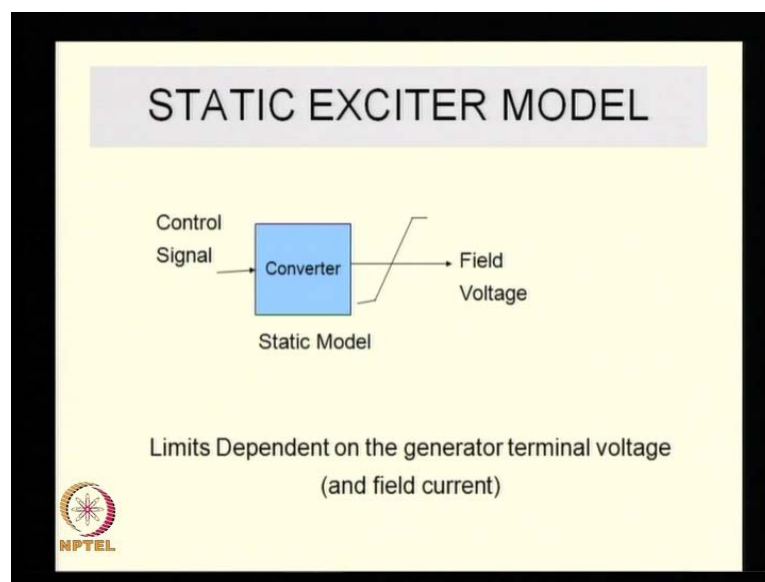


**Power System Dynamics and Control**  
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**Model No. # 01**  
**Lecture No. # 26**  
**Excitation System Modeling Automatic Voltage Regulator**

We now move on to the description of the controllers of an excitation system. So far, we have been discussing the issues related to the modeling of the power apparatus that is the exciter. But, in exciter in some sense is incomplete without this this a kind of discussion of those excitation system, is incomplete without the discussion of the controllers associated with the excitation system. In the previous class, we did the physical models of an excitation system. We did not actually derive in detail, the model, but, listed down the things which ought to be modeled. In today's class, in today's lecture we discuss one of the major control systems associated with the excitation system that is the automatic voltage regulator.

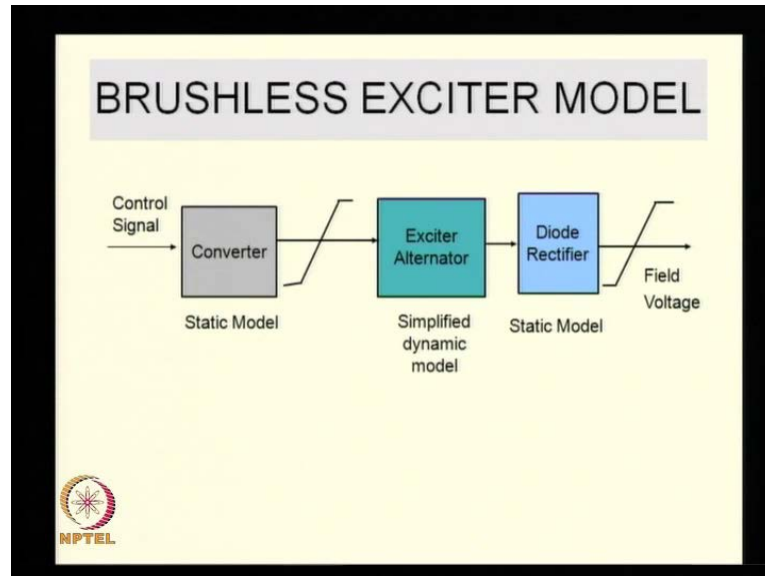
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So, in the previous class we studied two kinds of excitation systems; the static excitation system whose basic model is a static one with limit's, which are determined by the generator terminal voltage. The control signal here is effectively obtained from the

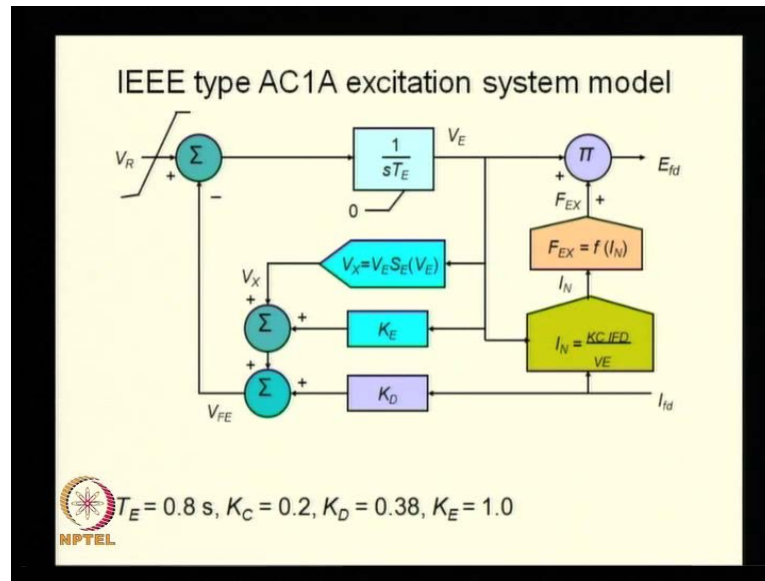
excitation system controllers. The brushless excitation system is a bit more complicated in the modeling aspects.

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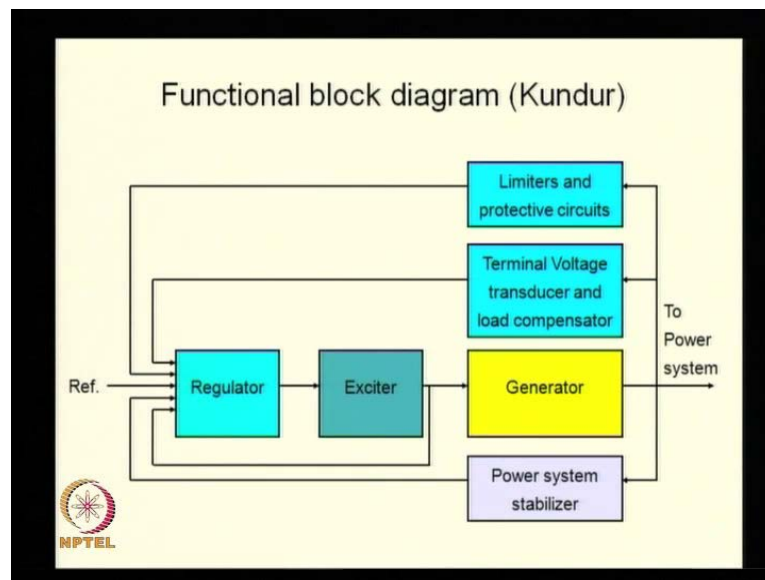
The main issue here of course, is that the **excite** exciter alternator has to be modeled by a dynamic model not a static model. And the diode rectifier which is an uncontrolled rectifier as got a limit that is its voltage cannot go negative. The output voltage to the field cannot go negative. So, there is a limit associated with that. The control rectifier associated with the brushless excitation system is modeled in similar way to a static excitation system. That is, it is a static model with limits that is given on your left hand side.

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Of course, IEEE has come up with standard excitation models. In fact, the IEEE type is A C 1 A model which describes the model of a brushless excitation system is given here.

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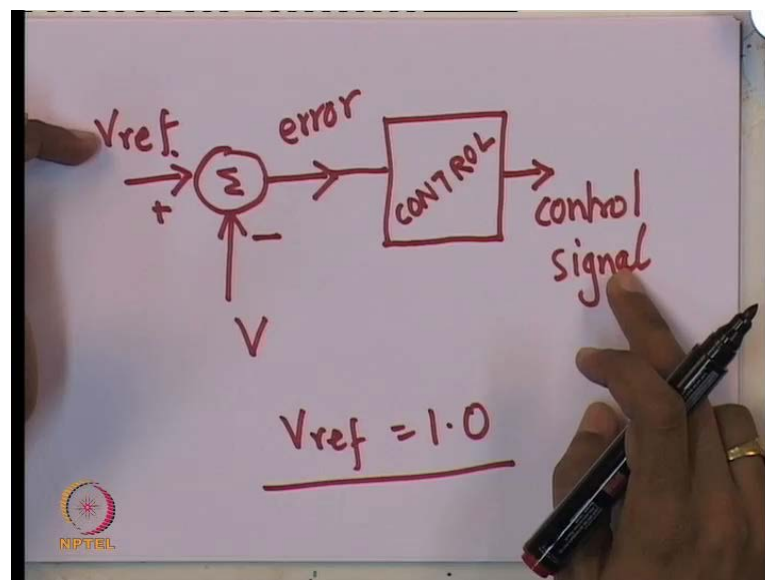
The functional block diagram of an excitation system including not only the power asperities but, also the regulators, limiters and stabilizers which effectively form the control system; remember control system is designed by us it is more of a mathematical logic which is implemented using some physical elements. So, you can have analog or

digital implementations of the mathematical logic which really determines the behavior of the controller.

