

Power System Dynamics and Control
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Lecture No. #20
Synchronous Machine Modelling
Short Circuit Analysis (Contd)

We now continue in this lecture, with the analysis of the short circuited generator. In the previous class, we just started upon what happens when you have the synchronous machine running at a constant speed under open circuit conditions, and we apply a step voltage at the field winding. The voltage which we apply at the field winding is such that under open circuited conditions, we will get one per unit line to line rms voltage at a terminals of the stator; assuming of course at the stator is connected as a star is connected in star. Now, we did not actually complete that part of the exercise at a written a small psi lab program to see the transient behavior of an open circuited generator. So, we continue with our discussion of an open circuited generator by the end of this lecture, we would have come to the short circuited generator, and a transient response of a synchronous machine which is short circuited.

So, actually what we will be doing is studying two transients, we will assume the machine is rotating at the synchronous speed, then we give a step change in the field voltage. So, that the voltage builds up in a generator, the generator gets excited and then we short circuit the generator, and see the transients which arise due to that.

Now, in this particular study we will of course, not be considering the interaction with the mechanical system. We will assume that the machine is running at a constant speed, we will assume it is running at the rated or the base speed. Of course, as we shall see later under short circuit conditions a non zero torque is created, whenever we apply short circuit. So, in fact the speed will be affected in a certain way, but for the time being we do not consider that that interaction with the mechanical system. We will of course, do that later in this course, in fact a large part of our course will be dedicated to studying electro mechanical oscillation which we shall see later.

If you look at the synchronous machine equations, in fact just the flux equation. Since, you have not considering the mechanical equation or the torque equations.

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$$B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_d'} \cdot \frac{x_d'}{x_d - z_d} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

NOTE

$$\underline{\dot{\psi}} = A_1 \underline{\psi} + A_2 \underline{i} + B_2 E_{fd}$$
$$A_3 \underline{\psi} = \underline{i}$$

The flux equations are in fact linear, and these transients in fact can be studied by a simple linear system analysis that is using Eigen values, Eigen vectors and the complete response can be characterized and we can plot the response. We do not have to do a numerical integration because this is a linear system and response of the system comes out to be in a nice closed form. So, that is what we discussed in the previous class. We just revise what we did then we have got the equations of the flux of the machine under constant speed it turns out that A_1 , A_2 , B_2 will be constant matrixes E_{fd} is a step of one per unit recall that E_{fd} is proportional to the field voltage in fact, we apply the field voltage so that we will have E_{fd} as one per unit or in other words in steady state; we will have the line to line voltage of a star connected synchronous generator to be one per unit.

So, that that is the voltage we apply at the field recall that we have formulated our model in such a way that we do not really specify the field voltage but we directly but we say we give the field voltage in terms of what effect it has on the open circuit voltage in steady state. So again let me repeat that we apply a field voltage such that E_{fd} is one or the open circuited line to line rms voltage of the synchronous machine is star connected is one per unit. So, we assume of course this is speed is constant and we order the states in this fashion i_d and i_q in fact you may wonder, what is where is v_d and v_q here? v_d and v_q are the voltages at the terminal of the generator. What we assume of course is that the terminal voltage of the generator is written in terms of the current.

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$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_L & 0 \\ 0 & R_L \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$R_L \rightarrow \infty$ - open ckt
 $R_L = 0$ - short ckt.

i_d and i_q and since, we are considering the open circuited case we will assume that R_L is a very large we will R_L as very large.

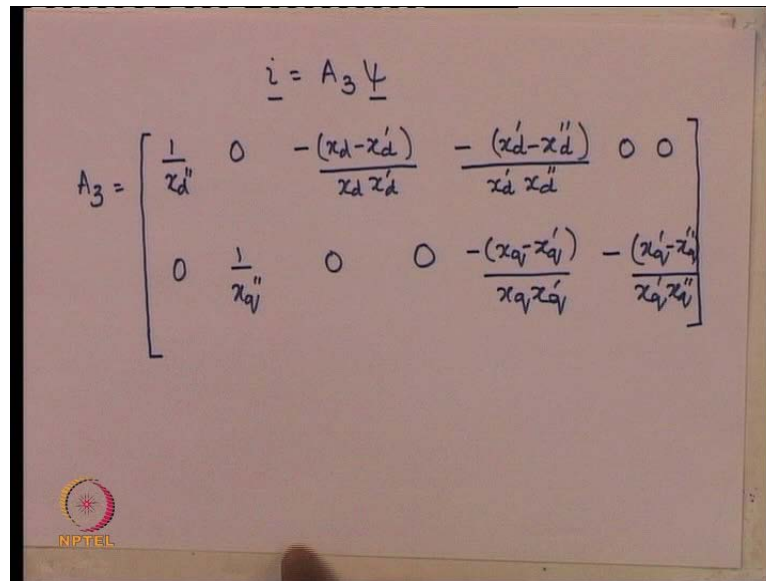
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$$A_1 = \begin{bmatrix} 0 & -\omega & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T_d'} & 0 & -\frac{1}{T_d'} & 0 & 0 & 0 \\ \frac{1}{T_d''} & 0 & 0 & -\frac{1}{T_d''} & 0 & 0 \\ 0 & \frac{1}{T_q'} & 0 & 0 & -\frac{1}{T_q'} & 0 \\ 0 & \frac{1}{T_q''} & 0 & 0 & 0 & -\frac{1}{T_q''} \end{bmatrix}$$

So, open circuited generator is represented as star connected load with the resistances values R_L as very large now. So, in some sense the effect of v_d and this v_d and v_q is got subsumed into A_2 since your expressed in terms of i_d and i_q . A_1 looks like this, we did it in the previous class, ω is the speed of the machine we will assume that in this study this speed is constant and equal to ω .

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$$\underline{i} = A_3 \underline{\psi}$$

$$A_3 = \begin{bmatrix} \frac{1}{x_d'} & 0 & -\frac{(x_d - x_d')}{x_d x_d'} & -\frac{(x_d' - x_d'')}{x_d x_d''} & 0 & 0 \\ 0 & \frac{1}{x_q''} & 0 & 0 & -\frac{(x_q - x_q')}{x_q x_q'} & -\frac{(x_q' - x_q'')}{x_q' x_q''} \end{bmatrix}$$


Base or the rated value of the synchronous generator i_d and i_q in fact are related to the fluxes, as I shown some time back the fluxes by a matrix of this kind, remember that this is this model is a per unit model. We are talking in terms of x_d dash x_q dash but remember that earlier, we are talking in terms of l_d dash and l_d double dash in per unit l_d dash l_d double dash is equivalent to x_d dash and x_d double dash and so on.

