

Power System Dynamics and Control
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Lecture No. # 16
Modeling of Synchronous Machines
Standard Parameters

In the previous class, we could draw two inferences about the behaviour of a synchronous machine, from its **constit** constituent equations. In fact, the analysis which we did was a steady state analysis. What we did last time was, we obtained the open circuit voltage, at the test terminals of a synchronous machine; when a voltage V_f is applied at the field.

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$$V_a = \omega_0 \sqrt{\frac{2}{3}} M_{af} \left(\frac{V_f}{R_f} \right) \sin \omega_0 t$$

$$= \omega_0 M_{af} i_f \sin \omega_0 t$$

O.C. $\theta = \omega_0 t$

$\sqrt{\frac{2}{3}} \omega_0 M_{af} \cdot i_f$

$$V_a = \sqrt{\frac{2}{3}} V_f \frac{M_{af}}{R_f} \sin(\theta - \frac{2\pi}{3})$$

The machine is of course, is rotating at a speed, say ω_0 in such a case, it was shown that the voltage, open circuit voltage which appeared at the terminals of a machine was given by V_a is equal to $\omega_0 M_{af} i_f \sin \omega_0 t$. This was with the assumption of course, the θ is equal to $\omega_0 t$ of course, remember, that i_f in steady state is nothing but V_f by R_f . So, the expression which we got was of V_a voltage, the voltage which appeared across the a winding. Of course, if it is the star connected winding, $\sqrt{2}$ by 3 times this, would be the line to line voltage; so line to line voltage magnitude would be $\sqrt{2}$ by 3 into $\omega_0 M_{af} i_f$. **I am sorry** this

should be root 3 by 2 Maf into i f. So, this would be the line to line voltage, rms magnitude, whereas the voltage across the a winding would be this.

The second and more interesting inference about, the steady state behaviour was a machine was that, if a machine was connected to a three phase balance voltage source. And we studied a particular situation, in which the rotor is aligned at an angle delta, at the time when the voltage source to which the synchronous machine is connected to undergoes a negative to positive 0 crossing.

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$$\underline{T_e' \cdot \omega_0} = V_{LLrms}^2 \cdot \frac{\sin 2\delta}{2}$$

$$T_m = T_e$$

$$+ \frac{V_{LLrms} \cdot (x_d f i_f)}{x_d} \cdot \sin \delta$$

open circuit voltage L_r rms

So, if that situation exists, T dash into omega naught is equal to V line to line rms square into sin 2 delta by 2 into this term, remember that this term is equal to 0, in case x d and x q are equal, so this is the saliency dependent term. The other term for in fact, this is the power if you multiplied by omega naught, it is the steady state power, the second term of this steady state power equation is V line to line rms of the applied voltage into x d f into i f x d f into i f as we saw sometime back, was nothing but the open circuit line to line rms voltage.

Please remember, that we did an open circuit analysis of the machine, and we got root 3 by 2 into omega naught into Maf into i f as a line to line rms voltage, magnitude. Now, root 3 by 2 into Maf is nothing but Mdf omega naught into Mdf is x df, so what we have here is, the steady state torque, torque of power expression looks like this, with this should have a sin (0) into sin delta term here, yeah which was missed out. So, this

particular term has got V line to line rms multiplied by the open circuit line to line rms voltage into $\sin \delta$ by x_d (Refer Slide Time: 04:11).

So, these were the two simple and important results of a synchronous machine, we now move on to one important part of our course, in fact in the previous class, we have previous lectures, we have obtained the synchronous machine equation, the differential equations both the flux equations and the torque equations. So, in fact, we have come to a point where we can analyze the machine completely, but there is one important engineering aspect, which we should go through, before we start actually applying these equations for the analysis of a synchronous machine. It may appear very trivial, especially for a theoretically inclined person, what we are going to do now.

Basic point, we are going to do is **is** try to obtain the parameters which are required for the analysis of a synchronous machine; so if I want to obtain certain parameters, then I need to do a certain amount of testing. And once I do testing, I will have to fit whatever I get as a test results into my model, and obtain the parameters for it. So, this is what is going to be the aim of this particular lecture; in fact, what we see in the literature, in the literature on synchronous machine analysis is that often, we will not be given the inductances of the machine.

For example, **(O)** M_{af} all these inductances are rarely given, what instead are given are some of the parameters obtained, **from this** from some test which applied to a synchronous machine. For example, you will be given things like the open circuit time constant of the synchronous machine or the short circuit time constants of a synchronous machine or the transient and subtransient reactance of a synchronous machine.

Now, do you correlate these **these** parameters or these variables which are given, I would not say variable, the parameters which are given as a result of testing of a synchronous machine. How do you correlate them to the model that we have derived so far, so that is basically the aim of this particular lecture. So, we will try to understand a synchronous machine, in terms which is written the equations of which are written in terms of what are known as the standard parameters, or the parameters obtained from standard tests.

So, this is our lecture today, the Modelling of Synchronous Machine, in terms of Standard Parameters. Now, suppose I have got of course, the synchronous machine equations, there the d q 0 flux equation, differential equations, then the f g h and k flux

equations, the differential equations, and there is an algebraic relationship between flux, a linear algebraic relationship between the fluxes and the currents. So, my equations are constituting in fact, the combination of differential and algebraic equations, there is nothing sacrosanct about, representing the algebraic equations.

You can always eliminate current by expressing it, in terms of flux, and ensuring that your **your** flux equations are in pure state space form, but generally people like to write the flux equation, in terms of differential equations in flux, which contain current and separately write current as **as** algebraically related to the fluxes. Now, let us say I want to obtain these parameters of the synchronous machine, they have several ways you can do it, one you can of course, **you know** do an electromagnetic analysis of a machine. And try to using **say** some computational methods for electromagnetic fields, accurately calculate the inductances and other parameters of the machine.

