

Power System Dynamics and Control
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Lecture No # 23
Synchronization of a Synchronous Machine

We started off the past 10 lecture or so, with the reasonably detailed model of a synchronous machine. In fact, we considered several windings on the rotor I mean, in fact one can imagine that the simplest model, which could give **most of the correct information** most correct information of how a synchronous machine were especially in steady state etcetera. Can be got without considering damper windings because, eventually the current goes down to 0. Of course, during transient condition as well, one it would be useful to work out some simple models of synchronous machine.

So, that well there are three reasons why one can look for simplified models. One of course, is that the data for detailed models may not available, we may not be get all the standard parameter some times in our studies, these this kind of data is not available. On the other hand, one would like to have all this data and use a detailed model. In some cases of course, you will find that you can model a generator by a **simple** more simple model than what we have considered. For example, hydro turbines it turns out can be represented by one less damper winding on the Q axis.

So, one can actually work with a simple simplified model, which is suitable for a hydro turbine. Of course, if you one wants to do a theoretical analysis and try to get information about phenomena without worrying too much about you know, getting a great deal of accuracy. In that case, one may wish to work with lower order models. They give; they seem to give better insight right, now of course, if we look at our synchronous machine model, it contains a large number of equations. So, you have got in fact, if you consider the 0 sequence equations as well you will have 7 flux equation, 7 differential equations of flux in addition to the electro mechanical equations. And as we shall, we shall see later in this course, when you consider an integrated power systems, you have got lots of synchronous machines and so on.

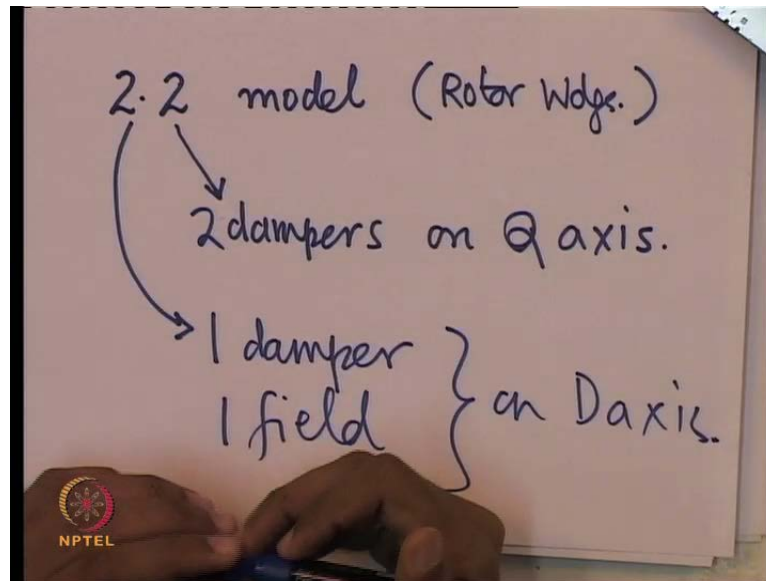
So, what you will find is that, the number of equations becomes quite large and one may not able to do you know any kind of theoretical or a kind of insightful analysis into a

synchronous machine without invoking fairly detailed and sophisticated numerical tools. So, I think it is worthwhile to look at somewhat simplified synchronous machine models, which ignore one or the other damper winding. In fact, actually we have already worked out one simplification, which is okay; When we study slow transients that is you know replacing $d \psi_d$ by $d \psi$ and $d \psi_q$ by $d \psi$ is equal to 0 and converting the corresponding equations into algebraic equations that was one simplification, which we did which was justifiable in case one used one **one** was really interested in only slow transients.

So, that is one thing which you have already done. What we will do now **in this** in today's lecture **come with** come up with more simple models in which, you will be neglecting one or the other damper winding or even we **we** may neglect all the damper windings and come up with a simple model. Now, one interesting thing which I want to do by the end of this lecture is, using the you know the model which we have derived using two damper windings of the Q axis and one on the D axis along with the field winding. I want to come to a point, where I can show you to that can get what is known as the classical model of a synchronous machine, which we have used to derive or rather understand some important phenomena right in the beginning of the course. So, what are the **approximation involved** approximations involved in getting in such a simplified model of a synchronous machine. So, that is something we will try to **you know try to** understand by the end of this **by this** lecture.

Now so, today's lecture is titled simplified synchronous machine models, just a one small point about the kind of terminology we will be using.

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We have derived, what is known as or we shall call as the 2.2 model of a synchronous machine. 2.2 model of a synchronous machine involves two damper windings on the Q axis, this of course, refers to the rotor windings and one damper winding and one field winding on the D axis. This is a fairly detailed model, it is a quite a respectable model to use especially for stream turbine driven generators, round rotor generators normally we would use such a full blown model.

Now, what if we want to get a simplified model in fact, before I go ahead with trying to reduce a number of rotor windings and getting simplified models with lesser number of rotor windings. Let us look at one of the approximations, we have already made just for the sake of the revision.


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q-axis Model - per-unit

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)$$

$$\psi_q = x_q'' i_q + \frac{(x_q' - x_q'')}{x_q'} \psi_K + \frac{(x_q - x_q') x_q''}{x_q x_q'} \psi_G$$

$$\frac{d\psi_q}{dt} = \omega \psi_d - \omega_B R a^i q - \omega_B v q$$


So, let us just first look at the Q axis per unit model. So, we will just look at what we have so far in the 2.2 model. We have got different two differential equations corresponding to the two damper windings in the Q axis. We have an algebraic relationship relating psi q to the current and psi K and psi G. And we have of course, a differential equation in psi q, d psi q by d t.


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d-axis Model - in pu (assuming $T_{dc}'' = T_d''$)

$$\frac{d\psi_H}{dt} = \frac{1}{T_d''}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T_d'}(-\psi_F + \psi_d + \frac{x_d'}{(x_d - x_d')} E_{fd})$$

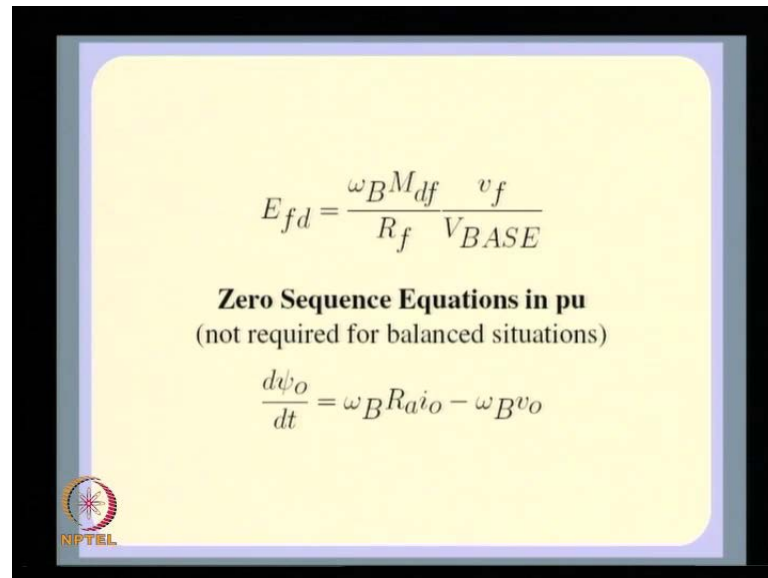
$$\psi_d = x_d'' i_d + \frac{(x_d' - x_d'')}{x_d'} \psi_H + \frac{(x_d - x_d') x_d''}{x_d x_d'} \psi_F$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - \omega_B R a^i d - \omega_B v d$$


Now, in the D axis with an assumption that T dc double dash is equal to T d double dash. We have similar model but, of course, one of the windings here, is you know is the field

winding. The field winding is of course, affected by what voltage you apply to the field. So, that **field winding** the effect to the field winding in **in** these per unit equations is captured by E_{fd} .


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$$E_{fd} = \frac{\omega_B M_{df} v_f}{R_f V_{BASE}}$$

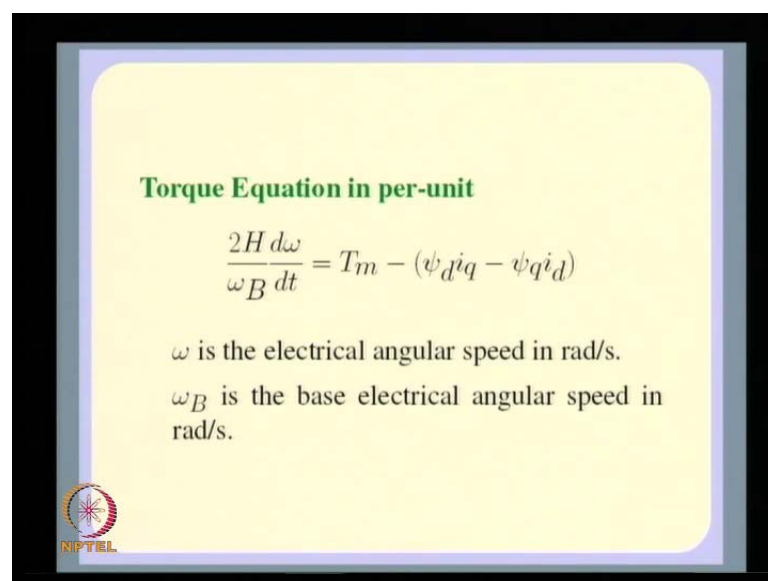
Zero Sequence Equations in pu
 (not required for balanced situations)

$$\frac{d\psi_0}{dt} = \omega_B R_{a0} i_0 - \omega_B v_0$$



So, if you look at what E_{fd} is it is of course, related to the field voltage, which is applied at the field winding. Of course, we have 0 **sequence** sequence equations as well, which you **you** may require to use in case, you are doing unbalance analysis.


