# **Power System Dynamics and Control Prof. A.M. Kulkarni Department of Electrical Engineering Indian Institute of Technology, Bombay**

## **Lecture No. # 17 Synchronous Generator Models using Standard Parameters**

Although in the previous lectures, we have developed a synchronous machine model using the basic parameters like the inductances, mutual inductances, resistances etcetera. In the previous lecture, we had introduced the concept of system identification or obtaining the parameters of the model from measurement; and we discussed one simple frequency response test, which could actually give us some parameters of the model.

Unfortunately, when we do test like a frequency response test, what we actually get are the coefficients of a transfer function that is obtained by fitting the frequency response obtained by measurement onto a transfer function of a certain order. So that was, what we get from measurement. And after we get that from measurement, we need back calculate the basic parameters from it, but the major issue there is that if you do not do do not have an adequate number of measurements; you may not be able to get all parameters required for the model. So, that was the basic point which I try to emphasize in the previous lecture.

Now, in today's class, we will try to get a synchronous machine model based on parameters obtained by measurement. And the title of today's lecture therefore is synchronous machine models using standard parameter. We will quickly recap our synchronous machine model.

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Flux and Current Equations on the d axis
(From Model - with k_d = \sqrt{\frac{2}{3}})
         \psi_d = L_d i_d + M_d f i_f + M_d h i_h\psi_f = M_{df} i_d + L_{ff} i_f + L_{fh} i_h\psi_h = M_{dh} i_d + L_{fh} i_f + L_{hh} i_h\frac{d\psi_f}{dt} + R_f i_f = v_f\frac{d\psi_h}{dt} + R_h i_h = 0
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If you recall the basic parameters of a synchronous machine were obtained as follows. So, you have essentially the flux current relationships psi d, psi f, psi h are the  $\frac{d}{dx}$  axis coils the flux in the d axis coils and they are related to the d axis currents. These of course this model of course, is in the basic parameters, and it is obtained with k d is equal to root 2 by 3. And there of course, two differential equations associated with the field winding and the h damper winding. We also have a differential equation, which is missed out being written here that is corresponding to the  $\frac{d}{dx}$  d axis flux psi d. Now, if you take the Laplace transform of this, this is what we did last time; and you get these equations (Refer Slide Time: 02:38).

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And from these, if you you know get rid of psi f of s, psi h of s and I f of s and I h of s we get a input output or transfer function relationship between psi d, I d and V f of s; we call that,  $V f s$  re V f of s is the field voltage, the voltage applied to the field winding. So, this is the nature of the transfer functions you get from this model (Refer Slide Time: 03:14).

Of course, in addition to these differential equations let me again emphasize, there is an additional differential equation corresponding to d psi d by d T, which you have not written here, but we shall write it down shortly; But if I want to get a transfer function relationship between psi d and I d by eliminating psi f of s, psi h of s, I f of s and I h of s I essentially use the algebraic equations.

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Laplace Transform of Flux and Current **Equations on the d axis**  $\Psi_d(s) = L_d I_d(s) + M_{df} I_f(s) + M_{dh} I_h(s)$  $\Psi_f(s) = M_{df} I_d(s) + L_{ff} I_f(s) + L_{fh} I_h(s)$  $\Psi_h(s) = M_{dh} I_d(s) + L_{fh} I_f(s) + L_{hh} I_h(s)$  $s\Psi_f(s) + R_f I_f(s) = V_f(s)$  $s\Psi_h(s) + R_h I_h(s) = 0$ 

The first three equations which are algebraic equations, the later two equations are also algebraic equations, but they are obtained effectively from differential equations on which, we apply the Laplace transform, so all of them in fact are algebraic equations. So, what we see that, it eventually leads us to a second order transfer function, L d of s and G dash of s.

And the nature of transfer function looks like this (Refer Slide Time: 04:11), of course the coefficients A and B and A D and B D and A G are in fact related to the basic parameters. So, if you this solve these algebraic equations and get a transfer function relationship of this form, you shall find that B N, A N, B D, A D, A G are related to the basic parameters by these equations (Refer Slide Time: 04:45).

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$$
B_N = \frac{L_{ff}}{R_f} + \frac{L_{hh}}{R_h} - \frac{M_{dh}^2}{L_dR_h} - \frac{M_{df}^2}{L_dR_f}
$$

$$
A_N = \frac{L_{ff}L_{hh}}{R_fR_h} - \frac{L_{fh}^2}{R_fR_h} - \frac{M_{df}^2L_{hh}}{L_dR_fR_h}
$$

$$
-\frac{M_{dh}^2L_{ff}}{L_dR_fR_h} + 2\frac{M_{dh}M_{df}L_{fh}}{L_dR_fR_h}
$$

$$
B_D = \frac{L_{ff}}{R_f} + \frac{L_{hh}}{R_h}, \quad A_D = \frac{L_{ff}L_{hh}}{R_fR_h} - \frac{L_{fh}^2}{R_fR_h}
$$

$$
A_G = \frac{L_{hh}}{R_h} - \frac{M_{dh}L_{fh}}{M_{df}R_h}
$$

So for example, A G is dependent on an L h h, R h, M d h, L f h, M d f and R h, so these are essentially what we get from the original model. So, from the original model we can get a transfer function, the transfer function is in terms of basic parameters.

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Equating with Transfer Function Obtained  
\nfrom Measurement, with 
$$
v_f = 0
$$
  
\n
$$
\frac{\Psi_d(s)}{I_d(s)} = L_d \frac{(1 + sT'_d)(1 + sT''_d)}{(1 + sT'_{d0})(1 + sT''_{d0})}
$$
\nGives us  $L_d$  and the following relationships:

\n
$$
T'_d + T''_d = B_N
$$
\n
$$
T'_d T''_d = A_N
$$
\n
$$
T'_{d0} + T''_{d0} = B_D
$$
\nThen, the following conditions are given by:

\n
$$
T'_{d0} + T''_{d0} = A_D
$$
\nwhere  $L_d$  is the following result.

