

Power System Dynamics and Control
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Lecture No: # 14
Modeling of Synchronous Machines
dq0 transformation (Contd)

We continue our discussion on the dq0 transformation on the flux equations of a synchronous machine. Recall, that the dq0 transformation is a time variant transformation and the basic reason why actually we go for dq transformation is to get a set of time invariant equations, as far as, the flux relationships are concerned.

In the previous lecture we began transforming even the differential equation, that is, the Faraday's law applied to the flux of the machine and we noticed, that there were, whenever we apply a time variant transformation in the flux equations in the new variables, we do get, what is known as a speed EMF term. We will just recap what we have done first before we proceed further in this lecture.

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d - q - 0 transformation

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = C_p \begin{bmatrix} f_d \\ f_q \\ f_o \end{bmatrix}$$

C_p is a function of " θ "

$$\begin{bmatrix} f_d \\ f_q \\ f_o \end{bmatrix} = C_p^{-1} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

So, today's lecture is we continue to see the implications of the d-q-0 transformation on the machine equations. Now, recall, that the d-q-0 transformation, essentially, transforms any a, b, c variables, the phase variables a, b, c using a transformation matrix C_p into f_d , f_q and f_o , which are known as the d-q-0 variables. C_p is a function of theta, theta is

of course, the electrical angle, which we have discussed before. Similarly, of course, f_d , f_q and f_0 are equal to C_p inverse of f_a , f_b , f_c . So, we assume, of course, C_p inverse does exist. In fact, we can verify that it does exist, remember that by doing a transformation.

We are just, re, we are going to reformulate the equations in the new variables. So, we are not writing any fundamentally new physical equations, but we are just reformulating the existing, we are just reformulating the existing flux relationships, as well as, the flux differential equations in the new variables. So, once we can do the analysis in the $d, q, 0$ variables, which is presumably going to be simpler. After we do the analysis, we can transform back to the a, b, c reference frame using this transformation. So, that is the basic idea of the transformation.

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A photograph of a hand-drawn matrix equation on a whiteboard. The matrix is labeled $C_p =$ and is enclosed in large square brackets. It is a 3x3 matrix with the following elements:

- Row 1: $k_d \cos \theta$, $k_q \sin \theta$, k_0
- Row 2: $k_d \cos(\theta - 2\pi/3)$, $k_q \sin(\theta - 2\pi/3)$, k_0
- Row 3: $k_d \cos(\theta + 2\pi/3)$, $k_q \sin(\theta + 2\pi/3)$, k_0

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

Now, C_p , of course, is this. This K_d , K_q and K_0 are non-zero constants, three non-zero constants, which are used in the transformation. Now, the reason why I have not put any numerical value on this K_d and K_q and K_0 is, that it is not essential to the, the values of K_d , K_q and K_0 can be arbitrary, but non-zero.

If they are 0, of course, you will not get C_p inverse, C_p inverse will not exist. So, K_d , K_q , K_0 are in fact, non-zero constants and whatever be their value, as long as, they are not zero, they satisfy our purpose of trying to convert the time variant flux relationships into time invariant flux relationships in the $d, q, 0$ variables.

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$$C_p^{-1} = \begin{bmatrix} k_1 \cos \theta & k_1 \cos(\theta - 2\pi/3) & k_1 \cos(\theta + 2\pi/3) \\ k_2 \sin \theta & k_2 \sin(\theta - 2\pi/3) & k_2 \sin(\theta + 2\pi/3) \\ k_3 & k_3 & k_3 \end{bmatrix}$$

$$k_1 = \frac{2}{3K_d} \quad k_2 = \frac{2}{3K_v} \quad k_3 = \frac{1}{3K_0}$$

C_p inverse, in fact, looks like C_p transpose. In fact, if you look at the structure, you will see, that you have got $\cos \theta$, $\cos \theta$ minus 2π by 3 and $\cos \theta$ plus 2π by 3 . These constants K_1 , K_2 and K_3 are related to K_d by this relationship, K_1 is equal to 2 by 3 times K_d , K_2 is equal to 2 by 3 times K_v and K_3 is equal to 1 upon 3 times K_0 . So, this is basically the inverse, you can just verify this, of course, at leisure by multiplying C_p with C_p inverse and just verifying that it does turn out to be an identity matrix, which will, of course verify, that C_p inverse is indeed the inverse as I have written it down here.

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$$\begin{bmatrix} y_s \\ y_r \end{bmatrix} = \begin{bmatrix} C_p & 0_{3 \times 4} \\ 0_{4 \times 3} & I_{4 \times 4} \end{bmatrix} \begin{bmatrix} y_{da} \\ 0 \\ y_r \end{bmatrix}$$

Now, the basic idea here is to convert the flux equations. So, what we will do is try to convert the a, b, c. The size, remember, is nothing, but psi a, psi b and psi c and psi R of course, is psi f, psi h, psi g and psi K psi dq. So, what we do is try to change the variables a, b, c to dq0. Of course, psi R we will keep unchanged. So, the transformation, if you look at it, is C p.

The sub matrix C P, the sub matrix of this larger matrix had got this element as C p, this is 3 by 4 null matrix, this is 4 into 3 null matrix and this is an identity matrix, which is 4 into 4.