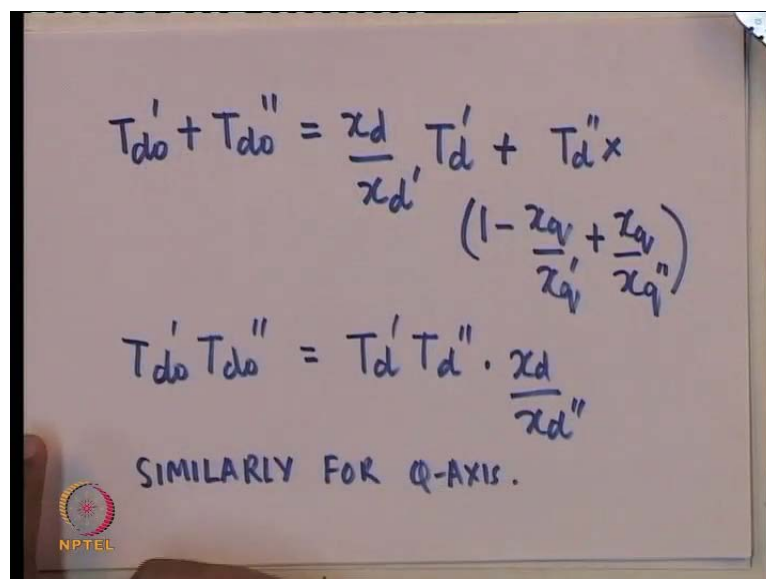
So, just remember that this is a per unit model B2 off course is this remember we are using model two with the assumption that t_{dc} double dash is equal to t_d double dash.

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$$T_{do}' + T_{do}'' = \frac{x_d}{x_d'} T_d' + T_d'' \left(1 - \frac{x_q}{x_q'} + \frac{x_q}{x_q''} \right)$$

$$T_{do}' T_{do}'' = T_d' T_d'' \cdot \frac{x_d}{x_d''}$$

SIMILARLY FOR Q-AXIS.




The data for this generator, I will just show it to you on a slide.

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Data for example

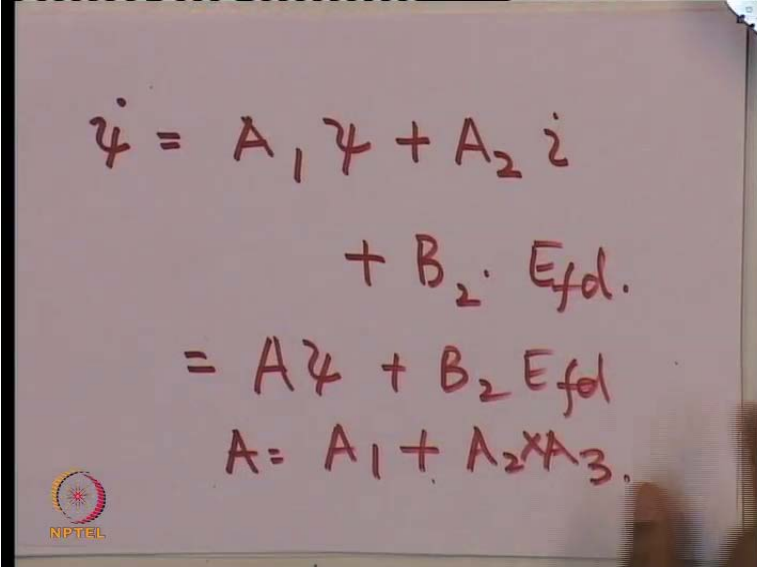

- Use Model II
- $x_d = 1.8, x_d' = 0.3, x_d'' = 0.23,$
 $T_{do}' = 5.0 \text{ s}, T_{do}'' = 0.03 \text{ s}$
- Q-axis : $x_q = 1.7, x_q' = 0.65, x_q'' = 0.25$
 $T_{qo}' = 1.0 \text{ s}, T_{qo}'' = 0.07 \text{ s}$

$\beta = 0.003$



So, the data for this is given in this slide we will be using model two of course, and T_{d0} will of course, take as 5 seconds not 8 seconds as given before. Now what we will do is of course, kind of a write a program to plot the values of the transient the transients seen in the voltages currents and so on for this particular transient. So, the transient we are considering is a step change in the field voltage we will of course, do the short circuit subsequently so right now we will of course, do the step change in field voltage recall that once we have got the equations of the machine in this form.

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$$\begin{aligned}\dot{\psi} &= A_1 \psi + A_2 i \\ &\quad + B_2 \cdot E_{fd} \\ &= A \psi + B_2 E_{fd} \\ A &= A_1 + A_2 A_3.\end{aligned}$$


Which is a purely state space form $\dot{\psi} = A \psi + B_2 E_{fd}$ remember that i is nothing but $A_3 \psi$ so finally, we get this $\dot{\psi} = A \psi$

plus B2 this is typical input output rather state space form of the machine equations. And we saw in the previous class that you could get the analytical expression for the response in this fashion.

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$$\psi(t) = e^{At} \psi(0) + A^{-1} [I_{6 \times 6} - e^{At}] B_2$$