Now, if we carryout a measurement with v f is equal to 0, this is something we discussed in the previous lecture. If we get carryout a measurement what we will effectively get or again the coefficients of the transfer function or equivalently we will get this time constants T d dash, T d double dash, T d 0 dash, T d 0 double dash and L d.

Now, of course A N, B N, B D and A D are related to this time constants by the four equations given there. So, what we have here is, some parameters obtained from measurement the A N, B N, B D, A D parameters obtained from measurements and the basic idea is that, I do not need back calculate the basic parameter from these.

Now, if we could calculate the basic parameters from these, we could actually use our model in realistic studies, so that is what the basic idea is. But you will notice that, there is already a load road block, if you just use this one measurement then, we have a problem, why is there a problem? Because, the number of basic parameters are more than the parameters obtained from measurements. So, what we obtained from measurement as simply L d, T d dash, T d double dash, T d  $\theta$  doub dash,  $\ln$  T d  $\theta$  double dash; which are the effectively give you the coefficients of the transfer function from which, we have to back calculate the basic parameters.

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Now, of course, you could do another measurement for example, you could obtain the transfer function, G dash of s; that is, you obtain a transfer function of **psi** d with respect rather psi d given V f s an input and obtain this transfer function given here with I d is equal to 0. So, effectively if you look at this transfer function, if I said I d is equal to 0 and say give V f s an input and take take out the transfer function  $V$  d of G dash of s; we will essentially obtain, M d f by R f and T d c double dash, so M d f by R f as a whole we will obtain and the time constant T d c double dash.

So, this will so this is what the transfer function will yield or the measurement will yield. Of course, it presumes that you  $($ ()  $)$  T d 0 dash and T d 0 double dash on the previous measurement also; so, this is what we get of course, this T d  $\theta$  dash and T d  $\theta$  dash double dash also can be obtained from this measurement itself by curve fitting the frequency response. So, this is what we can get if we carry out to measurements.

But unfortunately, in most of the synchronous machine literature, they do give us the parameters obtained from one measurement. So, we typically do not have this tran the value of T d c double dash and M d f by R f in most studies actually, it is not very difficult to get this; but often, the data set which is provided to you will not have the information relating to this M d f by R f as a whole and T d c double dash, this is typically not available, but in principle it could be.

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So, what I will describe to you is what typically will be available to you and what you can do with it. So, for example, I have got only one measurement and you are given these standard parameters L d, T d dash, T d double dash, T d 0 dash and T d 0 double dash; and the model parameters which you need to get are L d, M d f, M d h, L f f, L f h, L h h, R f and R h, so this is what you need to get from the standard parameters; But unfortunately, you cannot get all of them because, the number of parameters exceeds though the standard parameters So, the we cannot back calculate it, what we have effectively is from this.

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If you look at this, we can get five things five equations from here, we can get  $L$  d and we have got this four algebraic equation, which you can equate to A N, B N, B D and A D which are in fact these (Refer Slide Time: 09:34). But obviously, if the number of equations is less than the number of parameters then, you cannot back calculate all the basic parameters. The standard parameters are lesser than the number of basic parameters, so we cannot really get all of them. Of course, the solution to this is have more measurements, but I am described to you a situation, where you have only these parameters given to you.

So, one cannot get a unique solution for the model parameters with just one transfer function measurement, but this is what is typically available to you; of course, we also require stator resistance is something I did not did not emphasize, we also require stator resistance it is usually a very small value, but it can be obtained separately. So, whenever I talk of standard parameters implicitly also mean the stator resistance, which can be obtained easily from measurement.

So, we have got five measurements, five standard parameters here given here and we have to get the basic all the basic parameters, which is not possible to do that. So, we are actually having a kind of a situation, where we have we cannot get all the parameters required for the synchronous machine model, which we have derived. So, let us see, how we can work around this.

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Flux and Current Equations on the q axis
(From Model - with k_q = \sqrt{\frac{2}{3}})
         \psi_q = L_q i_q + M_q g i_g + M_{qk} i_k\psi_g = Mqgiq + Lggig + L_qk^ik\psi_k = M_{qk} iq + L_{gk} ig + L_{kk} i_k\frac{d\psi g}{dt} + Rgig = v_g\frac{d\psi_k}{dt} + R_k i_k = 0
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Now, if you look at a similar situation exists in the q axis winding, so this is the basic equations of the q axis excluding the differential equation corresponding to d psi cube by dt. The differential equation which tells us, how des psi d psi cube by dt is related to the other variable, this is missing from here, but we shall include it later.

Now, if you look at this also a similar situation exists (Refer Slide Time: 11:20), if you take out the transfer functions as before, this is what we get. Of course, there is all the damper windings are of course, shorted you do not have any voltage applied to the damper winding; So, you do not really have that additional transfer function G dash of s as in the  $\mathbf q$  d axis.

So, of course, the coefficients of this transfer function are related to the basic parameters by these equations (Refer Slide Time: 11:52). Again, we follow the same procedure to obtain the basic parameters, but we end up with the same problem that you have got eight basic parameters on the q-axis; and typically the standard parameters are only five, so getting back the basic parameters becomes a bit tricky.

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Now, so what is the way around this? There three possibilities. Now, one of the possibilities is use the state space model which requires fewer parameters you can do that, I mean we require eight parameters to get the original model in the original state that is the stator and rotor fluxes. But one can imagine that one can take out a state space model, which requires fewer **pamara** parameters and is in terms of other states. States, which are algebraically related to the stator and rotor fluxes, we discussed this in the previous lecture, where it was there is no unique way to obtain a state space model from a transfer function.

