

Power System Dynamics and Control
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Lecture No. # 15
Modeling of Synchronous Machines, Standard Parameters

We are now in the position to write down the equations of a synchronous machine together and try to draw some inferences from the equations which we have derived in the d q variables. Remember that we have done time variant and theta variant transformation of variables from the a b c variables or the phase variables to the d q 0 variables. The transformation which we have used is defined using the rotor position theta, that theta is of course, the electrical angle and to be instructive to just look at the transformation once.

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$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} K_d \cos \theta & K_q \sin \theta & K_0 \\ K_d \cos(\theta - \frac{2\pi}{3}) & K_q \sin(\theta - \frac{2\pi}{3}) & K_0 \\ K_d \cos(\theta + \frac{2\pi}{3}) & K_q \sin(\theta + \frac{2\pi}{3}) & K_0 \end{bmatrix} \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix}$$

So we can express f d f q and f 0 in terms of f a f b and f c and vice versa in this case of course, this transformation is what we refer to as cp and this of course, is cos theta cos theta minus 2 pi by 3 and cos theta plus 2 pi 3 and similar things for sine.

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Handwritten matrix equation showing the inverse transformation from abc variables to dq0 variables:

$$\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \begin{bmatrix} k_1 \cos \theta & k_1 \cos(\theta - 2\pi/3) & k_1 \cos(\theta + 2\pi/3) \\ k_2 \sin \theta & k_2 \sin(\theta - 2\pi/3) & k_2 \sin(\theta + 2\pi/3) \\ k_3 & k_3 & k_3 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

Below the matrix, the values of the constants are given:

$$k_1 = \frac{2}{3k_d}, \quad k_2 = \frac{2}{3k_q}, \quad k_3 = \frac{1}{3k_0}$$

An arrow points from the matrix to the constants, labeled C_p^{-1} .

Now, the inverse transformation of course, given by this. So, the basic idea which we have tried to get is that we can do the analysis in the d q 0 variables and after obtaining the results in the d q 0 variables we try to move on and see what are the values in the a b c variables.

So, why do of course, we go to the d q 0 variables the reason is that the differential equations which you get in the d q 0 variables are in fact time not theta dependent that is one major advantage of using theta dependent transformation of variables.

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Handwritten notes on synchronous machine equations:

(Prof. A.M. Joshi/Kanani)
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SYNCHRONOUS MACHINE EQUATIONS

STATOR FLUX EQNS. $k_d = k_q = \sqrt{\frac{2}{3}}, k_0 = \frac{1}{\sqrt{3}}$

(a) $-\frac{d\psi_d}{dt} - \omega\psi_q - R_a i_d = V_d$

(b) $-\frac{d\psi_q}{dt} + \omega\psi_d - R_a i_q = V_q$

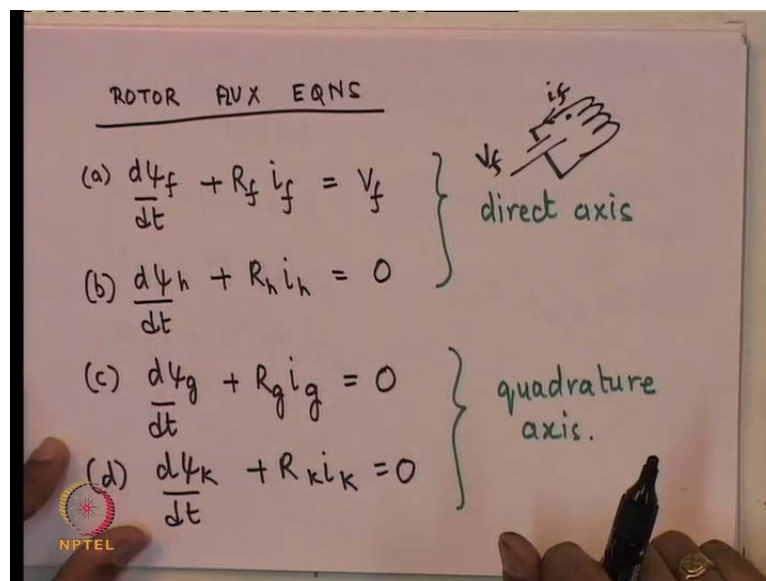
(c) $-\frac{d\psi_0}{dt} - R_a i_0 = 0$

$\omega = \frac{d\theta}{dt}$

So, what are those equations the basic equations of a synchronous machine in the d q 0 variables are as follows. So, the stator flux equations are given by this $\frac{d\psi_d}{dt} - \omega \psi_q - R_a I_d$ is equal to v_d and similar equations for ψ_q and ψ_0 ω here is of course, $\frac{d\theta}{dt}$ θ being the electrical angle and ω being the electrical angular frequency. Now, remember that these equations are obtained using the transformation as discussed before with k_d and k_q equal to $\frac{\sqrt{2}}{3}$ and k_0 is equal to $\frac{1}{\sqrt{3}}$ by doing this we make this transformation c_p and this inverse transformation c_p^{-1} transpose of each other that is $c_p^{-1} = c_p^T$.

Becomes equal to c_p^T for this choice of k_d k_q and k_0 of course, but of course, it is not necessary for k_d and k_q and k_0 to be these values in the sense that even if you choose a non 0 k_d k_q k_0 value absolutely arbitrarily still we will achieve one thing that is time variance of time invariance of the resulting differential equations. So of course what I will write here now, what we will do in this course will be for the specific choice of k_d k_q and k_0 .

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But you noticed the all the equations are time invariant now, the rotor equations recall that we have represented the rotor as 2 coils on the d axis and 2 coils on the quadrature axis h g and k are representing the effect of damper bars as well as steady current effects in the machine.

Now, v_f is the applied voltage on the field winding if we recall an earlier diagram. So, your this is I_f and this is v_f so, this is the applied voltage on the field winding now of course, the equations which relate if you recall if you look at these equations they are not in pure state space form for example, they are differential equations are in ψ , but you also have these terms i_d i_q and i_0 I_f I_g and I_k I_h I_g and I_k .