Now, the mathematical logic in fact is the mixture of, you know, differential, dynamical equations which have to be effectively numerically or implemented or implemented using analog blocks. Now the most important or the core function of an excitation system controller is regulation of the generator terminal voltage. So what it does is, essentially keeps the terminal voltage of the synchronous generator the main synchronous generator almost constant. Now, **so** this is shown in a functional block diagram here you can pay attention to that. There is a regulator; regulator is something which tries to bring if this case the voltage near or equal to a set point value which called the reference value.

But regulation is not the only function which is **which is** actually done by any excitation system controller. You also find that there are additional blocks corresponding to what is known as a power system stabilizer and limiters in protective circuits. So you can have the overall functional block diagram contain something more than a regulator. Now, we will try to understand each of these blocks one by one.

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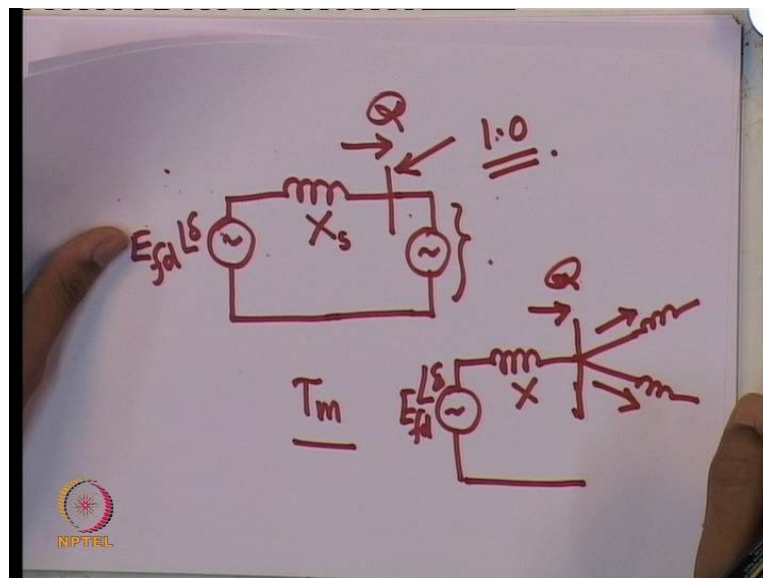


First of all is the regulation block. The regulation block essentially takes a reference or set point voltage. You compare it with the measured voltage **whether** measured terminal voltage of the synchronous generator and use this error signal to drive a controller which

in fact gives the control system which is used by the control rectifiers of the excitation power apparatus.

So, this is the controller. So, this is the controller or regulator you know. Now this controller of course, is a mathematical function. It is some mathematical function like something we will describe in a few moments from now. The voltage regulator effectively compares the set point which is given by us and the actual terminal voltage. Now the **one** one point about the set point itself, the voltage reference of a synchronous generator terminals is typically going to be 1 per unit. We are going to really try to keep the generator terminal near about its rated value. But, it is not sacrosanct **saying** that  $V_{ref}$  should be equal to 1. You must have done in course on any course on synchronous machines or electrical machines that, by changing the excitation system voltage. Remember the change in a voltage reference here finally affects the control signal and eventually affects the voltage which is going to apply to the field of the main synchronous generator. So, by changing this reference voltage say by increasing it, you are going to change effectively the field voltage and therefore, the reactive power output of the generator.

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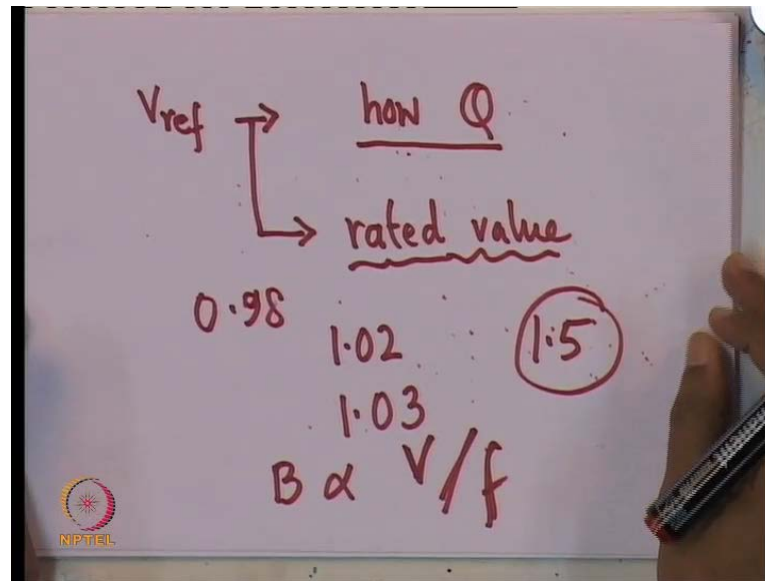
So, if you recall, a synchronous generator in its most, simplest steady state model can be represented in this fashion. This is the synchronous machine terminals. If the machine does not have saliency you can represent it by a synchronous reactant which is nothing

but, equal to  $X_d$  or  $X_q$  because they are equal. So, this is the kind this is the steady state model of a synchronous machine.

So, by changing  $E_f$  if I change the field voltage, I effectively can change the reactive power output of the generator. Remember, the real power output of a generator can be changed only by changing the mechanical power or in the mechanical torque of the machine. So, that is one thing you should keep in mind. So, if you look at this steady state model of non salient synchronous machine by changing  $E_f$  you can change the reactive power output. So of course, we do need to maintain the terminal voltage near about 1 per unit. But, in case you change, if you change try to keep this voltage slightly greater than this remember that, this although I have represent the rest of the system as a voltage source it actually is something like this.

So, the correct way of representing the rest of the system could be this is at an angle  $\delta$ . The rest of the network, **now the rest of the network** again consist of transmission lines etc etc. Simplified model of that could be inductances etc etc. So, if I change the magnitude of the voltage, the terminal voltage effectively the reactive power output of the machine will change. So,  $q$  of the machine can be changed by changing the reference value of the terminal voltage of a synchronous machine. So of course, we will not want  $V_{ref}$  to be very much different from 1 per unit. After all the voltage in a synchronous machine has to be maintained at roughly what its rated value is. If you have voltage is much larger than that we will have problems of insulation. And more importantly you may have over fluxing in the machine.

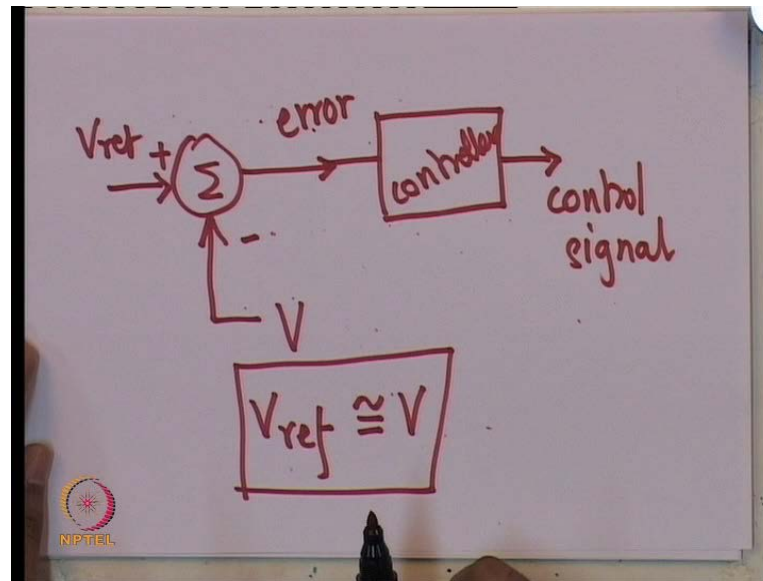
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So, one of the important issues here is what determines  $V_{ref}$ ? Well, one of the things is how much reactive power as the generator to be to supply. Of course, by reducing  $V_{ref}$  one can make the generator absorb reactive power. So,  $V_{ref}$  is determined by how much the reactive power output of the generator should be. So, a plant operator, power plant operator could quick around this  $V_{ref}$  near about the rated value of the synchronous machine. So you of course, we can except accept  $V_{ref}$  to be say 1.02 or 1.03. But, please do not excepted to be 1.5 or so. That would really be much larger than the rated value and that would take the insulation of the synchronous machine windings. And more over the flux in the machine, the fluxes in the machine would become greater. The flux density in the machine would exceed the saturation value in this case.

So remember, the flux density in the core are depended going to be dependent on the voltage of the machine divide by the frequency of the machine. So, in steady state  $B$  will be prepositional to  $V$  by  $f$ . So, one of the reason why you cannot exceed make  $V_{ref}$  too large is that, you will go into saturation. If you go into saturation the machine becomes non-linear. That is one of the, it will increase the flux value, increase it will increase the heating in the machine as well. So, that is one of the major points which you should remember when setting  $V_{ref}$ . Of course,  $V_{ref}$  can be less, you can have  $V_{ref}$  as point nine eight per unit. That is also feasible you know. You may, under low load conditions, want the generator to observe absorb reactive power. In that case you would reduce  $V_{ref}$  of the machine.