So, what we can do is, if you want to transfer function do not put a condition that you should write the equations in terms of the basic the old the original states, that is the stator and rotor fluxes; but you will write it in terms of states such that, the states space model requires fewer parameters, can you do it?  $Yes$  you can do it, we shall show you shortly. So, the first possibility is use a state space model which requires fewer parameters, but the states cannot be easily related to the original states, which are the stator and rotor fluxes, so that is model A.

The other possibility is, use the state space which requires fewer parameters, but while making certain approximations and assumptions, we try to obtain the states space model in terms of states which can be related very easily to the original states; that is so I write down my states space model and the model, which I get essentially is in terms of states which can be easily for example, it is proportional to some for example, we can have a state you know psi you know may be, psi f dash as I will show you shortly; which is proportional to the original state, which is the rotor flux psi f.

So, we can try to write down the equations in terms of states, which are related to the original states very easily. So, we of course, we would need to make some assumptions as said as I will mention shortly. So, we do of course, need to back calculate from the standard parameters, the parameters required for this state space model, so we will call this model I.

So, model A is a model without any approximations, but is a state space which requires fewer parameters, but the states cannot be easily related to the original states, which are the stator and rotor fluxes. The second possibility is, we will use the state space which requires fewer parameters, but it is an approximate model which make certain assumptions; but the good thing about the this model I is that, the states are very easily related to the original states.

The third possibility exists is to use the states space which requires fewer parameters and has all the properties of model I; but the good thing about the this model II, which I shall also talk about is that, you do not have to do this extra step of back calculating the basic parameters, you will write down the states spacing terms of the standard parameters itself. Now, this all may be sound a bit confusing to you, it is somewhat **cons** confusing, but as we go through the models, I am sure you will understand what I am trying to say. So, just let us look at model A. Suppose, I have got the standard parameters, so let us just go through model A.

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So, model A is a state space model which requires fewer parameters in fact, it will directly use the standard parameters, but the states cannot be easily related to the original stator and rotor fluxes. So, remember that, model A, model I, model II will have the same transfer function; but remember the main thing theme or main cracks of what I am trying to say is that, we try to take out the state space model, which use the standard parameters in some way.

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So, let us you know focus or attention first on q-axis, the q-axis transfer function is given by L q into this second order transfer function in fact rather I should say that, this the second order transfer function and it has got five parameters here, four time constants and one, overall you can call it again L q.

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q axis Model - Alternative Expression of  
Transfer Function  

$$
\frac{I_q(s)}{\Psi_q(s)} = \frac{1}{L_q} + \left(\frac{1}{L_q'} - \frac{1}{L_q}\right) \frac{sT_q'}{(1 + sT_q')}
$$

$$
+ \left(\frac{1}{L_q''} - \frac{1}{L_q'}\right) \frac{sT_q''}{(1 + sT_q'')}
$$

Now, one point one small diversion which we shall have right now is that, you can write the same transfer function in this form. How is this form different from this form? First of all, this is the transfer function of the psi q with respect to I q, psi q s upon I q s; I q s upon psi q s is simply the reciprocal of this transfer function.

Now, what I do is, I write this I q s upon psi q s in this fashion you know I rewrite it in this fashion, but you will notice that, instead of using T d 0 double dash and T d 0 dash, I am using now new parameters L q dash and L q double dash. So, this transfer function is the reciprocal of the earlier transfer function is written in terms of L q dash, L q double dash and the time constants T q dash, T q double dash and L q.

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**q** axis - Standard Parameters from Measurement  $L_q, T_q', T_q'', T_{qo}', T_{qo}''$ **OR**  $L_q, L'_q, L''_q, T'_{qo}, T''_{qo}$ **OR**  $L_q, T'_q, T''_q, L'_q, L''_q$ NOTE: Stator Resistance can also be obtained

Now, before you start getting a bit confused about what I am getting at, what I really want to say is that, instead of giving you L q, T q dash, T q double dash, T q 0 dash and  $T q 0$  double dash we I can justice while give you L q, L q dash, L q double dash,  $T q 0$ dash and T q 0 double dash. Now, what I mean to say is that, I can give you either the first set of parameters or the second set of parameters or the third set of parameters, but they all inter related.

So, they are basically five standard parameter these five, four time constants and L q, but you can also give instead three inductances that is L q, L q dash and L q double dash and two time constants with the understanding of course that, all these parameters are related to each other. So, the same transfer function can be written like this, where the time constants and the reactances are related to the each other in this fashion.

So, you may find sometimes, the data which you get you may find the first five **Para set** of parameters given to you or you may find the next set of parameters given to you or you may find L q, T q dash, T q double dash, L q dash and L q double dash given to you and of course, the stator resistance also will be given. Now, all these sets of data are equivalent in some way, because there is a relationship between L q double dash, L q dash and the time constants.

So, if you look at this relationship again it is this (Refer Slide Time: 19:53), this can be easily checked by equating you can actually write this in numerator, numerator polynomial by denominator polynomial form. And let us, take the reciprocal of this and equate it to this (Refer Slide Time: 20:08), if you do that you will get this inter relationship (Refer Slide Time: 20:15).

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q axis Model - Alternative Expression of **Transfer Function**  $\frac{I_q(s)}{\Psi_q(s)} = \frac{1}{L_q} + \left(\frac{1}{L_q'} - \frac{1}{L_q}\right) \frac{sT_q'}{(1 + sT_q')}$  $+\left(\frac{1}{L''_a}-\frac{1}{L'_a}\right)\frac{sT''_q}{(1+sT''_a)}$ 

Now, why did I have to get this suddenly into the picture, because the model A which I am going to describe to you can be conveniently written down in terms of reactance's and the time constants T q dash, T q double dash. Incidentally, I have not described to you, why what is inductance and time constants are called? L q dash and L q double dash in these equations or in fact called the transcend and sub-transcend inductances of the synchronous machine; and T q dash and T q double dash are called the short circuit time constants of the synchronous machine.