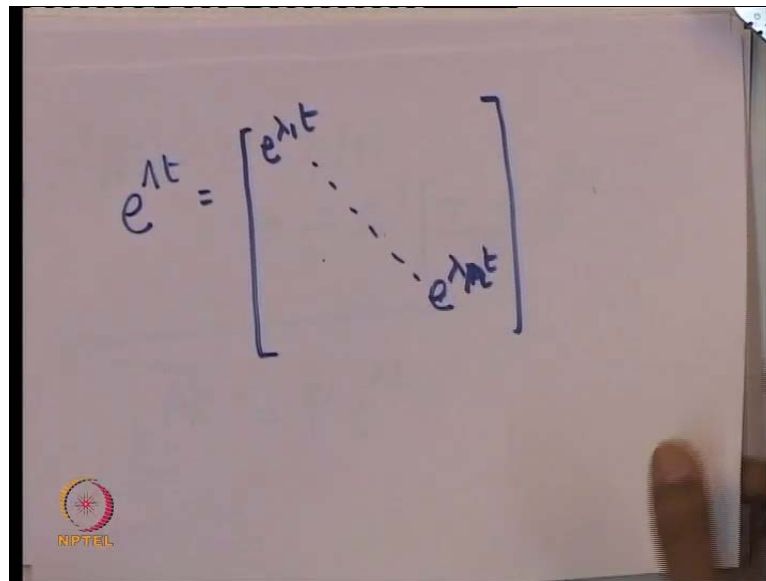
$$e^{At} = P e^{\Lambda t} P^{-1} \quad P \rightarrow e v$$

$$\Lambda \rightarrow \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \\ & & & \lambda_3 \end{bmatrix}$$

And that really boils down to evaluating this expression here. So, we do not do a numerical integration because it is not necessary we have got a close form expression, we will plug in the values of time in order to obtain a final response in this particular expression P is in fact the Eigen vector right. Eigen vector matrix which we have talked in the first you know, the first ten lectures of this course you have discussed analysis method for dynamical systems where define the right Eigen vector.

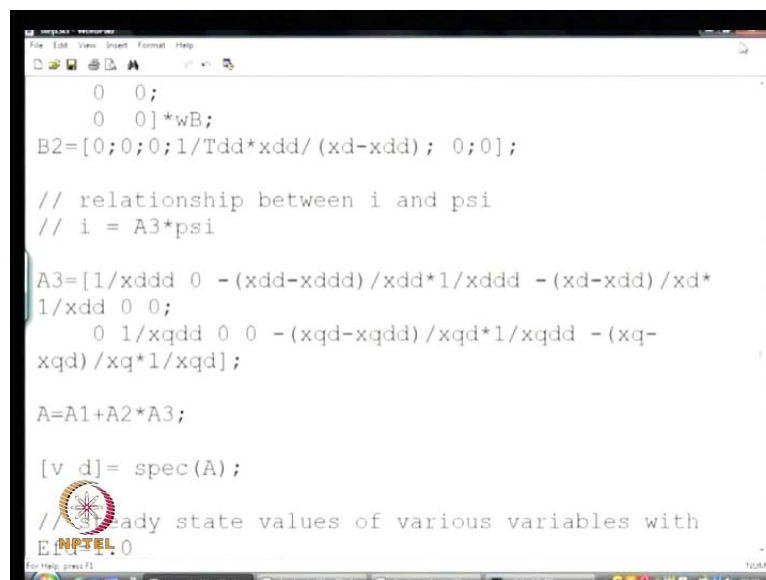
And this matrix here is a diagonal matrix in fact there are A is of six by six matrix. So, actually in this case you will have six Eigen values not three as shown here. And this is defined in this fashion.

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$$e^{At} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & \dots & \\ & & e^{\lambda_n t} \end{bmatrix}$$

So, this is where we were last time what we will do now is of course, I will show you the program corresponding to this particular transient. So, I have programmed this of course, using psi lab, and we had seen the program in the previous class we will just have a look at it fast, again and then run it.

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```
0 0;
0 0]*wB;
B2=[0;0;0;1/Tdd*xdd/(xd-xdd); 0;0];

// relationship between i and psi
// i = A3*psi
A3=[1/xddd 0 -(xdd-xddd)/xdd*1/xddd -(xd-xdd)/xd*
1/xdd 0 0;
0 1/xqdd 0 0 -(xqd-xqdd)/xqd*1/xqdd -(xq-
xqd)/xq*1/xqd];

A=A1+A2*A3;

[v d]= spec(A);
// ready state values of various variables with
E=NIPTEIL.0
```

So, let us go to that program. So, we are at the psi lab work space at the present time, I will show you the program it is called step dot sci. We have seen it last time we will just run through it again quickly. So I have enter the data it is it is quite intuitive and easy to follow, I will just rerun it again re through it of course, we have given the standard

parameters here we have been given the some the reactance's as well as the time constants.

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```
w=wB;
// Speed is assumed to be constant in this study
Ra=0.003
xd=1.8
xdd=0.3
xddd=0.23|
xq=1.7
xqd=0.65
xqdd=0.25
Td0d=5
Td0dd=0.03
Tq0d=1
Tq0dd=0.07

// determine Tdd and Tddd from Td0d and Td0dd
a=(1- xd./xdd + xd./xddd);
```

We have been given what are known as the open circuit time constants, why these are called open circuit time constants will become apparent soon. Of course the way we have formulated our equations.

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```
// To determine Tdd and Tddd from Td0d and Td0dd
a=(1- xd./xdd + xd./xddd);
b=-(Td0d + Td0dd);
c=(xddd./xdd) .*Td0d.*Td0dd;
Tddd1= (-b + sqrt(b.*b - 4*a.*c))./(2*a);
Tddd2= (-b - sqrt(b.*b - 4*a.*c))./(2*a);
Tddd= min(Tddd1,Tddd2);
Tdd = Td0d.*Td0dd.*(xddd./xd) ./Tddd;