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Torque Equation in per-unit

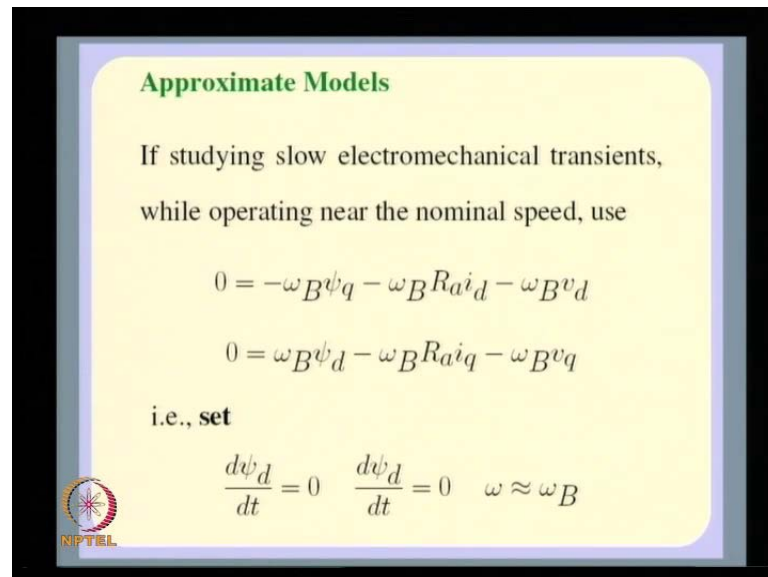
$$\frac{2H}{\omega_B} \frac{d\omega}{dt} = T_m - (\psi_d^i i_q - \psi_q^i i_d)$$

ω is the electrical angular speed in rad/s.
 ω_B is the base electrical angular speed in rad/s.



And we have the torque equation in per unit, that is relating the rate of change of the speed of the synchronous machine with a electromagnetic torque, which is a function of course, of the fluxes and current. So, this is where we are, this is a kind of Mount Everest of synchronous machine modeling. Now, we look we slide down **mount** Mount Everest.

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Approximate Models

If studying slow electromechanical transients, while operating near the nominal speed, use

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

i.e., set

$$\frac{d\psi_d}{dt} = 0 \quad \frac{d\psi_q}{dt} = 0 \quad \omega \approx \omega_B$$

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And we look at approximate models. One of the models, which you are already kind of beaten to death or other one approximation, which we have considered fairly in the past 2 or 3 lectures was replacing the $d\psi_d$ by $d\psi_q$ differential equation by algebraic equations. This was done simply by setting $d\psi_d$ by $d\psi_q$ equal to 0 and $d\psi_q$ by $d\psi_d$ equal to 0. So, in that equation, we just **just** replace $d\psi_d$ by $d\psi_q$ equal to 0 and $d\psi_q$ by $d\psi_d$ is equal to 0. So, this is an approximation, which is valid in case we are talking of slow electromechanical transient, slow electromechanical transient, the swings which you were discussing in some of the lectures.

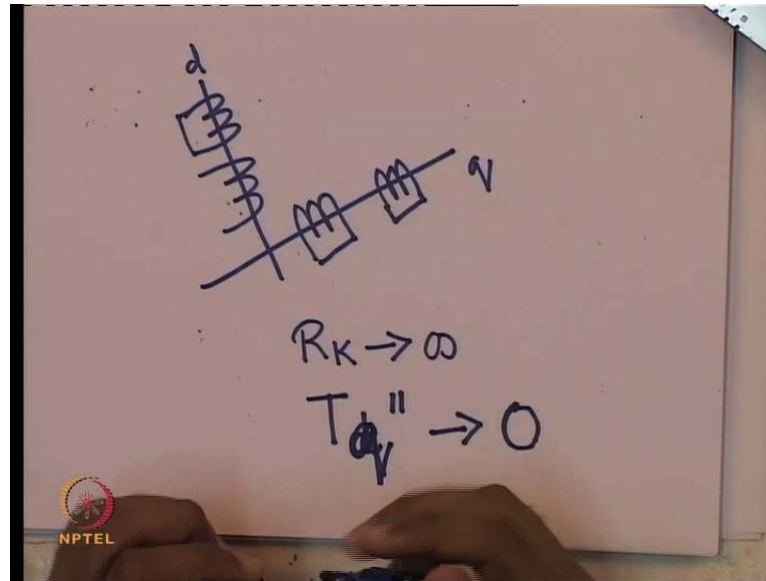
Now, while operating near the nominal speed, we could if we are all are electromechanical transients and in fact are if are taking place, if we are near the nominal speed, we could in this particular equation replace ω by ω_B . So, that becomes kind of a constant multiplication factor. Now, this kind of approximate model can be used, if you are of course, near the nominal speed, you cannot use this approximation of ω approximately being equal to ω_B , if you are trying to simulate or you know **you know** understand a synchronous machine right from the process of synchronization.

That is starting rolling **rolling** the generator in getting it near the synchronous speed. When this speed is not too close to the synchronous speed, we cannot make that approximation of ω , being approximately equal to ω_b in this particular equation.

But, as I mentioned, if we are talking of transients in which, you are not going to deviate too much from 50 hertz or 60 hertz whatever your nominal speed may be. Then of course, this equation is valid, if one is studying slow transients. So, this is one of the approximations that we have made and we saw that it did not make much of a difference during our short circuit study, in fact it made some difference alright. But, it did not affect the modes associated with this slow transient. So, that was the basic effect **of this** of this approximation, of course. Do not make this approximation, if you are going to study fast transients say, which **which** occur in time scales of one or two cycles of course, that would not be correct, it would not be right to set $\frac{d\psi_d}{dt}$ and $\frac{d\psi_q}{dt}$ equal to 0 in that case

So, this is one approximate model, which we have done. We move on to making approximations not in the stator coil ψ_d and ψ_q of course, are stator coils, what we will do is making **make** a few approximations as far as the rotor coils are concerned. So, the simplest thing we can do is, one of the damper windings we the effect of it is neglected. So, **for a** for doing that what we need to do is, consider one less damper winding in our original you know derivation of this synchronous machine model. But, rather than of course, redoing the whole derivation again, a simple thing which can be done is of course, open **the** that **that** particular damper windings.

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So, suppose you have of course, got the d and q windings. So, in the q axis rotor windings are of course, two the G and the K winding and there is a field winding and a damper winding on the d axis. What I need to do is, in this particular model in order to simplify it is, get rid of this effect of this winding. So, what you can do of course, is set the resistance of this winding. For example, this winding R K, if the winding R K I set the resistance to be a very large value in that case, it is as good as opening the damper winding in which case of course, no current will flow in the damper winding, that particular damper winding and its effect is in would get nullified.

Now, important point which you should note is that, when I said R K tending to infinity, what effectively happens to the time constant that is the something I leave as an exercise to you. But, you can show that in such a case, $T_{d''}$ tends to 0. So, if R K tends to infinity $T_{d''}$ tends to 0. How do I know that, well I know the equations relating the **time constant** time constants in this standard parameter model, standard parameters with the original inductances and resistances of the windings? Recall this, we have done it, somewhere in the between the 10th and the 20th lecture, where we were modeling a synchronous machines.

Now, **so**, setting R K tending to infinity would mean that, you are setting $T_{d''}$ double dash tending to 0. Now, if you do that, what happens?

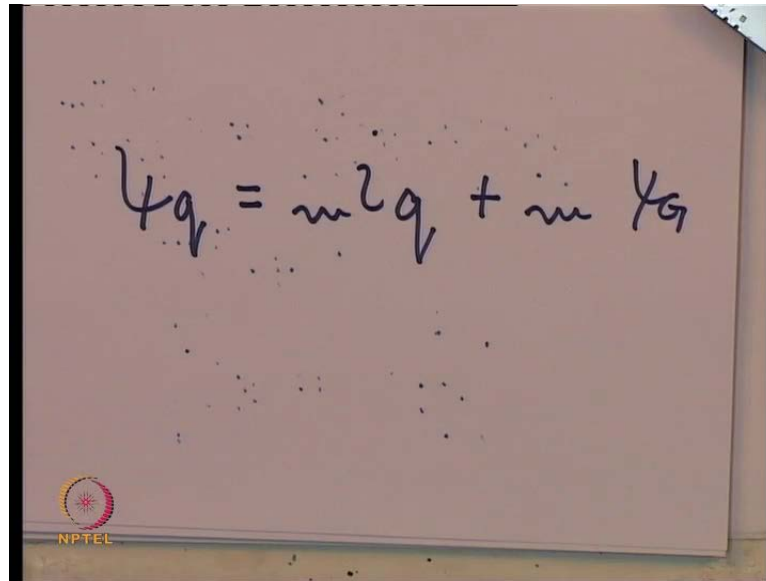
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$$T_q'' \frac{d\psi_k}{dt} = [-\psi_k + \psi_D]$$
$$T_q'' \rightarrow 0$$
$$\psi_k = \psi_D$$

So, if you look at the basic equation, which is there, you can write this in fact as T_q'' double dash is equal to minus ψ_k plus ψ_D . So, this is the equation, which is there of the flux **equation** of this particular winding and if, T_q'' double dash tends to 0. You can roughly say that, this term here becomes 0 and as a result of it ψ_k becomes equal to ψ_D . So, from this, you get this. Now, if ψ_k is equal to ψ_D , you can replace ψ_k by ψ_D in the algebraic relationship, which relates ψ_D sorry ψ_q , i_q , ψ_G and ψ_k .

