Power System Dynamics and Control

Prof. A M Kulkarni.
Department of Electrical Engineering.
Indian Institute of Technology, Bombay

Lecture No # 22
Synchronization of a Synchronous Machine (Contd.)

We have been studying the synchronization transient of a synchronous machine connected to an infinite bus. What we saw last time was that, you can have a near bump less synchronization of a synchronous machine to an infinite bus, which is essentially a fixed voltage source, the machine locks on to the frequency of the synchronization of the infinite bus. In case, it is synchronized well, now the thing we noticed last time was that, if the speed of the machine initially is almost the same as that of the infinite bus. And, at the instant of you know the interconnection, the voltage, open circuit voltage of the generator, the voltage phasor you can say is practically the same as the infinite bus voltage phasor, then you get a kind of a bump less transfer.

Of course, you cannot get an perfect transfer without transients, it’s practically not possible. There will be some transient, because you cannot might do this match, which I mentioned sometime back exactly. So, what you really see is that, when you connect a synchronous machine to an infinite bus, you do get a small synchronization transient. Most notably, you notice a kind of a what is known as a swing mode or a low frequency oscillation, which usually damps down and that really after that, synchronous machine is in synchronism with the voltage source. Thereafter, if you increase the mechanical torque to the synchronous machine, you find that the machine transfers power from from itself to the voltage source or the infinite bus.

So, what you notice there of course, is that as you go on increasing the mechanical power, you will find at that the rho the angle delta increases and in fact there comes a point after which if you try to increase the mechanical power the synchronous machine in fact loses synchronism. There is it kind of fails to reach a steady state, in case you go on increasing the mechanical power beyond the point.

Of course, the important thing is that we had in fact seen precisely such a transient in one of our lecture. In fact, the first lecture, I had shown you a small demonstration clip, it will be good to revisit that clip, you can go back to the first lecture and see that clip that is precisely what we have tried to stimulate, tried to stimulate in the 21 lecture. That is a synchronous machine connected to a infinite bus and there in thereafter, we go on increasing the mechanical power to point at which it loses synchronism

One of the things which we did not do, when we increase the mechanical power was the mechanical torque to the synchronous machine was that, we did not increase the field voltage simultaneously. In fact, a synchronous machine tends to lose synchronism very easily, in case you try to load it without a concurrent increase in the field voltage.

So, what we will do this, in this lecture is? We will first (()) revisit this earlier transient, in which we first give a small step, we first synchronize the machine then, give a small step in the mechanical power of 0.25 per unit and then we try to increase the mechanical power right up to its rated value of 1 per unit, and there we do not increase the field voltage, we will see that it loses synchronism. Thereafter, we will redo this simulation with, by increasing the field voltage concurrently, that is at the same time as we rather at this we increase the field voltage simultaneously, traps simultaneously the better word. We increase the field voltage simultaneously along with the mechanical torque. You will find that under such circumstances the the generators able to supply the rated power to the infinite bus.

Now, one small clarification of course, I hope you did not miss it last time was that, when we are simulating the equations of a synchronous machine, we are neglecting the stator transient or the stator flux transients. That is we have replaced d psi d by d t and d psi q by d t by 0. The reason is of course, that the transients associated with the these states psi d and psi q are very fast. So, what we have done essentially is, replace the differential equations which relate the psi d and psi q fluxes by algebraic equations. One thing you should note here is that, an implicit thing is that, when I am making this assumption, I am really interested in the slower electro mechanical transient. That is why; I can do without the psi d and psi q differential equation, replace them in fact by the algebraic equations and still get a reasonably correct result.

Now, you may ask that well, why do I do it anyway you did can just as well keep the differential equations corresponding to psi d and psi q, the reason why of course, I have done that is if I retain the differential equation in d psi in psi d psi q, you get what is known as stiff system, that is a mixture of fast and slow transients are exciting in that system and as a result of which, simple numerical integration scheme are likely to misbehave. So, what I have really done is, I have made this approximation of neglecting the fast transients, removed in some sense the stiffness of of the system and then used Euler method, which is a simplest method to simulate the system.

So, that is why I have done this. If I if of course, I had used I had retained the stiffness in the system, that is I had retained the d psi d by d t and d psi q by d t terms in the equations. In such a case, you would find that you would need to use method like trapezoidal rule or backward Euler method. Now the problem in doing that is, once you discretize the differential equations with trapezoidal rule or backward Euler method, remember these are non-linear differential equations. Then, what you really get is are non-linear algebraic equations. Once you discretize the system by by these numerical methods, you get non-linear algebraic equations and for every time step, I would need to solve numerically solve for every time step or numerically you have to solve the algebraic equations.

On the other hand, Euler method is simple and the per time step competition is very very straight forward. So, that is what we have done so far. Today, I will try to simultaneously increase the field voltage along with a torque increase. And, we can see that in such a case, you can in fact run the synchronous machine at rated power. So, let us just redo the transient we did last time and go head and simulate the transient with field voltage increase simultaneously.

So, let us go on to that. So today’s lecture of course, we will be revisiting the transient associated with a synchronizations of a synchronous machine. Now, what we will do is of course, like in the previous lectures, we will do a simulation of this system.

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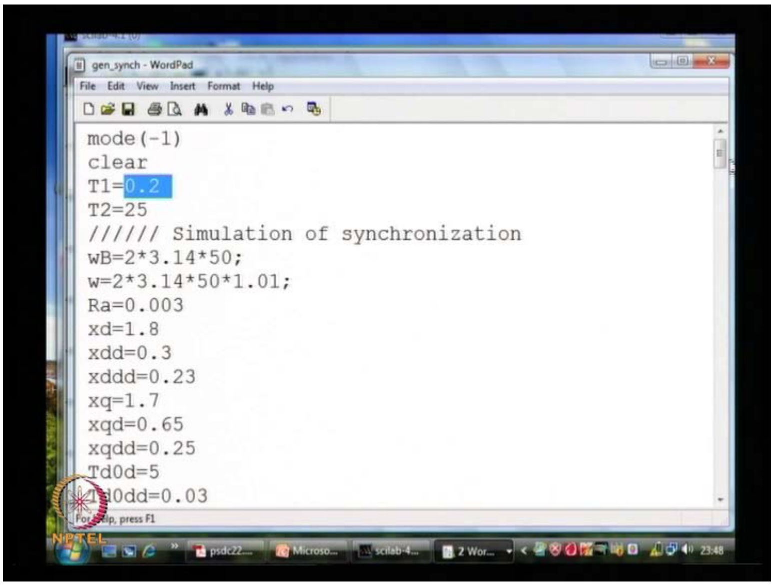

So, let me just show you the simulation file here. This is gen underscore sink is the file, I will be doing a 25 second simulation. I will be synchronizing the machine at the point two seconds. This is of course, the you see the data of the machine, this something you have done a quiet a few time before in this in these lecture.

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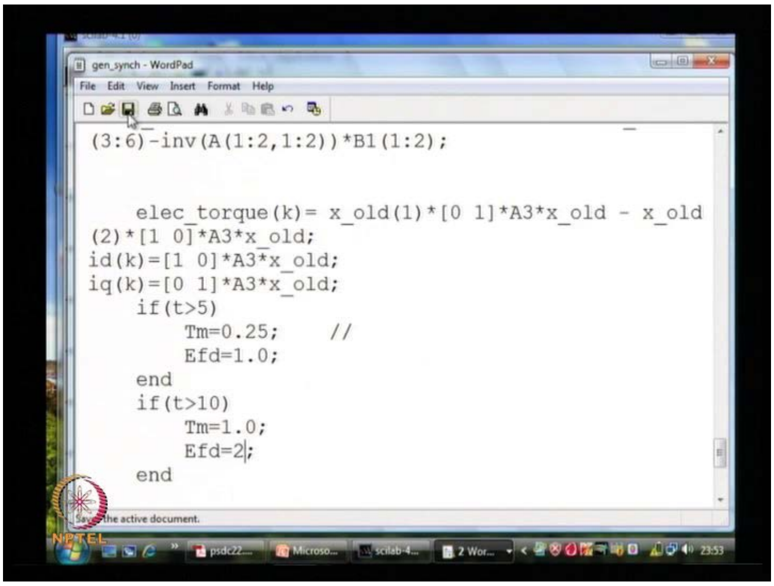

So, I will just quickly came through this. As the mechanical power is changed to one per unit, we also double the field voltage. So, simultaneously we double the field voltage and will redo this transient, redo this simulation here.

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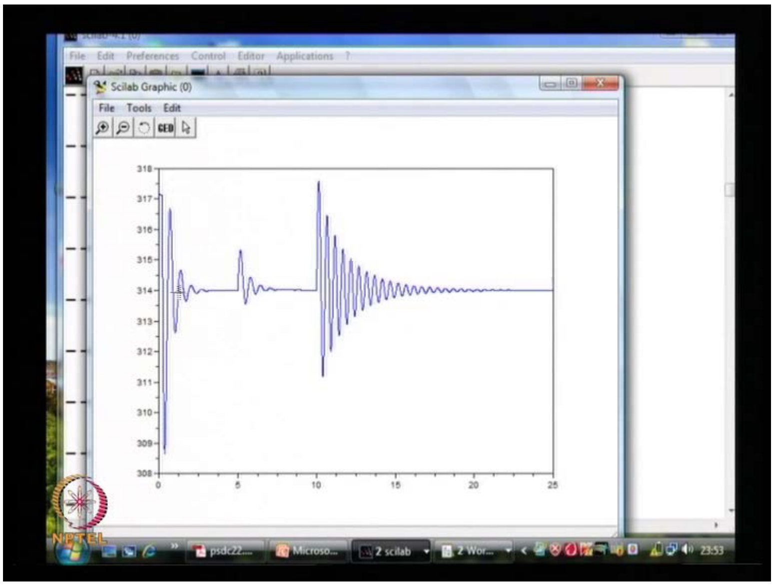

And I plot, now the speed. I notice that, in this particular case, the machine regains rather is still in synchronism.

So, by increasing the field voltage in some sense have insured that the machine is able to deliver the rated power. And the remain in synchronism and operates stably of course, these there is a electro mechanical oscillation or a swing, which is seen in the speed, whenever we give any such disturbs. Now, we can of course, have a look at how delta looks is well yeah.