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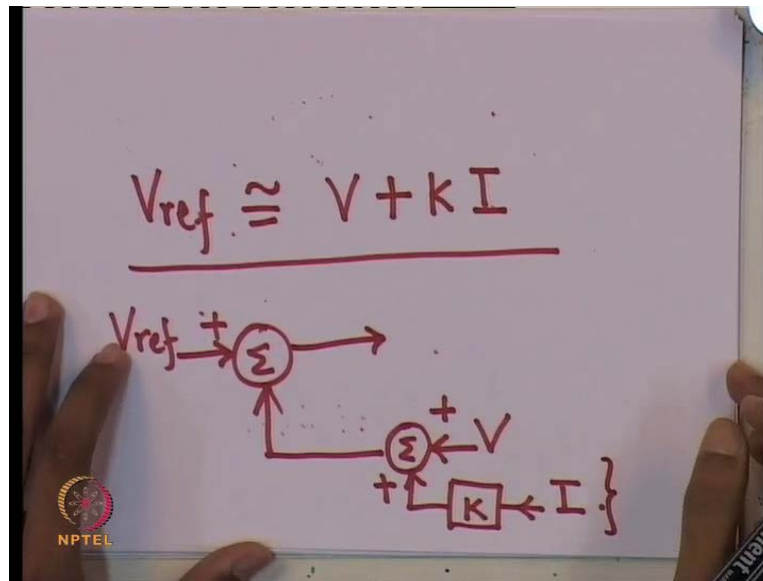


So, this is the basically what goes into deciding what  $V_{ref}$  of the machine is. Now, once you have got  $V_{ref}$ , you have decided what  $V_{ref}$  should be, the power planned operator as some freedom in deciding that. He compares it is compared with terminal voltage of the machine. Now, the question which you **the you** know, which needs to be asked is this controller whichever is there, is it going to drive this error to 0? That is, will it make  $V_{ref}$  exactly equal to be the answer is? Well, it depends on the mathematical function used in this controller. Now, another interesting variation which you see, other **of** interesting variation which you see is instead of maintaining  $V$  is equal to  $V_{ref}$ , in some machines or in some situations  $V_{ref}$  is you know, you try to maintain  $V_{ref}$  equal to  $V$  in most machines.

So, this error is driven either to 0 or near 0. That is what your controller should do. It depends on the mathematical function you are using. But, the aim of course, should be to reduce the error almost to 0. That is  $V_{ref}$  approximately equal to  $V$ . So, that is why it is called a regulator a voltage regulator. But, in some situations you would not necessarily try to keep  $V_{ref}$   $V$  almost equal to  $V_{ref}$ .



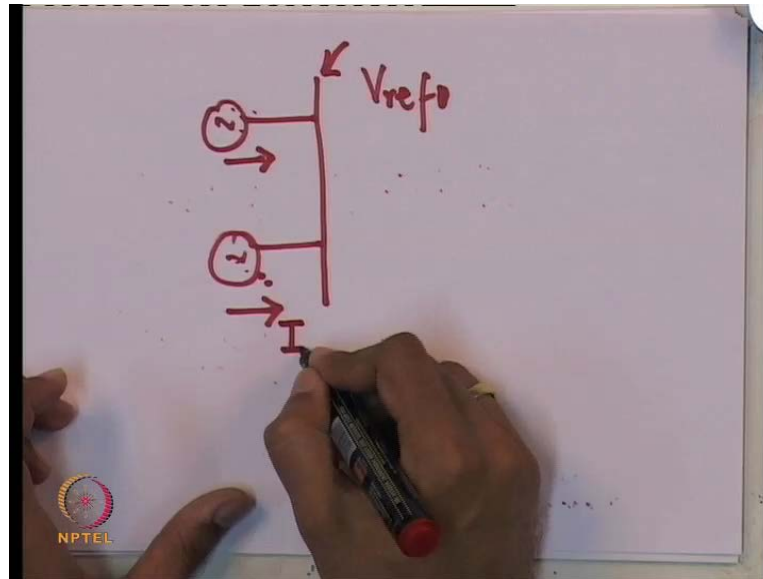
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But, you can have  $V_{ref}$  almost equal to  $V$  minus  $K$  into the magnitude of current going out of the machine. So, you can have situation like this is well. So, you can, instead of having control system like what I shown you before that is just having  $V$ , what you can have is, this is known as load compensator. So, the thing is that of course, I should, this should this usually is positive. The point is that, instead of keeping  $V$  equal to  $V_{ref}$  or almost equal to  $V_{ref}$  will keep  $V + KI$  is equal to  $V_{ref}$ .

So in case,  $I$  is large and depending on this value of  $K$  if it is nonzero; you will find at  $V_{ref}$  is not maintained at  $V$  is not maintained at  $V_{ref}$  but, slightly lower than  $V_{ref}$ . Now the question is why do this, add this additional complication here?

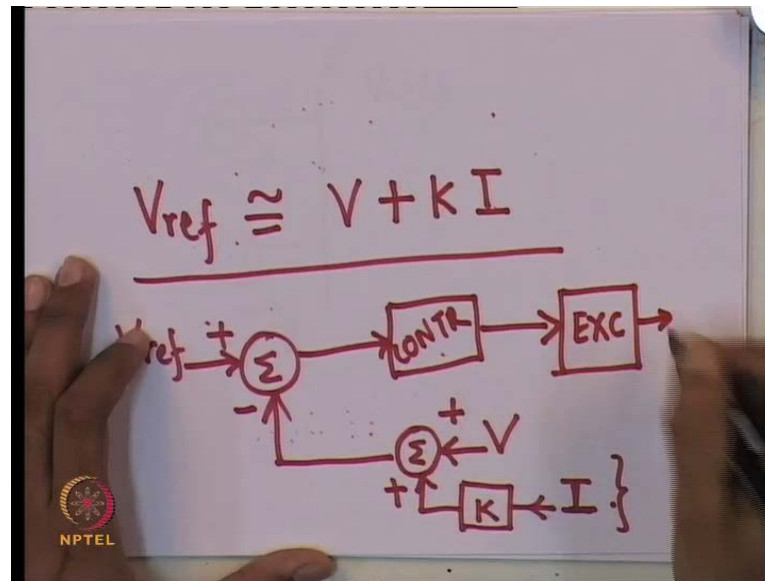
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The thing is that, if you have got two machines which are connected to the same bus. So, this is two synchronous machines connected to the same bus and both these machines have an excitation system. So, both these machines have an excitation system. Both are trying to maintain the voltage of this bus a constant at  $V_{ref}$ .

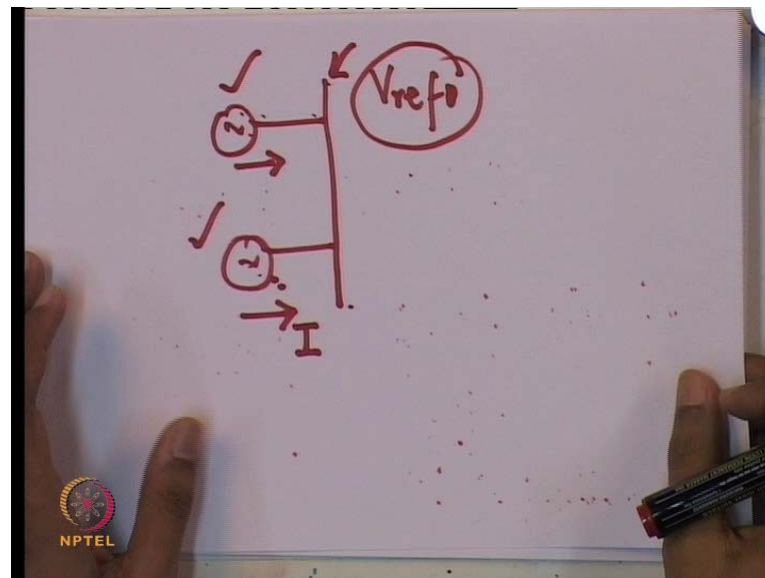
Now, what will happens is that, in case both these machines are trying to maintain the voltage here at  $V_{ref}$ . The question arises is how much which each individual machine contribute to keep in this voltage at  $V_{ref}$ ? Now, normally what we do is, we make this change to the automatic voltage regulator so that, if a machine contributes more current you know, or more load, more current. In that case this gets **modify** modified. This additional term essentially, reduces  $V$ , makes  $V_{ref}$  try to less than  $V$ . Rather makes  $V$  slightly less than  $V_{ref}$ . The reason **it is done** is that you can then impose a sharing between these two generators. If a one generator supply more current, automatically what is done is that, this becomes larger. If this becomes larger  $V_{ref}$  will not be maintained.  $V$  will not be maintained at  $V_{ref}$ . But, slightly lower value. So what I will, eventually happen is that, this synchronous generator will start if this  $I$  increases effectively what will happen?