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$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & M_{df} & M_{dh} & 0 & 0 \\ 0 & L_q & 0 & 0 & 0 & M_{qg} & M_{qk} \\ 0 & 0 & L_0 & 0 & 0 & 0 & 0 \\ M_{df} & 0 & 0 & L_{ff} & L_{fh} & 0 & 0 \\ M_{dh} & 0 & 0 & L_{hf} & L_{hh} & 0 & 0 \\ 0 & M_{qg} & 0 & 0 & 0 & L_{gg} & L_{gk} \\ 0 & M_{qk} & 0 & 0 & 0 & L_{gk} & L_{kk} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_h \\ i_g \\ i_k \end{bmatrix}$$

These are in fact related to the flux variables by this relationship which is of course, theta invariant. So, this is i_d i_q i_0 this is a diagonal matrix we have just shown the non 0 terms in bold this is of course, a symmetric matrix because we have chosen our k_d and k_q values such that we get this symmetry.

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$$M_{df} = M_{af}/k_d$$

$$M_{dh} = M_{ah}/k_d$$

$$M_{qg} = M_{ag}/k_q$$

$$M_{qk} = M_{ak}/k_q$$

$$L_d = L_{aa0} - L_{ab0} + \frac{3}{2}L_{aa2}$$

$$L_q = L_{aa0} - L_{ab0} - \frac{3}{2}L_{aa2}$$

$$L_o = L_{aa0} + 2L_{aa2}$$

In fact the special choice of k_d and k_q was made in order to get this symmetry and of course, interesting property for c_p and c_p inverse in c_p now remember that m_{df} here is related to m_{af} the mutual inductance coefficient by this formula. So, our m_{df} is m_{af} divided by k_d m_{af} is in fact the mutual inductance between the a winding and the field a winding and the field winding in fact is the max value of the inductance.

Of the a winding and the field winding divided by k_d of course, we have taken as k_d by root 2 similarly, we can define m_{dh} m_{qg} and m_{qk} and of course, remember that l_d itself is can be written down as l_{aa0} minus l_{ab0} plus 3 by 2 times l_{aa2} and similarly, l_{dq} and l_o are defined.

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$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & M_{df} & M_{dh} & 0 & 0 \\ 0 & L_q & 0 & 0 & 0 & M_{dg} & M_{dk} \\ 0 & 0 & L_0 & 0 & 0 & 0 & 0 \\ M_{df} & 0 & 0 & L_{ff} & L_{fh} & 0 & 0 \\ M_{dh} & 0 & 0 & L_{hf} & L_{hh} & 0 & 0 \\ 0 & M_{dg} & 0 & 0 & 0 & L_{gg} & L_{gk} \\ 0 & M_{dk} & 0 & 0 & 0 & L_{gk} & L_{kk} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ i_f \\ i_h \\ i_g \\ i_k \end{bmatrix}$$

In this fashion so, these are of course, the original self and mutual inductance terms when written down using the a b c variables, but of course, when you go to the d q 0 frame of reference or the d q 0 variables this relationship is like this and of course, does not depend on theta. In fact you can aggregate all the windings on the d axis that is the d f and h winding around the d axis the direct axis.

Q g and k are on the quadrature axis and psi 0 is of course, the sequence flux. So, if you relate the d axis fluxes they are only dependent on the d axis currents similarly, the q axis fluxes are dependent on the q axis currents alone and psi 0 is dependent on I 0 alone. So I will just re arrange the equation and they will look like this of course, we remember that the d q variables are coupled, but via this differential equation.

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The image shows handwritten mathematical derivations on a whiteboard. At the top, the differential equation is written as $\frac{2}{p} J \frac{d\omega}{dt} = T_m - \frac{P}{2} (\psi_d i_q - \psi_q i_d)$. Below this, $\omega = \frac{d\theta}{dt}$ is written. The next equation is $\frac{2H}{\omega_B} \frac{d\omega}{dt} = T_m - \frac{(\psi_d i_q - \psi_q i_d)}{(MVA_{base} / \omega_B)}$. At the bottom, the inertia constant is defined as $H = \frac{1}{2} J \omega_{mB}^2 / MVA_{base}$. A note on the left states $T = \frac{MVA_{base}}{\omega_m}$. A small logo for NIPTEIL is visible in the bottom left corner of the whiteboard image.

Remember these differential equation in psi d has got this psi q term what we kind of inferred to be the speed e m f terms which comes because of taking the derivatives of the transformation. We have done of course, his in the previous class now our discussion would not be complete unless we talk about how omega or theta vary in fact omega itself is d theta by d t this psi the electrical angle theta.

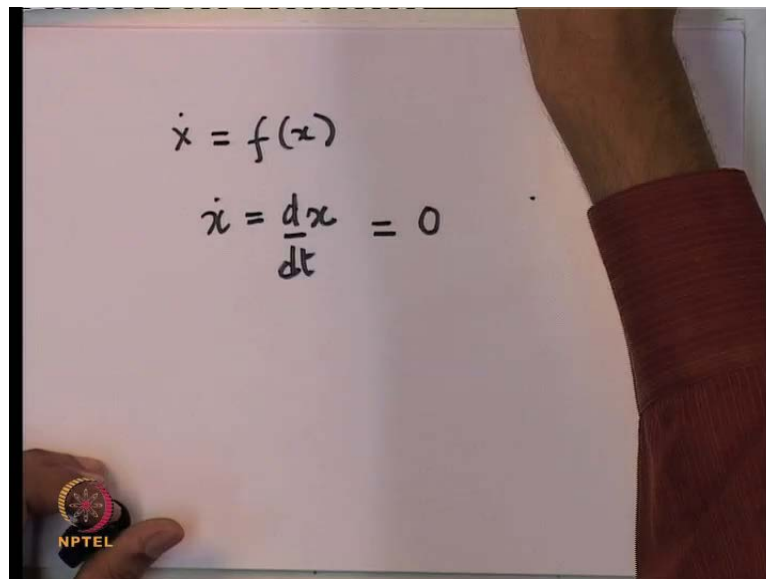
Omega is the electrical frequency so, we got this equation sometime back 2 by πp is the number of poles J d by d t J d is the number of moment of inertia is equal to $t m$ minus p by 2 into $\psi_d I_q$ minus $I_d \psi_d$ it is a very neat term for the $t e$ dash. So, π by two times the $t e$ dash is actually the electromagnetic torque which we get now, we can write these in per unit by dividing the whole both the sides by the torque base torque base is nothing, but $m v a$ base divided by the mechanical rotational angular frequency. So if you do that you can actually get this in this form where $t m$ is the per unit torque mechanical torque the prime mover torque.