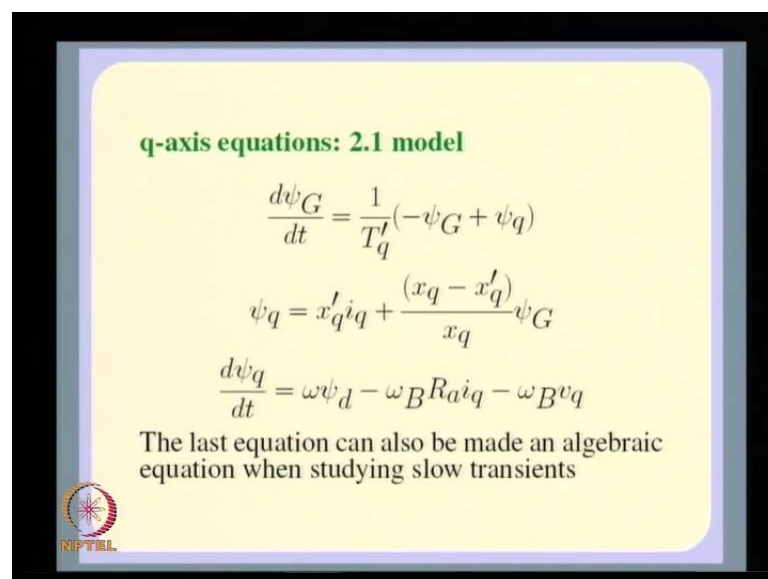
So, do you recall that equation, we will just have a look at it right away. So, what I am trying to say is of course, that you get ψ_k is equal to I **am sorry yeah** it should be ψ_k equals to ψ_q , this some small error which I have made while writing this **this** should have been ψ_q , this should have been ψ_q **yeah**. So, getting back to this slide which I have, you get ψ_k is equal to ψ_q . So, you can substitute ψ_k by ψ_q here in this algebraic equation, the third equation is an algebraic equation and then, if you do that you can write ψ_q as a function of **is a function of** i_q something into i_q plus something into ψ_G .

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$$\psi_q = m_2 \psi_q + m \psi_G$$

So, the effect of ψ_k get substitute in ψ_q . So, what are these coefficients here and here, you can work it out. I will just write down what you get eventually, what you will get eventually is if you look at the slide, your equations of the 2.1 model 2.1 because, now we have got just one damper winding on the q axis will be given by one differential equation corresponding to ψ_G one of course, is corresponding to ψ_q and the algebraic equation relating ψ_q i q and ψ_G becomes much simpler. And you will notice that, x_q double dash no longer appears in this equation.

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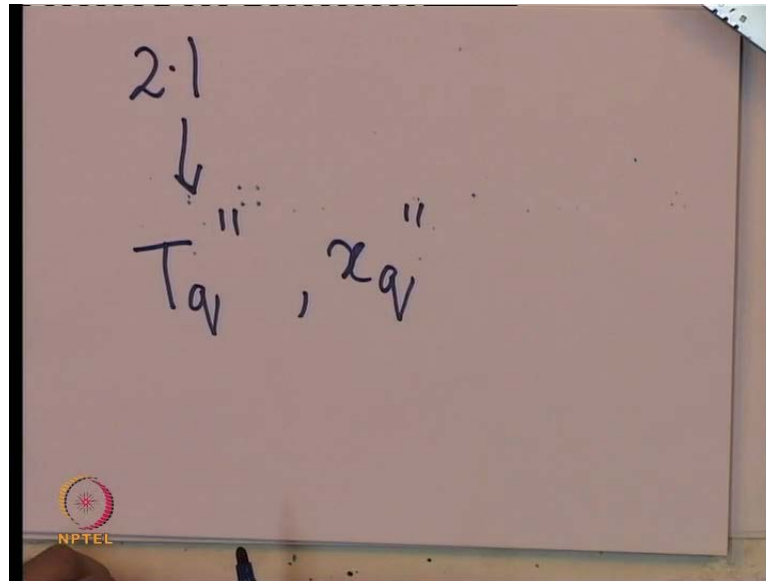


q-axis equations: 2.1 model

$$\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)$$
$$\psi_q = x_q' i_q + \frac{(x_q - x_q')}{x_q} \psi_G$$
$$\frac{d\psi_q}{dt} = \omega \psi_d - \omega_B R_{a'q} i_q - \omega_B v_{q'}$$

The last equation can also be made an algebraic equation when studying slow transients

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So, for this particular model, 2.1 model, you do not require one set of time constants and one set of one basically, you will not require one time constant and one reactants. So, it does not figure in the equations any longer.

If you look at the slides again, the last equation of course, is still the differential equation in the flux ψ_q remember that, you can make an approximation that you can replace $d\psi_q/dt$ equal to 0 and get back to the approximation, which you mentioned right at the **beginning of this course** beginnings of this lecture. That would be okay, if your studying slow transient. So, the last equation also could be simplified in case, you are talking about slow transients.

Now, one of the things which I should mention at this point is that the model 2.1 model, which I have just discussed so far is found to be adequate in the sense, that it can represent the transients, which occur in **hydro driven** hydro turbine driven synchronous generators. So, hydro turbine synchronous generators are often represented by 2.1 model and do not be surprised, if the data sheet of a hydro **hydro** turbine generator has one less time constant and one less reactants. So, that particular data will not be given, because it is adequate to model hydro turbine just by 2.1 model. So, this something you will ought to **keep in** keep in mind.

Now, an interesting observation here, which you may have noticed is that, if you look at the algebraic equation relating ψ_q and ψ_G and i_q of course, it does not contain x it does not contain x_q double dash, there is no need of having x_q double dash.

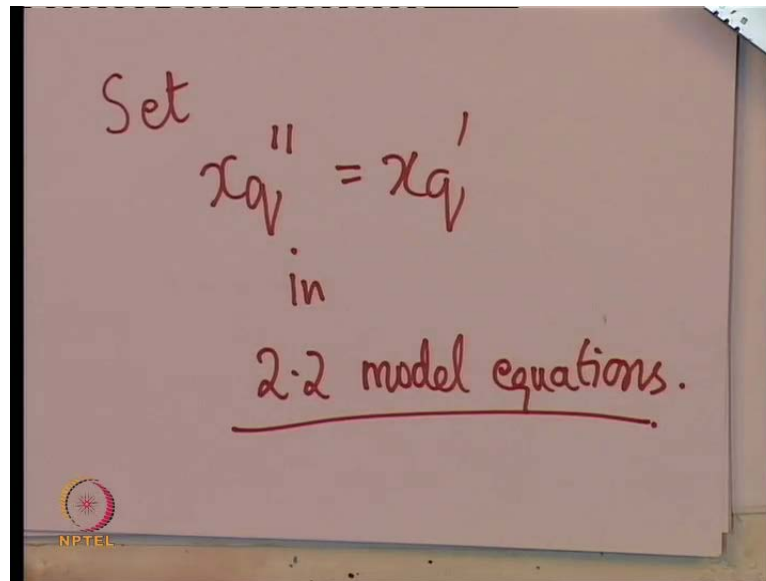
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The image shows a whiteboard with three equations written in black and red ink. The first equation is $\psi_q = \lambda_q' i_q + \frac{(\lambda_q - \lambda_q')}{\lambda_q} \psi_G$. The second equation is $\psi_q = \lambda_q'' i_q + \frac{(\lambda_q' - \lambda_q'')}{\lambda_q'} \psi_K$. The third equation is $+ \frac{(\lambda_q - \lambda_q')}{\lambda_q} \cdot \frac{\lambda_q''}{\lambda_q'} \psi_G$. There are red annotations: an arrow pointing from the first equation to the second, and the text $\lambda_q'' = \lambda_q'$ written in red.

But, if you look at the algebraic equation, so I will just write it down here, ψ_q is equal to x_q dash i_q plus x_q minus x_q dash upon x_q into ψ_G . If you at this particular equation and compare it with the algebraic equation, which we had for 2.2 model. So, if you look at this screen here, you will see that the equation which was there before, I will just write it a bit quickly out here well, it looks pretty complicated but, something you will probably able to suggest is that, if I had directly set x_q double dash is equal to x_q dash in this equation, in that case you would have directly got this equation.

Note that this whole **this** term would have disappeared, this is in fact dash. So this would be one. So, we would be really getting back to these equations. So, an interesting observation here is that, if I want to go from 2.2 model that is the original two dampers and the q axis model to one damper winding on the q axis, then what you can do actually is simply set **simply set simply set** x_q double dash is equal to x_q dash in the 2.2 model equations.

