

Power System Dynamics and Control
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Lecture No: # 13
Modeling of Synchronous Machines (Contd.)

In the previous class we derived voltage equations of a synchronous machine. In fact, we applied Faradays law and we got the relationship between voltage and fluxes. And of course, the relationship between fluxes and the currents also we tried to get the relationship. In fact, that relationship is the inductance matrix which turns out is position dependent that is theta dependent. Remember, an important thing is that theta is continuously changing in a synchronous machine. So, that inductance matrix is time dependent, it is dependent on the theta position.

Now, in today's lecture what we will do is the lecture is continuation of the modeling of a synchronous machine. We will introduce you to a transformation of variables called the T q transformation or parks transformation. In the previous lecture, we kind of concluded at a point where we derived the equation of torque in terms of the current; we will just redo that again at the beginning of the lecture just to refresh your memory and I will introduce the transformation which I just mentioned. Now, if you recall what we did last time for the mechanical equations of a synchronous machine.

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$$J \frac{d\omega_m}{dt} = T_m - T_e$$
$$T_e = - \frac{\partial W'}{\partial \theta}$$
$$W' = \frac{1}{2} \begin{bmatrix} i_s^T & i_r^T \end{bmatrix} L \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$
$$L = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}$$

So, if you have got a synchronous machine moving in this direction with the mechanical speed of ω_m and it has got electromagnetic torque which of course, opposes the motion, and the prime over torque T_m . So, if you look at the equations of this system you have J is the moment of inertia in kg meter square. So, this is the well known equation which describes the rotation.

And the electrical torque the basic electromagnetic energy conversion formula tells you the electromagnetic torque is given by partial derivative of the co energy with respect to θ . So, the torque in this direction and remember θ is measured in the anti clockwise direction. So, in that case you have this. So, the co energy is of course, the function of the currents. So, when I take the partial derivatives I should take the currents constant. The co energy is defined as half of $i_s^T i_r^T$; i_s and i_r are the L inductance matrix of the system. We already derived what is L matrix is in fact, the three of its sub matrices are functions of θ . So, L is made out of sub matrices L_{ss} , L_{sr} , L_{rs} and L_{rr} ; out of which L_{rr} is not θ dependent, but all the rest are θ dependent. So, this is what we have done before; i_s remember denoting the status current, i_r is the router current.

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The image shows a whiteboard with three equations written in black marker. The first equation is $T_e = - \frac{\partial W'}{\partial \theta_m} = - \frac{P}{2} \frac{\partial W'}{\partial \theta}$. The second equation is $T_e' = - \frac{\partial W'}{\partial \theta}$. The third equation is $T_e' = - \frac{1}{2} \left[i_s^T \frac{\partial L_{ss}}{\partial \theta} i_s + 2 i_s^T \frac{\partial L_{sr}}{\partial \theta} i_r \right]$. In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' below it.

Now, T_e which is nothing, but minus $d \omega$ dash by $d \theta$. In fact, only for the two pole machines remember that the correct formula in general is T_e is negative of the

partial derivative of $d w$ dash is the co energy with respect to the mechanical angle. So, please remember that this is the mechanical angle.

Of course, when we derived L , it is a function of θ where as this is θ_m . So, just remember that. So, what we have is θ_m which is nothing, but minus P by $2 d w$ dash by $d \theta$. Now, if we define T_e dash to be equal to minus of this we can show of course, that your T_e dash will be equal to minus of half $i s$ transpose. And as I mentioned last time, you would not find $L r r$ coming in these equations because the partial derivatives of $L r r$ with respect to θ will be equal to zero. And we have already made an observation that $L r s$ transpose is equal to $L s r$. So, we just get one of them you know this simplifies to this.

So, this is what we did last time; we got a torque expression for T_e dash.

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$$J \cdot \frac{2}{P} \cdot \frac{dw}{dt} = T_m - \frac{P}{2} T_e'$$

$$\frac{1}{2} \omega_{mB}^2 \cdot J \cdot \frac{2}{P} \frac{dw}{dt} = \frac{1}{2} \omega_{mB}^2 \left(T_m - \frac{P}{2} T_e' \right)$$

$$\frac{\frac{1}{2} J \omega_{mB}^2}{VA_{base}} \cdot \frac{2}{P} \frac{dw}{dt} = \frac{\frac{1}{2} \omega_{mB}^2}{VA_{base}} \cdot \left(T_m - \frac{P}{2} T_e' \right)$$

And once you of course, do that you can write your equations as J into $d \omega_m$ by $d t$. Just again one small point which I wish to emphasize this is the angular speed. So, you can write this as 2 times p times the rate of change of electrical speed is equal to T_m minus T_e which is nothing, but this is basically what we have and remember T_e dash is this.

