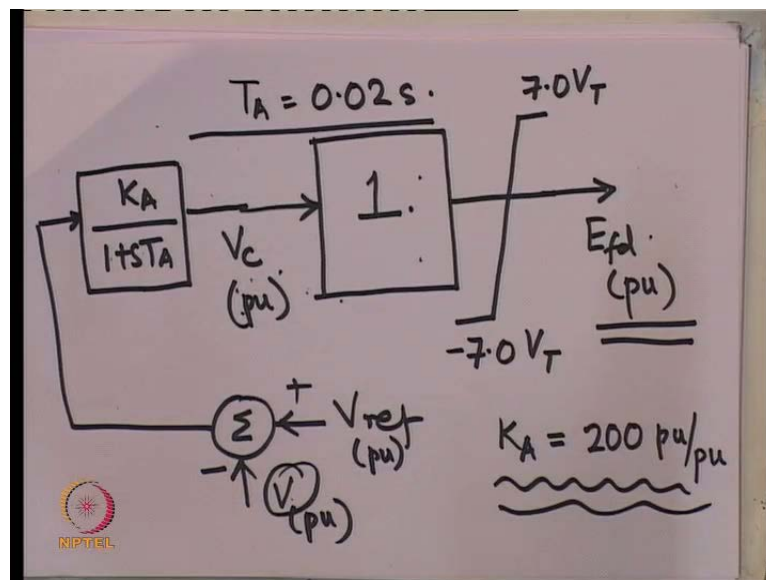


Power Systems Dynamics and Control
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Lecture No. #28
Excitation System Modeling Automatic Voltage Regulator (Simulation)

In today's lecture, we will simulate the generator with an AVR. In the last class, we had of course, discussed various transfer functions, which could be used with an AVR. We now, integrate the system. In fact, this is our kind of first experience of trying to simulate a control system, the power apparatus, and the external power system.

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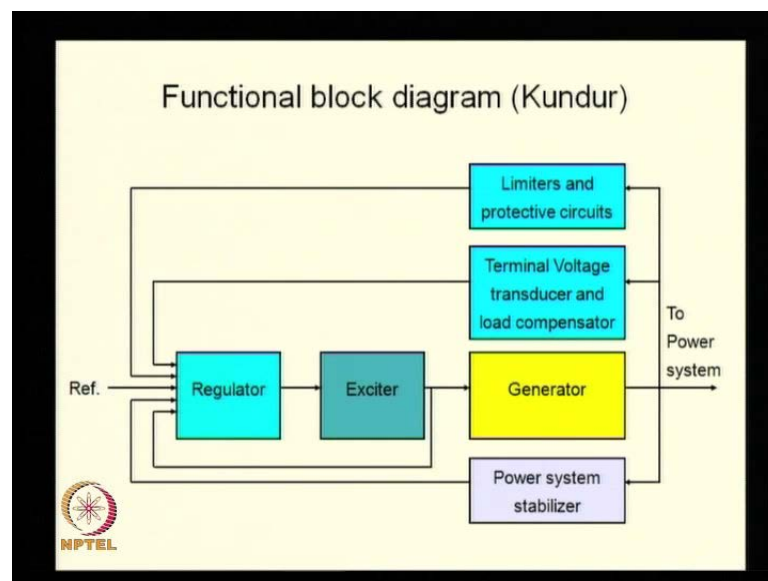
So, that is what we will do today. Recall that in the previous class, you can look at this paper. We had discussed the model of a simple static excitation system, we have chosen a static excitation system, because it is the model of the excitation power apparatus is very simple. The only complexity so to speak is the limits of the static exciter, which I dependent on the terminal voltage. Remember that if we normalize the field voltage, and the control voltage to their corresponding values when we get one per unit at the terminal of an open circuited generator running at rated speed, then the gain of this converter can be is effectively 1.

Because everything is normalized, the control signal also is normalized, and output also is normalized. If we have $V_{ref} - V$, the summing junction; V and V_{ref} and V

are expressed in per unit, then a typical or rather a transfer function, which we can use the simplest one, one can say is a proportional type static excitation system. T_A is very small, the gain K_A when we use normalized control output and also the per unit values of voltage here, in that case K is typically around 200, 300 or you know that it is in that range. So, coming back to what we will be doing in this lecture is to actually mathematically write down these equations. I will just show you simple simulation and we shall also try to study the behavior of a synchronous machine connected to an infinite bus or voltage source through reactance and with the AVR regulating the terminal voltage of the generator.

So, far, we have been considering constant field voltage or the field voltage, you can say manually increased in steps, but now we will have an automatic continuously acting feedback control systems. So, today's lecture will be focused on the simulation of a generator connected to an infinite voltage source with the AVR in action.

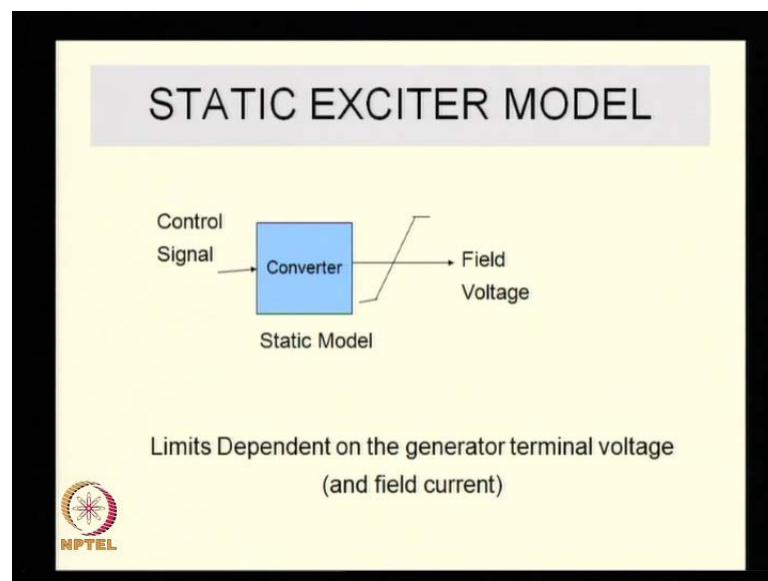
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So, this is of course, the starting point of our discussion yesterday. The only thing we will be simulating today, at least in the beginning would be the regulator function. We will not of course, be talking about the limiter or protective circuits or the power systems stabilizer, although I hope to give you a motivation for its use today itself, why we need a stabilizer. As I mentioned sometime back, we will using simple model of excitation system, that is a static exciter and a generator, which is connected to a power system and

of course, as I mentioned a couple of lectures back, we will be simply trying to regulate the voltage, that is the terminal voltage is measured and compared with the reference. We will not be putting any compensation for the load on the generator. So, we will not be having in the summing block any component corresponding to the current output of the generator. So, simply we will be having V_{ref} minus V at the summing junction of the AVR.


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So, this is our static excitation system and as I mentioned sometime back. So, this is the model we will be using for the system.


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Compact Form of Flux Equations

$$\frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = A_1 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} + A_2 \begin{bmatrix} i_d \\ i_q \end{bmatrix} + B_1 \begin{bmatrix} v_d \\ v_q \end{bmatrix} + B_2 E_{fd}$$



Now, when we want to simulate a synchronous generator connected to an infinity bus, we will again have to use the equations, which are given here. The fluxes of course, are something we have defined earlier. These are fluxes of the synchronous machine. i_d and i_q are the currents going out of the machine, v_d and v_q are of course, the voltages at the terminals of the machine and E_{fd} is the per unit value of the field voltage as defined earlier.

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$$A_3 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$


Now, remember the currents and fluxes are related by an algebraic relationship as given here.


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$$A_1 = \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T'_d} & 0 & -\frac{1}{T'_d} & 0 & 0 & 0 \\ \frac{1}{T''_d} & 0 & 0 & -\frac{1}{T''_d} & 0 & 0 \\ 0 & \frac{1}{T'_q} & 0 & 0 & -\frac{1}{T'_q} & 0 \\ 0 & \frac{1}{T''_q} & 0 & 0 & 0 & -\frac{1}{T''_q} \end{bmatrix}$$

And A 1 is of this form. Remember **show** you **you** can notice the omega dependence of A 1. That is an important thing you should remember.


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$$A_2 = \begin{bmatrix} -\omega_B R_a & 0 \\ 0 & -\omega_B R_a \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And A 2 of course, which relates the derivative of the flux of the current, is given by this. B 1 and B 2 are given by these, by these matrices.

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$$A_3 = \begin{bmatrix} \frac{1}{x_d''} & 0 & -\frac{(x_d - x_d')}{x_d x_d'} & -\frac{(x_d' - x_d'')}{x_d' x_d''} & 0 & 0 \\ 0 & \frac{1}{x_q''} & 0 & 0 & -\frac{(x_q - x_q')}{x_q x_q'} & -\frac{(x_q' - x_q'')}{x_q' x_q''} \end{bmatrix}$$

And A_3 , which relates the current to the fluxes, is a matrix. It is a 2 into 6 matrix. Now, of course, we are not considering 0 sequence, the implicit thing in all our discussion so far, have been that we are having a balance system. So, we do not actually have to model the 0 sequence. The thing is that the 0 sequence variables do not appear in the d q variable equations. So, this is complete decoupling, that coupling will occur in case synchronous machine is connected to an unbalanced network, but otherwise these equations are completely decoupled and the 0 sequence equations can be separated out. 0 sequence equations have no source of excitation. In case, you have got a balance system. So, you can practically say that all the 0 sequence variables are 0 and they are not coupled with the d q variable.

So, that is the reason, why we are not considering 0 sequence variables, but in case you do study unbalanced systems, you should remember to take into account the 0 sequence equations.