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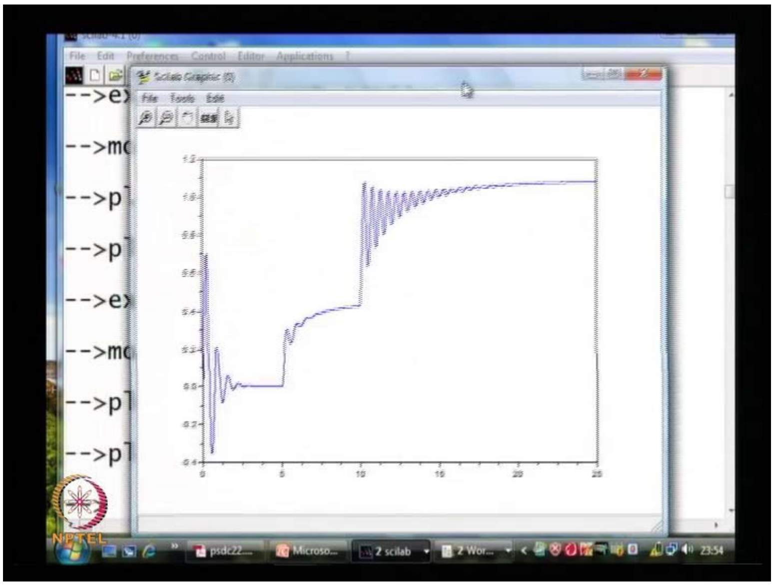

So, for the first disturbance of course, delta goes from 0 to 0.4 roughly, this is radians. And the second transient also it is stable, there is a swing and there is a gradual raise as well, rather kind of a oscillate decaying oscillation, which we see here and also decaying exponent, I meant there is also this kind of raise because of, which it settles finely near about one radiant.

So, this is basically how the system behaves in case we increase the mechanical power along with a change in the field voltage. If we do not change the field voltage along with the mechanical power, the system can lose synchronism. So, that is one important thing you should remember. In the next ah next few lectures, we will go on to talking about excitation systems in the sense, that we will try to understand how the field voltage is changed ah by by a control system or an a control system and an excitation system. Field voltage in a synchronous machine changes along with the loading. In fact, we generate in such a way that whenever the loading of a synchronous machine changes, the field voltage is also changed. It is in fact every necessary to do. So, this is something you should keep in mind.

Now, what we will do next is look at another form of analysis, what we will do is? Do a kind of small signal analysis of a synchronous machine. So far, what we have been doing here is, in fact looking at the a numerical simulation of the system. The reason why we do a numerical simulation is that the system is non-linear. Now, we can in fact do a linearized analysis around an operating point using Eigen analysis, the Eigen analysis tool. As we have done before, the only reference here is that (()) this is non-linear we have to first convert the system into a linearized system around an operating point.

So, what we will be really doing is (()) are non-linear system, we cannot inferred the non-linear behavior by Eigen analysis. That is not possible but, what we can do is, if you are at an operating point, that is an at an equilibrium point. If you give small disturbances, you can rewrite the equations in the linear form, which has valid only for small disturbances and inferred the behavior from the Eigen values of the resulting linear differential equations.

So, this is what we will do next at this point, we should look it, where are actually the nonlinearities in our equations. Let us have a look.

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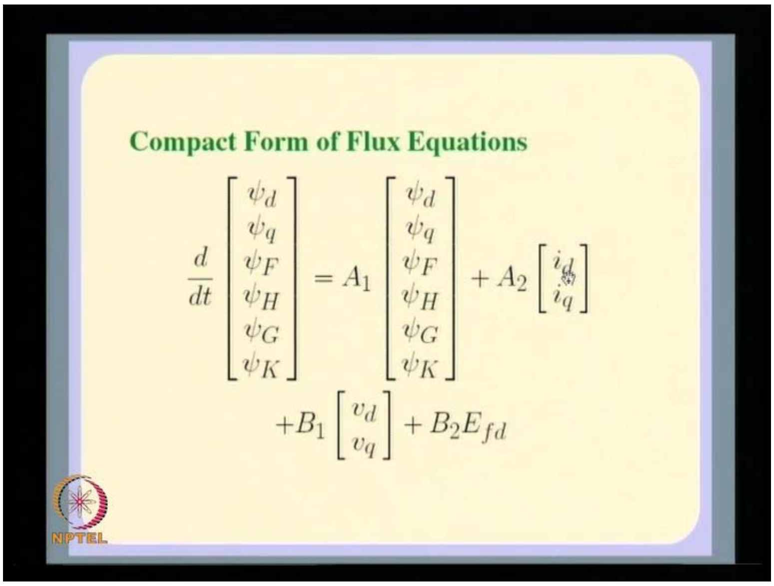

Now, if you look at the equations of a synchronous machine, we had we have been using these compact form of equations in our analysis, that is first of all we have written the flux equations, d by d t of the flux equations is equals to A 1 into the flux. So, this is the states space form of the flux equations. Remember that, A 1 is a function of speed. So, actually all though this is written, it looks almost as if it is linear. A 1 is actually a function of the state, of the states. Speed is the state of the system. Remember that, this is written in a kind of a composite form i d i q are in fact related to all the fluxes by another set of equations.

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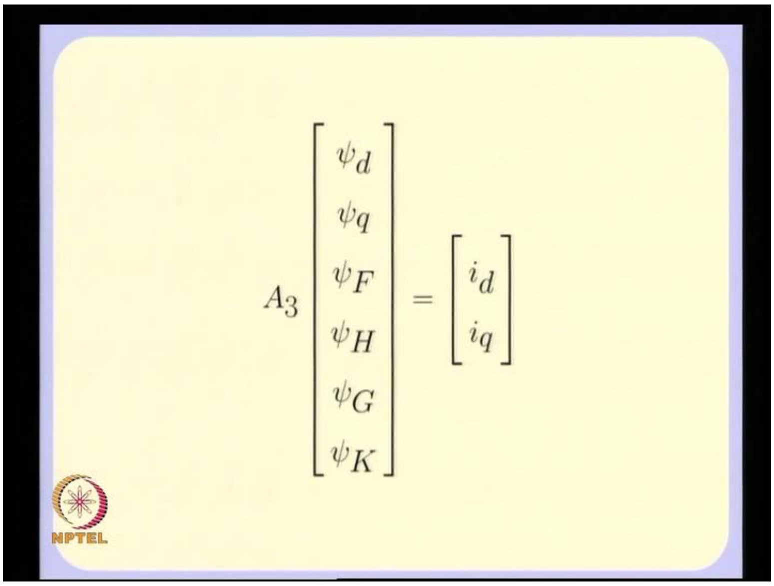

The in fact, the inductance effectively the inductance matrixes or the reactants matrixes in per unit.

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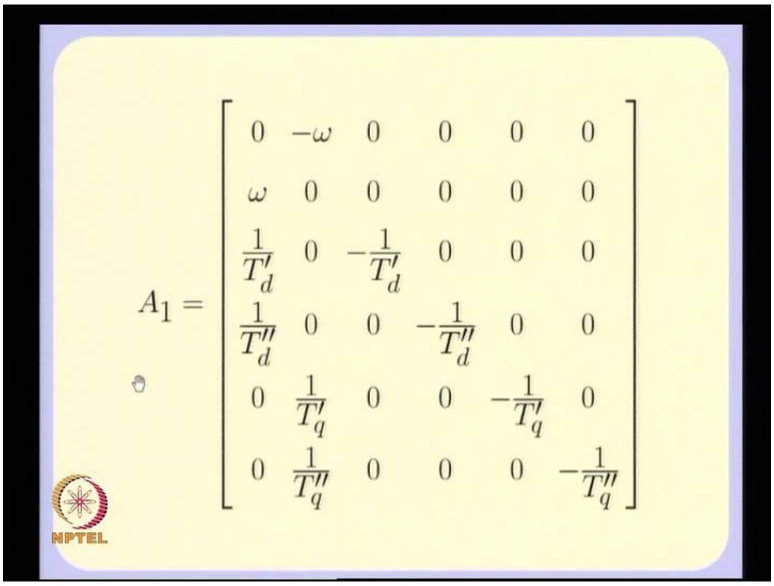

These equations are of course, in per unit form. A 1 looks like this, it is in fact linear non-linear because, A 1 itself is a function of the states. So, A 1 into psi involves effectively product terms of speed and flux. So, that is why it is basically a set of non-linear equations. So, there is nonlinearity here, in fact these are of course, the definitions of A 1 the A 1 B 1 B 2 A 3. So, one of the things you will notice here is of course, I think I have I have I have effectively written the torque equation, if you look at it in per unit, this also involves speed product terms.

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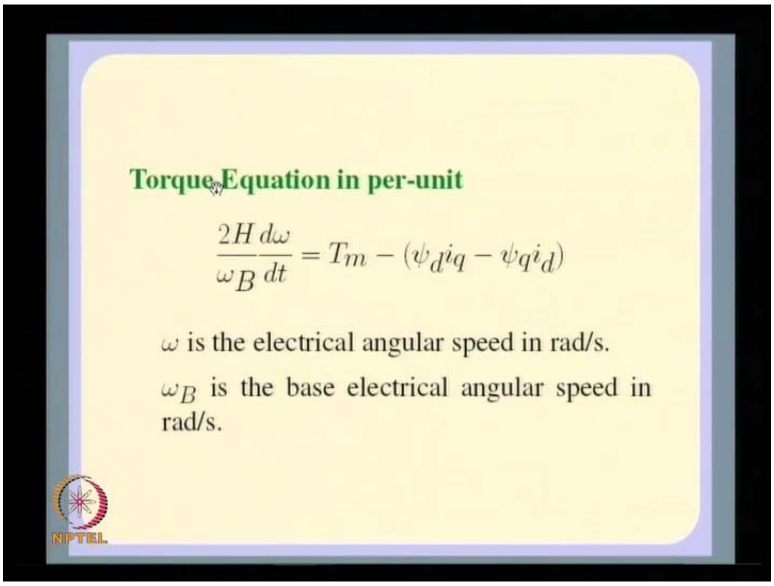

So, if you look at the basic compact equations of a synchronous machine. This is a product kind of term, even contains is the function of the states and also the torque equation in per unit is a non-linear equation. So, when we are trying to use Eigen analysis tools, remember that the mechanical and electrical equations are in fact coupled and you see the flux and current equation in the mechanical equations and in the flux equations, there is a omega dependants.

So, because of this nonlinearity, we cannot directly apply linear analysis. We will have to do a linearization around an equilibrium point. So, the basic idea is, suppose you are operating at a particular equilibrium point.