So, we have got the relationship between the currents and the torques. So, In fact, just to correlate something we have done some time back. Let me just multiply both sides by

half omega m square. So, I will do half into omega m the base value of the speed or the rate value of that speed into J into 2 by P d omega by d t. So, I have multiplied it there; is equal to you multiple this side also.

You will get half of omega m B square into T m minus P by 2 T e dash. Now, what I will do next is divide both sides by the volt ampere base of the machine. Remember, omega m base is the mechanical base frequency; angular frequency. So, it is usually the rated mechanical angular frequency of the machine.

So, that is one thing which you should remember. So, actually if you divide by volt ampere base you will get half J omega m B square divided by volt ampere base into 2 by P d omega by d t is equal to half omega m B square by volt ampere base into T m minus P by 2 T e dash.

Now, a few things this is what is commonly known as H or the inertia constant of the machine; and its units are joules per volt ampere or mega joules per mega volt ampere. So, this is called the H of the machine. So, you have basically if you look at this particular equation I will rewrite it.

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$$\frac{\frac{1}{2} J \omega_{mB}^2}{VA_{base}} \cdot \frac{2}{P} \frac{d\omega}{dt} = \frac{\frac{1}{2} \omega_{mB}^2}{VA_{base}} \cdot \left(T_m - \frac{P}{2} T_e' \right)$$

$$\frac{2H}{\omega_{mB}} \cdot \frac{2}{P} \frac{d\omega}{dt} = \frac{\omega_{mB}^2}{VA_{base}} \left(T_m - \frac{P}{2} T_e' \right)$$

$$\frac{2H}{\omega_{mB}} \cdot \frac{d\omega}{dt} = \frac{T_m}{T_{base}} - \frac{\omega_{mB} \cdot P \cdot T_e'}{2 \cdot VA_{base}}$$

So, you can actually write it as we will get 2 times H you get his two onto this side.

Two times H I got this 2 onto this side; I will take one of these omega m B is out of this side into P into d omega by d t is equal to omega m base divided by volt ampere base

into T_m minus P by $2 T_e$ dash. If you actually look at how these ω_m mechanical base into P divided by 2 would actually give you the electrical radians per second base value. You will have $2 H$ by ω_B ; ω_B is the radian frequency base value of the machine.

If you look at this, it is the mechanical torque base. So, you have got T_m is the mechanical torque divided by the torque base of the machine minus what you have is I will get rewrite it here. So, that it becomes easy what I am doing base into P by 2 into T_e dash; just check if it is volt ampere base into P by 2 into this.

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$$2H \cdot \frac{d(\omega/\omega_B)}{dt} = T_{mpa} - \left(\frac{T_e'(\theta)}{VA_{base} \omega_B} \right)$$

$$\frac{2H}{\omega_B} \cdot \frac{d\omega}{dt} = T_{m_{pu}} - \frac{T_e'(\omega)}{V}$$

Finally, what we get is the torque equation of the machine is $2 H$ by ω_B into $d \omega$ by $d t$ is equal to T_m per unit minus T_e dash divided by volt ampere base divided by ω_B ; the electrical base. So, this is the final equation of the torque of the machine. In fact, you will note that T_e dash will be the function of θ , you know the inductance matrix will be actually functions of θ whatever we have derived earlier functions of θ .