But, another simpler and better way of doing things is to actually physically test the machine, and obtain its parameters; so for example, one of the ways you can do it is, what is known as the, stand still frequency response test of a synchronous machine, so let us see what we can do.

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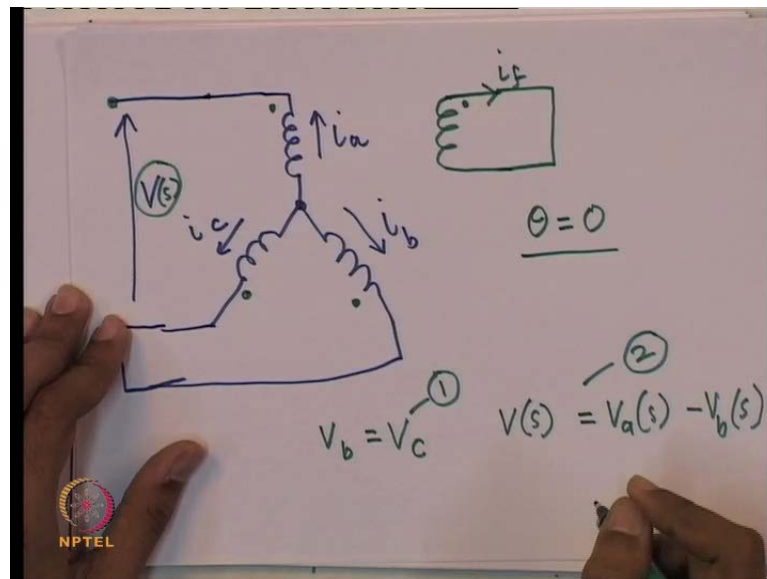
$\omega = 0$ $V_f = 0$
 $-\frac{d\psi_d}{dt} - R_a i_d = V_d$
 $-\frac{d\psi_q}{dt} - R_a i_q = V_q$
 ψ_0, i_0

Suppose, you have got a synchronous machine, which is at standstill, that is its speed is 0, and what we do is also for example, keep the field voltage 0, that is short circuit the field winding, in such a case, the stator flux equations, can be written down like d psi d

by $d t$ there is no speedy $m f$ term (No audio from: 09:26 to 09:46), because speed is 0. So, suppose you have a situation where speed is equal to 0, $V f$ is equal to 0, this one the equation of a synchronous machine, the stator fluxes are going to look like this.

Of course, if you assume, that the resistances of the synchronous generator are very small, for large synchronous machines, they are indeed very small resistances; then you get $d \psi d$ by $d t$ is equal to minus $V d$ and $d \psi q$ by $d t$ is equal to minus $V q$. We shall arrange our tests in such a way, that the fluxes the 0 sequence fluxes and currents are 0, so we shall arrange everything in such a manner, so we do not have to bother about the 0 sequence variables here.

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Now, suppose I excite the stator winding in a certain fashion for example, I take my stator winding, and I excited in this fashion, I connect the V a b c windings, in star keep the star point open and of course, this is i_a this is i_b and this is i_c . And I apply a voltage here, which is equal to V and I short this winding here, so I apply a voltage here V short this winding. So, this is what I will do, the other thing I will do is, I will align my field winding axis to the a winding axis, so what I will do is the field winding is aligned to the a axis. So, what is theta, so theta will be 0 see remember, its **its** also a stands still theta will always remain in 0. Now, what I will do is, apply voltages of various frequencies here, and try to take out the frequency responses, between the currents and the voltages.

In this case, since V_b is equal to V_c and V_s , s is the Laplace variable is equal to V_a of s minus V_b of s , this is condition number 1, this is condition number 2.

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$$I_a(s) = -I(s) \quad - (3)$$

$$I_b(s) = I_c(s) = \frac{1}{2} I(s) \quad - (4)$$

$$\theta = 0$$

$$I_d = \sqrt{\frac{2}{3}} \left[-I \cos 0 + \frac{I}{2} \cos(-120^\circ) + \frac{I}{2} \cos(+120^\circ) \right]$$

$$= -\sqrt{\frac{3}{2}} \cdot I$$

And there is a third condition I_a , I will call as I and I_b which will be equal to I_c will be equal to half of I of s , so if you look at this figure here, this is I , so the current will split equally between these two windings, and you will have these conditions, so there are 3 4 conditions, let me just write them down here V_b is equal to V_c and this (Refer Slide Time: 13:07). Now, since θ is equal to 0, we have I_d is equal to root 2 by 3 sorry this current field take in this direction current will take in this direction into the machine.

In that case, you will have minus $I \cos$ of 0 θ is 0 remember plus I by 2 \cos of minus 120 degrees plus I by 2 \cos of plus 120 degrees. So, this will be equal to minus root 3 by 2 rather I should say minus of yeah root of 3 by 2 into I .