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The image shows two equations written on a whiteboard. The top equation is:

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss}(\theta) & L_{sr}(\theta) \\ L_{rs}(\theta) & L_{rr} \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

The bottom equation is:

$$\begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix} = \begin{bmatrix} C_p^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} L \\ L \end{bmatrix} \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

An arrow labeled 'L' points from the top equation to the bottom equation, indicating the transformation of the inductance matrix.

So, what I will do is apply the transformation to these variables. Remember, that the flux current relationship is psi s is equal to L ss of theta L sr of theta L ss of theta into i s into L sr of theta into i r and similarly, psi r is expressed in this fashion.

Now, if I call this the L matrix, I will call this L, it is, it is easy to see, that psi, if you try to reformulate this particular equation, we are just reformulating it in the new variables. So, psi dq0 psi R will be this into L, L is nothing, but this matrix. Remember, of course, these are sub matrices, so this into L into this transformation into i dq0 and i r. Now, the good thing, which we saw in the last lecture was when we work out this, we did work out one particular term in this L matrix, but I encourage you to actually try out to evaluate the complete L matrix.

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$$\begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss}' & L_{sr}' \\ L_{rs}' & L_{rr} \end{bmatrix} \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

$$L_{ss}' = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix}$$

$$L_d = L_{aa0} - L_{ab0} + \frac{3}{2} L_{aa2}$$

$$L_q = L_{aa0} - L_{ab0} - \frac{3}{2} L_{aa2}$$

$$L_0 = L_{aa0} + 2 L_{ab0}$$

The complete L matrix, if one evaluates one will find, that the, of course this, you get this new relationship where this dash, I have called this L ss dash, L sr dash, **L ss**, L rs dash and L rr and L rr of course, remains unchanged. So, L ss dash, so this should be L ss dash, is nothing, but it turns out is a diagonal matrix, it is an interesting thing. It is a diagonal matrix where L d, L q and L 0 are not functions of theta. So, that is, the basic beauty of this transformation is that the relationships in the d, q, 0 variables are in fact, the flux current relationship in the d, q, 0 variables are in fact, independent of theta, not only that, as you see L d for example, if I just expand this, you will find, that psi d is equal to L d into i d plus 0 into i q plus 0 into i 0.

So, there seems to be decoupling in the flux relationship. That is also an interesting thing, which happens because of the fact, that this is diagonal.

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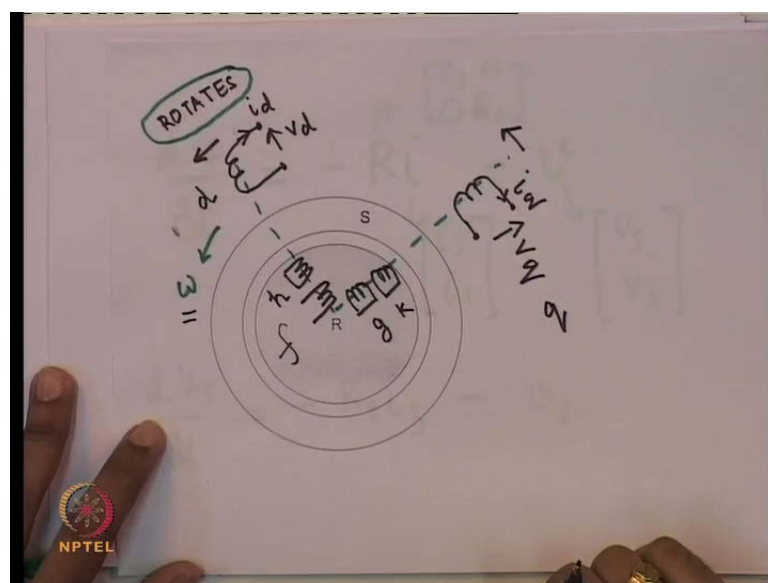
$$L_{sr}' = \begin{bmatrix} \frac{M_{af}}{K_d} & \frac{M_{ah}}{K_d} & 0 & 0 \\ 0 & 0 & \frac{M_{ag}}{K_q} & \frac{M_{ak}}{K_q} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$L_{sr}' = (L_{rs}')^T$ if $K_d^2 = \frac{2}{3}$
 $K_q^2 = \frac{2}{3}$
 $L_{sr} = L_{rs}^T$

In fact, L_{sr} is this, M_{af} by K_d , M_{ah} by K_d , 0, 0, 0, 0, M_{ag} by K_q , this is wrongly written, so I will write this as K_q and M_{ah} by K_q .

And the last column of course is, the last row is 0. Now, L_{sr} is equal to L_{rs} , only if this is true. So, it is not true in general. See, remember L_{sr} was equal to L_{rs} transpose, but this is true only if we choose K_d and K_q , such that they satisfy this relationship. K_d^2 is 2 by 3 and K_q^2 is equal to also 2 by 3.

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Now, what we see here is, of course, there is a complete decoupling between what are known as the d-axis coils. What are the d-axis coils? f, h and the stator coil, which is transformed into the d q frame. So, if you look at this will be f, h and d on this and g, k and q here on the q-axis. So, the basic idea here is that basic thing, which comes out is, that the fluxes in the coils of the d-axis including this, you know, this fictitious coil, we will call it the d, d-axis coil, is not dependent on the q-axis currents. So, that is a very interesting thing.