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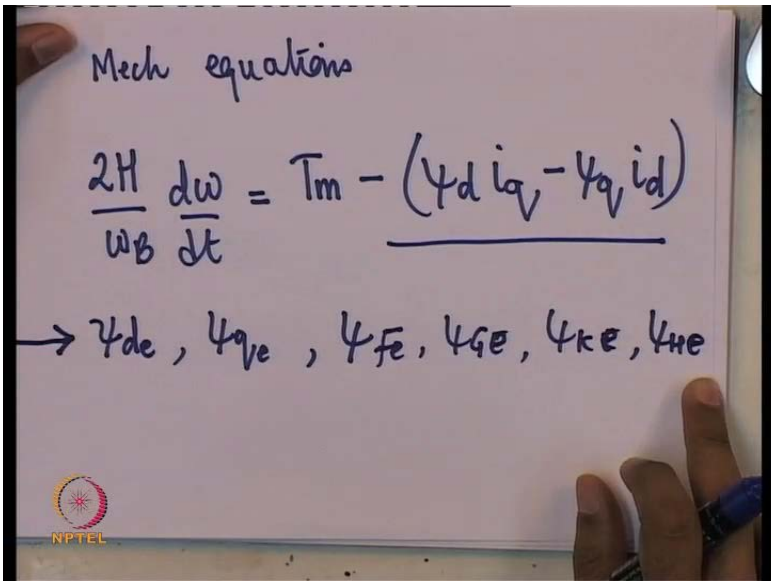

So for example, if you look at the mechanical equations, 2H by omega B d omega by d t is equal to T m minus psi d i q minus psi q i d. Now, if you look at this, this is a non-linear equation.

Now, if you are operating at a certain equilibrium point and the equilibrium values of psi d is psi d e similarly, the equilibrium value of psi q is psi q e and of course, i q and i d are dependent on the equilibrium values of psi F e in addition to these psi F e, psi G e, psi K e and psi H e. So, these are the equilibrium point. So, this e here is denoting equilibria. So, this the equilibrium value of these states. So, if you look at you rewrite rewrite these equations again. If your speed equilibrium speed of a synchronous machine connected to an infinite bus is well if you look at d delta by d t it is nothing but, omega minus omega 0, this is the frequency of the infinite bus.

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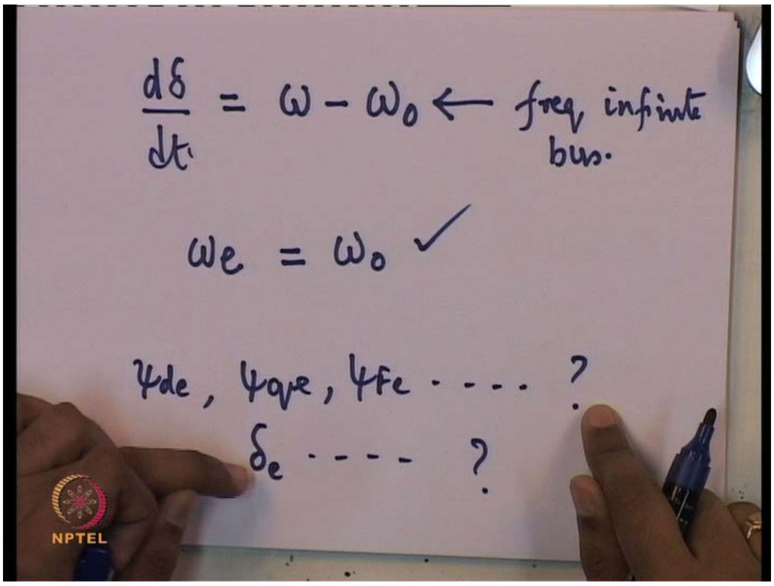

So, if look at what is the equilibrium speed of the machine? The equilibrium speed of the machine is omega e and by definition of equilibria, equilibria are the value of the states at which all the d by e t is become equal to 0. So, if you look omega e it should be equal to omega naught. So, the value of omega at equilibrium is omega e, it is equal to omega naught. So, these are essentially the equilibrium values of the states. So, equilibria are actually obtained by setting d delta by d t d omega by d t and all the d psi by d t is equal to 0. Now, what are the equilibrium values of, what are the actual values of psi d, psi q e, psi F e and so on. What is the equilibrium of value of delta e for example, what is the equilibrium value of delta? So, all these are all the states, so one of the equilibrium value we have of course got, then they are 6 flux states for which we have to get the equilibrium values and delta equilibrium value has to be obtained.

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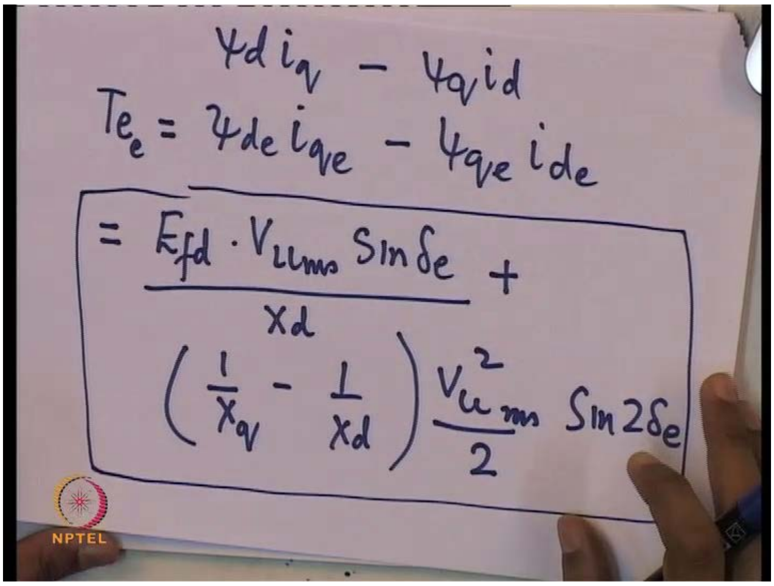

Now remember that, we had obtained the expression for the electrical torque, if you recall what is the electrical torque expression? The electrical torque expression was, in steady state of a synchronous machine was, electrical torque under equilibrium conditions is of course, it is psi d e i q e minus psi q e i d e. But, we can write it down as, we have derived this in some of our previous lectures, sin delta e upon x d plus 1 upon x q minus 1 upon x d into v line to line rms square pi 2 sin 2 delta e. So, the equilibrium value of the torque is nothing but E f d, this is depending on the field voltage, v line to line rms is the voltage line to line rms voltage of the voltage source to which the synchronous machine is connected to delta e of course, is the equilibrium angle.

Now, the important thing is that, this is the this if you look at psi d e psi d i q minus psi q i d, this is essentially an equilibrium rather the transient, this expression for torque is valid also in transient conditions. But, (()) remember this expression here which I have written down is only valid during steady state. So, please do not use it for transient conditions. So, under equilibrium conditions, this is true.

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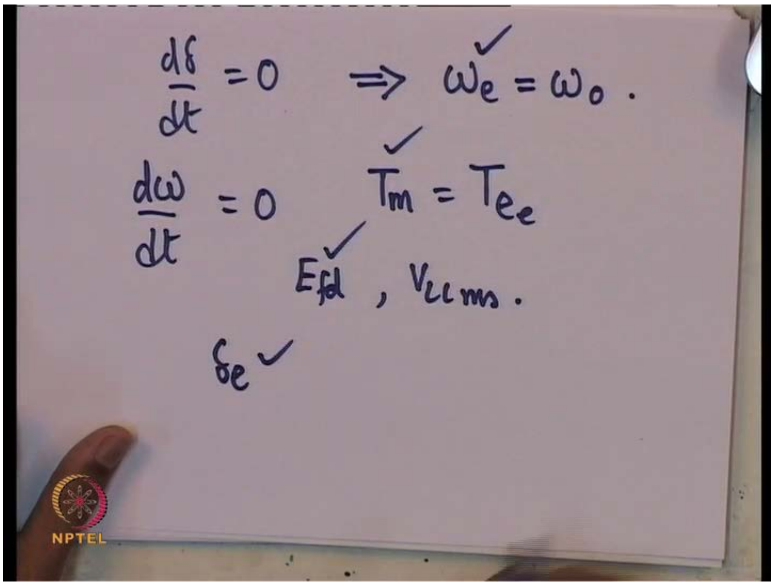

So, if I set d delta by d t is equal to 0, we we get the equilibrium value of omega should be equal to omega naught. Also, by setting d omega by d t equal to 0, we get T m should be equal to the equilibrium value of this. Now, if we know what e f d is and what V line to line rms is, from this we can easily back calculate what the value of delta e is? So, you I know what T e e is t m, so if I given you what the mechanical torque is and what the E f d is. In such a case, you should be able to compute delta e. So, first state, second state is computed. This is a equilibrium values of omega e and delta e.

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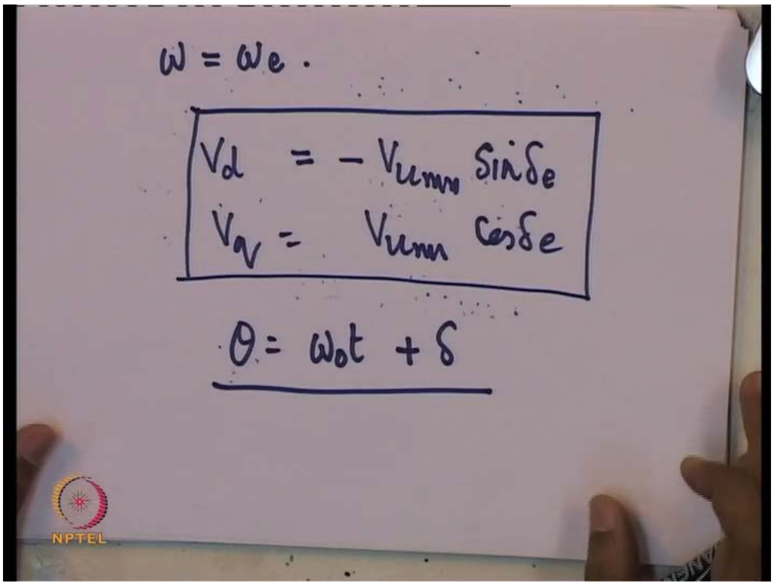

Thereafter, once if you know the equilibrium value of omega is omega e, also you have got the equations of the the differential equations corresponding to the fluxes. So, if you look at them here, I have written them down in compact form here.