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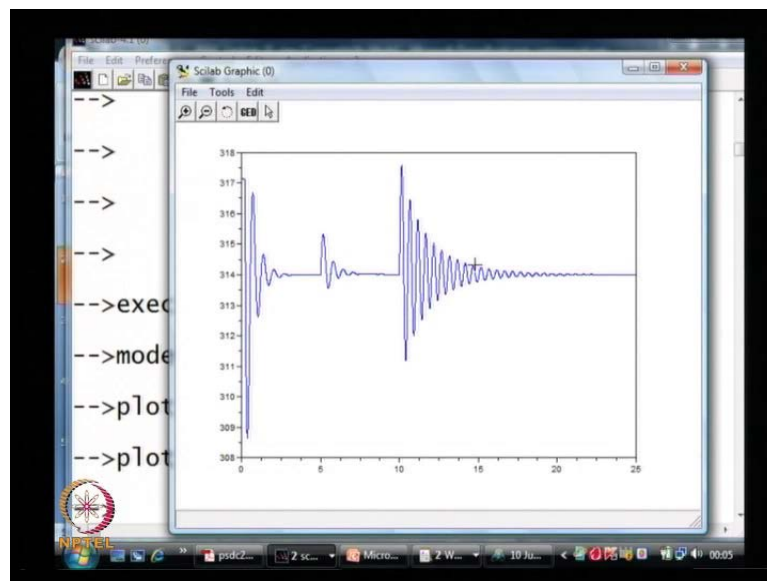


Now, why am I telling you to do this, well you may have already written a program or you know kind of a tool, which does Eigen value analysis or synchronization the simulation numerical integration of the differential equations in the 2.2 model. And somebody tells you suddenly well a particular generator has to model not by 2.2 model but, it has to model by 2.1 model. In that case you do not have to rewrite your program with lesser number of equations, you **you** would need to tinker with your program. Instead of doing that, you just set $x_{q''}$ is equal to $x_{q'}$ in the data. So, just look at these equations again, you will find that because of doing this, you will get rid of the effect of ψ_k of course, this equation would still be there. But, it could be kind of decoupled from every means; the effect of ψ_q would not be effectively seen by a synchronous generator stator windings at all.

So, there is no harm in, just setting $x_{q''}$ is $x_{q'}$ equal to $x_{q'}$ and removing the coupling, which ψ_k has with the rest of the equation. So, this is a pragmatic way of using the program or a tool, which **which** is programmed for 2.2 model **directly make it suitable for use** to directly make it suitable for use it 2.1 model, just set $x_{q''}$ is equal to $x_{q'}$. Of course, you need not do it, you can actually even do another thing that is reprogram everything for a lower order model, that is also possible. So, in case you do that you will have to use these equations, the lesser number of equations in your programming. So, this is what basically is you do when you neglect the effect of one damper on the q axis.

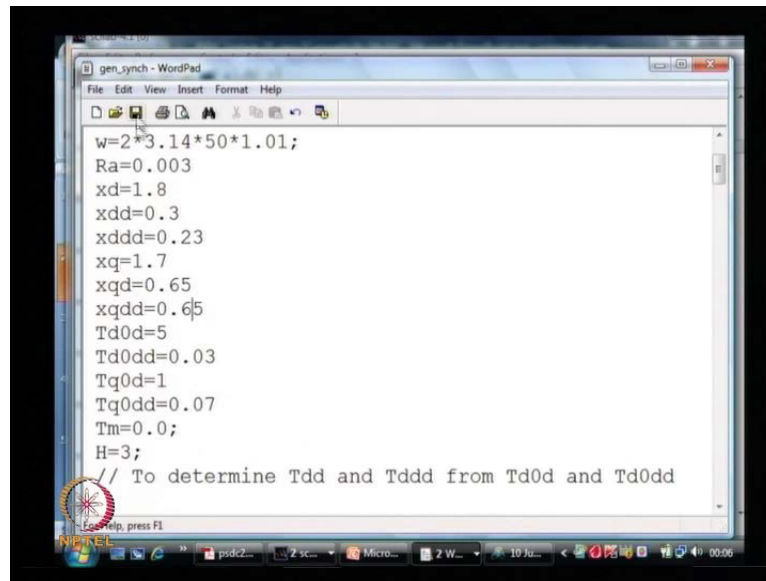
Now, before we go ahead let us just see what happens in **in** case, we try to simulate a system with a lower order model of a synchronous machine. So, what we will do is, take the data which we have been using for studying this synchronization transient, in fact the data is for a round rotor machine but, all the same we will still **still** use it for you know, just comparing what happens in case you take a simplified model. So, what I will do is, I programmed **programmed** the program for 2.2 model but, I will simply make this the simplification $x_q \text{ double dash is equal to } x_q \text{ dash}$ and effectively get 2.1 model. So, I will just do that.

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So, we will get back to our sci lab program and what I will do is of course, first of all show you the original speed transient under synchronization, we have been doing this in the past few lectures. So, I need not explain everything right from scratch, it is basically a transient which **which** shows the synchronization of a synchronous machine to a infinite bus, followed by torque and field voltage increases. So, this is increase in torque and field voltage. So, these are the transients, which you see. Now, in case you take this program.

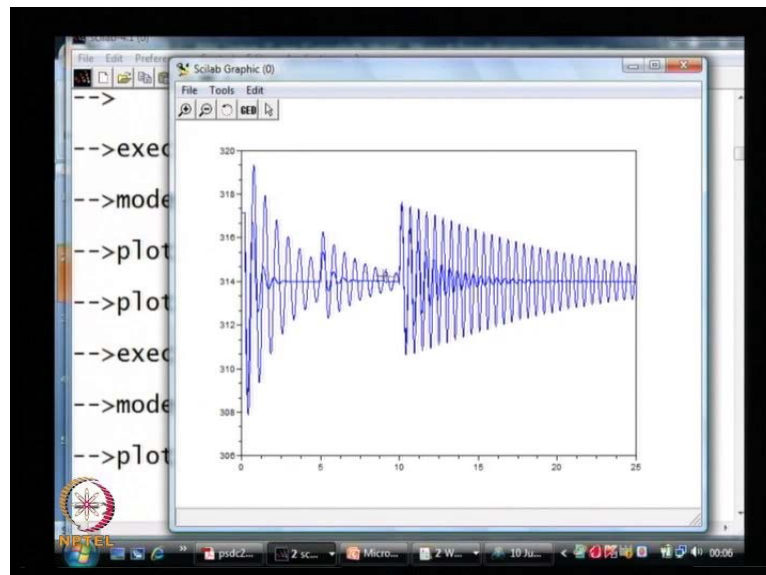
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```
gen_synch - WordPad
File Edit View Insert Format Help
w=2*3.14*50*1.01;
Ra=0.003
xd=1.8
xdd=0.3
xddd=0.23
xq=1.7
xqd=0.65
xqdd=0.65
Td0d=5
Td0dd=0.03
Tq0d=1
Tq0dd=0.07
Tm=0.0;
H=3;
// To determine Tdd and Tddd from Td0d and Td0dd
Help, press F1
```

And in that, set x_{qdd} which is here equal to x_{qd} , so, if you do that how does it affect. So, let us just try if we can do it of course, we have to hope and pray that. So, the program runs without any problem. It has just we have just change the data; we have not really changed anything in the program.

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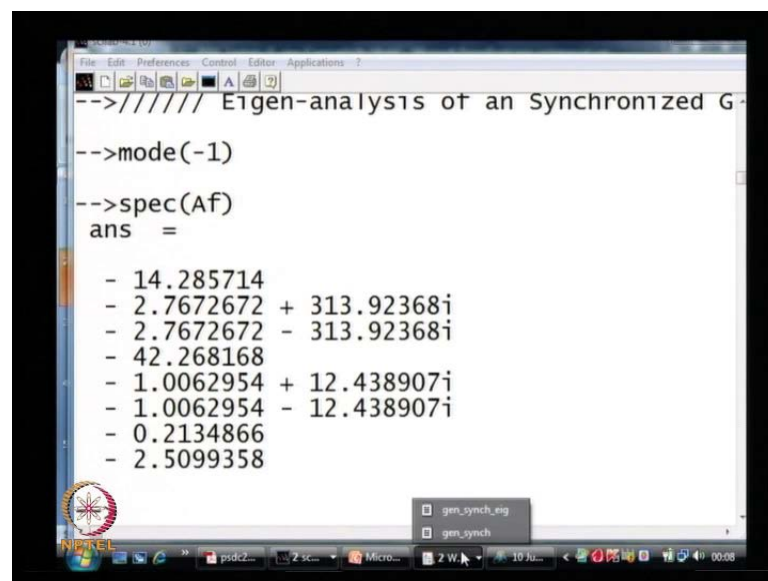


So, if I plot this now and look at what we get, well you notice something, well the original program, the damping was much higher, where is now in all the transients which are considered one thing you can notice is that the transient takes a longer time to die

down. In fact, in **in** a synchronous machine with a one damper winding less, it appears that the damping has reduced. This is not very surprising in fact, you know one may say that the damper winding derives its name in some sense from the damping effect that it has, so we actually reduce the amount of damping in that case, that is by removing one damper, it is not surprising that we get this larger time for decay of the transient.

So, this is something which is not at all surprising; this is effectively the response we will get the top one, which you see here. The one which is taking a long time to die is effectively the response without a damper winding, one of the damper windings. So, let us go back and also do the Eigen analysis. We will do exactly the same thing and do an Eigen analysis, Eigen value analysis using a linearized model, which we discussed in the previous class.

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```
-->//////// Eigen-analysis of an Synchronized G
-->mode(-1)
-->spec(Af)
ans =
- 14.285714
- 2.7672672 + 313.92368i
- 2.7672672 - 313.92368i
- 42.268168
- 1.0062954 + 12.438907i
- 1.0062954 - 12.438907i
- 0.2134866
- 2.5099358
```

Now, I will just run the program. So, I just take out the Eigen values and the Eigen values are these well not much can be understood from this unless we compare it with the Eigen values with the 2.2 model. So, what I just now showed you was the Eigen values with 2.1 model, I will rerun the program to see the Eigen values with 2.1 model.

