and show that this is same as this. So, what let me tell you what **we know that we know what what** we effectively get is this particularly equation. That is why it appears like this.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, it says  $1.0L0 \rightarrow E_{an}$ . Below this, it lists  $E_{bn}$  and  $E_{cn}$ . The next line shows  $E_{an}$  followed by an equals sign and  $\sqrt{\frac{2}{3}} \sin(\omega t)$ . At the bottom left, it says  $1.0L\theta$ . There is an NPTEL logo in the bottom left corner of the whiteboard area.

So, the point here is that you've got  $1 \text{ angle } 0$ . These are the voltages of the infinite bus  $E_{an}$ ,  $E_{bn}$  and  $E_{cn}$ .

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$$\begin{cases} V_{an} = \sqrt{\frac{2}{3}} \sin(\omega t + \theta) \\ V_{bn} = \sqrt{\frac{2}{3}} \sin(\omega t + \theta - \frac{2\pi}{3}) \\ V_{cn} = \sqrt{\frac{2}{3}} \sin(\omega t + \theta - \frac{4\pi}{3}) \end{cases}$$

$\theta \quad V_d, V_q$

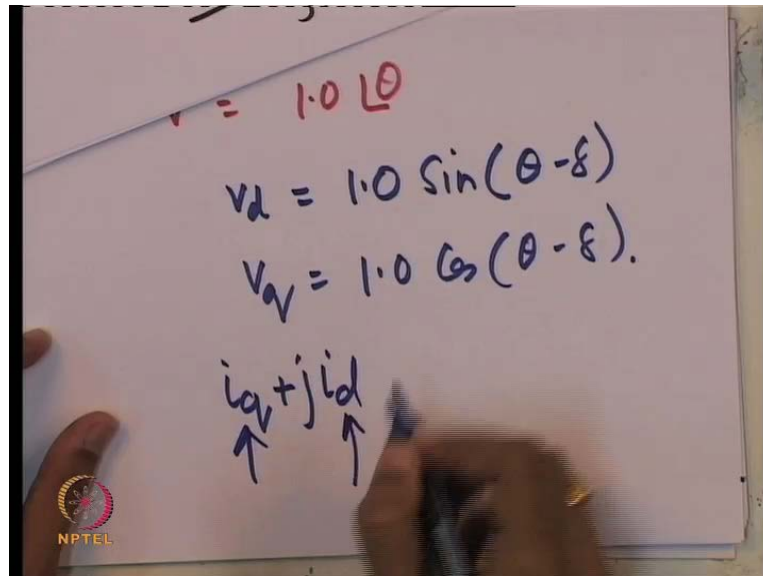
So, what does it mean that  $e_a$  is equal to  $\sqrt{2/3} \sin \omega t + \theta$  and  $e_b$  and  $e_c$  are of course, balanced counterparts of it. Your terminal voltage is 1 angle  $\theta$ . What does it mean? It means  $V_a$  is equal to  $\sqrt{2/3} \sin \omega t + \theta$  and  $V_b$  is of course,  $\sqrt{2/3} \sin \omega t + \theta - 2\pi/3$  and  $V_c$  is  $\sqrt{2/3} \sin \omega t + \theta - 4\pi/3$ . So, the point is that if I know  $\theta$  I can tell you what  $V_a$ ,  $V_b$  and  $V_c$  look like. As a result of it, we can get what  $V_d$  and  $V_q$  look like. That is the most important thing which you should get out of this.

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$$\begin{aligned} 1.0 \angle \theta &\rightarrow \begin{matrix} E_{an} \\ E_{bn} \\ E_{cn} \end{matrix} \\ \boxed{\begin{matrix} E_d = -E \sin \delta \\ E_q = +E \cos \delta \end{matrix}} \\ &= \sqrt{\frac{2}{3}} \sin(\omega t) \\ 1.0 \angle \theta & \end{aligned}$$

See if you know  $E_a$ , I told you that  $E_d$  is equal to minus  $E$  times  $E$  is 1 in this particular case  $\sin \delta$  and  $E_q$  this is plus  $E \cos \delta$ .