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Handwritten mathematical derivations on a whiteboard:

$$I_d(s) = -\sqrt{\frac{3}{2}} I(s)$$

$$I_q = \sqrt{\frac{2}{3}} \left[-I \sin 0 + \frac{I}{2} \sin 120 + \frac{I}{2} \sin(-120) \right] = 0$$

$$V_d = \sqrt{\frac{2}{3}} \left[V_a \cos 0 + V_b \cos(-120) + V_c \cos(+120) \right]$$

$$V_d(s) = \sqrt{\frac{2}{3}} V(s)$$

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So, we have got let just write it again, V_d I_d of s is minus root 3 by 2 into I and I_q , you can take this out very quickly, I will just write it down minus of I into sin of 0 plus I by 2 into sin of 120 minus plus of I by 2 into sin of minus 120, so this turns out to be equal to 0. Similarly, V_d is equal to root 2 by 3 into V_a cos of 0 plus V_b into cos of minus 120 plus V_c into cos of plus 120, and that comes out to be equal to root 2 by 3 times V . So, what we have is this, and V_d of s where s is a Laplace variable is this, so if we apply Laplace transforms here, on the basic time dependent quantities, these are the time dependent quantities, we will have this is of course, of s .

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Handwritten mathematical derivations on a whiteboard:

$$\frac{V_d(s)}{I_d(s)} = -\frac{2}{3} \frac{V(s)}{I(s)}$$

A circle is drawn around $\frac{V(s)}{I(s)}$, with an arrow pointing to $\frac{\psi_d(s)}{I_d(s)}$.

$$-\frac{d\psi_d}{dt} = V_d \quad -\frac{d\psi_{qv}}{dt} = V_{qv}$$

$$-\psi_d(s) = V_d(s)/s$$

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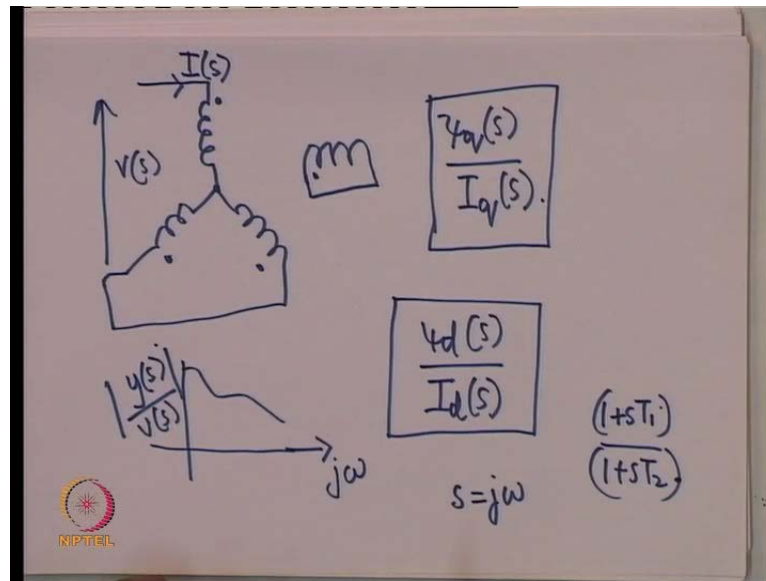
So, what we have here is V_d of s upon I_d of s is nothing but minus 2 by 3 times V of s by I of s , so what we will do is, in this set up, this test set up, I will apply a voltage V , this is voltage which I will apply V , and obtain the current I , what I will do is, I will do these for various frequencies of voltage, which are applied (Refer Slide Time: 17:00). So, what we will get, if some kind of a frequency response V s by I s , so this is what we will get if we do this test.

Since, V_f is shorted that is, the field winding is shorted here you do not have any term corresponding to V_f , so this test is done with the field winding shorted. So, what we can do is obtain V of s by I of s and recall from one of the equations, which we wrote previously, we have $\text{minus } d \psi_d \text{ by } d t$ is equal to V_d and $\text{minus } d \psi_q \text{ by } d t$ is equal to V_q this is if of course, resistance is neglected and the speed is 0; so these two things are obtained from those equations.

So, the point is once I get this transfer function, can I correlate it with what we get from actually, what I get from the model, what is the transfer function I get from the model, we shall see shortly. But basically what I wanted to say is since, ψ_d and V_d are related in this fashion, we can get the transfer function from this, because we have this relationship, and we also have this relationship, I will just write it in Laplace domain $\text{minus of } \psi_d \text{ of } s$ is equal to $V_d \text{ of } s$ upon s (Refer Slide Time: 18:53).

So, actually you can get the relationships ψ_d upon I_d of s by actually, doing a measurement of this transfer function. So, what again do is take out this frequency response of this transfer function, and try to fit it into the transfer function, I get from the model which we have, so that is what we can do.

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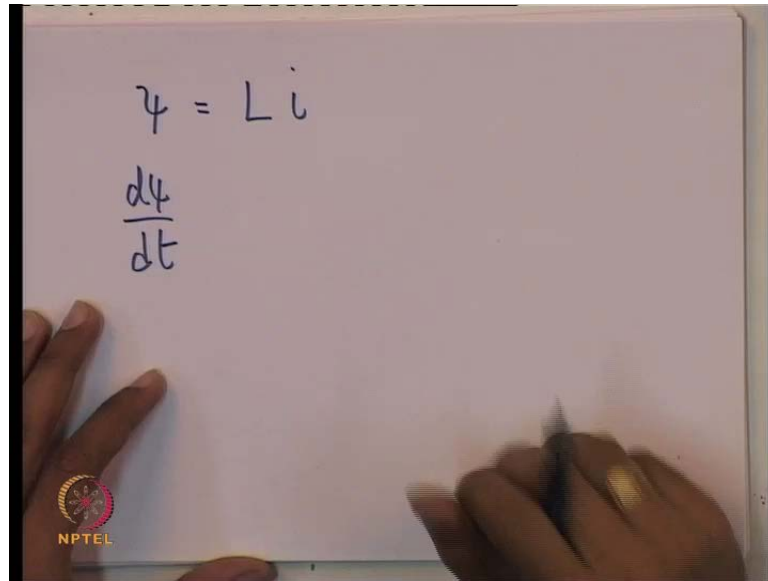
In fact, its an easy **easy** to see that in case, I align my field winding at theta is equal to 90 degrees **if I align my theta at 90 degrees**, in such a case, I can get this transfer function, the reason of course, is that un under these situations I d will turn out to be 0. So, what we will get effectively is the transfer function, which is of importance is actually going to be this. Now, its in fact, one important point which I missed out was, in case the field winding is aligned, axis is aligned to the a **a** winding V q is also 0; so this is something I did not show, so in fact that is something I did not explicitly show, but it is easy to see.