// To determine Tqd and Tqdd from Tq0d and Tq0dd
a=(1- xq./xqd + xq./xqdd);
b=-(Tq0d + Tq0dd);
c=(xqdd./xqd) .*Tq0d.*Tq0dd;
Tqdd1= (-b + sarrt(b.*b - 4*a.*c))./(2*a);
```

We have formulated them in terms of the time constant Td dash Td double dash etcetera. but remember you can get Td dash and Td double dash using these two equations. In

fact you get a quadratic which we have to solve in order to get T_d dash and T_d double dash and similarly, T_q dash and T_q double dash.

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Handwritten equations on a whiteboard:

$$T_{do}' + T_{do}'' = \frac{x_d}{x_d'} T_d' + T_d'' \left(1 - \frac{x_{qV}}{x_q'} + \frac{x_{qV}}{x_q''}\right)$$

$$T_{do}' T_{do}'' = T_d' T_d'' \cdot \frac{x_d}{x_d''}$$

SIMILARLY FOR Q-AXIS.

NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, that is what is shown on the screen at present from the parameters. We are in fact obtaining T_d dash and T_d double dash similarly, T_q dash and T_q double dash we of course, taken the states in this fashion and we have we taken $R L$ to be a very large value in order to mimic open circuit conditions.

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```

0 0;
0 0;
0 0]*wB;
B2=[0;0;0;1/Tdd*xdd/(xd-xdd); 0;0];

// relationship between i and psi
// i = A3*psi

A3=[1/xddd 0 -(xdd-xddd)/xdd*1/xddd -(xd-xdd)/xd*
1/xdd 0 0;
0 1/xqdd 0 0 -(xqd-xqdd)/xqd*1/xqdd -(xq-
xqd)/xq*1/xqd];

A=A1+A2*A3;

[v,d]=spec(A);
// ready state values of various variables with

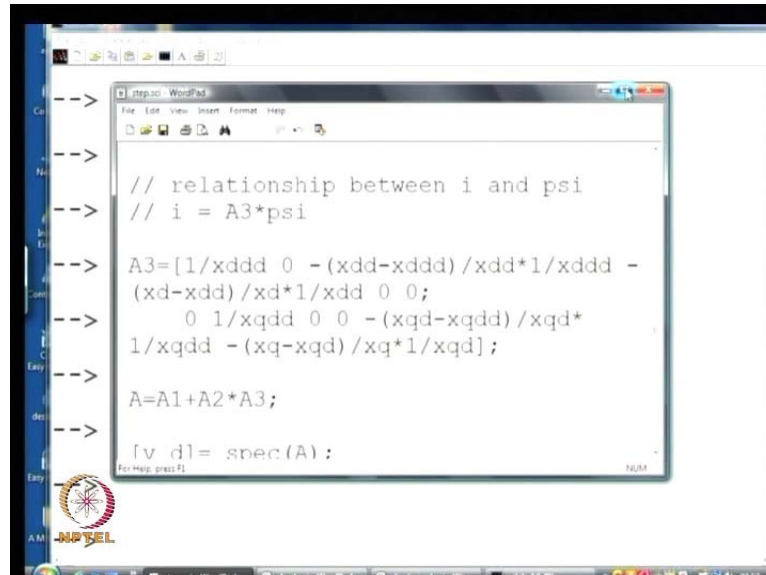
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And this is the A matrix, this is the A_2 matrix, this is the B_2 matrix, this is the A_3 matrix, this is the final state space matrix is A the command $v d$ is equal to spec of A is

in fact will obtain the Eigen values and Eigen vectors of A. The final steady state value of the states can be obtained by setting psi dot is equal to zero in which case of course, the steady state value of the fluxes are given by A, A inverse B2 which is implemented in this program here, I will just see, if I can move this slightly.

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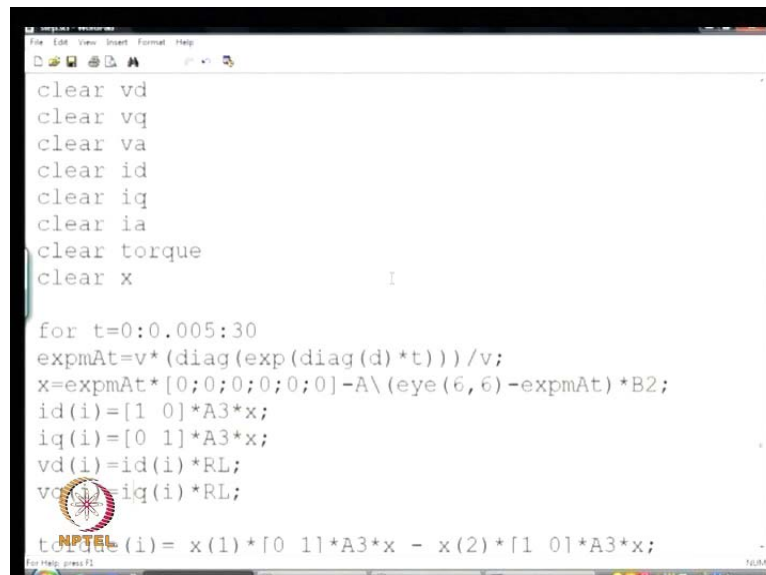


```

--> // relationship between i and psi
--> // i = A3*psi
-->
--> A3=[1/xddd 0 -(xdd-xddd)/xdd*1/xddd -
--> (xd-xdd)/xd*1/xdd 0 0;
--> 0 1/xqdd 0 0 -(xqd-xqdd)/xqd*
--> 1/xqdd -(xq-xqd)/xq*1/xqd];
-->
--> A=A1+A2*A3;
-->
--> [v d]= spec(A);
  
```

So, now we evaluate the time response of the states.

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```

clear vd
clear vq
clear va
clear id
clear iq
clear ia
clear torque
clear x
clear I

for t=0:0.005:0.005
expmAt=v*(diag(exp(diag(d)*t)))/v;
x=expmAt*[0;0;0;0;0;0]-A\eye(6,6)-expmAt)*B2;
id(i)=[1 0]*A3*x;
iq(i)=[0 1]*A3*x;
vd(i)=id(i)*RL;
vq(i)=iq(i)*RL;
end

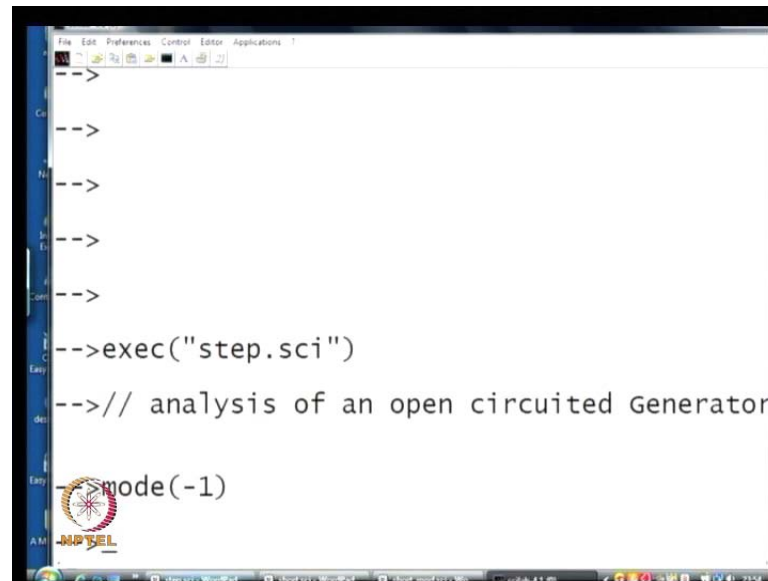
torque(i)= x(1)*[0 1]*A3*x - x(2)*[1 0]*A3*x;
  
```

So, actually it is a direct evaluation it is not a numerical integration because it is not necessary to numerically integrate to obtain the answers. We will assume the initial conditions before we apply the step are zero and we actually evaluate the expression

which I had just given some time back. Finally, of course I can get once I get the fluxes which are x give denoted by x here, I can obtain the currents and the voltages and the torques and so on.