So, you set this equal to 0. A 1 is a function of the time constants as well as omega but, you know that the equilibrium value of omega is omega e. So, you get an equation so, you can evaluate what a one is, this is set to 0. A 2 into i d i q, i d i q itself is a function of psi d and psi q, it is a linear function. So, A 2 into A 3 into psi basically, because i d i q itself is related by this expression. Thereafter, if you look at d 1, so b 1 is also a constant matrix. V d and V q in fact, if you look at what I am writing here, V d we have seen before by the definition of the voltage source, we have we wrote down the time, the voltage voltage of the infinite bus in terms of V a V b V c, as per that definition V d will come out to be minus V line to line rms into sin delta e and V q comes out to be V line to line rms into cos delta e.

So, this is of course, obtained we have obtained this before in our course, remember that the rotor position is nothing but, omega naught t plus delta. So, this is the definition of the rotor position, omega naught is the speed of the infinite bus and because of that and the definition of v a n, v b n and v c n the phase to neutral voltages of the infinite bus, we obtain v d and v q in this form. So, what I wish to stay what I wish to say here is that, first of all we have got the equilibrium value of the speed, then we have got the equilibrium value of delta e. From equilibrium value of speed and delta e, we can from the flux equations which are shown here, by setting the right hand side sorry the left hand side here equal to 0. That is put setting d by d t equal to 0, obtain the equilibrium values of psi d, psi q, psi F, psi H, psi G and psi K.

So, this is basically what is involved in getting first the equilibrium values. Once you have got the equilibrium values in order to do a small signal or a small disturbance analysis analysis around an equilibrium point. What we really need to do is? Reformulate the equations or linearized the equations, which is suitable for such small disturbance analysis.

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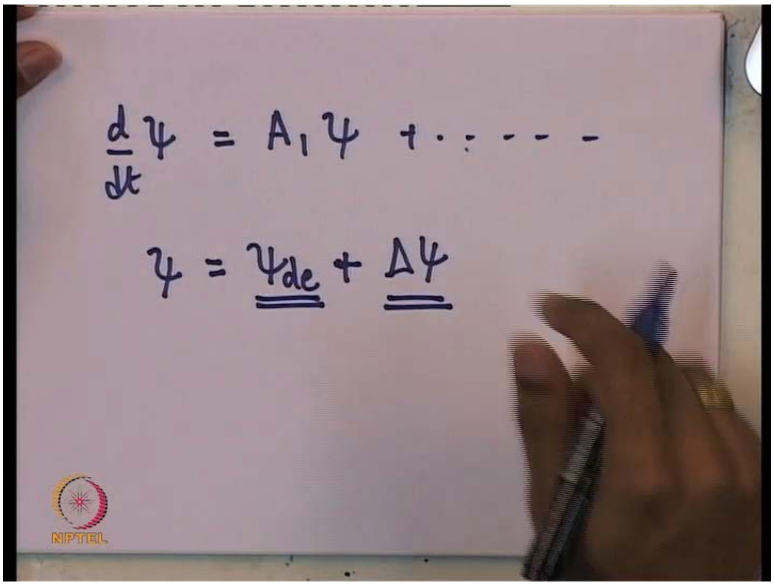

So for example, we have got d by d t of the flux is equal to A 1 into the flux and so on. Now, what we do is we assume that the fluxes, we are considering only small disturbances are from the equilibrium. So, we will assume that psi d e plus delta psi. So, this is the equilibrium value of the flux and this is the deviation from it.

So, we can reformulate the differential equations in terms of only the deviations around the equilibrium. Now, if these deviations are small, one can do a procedure called linearization.

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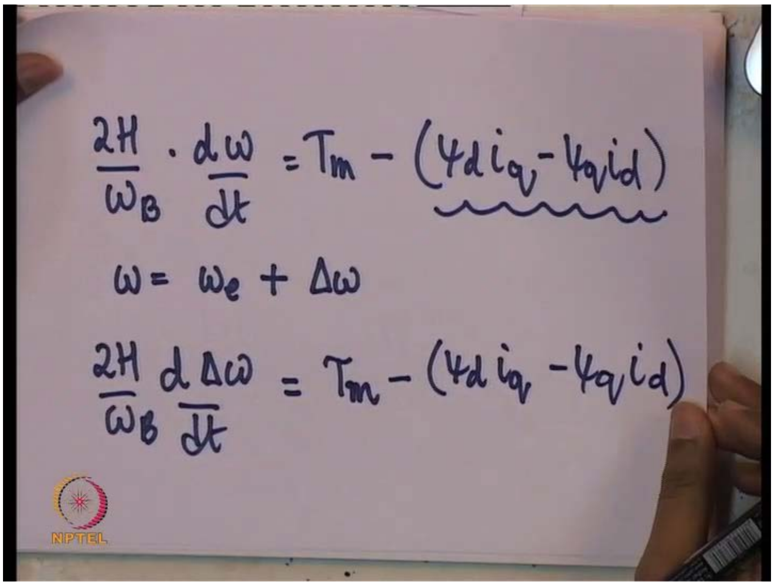

So, I will just show you the linearization procedure for a simple case, 2H we just look at this equation, you can actually linearize every equation equal T m minus psi d i q minus psi q i d. This is of course, a non-linear term, now we assume omega is equal to omega equilibrium plus the deviation from the equilibrium. This is a known quantity. So, we can reformulate these equations, you can write this as 2H by omega B d of omega e plus delta omega, omega is a constant, it is the equilibrium value of this speed. So, what we will do is, is equal to T m, we will assume that T m is a constant minus psi d i q minus psi q i d. Now, psi d i q minus psi q i d in fact can be written down as.

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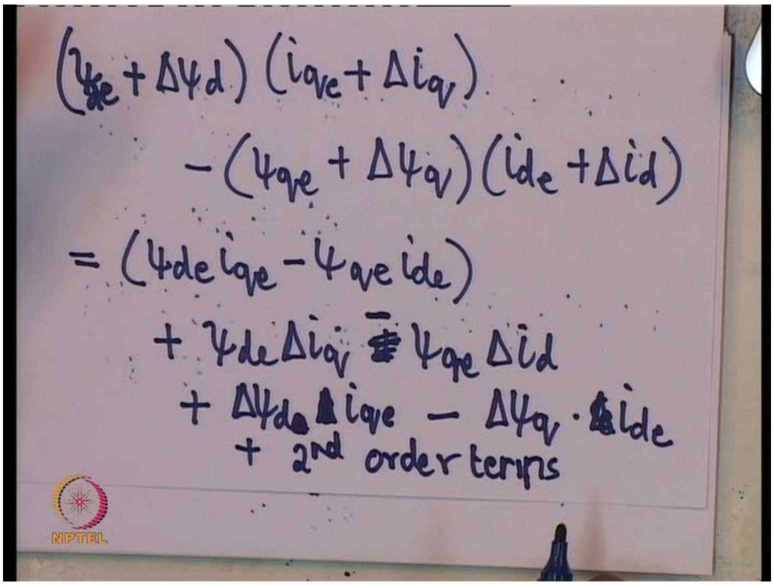

So, it is psi e psi d e plus delta psi d into i q e plus delta i q minus psi q e plus delta psi q into i d e plus delta i d.

So, this becomes if you neglect, well first let us write it down. So, what we can write it down is psi d e i q e minus psi q e i d e plus psi d e delta i q plus psi q e delta i d sorry this should be a minus, plus delta psi d e sorry I am sorry delta psi d into i q e minus delta psi q into i d e plus, what are second order terms? that is product of two delta kind of terms. Note that psi d e into i q e minus psi q e into i d e is in fact equal to the equilibrium value of the torque, electrical torque which is of course, going to be equal to the mechanical torque T m. So, psi d e into i q e minus psi q e into i d e is in fact T m.

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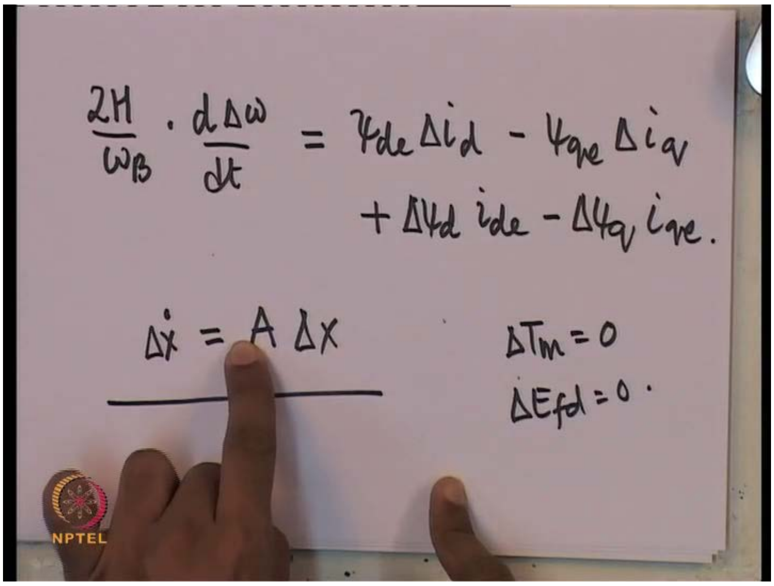

So, what we have essentially is the linearized equation 2H by omega B into delta, if you assume that T m is a constant in fact, you can write this as psi d e delta i d minus psi q e delta i q plus delta psi d i d e minus delta psi q i q.

So, this is just an example of how you can do linearization. So, you can in fact linearized the d psi d by d t equation and d c d psi by d t equation in a similar fashion. So, what you eventually get is, you know you have to couple of course; the mechanical and electrical equations the flux equations are coupled. So finally, what you will get is going to be delta x in the linearized states assuming of course, delta t m is equal to 0 and delta E f d is equal to 0. So, we will be assuming the inputs are constants, will be only giving changes in delta x that is giving the initial condition or initial disturbances to the states.

So, around the equilibrium point, the behavior of the equations for small disturbances can be studied using this. So, I have just shows you how you can do a linearization. So, I request you to go go back an just try to do the full system linearization and get it in the form delta x dot is equal to A into delta x. The properties of the transients for small disturbances around the equilibrium can be got just by analyzing the Eigen properties. That is the Eigen values and Eigen vectors of the resulting state matrix (()) of course, A now is dependent on the equilibrium point because, it really has terms like psi d e and so on. But, is a constant for that equilibrium point, it is the constant. But, it is a function of the value of the states at the equilibrium point.