In fact, it is tempting to think, think, that this transformation, what it essentially has done is represented the three stationary stator coils in the a, b, c frame for reference. It is, kind of, converted this three stationary a, b, c coils into two rotating d, q-coils, which are also rotating at this angular frequency omega. This is tempting, but as we shall see soon, the, when we write down Faraday's equation we will see, that the emf induced in this will be dependent on the fluxes caused in this axis.

So, you know, we should remember, that although this gives a nice picture of you know, the decoupling of the fluxes and the currents in the d and q axis, the emf equations of course, have a coupling. So, what do I mean by that?

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The image shows a whiteboard with handwritten mathematical equations. At the top, a matrix is written as $\begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix}$. Below it, the equation $\frac{d\psi}{dt} = -Ri - v$ is written. Arrows point from $\frac{d\psi}{dt}$ to a column vector $\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix}$, from i to a column vector $\begin{bmatrix} i_s \\ i_r \end{bmatrix}$, and from v to a column vector $\begin{bmatrix} v_s \\ v_r \end{bmatrix}$. Below this, the equation $\frac{d\psi_s}{dt} = -R_s i_s - v_s$ is written. An arrow points from ψ_s to a column vector $\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix}$. In the bottom left corner, there is a small NPTEL logo.

We did this in the last class; we have seen the d psi by dt. Psi, of course, is nothing, but psi s and psi r, is equal to minus of R i minus of v, this is what basically our flux equation were, which we did in, you know, about two classes back.

R of course, is the diagonal matrix, which has sub matrix R_s and R_r ; R_s and R_r , of course, are the, it is a diagonal matrix containing the resistances of the f, g, h and k coils; i of course, is i_s and i_r ; v is nothing, but v_s and v_r . So, what you have if we just take out the ψ equations? We will have $\frac{d\psi}{dt}$ is equal to minus $R_s i_s$ minus v_s . Of course, ψ , you know, the subscript s actually denotes ψ_s is nothing, but actually ψ_a , ψ_b and ψ_c .

And similarly, i_s and v_s are i_a , i_b , i_c and v_a , v_b , v_c .

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what is

$$-\frac{d(C_p \psi_{dq0})}{dt} = -C_p \frac{d\psi_{dq0}}{dt} + \left(-\frac{dC_p}{dt}\right) \cdot \psi_{dq0}$$

The image shows a hand pointing to the second term of the equation, $-\frac{dC_p}{dt} \cdot \psi_{dq0}$.

Now, if you look at this equation, we can substitute ψ_a , ψ_b , ψ_c as nothing, but C_p into ψ_{dq0} . So, our equations, in fact, become this, so I have just re-written the equations. These equations, these ones I have re-written it like this and the re-written equations are substituted for i_s , which is nothing, but i_a , i_b , i_c , I substituted it by $dq0$ variables, so v_s is nothing, but C_p into v_{dq0} , ok.

So, if you recall our discussion in the previous class, I had asked what is $C_p \psi_{dq0}$? In fact, it is minus of C_p minus of this is nothing, but minus of C_p . This is, you are applying the chain rule plus this extra term. Remember, that C_p , unlike some other transformations we did in the first few lectures of this course, C_p is in fact the function of theta. Theta, in a synchronous machine, is a function of time, it cannot be considered. Even in the steady state conditions you will find, that theta is, in fact, varying

continuously. So, dC_p by dt , in fact, had to be evaluated, we gave to apply chain rule. So, what we will get is this, in fact, is having two terms.

So, your flux relationship, so I will just, this has been rewritten again, this particular term dC_p by dt can be written as dC_p by $d\theta$ into $d\theta$ by dt . So, this extra term, as we discussed in the class, previous class, comes out because of being mathematically consistent while applying the transformation of variables, ok.

This extra term, in fact, denotes, see what we, you know, this rate of change of flux of course, is common with a, b, c equations, but we have got extra, what is known as **P d M f term**.

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$$\frac{dC_p}{d\theta} = \begin{bmatrix} -k_d \sin\theta & k_q \cos\theta & 0 \\ -k_d \sin(\theta - \frac{2\pi}{3}) & k_q \cos(\theta - \frac{2\pi}{3}) & 0 \\ -k_d \sin(\theta + \frac{2\pi}{3}) & k_q \cos(\theta + \frac{2\pi}{3}) & 0 \end{bmatrix}$$

$$= C_p \cdot P_1$$

So, if we look at this dC_p by dt , if we actually evaluate it, in fact, so we carry on forth from last time. dC_p by $d\theta$ is equal to, it is easy to see this, you take the derivative of the transformation, so you will get minus $K_d \sin\theta$ $K_q \cos\theta$ and of course, K_0 , when you take the derivative, you will get zero. So I will write 0 here and you will have minus $K_d \sin\theta$ minus 2π by 3. It is difficult to fit it in the paper, but we will try to do that. K_q into $\cos\theta$ minus 2π by 3 and this will be, sorry, this is, yeah, minus $K_d \sin\theta$ plus 2π by 3. And this term here is $K_q \cos\theta$ plus 2π by 3. So, this is basically what we have I will just read out term it might not be very clear here. It is, $K_q \cos\theta$ plus π by 3 and this is of course, a distinct term here.

Now, it is very easy to see, it is not too difficult, you know, how, what C_p looks like? C_p looks like this, sorry, this is C_p inverse, I am sorry, C_p looks like this.