So, in fact we will plot the values of torque and the voltages etcetera shortly. So I will minimize this and now I will actually run the program.

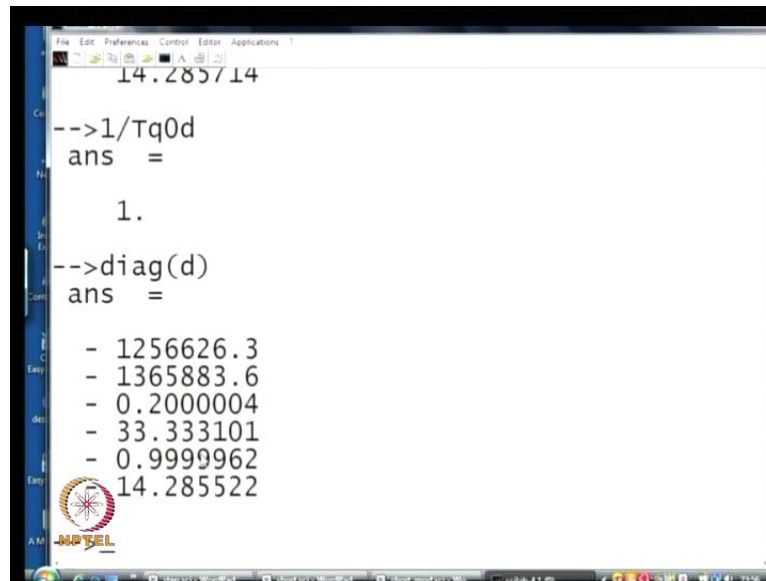
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```
File Edit Preferences Control Editor Applications
-->
-->
-->
-->
-->
-->
-->exec("step.sci")
-->// analysis of an open circuited Generator
-->mode(-1)
```

So, what I will do is run the program from the psi lab work space and in fact we are simulating it for more than thirty around thirty seconds. We shall see why we need to simulate this for thirty seconds? So, it is taking a bit of time because I have evaluating it at a relatively short time interval, I mean the if you look at time I am evaluating this for. So, I will just go down scroll down the program and I am evaluating it every five mille seconds for thirty seconds, so because of the that the program did take some it did take a while for it to simulate it in fact not simulate it evaluate it at various time instance.

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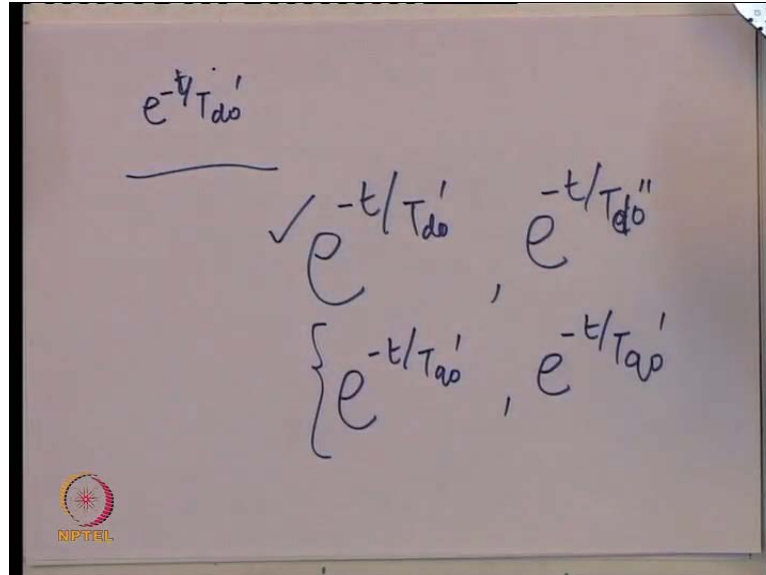
```
File Edit Preferences Control Editor Applications ?
14.285714
-->1/Tq0d
ans =
    1.
-->diag(d)
ans =
 -1256626.3
 -1365883.6
  0.2000004
 33.333101
 0.9999962
 14.285522
```

Now, the thing is before I go on and show you how the time response looks? like what we first, we will see are the Eigen values of the matrix A in fact, if you try to evaluate the Eigen values they are getting we are getting two large numbers these large numbers are indicative of something happening very fast in fact these arise because we have chosen R L to be a very large number. So, in fact R L being a very large number results in a very large negative Eigen value so this transient in fact dies down very soon it is like having a large and R L circuit with a large R L, so that is why you get these two large Eigen values.

The rest of the Eigen values on the other hand are not very large they in fact quite small and the one interesting thing is that if you look at one upon Td zero dash it comes out to be 0.2. So, in fact one of the Eigen values is 0.2 similarly, one by Td zero double dash is 33.33 and you find that the Eigen value also is minus 33.33. Similarly, TQ dash in TQ zero double dash. So, and TQ zero dash and TQ zero double dash, so if you look at Eigen values in fact they are very close or practically equal to the open circuit reciprocal of the open circuit time constant. Let me put this in perspective when you have a synchronous machine which is under open circuit conditions, the Eigen values are in fact the reciprocals of the open circuit time constant in fact, that is why they called the open circuit time constants in fact, I had waited for a very long time to actually explain this particular point about why they are called open circuit time constant? I hope it is clear now in fact, the Eigen values which you get you can even prove this analytically we have

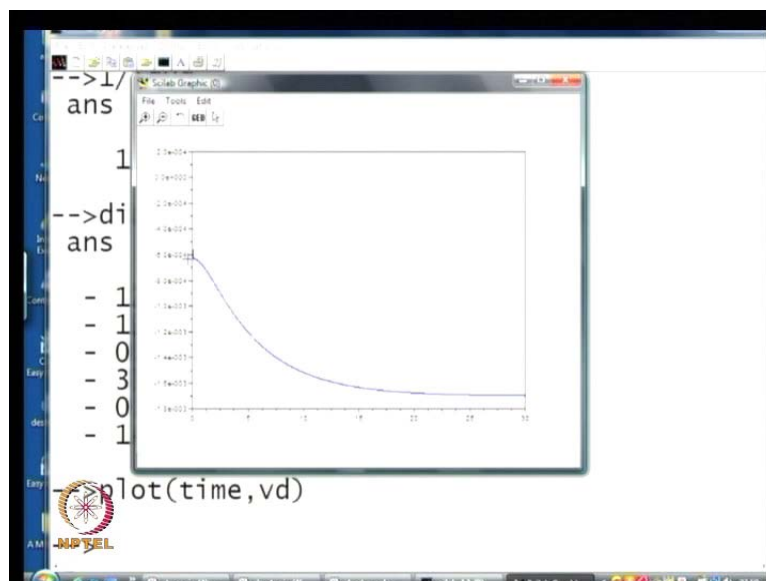
done it numerically but the Eigen values are in fact the reciprocals of the open **the open** circuit time constants which also means that the kind of responses.

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We are going to get are going to be of the kind t upon I will just write this again in case it was not very clear you can expect in the response to have of course, provided that is Eigen values are modes are observable in the output. You are likely to have a responses especially in the voltage to contain this and this in fact you can also expect but actually these are not observable in the voltage these two, these two modes. These two are in fact so that is what we can expect.

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This is something you can actually prove I will not prove it here but you can prove the two time constant will be visible in the voltage response. So, if I actually plot, plot time versus v_d , so I will just interpret this for you in case it is not very clearly visible on your screen this is 6×10^{-4} . So, the six into ten raise to minus four so actually it is zero in fact, v_d is a if you have under in fact it is showing it to be not exactly zero because R/L we have taken to be a large number which is not infinity, so it is not a perfectly open circuited condition but this is a v_d is extremely zero extremely small and it is practically zero.

So, v_d under open circuit conditions is practically zero even under transient situations in fact we have proved this about two or three lectures back where we had considered the open circuited steady state behavior of the machine, steady state behavior of the machine is in fact, something we did some time back and we did prove that v_d is equal to zero and v_q in fact, will be equal to E_{fd} which is the open circuit line to line r m s voltage in steady state.

So, the open circuited so since we have in fact said E_{fd} or we have given the field voltage, so that we will get E_{fd} is equal to one we should expect that v_q will settle down to one. So, let us just see that yes it does so it settles down to the value one starts from zero settles down to the value one and if you look at this time scale it is thirty seconds, so in thirty seconds it takes about thirty seconds to settle down in fact, if you see the settling time it is approximately thirty seconds the time constant recall of the open circuit time constant of a machine can be very large. In fact, here I have take it five seconds in fact you can have open circuit time constants of the other of ten seconds as well, in which case the settling time would be approximately fifty or sixty seconds.

So, I have taken it a time constant of five, so five, five za twenty five. So, approximately five times time constant is the settling time, so this $E \text{ raise to minus } t \text{ by } T_d \text{ zero dash}$ is very clearly visible as a slowly increasing you know exponentially rising a response in v_q of course, from v_d and v_q assuming that θ is equal to ωt ω zero t we can obtain v_a as well. So, if I plot v_a this is how it looks of course, it is a sinusoid it, it reaches sinusoidal steady state, so it also has a the envelop prices in the same way as v_q but this is sinusoidal in fact, if I try to zoom this let us see if I can this is in fact a fifty hertz sinusoid since speed is constant. So, this is a sinusoidal value I will just un zoom this of course, you notice that this is tending towards point approximately 0.8 and not one that is not surprising, because we have we assume that this is a star connected

winding and the line to line voltage r m s voltage is one per unit in fact you have chosen our v f to be such a value, so that the line to line r m s voltage is one per unit.