So, now I have got these transfer functions from these tests, the question is from these transfer functions, can I obtain the parameters of a synchronous machine. Now, the obvious thing which we ought to do, whenever we try to equate the transfer functions which are obtained by testing in fact, you will get what is known is the frequency response of this transfer function by testing.

So, suppose you get some frequencies of any transfer function like this, suppose there is a transfer function y s upon U s which is of this kind, for various frequencies you evaluate this transfer function by putting s is equal to j omega; and you get the magnitude of this transfer function, and you get the phase of this transfer function (Refer Slide Time: 21:51). What you need to do is, you can fit say a rational function like, 1 plus s T 1 upon 1 plus s T 2, and try to see by choosing appropriate values of T 1 and T 2, can I get a response which is identical to the one obtained from measurements.

The answer is that in general, in physical systems whenever, you take out these transfer functions actually, the frequency response of transfer functions by testing the **the** physical system. You will find that, it **it** usually will not match exactly with the transfer function obtained analytically via some model, because there are always some kind of approximations implicit whenever, we obtain a model of a synchronous machine.

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So, for example, if we take the model of a synchronous machine, that is we have got what are known as the flux, and the current relationships, we have got the differential equations of the **flux** fluxes that is you have heard equations in $d\psi$ by $d t$, we also have got of flux current relationship. The question is, can I get the same transfer function analytically, the answer is yes, you can; in fact, if you look at the rotor equation of a synchronous machine in the direct axis, the rotor equations are given by these two differential equations, in the quadrature axis by these two differential equations.

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ROTOR AUX EQNS

$$\left. \begin{aligned} (a) \frac{d\psi_f}{dt} + R_f i_f &= V_f \\ (b) \frac{d\psi_h}{dt} + R_h i_h &= 0 \end{aligned} \right\} \begin{array}{l} \psi_f \\ \text{direct axis} \end{array}$$

$$\left. \begin{aligned} (c) \frac{d\psi_g}{dt} + R_g i_g &= 0 \\ (d) \frac{d\psi_k}{dt} + R_k i_k &= 0 \end{aligned} \right\} \begin{array}{l} \text{quadrature} \\ \text{axis.} \end{array}$$

Moreover as I mentioned sometime back, the currents which appear in this differential equations are related to the fluxes by this relationship, so this is the relationship you have got for the fluxes.

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$$\frac{\psi_d(s)}{I_d(s)}$$

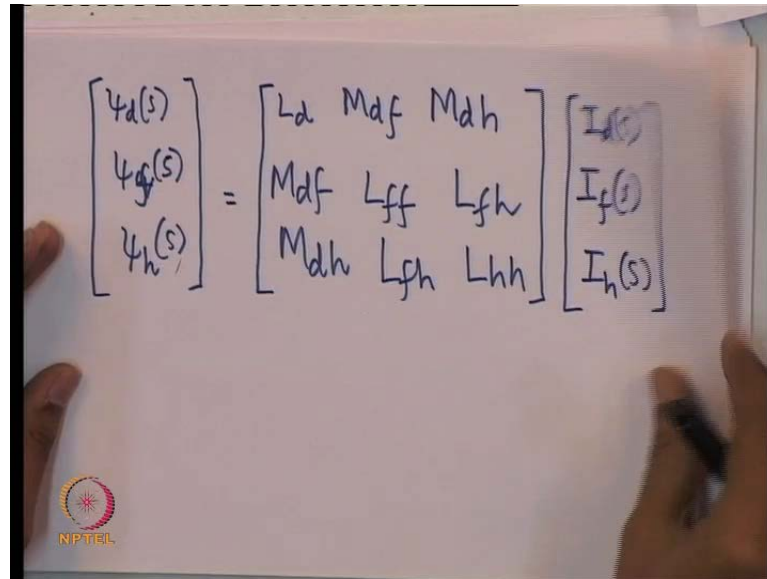
$$s \psi_f(s) + R_f I_f(s) = \underline{V_f(s) = 0}$$

$$\left. \begin{aligned} \psi_f(s) &= -\frac{R_f}{s} I_f(s) \\ \psi_h(s) &= -\frac{R_h}{s} I_h(s) \end{aligned} \right\}$$

So, if for example, I wanted to find out, the transfer function between ψ_d and this, what would I do, what I need to do is take out the apply Laplace transformation to the direct axis, differential equations as this is a result of which I will get s into ψ_f of s plus R_f into I_f of s is equal to V_f of s . Now, of course, if voltage applied to the field winding is

0, in that case you said this equal to 0, in such a case you will have ψ_f of S is equal to minus of R f by S into I f of S similarly, the other equation that in ψ_h of S differential equation in ψ_f of S, if you take the Laplace transform you will get minus of R h upon S into I h of S. Now, you have got these equations, and you wish to now obtain this transfer function.