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The image shows a whiteboard with the following handwritten content:

$$P_1 = \begin{bmatrix} 0 & k_{\omega r}/k_d & 0 \\ -\frac{k_d}{k_{\omega q}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{dC_p}{d\theta} = C_p \cdot P_1$$

There is an NPTEL logo in the bottom left corner of the whiteboard image.

So, it is very, by inspection you can really make out, that this is nothing, but C_p into matrix P_1 where P_1 is nothing, but $0, k_{\omega q}$ by $k_d, 0$, minus k_d by $k_{\omega q}, 0, 0$ and this is $0, 0, 0$. So, what I have, so let me just repeat dC_p by $d\theta$ is equal to C_p times P_1 , ok, where P_1 is this.

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The image shows a whiteboard with the following handwritten content:

$$C_p \frac{d\psi_{dq0}}{dt} - \dot{\theta} C_p P_1 \psi_{dq0} - R_s C_p i_{dq0} = C_p v_{dq0}$$

$$-\frac{d\psi_{dq0}}{dt} - \dot{\theta} P_1 \psi_{dq0} - R_a i_{dq0} = v_{dq0}$$

There is an NPTEL logo in the bottom left corner of the whiteboard image.

So, our final equations, flux equations, come out to be minus of $C_p d \psi_{dq0}$ by dt minus $d \theta$ by dt . So, I will just call this $\dot{\theta} C_p P^{-1} \psi_{dq0}$ minus R_s into $C_p i_{dq0}$. It is nothing, but C_p into v_{dq0} . So, your flux equation becomes, in fact, I will just multiply, pre-multiply both, both sides by C_p inverse, you will get minus $d \psi_{dq0}$ by dt minus $\dot{\theta}$ into P^{-1} into ψ_{dq0} minus R_s . Of course, remember, is a diagonal matrix containing R_a , R_b and R_c .

Of course, if all the coils are identical I can just say, R_a into i_{dq0} is equal to v_{dq0} , which leads us to, if I really write this down separately, you know, because it sometimes is not very evident, that what we are getting unless we write down all the equations separately. So, what I will do is I will write down the $dq0$ equation separately.

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$$\begin{aligned}
 -\frac{d\psi_d}{dt} - \omega \frac{K_q}{K_d} \psi_q - R_a i_d &= V_d \\
 -\frac{d\psi_q}{dt} + \omega \frac{K_d}{K_q} \psi_d - R_a i_q &= V_q \\
 -\frac{d\psi_0}{dt} - R_a i_0 &= V_0
 \end{aligned}$$

$\omega = d\theta/dt$

So, the $dq0$ equation separately turns out to be minus $d \psi_d$ by dt minus $d \theta$ by dt , in fact, or $\dot{\theta}$ is nothing, but, it is nothing, but ω , angular frequency, electrical angular frequency and this is K_q by K_d ψ_q minus $R_a i_d$ is equal to v_d . Minus $d \psi_q$ by dt plus ωK_d by K_q ψ_d minus $R_a i_q$ is equal to v_q and minus $d \psi_0$ by dt minus $R_a i_0$ is equal to v_0 , where ω is equal to $d \theta$ by dt or $\dot{\theta}$.

So, if it is not clear I am just panning this a bit. Now, one interesting thing you should see here is that there is what I mentioned sometime back, this $P d M f$ terms, in fact, although we saw, that there is complete decoupling between the d and q axis. So, recall, we had a complete decoupling between the d and q axis.

Recall this equation or the flux equations, in fact, we had, I had had mentioned sometime back, that ψ_d is dependent on i_d , i_f and i_h , but it is not dependent on i_q , i_0 or i_g and i_k . It is not dependent on the q-axis also, the flux current relationships. There is a complete decoupling between the d and the q-axis coils, so I call this the d-axis coil. So, the fluxes in the d-axis coil are not dependent on the q-axis currents, none of the currents, but importantly, when you look at the flux equations, there are speed emf terms. So, the flux Faraday's law, when you apply to ψ_d , ψ_d , you have to put this extra term, which comes because of applying the correct mathematics to the transformed equations.

So, although sometimes it is tempting to start from a kind of a physical model of rotating windings, when we come to obtaining the flux equations in the d coil or the flux equations in the q coils, remember that there are speed emfs in the d coil due to the flux in the q-axis. This cannot be explained by just from the starting of this model.

So, one of the things, which you should keep in mind is it is a good idea, as I mentioned in the previous class, to first work out the mathematics. The correct mathematics give you the, give you, gives you these equations where there are extra speed emfs terms, these speed emf terms come because we apply the derivative to the time varying transformation as well. These are the correct equations.

So, please remember, that there is coupling coming between the d and q axis coils because of these extra, what I call as, speed emf terms.

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$$\frac{d\psi_f}{dt} + R_f i_f = V_f$$
$$\frac{d\psi_h}{dt} + R_h i_h = 0$$

Now, these are, as far as, the stator equations are concerned, the rotor equations, of course, we know.