So, one can expect that the phase to new the voltage across each winding which is also the phase to neutral voltage will be square root of 2 by 3 into 1 into 1 which is 0.81 that is why this you see the peak value of this is going towards 0.8. So, you see this is going towards 0.8, so this is basically the open circuit response of a generator of course, the point is that somebody may ask well am not seeing the other time constant E raise to minus t by Td zero double dash in this actually t d zero double dash t zero dash is five and t zero Td zero double dash is in fact 0.03. So, in fact the reciprocal of 0.03 which is also an Eigen value as mentioned some time is much much much faster than one upon five.

So, that basically the other mode so to speak is not visible very clearly though it does exist the reason of course, being that it dies down much faster than the transient associated with t d zero dash which is a the open circuit time constant ok. Now t q zero dash and t q zero double dash related you know modes are not observable in the response, so this is something I request you to prove that you know for this transient in which I take zero initial conditions, remember I am taking zero initial conditions if you take zero initial conditions you will not see these two terms you know t this related to Tq zero dash and Tq zero double dash appearing in the response you do not observe this for the step response but you do see these two this is not clearly visible because it is very fast it dies down very fast I can call this the dominant mode.

So, we move on now from this point to the short circuit analysis of a machine, so what we need to do here now is consider that the machine is rotating at a constant speed as we have been doing so for then what we do is give a step change in v f, so that you know build up voltage once you build up the voltage and reaches steady state now give it give a short circuit to the generator.

So, what we will do is set R L equal to zero, so once the machine reaches its steady state we will assume that R L sudden we will take R L equal to zero suddenly changing R L of course, changes the value of the state matrix a that is because A2 changes if A2 changes then A also will change. So, you will that the state matrix changes in the Eigen values also changes, so the modes of the system the nature of the modes of the system are different under short circuit conditions as compared to the open circuit conditions as you

may guess well there is nothing to guess really once you have a short circuit there will be a current and there will be the voltage of course, will be zero at the terminals so that is what you will expect to happen.

Now, what we will do is of course, we will not simulate the complete open circuit conditions in fact we will not reevaluate the system under open circuit conditions what we will assume is that we have given the step and the system is now in a steady state corresponding to open circuit conditions then after about five second we will give the short circuit. So, we will not build up the voltage again we will assume that is already build up and the system is in steady state under open circuit conditions running at a constant speed and having E_{fd} as one per unit that is the line to line r m s voltage is one per unit ok.

So what we need to do is the system we assume just a movement we will just scroll back this is another program this is not the same program as I showed you some time back the initial part of the program is the same we assume that under open circuit conditions R_L is a large value what we do initially is obtain the steady state value of the states for E_{fd} is equal to one.

What we will do is assume that the machine is in steady state then evaluate the response assuming that the initial conditions are equal to the steady state values if we do that of course, if we are in steady state remember that the rates of change of the rates of change of the states are zero so we if we are in if we plug in the initial conditions corresponding to the steady state into our expression for the time response, we are just going to remain steady so after some time of course, this is for around five seconds we remain steady at the open circuit conditions after that we apply a short circuit remember when we apply a short circuit R_L becomes equal to zero that is what given here wonder if it is seen there R_L is equal to zero.

And then we of course, reevaluate A_2 and the state matrix we reevaluate the state matrix a and of course, from five seconds onwards we use the new a matrix so E raise to what we do is really E raise to the a new from t is equal to five seconds onwards ok. So, if you look at the analytical expression suppose a you have got the initial condition rather if you want to express x in \dot{x} is equal to Ax plus Bu , I will just write this again slightly in a large and you have got the value of x at five seconds is given in that case x of t after five

seconds is actually can be written as $e^{A(t-5)} x(5) + e^{A(t-5)} u(5)$.

So, this is basically if you know the initial conditions at five seconds and you want to evaluate what happens after at time is equal to five seconds onwards so what we will do know is actually evaluate the short circuit the short conditions ok. So, in fact if you take out the Eigen values of A under short circuited conditions the Eigen values turn out to be like this in fact if you look at the Eigen values there is a term here which is minus three plus or minus 313.95 this is of course, a reminiscent of a omega in fact in radians per second so if you look at imaginary part well it is reminiscent of that. We'll discuss this as a bit this issue in fact this point a bit further later the other time other Eigen values are minus two minus 34.1 minus 43.1 and minus one point two in fact if you look at the time constants in fact one by Td dash 1.2085 in fact the reciprocal of the Td dash is in fact one of the Eigen value of this system one by Td double dash is also and Eigen value ok.

So, the four in fact the last four Eigen values are the reciprocals of the time constants Td dash Td double dash Tq dash and Tq double dash in fact you will expect in the response to see terms like $e^{-Td \text{ dash } t}$ and $e^{-Td \text{ double dash } t}$ and in fact that is why these terms are called as short circuit time constants of the system, so that is the reason why the time constants are called the short circuit time constants.