So, what you notice here of course is that there is significant change in the real part of the Eigen value corresponding to the electromechanical oscillation, which is seen in the speed transient. So, this is consistent with what we have seen before in fact of course, we need to just check out the operating point, which we are talking of **yeah**. So, what we see

here is of course, that if I take 2.2 model, the real part of the Eigen value is 1.4 roughly whereas, the Eigen value here is one. So, you see that with 2.1 model, the damping of the swing mode is slightly lower. So, this is something we if **it seem** it seems to be consistent with what we see here, we see a slower rate of decay, when **when** we consider lower order model.

Now, remember here that we are not doing any close loop feed back control of the field voltage. The field voltage is practically a constant, we are either giving a step changes or we are keeping it a constant. We shall see later in the later part of this course, that the feed back control of the field voltage in a synchronous machine can again affect the damping of the electro mechanical or what is known as the swing mode that oscillatory mode, which is has a frequency of **around ten** around 10 radian (()) radian per second here.

Now, let us go one step ahead. Let us go to a model, now what I will do is go a bit faster and talk about, what next can be done.

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d-axis equations 1.1 model

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

$$\psi_d = x'_d i_d + \frac{(x_d - x'_d)}{x_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

The last equation can also be made an algebraic equation when studying slow transients

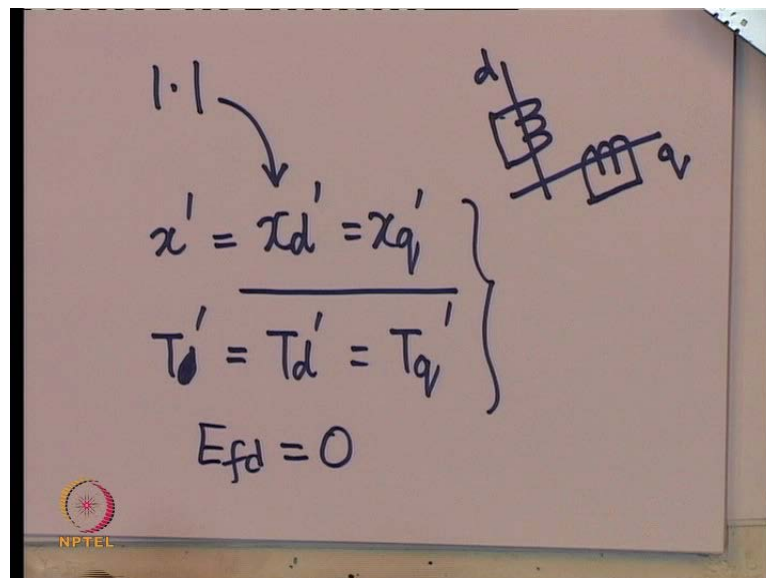
You can in fact, go to what is known as the 1.1 model. In 1.1 model we have done the corresponding thing with the damper winding H. So, what I have done is set R H tending to infinity and therefore, got rid of the differential equation corresponding to psi capital K and as a result of which, we get a simplified model with just one differential equation. And, as I mentioned last time, the same thing can be done here. You can get this lower

order model directly by setting $x_q = x_d$ is equal to x_t . If you have already programmed 2.1 model or 2.2 model to get 1.1 model, you all you need to do in that the program is to set x_t is equal to x_t .

Alternately, you can program a lower order model. So, both of the thing both these things can be actually done. We can go one step further, we can actually you know get rid of the last remaining winding, damper winding on the q axis, that is the G winding. But, at this point let us just take a small diversion not a very big one. If from this 1.1 model, a 1.1 model means you will be using these equations on the q axis and these equations on the d axis. So, that is 1.1 model, I will show it to you again. You will be using these equations in the q axis and these equations in the d axis.

Now, one question which I could like to ask you is, can we **can we** get the equations of an induction machine from 1.1 model of a synchronous machine.

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The answer is yes, from the 1.1 model **e** equate x_d and x_q , an induction machine can be modeled, a simple induction machine model could be by considering one damper winding on the d axis and one damper winding on the q axis. Remember, induction machine does not have any field winding. So, what you can do is of course, these windings are absolutely symmetric, if you look at a normal synchronous **mish ah** induction machine.

Both the d and q axis appears similar; there is nothing to distinguish d and q winding. So, need to set this I will call it x dash and what you need to do is of course, do this equate effectively what I have done is got from 1.1 model, simply by setting x d dash is equal to x q dash and t dash is equal to t d t dash is equal to t dash is equal to t q dash. You can in fact from the 1.1 model get a induction machine model. So, 1.1 model of a synchronous machine can directly yield to you the induction machine model you will of course, have to said E f d is equal to 0, this is something you need to do. And effectively equate the d and q axis parameters, set f d also equal to 0.

So, these are the things you need to do in case, you wanted to derive an induction machine model directly from 1.1 model of a synchronous machine. So, this is a interesting diversion, which this is (()) this point. Now, we can go ahead and get rid of even the remaining damper winding on the q axis. So, what we are going to now is talk about the simplified synchronous machine model, it does not have any damper winding at all.


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q-axis equations 1.0 model

$$\psi_q = x_q i_q$$

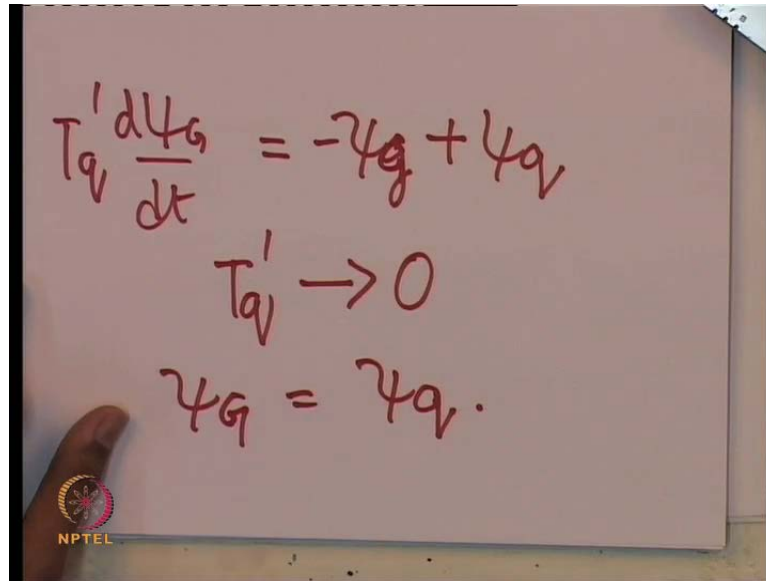
$$\frac{d\psi_q}{dt} = \omega \psi_d - \omega_B R_a i_q - \omega_B v_q$$

The last equation can also be made an algebraic equation when studying slow transients



So, this is **the** what you will get, effectively what you have done is set T q dash tending to 0. Basically, if you have R g tending to infinity, that is you are opening, the last wind remaining winding of the q axis in that case, T q dash tends to infinity and in that case.

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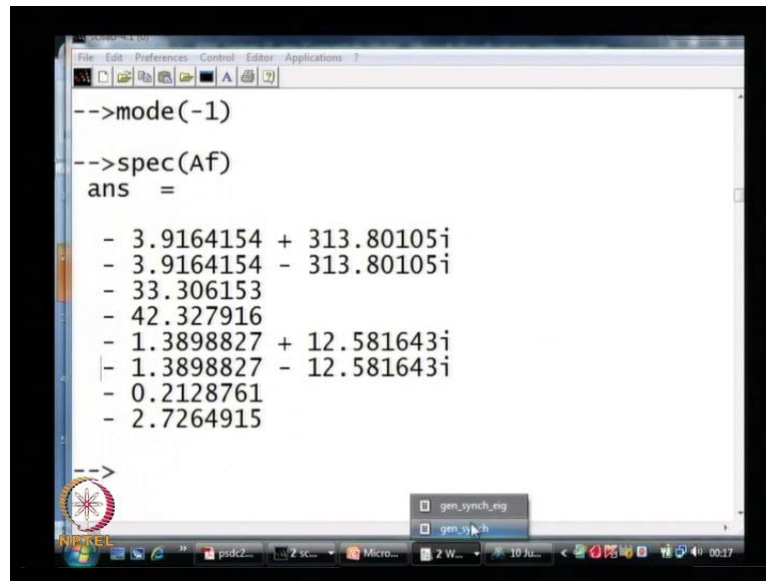

$$T_q' \frac{d\psi_g}{dt} = -\psi_g + \psi_q$$
$$T_q' \rightarrow 0$$
$$\psi_g = \psi_q.$$

We can kind of convert the differential equation corresponding to the g winding, that is $d\psi_g$ by dt T_q' is equal to ψ_g plus ψ_q .

And if you do that, if you set $d\psi_q$ T_q' tending to 0 in that case, you will get ψ_g is equal to ψ_q and if you use this fact in the algebraic equations, which you have seen earlier, see if you look at to 2.1 model, if you look at this algebraic equation here. If you set ψ_g is equal to ψ_q a few manipulations will get you to ψ_q is equal to $x_q \psi_q$. So, what you have in the q axis is just one algebraic equation and one differential equation corresponding to the stator winding that is $\text{sinh } q$. So, one differential equation and one algebraic equation in fact, the differential equation also can be removed by setting $d\psi_q$ by dt is equal to 0, in case you are studying slow transients.

So, this is what you get as 1.0 model in fact, 1.0 model equations will have **d the** d axis equation is looking like this and the q axis equations looking like this. So, that effectively tells you what is the 1.0 model of a synchronous machine is. In fact, this is you can say the last or the most basic of synchronous machine models, where you are now you have just the field winding and the stator windings and all the damper windings have been got rid of.