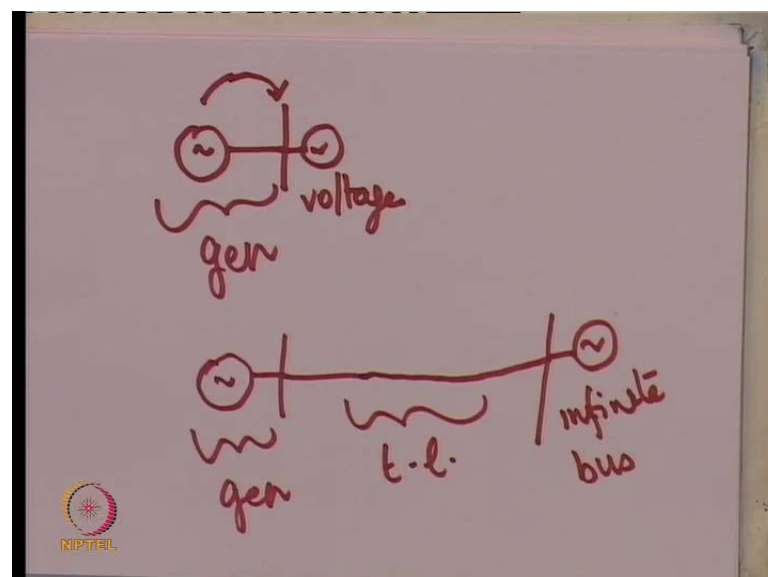
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Model of interconnection:

$$\begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{pmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} \end{pmatrix}$$


Now, coming to the system we are going to study. Now, in the last simulation, which I had shown you, I had got a synchronized machine which is synchronized directly to a voltage source and that is the kind of simple system I could have shown you this synchronization. In order to show AVR action of course, it does not make sense to connect a synchronized generator to a voltage source, because if you have connected a generator to a voltage source, which is a stiff or constant voltage source, there is no question of voltage regulation because the voltage is maintained by the voltage source itself.

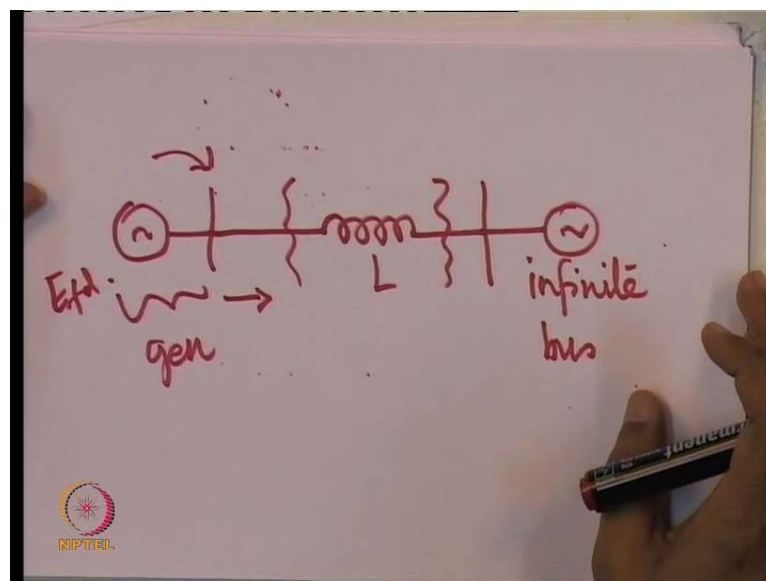
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So, voltage regulation makes no sense, in case you have got a generator directly connected directly to a voltage source in this fashion. Of course, this is a symbolic representation of the generator. The generator itself is not a voltage source. It is represented by the equation, which I have already shown you. Now, the aim at the AVR is trying to maintain the voltage of this, constant **constant**. So, constant or near constant. In fact, if you are using a proportional gain, it is near constant not perfectly constant. Now, as I said this does not make any sense, if you have got a perfect voltage source here to which this generator is connected. What makes sense of course to you know show the action of a AVR, is to connect it to a infinite bus or a large grid.

It is connected to a large grid. It is also called a infinite bus or constant voltage source, whose magnitude, frequency, phase angle all are all remain constant. What we are going to do is connect it via, what would you connect to via, a transmission line or a transformer or a transformer and a transmission line. So, this is the transmission line. Now, we have not gone into the modeling of a transmission line.

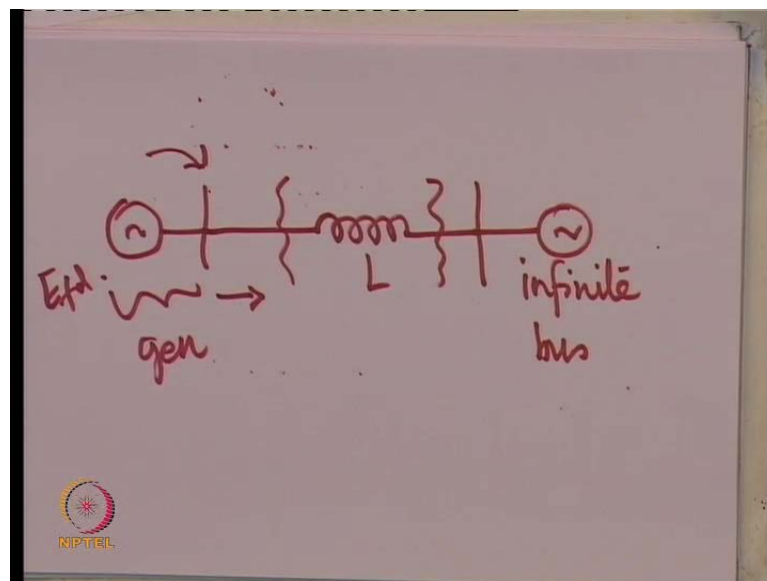
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So, we will not really dwell deeply into this modeling of the transmission line itself, but you will know that, well a kind of a simple model of generator connected via a transmission line and transformer. A very simple model could be a lumped reactor. It is a simplest model lets, you know which one can have. **You know in a**. In a laboratory, we could actually make this set up, we could actually connect via reactor.

But, right now we will assume that you have got a three phase inductance L , which connects the generator to the perfect voltage source that is the infinite bus. Now, the generator now can **you know** maintain the terminal voltage, try to maintain the terminal voltage because it tends to vary. How does it vary the, how does the terminal voltage vary. For example, the power output of this generator changes, in that case the current through this will change and you will find that the voltage here changes. The only way you can maintain this voltage here at a near constant value, is to have a closed loop feedback system, which changes E_f , the field voltage.

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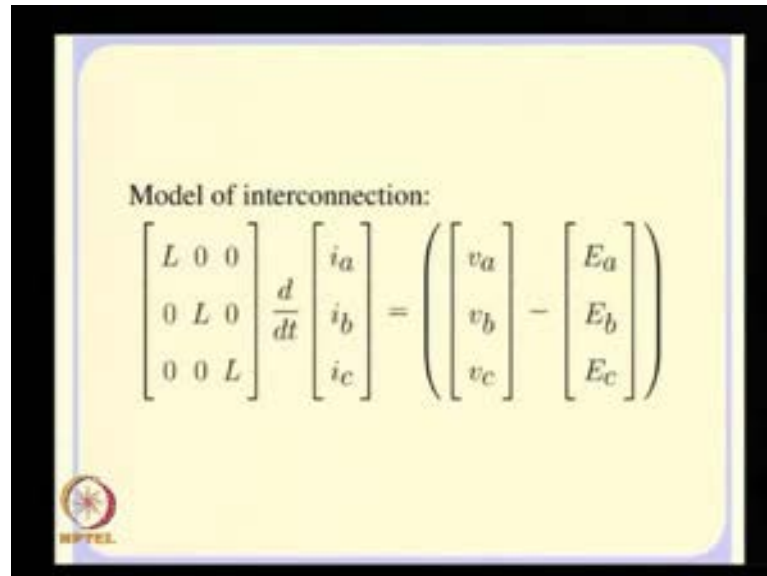
Now, in of course in steady state, the steady state representation of a non salient pole generator is simply like this. Please refer to a fairly early analysis of a **steady state** steady state analysis of a synchronous generator, which was done roughly 6 or 7 lectures back, **quite time** quite at some time back. So, steady state representation of a non salient pole synchronous generator is this.

So, this is the steady stated. Not, of course, please do not use this under transient conditions. Under transient conditions, you will actually have to use the differential equation model of the synchronous machine. So, do not use this for a transient representation of a machine, but we can see that if you have got a system like this infinite bus, at least in steady state it is easy to see that if I change E_f , I will be able to control or change the voltage here. So, this is the basic principle of course, of the AVR itself,

that you change E f d. Now, coming to the model of this transmission line, now this is a simple reactor, whose inductance is L, then the equations are given as shown in this slide.

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Model of interconnection:

$$\begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{pmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} \end{pmatrix}$$


That is, $L \frac{di}{dt}$ is equal to v_a minus E_a . E_a is of course, E_a and v_a are in fact, phase to the neutral voltages of the balanced infinite the generator as well as the infinite bus. So, remember that E_a , E_b , E_c actually represent the phase to neutral voltages of the infinite bus and synchronous generator. For simplicity, we will of course, assume that your infinite bus is simply E_a is nothing, but.

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Handwritten equations on a whiteboard:

$$E_a = \sqrt{\frac{2}{3}} E \sin(\omega t)$$
$$E_b = \sqrt{\frac{2}{3}} E \sin\left(\omega t - \frac{2\pi}{3}\right)$$
$$E_c = \sqrt{\frac{2}{3}} E \sin\left(\omega t + \frac{2\pi}{3}\right)$$

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So, for simplicity we will assume that E_a , the phase to neutral voltage of the infinite bus is nothing but $\sqrt{2/3}$ V line to line RMS, the line to line RMS of rather I should call it E . That would be notationally more easy to remember. This is $E \sin \omega t$. So, we will assume that you are seeing the infinite bus is of course this. The 120 degree is this plus 120 degrees. Now, so ω is the constant frequency of the infinite bus.

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Model of interconnection (d-q pu form)

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$
$$x = \frac{\omega_B L}{Z_{base}}$$

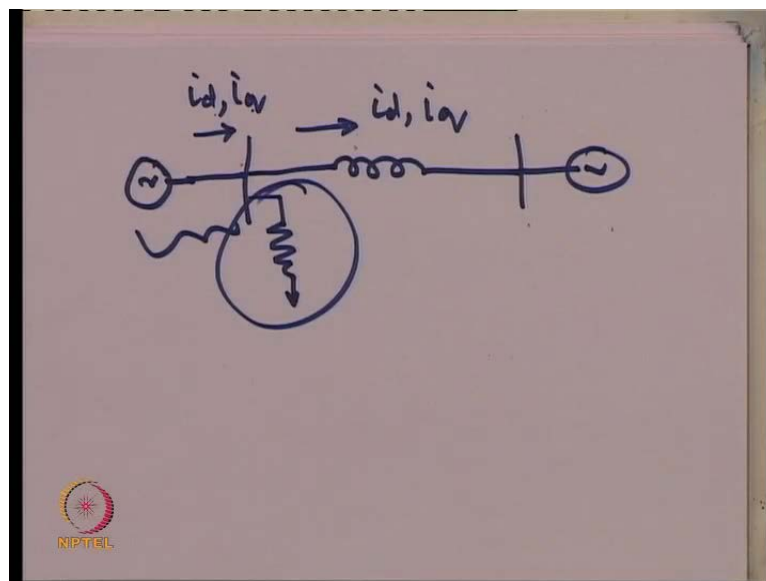
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So, this is what E_a means. Now, if you look at what will happen in case, you try to transform this into d q 0 variables. Now, transforming the d q 0 variables is quite simple

in this case. You know the d q transformation; we have talked about d q transformation or the Park's transformation. So, if you use the Park's transformation, you transform i_a, i_b, i_c into i_d, i_q and i_0 and by doing that, the equations are quite straight forward, but remember that you get these speed terms. You know or rather I should say omega dependence. Omega, remember is $d\theta$ by dt , where theta is the position instantaneous position of the rotor of the machine. So, if you do the d q analysis of this system, we can express this equation in this fashion. It is also in per unit.