So, one of the things you should remember is that a non-linear system, which is linearized and then we if we do non-linearized rather linearized analysis on it, Eigen analysis on on it, what we will see that the Eigen values which, we get for different equilibrium points are going to be different. So, the transient behavior around different equilibrium points in facts, is going to be different. It is not going to be the same. Did we actually notice this in our simulations, the answer is yes.

So, let us look at the simulations again carefully. So, back to the simulations, if you notice even one of the plots, which I had shown you, yeah maybe we can have a look at the speed transient. Well, you may say well, there is no difference in the kind of behavior around an equilibrium point, you have these swings. But, one of the simple things you can have actually look at look for rather is whether there is any change in the frequency of the swings. For example, the frequency of the swings swings, we will just do you do this again one second, we just get this back yeah. If you look at the frequency of swings around this equilibrium point yeah, I will just yeah.

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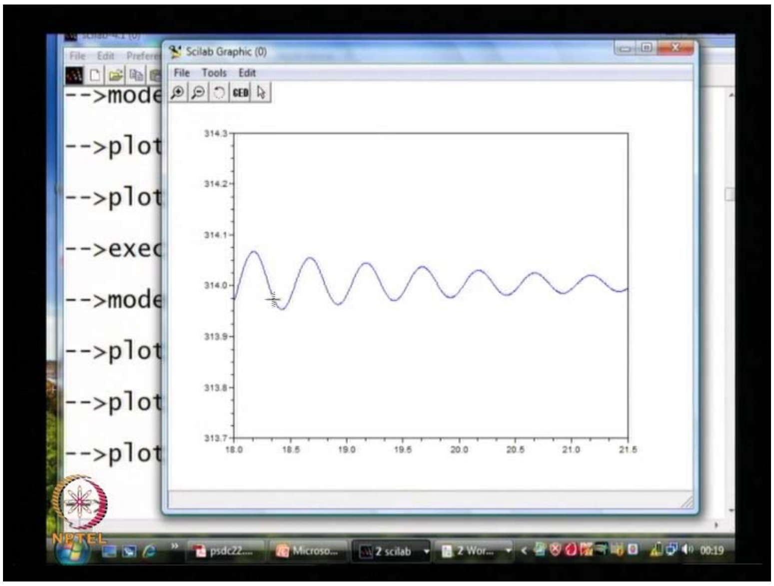

You look at this frequency of the swings around this equilibrium point, you will find it is well almost half a second. So, this is almost a 2 hertz oscillation, around the equilibrium point corresponding to I am sorry yeah, around the equilibrium corresponding point corresponding to T m is equal to 1 and E f d is equal to 2. The frequency of oscillations is in fact frequency of oscillations is in fact around 2 hertz. What about the frequency near about the equilibrium corresponding to T m is equal to 0.25 and E f d is equal to 1.

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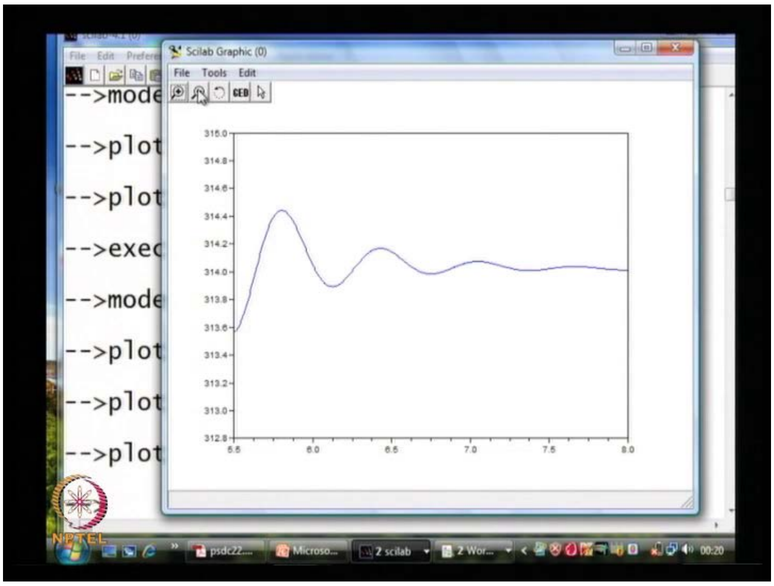

So, let us just have a look at that here. So, if you look at that, this is a roughly 6.1 seconds and this is roughly well, this is near about slightly more than the frequency, the period is more than 0.5. It in fact the frequency seems to be lesser near about T m is equal to 0.25 per unit and if you look at the behavior just of the synchronization, when mechanical power in fact 0.

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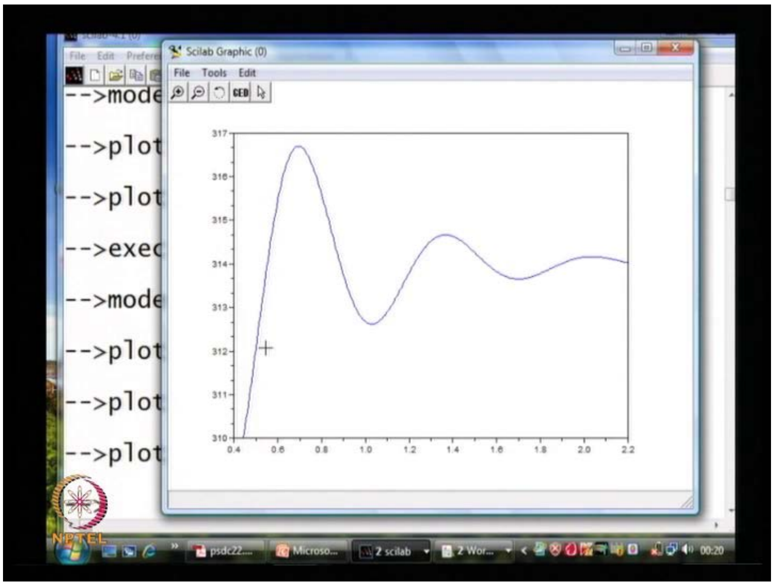

The frequency of oscillation is is again roughly 0.7 yeah. So, with T m is equal to 0 the frequency slightly lesser, in fact the period is larger and with larger values of T m with of course, a simultaneous increase of E f d, we see that the frequency in fact slightly increases, in fact it is higher here compared to here. So, as the operating point changes, we get a change in frequency.

That is not suppressing because, the Eigen values change because, the A matrix whose Eigen values, we are computing is the function of the equilibrium values of the states, which are obtain after linearization. So, that is basically why this happens, in fact one more striking thing, which you can observe from this plot. In fact all the plots revile a great deal of information. You will notice that the rate of decay of this oscillation is much much slower compared to the decay of oscillation here. In no time, you will find that this reaches equilibrium whereas; this takes several seconds to reach an equilibrium.

So, what one can expect after doing a linear analysis, an Eigen value analysis we will do that shortly of course, is that we can infer that whether, this frequency is going to change whether, the lamping is going to change and so on from that analysis itself.

So, let us just verify that, by actually writing down an analysis Eigen value analysis program. So, as I mentioned sometime back of course, it involve some some deal of effort in the sense that, you will have to linearized the set equations, form the A matrixes, the A matrix of the system, which is the function of the equilibrium point.

So, this is something you you will have to do. Now, one of the one of the points, which I need to of course, clarify this point the A matrix, which are going to get here, would be initially we will include the d psi d by d t and d psi q by d t transient. We will not convert d psi d by d t and d psi q by d t differential equation into algebraic equation. We will first consider all of them together.

So, what we are really looking at is a model, in which you are going to have this set of equations. So, this set of equations is what we are going to consider with the differential equations in psi d psi q retain, we are not neglecting or replacing d psi d and d psi q by d t equal to 0. But, we can always do that, in case we do that we will be converting two of these differential equations to algebraic equations, allowing us to eliminate two of these variables. So, two algebraic equations when we get, we can write psi d and psi q in terms of other states and get rid of them and reduce the number of differential equation differential equations.

We in fact did this, when we were doing the numerical simulation, in order to remove the stiffness. When you doing Eigen analysis, there is no compelling reason to remove the stiffness of the system, removing the the situation, where they are fast and slow transient that is what I mean, when I say removing stiffness. When you are doing Eigen analysis, there is no compelling reason whereas, for simulation if you want to use simple numerical integration methods, then you have to remove the numeric the stiffness in the equations.

So, let us now do an Eigen analysis of the system, I have written down a program again in (()) lab for doing the Eigen analysis.

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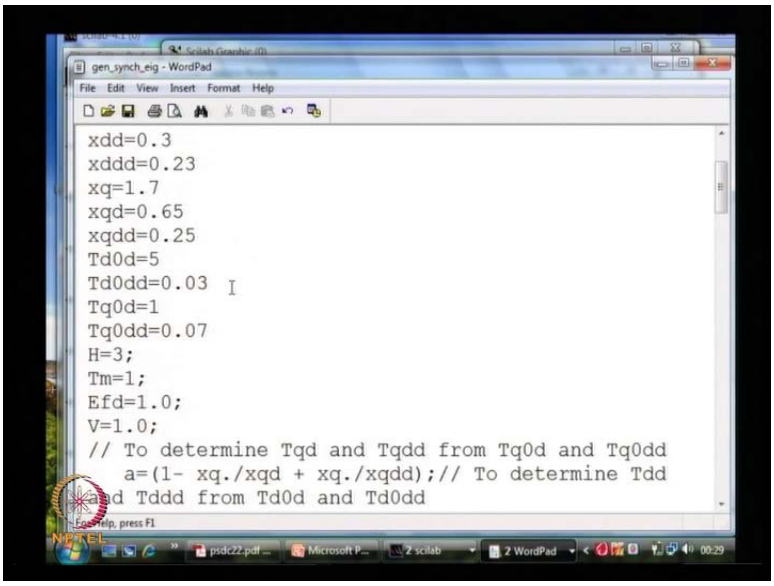

We will just run through it. The important differences between a Eigen value analysis program and a simulation program is that of course, we are talking in terms of a system operating initially at an equilibrium. So for example, if we look look at the equilibrium T m is equal to 0 and E f d is equal to one. From this, so I have saved this. From this, we could we would need to calculate the equilibrium value of the state corresponding to speed.