The short circuit transcend time constant is T q dash, this short circuit sub-transcend time constant is T q double dash. Similarly,  $T d 0$  double dash T re T  $d 0$  T q 0 dash is the transcend open circuit time constant and T q o double dash is the sub-transcend open circuit time constant.

So, we have got T q dash and T q double dash, which are the short circuit time constants. L q dash and L q double dash here, which are the transcend and sub-transcend reactance inductances and T q 0 dash and T q 0 double dash, which are the open circuit transcend and sub-transcend time constants, this is what they are called, why are they called. So,

this is this will become clear in a couple of lectures from now, when we do the short circuit and open circuit analysis of the synchronous machine.

The model A let us again get back to what we are getting at, we are trying to get a model based on the standard parameters which uses just the standard parameters; but I just introduced one small issue or point here that, the instead of giving you four time constants and the reactance and the inductance  $L$  q sometimes, you are given three reactance's and two time constants, but the point is that, they all inter related.

So, if you get this data do not get suddenly perturbed, you can get the time constant from the reactance's by this inter relationship. So, you are normally given these any of these three sets of parameters, they can be you know they are all the parameters are inter related by this (Refer Slide Time: 22:56).

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\mathbf{q}\text{-axis Model A}
$$
\n
$$
\frac{d\psi_G}{dt} = \frac{1}{T_q'}(-\psi_G + \psi_q)
$$
\n
$$
\frac{d\psi_K}{dt} = \frac{1}{T_q''}(-\psi_K + \psi_q)
$$
\n
$$
\psi_q = L_q'' i_q + \frac{(L_q' - L_q'')}{L_q'} \psi_K + \frac{(L_q - L_q')}{L_q} \frac{L_q''}{L_q'} \psi_G
$$
\n
$$
\frac{d\psi_q}{dt} = \omega \psi_d - Ra i_q - v_q
$$
\n
$$
\text{where } \psi_G \text{ and } \psi_K \text{ are linearly related to } \psi_g \text{ and } \psi_k.
$$

So, what I will do is now give you a q-axis model, it is a state space model which is using only the standard parameters instead of course, T q 0 dash and T q 0 double dash I am using the inductances L q dash, L q double dash, but remember that, they are they can be obtained from one another. So, really we are in fact using the five standard parameters itself.

Now, one of the points which you should remember here that psi, what I have done is psi q is of course, the state we know, psi G and psi K I will use the upper case subscripts to denote that, these are not the original psi lower case g and psi lower case k. These are states psi upper case G and psi upper case K are states which are linearly related to psi G and psi K; but that in fact, that inter relationship is something which we do not know, all we know is that, this is one model which will also give you this is also give you the same transfer function as before.

So, this is something you should this is what I want you really to get that; this particular transfer function which you are getting here this particular sorry this particular states space model which you are getting here is in fact using only the standard parameters; but psi upper case G and psi upper case K are we know that, they are related to psi lower case g and lower case k, which are the original rotor fluxes, but we actually do not know what that inter relationship is. All we know that, this is the states space model, which will yield the same transfer function as before.

Actually, this is not a really a big problem. In fact, psi G psi K psi upper case G and psi upper case K are states, which are related to psi lower case g and psi lower case k, but unless we are really interested in knowing the fluxes to the damper winding or the currents to the damper winding for that matter, it is ok to use this model.

So, psi q is of course, as before which is the flux through the q-axis winding, but psi G psi upper case G and psi upper case K, it is difficult to assign of you know they are related to psi g and psi k or lower case g and lower case k; but unless, we are really interested in knowing what the damper winding fluxes are, it is alright to use this model; because, as far as the stator is concerned as far as the transfer function we obtain from this model is exactly the same as can we model obtained from the original states space model.

So, this is an acceptable state space model, which yields the same transfer function as this transfer function let us show I will show show it you. So, you can actually work this out, it is a it is a bit of an exercise, but you can show that, this particular transfer function this particular states space model will give you the same transfer function relationship between psi q and i q as given here.

So, this is something which you should you can actually work it out, all you have to do is for example, take the Laplace transform of psi G the first differential equation and the second differential equation and the third algebraic equation; and you should be able to get this inter relationship. In this particular states space model I have also written down the differential equation for psi q, d psi q by dt. So, just to make things a bit clearer I will just indicate the steps you need to go through to verify that model A indeed gives you the transfer function, which I just shown you sometime back.

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 $\psi_{4}(s) = -\psi_{4}(s) + \psi_{4}(s)$ <br>  $\psi_{4}(s) = -\psi_{4}(s) + \psi_{4}(s)$ <br>  $\psi_{4}(s) = \frac{\psi_{4}(s)}{\sqrt{1 + s + 1}}$  $\psi_{k}(s)$  =

So, for example, what you need to do is, you take the transfer function of the first differential equation s into  $\frac{d}{d}$  psi G of s is equal to one upon, so this you can write as T q dash is equal to minus psi G of s plus psi q of s. And this is T q double dash s psi K of s this is upper case K just to differentiate this from the original state, so this is what we get. So in fact, you can from this you get psi G of s is equal to psi q of s 1 plus s T q dash and psi K of s is equal to psi q of s 1 plus s T q double dash.

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 $(5) =$ 

So and we also have psi q of s this equal to  $L q I q$  of s we will write this, like this  $I q$  of s plus L q dash minus L q double dash upon L q dash psi K of s plus L q minus L q dash on L q psi G of s. So, you can look at the slide what we I have essentially done is, obtained the Laplace transform of these equations, the first three equations and if you look at the if you follow the steps.