So, what you see is, x here is $\omega B L$ by Z base. So, so the best thing would be of course, to use a common base, which is the base of the synchronous machine itself. So, the impedance base of the synchronous machine is used to define the bases, sometime go in our course. Now, one small and interesting point, which we have here is the current through **the** this reactor, which is interconnecting the synchronous machine to the infinite bus is the same as the current through the synchronous machine.

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So, actually if you look at the synchronous machine, this is your synchronous machine, **this is this is** this is only the symbolic representation, this is not an electrical circuit. Remember, synchronous generator cannot be represented simply as a voltage source. This is a reactor and this is here. So, your current output of the machine i_d and i_q is the same as the current through this i_d and i_q . i_0 is also there, which the neglect of **which the neglect of** which I have already explained sometime back. Of course, if you

have got some load here, for example, if you got have you know resistive load here or any kind of load here, this will not be true.

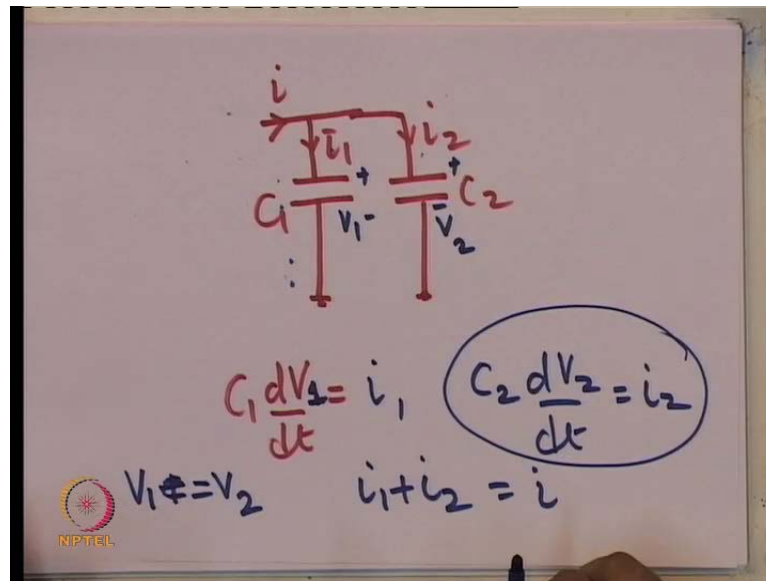
So, if I have got something here, then i_d and i_q are not the same for the generator and the inductor. Now, one of the interesting points, which you have here is, this is interesting theoretically also, is and also practical consequences, when you are trying to program the simulation of this system is that since, i_d and i_q are effectively determined are a function of the fluxes, they algebraically related to the fluxes we have seen that, but you also see that i_d and i_q are also determined by this differential equation, so; obviously, there is a kind of a interesting situation here, you have got two sets of one algebraic equations and one differential equation, which are both are trying to define the current.

So, this is actually you have to be consistent, you cannot have this differential equation; for example, telling you that the current is something else and algebraic equation is telling you something else. Remember the algebraic equations relate the current to the flux, which are fluxes, which are again independently determined by the differential equations.

So, this. So, the issue here is, that in some sense we have got these two differential equations here. They are quite redundant. In the sense, that i_d i_q is already determined by that, by an algebraic relationship with the fluxes. You really do not need to define to **you know** this extra set of differential equations and if you do use these differential equations as well, we have to be really care to be consistent. In the sense, you cannot give i_d and i_q here; for example, the initial conditions of this differential equations, which are inconsistent with what you would obtain, when you get the currents algebraically related to the fluxes.

So, all the initial conditions would need to be absolutely consistent. Now, the situation like this could arise, this is of course, a slight diversion from our main theme of power system dynamics, but it could be nice to just chew on this.

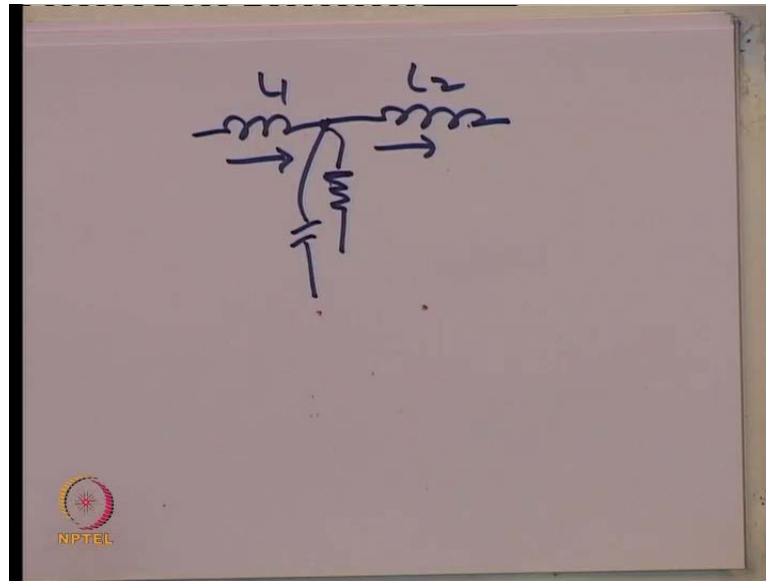
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Suppose, I have got two capacitors, which are in parallel. So, you can have, this is suppose, the current here is i , you have got dV by C . C_1 , C_2 is equal to i_1 , i_2 . So, we will call this V_1 and V_2 , the voltages V_1 , V_2 . So, you will have $C_1 \frac{dV_1}{dt}$ is equal to i_1 and $C_2 \frac{dV_2}{dt}$ is equal to i_2 and i_1 plus i_2 is equal to i and V_1 should be equal to V_2 .

So, actually, this in some sense, you can say that you know by choosing V_1 equal to V_2 , why do we require a differential equation again. Why do we need another law to define V_2 . So, this in some sense V_2 is again a redundant state in such a situation. So, when you are actually trying to simulate such a system, it is better to remove the redundant states. You know of course, that V_2 is dependent on V_1 , it is equal to V_1 and V_2 are equal. So, we only need to determine the current through this. Whatever voltage you have here will be the same as the voltage here. Then we have another situation when an inductor is connected to another inductor.


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So, there **there** to you know the question is how many states are there. You know you **you** can if you write two differential equations, this could be when taking one inductance splitting into two and getting into this problem. So, this is something, which this is similar situation, which you encounter here. Of course, this problem is solved. In case you got something connected in shunt, which is not an inductor. For example, a resistor or a capacitor you connect it, then these two states become distinct. You can have two different two sets of differential equations, which really for these inductors, which define give, separate important information. So, this is one interesting point, which you know you should chew upon.


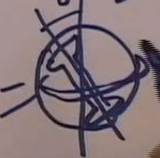
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Infinite Bus

$$E_d = -E \sin \delta \quad E_q = E \cos \delta$$
$$\frac{d\delta}{dt} = \omega - \omega_0$$
$$E = 1, \omega_0 = \omega_B$$


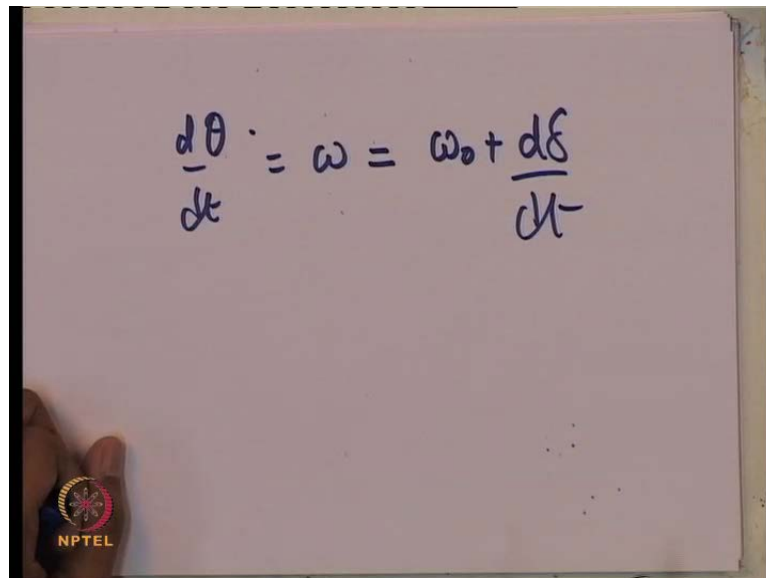
If you look at something about the infinite bus itself, we are a kind of progressing to finally getting our equations. We have not actually solved this earlier problem of redundant states. I have just told you that, these states are actually redundant because i_d and i_q are obtained by algebraic relationships with the states. So, how do you use this information usefully? So, that is something, which you need to chew upon. The infinite bus itself remembers its frequency is ω_0 and the phase angle since E_a , E_b and E_c are defined to be these.

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$$E_a = \sqrt{\frac{2}{3}} E \sin(\omega_0 t)$$
$$E_b = \sqrt{\frac{2}{3}} E \sin(\omega_0 t - \frac{2\pi}{3})$$
$$E_c = \sqrt{\frac{2}{3}} E \sin(\omega_0 t + \frac{2\pi}{3})$$
$$\theta = \omega_0 t + \delta$$


Now, if the rotor angle position theta, there is a position of the rotor theta is defined like this. If omega 0 is the frequency of infinite bus, rotor position is omega naught T plus delta, which also means that if you have got a two pole machine, we will just talk about two pole machine here. The a-phase winding, the axis of the a-phase winding and the axis of the rotor winding, the field winding that is, is delta. Whenever there is negative to positive 0 crossing of the sine wave if I take a snap shot of a synchronous machine, I will see that the rotor is at an angle delta. So, that is what delta means. If that is the case E d will be E sin minus E sin delta. This is something we defined before. This is just to obtain by applying d q transformation E a, E b and E c and d delta by dt from by **differentiate** differentiating theta.