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It will try to reduce. This is the exciter power apparatus, this is the controller. If this one of the generators takes on more load it tends to decrease. It will tend to decrease the final exciter voltage. And therefore, what happens is that, it prevents one generator from taking all a load. In fact, by choosing case appropriately for both machines.

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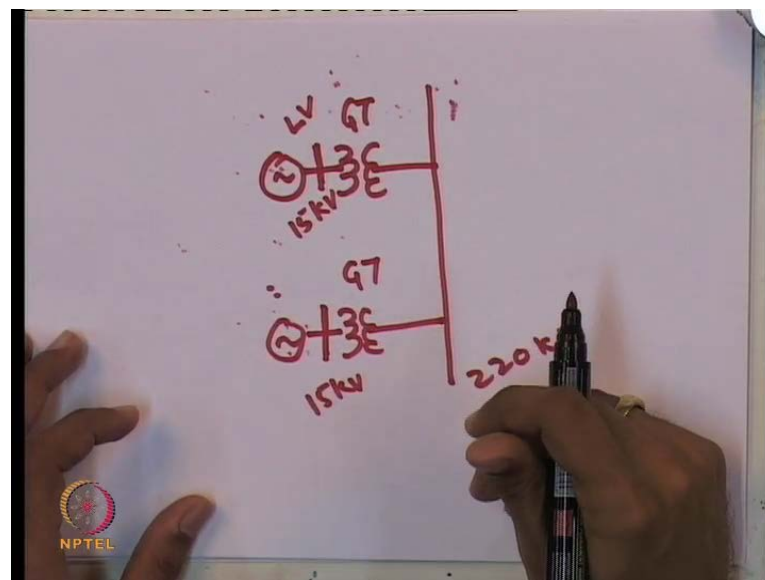


So, in both machines if I choose my K appropriately, I can actually impose a proper sharing agreement between these two generators. So, no one machine pushes in more current than what is necessary. So, what happens is that, you make the excitation system

or the exciter field voltage **of** a function of the load current. So, if you have got two alternators or three alternators which have bussed together or they are connected to the same bus and your automatic voltage regulator is trying to maintain the voltage of that bus at  $V_{ref}$ .

So each generator as got an independent voltage regulator which is trying to maintain one common bus voltage at  $V_{ref}$ . Then, to impose the sharing of the reactive power, one may have to put this additional component in the control system which makes the excitation field voltage dependent on  $I$ . So, if one generator starts pushing more current, it automatically self corrects. So, that is the basic idea. So, this sharing of the generators can be imposed by choosing appropriate values of  $K$  for each generator. Is it okay?

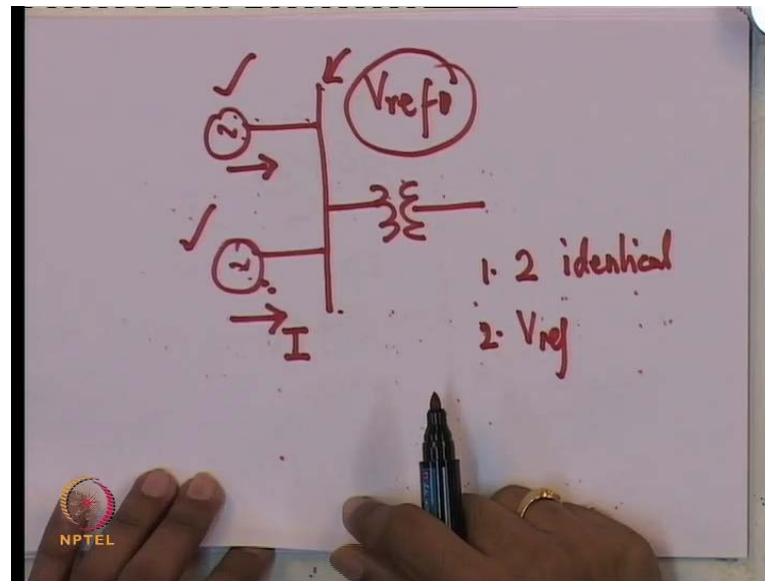
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But, of course, if the generators are not bussed together; in fact, typically what happens is **heat** each generator is connected to its unit transformer. It is called a generator transformer which steps up the voltage and the generator are bussed together at the high voltage side.

So, this is the low voltage say 15 K V **15 K V** and this is 220 or 440 or 400 kilowatts say. So, if you got situation like this, each generator maintains its own terminal voltage constant. It is not maintaining a common bus voltage at  $V_{ref}$ . So, in this case it is not necessary to have load compensator for **two** sharing of, because you are not maintaining common bus voltage at  $V_{ref}$ .

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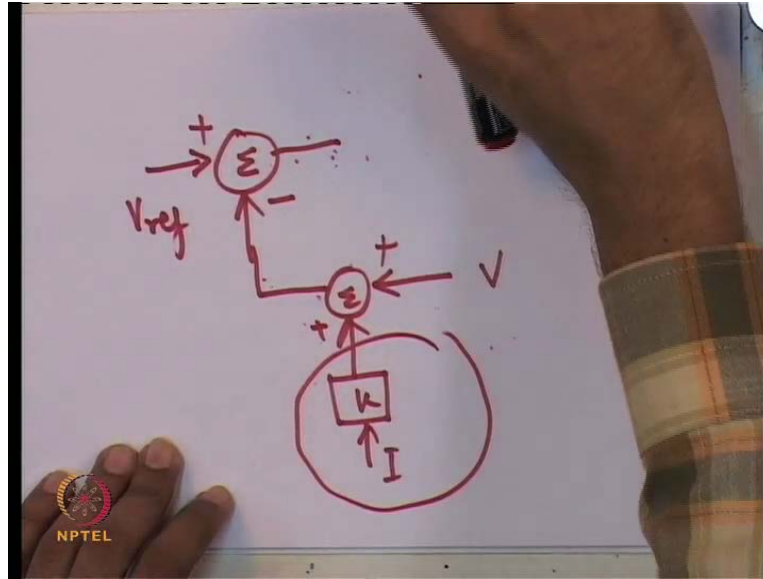


So but, in case in case two generators are bussed at common point directly, the terminals of the generator are connected to a common bus and then you step it up, side or use it for other purposes. In that case you need to have a load compensator to **insure** ensure that while trying to meet a common objective, the sharing of the load is done as per our design, so, this K value.

I will leave it as **an d** an **the** exercise for you to show that indeed if I choose K 1 and K 2 appropriately for say, two identical generators I can or two different generators. For that matter a choosing the K value appropriately, I can impose a sharing of reactive power between the two generators. So, this is something I request you to try to prove. So, the steps are, just to make things simpler assume two identical generators, assume that there are supplying the same amount of real power, they have got same mechanical power input say, that is the values are equal. Let us assume that the V ref for the both the automatic voltage regulator independently I mean they two independent automatic voltage regulator is this same.

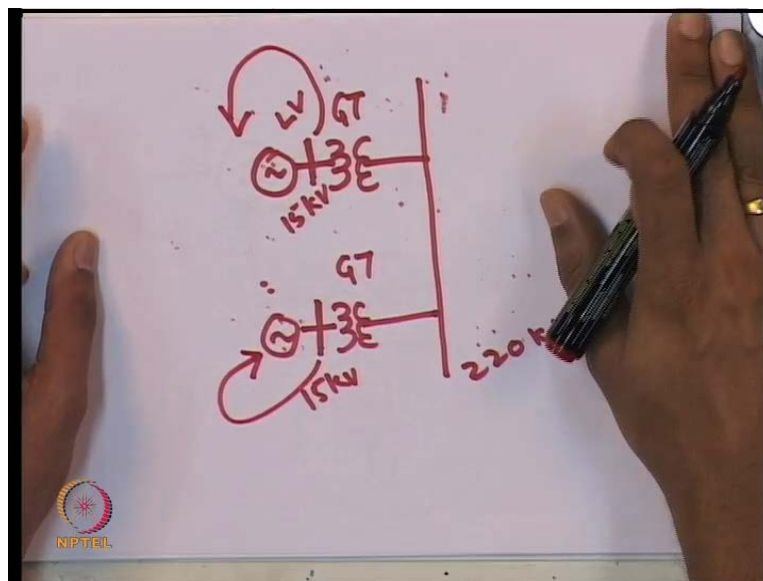
And these two machines are bussed together at a common point. Then by choosing K 1 and K 2 appropriately the reactive power sharing can be imposed as desired by us. So, you can choose K 1 K 2 as per you have reactive power sharing requirement in this situation.

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So, this particular function of  $V_{ref}$  being modified, this summing point being modified by applying an additional correction signal here, you can say which is dependent on the loading of the machine alters the characteristic of the machine, alters the voltage. You are **not may not** no longer maintaining  $e$  is equal to  $V_{ref}$  but, maintaining  $V$  plus  $K I$  approximately equal to  $V_{ref}$  so this is called as load compensator.