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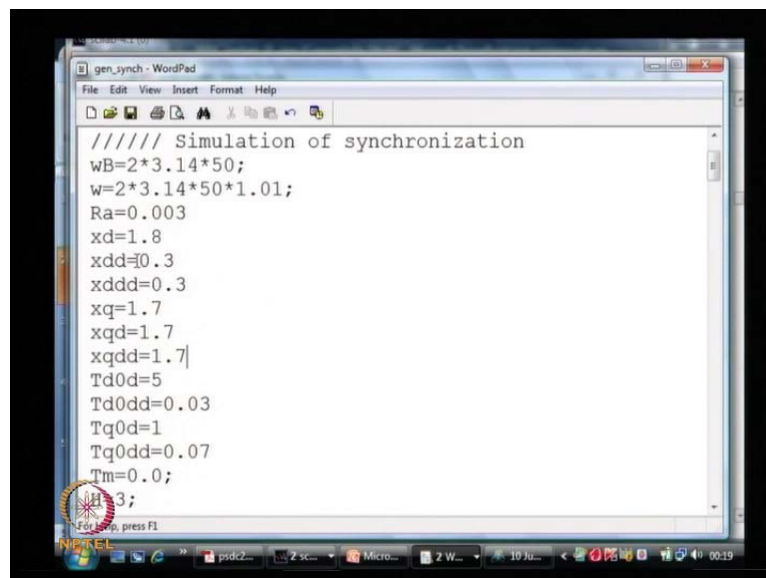
```
-->mode(-1)

-->spec(Af)
ans =

- 3.9164154 + 313.80105i
- 3.9164154 - 313.80105i
- 33.306153
- 42.327916
- 1.3898827 + 12.581643i
- 1.3898827 - 12.581643i
- 0.2128761
- 2.7264915
```

Now, if you look at the behavior of a 1.0 model synchronous machine. So, what I do is in this generators program, I set to get 1.0 model **what will I** what I need to do is, set first x t double dash is equal to x t dash, that is what I have done first. The second thing you can do directly, something which I have not mentioned here is that, in the 2.1 q axis equations if I directly set x q dash is equal to x q, you can get rid of you can really come directly to this algebraic equation and 1.0 models.

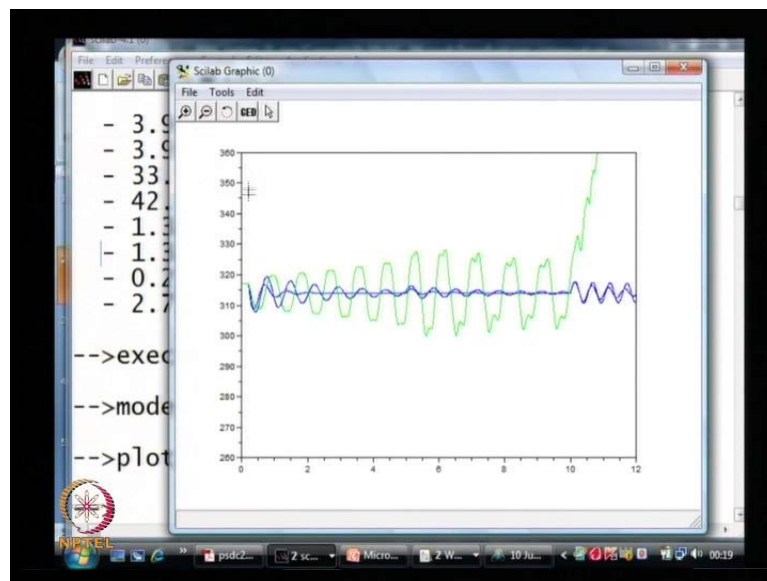
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```
gen_synch - WordPad
File Edit View Insert Format Help
///// Simulation of synchronization
wB=2*3.14*50;
w=2*3.14*50*1.01;
Ra=0.003
xd=1.8
xdd=0.3
xddd=0.3
xq=1.7
xqd=1.7
xqdd=1.7|
Td0d=5
Td0dd=0.03
Tq0d=1
Tq0dd=0.07
Tm=0.0;
3;
```

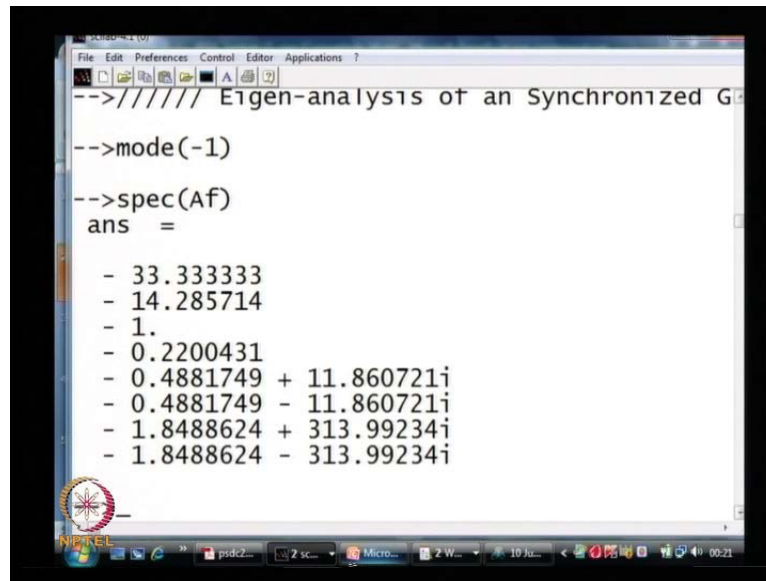
So, that is what I will do right now. What we will have here is, I will set $x_{q\dot{}} = x_{q\ddot{}}$ and of course, I also need to **set** $x_{q\ddot{}} = x_{q\dot{}}$. So, that we get 1.0 model. So, this is 1.0 model $x_{q\ddot{}} = x_{q\dot{}}$. So, by doing this we are directly modifying a program, we are not without modifying the program of 2.2 model, we can effectively get the response of the simplified generator model and I have also set $x_{d\ddot{}} = x_{d\dot{}}$.

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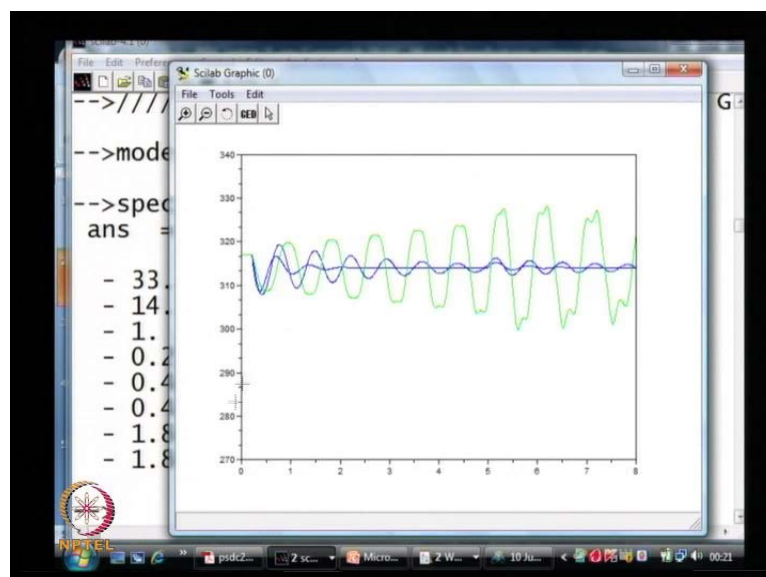
So, let us try to simulate this in sci lab **yeah**. And if I plot this now, **yeah** you get a totally different response in fact, what you are seeing is that the system is losing synchronism. So, in case you just the response is completely different. So, of course, one of the reasons why this could be happening that means just expand this p_u is a numerical reason, one of the reasons why this could be happening is a numerical reason. That is, see if you look at the first curve here, it is 2.2, this one is with 2.1. Now, we have gone to 1.0 and you see growing oscillations. Now, is this correct or not? That we can say by actually finding out the Eigen value of this system. For example, we try to **try to** find out the Eigen values of this system for 1.0 model **yeah**.

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So, I have modified the data, so that **yeah** and we will take out the Eigen values of this system. Now, what you notice is the real part of the Eigen value has become quite small, **it is** it is minus 0.4. Now, one of things you should remember here is that, we are using Euler method to simulate this system and if the damping that the real part of an Eigen value is low. Euler method may actually show it to be not a damped oscillation but, an increasing oscillation. And this is what exactly is happening here. So, all though Eigen analysis shows that the system is in fact stable, what you get here is a growing oscillation.

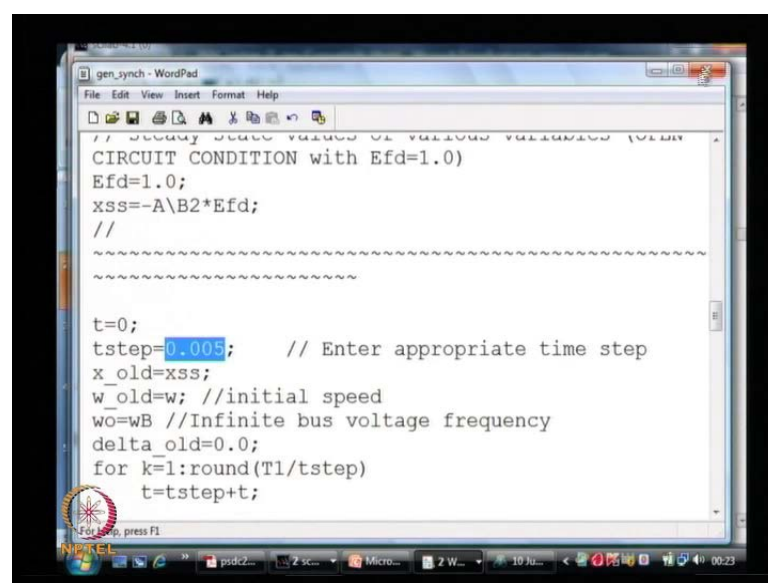
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So, growing oscillation of course, will eventually cause a loss of synchronism. So, this is kind of spurious but, remember one thing that if we neglect the effect of all damper windings in 1.0 model, the damping of the swing mode may come down to a low enough value, not necessarily to make the system unstable, as seen in this Eigen analysis. But, which may cause misbehavior or other wrong information to be displayed, in case we do a simulation using Euler method. Recall, in the first 10 lectures of this course, we have discussed numerical integration techniques and there we did find, we did discuss this that in case you use Euler method on a poorly damped system with a poorly damped oscillation in that case, you can in fact get wrong information.

Now, the solution this of course, is to keep on **keep keep on** reducing your time step of the simulation till Euler method starts giving reasonably correct results. But of course, that will take a long **long** time to simulate, if I really go on reducing my time step. So, one thing which you remember now, it is important to as it shown to distinguish between two issues we are talking here. One is that, with a lower order synchronous machine model 1.0 model, the amount of damping or the amount of rate of decay of swing mode has come down substantially, that is one aspect that is the physical aspect. One more aspect is about the analysis the damping has really come down to such extent that, if you try to simulate **Euler** using Euler method with the time step of, we just I just forgot what the name of the variable is for time step, I will just get that.

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```
gen_synch - WordPad
File Edit View Insert Format Help
// Steady state values of various variables (only
CIRCUIT CONDITION with Efd=1.0)
Efd=1.0;
xss=-A\B2*Efd;
//
~~~~~