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$$\begin{bmatrix} \psi_d(s) \\ \psi_g(s) \\ \psi_h(s) \end{bmatrix} = \begin{bmatrix} L_d & M_{df} & M_{dh} \\ M_{df} & L_{ff} & L_{fh} \\ M_{dh} & L_{fh} & L_{hh} \end{bmatrix} \begin{bmatrix} I_d(s) \\ I_f(s) \\ I_h(s) \end{bmatrix}$$

So, what you need to do is, use this equations in conjunction with the algebraic relationship, which is again **sorry** (No audio from 26:06 to 26:52), so you got this relationship too, so you can substitute, what we have got some time back this, into this (Refer Slide Time: 26:58).

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$$\psi_d(s) = L_d I_d(s) + M_{df} I_f(s) + M_{dh} I_h(s).$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} M_{df} \\ M_{dh} \end{bmatrix} I_d(s) + \begin{bmatrix} L_{ff} + \frac{R_f}{s} & L_{fh} \\ L_{fh} & L_{hh} + \frac{R_h}{s} \end{bmatrix} \begin{bmatrix} I_f(s) \\ I_h(s) \end{bmatrix}$$

So, just to cut a long story short, what you will have is, if you substitute for ψ_f and ψ_h here, you will finally get ψ_d of S is equal to $L_d I_d$ of S plus M_{df} into I_f of S plus M_{dh} into I_h of S . And as far as these two equations are concerned, in ψ_f and ψ_h , its easy to see that you will have (No audio from 27:54 to 28:39), this is by simply transferring the or expressing ψ_f of s in terms of I_f and I_h (Refer Slide Time: 28:46).

So, this is basically, what you get of course, this is multiplied by (O) you toward capital, so as to denote these are Laplace variables, Laplace transformed currents; now, what you can do is, express I_f and I_h of S , in terms of I_d of S and then what you do is effectively eliminate them from the first equation. So, you can write get ψ_d of s wholly in terms of I_d of s ; so what you effectively have to do, if you focus on this equation, which I have to get I_f and I_h in terms of I_d , I have to take this **this** term on to this, side then invert this matrix, and write I_f and I_h in terms of I_d of s and substitute here, so I has to substitute for I_d and I_h (Refer Slide Time: 29:43).

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The image shows a handwritten derivation of the transfer function $\Psi_d(s)$ in terms of the input current $I_d(s)$. The derivation is as follows:

$$\Psi_d(s) = L_d \frac{(1+sT_{d'}) (1+sT_{d''})}{(1+sT_{d0'}) (1+sT_{d0''})} I_d(s)$$

$$= L_d \frac{1 + s(T_{d'} + T_{d''}) + T_{d'} T_{d''} s^2}{1 + s(T_{d0'} + T_{d0''}) + T_{d0'} T_{d0''} s^2}$$

$$= L_d \frac{1 + B_N s + A_N s^2}{1 + B_D s + A_D s^2}$$

A small logo for NIPTEEL is visible in the bottom left corner of the handwritten page.

So, I will cut a very long story short, and directly tell you that ψ_d of S will have the following form in fact, it will be L_d into a transfer function, it will have this form, this is of course, obtained provided you keep the field winding shorted otherwise, you have another term in V of S . So, you have got a what is known as a second order numerator, and a second order denominator polynomials, is relate ψ_d of S to I_d of S .

Now, just will do it once, so just take a deep breath, I will just actually write down, the equations which you get, if you use actually evaluate this term, which I have been mentioning sometime back, that is you will get, if you actually evaluate this, you will get this $T_{d'} T_{d''} T_{d0'} T_{d0''}$ in terms of the original variables, that is M_{ds} M_{df} M_{dh} L_d L_{ff} L_{fh} and L_{hh} and of course, R_f and R_h (Refer Slide Time: 31:33).

So, this particular equation, so in fact you will get, this as L_d into $1 + s T_{d'} + T_{d'} T_{d''} s^2$ plus $T_{d0'} T_{d0''} s^2$ (No audio from: 32:13 to 32:22) (Refer Slide Time: 32:13), so they general form into S square **square**, so this is the general form. But the point is I have defined this new time constants, we shall see the significance of these time constants a bit later, but the fact is that transfer function which you will get, when you actually evaluate this, **you know** when you actually substitute for I_f and I_h here, you by using this particular equation, will look like this where I will call this as (No

audio from 33:03 to 33:51) A D of S square. So, what is A D and B D A N and B N well hold your breath, this is what it will look like.

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$$A_N = \frac{1}{R_f R_h} \left(L_{ff} L_{hh} - L_{fh}^2 - \frac{M_{af}^2 \cdot L_{hh}}{L_d} + 2 \frac{M_{af} M_{dh} L_{fh}}{L_d} - \frac{M_{dh}^2 \cdot L_{ff}}{L_d} \right)$$

So, A N is nothing but this it is a very complicated looks, a very complicated equation, in terms of the basic parameters of the machine.