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$$\frac{d\psi_g}{dt} + R_g i_g = 0$$
$$\frac{d\psi_k}{dt} + R_k i_k = 0$$
$$\begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss}' & L_{sr}' \\ L_{rs}' & L_{rr} \end{bmatrix} \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

And $d\psi_g/dt$ is equal to, sorry, plus $R_g i_g$ is equal to 0. So, just remember these are the remaining equations. In fact, if you know the flux, if you know the flux and current relationships and these equations relating to the emfs, the rotor flux equations on the d-

axis and the q-axis, we, in fact, have got the complete flux description, so if I know of course, I should, the relationship ψ_{dq0} ψ_r .

I know this relationship as well, ψ_{dq0} and i_r and this is nothing, but L_{sd} L_{sr} L_{rs} and L_{rr} . So, we have in fact, got a complete picture of the system. In fact, you can substitute for i_g , i_k , i_f , i_h , as well as, i_{dq0} by ψ_{dq0} and ψ_r using the relationship and you will get finally, things in the state space form. We will do it on a separate sheet.

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$$\dot{\psi} = "A" \psi + B \ddot{v}$$

$$\psi = \begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix}$$

ω

So, you will get, you can, of course, write it as $\dot{\psi}$ is equal to some A matrix into ψ plus B into v . So, you can get it into this, this form, where ψ is nothing, but ψ_{dq0} and ψ_r . This A, in fact, does not have time coming in explicitly; you do not have time or theta coming in explicitly. Of course, A does contain omega, remember that, because of the fact, that the equation flux have the speed emf term.

A is a function of omega, but if omega is a constant, then this A becomes linear, this particular equation becomes linear time invariant. So, that is the beauty of applying the dq0 transformation. It consists speed, of course, this becomes a linear time variant equation, there is no explicit dependence on theta, which itself changes with time. So, this is one of the main, you know, important thing, which really comes out of applying the dq transformation.

Now, in general of course, whenever you have linear time variant system, it is not obvious, that by transforming it in a certain way you will get a linear time invariant system. It turns out, that the machine equations have special, special structure, which permit the use of this transformation C_p , ok, which make the system time invariant.

Now, one of the things, which we have not discussed now, we will shortly again discuss it, this feature of making the, you know, equations of, flux equations of a machine's time invariant, is not dependent on the specific values of K_d , K_q and K_0 , it could be absolutely arbitrary choice. Of course, K_d , K_q , K_0 should not be 0; none of them should be 0. So, there is some flexibility in the choice of K_d , K_q , K_0 .

Now, before we go on to discussing this particular point again, let us just look at what happens to the torque equations. Now, if you look at the torque equation, just recall what was the equation for torque?

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The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$T_e' = -\frac{1}{2} \left\{ [i_{dq0}^T] \times C_p^T \times \frac{\partial L_{ss}}{\partial \theta} \times i_{dq0} + 2 i_{dq0}^T C_p^T \frac{\partial L_{sr}}{\partial \theta} i_r \right\}$$

The bottom equation is:

$$\frac{\partial L_{ss}}{\partial \theta} = -2L_{aa2} \begin{bmatrix} \sin 2\theta & \sin(2\theta - 2\pi/3) & \sin(2\theta + 2\pi/3) \\ \sin(2\theta - 2\pi/3) & \sin(2\theta) & \sin(2\theta + 2\pi/3) \\ \sin(2\theta + 2\pi/3) & \sin(2\theta + 2\pi/3) & \sin 2\theta \end{bmatrix}$$

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

If you recall what we did some time back, I will just show you the equations first, so you can recall what we have done. This was the torque equation in the previous lecture where T_e is nothing, the minus of the partial derivative of the co-energy with respect to the mechanical position.

Torque T_e is, of course, in this direction, θ is measured in this θ M and θ_e are both measured in this direction, the anticlockwise direction. Here, the co-energy is

expressed as a function of currents. You have got this particular formula and of course, eventually we did get this, so T_e , which is the electrical torque, is nothing, but the minus of P by 2 into P is the number of poles into d by derivative, partial derivative, of co-energy with respect to θ , which is the electrical angle.

So, we call, of course, T dash as this. So, T dash turns out to be this, so this is what our equations are. I basically took out the derivative, partial derivative of co-energy with respect to this. So, now, if I want to get this expression for torque T_e dash, remember T_e , the actual torque is actually minus P by 2 times this. So, we will have T_e is equal to T_e dash, the actual torque. Remember, it gets P by 2 minus P by 2 times T_e dash. So, T_e dash is nothing, but 2 by P times T_e , half, so I substitute here. So, I have got i_s transpose here, so I will have to write, i_s remember is $i_a i_b i_c$, so I can write this as i_{dq0} transpose into C_p transpose into the partial derivative of L_{ss} with respect to θ .

L_{ss} , remember, is the sub matrix of the original L matrix into, there is not much space here, so I will just, we can write it here I guess, i_{dq0} . i_s here is C_p into i_{dq0} , inadvertently we have just written i_{dq0} . So, remember, i_{dq0} is row, rather a column i_d, i_q, i_0 , so this is a neat way of writing it, plus, plus, in fact, this is 2 times i_{dq0} transpose C_p transpose dL_{sr} by $d\theta$ into i_r , closing of curly bracket.