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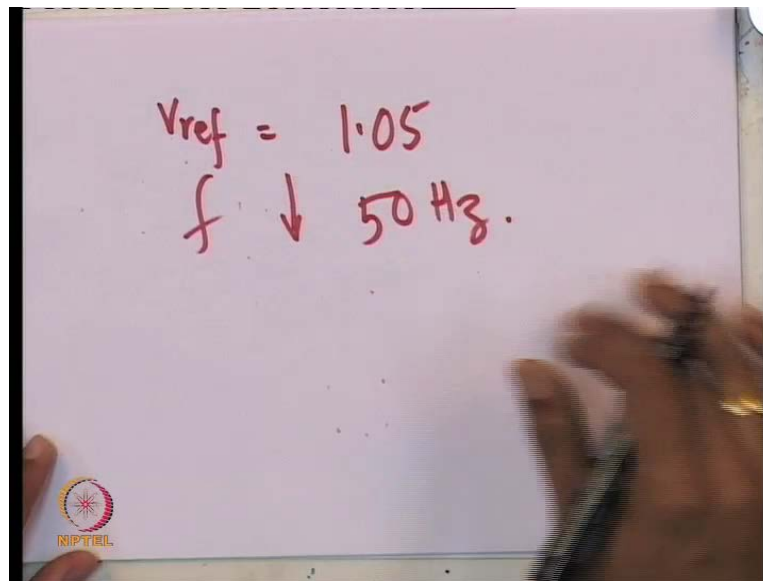


It is usually not used in situation where a common objective is not being met by two excitation system. For example, in this case when the machines are bussed together at the

high voltage side and your voltage regulator is trying to maintain these voltages constant, it is not necessary to apply this load compensator and in such situations it may be absent.

So please **please** chew upon this point. Now, if you look at the other function which I showed in this block diagram, there limiters and a stabilizers. But, will come to that a bit later. In fact, I **will** have already given you an hint where this limiter may come into **into** play.

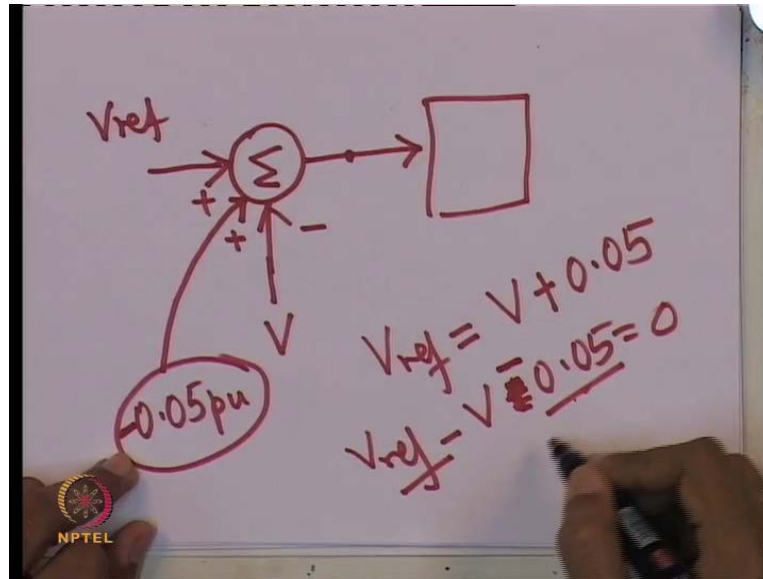
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For example, if **I** given  $V_{ref}$  1.05 and you also see that the frequency of rotation of the machine has come down. It is less than normal say 50 **yards** hertz. The frequency has come down. In that case, the flux level in the core may be higher than what is sustainable by the core in sense the core is start heating up **so** and will also get saturated.

So, in such situation you may require to **has** have some overriding functions which prevents the flux in the machine from becoming too large. So, what you have to do is if effectively under such situation if you see that the flux limit is being exceeded; what you would do is override what you want to have a, rather override the regulator in the sense that do not aim to achieve  $V_{ref}$  is equal to say 1.05. But, reduce this  $V_{ref}$  to a point at which the flux is not exceeding its rated value. So, this kind of protective and limiting functions can be **there** rare. Basically, what they do is override  $V_{ref}$  or at least change  $V_{ref}$ . So, what you have usually is these stabilizing and limiting functions.

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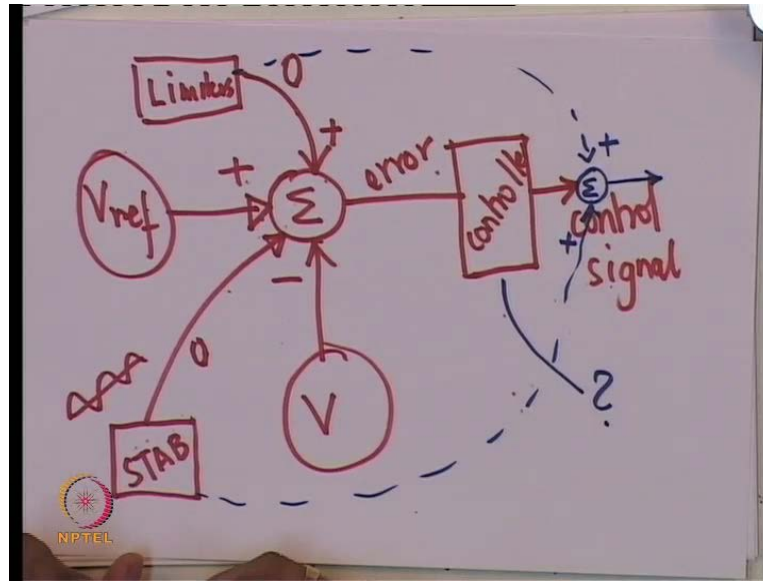
So, what you have normally is,  $V_{ref}$  in case the generators are not bussed together right at the terminals. This is the structure of the summing point of the voltage regulator.

But, **we**  $V_{ref}$  itself may be overridden in the sense that, some additional control if you for example, if I put some additional input here say, 0.05 per unit I added this point. So, what will happen is that, if I if the controller here the controller here drives this two 0; what you will have is,  $V_{ref}$  is equal to  $V$  minus 0.05 or in other words  $V_{ref} - V + 0.05$  is equal to 0. So, what infect by doing this is, I have done is increased  $V_{ref}$ . So, if you have any additional signals adding on to  $V_{ref}$ , the infect compromise on this voltage regulation function for **we**  $V$  will no longer be equal to  $V_{ref}$ . So, for example, if there is an over flux in situation you may adjust; this becomes minus. You may adjust this  $V_{ref}$  by an additional signal which adds on to these summing points.

So, I may add on minus 0.05 so as to, so I augment this summer what comes out of this summer. So finally,  $V_{ref}$  we will not be equal to  $V_{ref}$  but, slightly less than  $V_{ref}$ .



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So, this is what effectively limiting function does. The other function which we have seen here is, so I actually read out this. So, we got  $V_{ref}$ . This is the structure of a voltage regulator summing point without load compensation. So, we just have  $V$  here. This is what is normally required. You may have overriding additions or subtractions from limiters and you may even seek to modulate. You may wish to modulate **you may wish to modulate** this voltage reference by some external stabilizers. Now, we shall discuss these stabilizers later in the course. But, remember you can modulate these things. Now, one of the important things about limiters in stabilizers is, which you should be very clear about is that, limiters normally give zero output if the machine has not hit any limit. Similarly, a stabilizer gives zero output when the machine is in steady state.

So, the overall regulation function of this voltage regulator eventually falls down to this and this. The summing point in this regulation function falls down to taking the difference between  $V_{ref}$  and  $V$ . In steady state if no limit is violated. So, the voltage regulator in some sense gets in it is completely enabled when no limiter or no limit violation is taking place. However, if any limit violation is taking place some input will manifest at this point due to the limiting function. And **we**  $V$  will no longer be equal to  $V_{ref}$ . It will get overridden in some sense by this additional superimposed signal. During transients, **it is stabilizers** stabilizing function augments this  $V_{ref}$ . Say you can during transients you could be sending in modulation signal. How to drive this modulation signal is something we will discuss later. We may modulate this point.

Why we need to do it? Well, by using this modulation function we accept that we will try to improve the dynamic performance of this synchronous machine. So, this is what essentially is the things you will see at any summing point. One of the questions you may ask is, well you have got this controller or regulator which takes this error **that is the** does some mathematical function on it and drives the controls signal which goes to the power apparatus. It is excitation power apparatus. The question is can we not shift this limiting functions to this point? So, can we not instead of, changing the summing point here can we have this overriding functions acting here? The answer is yes you can.

So, in some implementations of regulators is limiting the output of the limiting functions and stabilizing functions are in fact added at the control signal level, the control signal which is sent to the excitation power apparatus. That is to the control rectifier of the power apparatus, excitation power apparatus. So, you can have this kind of alternative structure where you are having this summing junction here for the limiting and **it is** the stabilizing function. Now, before now the main important point here is, now what is the structure or mathematical function which are typically use in the controller block which is shown here and also what are the mathematical functions which I use to implement this limiters in stabilizer.