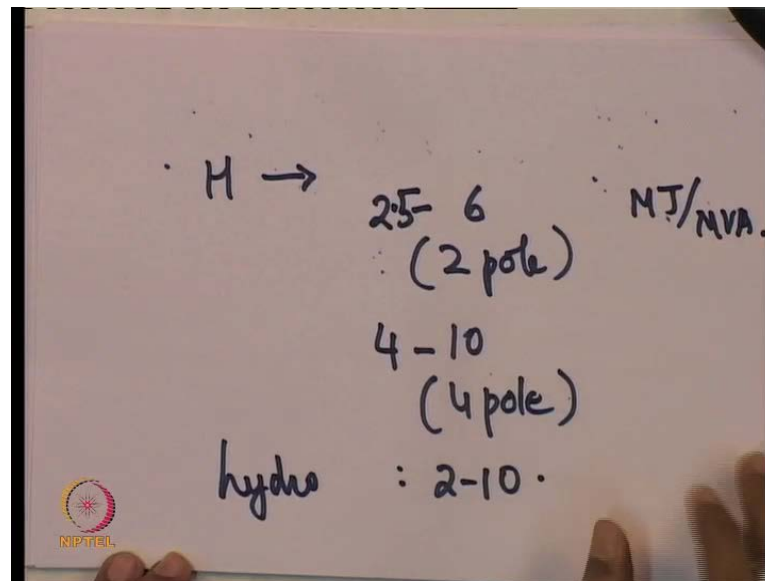
So, in fact, if you look at the interesting thing about the number of poles, it does not appear in this equation. So, when you write this thing per unit you will not find the number of poles. In fact, we will express this T_e in terms of fluxes and the currents and the inductance is the function of θ volt ampere base and ω_B , but you will not

get P exquisitely. Actually, when you go to the per unit system normally you can actually be blind to the number of poles working into the per unit system.

Of course, if you actually want to get the mechanical speed; if you know the electrical speed then you do the required the number of poles. So, that is of course, something which you should remember. In fact, you can write this $2H$ into $d\omega$ by ωB . So, in fact, I call this as a per unit equation; everything is in per unit.

Rather we have not really this T is not in per unit, but divided by volt ampere base by ω base. So, eventually of course, after sometime I will show how everything in this also can be expressed in per unit. So, this is the final equation we used sometime back in our earlier lecture that this is the original of this particular equation.

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Now, moving on a bit of course, move on let me tell you know the typical values of H for at thermal unit can be between 2.5 to 6. This is of course, for a two pole machine these values I have taken form Kundurs book for a four pole machine. And for hydro units it is usually 2 to 10. And of course, I have not written the units they are mega joules per M V A or joule per volt ampere.

So, the unit of H is mega joule per M V A. Now, one of the things if you recall the equations of the synchronous machine you will get the rate of change of flux is you know dependent on v .

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$$= -RL^{-1}\psi - v.$$

$$\frac{d\psi}{dt} = -Ri - v.$$

$$\psi = Li$$

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_f \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\psi = \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} \quad i = \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

In fact, if you recall the formula which we had written last time; the general form is $\frac{d\psi}{dt}$ is equal to minus R into i minus v and if you recall ψ was nothing, but L into i ; ψ is of course, made out of ψ_s and ψ_r and i is also made out of i_s and i_r ; and v is nothing, but v_a, v_b, v_c then minus v_f is made out of the rotor end state voltages zero zero zero.

So, this is something we did in the previous class. In fact, you can replace instead of writing it as this way; you can replace this by minus $RL^{-1}\psi$ minus v . So, the only complication when we are trying to solve this equation is this actually looks like a linear equation.

Even if you assume that rate of change of θ is a constant, speed of a synchronous machine is constant. Remember that this L will be time varying. In fact, if you just look at these flux equations they are linear time variant equations. So, if I want to get the solution of this particular equation, well I cannot use directly the Eigen analysis techniques to solve this equation.

Again I repeat this inductance matrix is time varying, it is not a linear time invariant system; it is a function of θ . Even if you assume that θ rather the speed of the machine is constant still you will get a time variant set of equations. So, that is why it will be good to see how you can explore ways to analyze this.

Now, one of the most powerful ideas which have you used before is to use the idea of a transformation. We have used transformation; transformations is like analyzing a system \dot{x} is equal to Ax ; we tried to find out the transformation which would diagonalize the matrix. The idea was of course, diagonal matrix implies there is no coupling between the states.

So, if I have got \dot{x} into diagonal matrix into \dot{y} , there is no coupling between the coupling states and you can easily get the solution in terms of the exponential functions. Now, here we can use again the idea for transformation, but right now we limit ourselves to try to make these time variant set of equations into time invariant set of equations. So, what we will do is try to get a transformation of variables. So, that we will get this as our objective. Now, of course, how do you get the transformation? What I will do is introduce the transformation directly and see what its consequences are; I am not going to derive the transformations for you.