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$$B_N = \frac{(L_d L_{ff} R_h + L_d R_f L_{hh} - M_{dh}^2 R_f - M_{df}^2 R_h)}{L_d R_f R_h}$$

$$A_D = \frac{L_{ff} L_{hh}}{R_f R_h} \quad B_D = \frac{L_{ff} R_h + R_f L_{hh}}{R_f R_h}$$

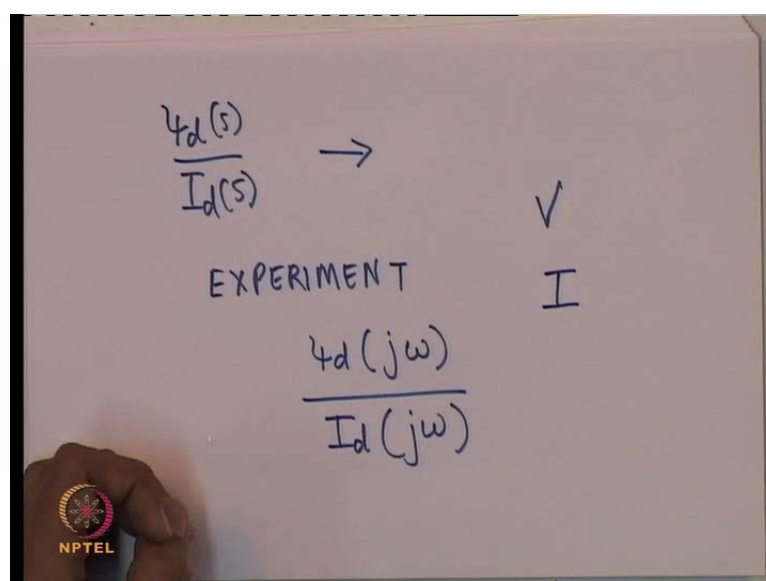
And B N in fact, looks like this, its again looks very complicated A D equals Lff Lhh upon R f R h, and B D is equal to Lff R h plus R f into Lhh upon R f R h. The expression for A D is wrongly written, the numerator should be Lff into Lhh minus () square, so

please note that this error. So, although these expressions look very complicated, they can actually be found out by applying by in fact, substituting for the field and h damper winding currents, in terms of I_d of S . So, let me just tell you that, if so the point is that this second order transfer function we have got, the coefficients $B N A N B D$ and $A D$ can be written in terms of these, but the general you can factorise them, in this form and I define this time constants which will come out as a result of this factorization, as T_d dash T_d double dash for the numerator polynomial and $T_d 0$ dash in $T_d 0$ double dash in the denominator polynomial.

So, let me just summarize what I have done, I have got the transfer function in terms of the basic variables of the machine $R_f R_h L_{ff} L_{fh} L_{hh}$, so these are the basic parameters of the machine. It turns out that ψ_d by I_d from the model which I have used will give you a second order, you will have the form, in which you have got a second order polynomial in (s) divided by second order polynomial in the denominator, both in the numerator and the denominator you will find that a second, they are second order polynomials.

So, I can actually what I can do is, I can do an experiment, do a test on the machine, obtain the frequency response of the machine; that is obtain the frequency response of ψ_d of S upon I_d of S for various ω .

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That is I do an experiment, obtain ψ_d in fact, I should say $j\omega$ upon I_d this is basically a transfer function, so if I apply various the voltages, voltage as I showed some time back at various frequencies, and I compute this transfer function or rather I obtain this transfer function from the measurements, which I take. So, what I need to do is actually measure V , the magnitude and phase of V and measure the magnitude and phase of I_f of various frequencies for the test, I described sometime back.

So, once I do that, I can actually try to correlate the frequency response with the transfer function, which I have got. In particular, I could get the parameters of the transfer function, which are theoretically obtained by comparing it with the transfer function I obtained by measurement. So, I have to in some sense fit the model to the experimental data, now one important point which I should mention here, that I took when I model the synchronous machine, I model in the q axis.

For example, two damper windings, it may so happen especially for, if you look at hydro turbine driven generators, that you will be able to fit the data obtained from experiment to the model very easily, by just one damper winding, on the q axis, so that that can happen. So, what we have assumed is kind of a model, in which the two damper windings on the q axis, one damper winding on the d axis, and the field winding, and obtain the transfer function, and the transfer function show obtained when we correlate with experiment, it may so turn out that we get a lot of error or other we are not able to fit the experimental data to the model which we have got.

So, this may indicate that our is assumption which we have made right in the beginning, of having two damper windings on the q axis, and one field winding in one ampere on the d axis, may require revision you may require more windings or in some cases you can even model the synchronous machine, adequately with just one winding on the certain axis. So this does happen, but what we will take right now, what has been found in the literature in or reported in the literature is that, this two damper windings on the q axis.

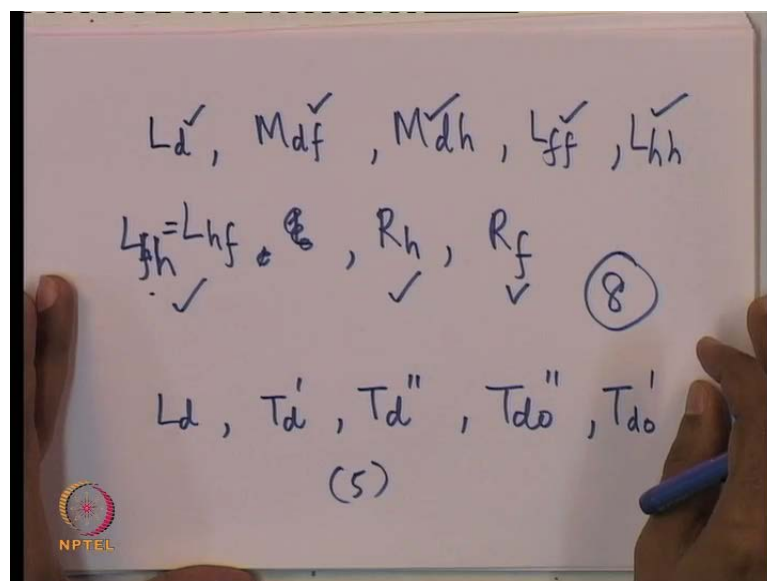
And one damper winding on the d axis, with along with the field winding is adequate or other models a steam turbine driven generator quite well. For a hydro turbine genera driven generator, you can in fact show that you can in facts, observe that one damper winding on the q axis is adequate to model the hydro turbine driven generator. In fact

even, if you have model the higher order machine **you know** machine with more number of windings one can always reduce the order of the **or rather reduce the order of the** model by open circuiting one of the damper windings. For example, if I set **set** R_h equals to infinity are a very **very** large value it is equivalent to opening that damper winding. So, depending on what experimental data we get, we will be able to in most cases or most practical generators fit **you know** the experimental data to the theoretical transfer functions.