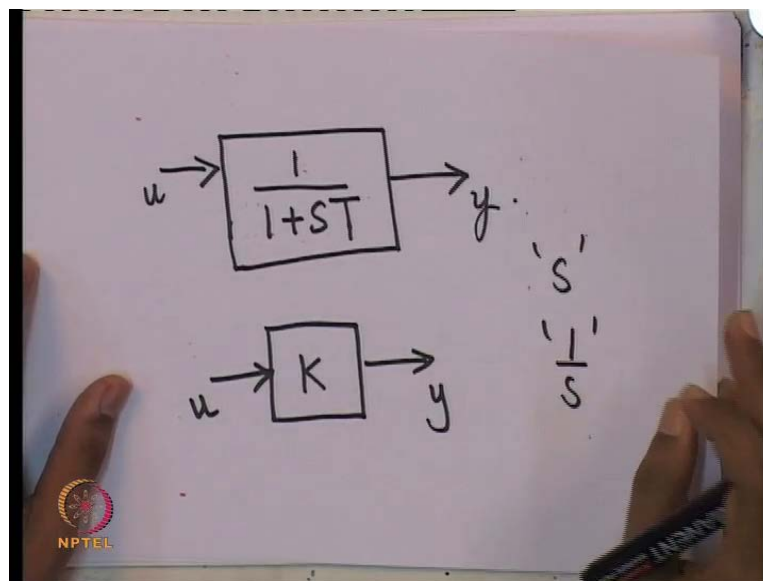
Now, to understand this, we will actually have to understand what are known this Transfer function blocks. Well, you do not have to understand this in terms of what are known as Transfer function blocks. But, most of the excitation system control system representation which are available in the literature comes in the form of block diagram. So, you should be able to write down the dynamical equation of the differential equation or the mathematical functions which really correspond to this block diagrams. So, mainly there are what are known as Transfer function block diagram. In fact I have already shown you a transmission block diagram. You call this **this** IEEE model of brushless excitation system is in fact a Transfer function block diagram. You see this S function here; well this S function is in fact the Laplace variable. We have use of course, we have without actually explicitly defining these we have already used this summation blocks. This is the product block.

So, these are what are known as typical block diagrammatic you know, components which are use in trying to model an excitation system. Of course, **this trans** this block diagram here which uses Laplace variable S in fact, one upon S is what is known as is

essentially an integrator in integration function. So, this one upon  $S$  here represents integration function. Now this of course, was the model of the power apparatus. So what we did was roughly model the excitation alternator by a differential equation and then converted it into block diagrammatic representation of this kind. We now move on to the block diagrammatic representation not of the power apparatus but, of the control system.

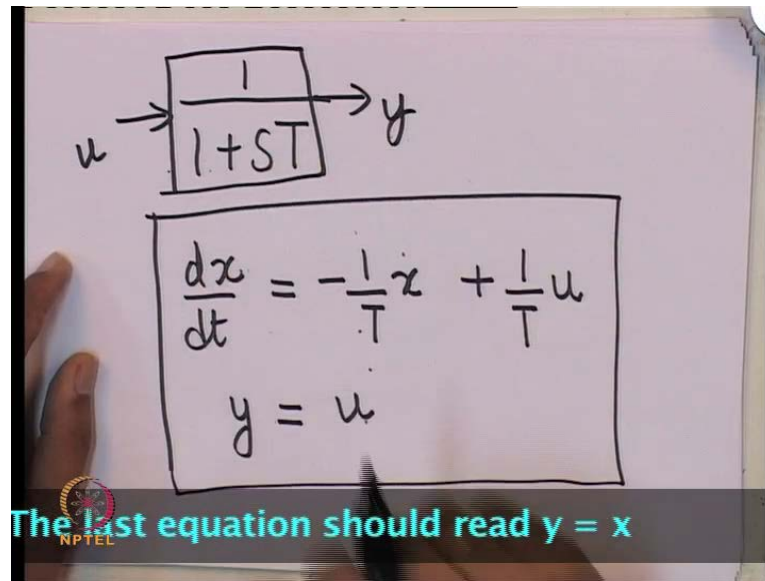
Now, control system as something which we will be implementing. Now what should be the structures of any control system? Now, it is usually made up of gain blocks, integration blocks, summation blocks and so on and compactly written down in terms of input output relationship or what are known as Transfer function relationships. So, let us just **just** take minor diversion, we need to build up this kind of expedites and try to understand some basic Transfer function which you will see.

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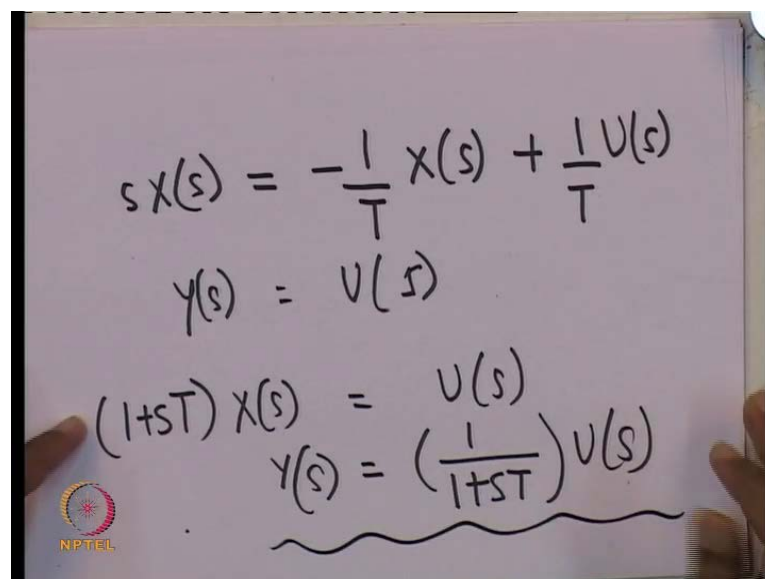
For example, consider the block  $1$  upon  $1$  plus  $S$   $T$ . This is the Transfer function block. In fact the simplest block one can have is of course, gain block. A gain block is a simple simply multiplies  $u$  by  $K$  to get  $y$ . A Transfer function block on the other hand means something more. In fact in a Transfer function block inherently or you know implicitly there is some numerical or actual implementation, integration being there. So, this  $S$  in fact is a Laplace variable.  $1$  over  $S$  represents an integration function and  $S$  of course, is a derivative function.

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So, what is  $1$  upon  $1$  plus  $S T$  derived from? In fact if you look at the dynamical equations; if you look at these dynamical equations, **so** I have written them in, in fact this dynamical represents the dynamical model using differential equations is actually how this as to be interpreted.  $y$  is well,  $y$  this mean this? It is not very difficult to see. Suppose, I do replace  $d x$  by  $d t$  here by it is appropriate last Laplace transform value what you will get is, if applied Laplace transform on both sides of the equation; you will have  $S$  into  $X$   $S$  is equal to minus  $1$  upon  $T$   $X$  of  $S$  plus  $1$  upon  $t$   $u$  of  $S$  and  $y$  of  $S$  is equal to  $u$  of  $s$ .

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So, what you will have here is, you get on to this side you will have  $X$  of  $S$  into  $1$  plus  $S$   $T$  is equal to  $u$  of  $S$  and as a result of it, of these equations you will have  $y$  of  $S$  is equal to  $1$  upon  $1$  plus  $S$   $T$  into  $u$  of  $S$ . So, that is why this differential equation after **going** doing Laplace transform the transformation of this differential equation, you get effectively this input output relationship. In fact input output relationships have been discussed in the past. In fact, **our** when we talk about the modeling of synchronous machine, when we actually core related what we get from measurement for frequency response obtained from measurement, we did use a Transfer function representation of the synchronous machine. There in also, the basic differential model, differential equation model of the synchronous machine was reduced to input output relationship in by using this laplace transform variable  $S$ . So, remember that  $1$  upon plus  $S$   $T$  represents this Transfer function. Of course, one should be a bit careful and precise well there is no one to one mapping between this and this. In the sense that, different model of, you can have different state space or different differential equation model to get this input output relationship.

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$$\frac{dx}{dt} = -\frac{1}{T}x + u$$

$$y = \frac{1}{T}u.$$

$$SX(s) = -\frac{1}{T}X(s) + U(s)$$

$$Y(s) = \frac{1}{T}U(s)$$

For example,  $\frac{dx}{dt}$  is equal to minus  $1$  upon  $T$   $X$  plus  $u$  and then  $y$  is equal to  $1$  over  $T$  of  $u$  will also give you the same Transfer function. Can we verify that? You will get  $S$  of  $X$  is equal to minus  $1$  upon  $T$   $X$  of  $S$  plus  $u$  of  $S$   $y$  of  $S$  is equal to  $1$  over  $T$   $u$  of  $S$ .