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$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = C_p(\theta) \begin{bmatrix} f_d \\ f_q \\ f_o \end{bmatrix}$$

$$\begin{bmatrix} f_d \\ f_q \\ f_o \end{bmatrix} = C_p^{-1}(\theta) \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

So, just let me introduce it to you directly without any further delay. This transformation is called park's transformation. So, instead of looking at the differential equations in the variables i_a, i_b, i_c or v_a, v_b, v_c or $s_i a, s_i b, s_i c$; this f could denote any three phase variable; I will look at the variables f_c, f_q and f_o using the transformation C_p , this is a matrix which is dependent on theta.

So, that is an important point you should keep in mind. Now, what is this transformation? Of course, this whole key is what is this transformation? It turns out that this is really a very useful transformation.

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$$C_p = \begin{bmatrix} K_d \cos \theta & K_q \sin \theta & K_0 \\ K_d \cos(\theta - \frac{2\pi}{3}) & K_q \sin(\theta - \frac{2\pi}{3}) & K_0 \\ K_d \cos(\theta + \frac{2\pi}{3}) & K_q \sin(\theta + \frac{2\pi}{3}) & K_0 \end{bmatrix}$$

So, C_p is equal to I will just define these transformations; it is a three by three matrix. Something you may have guessed that I am applying transformation only to a b c variable and not to the field you know d, f and j variables. In fact, the a b c quantity we are applying this particular transformation. Now, in fact, just recall it is just the interesting correlation which you should recollect the currents on the rotor become constants in steady state; where as you know those currents in the stator or voltage in the stator or three phase sinusoids. In fact, for balanced condition we get three phase balanced sinusoids.

So, if I am going to try to make what do you call in a time variant set on equations one can kind of guess very intuitively that you have to apply transformation only to the a b c variables. So, indeed that is the case. So, this particular transformation which I am going to talk about actually is applied only to the stator variables a b c variable. So, the interesting thing about the transformation even without going further is that it is inverse

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$$C_p^{-1} = \begin{bmatrix} K_1 \cos \theta & K_1 \cos(\theta - 2\pi/3) & K_1 \cos(\theta + 2\pi/3) \\ K_2 \sin \theta & K_2 \sin(\theta - 2\pi/3) & K_2 \sin(\theta + 2\pi/3) \\ K_3 & K_3 & K_3 \end{bmatrix}$$

$$K_1 = \frac{2}{3K_d} \quad K_2 = \frac{2}{3K_q} \quad K_3 = \frac{1}{3K_0}$$

C_p inverse always exists unless you choose K_d , K_q , K_0 you know any of them zero. So, K_d , K_q and K_0 are non zero then you can actually get a transformation C_p inverse which is we find like this. It has this form $K_1 \cos \theta + 2\pi/3$; we will show this side. This is $K_2 \sin \theta - 2\pi/3$ K_3 and $K_1 \sin \theta + 2\pi/3$; this is K_3 . Where K_1 is equal to $2/3 K_d$; this is something which you can work out I am not working out for you, but in fact, one simple thing you can do is just multiply C_p and C_p inverse and just verify that it is actually giving an identity matrix. Note that there is a minor error here; the two three component of C_p inverse should be K_2 and not K_1 . C_p inverse comes out to be this. Why we need C_p inverse? The point is whenever we need any kind of transformation analysis remember that if I define a transformation of variables, if we want to know what the old variables were eventually I would need the inverse transformation.

So, in fact, f_d , f_q and f_0 is equal to C_p inverse θ . So, actually a transformation makes sense only if you can go back and forth between the old and the new variables. So, it is important that C_p should be invertible, but it turns out the C_p is actually invertible unless you choose K_d , K_q and K_0 is equal to zero. So, C_p inverse turns out is invertible. Another interesting thing you will notice before we go ahead is that the structure of C_p and C_p inverse looks similar; I mean if you look at the structure of C_p and C_p inverse, look at C_p this constants along this columns sine thetas along this cos

theta here of course, the constants are K_d , K_q and K_0 whereas, if you look at this the cosine functions are here; the sine functions are here; this is constant.

So, you can guess that for some you know the structure it looks like transpose C_p inverse. In fact, for special values of K_d , K_q and K_0 , this is indeed true. So, for you actually let you out one this K_d , K_q and K_0 is root two by three and K_0 is 1. So, if K_d is equal to root two by three and K_0 turns out to be one by root three. In that case, C_p will be equal to C_p transpose. So, that is one important result which you should keep in mind.