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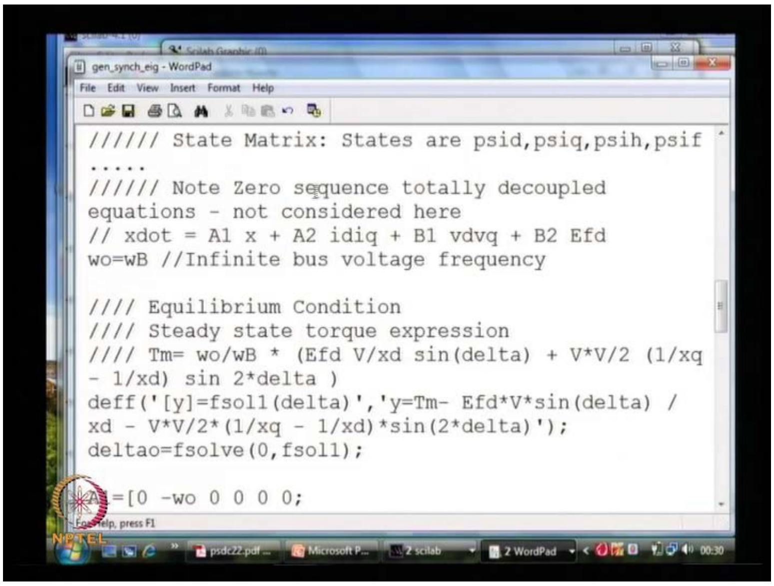

But, we know that omega e is equal to omega naught, which is equal to the infinite bus frequency. That is of course, the equilibrium value of the speed.

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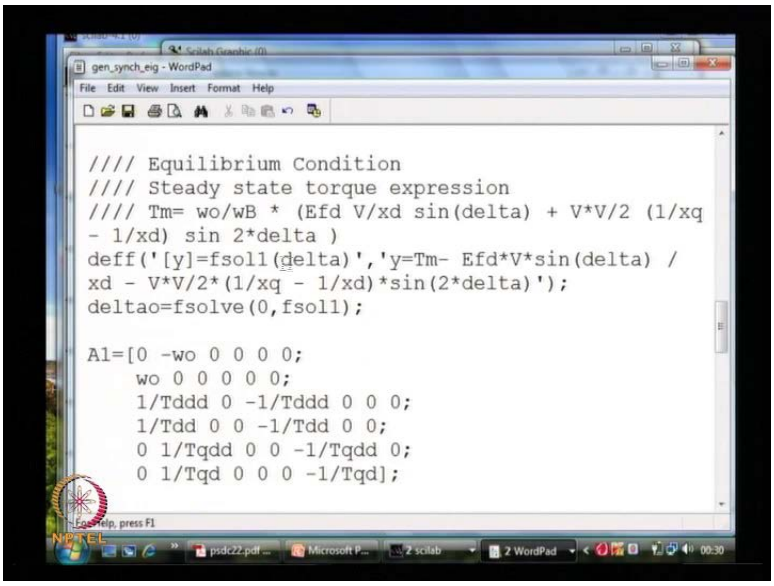

The equilibrium value of delta has to be obtained by solving this equation. Omega naught and omega B will of course, assume to be the same, the speed of the infinite bus we will assume to be equal to the base value of the frequency.

So, we can solve for delta. \Once we solve for delta we can also using the algebraic equations using the algebraic equations solve for fluxes and once we solve for fluxes, we can get the equilibrium values. Once we get the equilibrium values, we have to form the A matrixes from an linearized equations.

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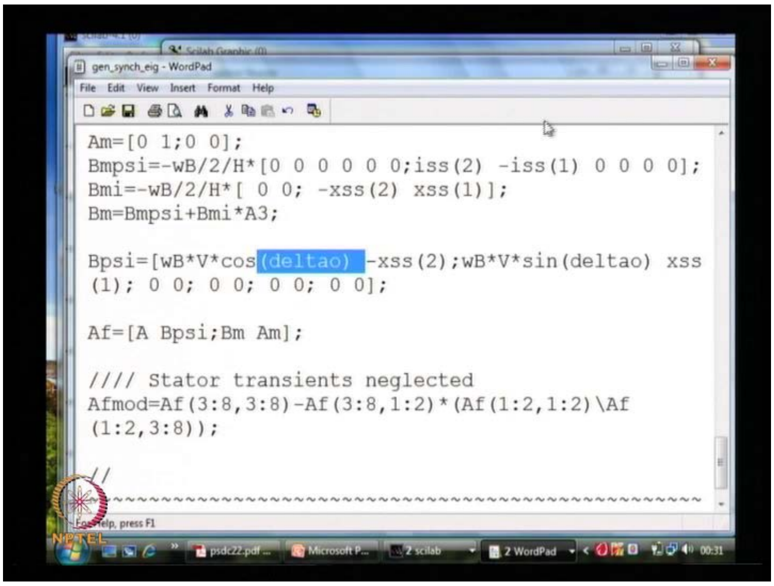

So for example, if you look at this equation, let me just in this equation, you realize that the components of the A matrix eventually, what I call is A f is consisting of the equilibrium values delta 0. So, remember that linearized matrixes are a function of the states, the equilibrium values of the states. So, what we will do is, now run this Eigen analysis program. I encourage you to try it out yourself, you try to write a program to do the linearized analysis. So, I will just run it for you here, (no audio 40:21 to 40:39). just one second. Remember that, we are doing the Eigen analysis for system around T m is equal to 0 and E f d is equal to 1.

So, one thing what we will do is, what is the equilibrium value of delta 0, the answer is 0. Of course, if mechanical power is 0 T m is equal to 0, you can directly infer from the steady state torque equation that delta 0 is equal to 0. Now, if I find out the Eigen values of the system, the Eigen values of a system linearized around the equilibrium point corresponding to T m is equal to 0.

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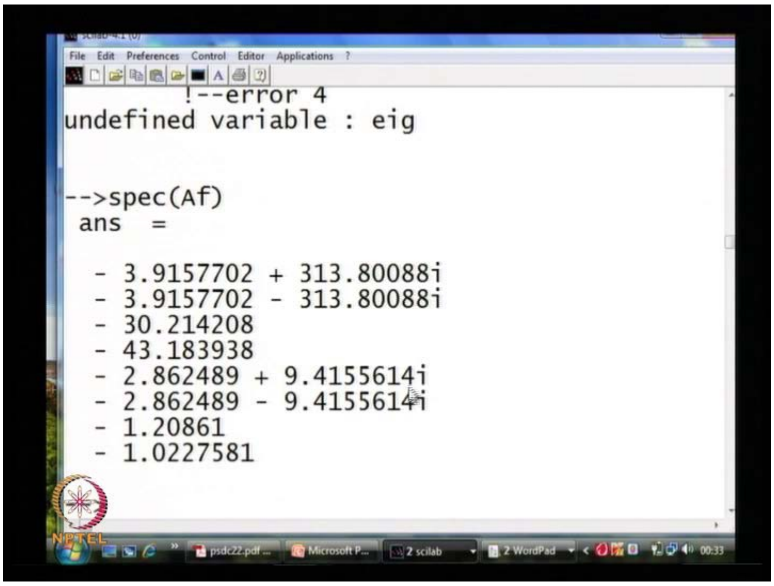

What we find is what we find is, the Eigen values are as seen here. What we see is? You have got these two Eigen values, which are very large in magnitude. They almost 314 radians per second, as discussed in the short circuit analysis of of a synchronous machine, these are actually corresponding to the d psi d by you know they are very much associated with the psi d and psi q states. You will find that these two sort of new Eigen values are coming, because of the inclusion of the electro mechanical equations. The mechanical equations contribute the equations corresponding to omega and delta.

So, basically you get these two extra Eigen values compared to the Eigen value analysis of a short circuit synchronous machine. These two extra Eigen values in fact, indicate that you should be having a damped oscillation of frequency 9.4 radians per second. So, 9.4 radians per seconds corresponds to divided by 2 pi, corresponds to 1.49 hertz.

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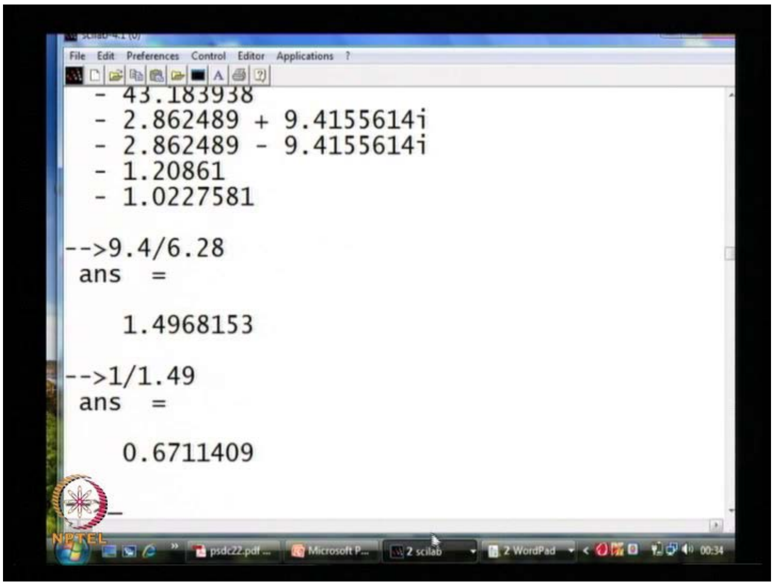

So, one upon 1.49 hertz is in fact 0.67 seconds, which is the period of the oscillation corresponding to T m is equal to 0. So, if you look at this again, this frequency out here should be around 0.7, that is what Eigen value analysis predicts. So, is it actually true, look at it again yeah, this is around 1.56 and this is roughly 2.4. So, that is around near about 0.7 to 0.8 second.

So, actually what we are getting through Eigen analysis is in fact, quite accurate. Also, you can see the rate of change of the peaks, rather you can see that the peak value of this oscillation keeps decaying with time. I leave this as an exercise to you to verify that the real part of this Eigen value corresponds to the rate of decay, which is seen in this figure. We redo the Eigen analysis with E f d is equal to 2 and T m is equal to 1, so we save this and rewrite redo it. Oops So, what we need to do is of course, yeah. So, if you take out the Eigen values, you will find that the Eigen values you the most striking thing you will notice is that the frequency of oscillations has increased, the damping has come down. This real part, negative real part magnitude has come down. So, what one one can get from the simulation essentially is what one should see in the simulation is that the damping is this is of course, the plot of delta, you see that the damping is much faster near the T m is equal to 0 case, as compared to the damping near T m is equal to 1.