t=0;
tstep=0.005; // Enter appropriate time step
x_old=xss;
w_old=w; //initial speed
wo=wB //Infinite bus voltage frequency
delta_old=0.0;
for k=1:round(T1/tstep)
    t=tstep+t;
```


With the time step of 0.005, a system with a poorly damped oscillation like this one, is likely to show. So, you let us just see this again t step, **Yeah**. A system which has poor damping is likely to show, is likely to cause incorrect information being shown in a simulation in which, Euler method is used with a time step of this kind. So, you really what to need to do as I mentioned sometime back is go on reducing the time step or use a better numerical integration method. But, of course, we are been using Euler method simply because it is an easy method to use. It is sacrilegious to use Euler method for any realistic or you know practical or industry grade program. I mean, because of this particular problem that it is not very accurate, it can give wrong information. So, this is another illustration of this.

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```

- 33.333333
- 14.285714
- 1.
- 0.2200431
- 0.4881749 + 11.860721i
- 0.4881749 - 11.860721i
- 1.8488624 + 313.99234i
- 1.8488624 - 313.99234i

-->tstep
!--error 4
undefined variable : tstep

-->exec("gen_synch.sci")
mode(-1)

```

So, all though the system is **is** stable remember, all Eigen values for all operating points this is of course, the Eigen values for a particular operating point, the Eigen values have negative real parts. So, it is a stable system but, simulating it may result in problems, if you are using methods like Euler methods. So, that is the summary of what we have been discussing so far.

Now, 1.0 model in some sense is the limit of what simplification we can have in a synchronous machine. If we want to you know get even in a realistic picture in study state, you have to use at worse, you will use this simplified model. Of course, for theoretical studies involving the swing mode, we can use a simplified model which

includes no differential equation, no electrical differential equation no equation differential equation corresponding to the flux. How do we get to that model, well this is not a very respectable model.

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Classical Model: 0.0 model

$$R_f \rightarrow 0 \Rightarrow T'_d \rightarrow \infty$$

$$\frac{d\psi_F}{dt} = 0$$

$$\psi_d = x'_d i_d + \frac{(x_d - x'_d)}{x_d} \psi_F$$

$$\frac{d\psi_d}{dt} = -\omega \psi_q - \omega_B R_a i_d - \omega_B v_d$$

Because, so I will just tell you how you can get to what is known as 0.0 model, which is what is known as a classical model. A classical model is not a respectable model for getting realistic and quantitatively correct answers as far as a synchronous machine behavior is concerned. For example, you cannot use classical model to understand the numbness of what happens during a short circuit in a synchronous generator.

So, there are very great limitations in applying this classical model. But, all the same I will just tell you what it is and how it is obtained and what kind of assumptions are inherent, when you get to classical model. Remember that, right in the beginning of the course when I was talking about our introducing you to things like loss of synchronism and the **the** you know the origin of swings, power swings or rotor angle swings in power system. I had used in **in** fact the classical model, it is not a respectable model, I called it a toy model then I will call it a toy model even now.

But, let us just see what really was involved in getting to this classical model. What you need to do is from 1.0 model, you can look at the screen again. From 1.0 model, 1.0 modeling in requires these equations on the d axis and this equation in the q axis, what you do is, you assume that the field winding resistance is very small. As a result of which

you can show using the basic equations, which relate the standard parameters and the resistance and inductance parameters of a synchronous machine. You can show that T_d becomes a very large value, it becomes tending to infinity.

If T_d tends to infinity, you are in effect saying that the field winding flux does not change. So, ψ_f here is 0, $d\psi_f/dt$ is equal to 0. So, ψ_f becomes a constant. So, what you have is, you have got rid of the differential equation, the last remaining differential equation of the rotor field winding. You have effectively set that ψ_f is simply a constant. Now, a further simplification of course, which we have being, which I talk to right in the beginning was you set $d\psi_d/dt$ equal to 0 as well. If you are going to slow study slow transients, this is an approximation to make.

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Classical Model

$$\psi_d = x'_d i_d + E'$$

$$0 = -\omega_B \psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$\psi_q = x_q i_q$$

$$0 = \omega_B \psi_d - \omega_B R_a i_q - \omega_B v_q$$

Further Approximations : $R_a = 0, x_q = x'_d$

So, what you have eventually is, for the classical model setting $d\psi_d/dt$ and $d\psi_q/dt$, we have got this model.

So, this is the classical model of a synchronous machine, which does not have any differential equation. No electrical differential equation, we have got the mechanical differential equations but, there is no electrical differential equation and a further simplification, which you can make is R_a is equal to 0 and x_q is equal to x'_d .

Now, this is an absolutely an (()) assumption. This is no, you know kind of justification which I can give you. So, classical model is obtained by a large number of approximations. So, if you want to get from 2.2 model to classical model, you have really made a huge a large number of approximations.

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$$\psi_d = x_d' i_d + E'$$

$$\psi_q = x_d' i_q$$

$$0 = -\omega_B \psi_q - \omega_B V_d.$$

$$0 = \omega_B \psi_d - \omega_B V_q.$$

So, what you will get of course, if you just go ahead with what you have got, ψ_d is equal to $x_d' i_d$ plus E' , E' is that is actually proportional to ψ_f , which is assumed to a constant. So, E' is also a constant and you also have ψ_q is equal to $x_d' i_q$. This is got by approximating x_q is equal to x_d' and absolutely (()) assumption.

We all from, we also have 0 is equal to this is by neglecting $d \psi_d$ and $d \psi_q$ by $d t$ terms, this is by neglecting $d \psi_d$ by $d t$. We also make R_a equal to 0 . So, we will have minus $\omega_B V_d$ and ψ_q we will also have is equal to $\omega_B \psi_d$ minus $\omega_B \psi_q$. And, what you can do is effectively substitute for ψ_d and ψ_q in these equations. So, what you will have eventually is, if you do that you will have ω_B , we will.

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$$0 = -x_d' i_q - v_d \checkmark$$
$$0 = x_d' i_d + E' - v_q \checkmark$$

$$\frac{2H}{\omega_B} \frac{d\omega}{dt} = T_m - (\psi_d i_q - \psi_q i_d)$$

So what we will have, 0 is equal to minus of x_d' i_q minus of v_d and what we will have here is, x_d' i_d plus E' minus v_q .

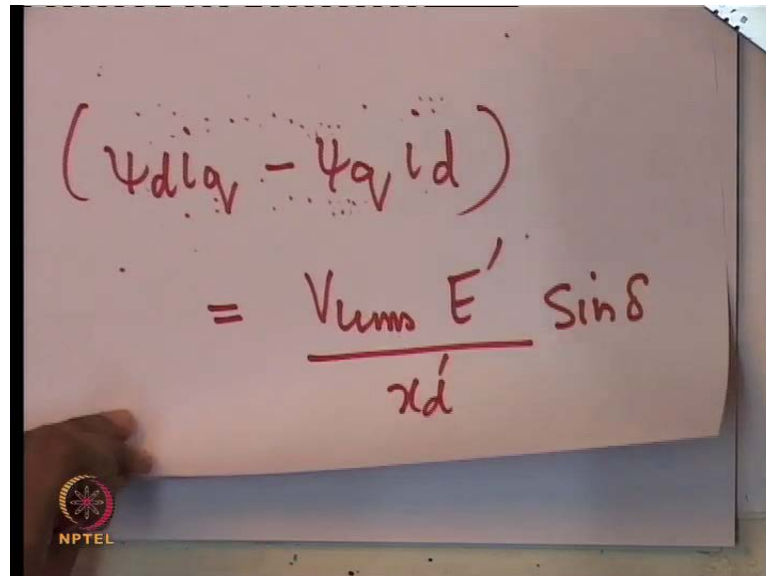
Now, we also have $d\omega$ by dt , $2H$ into $d\omega$ by dt by ω_B into $d\omega$ by dt is equal to T_m minus $\psi_d i_q$ minus $\psi_q i_d$. Now, if you look at so, you have got these two algebraic equations and this differential equation. Now, if you are talking of a synchronous machine connected to an infinite bus or a stiff voltage source as the one, which we have used for all our study, so far.