And h is nothing, but half of $J \omega_{mB}^2$ divided by $m v a$ base. So, this is basically if you divide both sides by the torque the mechanical torque base so, torque base is nothing, but note that we have not written down the synchronous machine equations in pure state space form that is \dot{x} is equal to $A x$ we have actually written it in a composite form.

We have written $\frac{d\psi}{dt}$ is equal to in terms of ψ if we for example, take fluxes as the states then we also use the auxiliary variables i_q and i_d and similarly, if i_h and i_k , but then we relate the fluxes and current through an inductance matrix. So, we are not written this down in a pure state we are now in a position we are now in a position.

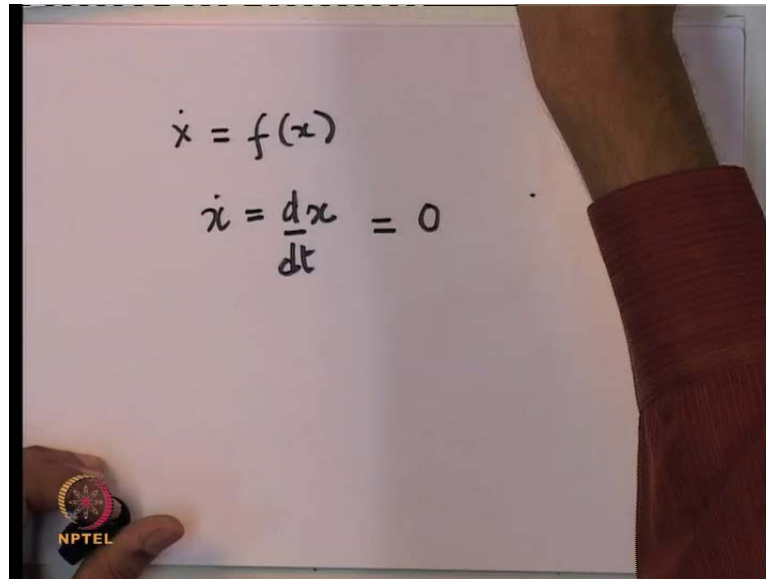
To actually draw inferences about the behavior of a synchronous machine that is how what we have doing this modeling after all we are trying to get how the synchronous machine behaves. So, what we will do is we will keep our ambition bit low at the present we will use these flux equations all the equations which you have written do to actually at least first infer the steady state behavior of a synchronous machine.

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$$\dot{x} = f(x)$$
$$\dot{x} = \frac{dx}{dt} = 0$$

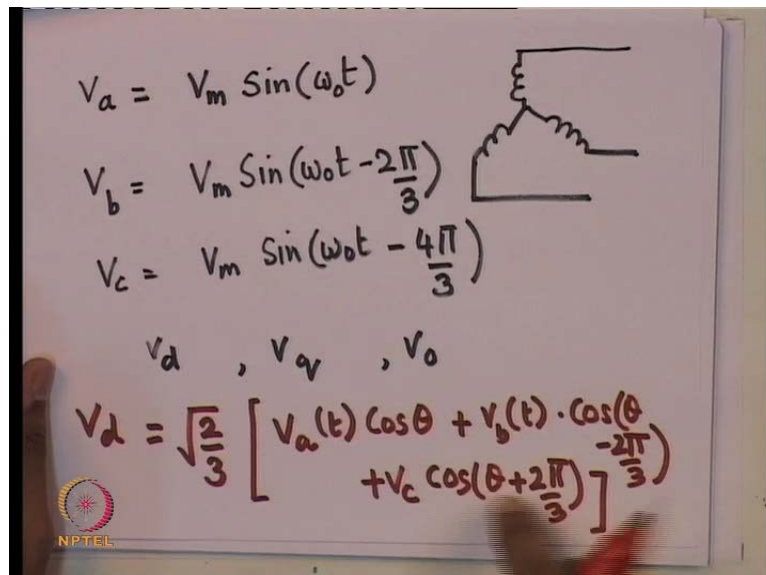
So, how do I infer the steady state behavior of a synchronous machine now the thing is that when do you say that a system \dot{x} is equal to f of x is in steady state where \dot{x} is equal to $\frac{dx}{dt}$ we would say of course, the system is in steady state when this equals to 0 now.

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$$\dot{x} = f(x)$$
$$\dot{x} = \frac{dx}{dt} = 0$$

The interesting thing about this whole set of equations in the d q frame d q 0 frame of references that although in the a b c frame. When we say we are in the steady state psi a psi b psi c are sinusoids it turns out if we are having a balanced system then psi b psi q and psi 0 in steady state become constant let me just give you an example.