So, this is the plot of delta remember. The frequency of oscillations here is higher than the frequency of oscillations here. So of course, we can also see this in omega, the the simulation of omega so, I just redo this and plot and plot omega. So, I just remove this and plot omega yeah. So, the same thing is of course, seen omega, remember the same modes are been seen in different all all almost all the states. So, what you see in delta are the same model characteristics are seen in omega. So, you see that the frequencies at different equilibria are different and the damping also is is different. I mean this takes longer to damp out compared to the damping here.

So, the real part of the Eigen value near T m is equal to 0, can be excepted to be larger than the real part, negative real part of course, the magnitude is lower in this. This is actually true. So, this is basically a good correlation between Eigen analysis and the simulation.

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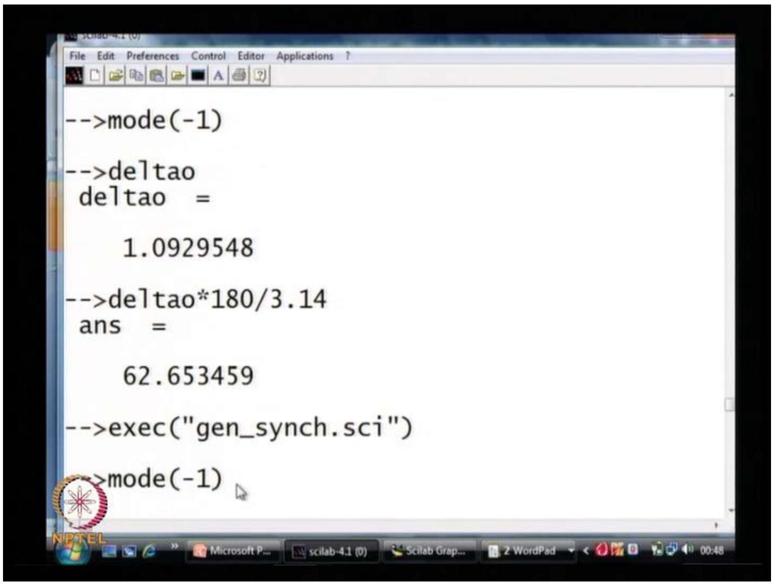

So, this is what we see here, in fact if you look at the value of delta 0 here, from the Eigen analysis sorry the computation of I have to redo the Eigen analysis program. So, the value of delta, equilibrium value of delta, when T m is equal to 0 and E f d is equal to 1, is 1.09 radiant. So, that actually we can convert this radiant value to a degree value, in fact it is 62 degrees. So, in fact if you look at the simulation, we will redo the simulation and plot the value of delta.

So, what we will do is, this of course, omega we will plot the value of delta with T m is equal to, so if you look at the simulation, the simulation also if you look at the value of delta which, to with this settles down is also near 1.09. So, what you are really seeing is a good correlation between the numerical simulation as well as the Eigen analysis of a system linearized around an equilibrium point. So, this is an interesting study in which, we have actually done the linear analysis also of the synchronization transients.

So, I hope you got the hang of what was what we are trying to do. We could really, it is very interesting, you know when you do a simulation, you get a certain time response plot but, you feel a great deal of joy, when you are able to correlated it with Eigen value analysis, which one many of us feel that is the more analytically and it gives better insight. So, it is important whenever you do a numerical simulation, whenever you study a transient, it is very important to interpret the results, which are coming correctly and you should be able to correlate it with the kind of inferences you get from other tools.

So, I hope you got a flavor of this, in this lecture. In the next lecture, we will try to do a simplified, I will I will try introduce you to some simplified models of a synchronous machine. In fact, we have used what is probably a full blown model in all our analysis so far. We will try to do the reverse thing, what we will do is try to make more and more simplifications and come down to a bear bound model or rather the classical model with which, we had actually done some very simple studies right at the beginning of our course. So, we will really know by looking at what kind of simplifications we have done. We can arrive at the toy model, which we considered right at the beginning of the course, that itself is a nice exercise, where we come to know what all we have actually brushed under the carpet in order to get a simple toy model.

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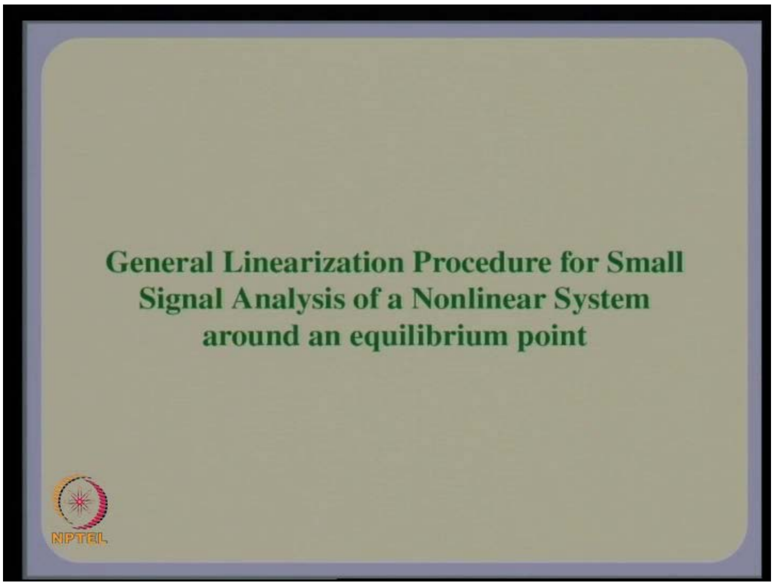

Before we move on, we will discuss again the general linearization procedure for small signal analysis of a non-linear system around an equilibrium point. So, although I have done the linearization of this system, it would to be good for you to be familiar and and at e is with the general linearization procedure.

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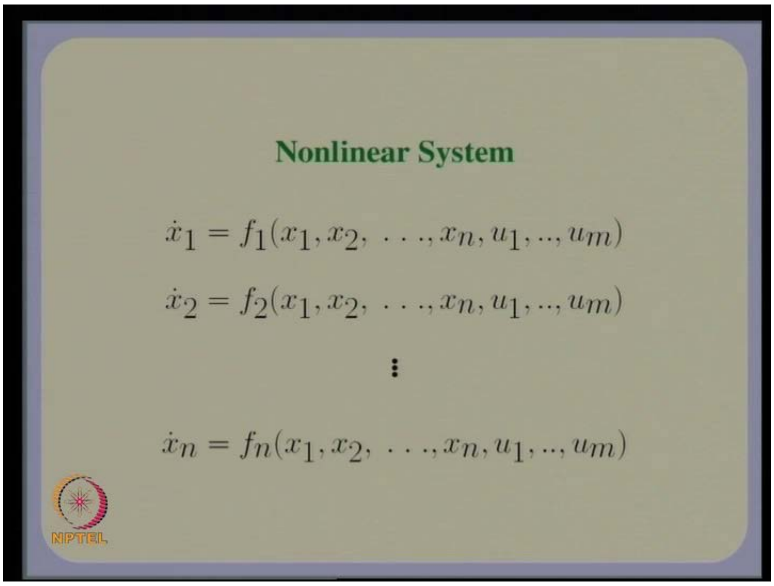

So, if you if you have say a non-linear system of this kind, x dot x 1 dot is nothing but, d x 1 by d t and you have got n variables of this kind, then the general non-linear equations are given as shown in this screen on the screen.

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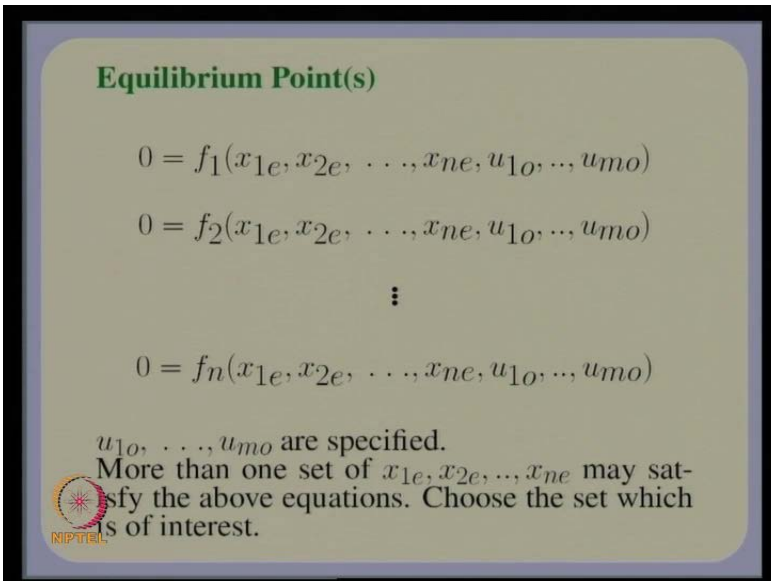

Now, the equilibrium points for this system are obtained by setting x 1 dot x 2 dot and so on equal to 0. Of course, in this non-linear system, u denotes the inputs. So, if I given the inputs that is u 1 0 u n 0 are specified, those are the nominal inputs which are given for this equilibrium point, then the equilibrium values of the states may be obtained from these algebraic equation, these again are non-linear algebraic equations. The equilibrium points are obtained by solving these non-linear algebraic equations. Remember that, more than one set of equilibrium states may satisfy the above the equations. So, you have may have more than one set.

Now, in the general linearization procedure, we will try to or we will choose the set, which is of interest. I mean, you are really going to do small signal analysis around an equilibrium point. So, you have to decide the equilibrium points. So, choose the set which is of interest.

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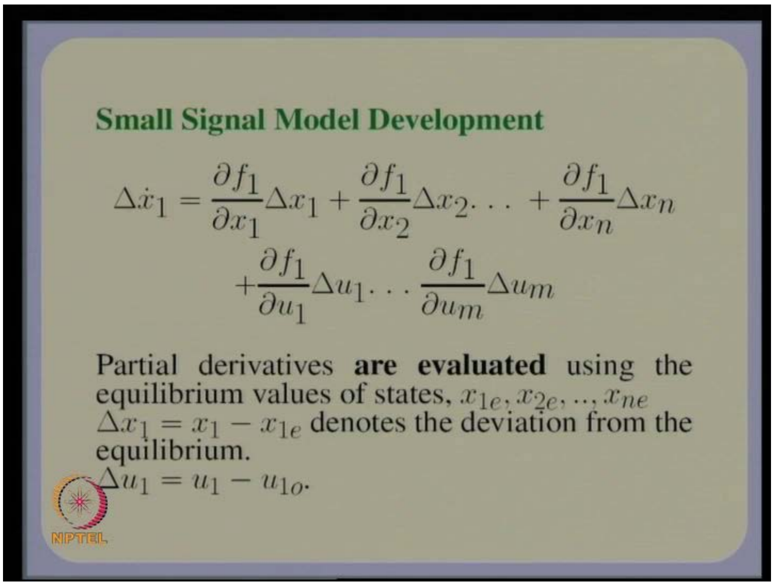

After that, you need to take each differential equation, each non-linear differential equation and linearized it. So, what this involves I mean, what what eventually you need to do is, obtain the partial derivatives of f 1 with respect to the states x 1 x 2 to x n as well as the into u 1 to u m. Now, remember that these partial derivatives are evaluated using equilibrium values of the states. So, these partial derivatives are essentially constant.