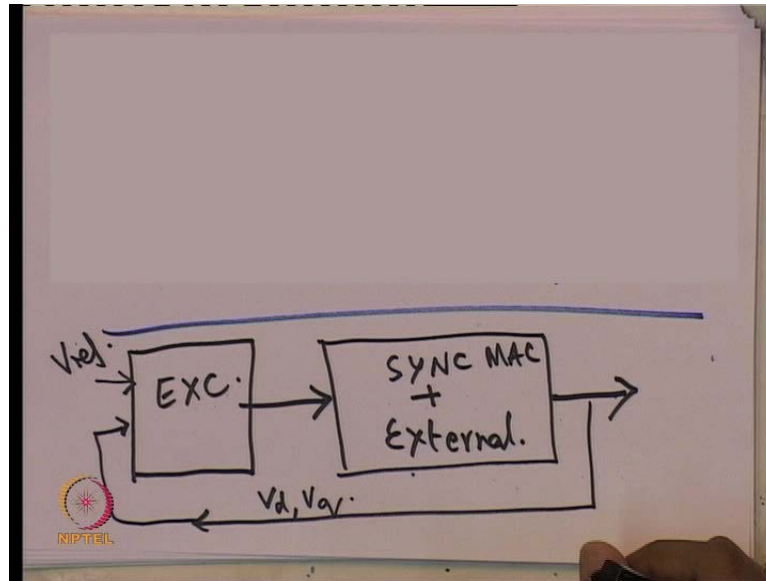
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$$\frac{d\theta}{dt} = \omega = \omega_0 + \frac{d\delta}{dt}$$

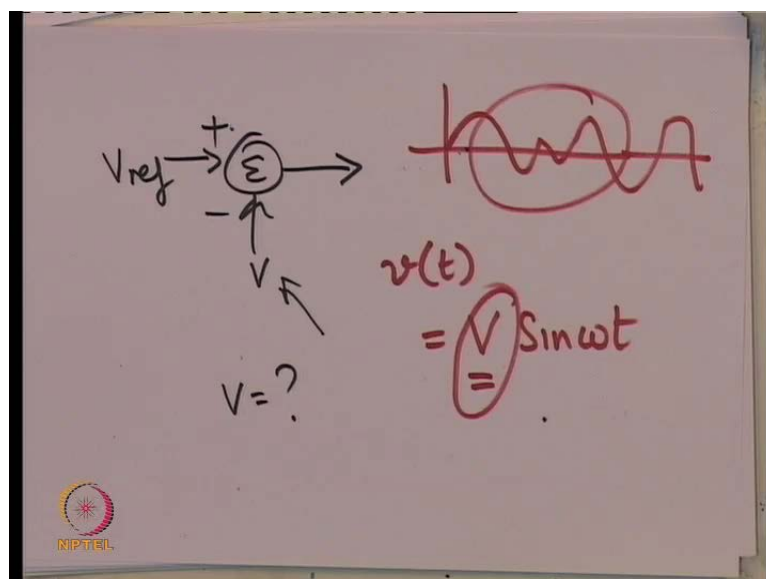
Theta, d theta by dt is nothing, but the speed of the machine, the instantaneous speed of the machine, which **which** is nothing but omega 0 plus d delta by dt and that is how I get d delta by dt is equal to omega minus omega 0. Let us assume that E is equal to 1 and a frequency of the infinite bus is equal to the base frequency. So, this is a simplifier or analysis.

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So, what we have now is the synchronous machine and plus the equations of the external network. Now, what remains to be done of course, is the exciter equations themselves. The excitation system essentially takes a feedback of the voltage, the terminal voltage V_d and V_q and the set point is what is given by us and this is how your system looks like. Now, one of the things is that for the excitation system, we really require to know the magnitude of the voltage and that is something we will spend little bit of time on now. What is the terminal voltage magnitude? In fact, I have been using this somewhat nebulous kind of concept of a voltage magnitude.

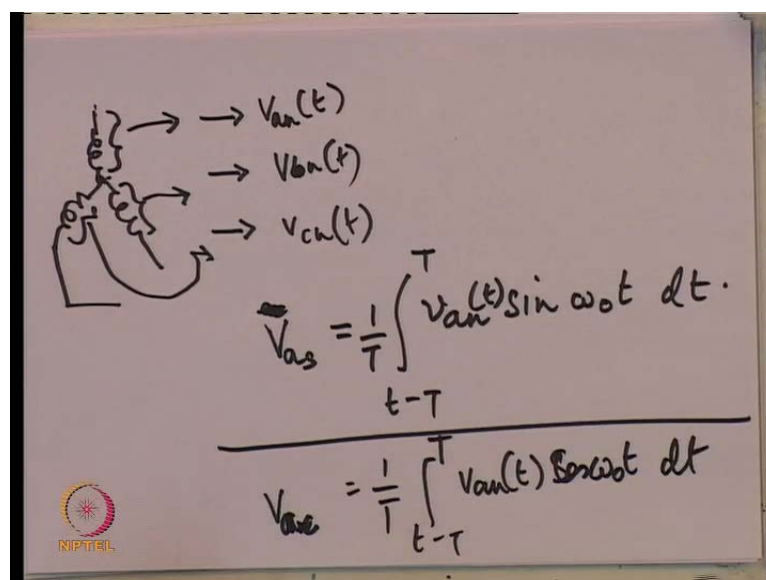
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This is the voltage magnitude of the terminal. **You know** what is V? V is the magnitude of the voltage at the terminal of a synchronous machine, but you will immediately recognize, is the fact that voltage magnitude in transient. What do we mean **you know** if you have got a pure sinusoid, then getting the magnitude is very simple. **You know** you can for example, take the peak value and you can divide **you know** kind of from the peak value, you can find out line to line RMS value of voltage. So, you have got balanced three phase sinusoid, you can get the peak value of any phase and then, get the line to line RMS magnitude.

Now, mere magnitude of course, is in some sense coefficient of the sinusoidal term, when we write down the time relationship. So, when I say magnitude, I usually mean something like this. If I want magnitude of a sinusoid, I mean that I usually mean that the coefficient here is the magnitude, but this assumes that you have got a sinusoid. Now, if I give a waveform, which is like this and I tell you well find out the magnitude of the sinusoid here in this case. That is a bit of **a bit of a** question mark because this is no longer sinusoid. So, what do I represent this as and how do I get this magnitude as coefficient of a sine function. You know when this is a transient kind of behavior, where you cannot represent it as a pure sinusoid.

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The whiteboard shows a diagram of three phase voltage waveforms labeled $v_{an}(t)$, $v_{bn}(t)$, and $v_{cn}(t)$. Below the waveforms, the average value V_{as} is calculated as:

$$V_{as} = \frac{1}{T} \int_{t-T}^T v_{an}(t) \sin \omega_0 t \, dt.$$

$$V_{as} = \frac{1}{T} \int_{t-T}^T v_{an}(t) \cos \omega_0 t \, dt$$

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So, this is an interesting point, in practice whenever I am getting a magnitude of V, what could I do? One of the things I could do is, you take the three phase voltages V_a , V_b , V_c .

So, for example, you have got V_a is the voltage across the winding of a synchronous machine, I could connect them in star and I could take the phase to neutral voltage. This is the voltage across the y_a winding, give it here, sense it.

So, I will get 3 sine 120°, in steady state I will get three sinusoids, but otherwise of course, I will get simply the instantaneous values V_{a_n} , V_{b_n} and V_{c_n} . Now, one of the ways you could **you know** define magnitude of a voltage is to take the Fourier component of V_a , V_{a_n} , V_{b_n} and V_{c_n} . For example, of course, your data is just coming in, you know it is not a fix signal, but you know you are kind of continuously getting. If this is a digital signal system; for example, you will be continuously getting samples of V_{a_n} , V_{b_n} and V_{c_n} .

So, how do you actually take out the magnitude or you can say **Fourier coefficient of this** fundamental Fourier coefficient of this well. So, I could define V_a as something like this, t minus T to t . So, this is the one way to defining it. So, you try to get magnitude of V_a in this fashion. As you get this instantaneous values, we evaluate this integral using some kind of function, you will have to actually implement this using; for example, digitally you can implement a numerical integrator using the discrete samples of V_{a_n} . This is what you could do and get a kind of Fourier coefficient. Of course, during transients you will get something, which seems reasonable to assume what you will get is the Fourier, what you call the magnitude of V_a .

So, this is one way of doing it. In fact, you will have to get a sine component as well as the cosine component and then, whatever you get let us call this sine component and this is the cosine component, this is of course, a real number and then, you can use this root of $V_{a_s}^2$ plus $V_{a_c}^2$, to get what we can call or define the magnitude. So, if **you** you should remember that, when it comes to transients, you have to kind of define what is voltage magnitude. There is **there is** no definition. By definition you cannot have a magnitude of rather naturally or inherently there is no meaning to having a magnitude of a non sinusoidal wave.

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Handwritten notes on a whiteboard showing the derivation of the magnitude of a three-phase voltage vector V . The notes list V_a , V_b , and V_c , then show the formula $V = \sqrt{V_a^2 + V_b^2 + V_c^2}$. Below this, it states $V = \text{const}$ and $V = \underline{\underline{LL \text{ rms}}}$.

So, another easy way of doing things is, you take this V_a , V_b and V_c and you compute root of V_a^2 plus V_b^2 plus V_c^2 . Now, this also I can call as the magnitude of voltage. Remember that if V_a , V_b and V_c are balanced sinusoids, I leave it to you to prove that V will be a constant and equal to the line to line RMS magnitude. So, if V_a , V_b and V_c we take, evaluate this instant, an instant by instant. So, this is an instantaneous value. If V_a , V_b and V_c are pure sinusoids, balanced sinusoids, balanced set of sinusoids, you can show that V is a constant and equal to the line to line RMS voltage under these circumstances.

So, what one extension I can do is, that even during transients **even during transients** treat this V as if it is a voltage magnitude. So, this is one way of doing things and this is a simple way of doing things. So, you can either use this kind of method of finding the Fourier components, even during transients by using this kind of numerical integration or you can take the instantaneous values of V_a , V_b , V_c square them and square and add them up, get this value of V and treat it as if it is voltage magnitude, even during non sinusoidal and transient conditions. Of course, V is not constant in case the system is not balanced or during transient conditions, but eventually V settles down to a constant value, if transient dies down and we reach a balanced sinusoidal steady state. So, this is one important point.