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$$V_a = V_m \sin(\omega_0 t)$$
$$V_b = V_m \sin(\omega_0 t - \frac{2\pi}{3})$$
$$V_c = V_m \sin(\omega_0 t - \frac{4\pi}{3})$$

v_d, v_q, v_0

$$v_d = \sqrt{\frac{2}{3}} \left[V_a(t) \cos\theta + V_b(t) \cdot \cos\left(\theta - \frac{2\pi}{3}\right) + V_c \cos\left(\theta + \frac{2\pi}{3}\right) \right]$$

Suppose you have got a set of voltages v a, v b and v c and these are a balanced set of voltages. So, what I can write down v a as example of v a is equal to v max the phase to neutral max suppose have got a star connected machine.

The stator is connected to the same star then this is v sine and it is say the voltages applied to the stator have a frequency ω then v_b is equal to v_m . Suppose this is a balanced set of voltages in that case what is v_d and v_q and v_0 v_0 is easy to get remember the transformation is.

This of course, we are taking special values of k_d and k_q and k_0 k_d and k_q k are root 0 by 3 and k_0 is equal to 1 by root 3. So, I will just write down the expressions for v_d so v_d will be root 2 by 3 v_a of t into **I am sorry** \cos of θ plus v_b of t into \cos θ minus 2π by 3 plus v_c into \cos θ plus 2π by 3 plus 2π by 3. So, you have got v_d is equal to this so of course, v_a of t and v_b of t and of course, t are like this the question is what is v_d so what is v_d .

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$$v_q = \sqrt{\frac{2}{3}} \left[v_a \sin \theta + v_b \sin \left(\theta - \frac{2\pi}{3} \right) + v_c \sin \left(\theta + \frac{2\pi}{3} \right) \right]$$

$$v_0 = \frac{1}{\sqrt{3}} [v_a + v_b + v_c]$$

if $\theta = \omega_0 t$

And similarly, we have got v_q is equal to root 2 by 3 v_a v_0 is equal to. So, if the speed of rotation of the machine θ is equal to $\omega_0 t$ itself suppose the speed of the machine is $\omega_0 t$. So, if θ is equal to the speed of the machine is ω_0 and θ is equal to $\omega_0 t$.

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$$\begin{aligned}v_0 &= 0 \\v_q &= V_m \sqrt{\frac{2}{3}} \left[\sin^2 \theta + \sin^2 \left(\theta + \frac{2\pi}{3} \right) + \sin^2 \left(\theta + \frac{4\pi}{3} \right) \right] \\&= V_m \cdot \sqrt{\frac{3}{2}} = \underline{\underline{V_{llrms}}} \\v_d &= 0\end{aligned}$$

The whiteboard also features a small circular logo with a starburst pattern and the text "NIPTEEL" in the bottom left corner.

In such a case we will have v_0 will be equal to 0 that is almost equal to v_s its balanced set of sinusoids. So, you will have $v_a + v_b + v_c$ is equal to 0 and well have v_q will be equal to $\sqrt{2/3}$ into sine square theta plus so, it will be v_m sine square theta plus sine square theta minus $2\pi/3$ plus sine square theta plus $2\pi/3$ and that will give you in fact this remember there is an identity which we discussed sometime back.

This this sum of these 3 terms is actually $\sqrt{3/2}$ this is a trigonometric identity of course, multiplication factor of v_m here we well have v_m into $\sqrt{3/2}$ and it is easy to show that v_d will come out to be 0. So, in case v_a, v_b, v_c are like this v_0, v_q and v_d are equal to 0 and in more importantly v_0 is equal to 0 v_d is equal to 0 but, v_q comes out to be v_m into $\sqrt{3/2}$.

In fact this v_m is nothing, but the maximum of phase the maximum of the phase to the neutral voltage for a star connected system here so, maximum of the phase to neutral voltage. So, it is easy to see that v_q in this case is equal to the line to line r m s voltage of the system. So, actually what I wanted to show here is that if I got a balanced set of sinusoids then it turns out that v_0, v_q and v_d are in constants.

So, if you have got a if you are steady state in your system is a balanced 3 phasesinusoidal steady state it turns out that $d, q, 0$ variables act like constant. So, this is just simple example to prove that or show that actually now let us actually try to get

some inferences from of the steady state behavior from our differential equation. So, one thing I showed you is that if you are in a balanced 3 phase sinusoidal steady state.

Then the d q variables become constant and if they become constants of course, the d by dt's of all the d variables are going to be 0 more over the field winding of a synchronous machine as well as the damper winding currents also, become constants in steady state in fact the damper winding currents are 0 so, I_g I_h and I_k are 0 in steady state.

So, if you have all these conditions it should be possible to infer the steady state behavior of a machine. So, what the first thing we will try to do is what if we want to see this what happens in case we have got a field voltage v_f applied to the machine at the field windings in steady state what are the voltages induced on the stator so that is a very basic study that we can do so, what we do is take the differential equations of the synchronous machine set all the d by dt's equal to 0. So if you do that I will just draw few d by dt's, so if for example, if we take the stator flux equations.

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$$\begin{aligned}
 -\omega \psi_q - R_a i_d &= V_d \\
 \omega \psi_d - R_a i_q &= V_q \\
 -R_a i_0 &= 0 \Rightarrow i_0 = 0 \\
 i_g &= i_h = i_k = 0 \\
 \frac{d\psi_h}{dt} + R i_h &= 0
 \end{aligned}$$

Since, d by dt is equal to 0 I have $\omega \psi_q - R_a i_d$ is equal to v_d and you will have $\omega \psi_d - R_a i_q$ is equal to v_q and of course, $-R_a i_0$ is equal to 0 ω is of course, the electrical angular frequency please remember that now similarly, we have got i just mentioned some time ago so, this just implies that i_0 is equal to zero.

You also have ψ_h these are the damper winding currents i_h is equal to i_k is equal to 0 this is of course, simply obtained by setting the $\frac{d\psi_h}{dt}$ is corresponding to the field windings equal to 0. So in fact what you have is $\frac{d\psi_h}{dt}$ so, if we set this to 0 it automatically means that i_h is equal to 0. So similarly, we can show that i_g and i_k also equal to 0 i_f is not equal to 0 in steady state so, what we have is these two equations here and then we'll have.