So, if i actually plot v_a or so if i plot v_a initially of course, the system is under steady state and under open circuit conditions at time t is equal to five you find that this becomes zero. So, you find that t time t is equal to five seconds the voltage v_d , v_a for phase a become zero in fact it becomes zero for phase b and c also which am not showing right now if you look at v_q , v_q also becomes we will have to close this figure first and i have plot it again.

So, actually it is not seen very clearly it goes from one to zero at five seconds, so since there is a short circuit this happens v_q in fact so if you look at v_q it becomes zero from one at five seconds. So, in fact if you plot i_a on the other hand for example, first what we will do is plot the current i_d if you look at plot of current i_d this is how i_d looks like so it is of course, zero in initially because we are under open circuit conditions and then there is a big jump it in fact the current becomes the very large negative value and then slowly it settles down to a value here which is approximately steady state values in fact approximately 0.5.

Whereas, so this is the response under short circuit conditions so what you notice of course, is the response consists of several modes in fact some the modes of course, are not clearly seen all over the modes are not very clearly seen but one of the dramatic things you see is that there is an oscillation this is not surprising because one of our Eigen values is a complex Eigen value with a imaginary part equal to ω in fact three one three you recall that Eigen value.

So, this is what is it is manifest here you also see that there seems to be some kind of an exponential decay, so you know you find an oscillation which decays so if you just look at me here you will find that once this is step, there is a step there is an oscillatory value which decays but also there is a kind of a there is another exponential you know decay here so there is a decay of this kind and there is a decay of an oscillation as well. So, actually this is not very surprising because we have really seeing several modes it is a response is super position of several modes and the key modes of course, in this case are one by T_d double dash and one by T_d dash and of course, the oscillatory Eigen value which we have seen some time back.

So, there is an oscillatory mode we will just get the Eigen values again so you can expect to see these things, so you're getting going to get a complex pair of modes of course the whole system is stable because it is got Eigen values with negative real parts. So, in fact if you plot i_a which is the phase current phase current you will find that, we will just re draw this **we will re draw** this i_a . So, what you see in i_a is in fact this is the phase current remember in the phase current you see that all though in steady state if you look at things in steady state it is absolutely symmetric the way form is symmetric it is a symmetric sinusoid as one would expect in transient during transients you see a dc offset as well as the envelope of this i_a is large.

So, what you have as the response of i_a if you look at me you will find that the envelope of i_a decays with time, the envelope of i_a decays with time but also the envelope itself has got a dc offset, so you see dc offset like this and you see an envelope which is also dying with time. So, you have a huge envelope in the beginning that envelope itself decays and also the envelope is offset and it is coming down like that, so that is the typical response of the phase currents in case you suddenly short circuit a generator, so this is what is a typical response of a short circuited synchronous machine.

Now, one of the things which we need to which we can of course, see here is that we have done this before the steady state value of i_d is nothing but minus of E_{fd} in steady state divided by X_d , you know this is what one can easily prove for a short circuited generator in steady state this I will leave this as an exercise and i_q in steady state is going to be equal to zero this is something which you can prove very easily.

So, let us just verify that this is true, so we will just check out that i_d if you look at the value of i_d all though there is an initial transient. We will have to we will just close this and redo it, so the steady state value of i_d is approximately minus five five and if you actually look at E_{fd} is in fact one. So, one divided by x_d is also 0.55, so what we are seeing is that this of course, the transient finally, becomes the steady state and in steady state x_d is minus E_{fd} by x_d . If you look at i_q the steady state value is zero does not mean of course, that the transients values are zero and you see this here you see that there is some transient and this becomes finally, zero in steady state.

Now, we move on to try to understand a some other aspects of a short circuited generator in fact if you look at the initial value just after the transient has began that is you know just start the time of short circuit the currents are very large if you look at i_d or i_q is of course, zero but i_d in fact is quite large you look at this the steady state value is only 0.55 but if you look at the initial transient you see the peak value goes right up to minus eight.

So, the initially the current is extremely large so this is in fact called as sub transient period this initial few cycles of the fault just after the short circuit has occurred is called as sub transient period of the machine. In fact if you look at you see that the peak value becomes as low as 0.8 and the mean value is approximately, if you look at this figure is the mean value you can see this is the oscillation I will talk in terms of a mean value, the mean values approximately minus three in fact if it is approximately like this so if you take out a mean value something like this.

So, if you look at if you look at the curve of i_d you will find that it is initially zero and then it becomes and then there is an oscillation the envelope of the oscillation dies down this is transient and then it settles down to the steady state value which is E_{fd} / x_d in fact the mean value of this initial transient, let us call it this is roughly E_{fd} / x_d double dash. So, you will let us just verify that what we are seeing from this figure here is approximately 3.5 minus 3.5 this is approximately minus 3.5. So, actually let us see what

one divided by $x d$ double dash is in fact this minus it is 4.3 this is of course, because i cannot make out the mean value very easily from this figure but what you notice is that in the initial period the current has got an oscillatory component and the mean value is approximately three to four minus three to four id value is minus three to four. This is because of this in fact is the initial behavior just after a short circuit in fact, just after a short circuit one can say when the machine currents are determined roughly by the sub transient reactance of the machine in fact $x d$ double dash is called the sub transient reactance of the machine.