Remember of course, that delta x 1 here, is a small disturbance from equilibrium. So, delta x 1 is equal to x 1 minus x 1 e or the deviation from the equilibrium. Similarly, delta u is equal to u 1 minus u 1 0 u 1 0 is a nominal input. So, this is what you get as a linearized equation. Now, in my previous development, we we just for example, linearized psi i d i q minus psi q i d. You may say that well this looks different from that. No, actually if you take out a partial derivatives of the of the function i d i q minus psi q i d and plug in the equilibrium values you are going to get exactly this same linear model as we got some time back.

So, in fact this is the more general and direct you know representation of what we will get, if we apply this linearization with delta x and delta u very small. So, this is how you would normally linearize a set of non-linear differential equations around an equilibrium point.

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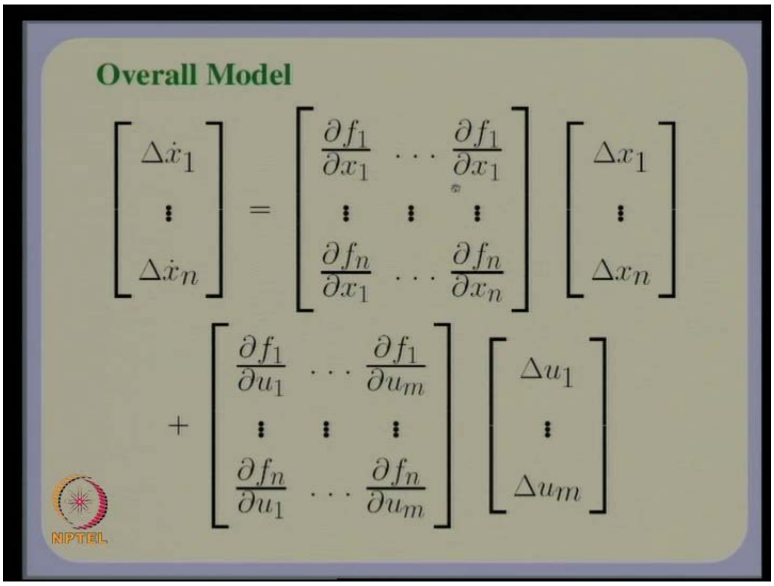

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \vdots \\ \Delta\dot{x}_n \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_1} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \vdots \\ \Delta u_m \end{bmatrix}$$

So, this fact will become cleared shortly remember of course, that you have to linearized each differential equation right from x 1 to x n and therefore, eventually you will get in fact a Jacobean matrix. So, you know you do this for x 1 but, you can also do it for the other x x 2 x 3 to x n and eventually you will get this delta x is equal to A into delta x plus B into delta u. This A and B of course, are these matrixes.

Note that, these are effectively constant matrixes because; these partial derivatives are evaluated at the equilibrium points. So, equilibrium point essentially is the denotes the values of the states. So, you have to plug in the values of the states. Now, this will become clear.

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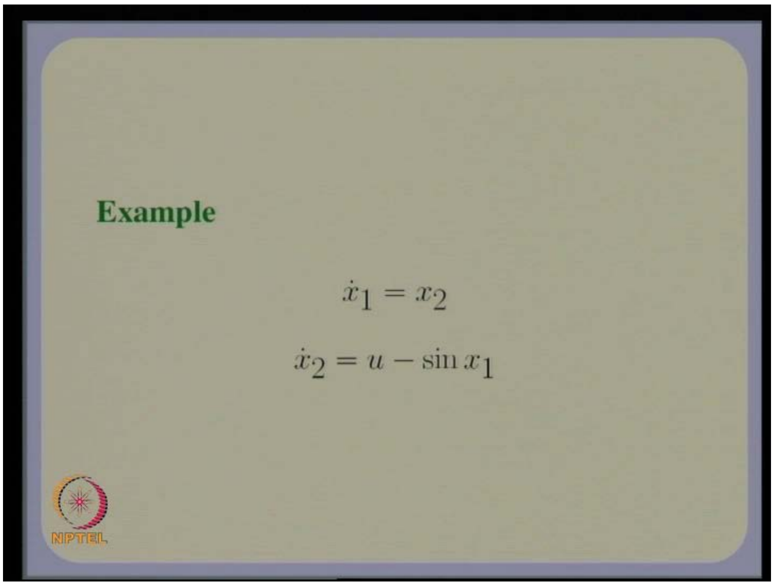

If you consider this example, x 1 dot is equal to x 2 and x 2 dot is equal to u minus sin x 1. So, this is a non-linear set of equations, the nonlinearity obviously is because of the sin term.

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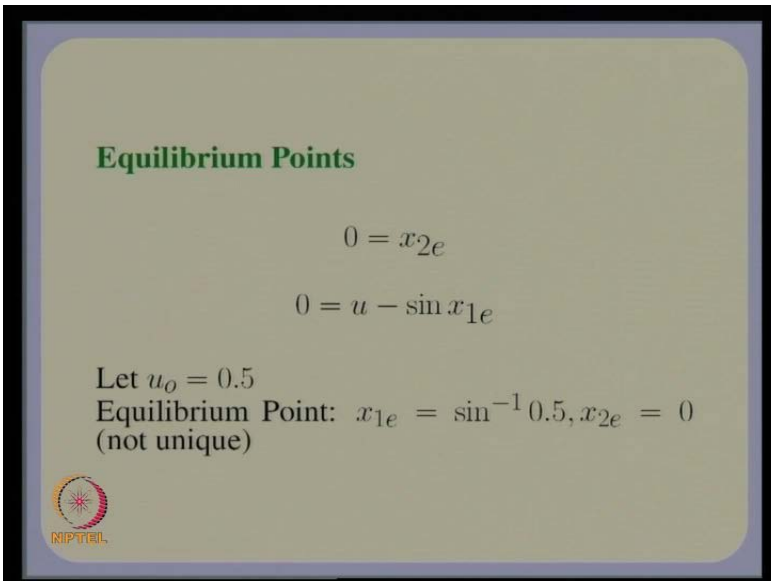

So, the first step in doing this is, find the equilibrium points for the nominal input 0.5, the equilibrium point is sin inverse 0.5 or sin inverse half, which is 30 degrees and x 2 e is equal to 0. So, x 1 e is in fact 30 degrees this is of course, the equilibrium points are not unique. For example, x 1 e could also be pi minus that is 180 minus 30 degrees. That is also in equilibrium point.

So, remember that there is in the non-linear systems, they may be more than one equilibrium point. Let us say the equilibrium point corresponding to sin inverse sin inverse 0.5 is equal to 30 degrees and x to e is equal to 0, is the equilibrium point of our interest. So, equilibrium point of our interest is 30 degrees and 0. Let us say, suppose this is our equilibrium point of interest.

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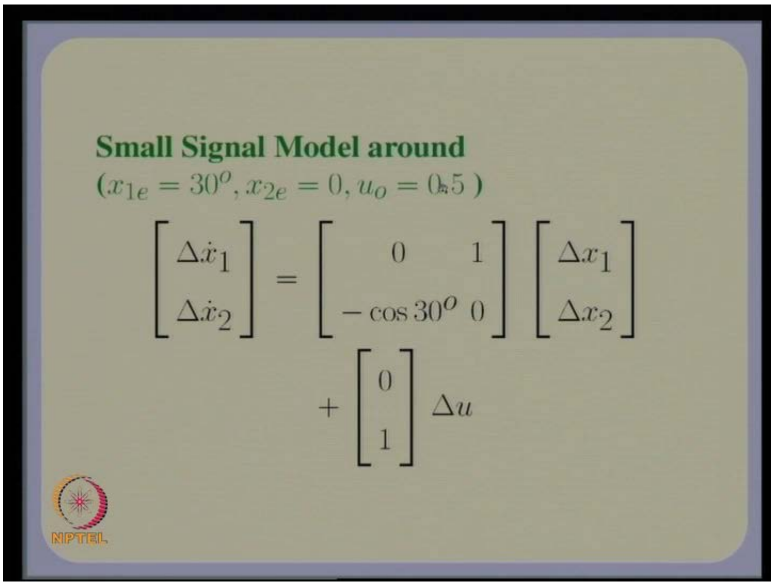

So, you can easily obtain the small signal model, this small signal model is essentially a linear model around this equilibrium point. So, this is eventually the small signal model of the system. One could also in fact, try to find out the equilibrium point; you know the linear model around another equilibrium point. So for, in this case for example, the other equilibrium point is x 1 e is equal to 150 degrees, x 2 e is equal to 0 and u naught of course, is the nominal value 0.5. And interestingly, the properties of the the small signal stability properties of different equilibrium points can be different. So, that is the very interesting and important thing, which you should keep in mind.

So, this is the general procedure, when you want to linearized a non-linear system around an equilibrium point. So, you can directly take partial derivatives and from the Jacobean matrix. Now, I wish to point out again that, in the example we have considered and where I have shown you how to linearized particular differential equation or this you know the the torque equation effectively. Have linearized the electrical torque but, it may not be directly evident that you are in fact you are doing exactly this. In fact, I have in fact taken out the partial derivative of the expression of t e with respect to psi d psi q i d and i q and plugged in the equilibrium value.

So this is but, a more general way of be general way of putting it. That is you take out the partial derivatives or evaluate this Jacobean at the equilibrium values of the states. So, eventually what you get is a linear model of a system with a constant coefficient matrixes and another point to be noted is of course, that is the multiple equilibrium points, you could have different small signal models around different equilibrium points and of course, the properties the stability properties of the A matrix a matrixes are around different operating points are equilibrium points could be different. So, this is something you should keep in mind. Now, we go back to our example again of a synchronous machine.