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$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

↓
0

$$\frac{v_f}{R_f} = i_f$$

Open circuit
 $i_d = 0 = i_q$

So, we set this equal to 0 under steady state conditions so, that means that v_f by r_f is equal to i_f . So, this is our second equation of importance now we also have from the flux current relationship if you look at this flux current relationship which we have we have got now this equal to 0 this equal to 0 and this equal to 0 and only non 0 currents are i_d i_q and i_f .

But of you are also we are actually taking the open circuit voltages induced on the stator of a synchronous machine then we also have i_d equal to 0 and i_q is equal to 0. So, what we have under open circuit conditions the differential the algebraic equations which are there are very simple.

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$$\begin{aligned}
 -\omega\psi_q &= V_d \\
 \omega\psi_d &= V_q \\
 V_f/R_f &= i_f \\
 \psi_d &= M_{df} i_f \\
 \psi_q &= 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} -\omega\psi_q &= V_d \\ \omega\psi_d &= V_q \\ V_f/R_f &= i_f \\ \psi_d &= M_{df} i_f \\ \psi_q &= 0 \end{aligned}} \right\}
 \begin{aligned}
 V_d &= 0 \\
 V_q &= \frac{M_{df} V_f}{R_f} \\
 V_0 &= 0
 \end{aligned}$$

So, you have got $\omega\psi_q$ is equal to V_d $\omega\psi_d$ is equal to V_q V_f/R_f is equal to i_f then we have ψ_d is equal to $M_{df} i_f$, but of course, i_d is equal to 0. So, the only term which comes is $M_{df} i_f$ all the other currents are 0 ψ_q turns out to be 0 because it is $i_q \psi_q$ is nothing, but i_q into i_q plus $m_q g$ into I_g plus $m_q k$ into I_k , but all these are in fact this is also 0 and this is also 0 under open circuit conditions i_d and i_q are also 0 under open circuit conditions so, what you have here is this is also zero.

So, under circuit conditions you will have from all this you can infer that you will have V_d equal to 0 and V_q is equal to $M_{df} V_f / R_f$ into V_f there is a small correction here I have forgotten to multiply the term V_q is equal to $M_{df} M_{df} V_f / R_f$ into V_f I forgotten to multiply it by ω .

Which is the speed of the rotational speed of the synchronous machine. So, V_q the expression here $M_{df} V_f / R_f$ into V_f has to be multiplied by ω not so, this is V_q and of course, V_0 turns out to be equal to 0 is to so, what we have here.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $v_a = \sqrt{\frac{2}{3}} \cdot 0 \cdot \cos\theta + \sqrt{\frac{2}{3}} \cdot \frac{Mdf}{R_f} \cdot v_f \sin\theta + 0$. The second equation is $v_a = \sqrt{\frac{2}{3}} \frac{Mdf}{R_f} \cdot v_f \sin\theta$. To the right of the second equation, it is noted that $\theta = \omega_0 t$. In the bottom left corner, there is a small circular logo with the text 'NPTEL' below it.

Eventually is a situation where we can find out v_a from the value of v_d and v_q . So, what is v_a from the transformation which we have defined v_a is nothing, but $\sqrt{2/3}$ into v_d so, v_d is equal to $0 \cdot \cos\theta$ plus $\sqrt{2/3}$ into $m d f$ by r_f into v_f into $\sin\theta$ plus 0 so, if you have this.

It turns out that v_a is $m d f \sqrt{2/3} m d f$ by r_f into $v_f \sin\theta$. So, actually $\sin\theta$ is θ if the machine is rotating at a singular frequency of ω_0 and suppose ω_0 is equal to $\omega_0 t$ what you have is under open circuit conditions you will have a voltage induced which is nothing, but a sinusoidal voltage and this is the coefficient corresponding to it.

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$$v_a = \sqrt{\frac{2}{3}} \frac{M_{af}}{\sqrt{\frac{2}{3}}} \cdot \left(\frac{V_f}{R_f} \right) \cdot \sin \omega t$$

$$= M_{af} I_f \sin \omega t$$

D.C.

$$v_b = \sqrt{\frac{2}{3}} V_f \frac{M_{bf}}{R_f} \sin \left(\theta - \frac{2\pi}{3} \right)$$

Now, if you look at this expression v_a is nothing, but $\sqrt{\frac{2}{3}} \frac{M_{af}}{\sqrt{\frac{2}{3}}}$ is nothing M_{af} by k_d , $\sqrt{\frac{2}{3}}$ itself into V_f by R_f V_f by R_f is the current of course, into $\sin \omega t$. So, if you look at this particular equation this is nothing, but M_{af} into I_f into $\sin \omega t$. The expression for v_a is M_{af} into I_f into $\sin \omega t$ has to be multiplied by ω , there is a minor correction here.

v_a the expression for v_a which is given as M_{af} into I_f into $\sin \omega t$ has to be multiplied by ω not so, actually it is understandable why this is true. So, if you have got a current I_f in your field winding M_{af} I_f is the flux link with the a winding because of the field winding and because of that we have got we are having this form.