Some special values of K_d and K_q , C_p transpose and C_p inverse are equal. Now, the main utility of the transformation can be seen if you look at the transformation of flux linkages.

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The image shows two equations written on a whiteboard. The first equation is:

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

The second equation is:

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & I_{4 \times 4} \end{bmatrix} \begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix}$$

In the second equation, the ψ_{dq0} term is underlined and has a dashed line below it, indicating it is a vector of three components. The identity matrix I is labeled as 4×4 .

For example, if you have got you know we have done this before i_s and i_r is equal to these are functions of theta. Now, if I use a transformation remember I only have to transform the stator fluxes and the stator currents.

So, suppose I define as before this is three by four matrixes full of zeros and this is an identity matrix I ; this is four by four identity matrix; so I will call this dq and 0 and i_r . So, what I am doing is I am not transforming i_s this is just say i_r is equal to i_r I am just transforming i_a , i_b , i_c into i_d , i_q , i_0 . So, that f variable I mean I have applied

whatever transformation to flux ψ_a ψ_b ψ_c . So, if I define a transformation of this kind in fact, similar transformation can be applied to i_s and i_r .

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The image shows a whiteboard with handwritten mathematical equations. The first equation is:

$$\begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix} = \begin{bmatrix} C_p^{-1} & 0 \\ 0 & I_{4 \times 4} \end{bmatrix} \times \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}$$

The second equation is:

$$\times \begin{bmatrix} C_p & 0 \\ 0 & I_{4 \times 4} \end{bmatrix} \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, in that case it is easy to see that the final equations relating ψ_{dq0} and ψ_r and i_r are ψ_{dq0} ψ_r is equal to L_{ss} , L_{sr} , L_{rs} and L_{rr} into C_p matrix is zero zero and this is identity matrix of size four into four into i_{dq0} and i_r . So, what I have actually done is I have replaced i_s and i_r by this i_{dq} and i_r . So, i_s and i_r is equal to C_p into i_{dq0} ; i_r is equal to i_r . So, I have just replaced this by this then you have got L_{rr} , then you have got this L matrix. So, I will just repeat what I said; I have replaced i_s and i_r by this. Then of course, L_{ss} , L_{rr} also comes here well show this again. So, this is what we get and of course, we get C_p inverse here zero zero $I_{4 \times 4}$ matrix. So, actually if you get it to this side you get i_s into i_r . So, basically what I have done is I have taken this equation here you have got a b c a b c this has been replaced by $dq0$ here. So, what you need to do is of course, I will re write this well get

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$$\begin{bmatrix} v_{dq0} \\ v_r \end{bmatrix} = \begin{bmatrix} C_p^{-1} L_{ss} C_p & C_p^{-1} L_{sr} \\ L_{rs} C_p & L_{rr} \end{bmatrix} \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

So v_{dq0} is equal to finally, $C_p^{-1} L_{ss} C_p$, $C_p^{-1} L_{sr}$, $L_{rs} C_p$ and L_{rr} into this is multiplied by i_{dq0} and i_r . So, this is what we get finally. Now, let me just give a sneak preview of what we are going to get? What we intend is we intend to get try to see if this is actually not going to be function of theta. So, what we are trying to do is see the relation between i_{dq0} and i_{dq0} and i_r ; and v_{dq0} and v_r . Now, a nice thing it would be nice to find if all these terms came out to be independent of theta. So, in fact, let me just try to show it at least for one term for example, if I want to do I will just show it for one term I request you to go and do it for each and every term.

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$$C_p^{-1} L_{sr} = C_p^{-1} \begin{bmatrix} L_{sr}^d & \dots & L_{sr}^q \end{bmatrix}$$

$$L_{sr}^d = \begin{bmatrix} M_{af} \cos \theta & \times \\ M_{af} \cos(\theta - 2\pi/3) & \times \\ M_{af} \cos(\theta + 2\pi/3) & \times \end{bmatrix}$$

So, what it turns out is that for example, $C_p^{-1} L_{sr}$. So, let me try to compute this. So, what do you mean by L_{sr} recall what L_{sr} is L_{sr} is C_p^{-1} ; please look into sometime what we did last class L_{sr} is equal to L_{sr}^T this is nothing, but L_{rs} . If we partition this into two parts $L_{rs} = d L_{rs} = q L_{rs}$ is this as this form and $L_{rs} d$ has this form. So, for example, if I use $C_p L_{sr} d$ and $L_{sr} q$. So, if I actually apply this transformation what is $L_{sr} d$? I will just write it down again $M a f \cos \theta$, $M a f \cos \theta - 2\pi/3$, $M a f \cos \theta + 2\pi/3$ then you have got another column; I will not write down that column this is nothing, but $L_{sr} d$.