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$$
4q(s) = L_{q}'' T_{q}(s)
$$
  
+ 
$$
\frac{L_{q} - L_{q}''}{L_{q}} \cdot \frac{4q(s)}{(1+sT_{q})}
$$
  
+ 
$$
(\frac{L_{q} - L_{q}'}{L_{q}}) \cdot \frac{L_{q}''}{L_{q}} \cdot \frac{4q(s)}{(1+sT_{q'})}
$$

Now, it is quite straight forward what we need to do is of course,  $\frac{\pi}{10}$  of s sorry psi q of s is now equal to L q double dash I q of s plus L q dash minus L q double dash upon L q dash into psi q of s, now psi q of s, psi K of s psi upper case K of s is nothing but psi q of s upon 1 plus s T q double dash plus L q minus L q dash upon L q L q double dash on L q dash into psi q of s upon 1 plus s T q dash. So, what we are really doing here now is must be quite apparent to you.

We are trying to get the transfer function relationship between psi q s and I q s of course, of the next step would be take this on to this side and this this whole term also on to this side and there by get relationship between I q of s and psi q of s. And what you need to verify is that, this yields this this particular equation eventually will lead you to this, which is equivalent to this (Refer Slide Time: 30:50). So, this is what I need you to just work out.

So, our q-axis model A we can actually use this model the q-axis model note that, it also includes the differential equation of psi q, d psi q by d t is equal to omega psi d minus R a into i q minus v q; this model can be used directly now for the q-axis, if you have got the standard parameters given to you just use this model.

If somebody asks you what is for example, tell me the flux that flux in  $((\ ) )$  damper wind G damper winding, the answer is I cannot find it out why, because? The states psi G and psi K, the upper case states are valid states of course, but they are related to psi lower case g, psi lower case k which is the original rotor fluxes; but that relationship is not known, this is something I am not giving you.

All I am assuring you is that, by using this particular state space model, you can get the same transfer function as before. This is the valid state space model of which yields the same transfer functions; so, you can use it you can use it, provided you do not require to know what actually  $\frac{\partial u}{\partial s}$  psi the actual G and K damper winding fluxes are currents are just remember that.

But  $($   $($   $)$ ) in most studies, we do require do not require to know exactly these fluxes, you only need to know what effect these fluxes have on the stator side in that sense, this is the valid model, because it gives you the correct transfer function relationship between the  $(())$  on the stator side that is psi q and I q, so that is also important point.

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In the d-axis if you remember that, the transfer function relationships are as shown here. Remember that, there is an additional transfer function here, because you have got an input the input is of course, the field voltage. Again instead of specifying four time constants or writing down the transfer function in terms of four time constants, I can write them in terms of three react three inductances and two time constants that is ok; because, the time censes time constants and these inductances in fact related by the relationship which is shown here.

So, this is something you can work out, it is not very difficult to do that, what you need to do is of course, equate this transfer function to the reciprocal of this, the first transfer function given in this equation (Refer Slide Time:  $33:41$ ), so it is not it is not very difficult to verify that this relationship holds.

So, if you are given L d, T d dash, T d double dash, T d 0 dash and T d 0 double dash you can in fact get the three inductances, L d dash and L d double dash. So, your standard parameters could be in any form either the first set, the second set or the third set showed in shown in this slide; but with this we have got essentially whatever you require, you do not have to because of the inter relationships which exists.

So, you will be given either of these set of the either this the first set or the second set or the third set, but using this any of these sets you could get the model you desire, because there is a inter relationship between reactances and the time constants. Stator reactances can also be obtained from measurement, so I am not explicitly of course mentioned this, but yes you can obtain stator resistance also from measurement.

Now, if you look at the d-axis model remember that, the state space model will have an input the rather I should say, the rotor differential equations corresponding to the rotor fluxes rather I should say,  $Rota$  the differential equations corresponding to the rotor fluxes will have an input term; this is not a unlike the q-axis, in the q-axis the damper windings have no voltage input. The q  $\alpha$  winding of course, thus have an input v q, but there is no equivalent of a field voltage in the q-axis damper windings.

So, damper windings are simply shorted, but in the d-axis one of the damper winding is shorted. The field winding has an input v f and of course, v d is an input to also to an input to the d winding.

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So, if you look at the inputs you have got v f and v d. Now, what you notice is, this is the state space model, this I am directly writing it down I am  $\overline{I}$  am just stating it without proof that this is the valid state space model, which yields the transfer function relationship this (Refer Slide Time: 35:59). So, all you need to do here of course, to verify this is take the Laplace transform of the first three equations, and get the interrelationship between psi d and I d, psi d of s and I d of s.

You can verify that you will get exactly the same transfer function as given here. So, this is the nature of the model on the d-axis, but remember there is one  $($ ()) here model A, the state psi upper case H and the state psi F upper case F are not equal to psi lower case h psi lower case f. Remember that, psi lower case h and psi lower case f are in fact the fluxes to the h and f winding respectively.

But, if you look at the differential equation written here, they are not they are using the upper case subscripts just to indicate that, these states are not the same as the original states. In fact, the relationship exists a linear relationship you know you can transfer from these states to psi h to psi f lower case, but that inter relationship is not known.

And if you notice this state has some in you know you know some contribution of the input also included. So obviously, this cannot be the original damper winding flux and this cannot be the original field winding flux because, now v f is kind of distributed among these two; you know in some sense, there is the input affects directly affects psi upper case H and psi upper case F. So, these states are not the original states, but remember what I am trying to get that, this is the valid states space representation, which yields the same transfer function.