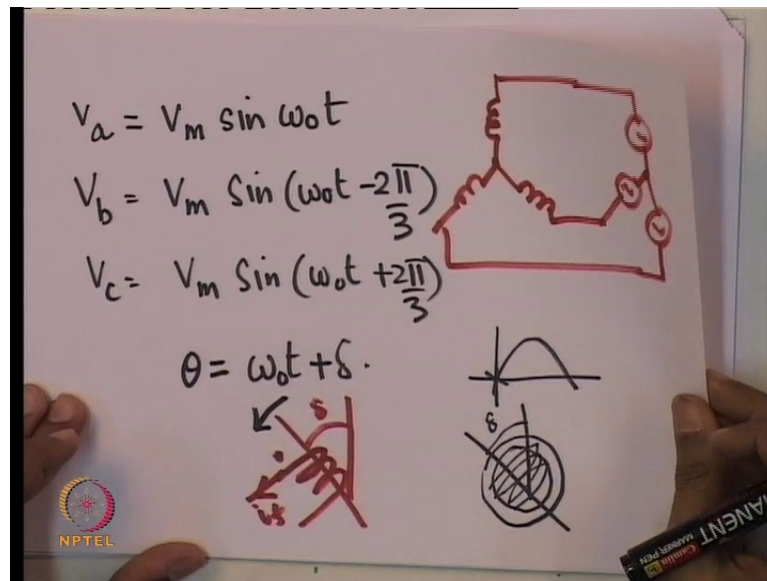
So, this is the open circuit voltage on the a winding of course, v_b is equal to nothing, but $\sqrt{\frac{2}{3}}$ in fact it is $\sqrt{\frac{2}{3}}$ by M_{bf} by R_f into \sin into V_f $\sin \theta$ minus $\frac{2\pi}{3}$ by $\frac{2\pi}{3}$ and similarly, we see it will be phase shifted by $\frac{2\pi}{3}$ so this is what we get when we apply the equations under steady state under open circuit conditions when V_f is applied to this stator. So, let me just revise what are the steps we just put d by d t equal to 0 why do we put them to be equal to zero.

Because under balanced 3 phase sinusoidal conditions d q variables become constants in steady state. So, if they are constant d a by d t equal to zero remember d ψ_a by d t is not constant in steady state. So, in fact the a b c variables are in sinusoidal steady state

corresponds to the d q variables being in you can see d c steady state means the values become constant.

So, by d by d t has become equal to 0. So, that is one thing is one important thing now, well go one step further let us see what happens. When we connect the machine to a voltage source we are not having an open circuited machine we have connecting to a voltage source so, let us assume we have got a star connected machine.

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So, the a winding b winding and c winding are connected to the voltage source there is a voltage source I have got a voltage source of this kind and let us assume that this voltage source is v_a v_m this is a constant voltage source three phase voltage source and a balanced one that too.

And v_c is equal to $v_m \sin \omega_0 t + \frac{2\pi}{3}$. So, if you have got this voltage source balanced voltage source connected is a three phase balanced source connected here and let us assume that θ is equal to $\omega_0 t + \delta$ now, what I am trying to say here that, we let us suppose we are operating at a situation suppose we have we have got a steady state situation in which whenever this voltage v_a has a 0 crossing.

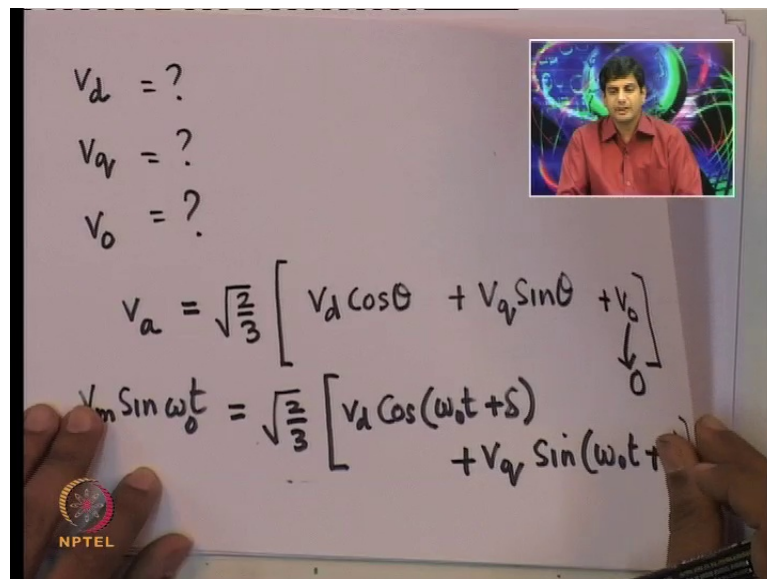
And I just take a quick snap shot of the machine I find that the field winding which is here is at an angle δ let me repeat what I said suppose I am operating at such a steady

state such, that if I take a snap shot of the machine when the phase voltage v_a is crossing it is 0 has a 0 crossing here take a snap shot at that point I see that the field winding.

The field winding is aligned at an angle δ with the a axis winding. So, if this is your field winding remember this is your field winding. So, this is something I am telling you it is not something which is obvious I am just telling you that a situation I am studying is such that when the v_a is having it's 0 crossing this is at an angle δ that basically the speed of the machine is also ω not t ω not.

So, the voltages which are applied and the rotational speed of the machine are in fact the same only thing is that at time t is equal to 0 or whenever, v_a hits the 0 if I take a snap shot this is at an angle δ so, such a situation suppose exists.

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In such a case what will be v_d , v_q and v_0 . So, we can apply of course, this transformation this transformation which you have studied with of course, k_d and k_q as $\sqrt{2/3}$ and k_0 is equal to $1/\sqrt{3}$, but of course, I will not go through the complete manipulations let me try to just show by just show by inspection what you will get now you know that v_a is equal from the c p transformation is nothing, but $\sqrt{2/3}$ times.

$V_d \cos \theta$ plus $v_q \sin \theta$ plus v_0 not of course, is 0 because v_a plus v_b plus v_c is equal to 0 well this is 0 also since, v_a is equal to $v_m \sin \omega t$ is equal to t

is equal to root 3 by 2. So, we have $v_d \cos \theta$ I have told you is $\omega t + \delta$ plus $v_q \sin \omega t + \delta$ so, can you guess what v_d and v_q will be.