So, what we will get in fact, after doing all these measurements is the parameter values L_d T_d T_d' T_d'' by fitting it, in fact this fit may not be exact, but you can always tune these values of T_d T_d' T_d'' and T_{d0} T_{d0}' T_{d0}'' , tune it in such a way, that they match with what is obtained **in** by doing the experiment. So, we have got the transfer function $\psi_d(s)$ upon $I_d(s)$ from experiment, and we are tuning in the parameters of the model, so that the model and the experimental data matches.

So, that is done by what is known as the stand still frequency response test, which I mentioned sometime back. Now, there is only one issue I will get L_d T_d T_d' T_d'' **T_{d0} double** T_{d0}' T_{d0}'' by fitting model to the experimental frequency response. Now, once you have got **these transfer** these time constants can you for example, back calculate all the parameters of the original model.

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For example, what was the parameters in the d axis of the original model, you have got L_d , M_{df} , M_{dh} , L_{ff} , L_{hh} , L_{hf} and **yeah** that is it, L_{fh} and L_{th} this is of course, equals, so I should not call L_{fh} is equal to and then you have got R_h and R_f . Of course, there is a parameter R_a , if the resistance of a synchronous machine and that we assume also has been **measured can be** measured separately. The resistance of the state of winding is something you can measure separately, so you will not worry about resistance of the state of winding.

Now, so you have got 1, 2, 3, 4, 5, 6 these are equal, so 6, 7, 8; 8 parameters in our original synchronous machine model; by doing the tests, we can fit the experimental data and obtain these rather tune the time constants (Refer Slide Time: 43:58). So, that they fit exact more or less they fit very well with the experimental data, so we will try to tune these parameters. So, we here you have got 1, 2, 3, 4, 5, so actually you have got from the tests you will get, the parameters of these parameters **these parameters** of the transfer function, so they have only 5.

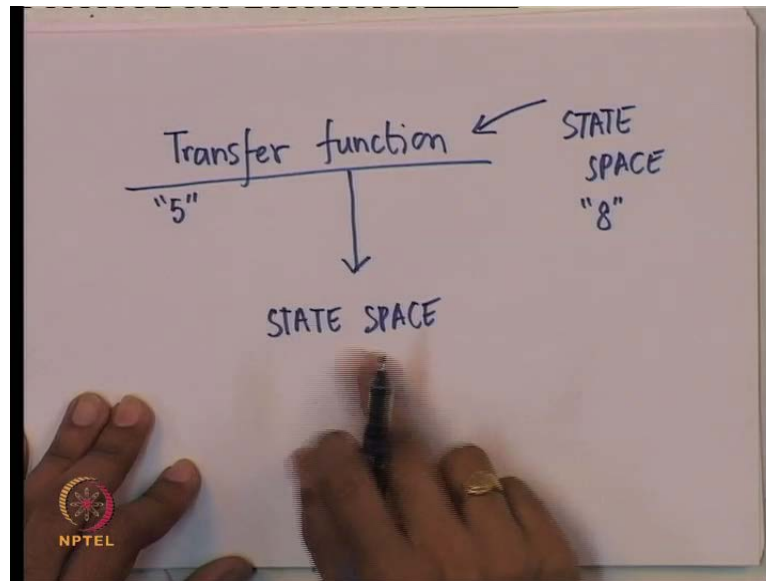
So, if you just know obtain these parameters, you will of course, manage to replicate **in the model** using the model at transfer function, which gives almost the same responses that obtained experimentally. But you will not be able to get all the parameters of the original model. Now, this is not very surprising, its **its** in fact a transfer function model is some times since, it is collapsing, the whole state space model into a input output relationship.

And some of the nuances, which are there present **you know** the nuanced information which is present in the states space model, is in some sense destroyed, because of this. So, just by doing this frequency response test, **this thus** this one test **you know** of the frequency response, we are not going to get all the parameters required of the original state space equation or let me put it this way will not get all the parameters specified in the original state space; in which the states are the of damper winding field winding fluxes.

So, before we go into a more deeper discussion **in** into this particular aspect, let me just tell you one thing that if I obtain these parameters, L_d T_d T_d' T_d'' T_d''' and T_d''' . Am I going to get a workable state space model of a synchronous machine, the point is I cannot get the original model of a synchronous

machine, because all these parameters are not obtainable from these. But it will be useful to understand what exactly can I obtain from this, these measurements we have got this time constants from this time constants, I cannot back calculate 8 values from these 5 values, remember of course, that (No audio from 46:37 to 47:13), whenever we have a transfer function, can we obtain the state space representation.

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So, if I got a transfer function representation of a dynamical system, in fact that is what I have got right now, if I actually do these frequency response test, I can infer the transfer function of a synchronous machine. A particular transfer function of a synchronous machine on the d axis with the field winding shorted. The question is from the transfer function, can I get back, the states space with reference to our previous discussion, I just one can say that obviously one cannot get the original states space, because the number of parameters obtained from this test are not adequate.