So, in fact, there is another column I have not written it down. So, you have got terms here and here also. So, if I do C_p^{-1} of $L_{sr} d$ you know what C_p^{-1} is? This is what C_p^{-1} looks like have a good look at this because we need to fit all our manipulations on a small sheet of paper. So, if you have got C_p^{-1} into L_{sr} . So, C_p^{-1} into $L_{sr} d$; I am not showing the derivations for all terms I will just showing it for one term. So, you do C_p^{-1} into $L_{sr} d$.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, it says $C_p^{-1} L_{sr} d (1,1) =$. Below this, there is a row of three terms in square brackets: $[k_1 \cos \theta \quad k_1 \cos(\theta - 2\pi/3) \quad k_1 \cos(\theta + 2\pi/3)]$. To the right of this row is a multiplication sign \times followed by a column of three terms in square brackets: $\begin{bmatrix} M a f \cos \theta \\ M a f \cos(\theta - 2\pi/3) \\ M a f \cos(\theta + 2\pi/3) \end{bmatrix}$. In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' below it.

The first component of $C_p^{-1} L_{sr} d$ that is one one component you can easily find out that it will be basically the first row of C_p that is $k_1 \cos \theta$ $k_1 \cos \theta - 2\pi/3$ and $k_1 \cos \theta + 2\pi/3$.

So, the first term of this complete matrix is nothing, but the first row of C_p matrix into the first column of $L_{sr} d$. So, the first column of $L_{sr} d$ is $M a f \cos \theta$, $M a f \cos$

theta minus 2 pi by 3 and M a f plus cos theta 2 pi by 3. So, if you multiply these two you will get the first term of this. I am just doing it only for the first term; I hope you will be follow up on the other terms.

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$$\begin{aligned}
 C_p^{-1} L_{sr}^d(1,1) &= K_1 M_{af} \left[\cos^2 \theta + \cos^2 \left(\theta - \frac{2\pi}{3} \right) + \cos^2 \left(\theta + \frac{2\pi}{3} \right) \right] \\
 &= K_1 M_{af} \times \frac{3}{2} = \frac{2}{3K_d} \cdot M_{af} \cdot \frac{3}{2} \\
 &= M_{af} / K_d
 \end{aligned}$$

So, what you will get if you that will be C p inverse L s r d the first term is nothing but I will just work it out; you will get K one which is going to be common.

M a f which is going to be common and you will get cos square theta plus cos square theta minus two pi by three plus cos square theta plus two pi by three. So, what you have got is this is the first term of C p L s r d. So, remember just before you lose track; let me just recall what we are doing? This is the final relationship between the d q fluxes and d q currents and of course, the rotor currents. This has to be evaluated remember that C p is the function of theta; L s r, L r s, L s s are functions of theta.

What I am just trying to show you is the derivation of one term that is first term rather the one comma one term of this matrix. And I leave of course, the computation of all other terms to you as an exercise; so, I will just do it for one term. So, that term comes out as to be K one M a f into what should it be? Now, this you must have learnt trigonometric identity cos square theta minus cos square theta minus two pi by three cos square theta plus two pi by three is a constant. In fact, it is equal to three by two. So, what we find here effectively is that the first term is this. In fact, we have already defined what K 1 is; K 1 is nothing but two by three into K d; K d is of course, defined in the

transformation C_p into $M_a f$ into three by two. So, what you get is a very convenient kind of number that is equal to $M_a f$ divided by K_d .

So, this is the first term of this matrix. So, let me just show it this to you again. So, what we are doing is we have got the first term of this. In fact, it is not a function of theta right $M_a f$ by K_d , it is the constant.

So, it turns out all the terms of this matrix are in fact, not functions of theta. So, that is one very interesting effect of this transformation. So, if I look into the flux and current relationships in these new variables in the dq variables you will find that. In fact, they are not functions of theta.