So, the sub transient period the machine experiences a much larger current compared to what it see's in steady state in steady state in fact if the current is 0.55 it is even lower than one per unit the rated current of the machine but under sub transient conditions it can be quite high. Now one of the things you will notice in the Eigen value is corresponding to this we will just see the Eigen values again you find that there is a clear separation of Eigen values, if you look at the Eigen values of the first two have a very large magnitude in fact the large they have a large magnitude and this omega the imaginary part is in fact equal to omega the rated speed of the machine it is in fact roughly three hundred and fourteen.

So, that is the radiant frequency corresponding to fifty hertz now it is not surprising in fact that the magnitude of it is quite large compared to the other Eigen values the magnitude of the first two Eigen value is the complex pair, so in fact can we you know do a fast and slow state decomposition of this particular system it is an interesting possibility right. So, if you look at the state matrix A1 in fact you have got omega minus omega and omega here, so you can guess I mean it is just a guess that the Eigen values corresponding to the first two the first two Eigen values in fact which have got an imaginary component omega are probably associated very much with the first two variables that is psi d and psi q in fact you can formalize this by looking at the participation factors corresponding to the states we discuss this in the first in around the 8th or 9th lecture of this course.

So, you can look at the participation factors corresponding to a these Eigen values see the participation of various states but we can guess we will not get into the a complete you know study of the participation factors but you can guess that psi d and psi q are primarily associated with this Eigen values corresponding to the complex pair of Eigen values the first two ones. And that is because just a bit of guess work if you look at the

state matrix it is got in fact for the first two states this ω minus ω and ω in the half diagonal elements of the first two states.

So, that is just the guess but it is an interesting exercise to actually prove this formally using participation factors and I encourage you to really do it, so actually an interesting possibility is of course, to do a simplified analysis and assume that these two states you know you set to zero you know you have got in fact we will call these as the fast states because they are associated with we guess that they associated with the Eigen values of large magnitude and these in fact are the slower states. In that case we can do a fast and slow decomposition as I had mentioned in the first ten lectures when we are studying the dynamic analysis of the analysis of dynamical systems, we had discussed this issue of fast and slow states and one another thing that I told you that if you have got a system consisting of fast and slow states X fast and X slow.

Then one can actually get the response of the slow state without much error, if one assumes that you can set this equal to zero, you set this equal to zero. So, you can in fact express X_f in terms of X_s and finally, \dot{X}_s will be $A_{ss} x_s - A_{sf} x_s$, $A_{ff}^{-1} A_{fs}$.

So, you can in fact study this system as a lower order system of the slow states in fact you can we do this in this case yes you can. In fact I will just run through the code in which I show you that this I have just programmed it, we will try to understand it a bit more in detail in the next lecture. I will show you the code for it I have basically am doing the simulation of a reduced order system in which I have treated ψ_d and ψ_q as fast variables and basically converted the differential equation corresponding to these fast equations into algebraic equations and then studied a lower order system.

So, we will just look at the code which does this same code what I have done is we will scroll down it is the same code except under short circuited conditions what I do is? I form a reduced a matrix by neglecting the fast transients, so this is something I do. So, I call this A_{old} , as I just tore the old A matrix in A_{old} and I form the reduced A matrix this is just a four by four matrix in the slow variables which are the variables from states three to state six that is from ψ_h ψ_f ψ_g and ψ_k and I do the simulation of this system.

So, I will just slowly go through this code and I will run it once and we will discuss it again in the next class, so what am doing is using a reduced order system to do the simulation I would not call it a simulation the evaluation so for example, I can execute

this program we just modify short circuit program with a reduced order state space by eliminating the state equations corresponding to the ψ_d and ψ_q , so wait for a while the Eigen values of the reduced model are these.

In fact they are not very much different from what we had got earlier from the larger model. So, our fast and slow decomposition and simplification does not cause a very large loss of accuracy as far as the slow states are concerned. And of course, if you plot for example, time versus i_d what you notice is we will of course, have to give the old figure I will just redo it again. We get a response as a shorn of the oscillatory transient which we had seen earlier, so we actually get the same responses before you initially recall that there was a oscillation of frequency ω_b now we do not see it.

We shall use this model in the next lecture, we will just start of the next lecture with this model with this reduced order model we will relook what results we have got and there after we will consider a few more cases of a synchronous machine. So, with this we stop today's lecture we have really this is in fact the first real power system analysis dynamic analysis we have done in this course using the model which we have developed.

So, we will of course, continue on this **on this** theme in the next class with the reduced order model. There after we will also in this course go, and model a few more elements before we can actually talk in terms of doing an analysis of an integrated system, and mind you the integrated system though it will be much much more complicated, it is amenable to you know sustain in scientific attack, as we have done for the short circuited synchronous generator.