In so far, as the stator winding stator effects that is the relationship between psi d and I d and the effects on the stator winding are concerned, you will get the same answer. But if somebody ask you the question, what is the field winding flux or what is the field winding current or what is the h-axis damper winding flux you will not be able to answer this question, if you use this model.

Because, these states are not directly or easily related to the original states or rather, I should say this the relationship is not known, all I can say is this yields the same transfer function as yields the correct transfer function or yields the correct inter relationship between the stator flux and the stator current  $\frac{1}{1}$  psi d and I d, but just remember this point, this is the valid state space model.

Now, beta 1 and beta 2 of course are other complicated expressions, this is something you can verify at leisure. So, this is something I state without proof, this is a directly daxis model, so in fact if if you do not want if you do not want to know, if you do not want to know, what the stator, what is the rotor flux rotor h winding flux or the field winding flux, you can still use this model for understanding the effects on the stator. So, this is one thing which you should keep in mind, so this is the valid model model A.

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Now, we go to model I (Refer Slide Time: 39:38), now just because it is possible that, this is this kind of gets a bit T d s, so let us is go back a few slides and remember what we are trying to do? We have got these parameters just from one measurement, if you had more measurements we could call back back calculated all the basic parameters required for the original model, which is in terms of rotor and stator fluxes.

But, what we are doing, what I shown you just now is model A, which requires fewer parameters, but the states in model A cannot be directly related to the original stator and rotor flux. In fact, stator fluxes I must make a small correction here, the stator flux psi q and psi d are retained in the models model; but the rotor flux is in fact you know are something I am not retained in model A. So, model A is a  $(())$  you can use this model, the d and the q-axis model, but you will not be able to answer the question of any question about what the flux in the field winding and h winding individually are.

Model I is model with approximation. So, now let us try to understand what we are going to do, we cannot get a model in the original parameters original basic parameters because, we do not we often do not we have not given adequate number of parameters from measurement. So, what we are going to do now is? Use model introduce to you a model called model I, which will use some assumptions and approximations; and what we will do is, we will create a model using the original states or a small you know like a states which is a proportional to the original states; and what we will do is, back calculate the parameters of this model from the standard parameters. So, this may again appear a bit confusing it is a bit confusing, but I am sure after you we go through the model you will understand, what I am getting at. So, let us talk about model I.

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Now, remember that the original equations on the d-axis, this is the original model in terms of the basic parameters.

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Now, what I will do is? I will try to obtain the same states space model in terms of new variables, so the new variables are psi a, psi f, psi h dash, so or prime, so we are going to use psi s prime, psi h prime, i f prime and i h prime, which are related to the original states. But remember that, this inter relationships which I am going to talk about is very straight forward is simply a proportional relationship.

In model A, which we just discussed I told you that, there exists the relationship between the upper case of the new states and the lower case states; but that, inter relationship is not given, it is not clear what does this  $((\ ) )$  it is a linear transformation from one states to the other.

Here also we are in fact using the linear transformation we are just, but but the interesting thing is, psi f dash is just dependent of psi f, so there is a direct kind of direct relationship between the new state variables which I am going to use and the old state variables. So, this relationship is direct or easy to understand, it is simply a proportionality relationship.

So, if you look at the new d-axis variables, these are in terms of the old variables, but the combination or the relationship is very straight forward, when I say straight forward what I mean is, psi f dash is just dependent on psi f. In model A in fact, this is not true psi F the psi upper case F would have been dependent on psi lower case f as well as, psi lower case h; but in this particular model we have got a direct relationship.

And so, you can even look at this inter relationship as, if it is like referring the variables of the flux to the stator side. So, you have got distinct windings here f and h; and what we are doing is, we are not bothering about what psi f and psi h are, but we will refer them, we are not bother we are not bothered in the sense that, we are seeing what it its effects are as, when they referred to the stator winding stator side. So, what we are doing is, doing a kind of turn's ratio kind of transformation.

So, it is similar to referring the variables to one side of the transformer. Now, you have got psi f, psi h, i f and i h, so these are in fact the inter relationships, which we are going to use.

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Equations in New Variables  
\n
$$
\psi_d = L_d i_d + M'_{df} i'_f + M'_{dh} i'_h
$$
\n
$$
\psi'_f = M'_{df} i_d + L'_{ff} i'_f + L'_{fh} i'_h
$$
\n
$$
\psi'_h = M'_{dh} i_d + L'_{fh} i'_f + L'_{hh} i'_h
$$
\n
$$
\frac{d\psi'_f}{dt} + R'_{f} i'_{f} = v'_f
$$
\n
$$
\frac{d\psi'_h}{dt} + R'_{h} i'_h = 0
$$
\n
$$
\frac{d\psi_d}{dt} = -\omega \psi_q - R_d i_d - v_d
$$

So, what if what if I use this inter relationship? Now, what will happen is, since I change the variables your equations in the new variables will look like this, so all the variables kind of have this prime except psi d i d, which you are going to remain as it is. But all the other variables are in fact are replaced by the corresponding prime variables. Now of course, M d we have not defined what M d f dash prime or M d h prime and so on are, so let us just define them. Remember, the psi s prime, i f prime, psi h prime and i h prime are defined by these variables these relationships.

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Equations in New Variables  
\n
$$
M'_{df} = \alpha_f M_{df}, \quad M'_{dh} = \alpha_h M_{dh}
$$
\n
$$
L'_{ff} = \alpha_f^2 L_{ff}, \quad L'_{fh} = \alpha_h \alpha_f L_{fh}
$$
\n
$$
R'_{f} = \alpha_f^2 R_{f}, \quad v'_{f} = \alpha_f v_{f}
$$
\n
$$
R'_{h} = \alpha_h^2 R_{h} \quad L'_{hh} = \alpha_h^2 L_{hh}
$$

So, of course, it is easy to find see it is easy to find that, the new coefficients of these algebraic equations are actually given by these relationships, it is not very difficult to see this. This must be appearing to you, similar to referring variables or you know resistances and inductances to one side of a transformer, so in fact it is similar this whole operation is in fact similar.