So, what we get is this, you can just work out, you have already, about two lectures we took out, what L_{ss} is actually. So, I will just write down what L_{ss} by $d\theta$ is nothing, but minus of $2L_{aa}$, this is a time varying part or the θ varying part, into \sin of 2θ \sin of 2θ minus 2π by 3 \sin of 2θ plus 2π by 3 \sin of 2θ minus 2π by 3 \sin of 2θ plus 2π by 3 here \sin 2θ . And of course, this is, this will be, I just, this is, it may not be clear, I will just read it out, \sin of 2θ plus 2π by 3 , it is not very clear here, so maybe, we will just write it down again.

(Refer Slide Time: 33:08)

$$\frac{\partial L_{ss}}{\partial \theta} = -2L_{a22} \begin{bmatrix} \sin 2\theta & \sin(2\theta - \frac{2\pi}{3}) \cdot \sin(2\theta + \frac{2\pi}{3}) \\ \sin(2\theta - \frac{2\pi}{3}) & \sin(2\theta + \frac{2\pi}{3}) & \sin 2\theta \\ \sin(2\theta + \frac{2\pi}{3}) & \sin 2\theta & \sin(2\theta - \frac{2\pi}{3}) \end{bmatrix}$$

$$\frac{\partial L_{sr}}{\partial \theta} = \begin{bmatrix} \frac{\partial L_{sr}^d}{\partial \theta} \\ \vdots \\ \frac{\partial L_{sr}^q}{\partial \theta} \end{bmatrix}$$

This is (()) theta is equal to minus, sine of theta, sine of 2 theta sine of...

No audio 33:19 to 34:02

This is this, L ss by L theta d L sr by dt will partition into d L sr d by d theta and d L sr q by d theta.

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$$\frac{\partial L_{sr}}{\partial \theta} = \begin{bmatrix} -M_{af} \sin \theta & -M_{ah} \sin \theta \\ -M_{af} \sin(\theta - \frac{2\pi}{3}) & -M_{ah} \sin(\theta - \frac{2\pi}{3}) \\ -M_{af} \sin(\theta + \frac{2\pi}{3}) & -M_{ah} \sin(\theta + \frac{2\pi}{3}) \end{bmatrix}$$

$$\frac{\partial L_{sr}}{\partial \theta} = \begin{bmatrix} M_{ag} \cos \theta & M_{ak} \cos \theta \\ M_{ag} \cos(\theta - \frac{2\pi}{3}) & M_{ak} \cos(\theta - \frac{2\pi}{3}) \\ M_{ag} \cos(\theta + \frac{2\pi}{3}) & M_{ak} \cos(\theta + \frac{2\pi}{3}) \end{bmatrix}$$

So, that turns out to be, Maf sine theta minus Mah sine theta...

No audio 34:42 to 35:47

So, better book keeping is required, but as we shall see soon, that finally, what we get is the torque expression after using these formulae is quite neat. So, this is what if we know dL_{sr} is this, so we have found out the sub matrices, which I mentioned some time back.

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$$\frac{\partial L_{ss}}{\partial \theta} \cdot C_p = -3 L_{aa2} C_p P_2.$$

$$P_2 = \begin{bmatrix} 0 & \frac{k_q}{k_d} & 0 \\ \frac{k_d}{k_q} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_p^T \frac{\partial L_{sr}}{\partial \theta} = \begin{bmatrix} 0 & 0 & \frac{3}{2} k_d M_{ag} & \frac{3}{2} k_d M_{ak} \\ -\frac{3}{2} k_q M_{af} & -\frac{3}{2} k_q M_{aq} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, you can see, that dL_{ss} by $d\theta$ into C_p is nothing, but minus of $3 L_{aa2}$ into C_p into P_2 , where P_2 is $0, \frac{k_q}{k_d}, 0, \frac{k_d}{k_q}, 0, 0$ and $0, 0, 0$. So, you can rewrite this. So, if you look at structure of dL_{ss} by $d\theta$ if you look at the structure of it you can write it as, in this fashion.

Therefore, and further, you will find, that C_p transpose of dL_{sr} by $d\theta$ is nothing, but in fact, it gets stripped of all the thetas eventually, $0, 0, \frac{3}{2} k_d M_{ag}, \frac{3}{2} k_d M_{ak}$ a k and this becomes minus $\frac{3}{2} M_{af}$ minus $\frac{3}{2}$, sorry, this is k here, so there is a $\frac{k_q}{k_d}, \frac{k_q}{k_d} M_{aq}, 0, 0$ here, $0, 0, 0$ and 0 .

So, it is a bit complicated, but this is how it comes out to be, we can verify this.

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$$T_e = \frac{3}{2} K_d K_q \left\{ i_q \left(\frac{M_{af} i_f + M_{ah} i_h}{K_d} + \frac{3}{2} L_{aa2} i_d \right) - i_d \left(\frac{M_{ag} i_g + M_{ak} i_k}{K_q} - \frac{3}{2} L_{aa2} i_q \right) \right\}$$

$$\psi_d = L_d i_d + \frac{M_{af} i_f + M_{ah} i_h}{K_d}$$

$$\psi_q = L_q i_q + \frac{M_{ag} i_g + M_{ak} i_k}{K_q}$$

So, eventually, we get, we will just, I will just write down the torque equation. T_e comes out to be $\frac{3}{2} K_d K_q$ into i_q into M_{af} by K_d i_f plus M_{ah} by K_d i_h plus $\frac{3}{2} L_{aa2} i_d$. So, I close this bracket, then continue, minus i_d times M_{ag} by K_q i_g plus M_{ak} by K_q i_k minus $\frac{3}{2}$ times $L_{aa2} i_q$, close this bracket and close this bracket. So, this is the expression for torque.