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The image shows a whiteboard with handwritten mathematical equations. The top equation is $V = \sqrt{V_a^2 + V_b^2 + V_c^2}$, which is circled in blue. Below it is the simplified equation $= \sqrt{V_d^2 + V_q^2 + V_0^2}$. At the bottom, there is a partial equation $V = \sqrt{V_d^2 + V_q^2}$ with a closing curly brace. An NPTEL logo is visible in the bottom left corner.

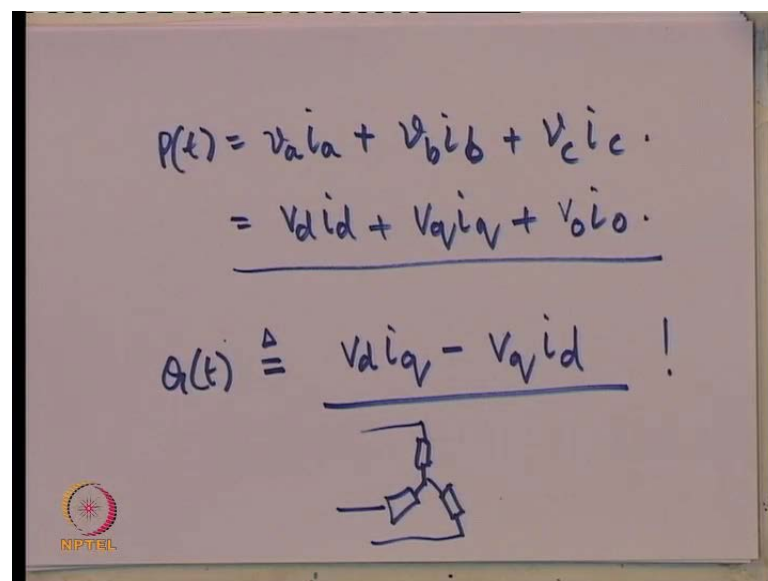
So, if V is equal to root of V_a square plus V_b square plus V_c square, you can show that this is equal to. So, you can actually **actually**, it is plus V_0 square, but we of course, assuming that using absolutely balanced system. So, V_0 is 0. So, V can be defined as root of V_d square plus V_q square. In fact, what it means really is that we can use our equations in the dq form and model the summing junction of an AVR, the inputs of the summing junction of the AVR by simple dq variables themselves.

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The image shows a whiteboard with handwritten mathematical equations. The top equation is $V = \begin{Bmatrix} [V_a & V_b & V_c] \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \end{Bmatrix}^{1/2}$. An arrow points from this equation to a second equation below it: $\begin{Bmatrix} [V_a & V_b & V_c] \mathbf{I} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \end{Bmatrix}^{1/2}$. An NPTEL logo is visible in the bottom left corner.

Now, can you prove this? I will leave it to you to prove. One of the things I can just hint to you is, V is equal to $V_a V_b V_c$ into $V_a V_b$ and V_c and of course, the square root of it. So, raise to half and remember that $V_a V_b V_c$ into, this can be treated as identity into $V_a V_b V_c$. This identity matrix can be written as C_p^T into C_p , the Park's transformation and that is how you will get this relationship. You can just work it out. There are some other interesting points in this stage, which I must tell you is that what is real power. Real power in terms of at a bus, injected at a bus. What is the real power in terms of the $d q$ variables?

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$$P(t) = v_a i_a + v_b i_b + v_c i_c$$

$$= v_d i_d + v_q i_q + v_o i_o$$

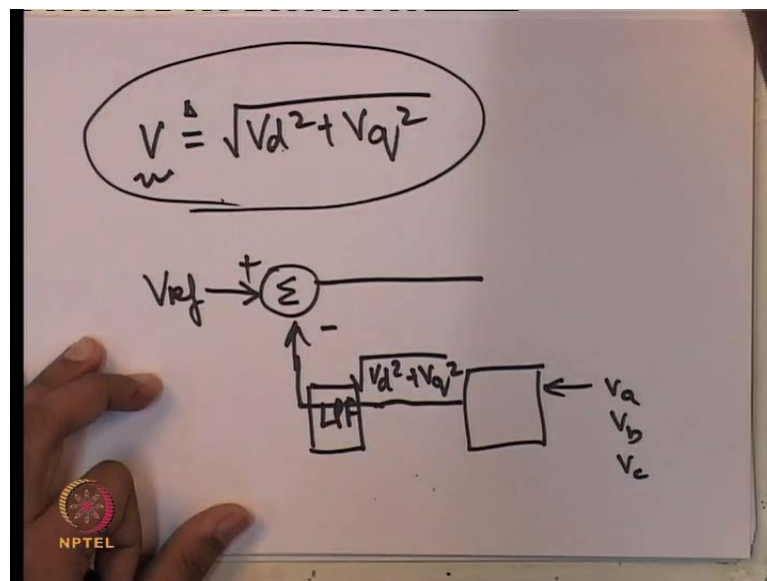
$$Q(t) = \frac{v_d i_q - v_q i_d}{2}$$

Now, instantaneous power is defined as in three phase balanced set up is this, this instantaneous power. Now, you can show that this is nothing, but if you use the transformation, which have you used sometime back. So, that is the Park's transformation with the appropriate values of K_d and K_q . This is in fact equal to this. Remember of course, that in sinusoidal steady state, this product is a constant; balanced sinusoidal steady state is productive is constant. Reactive power, what is the definition of the instantaneous reactive power? Well now, we have to be a bit careful. Reactive power probably makes no sense at this, I cannot make much sense of out of reactive power defined on an instantaneous basis, but if you for example, you can show that Q_t this, the reactive power instantly can be defined.

So, it can be defined as under balanced situations of course, you can show that this in fact, boils down to the normal definition of reactive power. So, this is something, please think over; if you have got a 3 phase sinusoidal circuit with you can just take a simple star connected circuit and prove these things. That at least in sinusoidal steady state this is true. This matches with sinusoidal balanced steady state. This particular expression matches with our classical expression of q , but q instantaneous reactive power can be defined in such a fashion. Remember that it does not really make sense to define the reactive power is a kind a of steady state concept. It is a sinusoidal steady state concept.

So, again it is a bit, you should remember that whenever you say instantaneous reactive power is this, this is only a mathematical artifice and it really does not have any physical meaning, but in steady state of course, this boils down to what is our classical definition of reactive power, which indeed has physical, make some physical sense. So, please this is something you should just think over, it is an interesting problem in itself.

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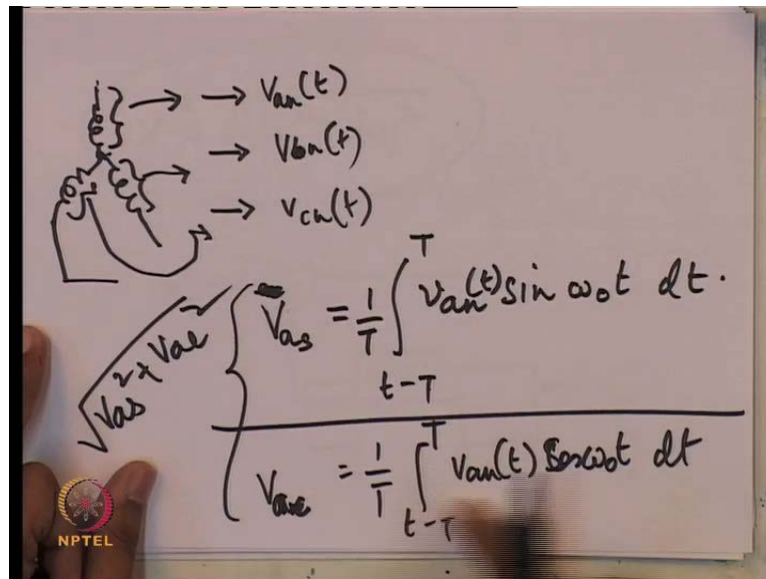
Now, if I am measuring V as root of V_d square plus V_q square, I am getting instantaneous values of magnitude or this is the definition of the instantaneous value of magnitude, which is consistent with what we get in steady state.

So, this will be equal to the line to line RMS voltage magnitude in steady state. So, this is a nice definition. Of course, if you are under unbalanced conditions, it is not difficult to show that V_d square plus V_q square, square root of that is not a constant. So, whenever

we are making; for example, could have V_{ref} and we could be calculating the magnitude, say by taking V_a, V_b, V_c from V_a, V_b, V_c we may be getting by square root of V_d^2 plus V_q^2 as the magnitude. Normally, we will not use this without any kind of filtering. We normally pass it through a low pass filter.

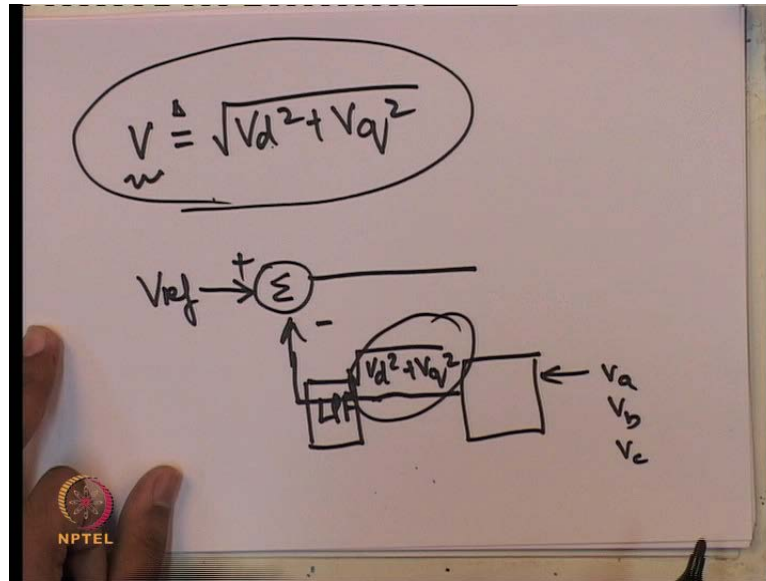
So, this is something, which you should remember, but this low pass filter would be basically design only to reject high frequency transients. It is not that slow transients. High frequency noise are unbalanced, which will cause V_d^2 and V_q^2 to keep varying will be removed by this low pass filter. So, this is how a summing junction would look like. Now, another interesting point is that if I do not use this d, q definition of voltage magnitude, but instead use this square root of V_s^2 plus V_c^2

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This is for the A phase. Square root of; as the magnitude and this is how I define and compute this from the instantaneous values. In that case, remember that since we are doing integration here, this is a moving kind of integration from t minus T . So, this is over a window of T , then in this case there is an kind of inherent filtering effect, which is their because of this integration. So, just think over it is an interesting problem of computing instantaneous magnitude and so on.