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$$V = 1.0 \angle \theta$$
$$V_d = 1.0 \sin(\theta - \delta)$$
$$V_q = 1.0 \cos(\theta - \delta)$$
$$i_q + j i_d$$

So,  $V_d$   $V_q$  is therefore, if  $V$  is equal to 1 point 0 angle of theta, what it follows is that  $V_d$  is equal to and  $V_q$  is equal to 1.0 **yeah**  $\sin$  of delta minus delta and 1.0  $\cos$  of theta minus delta. So, we've got  $V_d$  and  $V_q$  and  $E_q$  and  $E_d$ . How did you get  $V_d$  and  $V_q$ ? We computed theta. Theta is obtained from the initial condition. So, in effect we've obtained what  $V_d$  and  $V_q$  should be. If I know  $V_d$  and  $V_q$ , I can get  $i_q$  plus  $j i_d$  or equivalently  $i_d$  and  $i_q$  separately from this. So, if I have got  $i_d$  and  $i_q$   $V_d$   $V_q$  and  $E_d$   $E_q$ , the next step is to try to back calculate what is the value of delta, omega and all the other fluxes.

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The image shows a whiteboard with handwritten mathematical notes. At the top, the variables  $\delta$ ,  $\omega$ , and  $\psi$  are listed. Below them, the equation  $\omega = \omega_0$  is written and enclosed in a rectangular box. Underneath the box, the equation  $\psi_d = x_d i_d + E_{fd}$  is written. At the bottom, there are two equations:  $\frac{d\psi_F}{dt} = 0$  and  $\frac{d\psi_H}{dt} = 0$ . The variables  $\psi_F$  and  $\psi_H$  are written to the left of these equations, with curly braces indicating their association. An NPTEL logo is visible in the bottom left corner of the whiteboard.

Now, delta omega is equal to omega naught in steady state. So, if we are talking of an equilibrium point, omega will be equal to omega naught. So, we are taking out an equilibrium, we are doing an equilibrium analysis, we are back calculating the equilibrium conditions. Under equilibrium conditions psi d is equal to x d into i d plus E f d. How does one get that? Well, what we have to do effectively is set d f dot d psi F by dot by d t is equal to 0 and d psi H by d t is equal to 0. So, if you set this equal to 0 you will get psi f and psi H in terms of psi d. After that, replace it in the algebraic equations which relates psi d with i d. You will effectively get this particularly equation. Similarly, in steady state this is not true in general. This is true only in steady state.

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$$\psi_q = x_{qi} i_q \} \text{ s.s.}$$

$$0 = -\omega_0 \psi_q - \cancel{\omega_B R_a i_d} - \omega_B V_d$$

$$0 = +\omega_0 V_d - \cancel{\omega_B R_a i_d} - \omega_B V_q$$

$$\underline{V_d = -\psi_q \quad V_q = \psi_d.}$$

Please remember,  $\psi_q$  is equal to  $x_{qi} i_q$ . Similarly, you will get  $\psi_d$ , so under steady state conditions this is what you get. Now, we know that in steady state condition  $d\psi_d/dt$  is equal to 0. So, what you'll get is 0 is equal to  $\omega_0 \psi_q$  under equilibrium condition  $\omega_0$  is equal to  $\omega_B$  is equal to base value minus  $\omega_B R_a i_d$  minus  $\omega_B V_d$ . Similarly, you will have  $\psi_d$  this should be minus  $\omega_B R_a i_d$  plus  $\omega_B \psi_d$ . This is got by putting  $d\psi_d/dt$  and  $d\psi_q/dt$  equals to 0 which is to under steady state conditions. This is minus  $\omega_B R_a i_d$  this should be minus  $\omega_B V_d$  minus  $\omega_B V_q$ .