But we know that, so I will just partition this here, we know, that ψ_d is nothing, but $L_d i_d$, this comes out from the flux equations, ψ_d is equal to $L_d i_d$ plus $M_{af} i_f$ by K_d plus M_{ah} by K_d i_h and of course, ψ_q is nothing, but $L_q i_q$ plus M_{ag} by K_q i_g plus M_{ak} by K_q i_k .

So, what do we understand from this?

(Refer Slide Time: 41:06)

The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$T_e' = 3 K_d K_q \left\{ i_q \left[\psi_d - \left(L_d - \frac{3}{2} L_{aa2} \right) \times i_d \right] - i_d \left[\psi_q - \left(L_q + \frac{3}{2} L_{aa2} \right) \times i_q \right] \right\}$$

The bottom equation is:

$$T_e' = \frac{3}{2} K_d K_q \left[\psi_d i_q - \psi_q i_d \right]$$

A small NIPTEL logo is visible in the bottom left corner of the whiteboard image.

If you look at these equations, look at this and you look at this, so one can directly infer from this, that T_e is nothing, but 3 by 2 times $K_d K_q$, again we will put a curly bracket here, i_q into ψ_d minus L_d minus L_{aa2} , we will rewrite this again.

You can rewrite, T_e is equal to 3 into K_d into K_q into, a curly bracket here, into i_q into, we will put a square bracket here, ψ_d minus L_d minus $\frac{3}{2} L_{aa2}$ into i_d and I close the square bracket minus i_d into ψ_q minus L_q plus $\frac{3}{2} L_{aa2}$ into i_q . So, I close this bracket and I close this bracket. So, if you, from this it is very easy to see, you know, I, you can cancel off this term here you will get really after all the manipulations, finally it yields, 3 by 2 times $K_d K_q \psi_d i_q$ minus $\psi_q i_d$. In fact, this is small correction here, so I should be calling this T_e dash, T_e dash, so that is nothing, but T_e dash; so, T_e dash is this.

So, what we have here is basically a very neat expression, which is again, devoid of any of the thetas, they are all in term, so for d and q variables only. So, that is one interesting thing. So, our equations are turning out to be extremely neat. Now, in fact, if you write down the equation in d, q variables, you will find, that your equations will no longer have theta explicitly appearing in them, they are much easier to handle. The flux current relationship is brings out the de coupling between the d and q -axis, fluxes and currents.

The flux differential equations, of course, bring out the coupling between the d and q axis fluxes, there is a coupling. We now get the torque equation, remember what the

torque equation was? If we recall where we were sometime back, this, so T_e we have actually found out right now, which is, which I said was the function of theta, but of course, because we applied the d-q transformations, it turns out, that it is not explicitly coming, the theta does not come out explicitly. If you express T_e in terms of the d-q variables, so it is a very, very interesting, you know, very neat expression we get for torque.

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The whiteboard contains the following handwritten equations:

$$k_d^2 = \frac{2}{3} \quad k_q^2 = \frac{2}{3}$$

$$L_{sr}' = (L_{rs}')^T \quad k_3 = \frac{1}{3k_0}$$

$$C_p^{-1} \rightarrow k_1 = \frac{2}{3k_d} \quad k_2 = \frac{2}{3k_q}$$

Now, one thing about the choice, the last topic in this particular lecture, about the choice of K_d and K_q . We saw of course, that if we put K_d^2 is equal to 2 by 3 and K_q^2 is equal to 2 by 3, one interesting thing happens, that is, L_{sr}' becomes equal to L_{rs}' transpose.

Also, if in fact, we saw some time back, that when we take out C_p inverse, it is in terms, it has got elements K_1 , K_2 and K_3 , where K_1 is nothing, but 2 by 3 times k_d and K_2 is 2 by 3 times K_q . In fact, K_q is in the denominator and K_q is also in the denominator and K_3 is equal to 1 upon 3 K_0 . So, these are the coefficient of the sines and cosines in the C_p inverse matrix.

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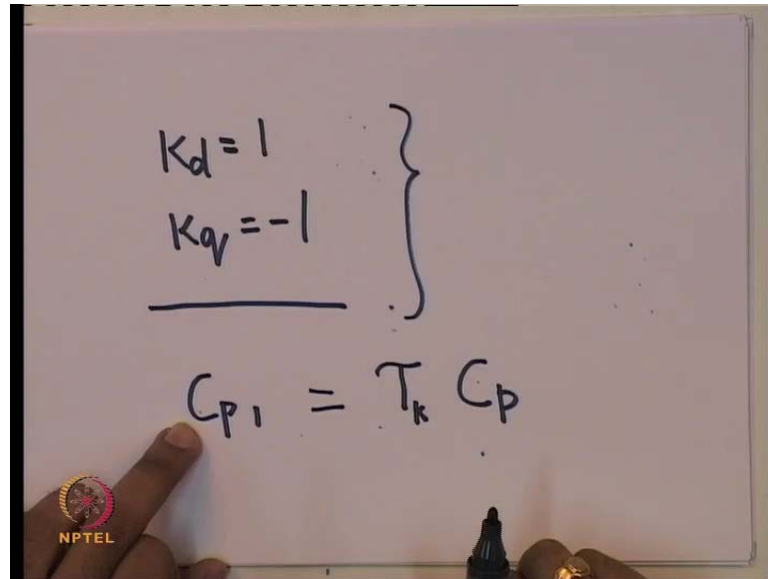
The image shows a whiteboard with three handwritten equations. The first equation is $K_d = K_q = \sqrt{\frac{2}{3}}$. The second equation is $L_{sr}' = (L_{rs}')^T$. The third equation is $C_p^{-1} = C_p^T$. There is an exclamation mark to the right of the third equation. In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' below it.