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So, in this particular course, we shall assume that this is when I say magnitude, it is this. Root of V_d square plus V_q square the square root of it. So, that is what is what we have? So, if I want to write down the model of AVR and exciter, this is what we need to do.

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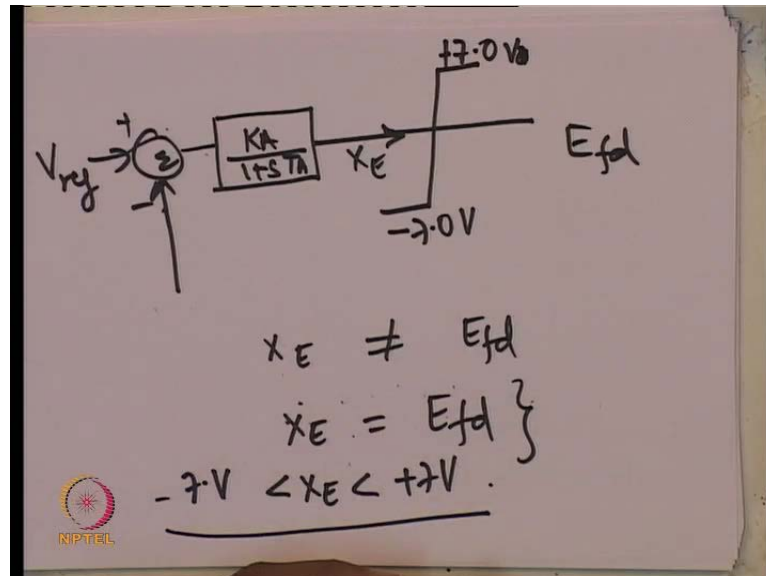
Simple Static Exciter Model

$$\frac{dX_E}{dt} = \frac{1}{T_A} (-X_E + k_A (V_{ref} - V))$$
$$V = \sqrt{v_d^2 + v_q^2}$$

So, if you take a simple static exciter model, which is suitable for slow transients. Slow transients I mean typically associated with electro mechanical phenomena like swings or

low frequency transients of around one between one or two hertz and which involves oscillations of one or two hertz, that is what I will define as a slow transients.

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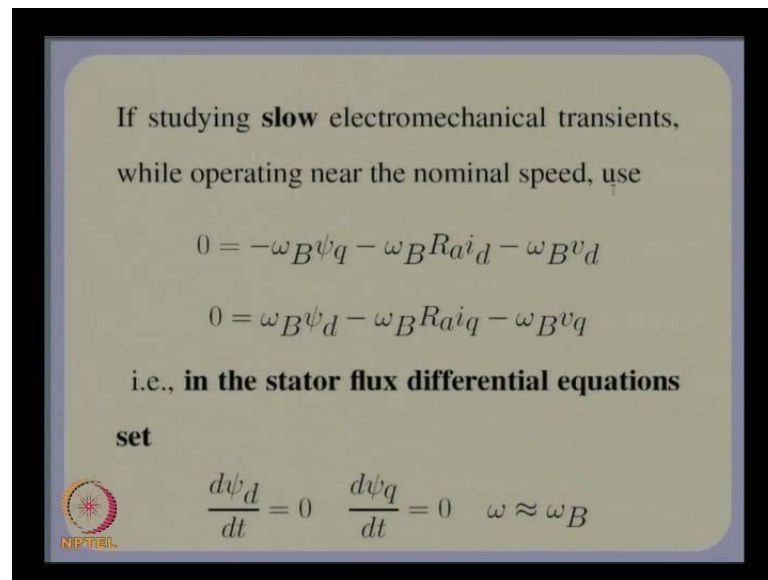


In that case, this is a simple model. That is the AVR is $K / (1 + sT_A)$. Then you have got this limit and you are got E_{fd} . So, the differential equations, which you get or let us call this state associated with this transfer function is X_E , we have already seen in the previous class that $1 / (1 + sT_A)$ can be written down in terms of state equations. So, in such a case, what we have effectively is dX_E / dt is equal to $-1 / T_A X_E + k_A (V_{ref} - V)$ and as I defined sometime back the voltage magnitude is the root of $V_d^2 + V_q^2$, the terminal voltage of the generator.

Now, X_E itself is not E_{fd} . Well not always. X_E is equal to E_{fd} , only if the value of X_E lies within the limits. For example, we could have plus as I mentioned in the previous class we could be plus 7.0 V and minus 7.0 V. This is essentially a modeling the limits of the converter. In the sense that, it is the output of the converter will be limited by the AC input to it. So, X_E will be equal to E_{fd} , only if X_E is between minus 7 times V and plus 7 times V. This is not volts; this is the magnitude of the terminal voltage. So, this is what we get. If we had used a brushless excitation system, remember that you would get much more complicated equations for the excitation system.

So, if I used a brushless exciter in fact, the output would depend there would be dynamical equations associated with excitation power apparatus as well. So, as I mentioned in the previous class, you need to look at the IEEE standard or several books, which really describe a brushless excitation system modeling in detail, but if you are talking about a static excitation system is practically only the limit, which has to be modeled. The AVR of course, is a simple transfer function; it is a simple proportional controller. In practice, you may have something more complicated, you may have lead lag block also in series with it, but we will not really go into modeling that much in detail. We will do simple a simulation to a show you the effect of the static excitation system.

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If studying **slow** electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., **in the stator flux differential equations set**

$$\frac{d\psi_d}{dt} = 0 \quad \frac{d\psi_q}{dt} = 0 \quad \omega \approx \omega_B$$




If studying slow electro mechanical transients while operating near the normal speed, we can as an approximation set $\frac{d\psi_d}{dt}$ is equal to 0 and $\frac{d\psi_q}{dt}$ equal to 0 and ω approximately equals ω_B .

So, this is of course, if you are operating in the normal speed and we are interested in the slow transients. As we have just discussed in our previous treatment of this system. Now, one more. So, we have two algebraic equations here, instead of differential equations.

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Model of interconnection (d-q pu form)

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$
$$x = \frac{\omega_B L}{Z_{base}}$$


On the other hand, our model of the interconnection, I have mentioned this sometime back in the context of the presence of redundant states. The model of the interconnection is given by this differential equation. Now, as a logical extension to the approximation, which you have just made, that is $\frac{di_d}{dt}$ is equal to 0 and $\frac{di_q}{dt}$ is equal to 0 and ω approximately equal to ω_B .


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A logical extension of the approximation ...

set

$$\frac{di_d}{dt} = 0 \quad \frac{di_q}{dt} = 0 \quad \omega \approx \omega_B$$

Therefore the differential equation for the inter-connection becomes an algebraic equation.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_B \\ \omega_B & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$


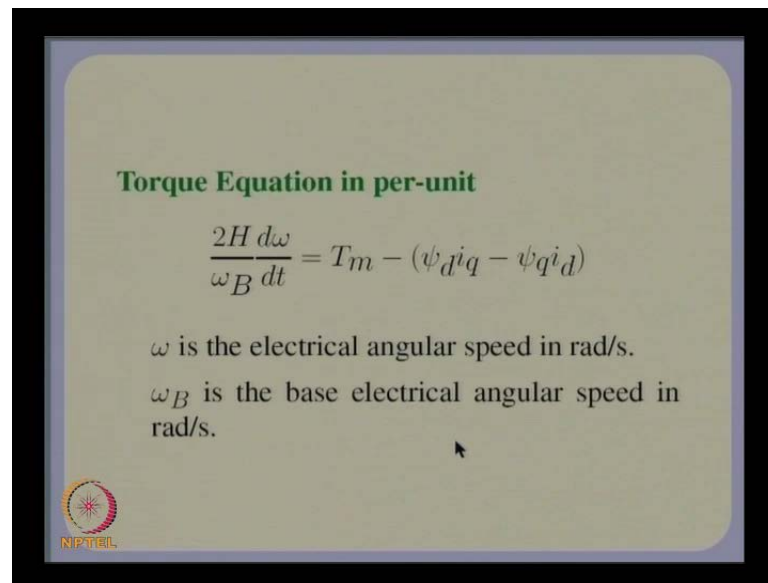
It makes sense to set $\frac{di_d}{dt}$ is equal to 0 and $\frac{di_q}{dt}$ is equal to 0 as well in these equations. So, if you do that of course, the differential equations get converted to

algebraic equations. The differential equations, which I just showed you in the previous slide get converted to algebraic equations, which is shown here. So, of course, what is the logic in doing this? Remember that in case we are neglecting fast transients by setting $\frac{d\psi_d}{dt} = 0$ and $\frac{d\psi_q}{dt} = 0$, there is really no point in retaining rather describing i_d and i_q by a differential equation, because if I do retain this differential equation, then we are not really getting the need of fast transients. You can show this is something I am not proving here, but you can show that If I retain this is a differential equation while setting $\frac{d\psi_d}{dt} = 0$ and $\frac{d\psi_q}{dt} = 0$, I am not really getting rid of the fast transients because of this differential equation, you will still have fast transients and the system will be still stiff.

So, in case if you are studying slow electromechanical phenomena, it makes sense; not only to set $\frac{d\psi_d}{dt} = 0$ and $\frac{d\psi_q}{dt} = 0$, but also $\frac{di_d}{dt} = 0$ and $\frac{di_q}{dt} = 0$. As a result, you get these algebraic equations. So, now, we have in fact, if you **if you** have noticed got rid of four differential equations and have algebraic equations in their place. Now, let us look at the other differential equations of the system incidentally before we go ahead. Remember that the redundancy of states, which I was just discussing sometime back in this lecture, that problem in some sense gets also solved.

Because once we set $\frac{di_d}{dt} = 0$ and $\frac{di_q}{dt} = 0$; $\frac{d\psi_d}{dt}$ and $\frac{d\psi_q}{dt} = 0$, they are no longer states and then we do not have to give initial conditions to them and if we do not give initial conditions to them, we do not have to worry about giving consistent initial conditions to these variables. Now, in fact, to some extent you will notice that rather I should say that since you have got i_d , i_q , ψ_d , ψ_q as algebraic variables, which are really dependent on other **other** variables in the system, we do not have to bother about the redundancy in states problem anymore.