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$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \sin(A+B) &= \cos A \sin B + \cos B \sin A \end{aligned}$$

$$\begin{cases} v_d = \sqrt{\frac{3}{2}} V_m (-\sin \delta) \\ v_q = \sqrt{\frac{3}{2}} V_m \cos \delta \\ v_0 = 0 \end{cases}$$

So, the question I am asking you is if I give you the **right** hand side and I tell you the left hand side is this can you find out what v_d and v_q are please recall the trigonometric identity $\cos a + b$ is equal to $\cos a \cos b - \sin a \sin b$ and $\sin a + b$ is equal to $\cos a \sin b + \cos b \sin a$ if you if you know this it is very easy to see that v_d is if the left hand side and **right** hand side are to be equal then v_d will be root 3 by 2 v_m into minus sine delta.

And v_q is equal to root 3 by 2 times $v_m \cos \delta$. So, v_d and v_q become this in fact v_m remember is the maximum voltage of the phase to neutral. So, if its star connected winding in fact this is the line to line rms voltage this taken together is the line to line rms voltage anyhow now that we have got v_d , v_q and v_n .

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$$\text{Set } \frac{d}{dt} \psi = 0 \text{ for all flux}$$
$$i_g = i_h = i_k = 0$$
$$\underline{v_o = 0} \quad \frac{d\psi_o}{dt} = 0 \quad i_o = 0$$
$$\boxed{v_f / R_f = i_f}$$

Can we take out again the steady state situation values of current and so on. Well in fact again we set the d by dt is equal to 0 set the d by dt's equal to 0 for all fluxes because they become constants in steady state. So, remember of course, by doing this we will again get the same condition by damper winding.

And the fact that v_o is equal to 0 v_o is also equal to 0 and $d\psi_o$ is equal to 0 also means i_o is equal to 0 well also have from the flux differential field flux differential equations v_f by r_f is equal to i_f . So, v_f by r_f is equal to i_f this is another equation we have this is another important equation.

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$$\begin{aligned} -\omega_0 \psi_{qv} &= V_d & \theta &= \omega_0 t + \delta \\ \omega_0 \psi_{qd} &= V_{qv} & \frac{d\theta}{dt} &= \omega_0 \end{aligned}$$

$$\psi_d, \psi_{qv}$$
$$\begin{aligned} \psi_d &= L_d i_d + M_{df} i_f \\ \psi_{qv} &= L_{qv} i_{qv} \end{aligned}$$

And if we neglect resistance of the stator then we also have minus $\omega \psi_q$ is equal to v_d and $\omega \psi_d$ is equal to v_q now, one of the things I mentioned sometime back is that θ is equal to $\omega t + \delta$. So, it follows of course, that this should be ω because $\frac{d\theta}{dt}$ is equal to ω is it.

I just discovered a small error in what we have done slightly earlier I will just write that down here too θ we have taken as ωt this is the earlier example. So, in fact this should be into $\omega t + \delta$. So, this is a small error please note this small error what we did previously.

We continue our example of a synchronous machine connected to a voltage source. So, if we neglect the resistance of the stator in fact get this particular set of equations and because of this now we have got we can get the value of ψ_d and ψ_q how see remember that ψ_d equal to $L_d i_d + M_{df} i_f$.

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$$T_e' = 4d i_q - 4v_d i_d$$

$$\frac{p}{2} T_e' = T_e$$

$$T_e' = \frac{v_q}{\omega_0} \cdot \frac{4q}{L_q} - \left(-\frac{v_d}{\omega_0} \right) \cdot i_d$$

$$= \frac{v_q}{\omega_0} \cdot \frac{4q}{L_q} + \frac{v_d}{\omega_0} \cdot \left[\frac{4d - M_d f i_f}{L_d} \right]$$

ψ_q is equal to $l_q I_q$ and of course, it is not related to since the other currents on the q axis are all 0 you do not have any other term. So, what we can do from all this is actually obtain the torque so, torque is equal to if you recall **I am sorry** electrical torque that is actually t dash actually is equal to $\psi_d i_q - \psi_q i_d$ and this is in fact the electrical torque for the two pole machine for in general the electromagnetic torque is p by q times.

T dash this is something you should remember. So, if you are working with the 2 pole machine then of course, we finally, have if you want to take out t dash it will turn out to be $\psi_d i_q - \psi_q i_d$ is nothing, but t q divided by ω_0 not v_q divided by ω_0 not into $I_q I_q$ q is nothing but, ψ_q by l_q and ψ_q minus $I_q \psi_q$ is nothing but.

Minus of v_d by ω_0 you can actually have a look at this these equations which we have just done in the previous slide into $I_d I_d$ is nothing, but I will just rewrite this t dash is equal to v_q by ω_0 not into ψ_q by l_q minus or we say we can call it plus v .

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Handwritten mathematical derivation on a whiteboard:

$$= \frac{V_q \cdot V_q}{\omega_0 L_q} + \frac{V_d}{\omega_0} \cdot \left[\frac{V_d - M_{df} i_f}{L_d} \right]$$

$$T_e' = \frac{V_q \cdot V_q}{\omega_0 L_q} + \frac{V_d}{\omega_0} \left[\frac{V_d - M_{df} i_f}{L_d} \right]$$

The derivation shows the simplification of the equation for T_e' . The first term is $\frac{V_q \cdot V_q}{\omega_0 L_q}$ and the second term is $\frac{V_d}{\omega_0} \left[\frac{V_d - M_{df} i_f}{L_d} \right]$.

So, if we just continue with these manipulations we'll get T_e' is equal to you can substitute for ψ_q again here. So, well that is all v_q into ψ_q ψ_q is nothing, but minus v_d by $\omega_0 L_q$. so, plus v_d by $\omega_0 \psi_d$ is nothing. So, if we gather all these terms again it is turning out to be quite an avalanche of equations.

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Handwritten mathematical derivation on a whiteboard:

$$T_e' = \frac{-V_q V_d}{\omega_0^2 L_q} + \frac{V_d V_q}{\omega_0^2 L_d} - \frac{V_d \cdot M_{df} i_f}{\omega_0 L_d}$$

$$V_d = \sqrt{\frac{3}{2}} V_m \sin \delta \quad V_q = \sqrt{\frac{3}{2}} V_m \cos \delta$$

The derivation shows the final simplified equation for T_e' and the expressions for V_d and V_q .