So what happens is, if you assume resistance is very small of course, you have the value there is no harm. If you assume that it is very small, what you will get is  $V_d$  is equal to  $\psi_q$ ,  $V_d$  is equal to  $\omega_B$  and  $\omega_0$  the same. We will assume that the speed of the infinite bus is the same as the base speed. In such a case  $V_d$  is equal to  $\psi_q$  and  $V_q$  is equal to  $\psi_d$ . So, this is one important thing which we get.

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$$V_d = -x_q i_q$$
$$V_q = x_d i_d + E_{fd}.$$

---

$$(V_q + jV_d) = E_{fd} + jx_d(i_q + j i_d) - jx_q i_q + jx_d i_d$$

So, what we have is  $V_d$  is equals to minus  $x_q i_q$  and what we have here is  $V_q$  is equals to  $x_d i_d$  plus  $E_{fd}$ .

So, if you have this particular equation; what a simple thing we can do is, we have got  $V_q$  plus  $j V_d$  is equal to **yeah**  $E_{fd}$  plus **is**. We can write this  $j$  times  $x_d$  yeah into  $i_q$  plus  $j i_d$  minus **yeah**. So, this should be minus here.  $E_{fd}$  minus  $j$  into  $x_d$  into  $i_q$  plus  $j i_d$ . So, what I have done is just added up this equation then written them in a particular compact fashion. So, what we have here is, **so what we have here is**  $j$  and this is plus  $j x_d i_d$ .

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A hand-drawn equation on a whiteboard. The first line shows the equation  $(V_q + jV_d) = E_{fd} - jx_d(i_q + jid)$  with the left side underlined. The second line shows the expansion:  $-jx_q i_q + jx_d i_q$ . The third line shows the result:  $= E_{fd} - jx_q(i_q + jid) + (x_d - x_q)i_d$ . An NPTEL logo is visible in the bottom left corner.

So, what we have is  $V_q + jV_d$  under steady state is equal to  $E_{fd} - jx_d i_q + jx_d id - jx_q i_q + jx_q id$ . An alternative way of writing this is  $E_{fd} - jx_q(i_q + jid) + (x_d - x_q)i_d$ . And instead of  $x_d$  you write it as  $x_q$  and as a result of which the additional term you will get is.

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A hand-drawn equation on a whiteboard. The first line shows the equation  $(V_q + jV_d) = E_{fd} - jx_q(i_q + jid) + (x_d - x_q)i_d$ . The second line shows the rearrangement:  $E_{fd} + (x_d - x_q)i_d = (V_q + jV_d) + jx_q(i_d + jid)$ . An NPTEL logo is visible in the bottom left corner.

So, you can write it in this fashion also. The basic **cracks** crux of the matter is that, if I know  $V_q + jV_d$ , so I will just write this down again. We will have  $V_q + jV_d$  is equals to  $E_{fd} - jx_q(i_q + jid) + (x_d - x_q)i_d$ . So, just **yeah** so what we get from

this is that, if I know this and I know this, so if I know this and I know this, I can get this. But, remember I do not know delta. Remember I do not know delta, so I cannot actually find out what  $V_d$  and  $V_q$  is even if I know what theta is. Similarly, I cannot get  $E_d$  and  $E_q$ .

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The whiteboard shows the following derivation:

$$[E_{fd} + (x_d - x_q) i_d] e^{j\delta}$$

$$= \frac{(V_q + jV_d) e^{j\delta}}{\theta} + jx_q (i_q + j i_d) e^{j\delta}$$

Below the equations, there are handwritten notes: a theta symbol, a box containing  $V_q$  and  $V_d$  with a diagonal line through it, and a box containing  $V_q$  and  $V_d$  with a diagonal line through it. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

So what I do is very interesting thing,  $E_{fd}$  I am multiplying by  $e^{j\delta}$ . So, I will get  $V_q + jV_d$  into  $e^{j\delta}$ . On both sides I just multiply  $e^{j\delta}$ . I multiplied by  $e^{j\delta}$ . Now, the question is, do I know this? The answer is yes because I know theta.