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Torque Equation in per-unit

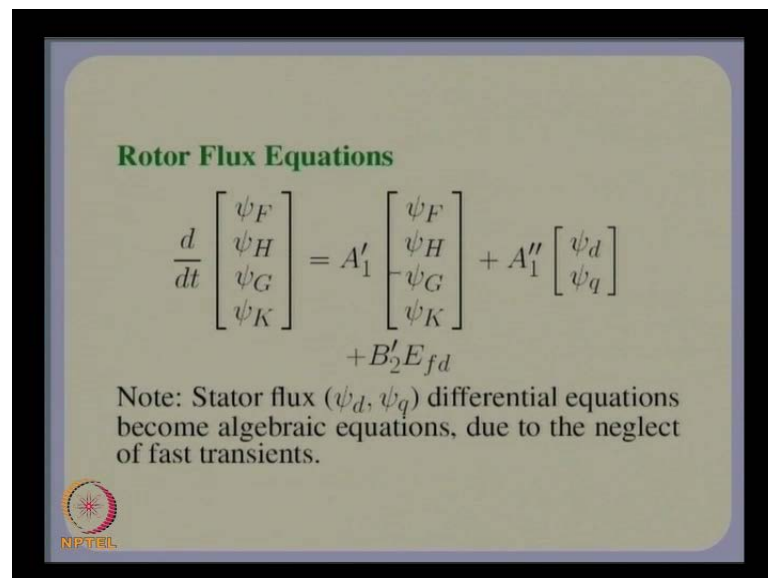
$$\frac{2H}{\omega_B} \frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.
 ω_B is the base electrical angular speed in rad/s.



Looking at the differential equations let just scan through all the equation again. This is a torque equation in per-unit.


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Rotor Flux Equations

$$\frac{d}{dt} \begin{bmatrix} \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = A_1' \begin{bmatrix} \psi_F \\ \psi_H \\ -\psi_G \\ \psi_K \end{bmatrix} + A_1'' \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + B_2' E_{fd}$$

Note: Stator flux (ψ_d, ψ_q) differential equations become algebraic equations, due to the neglect of fast transients.

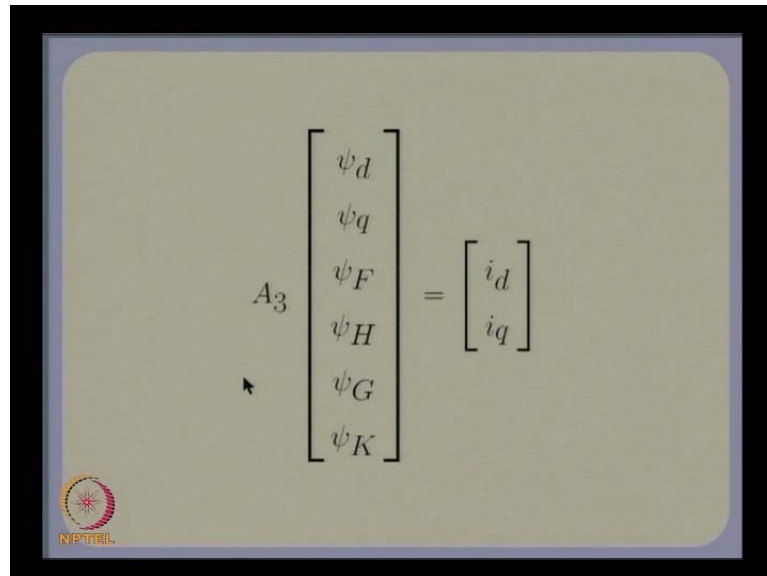


These are the rotor flux equations. Remember that psi d, psi q no longer being states, we do not have to write differential equations for psi d and psi q.

So, the only differential equations as far as the flux equations are concerned and the rotor flux equations. So, let us just. So, we have got one differential equations of the rotor speed, four differential equations corresponding to the rotor flux equations. Later on

missile also, see that there is one differential equation corresponding to the rotor angle delta. A 1 dash A 1 double dash and B 2 dash are given by these equations.

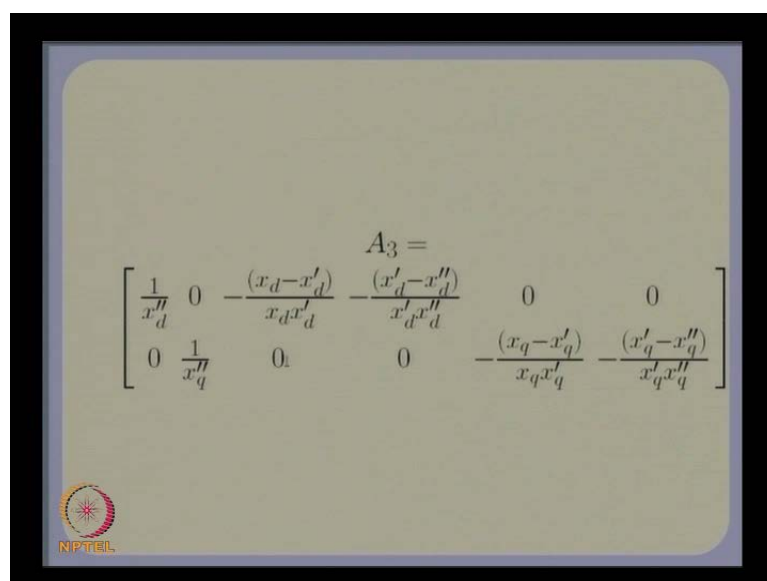
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$$A_3 \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_F \\ \psi_H \\ \psi_G \\ \psi_K \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

There is also an algebraic relationship, which relates i_d and i_q to ψ_D , ψ_Q , ψ_F , ψ_H , ψ_G and ψ_K . So, there are two algebraic equations **the two algebraic equations**, which really relate i_d and i_q to the rotor and stator fluxes. So, these are algebraic equations, not differential equations.

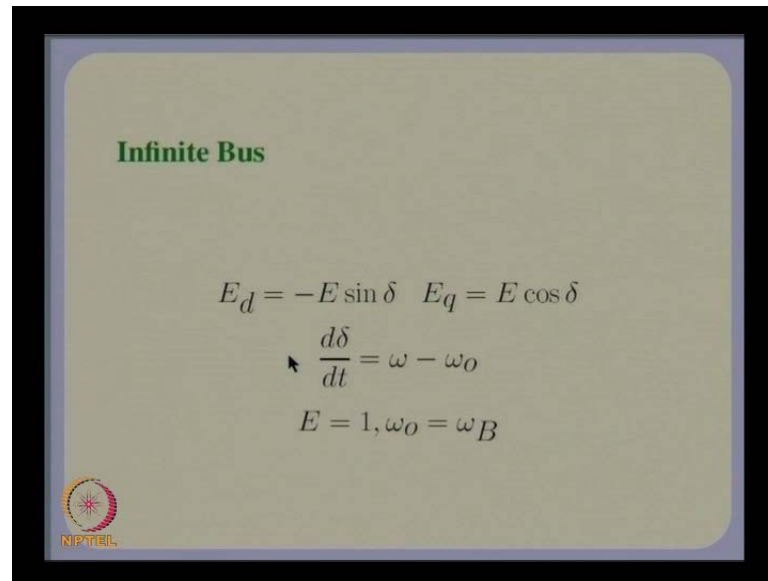
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$$A_3 = \begin{bmatrix} \frac{1}{x_d''} & 0 & -\frac{(x_d - x_d')}{x_d x_d'} & -\frac{(x_d' - x_d'')}{x_d' x_d''} & 0 & 0 \\ 0 & \frac{1}{x_q''} & 0 & 0 & -\frac{(x_q - x_q')}{x_q x_q'} & -\frac{(x_q' - x_q'')}{x_q' x_q''} \end{bmatrix}$$


So, over all incidentally A_3 is given by this. So, in this equation A_3 is given by this.

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Infinite Bus

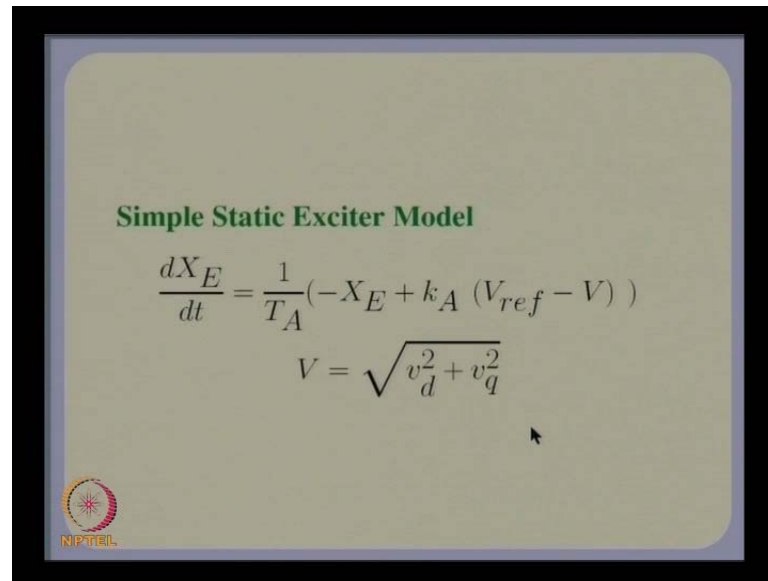
$$E_d = -E \sin \delta \quad E_q = E \cos \delta$$
$$\frac{d\delta}{dt} = \omega - \omega_0$$
$$E = 1, \omega_0 = \omega_B$$



We also see that E_d and E_q , which appear in these equations. In fact, E_d and E_q appears in the algebraic relationship of i_d and i_q , which we discussed sometime ago. So, E_d is nothing but $E \sin \delta$ minus $E \sin \delta$ and E_q is equal to $E \cos \delta$. This is by applying Park's transformation to E_a , E_b and E_c , which has phase to neutral voltages of the infinite bus. This is one differential equation here. Remember that in this **in this** system we assume this data E is equal to 1 and the frequency of the ω of the infinite bus is equal to the nominal frequency or the base frequency. This is of course, data which is given to us or rather I am giving it to you. This could be this is for example, E could be 1.1 also or the frequency of infinite bus could be slightly higher or lower than the nominal speed.


But, let us first for simplicity, let us assume E is equal to 1 and ω_0 is equal to ω_B . So, we are assuming that the infinite bus frequency is the nominal bus nominal **nominal** frequency of the synchronous machine.