If you look at, you know, these relationships, it appears, that it is not a bad idea to choose K_d equal to K_q is equal to root 2 by 3. So, that is an interesting and important point here. Of course, there is absolute freedom of choice of K_d and K_q in so far as our main objective is of getting a time invariant set of equations. Remember, that K_d , K_q do appear in the flux current relationship, as well as, the flux differential equations, as well as, in the torque expression here. Sorry, this should be K_q , but the fact remains, that although K_d , K_q do appear, θ does not appear. So, whatever be the value of K_d and K_q , as long as, they are non-zero, we do end up with what we have set out to do, that is, get a neat set of equations, which are invariant with respect to θ . So, if we choose K_d and K_q root 2 by, it is a very special choice, it will lead us to L_{sr} dash being equal to L_{rs} dash transpose, it is just convenient; it is very convenient.

Also, we will find, that C_p inverse will be equal to C_p transpose. So, it is easy, of course, it is easy to remember would be one advantage of using k_d and k_q in this fashion. So, it is easy to remember, that C_p inverse is equal to C_p transpose.

But let me repeat, K_d and K_q could be arbitrary in so far our main objective of getting time invariant equations as concerned. So, let me just summarize, but before I do that, it is K_d is equal to K_q is equal to root 2 by 3 is a kind of a, is our, it is a special choice and that is the choice, which I will really follow in this particular course.

(Refer Slide Time: 49:15)



The image shows a whiteboard with handwritten mathematical expressions. At the top, $K_d = 1$ and $K_q = -1$ are written, with a large curly brace to their right. A horizontal line is drawn below these two equations. Below the line, the equation $C_{p1} = T_k C_p$ is written. In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' underneath it.

It is, in fact, the, the same choice is used in K R Padiar's book. In fact, I, we look at the other literature the literature or many industry papers. They follow dimension K_d is equal to 1 and K_q is equal to minus 1, so they, they follow this choice. In fact, of course, I did not talk about K_0 , K_0 , if I want this to be true. In fact, I should have told you before.

You have to choose K_0 is equal to 1 by root 3. So, K_0 also gets refined if I want to have this to be true. So, just remember this again, that many industry papers, as well as, you know, in books by Kundur and many other books they follow the convention, k_d is equal to 1 and K_q is equal to minus 1. In this particular course, I shall follow Padiars convention, which is or the IEEE convention actually, which is k_d is equal to K_q equal to root 2 by 3 because it gets us some benefits.

In fact, you will notice, that in this torque equation if I choose K_d and K_q as root 2 by 3, this, this you know, this whole coefficient gets becomes equal to 1. So, I do not have to remember these. Coefficients because this, this if I put K_d equal to root 2 by 3 and K_q equal to root 2 by 3, this coefficient effectively becomes 1. So, there are certain advantages in choosing this, but, as I mentioned back, the choice is arbitrary, it is not hampered of, referred to get a time in variant set of equations.

Of course, the question then arises, if I, if we have got industry papers or papers or books, which follow this convention, I will call this people who follow this convention, I

will call this, people who are using this as a transformation using a transformation C_P . And we, in fact, apply a transformation C_p where K_d and K_q are root 2 by 3.

In such a case one can transform the equations, which they have used the people who are using these this set of values of K_d and K_q by again a kind of matrix T_k . So, I can, actually the variables, which they have defined are different from my variables because the coefficients k_d k_q have been chosen by one have been 1 and minus 1 And we are going to choose root 2 by 3.

So, whenever I am going to use their equations and I get answers with their variables, I would need to use this transformation matrix T_k to transfer from their variables to my variables. This will leave this as an excessive to find out what is T_k is T very simple constant matrix.

So, to summarize this particular lecture finally, obtain the equations of the synchronous machine in the d-q frame of reference and the equations are neat in a sine theta, they are not functions of theta. In fact, you can, if in fact speed of the machine is constant, $d\theta/dt$ is equal to constant, the flux equations, the flux differential equations are linear time invariant equations. Of course, if speed is not constant, in fact, in general, it need not be a constant.

You have a coupling between the mechanical equations and the flux equations, flux differential equations. In fact, the coupling is non-linear. Remember, the torque is $\psi_d i_q - \psi_q i_d$. So, there is a product terms, there also there is a product when you are talking about the speed emf terms in the flux differential equations, therein also, there is a product.

So the mechanical equations and the flux equations are in fact coupled in a non-linear fashion, but the flux equation themselves for a constant speed are in fact, time, linear and time invariant, that makes at least some of our analysis much, much simpler.

So, now we move on to, to, you know, trying to interpret the equations which have come, correlate with parameters obtained by measurement and thereafter, the most important thing of course, in this course is to draw inferences, what equations you have got and correlate to actual power system.