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$$v_d = -V_{LL} \cos \delta$$
$$v_q = V_{LL} \sin \delta$$

In that case, V_d is equal to V line to line rms of that source into $\sin \delta$ minus of it. So, this is the source, whose characteristics I have discussed in the previous lectures.

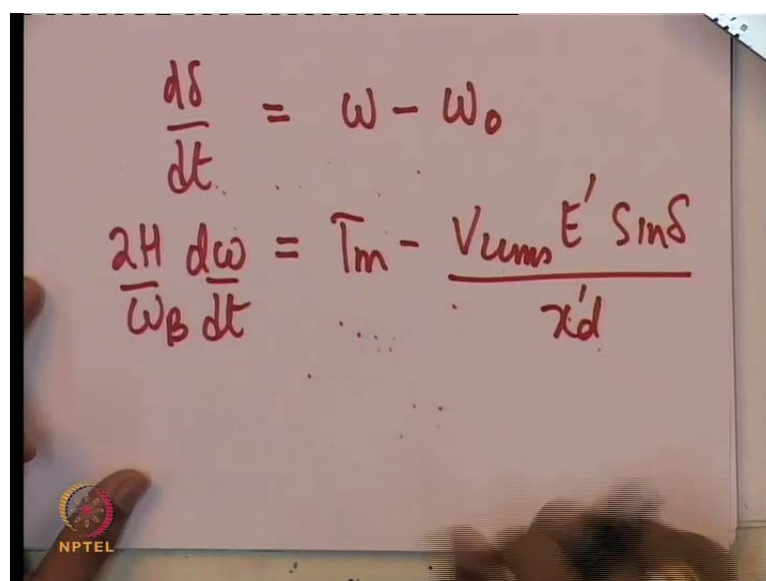
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$$\begin{aligned} & (\psi_d i_q - \psi_q i_d) \\ &= \frac{V_{\text{line rms}} E' \sin \delta}{x_d'} \end{aligned}$$

So, if this is true then it is easy to show, is quite easy to show that $\psi_d i_q$ minus $\psi_q i_d$ from all the equations, which we have got. From these equations and these equations, what you will get eventually is, this is the electromagnetic torque is nothing but, V line to line rms by x_d' E' $\sin \delta$.

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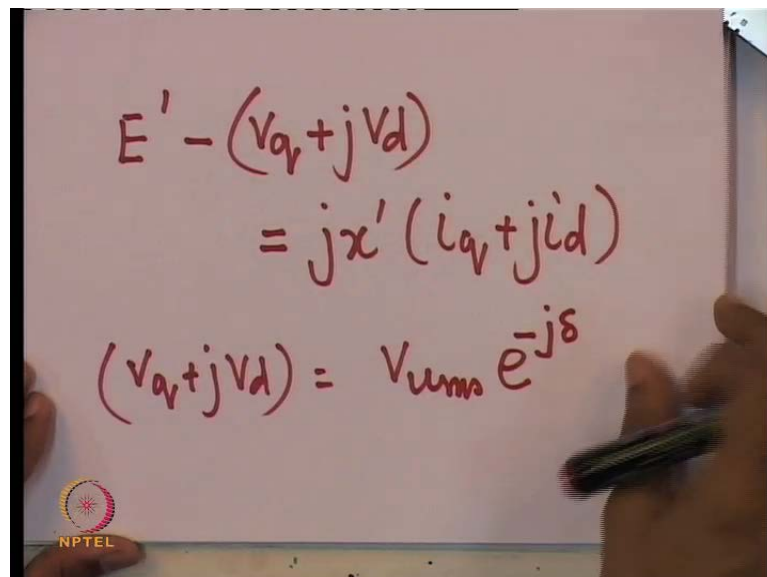


$$\begin{aligned} \frac{d\delta}{dt} &= \omega - \omega_0 \\ \frac{2H}{\omega_b} \frac{d\omega}{dt} &= T_m - \frac{V_{\text{line rms}} E' \sin \delta}{x_d'} \end{aligned}$$

So, from this and this **this** is what we have. So, the classical model effectively, the **torque** **the** torque equation or the electromechanical equations are $\frac{d\delta}{dt}$ is equal to $\omega - \omega_0$, ω_0 is the frequency of the infinite bus and you have got $\frac{2h}{\omega_0}$ and $\frac{d\omega}{dt}$ is equal to $T_m - \frac{V_{LL}^2}{X} \sin\delta$. This is transient reactant of the generator; we call it just X_d .

Now, another interesting thing is of course, from these algebraic equations, you look at these algebraic equations, you can write this very compactly.

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$$E' - (V_q + jV_d) = jx'(i_q + j i_d)$$

$$(V_q + jV_d) = V_{LL} e^{-j\delta}$$

We will have $E' - V_q + jV_d$, this is just multiplying the second equation by the complex number j , $E' - V_q + jV_d$ is equal to $jx' (i_q + j i_d)$. We need not apply the subscript any longer because, we have equated the d and q axis completely. This is what we get.

And you know from what we here, we have $V_q + jV_d$ can be written compactly as $V_{LL} e^{-j\delta}$. **yeah** e raise to its simply that just have a look at it.

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$$E' - (V_{lrm}) e^{-j\delta} = jx(i_q + j i_d)$$

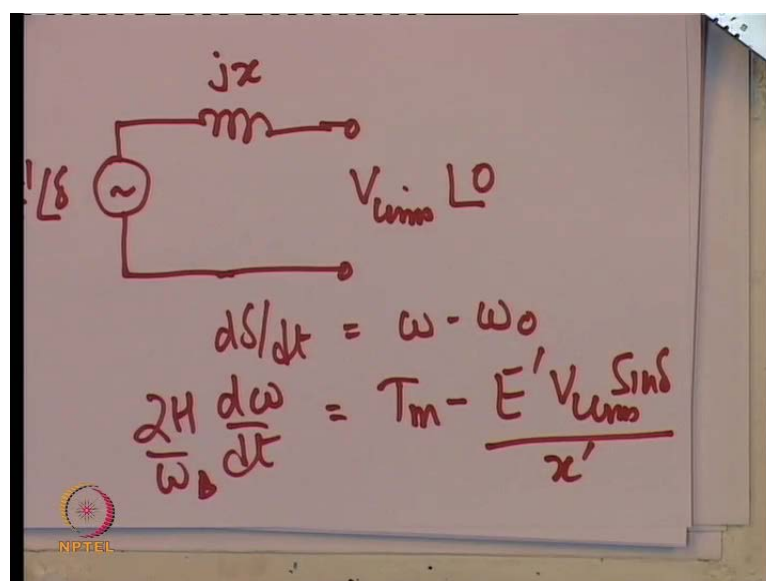
$$E' e^{j\delta} - V_{lrm} = jx(i_q + j i_d)$$

$$(i_q + j i_d) = (i_q + j i_d) e^{j\delta}$$

So, what we have here is, eventually $e^{-j\delta}$ into V_{lrm} into $e^{j\delta}$ is equal to $jx(i_q + j i_d)$. And, what we have from here effectively is $E' e^{j\delta} - V_{lrm} = jx(i_q + j i_d)$ where, $i_q + j i_d$ is nothing but, $i_q + j i_d$ into $e^{j\delta}$.

So, what we have here is effectively, a synchronous machine model in which, the electrical equations are simply given by this or effectively an electrical circuit.

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So, what we have effectively is an electrical circuit. So, if you want to represent the synchronous machine electrical equations is simply a phasor that is an algebraic equation. This is V line to line rms of the infinite bus, this is $E \angle \delta$ and the differential equations are $\frac{d\delta}{dt}$ is equal to $\omega - \omega_0$, ω_0 is the frequency of the infinite bus and $\frac{d\omega}{dt}$ itself is nothing but, T_m minus $E \angle V$ line to line rms $\sin \delta$ by x .

So, this is exactly the toy model, which we used in this in fact, in the second lecture where kind of model, which we have used here. In fact, this model predicts an oscillatory response for δ and ω , whenever there is an disturbance. So, what we have with 2.2 model is not only this electromechanical swing in fact, if you recall whenever you use a higher model with damper windings, **the Eigen values** the Eigen values are these. So, there are many many modes here. Amongst them, we have got what is known as swing mode. If you take the classical model directly, what you will have is simply just because of the two differential equation just one mode and that too really represent as we have seen right in the beginning, where we **we** did analyze this toy model, we do get the electro mechanical oscillations.

So, classical model is just to theoretically highlight, the fact that you do have such a mode, which is mainly associated with the electromechanical variables δ and ω . So, just a look at this classical model again, **yeah**. This should have been x dash here, these are the differential equation.

So, this is the classical model, which is **okay** just okay for theoretical studies. But, please do not **it probably will give you, fairly** it probably give you a wrong quantitative answers, wrong in the sense, highly imprecise answers. In case, you try to use it for practical studies. But, none the less just using the classical model, we can in fact show that a phenomenon called swings occurs, this is the evident in the 2.2 model but, the 2.2 model will also bring into picture many other modes, which are present.

So, this brings to an end practically an end our discussion of synchronous machine. We will just revisit a few minor points tomorrow in the next lecture. And thereafter, we will move on to another physical sub system, which is of importance in a power system that is the excitation system of a synchronous machine. We will discuss it is modeling and also discuss how it looks physically.