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Simple Static Exciter Model

$$\frac{dX_E}{dt} = \frac{1}{T_A} (-X_E + k_A (V_{ref} - V))$$
$$V = \sqrt{v_d^2 + v_q^2}$$



The static exciter is modeled for the first order differential equation. Now, X_E is not the same as E_f . In fact, X_E is same as E_f only if X_E is within the limits of the excitation system. Of course, if it exceeds the limits, then X_E is clipped to the maximum or the minimum value of E_f as defined by the exciter model. So, actually this is a simple static exciter plus voltage regulator model.

So, in fact, I should write here static exciter plus automatic voltage regulator model. So, it is just defined by one differential equation. V is equal to square root of V_d square plus V_q square as discussed sometime previously in this lecture.

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States, Inputs

1. States: $\delta, \omega, \psi_F, \psi_H, \psi_G, \psi_K$
2. Number of differential equations: 6
3. Other variables: $\psi_d, \psi_q, i_d, i_q, v_d, v_q$
4. Algebraic Equations : 6
5. Inputs: T_m, E_{fd}

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So, the number of states are 6. In fact, there are 6 differential equations corresponding to these 6 states. The other variable ψ_d, ψ_q, i_d and i_q, v_d and v_q are really not states. We have set $d\psi_d$ by dt and $d\psi_q$ by dt, di_d by dt and di_q by dt equal to 0, because of which we have got rid of differential equations and redundancy of states. There are 6 algebraic equations of course and inputs to the system are T_m and E_{fd} . E is of course, the infinite bus voltage, which also has to be given to you and speed of the infinite or the frequency of the infinite bus also to be given to you.

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$\psi_d, \psi_q, i_d, i_q, v_d, v_q$ can be obtained in terms of the states using the 6 LINEAR algebraic Equations

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_B \\ \omega_B & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\omega_B}{x} \left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} \right)$$

$$A_3 [\psi_d \ \psi_q \ \psi_F \ \psi_H \ \psi_G \ \psi_K]^T = \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

NIPTEEL

Remember, that the 6 others **other** variables ψ_d , ψ_q , i_d , i_q , V_d , V_q , which are no longer states can be obtained in terms of the states. The states are δ , ω , ψ_F , ψ_G , ψ_H , ψ_K . By using these 6 linear algebraic equations, linear simplifies our job. So, because the solution can be got in one shot without any numerical iterative procedure. So, we can directly write ψ_q , ψ_d and i_d , i_q and V_d , V_q in terms of δ , ω ; ω in fact, does not appear here, because we have taken ω approximately equal to ω_B , but δ appears in this E_d and E_q term and of course, ψ_F , ψ_H , ψ_G and ψ_K appear here.

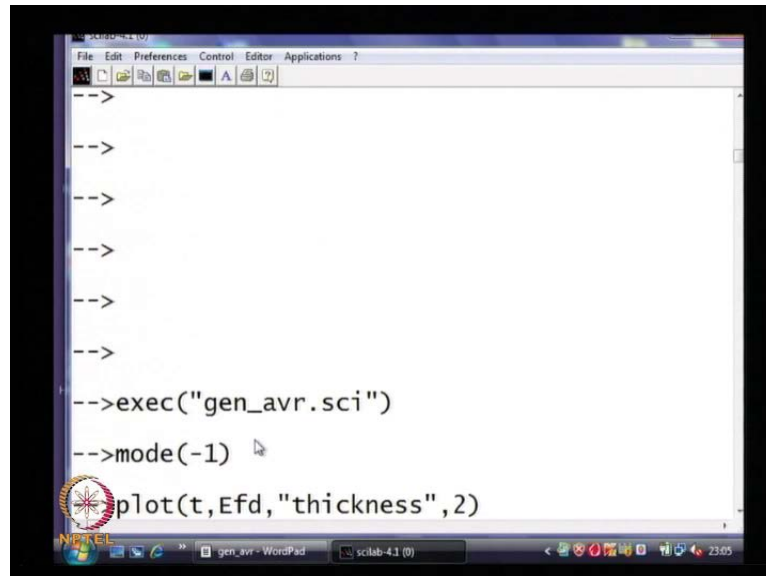
So, this is how we obtain all the equations, the mixture of 6 differential equations and 6 algebraic equations. The algebraic equations allow us to get rid or other better word would be to get to write V_d , V_q , i_d , i_q in terms of the states. So, in fact, V_d , V_q is required by the differential equations and ψ_d , ψ_q , i_d , i_q are required to differential equations. These are in fact, in terms of the states itself themselves ψ_F , ψ_H , ψ_G , ψ_K , δ and ω . So, it is a fairly trivial matter to write this ψ_d , ψ_q , i_d , i_q , V_d , V_q in terms of the states and in some sense, eliminate them from the differential equations.

So, before we close today, let me just give you a flavor of simulation. We will **will** not be able to explain all the aspects of the simulations today. What I do is, I will synchronous the generator right at time T is equal to 0. It will be a bump less synchronization. Thereafter, I will increase the torque of the synchronous machine. I will load this synchronous machine and I will show you that the terminal voltage remains more or less constant in case you have got an AVR and after a 15 seconds of course, the step in real power will be given at 5 seconds, after 15 seconds we will give a step change in the reference value of the AVR.

So, that is what we simulate and close this lecture thereafter. So, if I simulate this, I am simulating about 25 seconds with Euler method with a time step of 5 milliseconds. So, that is why it is taking fairly large amount of time. Remember one more problem which is encountered with Euler method is that one problem, which is encountered with Euler method; it is not a very numerically stable way of simulation. I have just used it for simplicity of simulation. So, it is often said that Euler method is only taught. It is never used really in practice.

So, anyway by making the time step very small and removing the stiffness and none the less I am able to use Euler method in this particular situation.

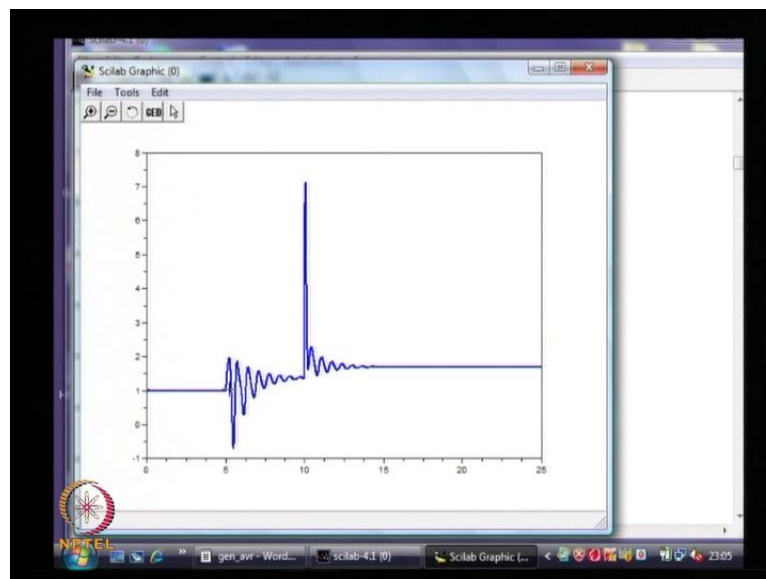
(Refer Slide Time: 52:45)



```
-->  
-->  
-->  
-->  
-->  
-->  
-->  
-->exec("gen_avr.sci")  
-->mode(-1)  
-->plot(t,Efd,"thickness",2)
```

I encourage you to try to use some other method. Now, let us plot how a $E f d$ looks. Now, remember at 5 seconds I have increased the torque and at fifteen seconds I have increased the reference value of AVR, v_{ref} .

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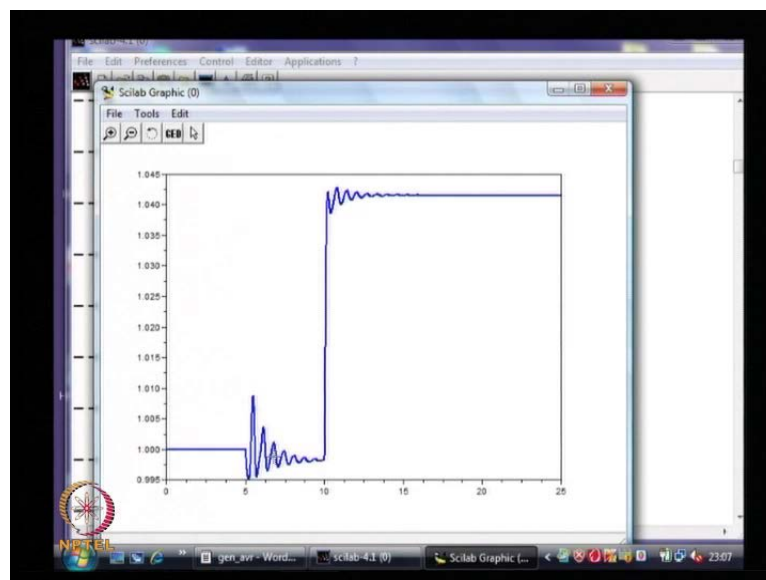


In both situations, you will notice that whenever there is change in the loading of the machine at 5 seconds, the $E f d$ changes from 1, which is the value under the no load

conditions. It changes automatically to around 1.5 and if I change the v_{ref} , that is the reference voltage of the synchronous machine, again E_f changes. Now, E_f is able to take on the value as high as 7 because the limits are very high, excitation system of this kind have high ceilings.

So, we are able to force the field to a very high value. That is of course, as I mentioned sometimes back, because the field winding is a very slow acting system and you really need to push it a lot in order to make it work faster. So, this is the way how field voltage changes. The field voltage is no longer constant because it is being changed by the automatic voltage regulator.

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And if you look at V itself, it is v_{gen} . I have changed the f_c , the reference value is initial was 1 per unit, near about one per unit. As I loaded the machine, the reference value went down slightly or rather the terminal voltage magnitude went down slightly, initially it was 1 per unit, if I load the machine its goes down.

Now, the question is, why does the voltage magnitude of the synchronous machine go down if it is regulated. Here, remember at 5 seconds I have applied the load. Now, while if I have applied load, why should the terminal voltage magnitude change if it is being regulated by the AVR. That is one question, which we need to ask ourselves. We will try to answer that question next time. Of course, if I give a step change in the automatic

voltage regulator at 10 seconds of course, it was 10 not 15 seconds, at 10 seconds the terminal voltage of the machine goes up.

So, it regulates it. It changes the value according to the set point. So, it is gone to around 1.045, you know roughly. So, this is how the AVR behaves, there are many **many** interesting points, which I need to discuss with you. There is not time for that in this lecture. So, we revisit this point in the next class, redo this simulation and try to bring out some of the nice interesting points, which come out of the simulation. So, for that we will meet again in the next class.