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$$V = 1.0 \angle \theta$$

$$v_d = 1.0 \sin(\theta - \delta)$$

$$v_q = 1.0 \cos(\theta - \delta)$$

$$(v_q + jv_d)e^{j\delta} \rightarrow 1.0 \cos \theta + j1.0 \sin \theta$$

So, this you can just try to prove that if I know  $V_d$  and  $V_q$ , **sorry** if I know that  $V_d$  and  $V_q$  have this form  $V_q + jV_d$  will e into e raise to j delta will simple give me what? 1.0 **yeah** and theta is something I know.

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$$[E_{fd} + (x_d - x_q) i_d] e^{j\delta} = (v_q + jv_d) e^{j\delta} + jx_q (i_q + j i_d) e^{j\delta}$$

So, what we have essentially is that I know this, I also know this. So, if I know when I know this I mean, I know this complete quantity here. Snd I know the complete quantity here. As a result of which I know this complete quantity. Now, this is a real number. So

if I take out the magnitude of this **this** something I know and this is something I know, I can take its magnitude an angle and equated to this.

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$$[E_{fd} + (x_d - x_v) i_d] \checkmark$$

$$\delta \checkmark$$

$$(V_q + jV_d) e^{j\delta} \rightarrow 1.0 \angle \theta$$

$$(E_q + jE_d) e^{j\delta} \rightarrow 1.0 \angle 0$$

$$(x_q + jx_d) e^{j\delta} =$$

As a result of which I will be able to get  $E_{fd}$  and I will also be able to get what  $\delta$  is. So, that the thing which is important to notice  $V_q + jV_d e^{j\delta}$  is nothing but, 1 angle **that**  $\theta$ . Similarly, actually if you look at that infinite bus voltage also, its very easy to show that this will be 1 angle 0 as per our initial situation which we have talked of. Similarly,  $i_q + j i_d$  into  $e^{j\delta}$  is nothing but, this minus this divided by  $j x$ .

So, we know this complete term and this complete term. We know the value of  $x_q$ . As a result of which we can back **cab** back compute the equilibrium value of  $\delta$  so you **you** can directly get what the equilibrium value of  $\delta$  is.

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$$(i_q + j i_d) e^{j\delta}$$
$$= \frac{1.0 \angle \theta - 1 \angle 0}{jX}$$

$(i_q, i_d)$  ✓

Once you got the equilibrium value of delta, the next step is get  $i_q$  plus  $j i_d$ . Remember we know what  $i_q$  plus  $j i_d$  into  $e^{j\delta}$  is. What is that equal to? 1 point 0 angle theta minus 1 angle 0 divided by  $jX$ . So, if I know delta I can actually find out what  $i_q$  and  $i_d$  are. Once I find  $i_d$  and  $i_q$ , the next step is from  $i_d$  you know this value, the magnitude of this by equating these two compute  $E_f d$ .