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$$\begin{aligned}(1+sT)X(s) &= T U(s) \\ Y(s) &= \frac{1}{T} U(s) \\ Y(s) &= \frac{1}{1+sT} U(s) \\ \left\{ \frac{(1+sT)}{T} \right\} Y(s) &= U(s)\end{aligned}$$

So, you will have, so what you will get eventually is, 1 plus S T of X of S is equal to t times u of S y of S is equal to 1 upon t u of S. So, from these equations what we can get effectively is, y of S is equal to 1 over 1 plus S T u of S. Is that okay? Y of S is equal to 1 plus S T upon t into u of s.

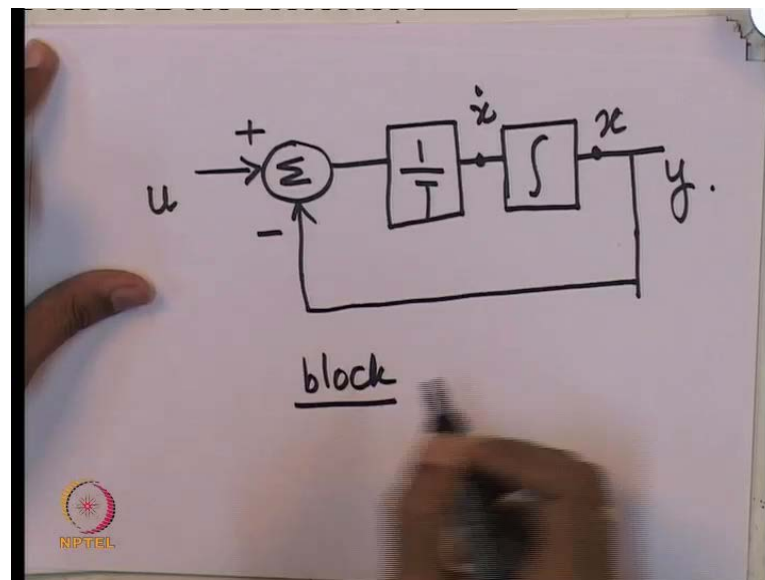
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$$\begin{aligned}\frac{U(s)}{T} &= Y(s) \\ &= \cancel{\phi}\end{aligned}$$

So, you will have S equal to **I am sorry this one** So, u of S by T is equal to y of S is equal to 1. **you** We will just redo this. So, what you will have here is, y of S there was one small error which I did in our previous, this should have been X. This should have been

X. So, please note this error. So, this should be X of S. In that case of course, you will get this particular relationship and just to recall what I doing just now; I will just trying to show that this is an alternative representation of the same Transfer function. So, what you will have is, it should be X of S, it should be X of S and as a result of this we will get, so what I have wish to say here is, this is one possible representation of the Transfer function. But, this is another representation of transformation. So there is no unique way of representing Transfer function by state space. So, this is 1 example of that but, usually in this particular course, I will just take this particular representation. It is very convenient of 1 upon 1 plus S T. In fact, I can split up this. If you look at this particular differential equation this is one possible representation of this Transfer function.

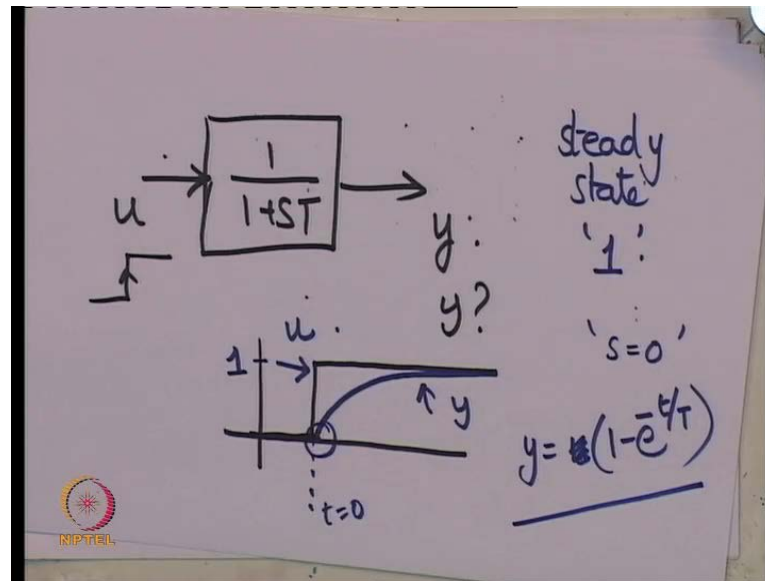
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So if you look at this differential equation; I can rewrite it as a kind of integrator. This is  $u$  and this is  $y$ . This is  $x$ ,  $x$  is equal to  $y$ . This is  $\dot{x}$ . If you integrate  $\dot{x}$  you get  $x$  dot is equal to  $1/T u - x$ . So, this is what exactly this says; the  $\dot{x}$  that is  $dx/dt$  is equal to  $u - x$  divided by  $T$ .

So, this is what effectively is block diagrammatic expansion you can say of this Transfer function. So if you see this, remember this.

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Now, see this is what we have drawn. So, right now is actually quite interesting thing the Transfer function itself. That is why I mean 1 upon 1 plus S T is an interesting Transfer function if you look at some simple things it does. If you got  $u$  and  $y$  here; if I give a step change in  $u$ , what will  $y$  be? In fact you can prove this by actually solving the state space equation and prove this. But, if you use step and  $y$  is going to be something like this; this is  $y$  and this is  $u$ , so this step change have some interesting features you see that at the incident of the step. The Transfer function as an output 0. It does not change. Suppose the initial condition associated with this state **state**  $x$  is 0, then if I give step change in the Transfer function will find that  $x$  remains 0 and as a result  $y$  also remains 0. So, there is an instantaneous change in  $y$  given in a step changing  $u$ . So  $x$  is  $y$  just remains where it is. But, as time passes you will find it  $y$  tends to be become equal to  $u$ . So, the steady state gain of this Transfer function is one that is  $y$  becomes equal to  $u$ .

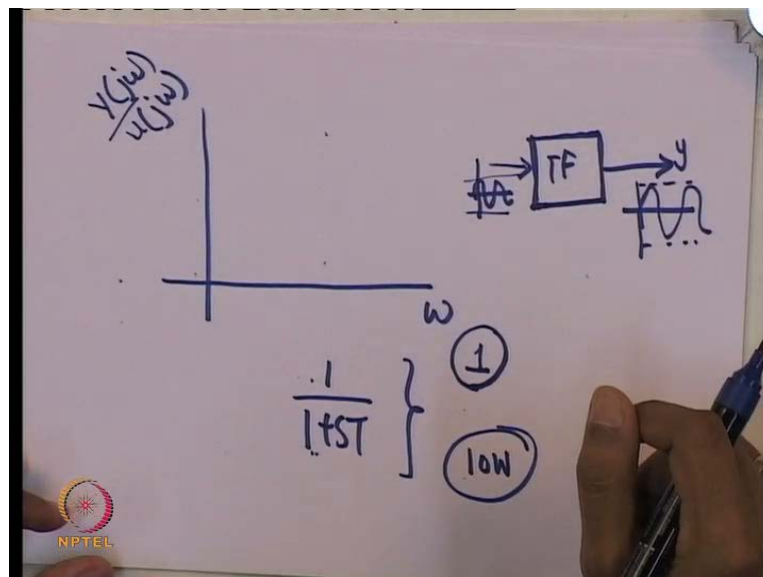
Now of course, the same thing can be got. The same you know inference can be obtained directly by setting  $S$  is equal to 0 in this Transfer function. So, shortcut method of finding the steady state gain of any Transfer function is to put  $S$  is equal to 0 in the Transfer function. So, if you give step input the steady state value or the steady state gain for a step input can be obtained by simply putting  $S$  is equal to 0. So, you can say that this transient gain or the high frequency gain of this Transfer function is actually 0. It does not instantaneously respond but, in steady state it takes on the value. Now, what is the nature of this curve? In fact you will find at  $y$  is equal to  $u$  1 minus  $e$  rise to  $t$  by



capital T. So, if this step occurs at time  $t$  is equal to 0, so this is the step change of magnitude one occurring at time  $t$  is equal to 0, the response is like this.

The time constant  $t$  appears here in the response. So, it is good idea whenever you come across any Transfer function block, you should be able to correlate it with its time response under various test inputs. If you look at another interesting and important way of looking at Transfer function, are what interpreting how they behave; is to look at the transient behavior. So, the transient behavior or what you called frequency response, we actually see in the transient behavior for a step change.

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The frequency response of the Transfer function can also be obtained. That is you vary the frequency and see the gain. That is  $y$  upon  $u$  in steady state. So what do I mean by a frequency response of any Transfer function? You take this block, feed by inputs of varying frequency. In steady state if it is a Transfer function, a normally a Transfer function in steady state would at the output if it is actually, a stable if it is stable system that is, it is associated with the state space which Eigen values on the left hand side of the imaginary axis; then what you will see is the output you will get also a sign wave. But, it **it** is magnitude would have change and it could have also got some **pay** phase shift. It could have got some; it can have some **pay** phase shift.