They are not adequate to obtain in fact, all the parameters of the original state space model. So, what we have done in fact, let me just tell you from the original states space which had in fact, 8 parameters which I just listed down; I got a transfer function, in fact I took at I got this transfer function by shorting the field winding. I got a transfer function in 5 parameters, now I cannot get back of course, this state space of 8 parameters using, these 5 parameters of the transfer function. But the question is can I get to a state space in which which is 5 parameters, the answer is yes, it is possible.

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Handwritten equations on a whiteboard:

$$\left. \begin{aligned} \dot{x} &= Ax + BU \\ y &= Cx \end{aligned} \right\}$$
$$\frac{Y(s)}{U(s)} = C (sI - A)^{-1} B$$
$$x = Rz$$

The whiteboard also features an NPTEL logo in the bottom left corner.

So, the key to this is to remember, that if you have got a states space representation of a system like this, the transfer function Y upon U of S is nothing but C into $(sI - A)^{-1}$ B . Now, **one thing** one interesting thing is, if I use a transformation of variables X is equal to R into z in such a situation, you will have the same system.

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Handwritten equations on a whiteboard:

$$\dot{z} = R^{-1}ARz + R^{-1}BU$$
$$y = CRz$$
$$\frac{Y(s)}{U(s)} = CR (sI - R^{-1}AR)^{-1} R^{-1}B$$
$$= C (sRR^{-1} - R(R^{-1}AR)R^{-1})^{-1} B$$
$$= C (sI - A)^{-1} B$$

The whiteboard also features an NPTEL logo in the bottom left corner.

When written in terms of the z variables will be \dot{z} is equal to $R^{-1}ARz$, the same system plus $R^{-1}BU$, and y is equal to C into x which is nothing but C into R into z ; the transfer function of this system is **is is** C into R into $sI - A$, of course, is

an identity matrix into $R^{-1}AR^{-1}B$, which is nothing but C into S into R^{-1} , this will be $R^{-1}AR^{-1}B$ into $R^{-1}AR^{-1}B$ inverse. So, this is of course, using simple rules of matrix algebra, so you will get C into $S I - A^{-1}B$; so what you have here is the same transfer function, this transfer function and this transfer function, in fact match exactly. So, the same system rather, if you have got to transfer function, you can either get to a state space of this kind or you can get a state space of this kind, and both are in fact valid representations of your system.

But, the point we should remember is that the original variables and the new variables are related by this relationship here. So, let me get back to our original issue from the state space of 8 variables, we got into transfer function of using 5 parameters, so using these 5 parameters of course, it will not be able to back calculate, all the 8 variables of the original state space equations.

But, you can if you so wish write down the state space equation using only 5 parameters, but the state variables of this state space system using 5 parameters, is not the same as the state variables here. Then in fact, to be some kind of linear combination of the states in this, so your state space equations here and the state space equation here are written in terms of different variables; so that is one important thing you should keep in mind. So, using these transfer I can get a state space representation using just these 5 parameters.

In fact it is a nice thing, if you can write down the state space with lesser number of parameters, but its important that the state space equations which you **you** will get will be in terms of states, which are related in some way by some linear transformation to the original state space variables. That is $\psi_f \psi_h \psi_d$, but of course, which is required lesser number of parameters.

Now, in power system analysis it is often required to at least **you know** have a nice neat interpretation of the states, so often people would insist that well I do not want a state space which use only which uses only 5 parameters, but the states are not easy to interpret. **You know** the states for example, ψ_f and ψ_h for example, are variable which are easy to interpret, they are in fact the fluxes link with the f and g y_f and h winding.

But of course, if I write the states space in terms of only 5 parameters, the states in this state space representation will be a linear combination of these states, and it may be difficult and is not really very nice to have. For examples, state space equations in which one of the states is say, 5 times the field flux plus 3.5 times the **you know** the damper winding flux. So, there is a in fact problem here and power system engineers have tried to solve this problem in a bit round about fashion, in fact the solution to this problem is to in fact **try to do** try to in fact, obtain more transfer functions by various other tests.

May be similar, setups some more tests and get **get** more parameters, which can be correlated to the original 8 parameters which have already said. So, if I through more test obtain more parameters, then I would should be able to obtain the original 8 parameters, which have mentioned by back calculating. Remember, that all the transfer functions which we have for example, if you recall, this is the coefficient of the numerator polynomial in its (Refer Slide Time: 55:44).

So, if you have got in fact in a just T d dash and T d double dash one will not be able to get all these parameters, if you have got for example, **sorry** T d dash T d double dash T d 0 dash and L d and T d 0 double dash, you are not going to get all these 8 parameters **which are** which are a part of this. So, its not possible to back calculate 8 parameters, but one can take out other transfer functions, you can **you know can con you know** try to think of other tests for example, you do these test to the field winding open.

In that case, you will get another transfer function, you can actually obtain the same transfer function using the analytical model which we have, then correlate both of them. So, you can actually have many more tests and actually get all the parameters, but unfortunately power system engineers have with a limited number of tests, and limited number of parameters, attempted to get an approximate synchronous machine model. And **there of** because of that we will in fact, have to make certain approximations in the kind of state space equations.

We are finally, going to get which are in terms of what are known as the standard parameters. So, there is a only one way of getting meaningful state space representations with a larger number of parameters than what are obtained by measurement, and that is by making certain approximations. So, we will redo or rather recap what we have done today in the next lecture, and try to obtain a state space representation of a synchronous

machine, a meaningful state space representation of asynchronous machine; may be with a few approximations in fact, with the few approximations, which will be in terms of states, which we can directly interpret. So, there will be not some transform states, which use lesser number of parameters, but we will try to get approximate states space equations, with a few approximate with lesser number of parameters; so this is what we will do in the next lecture.