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Now, what we will do is, make some simplification let us call them approximations or assumptions to reduce the number of parameters. Now, if you look at this model which we have actually we have not reduced any parameters, the differential equation on the states space model in fact looks just as a complicated as before.

Now, to reduce the number of parameters, what we will do is? We will choose this alpha h, choose it, so this is the very important word we choose alpha h. So that M d h dash is equal to M d f dash, so what we are going to do is, choose alpha h. Now so, if I am choosing it so that, this relationship is satisfied it means that I am using it in fact, alpha h is something I do not know, but I am choosing it; so that, this relationship is being satisfied.

Now, this also means that, since I am choosing it to you know to reduce the number of parameters its original meaning is it, turns ratio is no longer varied. So, **psi** alpha h is no longer they exact turns ratio between the **damper winding** d h damper winding and the stator winding, it is been chosen so that, we are reducing the number of parameter; This is ok, because with this is if we do not want to eventually know, what exactly the current in the damper winding is in amperes.

If somebody ask you, what is going to be the damper winding current in amperes, if you make the first approximates the first you know, if you have chosen alpha h such that, M d h is equal to M d f M d h dash is equal to M d f dash, we will not be able to tell eventually what actually the damper winding current in amperes is because, I have chosen alpha h to satisfy this criterion rather than, you know using the actual turns ratio. So, I am imposing a condition you know I am using alpha h not the actual turns ratio, but a value which will yield this.

Now, the second point here is very important, if alpha f is the actual turns ratio between the stator and the field winding, then M d f prime is L d minus L l, so we will talk of another parameter in fact, it is an **additional** addition to the standard parameters, this L l is the leakage reactance leakage inductance. So, if alpha f is an actual turns ratio, if I use actually the turns ratio between the state of winding and the field winding then, we shall see that in fact, L l is a leakage inductance.

Now, the third thing which I am using here is, assume M d f dash is equal to L f h dash, so actually by assuming this, I am reducing the need for one parameter. So, In fact this is an  $(())$  assumption without any justification we have not given any justification for this in fact, they are **leakage** leakages which have to be accounted for, so the third assumption is an  $(())$  assumption that is not based on some very realistic or very corrects kind of reasoning; but just an assumption made to reduce the number of  $(())$ .

So obviously, now we are talking in terms of approximations, so what we are going to get is, an approximate model, it is not the exact model. Model A was an exact model, it was an valid state space model, this is also going to be a valid state space model; but what we have made an assumption here, so it is an approximate state space model, alpha h is chosen based on trying to equate to mutual inductances. So, it is not going to be actually the turns ratio, you have chosen alpha h, so that this is satisfied.

So, we qui quick the value of alpha h, so that M d h dash and M d f dash are equal, alpha f is the actual turns ratio between the field winding and the stator winding, so M d f dash prime is actually L d minus the leakage, so this is fine. So, the first and second points which have mentioned in this slide here, are in fact in the sense, there is nothing wrong in what we have done here; but the third thing is certainly an assumption which will make our model approximate.

So, let us get recap what we are doing, model I is a state space model using certain assumptions, so the state space is in terms of states that can be related to the original states easily, what do I mean by that, this is an easy relationship. Alpha h and alpha f are in fact, can be looked upon as turns ratios, they are simply referred things to one side of the transformer, so that is essential.

So, what we have done is, writing down this state space equations in terms of referred states, but since referred states are simply proportional to the original states, this is not really a very, this is a kind of a reasonable or useful approximation to make. Alpha f we will keep as actual turns ratio, so psi f dash is in fact, going to give you the referred field winding flux sorry the turns alpha f is in fact, the turns ratio will use it as a turns ratio. Alpha h is something we choose, so that we make two mutual inductances equal, so alpha h need not be the original turns ratio between the damper winding and the stator winding. So, this is something, which we should remember alpha h is chosen by us.

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One interesting thing is that, if I write down my transfer functions in terms of the new states, the original transfer function does not get changed, the form is exactly the same of course, it is in the equations also looks similar.

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$$
B_N = T'_d + T''_d = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h} - \frac{{M'_{dh}}^2}{L_dR'_h} - \frac{{M'_{df}}^2}{L_dR'_f}
$$
  

$$
A_N = T'_dT''_d = \frac{L'_{ff}L'_{hh}}{R'_fR'_h} - \frac{{L'_{fh}}^2}{R'_fR'_h} - \frac{{M'_{df}}^2L'_{hh}}{L_dR'_fR'_h}
$$
  

$$
-\frac{{M'_{dh}}^2L'_{ff}}{L_dR'_fR'_h} + 2\frac{{M'_{dh}M'_{df}L'_{fh}}}{L_dR'_fR'_h}
$$

The only using primed quantities; everything else looks the same, which is not surprising.

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$$
B_D = T'_{do} + T''_{do} = \frac{L'_{ff}}{R'_f} + \frac{L'_{hh}}{R'_h}
$$
  
\n
$$
A_D = T'_{do}T''_{do} = \frac{L'_{ff}L'_{hh}}{R'_fR'_h} - \frac{L'_{fh}^2}{R'_fR'_h}
$$
  
\nThus, one may obtain  $L'_{ff}$ ,  $L'_{hh}$ ,  $R'_f$ ,  $R'_h$ , given  
\n
$$
L_d, T'_d, T''_d, T'_{do}, T''_{do}, L_l
$$

But the important thing is, because of the assumptions we have made in fact, not the assumption there is one assumption we have made; and one additional parameter, which we have to obtain from measurement we effectively get six parameters from measurement that is L d, T d dash, T d double dash, T d 0 dash, T d 0 double das h and L l and the only parameters we need to get are L d, L f f dash, L h h dash, R f dash and R h dash. So, we now actually can actually compute this is what I wanted to say actually.