So, you take any Transfer function. This is how you would measure the steady state frequency response. So, whenever people say frequency response it is implicit that they

talking of the response under sinusoidal steady state conditions for input sinusoids, input  $u$  which are at different frequency. So what do you do? This experiment which I shown you on the sheet here, at a particular frequency record the gain that is, how much is the amplitude with respect to the input amplitude. So, that is the gain. And also measure the **pay** phase shift, so the phase shift of this. There will be some **pay** phase shift between this signal and that. So, you measure the **pay** phase shift.

So, that is what will give what is known as a frequency response. No more frequency response implicitly the steady state frequency response of this system. So, you will get a gain which is a function of frequency that is one important thing. So, the gain is function of frequency, phase also is a function of frequency. It turns out for  $1$  upon  $1 + sT$  the steady state gain is one. Steady state frequency, what you call the  $0$  frequency when I say steady state, actually it means  $0$  frequency. The  $0$  frequency gain is one where as a high frequency gain is low. How do I know this phase shift and the gain? Well, you can do every simply using a Transfer function.

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$$\frac{1}{1+sT} \quad s=j\omega$$

$$\frac{Y(j\omega)}{U(j\omega)} = \left( \frac{1}{1+j\omega T} \right) \quad \text{-ve shift}$$

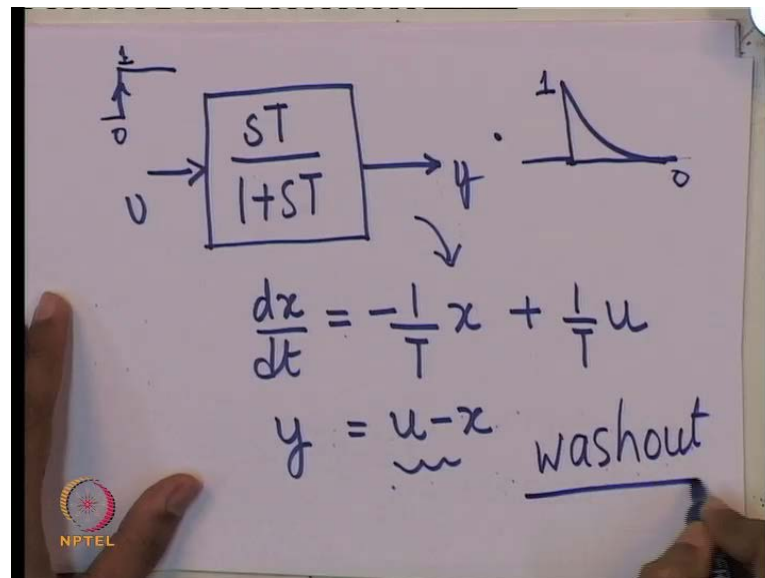
$$\begin{array}{l} 1 \leftarrow \omega=0 \rightarrow 0^\circ \\ 0 \leftarrow T\omega \gg 1 \rightarrow -90^\circ \end{array}$$

If I give you Transfer function, if I want to know what is it is gain and **pay** phase shift? All I need to do is put  $S$  is equal to  $j$  omega. Omega corresponding to that frequency which you are interested in getting the frequency response and you can get the answer. So, this something of course, you have done before. But, we have just revising that. So, this is the how it is represented. So, this is called a frequency response rather the

Transfer function evaluated at this frequency. So, what is the magnitude gain? Well, that can be obtained simple by obtain the magnitude of this complex number.

So, that is what will give you the gain and the **pay** phase shift is again obtained by looking under argument of this complex number which results. So, what one thing you will notice is that, this will always give negative **pay** phase shift. The Transfer function as a negative **pay** phase shift. In fact at omega is equal to 0. The **pay** phase shift is 0. **pay** Phase shift is 0 degrees at omega tending to a very large value. A large value in this **contacts** context means at omega T is much greater than 1. In such case you will find that omega T omega T or T omega much greater than 1 you will find I the **pay** phase shift is minus 90 degrees of this Transfer function. Similarly, at omega is equal to 0 the gain is 1 but, high frequency gain is 0 which correlates well with the step response. Just at insensitive step is given you can say that the input signal in some sense is kind of changing very fast or it is got higher rate of change or it as got very high frequency component. The Transfer function does not respond very well or it as very low gain to high frequency.

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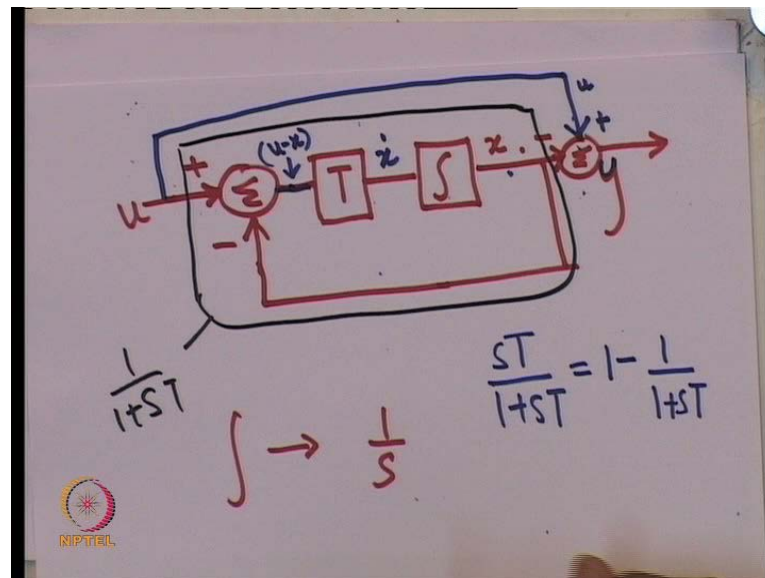


We will move on to one more Transfer function. That is, it is a kind of important Transfer function just as the previous one was. This is S T upon 1 plus S T. This is an interesting Transfer function again. You can verify that a steady state space

representation of this is  $y$  is equal to  $u$  minus  $x$ . So, this is the change. So, this is a steady state representation of this system or rather states space representation of this system.

So if you look at this, can you guess what is going to be the response for a step change in input? Well you can actually work it out but, it is not difficult to see that first thing is that the high frequency gain of this is going to be one. If you look at the frequency response with  $\omega$  large the gain of this is going to be one. For low frequencies, the gain is given going to be equal to 0. So, in fact will not work it out but, a step change, for a step change the response of this  $y$  this going to be like this. The final value is going to be 0. Initial value is going to be equal to whatever this value. Suppose, this is 1 then this is also one. So, this is known also as washout Transfer function. It is called a washout Transfer function. **the [re] the characteristic is that** It has got a characteristic of in a sense a high pass filter. It prevents slow changes from going through but, it allows fast changes to go through. Its steady state gain is 0, its high frequency gain is one and this is the state space representation or from state space representation one can easily get block diagrammatic representation.

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So it is quite simple. So, if you look at this, it is simply what it was before. The only thing is, this is an integrator. You can represent the integrator in Laplace domain is represented by  $1/s$ . So, this is  $u$  this is  $y$  and **oh** this is not  $y$ .  $Y$  in this cases **this** is

subtracted  $u - x$ . This is  $x$  does not additional two  $y$  is equal to  $u - X$  this is  $y$   
this is  $u$  this is  $x$  this is  $x$  dot.

And this is  $u - x$ . So, this is known as washout block. Remember this extra thing here completely changes what you get. In fact, you can easily see that  $S T$  upon  $1 + S T$  is equal to  $1 - 1$  upon  $1 + S T$ . So, it is not **it is not** surprising you have got block diagram of this kind in which, in fact this is nothing but, this whole thing is nothing but,  $1$  upon  $1 + S T$  either  $1 - 1$  upon  $1 + S T$ .

So this is as far as one of another of the Transfer function it goes. In the next class, will look at a few more Transfer functions, talk about what are known as what is  $pI$ , or propositional integral regulator and propositional regulator. We look at as I mentioned sometime back the lead lack blocks. These are also special Transfer functions and we will also look at the effect of limiters. There two kinds of limiters. So far I have implicitly assumed you know what a limiter is. But, you can actually have some variations of limiters themselves and they eventually affect how the system response is going to be. Typically, a control system will be made out of either plane gain blocks in addition to summers, multipliers and so on and Transfer function.

When I say Transfer function blocks; the Transfer function blocks themselves represent linear system, linear state space. Of course, if you put limiters in the Transfer function, in that case the limiters in transfer function blocks which are shown in fact in certain limiter it no longer a linear system. But, if you just remove that little bit complicity, Transfer function implicitly or obtained for linear systems or linearise systems, non-linear system. They linearise around an equilibrium point. So, using these blocks or these mathematical functions or these you can say mathematical dynamical functions, we can actually try to design a control system which meets our objectives. So, I am not, I have just discussed some of the control system which is used. Of course, rather the Transfer function block diagrams which I used, you can use this block diagram selectively to get the dynamical as well as steady state response that we desired. So, that is what we will do in the next class.