We just be steady. So, minus $v_q v_d$ by ω_0 I think there is ω_0^2 is there L_q plus $v_d v_q$ by $\omega_0^2 L_d$ minus v_d by $\omega_0 L_d$ into $M_{df} i_f$. So, these are the expressions which we will get finally, form this actually v_d and v_q

have already computed from previous manipulations there nothing, but v_d is equal to $\frac{\sqrt{3}}{2} v_m \sin \delta$ minus of it and v_q is equal to $\frac{\sqrt{3}}{2} v_m \cos \delta$ cos delta.

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The image shows a handwritten derivation for the electrical torque T_e' . The equation is written as follows:

$$T_e' = \frac{3}{2} \cdot \frac{1}{X_q} \cdot \frac{V_m^2 \sin \delta \cos \delta}{\omega_0}$$

$$- \frac{3}{2} \cdot \frac{1}{X_d} \cdot \frac{V_m^2 \sin \delta \cos \delta}{\omega_0}$$

$$+ \sqrt{\frac{3}{2}} \frac{V_m \sin \delta}{X_d} \frac{X_{df}}{\omega_0} \cdot I_f$$

In the bottom left corner of the slide, there is a logo for NPTEL (National Programme on Technology Enhanced Learning).

So, we can substitute this onto this and finally, obtain the electrical torque. So, what we get essentially T_e' is equal to its it will be $\frac{3}{2} \frac{1}{X_d X_q} \omega_0^{-1} I_q$ is X_q you have into v_m square sine delta cos delta plus that will have here by 2 times 3 by 2 times one upon X_d into v_m square sine delta cos delta divided by ω_0 minus.

Root 3 by 2 it is actually plus we are evaluating this expression please remember that, so i am evaluating now, this expression here. So, it will be $\frac{\sqrt{3}}{2} \frac{v_m \sin \delta}{X_d} \frac{X_{df}}{\omega_0} I_f$ its d into X_{df} by ω_0 into I_f into sine delta now does this expression look familiar to you this expression in fact is something we probably done in our under graduate here.

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$$\underline{T_e' \cdot \omega_0} = V_{LLrms}^2 \cdot \frac{\sin 2\delta}{2}$$

$$T_m = T_e$$

$$+ \frac{V_{LLrms} \cdot (x_d i_f)}{x_d} \cdot \sin \delta$$

open circuit voltage
L & r.m.s.

What we are having here I will just re write this is nothing ,but t dash is nothing, but t dash or t dash into omega not this is power you can say. So, if you look at the power it is nothing, but 3 by 2 times v m square so, if you look at this 3 by 2 v m square we actually v m is the phase to neutral peak value so, 3 by 2 times the phase value of the peak by neutral is the fact is rather root 3 by 2 times the peak of the phase to neutral is in fact the line to line r m s voltage.

So actually we can write this as v line to line r m s square into sine 2 delta by 2 cos delta sine delta the product is sine delta by 2 into one upon x q into minus one upon x d plus v line to line r m s into by x d into x d f into i f, so in fact x d f into i f is the open circuit voltage I should say line to line r m s voltage we have just done this few minutes ago in fact when we had.

Obtained our open circuit voltages formula in fact v a is nothing, but omega not m df into I f sine omega t which is which also means open circuit line to line voltages is this this should have sine delta into sine delta here which was missed out.

So, what we have obtained is the steady state let us say it is called a power angle characteristic. So, the power angle characteristic of a steady state characteristic is given by these equations, but of course, if you are going to do only a steady state analysis to some extent that is keeping our ambitions too low as far as this course is concerned what we will do in the next lecture is try to build a ground for doing.

Transient analysis of a synchronous machine we will be not setting the derivatives of the d q 0 as well as, the field flux and the damper winding fluxes equal $t = 0$ well we will not be doing that, but one step which we need to take before we actually start getting the transient analysis of a synchronous machine is try to you know kind or rephrase or write your synchronous machines in a more user friendly or engineer friendly format.

What I mean is that that the synchronous machine equation as they have been written now are in terms of inductances and resistances these often are not easy to obtain the values of the inductances and resistances themselves are not easy to obtain by simple tests what we instead can do is prescribe a few tests from which we can back calculate these inductances and resistances.

So, let me repeat although we have obtained the synchronous machine equations we need to obtain synchronous machine parameters from a real machine. So, if I want to do the analysis somebody has to give me the parameters. So, I should define some tests and methods to back calculate the inductances in fact.

You may argue of course, these inductances can be analytically derived well while that is to some extent true from electromagnetic analysis you can actually compute fields and compute the mutual inductances and self inductances and so on. It is in general quite a difficult process more over remember that we have actually represented a synchronous machine.

By windings and sometimes the windings are actually representing the effect of eddy currents so, they are not actual they are actually no windings in some cases in such a case to obtain the mutual of the self inductances analytically it is going to be tough. So, as I mentioned back we have to prepare a ground for inferring these values from measured tests. So, whatever, tests we do we try to infer from these values and we will find out a way to fit the measured you know responses into this model remember that whatever model we make this is obviously something which is very important in engineering whatever model we have made it is based on a some assumptions, so we for example.

As I mentioned, in the beginning of synchronous machine modeling that our assumption that we can represent the synchronous machine effects by 2 windings on the rotor and 2 windings on the rotor and d axis and 2 windings on the q axis in addition of course, to

the stator winding was an assumption we assume that we will be able to fit our results or rather we will be able to fit the results in obtained tests to our model.

That is the basic point which is implicit in all our model derivation. So, in the next class we will try to actually take this a bit forward try to see how we can measure parameters and fit them into our model or rather obtain the parameters in our model from the measured tests.