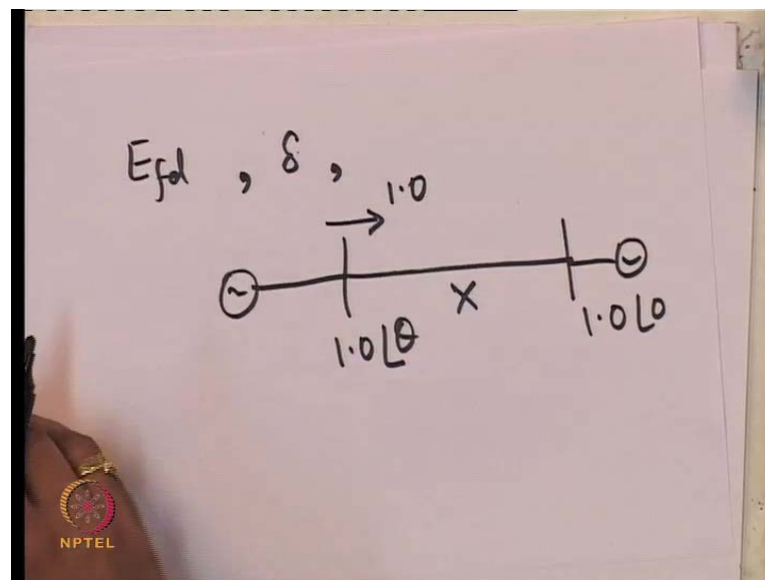
So we can compute  $E_f d$ . Now, we got  $E_f d$  initial value, delta. And once you know  $E_f d$  and delta in fact you can compute all the equilibrium conditions. So, what we have is let me just recapitulate what we have done. It may sounded a bit complicated but, it isn't that as complicated as its sounds. We will just **recapsulate** recapitulate quickly.

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$E_d = -E \sin \delta$   
 $E_q = +E \cos \delta$   
 $1.0 \angle \theta \rightarrow \begin{matrix} E_{an} \\ E_{bn} \\ E_{cn} \end{matrix}$   
 $= \sqrt{\frac{2}{3}} \sin(\omega t)$   
 $1.0 \angle \theta \quad (E_q + jE_d) e^{j\delta}$   
 $= 1.0 + j0.0$

If  $E_{an}$  is this, it means of course, that  $E_d$  and  $E_q$  are like this, which also means  $E_q$  plus  $j E_d$  e raise to  $j \delta$  is nothing but,  $1.0$  plus  $j 0.0$ . Similarly,  $V_{an}$  is this.

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So what you get essentially is  $\theta$ .  $\theta$  is something you get from the condition that, 1 per unit power is flowing through a reactance of  $X$ . The terminal voltage here is 1 point angle 0 and terminal voltage here is angle  $\theta$ . So, this  $\theta$  is obtained that way.

So, once you got  $\theta$  the next step is of course, getting the equilibrium values of  $\delta$  extra etc. Remember that, once have got  $\theta$  you know this. You do not know what

$\delta$  is but, interestingly  $V_q + j V_d$  into  $e^{j\delta}$  is nothing but, a function only of  $\theta$ . So, if you know  $\theta$  you know  $V_q + j V_d$  into  $e^{j\delta}$ . So, this is how you get  $V_q + j V_d$  into  $e^{j\delta}$ .  $i_q + j i_d$  into  $e^{j\delta}$  is nothing but, this so we get the values of  $i_d$  and we can get because of this, from this particular equation. Because we know this and this and the value of this parameter  $x_q$ , we get this value that is the magnitude and the angle can be obtained. So, we know the magnitude of this is we equated to the magnitude of the left hand side, the right hand side and left hand side magnitude equate. So, we know what  $E_f + x_d + x_q$  into  $i_d$  is. Thereafter we also know from the angle of this, what  $\delta$  is. Once we get this  $\delta$ , we can compute, we know what this is  $i_q + j i_d$  into  $e^{j\delta}$  we know. Once we know  $\delta$ , we can find out what  $i_q$  and  $i_d$  are so in that fashion. We can get the equilibrium values of all the states corresponding to this situation, whether terminal voltage of this is 1, the terminal voltage of the infinite bus is also one, the voltage of the infinite bus is also one.  $T_e$  is one also we can back calculate all the values of the states.

So that you get this particular situation once if computing, back calculated all the values of the states. You can actually do a linearise analysis by plugging in the equilibrium values wherever necessary in the linearized state space equations. Once you do that, one can get the Eigen values. There is too **it is** little time for us to actually compute the Eigen values. And I show you everything that are the important points corresponding to these Eigen values. So, we will do that in the next class.