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The parameters for this model are these and the parameters from measurement are the once given below, so we can actually get the parameters required for this model. Of course, you may say where is the M d f prime its missing from here, where is the M d h prime; but recall what we have done M d f in fact, is L d minus L l, M d h prime is equal to M d f prime and M d f prime is equal to L f h prime. So, what we have effectively got here now, we have reduced the number of parameters by the one assumption we have made, by the choice of alpha h the choice of alpha h remember and this extra leakage measurement, which will be required.

So, this is model I, now alpha f is the turn's ratio, but actually when we write down this model will not require alpha f at all provided v f dash is used in all the calculations, the referred voltage. So, although I have introduced this concept of alpha f, rather this turns ratio alpha f, it is not required in any of the calculations provided of course, in all my calculations and all my studies, I am going to use v f dash, which is a referred voltage. So, I will never specify what the field voltage is, but I always specify what v f dash, which is the referred voltage.

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**Summary: Model I (d axis)**  $\psi_d = L_d i_d + (L_d - L_l) i'_f + (L_d - L_l) i'_h$  $\psi'_{f} = (L_d - L_l)i_d + L'_{ff}i'_{f} + (L_d - L_l)i'_{h}$  $\psi'_{h} = (L_d - L_l)i_d + (L_d - L_l)i'_f + L'_{hh}i'_{h}$  $\frac{d\psi'_f}{dt} + R'_f i'_f = v'_f$  $\frac{d\psi'_h}{dt} + R'_h i'_h = 0$  $\frac{d\psi_d}{dt} = -\omega \psi_q - Ra i_d - v_d$ 

A similar thing can be, so the summary of the model in the model I on the d axis is this, so this is the model which you will use L f f dash, L h h dash, L d, L l, R f dash and R h dash are going to be calculated from the standard parameters using the inter relationships between which we have discussed some time back this (Refer Slide Time: 54:38). So, using these you can back calculate all the parameters required by this model. So, actually this is an interesting and important and in fact, most books and the literature follow this kind of model, so this is an important model, the model on the d axis.

Actually this is something which we have mentioned sometime back, the q axis can be similarly found, so model I is the model on the d axis and the q axis, this uses parameters back calculated from the standard parameters; but it does make some approximations in fact, once we if we use these model on the d axis and the q axis and we get out answer, one thing we know psi G dash, which we get and psi K dash, psi s dash and psi h dash are going to be proportional to the field winding fluxes and the damper winding fluxes; So, there is a kind of a direct and direct relationship between the states the prime states and the original states. So, this is the model satisfying kind of model because, the new states are in fact related to the old states.

Now, what about model A, is it model A valid, the model A is also valid; but in model A, it is difficult or not possible to tell what the actual field flux or the damper winding flux is going to be, not even the referred value. In model I, we at least know the referred value of these fluxes and currents you can do that, model A though an exact model obtained from standard parameters, you cannot do this.

So, this is the difference between model I and model A, model A is correct exact uses standard parameters, model I is approximate model, but the fluxes here and rather I should say, the states here and the currents here are easily related to the original states.

Now, in the next class, what shall we shall do is talk about another model model II, which uses a distinct approximation, but it has an advantage is that, this back calculation step will not be there; you know, if you look at model I, there is a back calculation step involved in the sense that, you got a standard parameters, but you still need to back calculate, if you look at this slide, you still need to back calculate L f f dash, L h h dash, R f dash and R h dash and L l.

This is not a problem really, but we say just look at another model model II, which is more convenient, so this back calculation step we can avoid. So, let me just clarify again what we are trying to do and summaries today's lecture. We have got these standard parameters obtained from measurement. If you do not have adequate number of measurements have given a limited amount of measurement data, which is typically the case will be given limited amount of measurement data.

We have to build our model, we can build in fact, the model the states space model, which uses fewer parameters; but we are struck with the problem that, the rotor fluxes there and the rotor currents there in that model cannot be easily related to the original rotor fluxes and currents.

On the other hand, if we use the model I we call this model I, which is most popular model in in some ways. We can use we are using fluxes and currents, which are related very simply in a simply by proportional relationship or the referred relationship with the original states. But of course, model I does involve an approximation we shall in the next lecture introduce to you model II another model, which uses the distinct assumption, but does not require this back calculations step, which is required as shown in this slide here, you need to back calculate these parameters from the standard parameters using the relationships these relationships. Remember that, model I involves an assumption.

The i f dash and psi f dash obtained from this model i f prime and psi f prime rather, which you get from this model and directly going to give you the referred value of the field winding field winding flux and currents i h dash and psi h dash, i h prime and psi h prime are in fact going to be proportional to the damper winding flux and current; but the exact proportionality relationship cannot be obtained from this model.

In fact, if somebody ask you the question, if I use this model can you tell me what this damper winding current in amperes is going to be, he will not be able to answer this question. If somebody ask you, what is the field current going to be in amperes still cannot answer this question, but if somebody actually tells you the turns ratio with in the field winding and the stator winding **yes** you can answer that question. So, this is the important thing to be kept in mind.

So, we can have d axis and the q axis model I, which is the most popular model, which is used in the literature. In the next class, we shall do model II and shall also discuss equivalent circuits and the per unit model, so using this, we will be all set to study the synchronous machine; and do in fact, realistic studies, because these these are the **parameters** standard parameters are the parameters, which will be available from measurement, is it ok. So, we will continue our discussion in the next lecture.