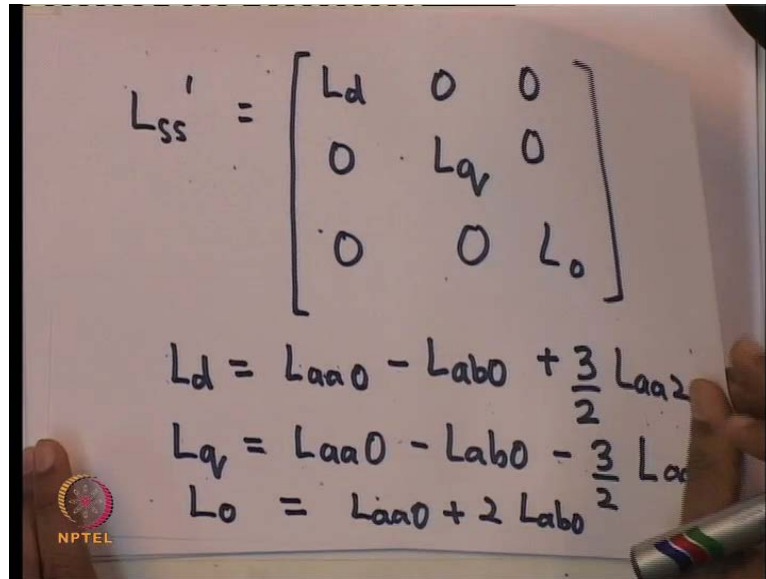
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$$\begin{bmatrix} \psi_{dq0} \\ \psi_r \end{bmatrix} = \begin{bmatrix} C_p^{-1} L_{ss} C_p & C_p^{-1} L_{sr} \\ L_{rs} C_p & L_{rr} \end{bmatrix} x$$

$$\begin{bmatrix} L_{ss}' & L_{sr}' \\ L_{rs}' & L_{rr}' \end{bmatrix} \quad \begin{bmatrix} i_{dq0} \\ i_r \end{bmatrix}$$

So, if you look at this particular matrix here I will just may be call it some name; let us just say it is terms are L_{ss}' ; this is L_{sr}' , L_{rs}' and L_{rr}' of course, L_{rr} and L_{rr} there is no dash here; this is simply L_{rr} , because L_{rr} does not get transformed at all. Then it turns out that all this sub matrices are in fact, not functions of theta.

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$$L'_{ss} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix}$$
$$L_d = L_{aa0} - L_{ab0} + \frac{3}{2}L_{aa2}$$
$$L_q = L_{aa0} - L_{ab0} - \frac{3}{2}L_{aa2}$$
$$L_0 = L_{aa0} + 2L_{ab0}$$

So, an interesting if you really sit and derive it you will get L'_{ss} is equal to in fact, it turns out to be a diagonal matrix where of course, L_d is nothing, but written in terms of the what I can call as the primitive parameters L_{aa0} , L_{ab0} plus three by two L_{aa2} . What are these L_{aa0} , L_{ab0} etcetera? Please recall that our L_{ss} was made out of you know this L_{aa2} , L_{aa0} 's and L_{ab0} 's; this whole L_{ss} matrix was made like this.

So, L'_{ss} which is the relationship between the transformed flux and currents are in fact, related by this and L_q is equal to L_{aa0} minus L_{ab0} minus three by two L_{aa2} and L_0 is nothing, but L_{aa0} plus two L_{ab0} .

What you notice here? Of course, if L_{aa2} is equal to zero; L_d and L_q are equal. So, in fact, when does this occur this occurs if there is no saliency? So, in fact, L_{aa2} will become equal to zero if there is no saliency. So, what you get is L_a , L_d and L_q will become equal of course, in general it is not true because this is non zero.

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$$L_{sr}' = \begin{bmatrix} \frac{M_{af}}{K_d} & \frac{M_{ah}}{K_d} & 0 & 0 \\ 0 & 0 & \frac{M_{ag}}{K_q} & \frac{M_{ak}}{K_q} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$L_{rs}' \neq (L_{sr}')^T$ in general.

So, one of the important things are it is not only time, it is not a function of theta; L_{ss} is not a function of theta, but it is also diagonal. So, that makes it very neat. Similarly, L_{sr} dash is nothing, but M_{af} by K_d , M_{ah} by K_d , zero and zero and you will have zero M_{ag} by K_q , M_{ak} by K_q and zero zero zero and zero. And L_{rs} dash is equal to L_{sr} dash transpose. In general, it is not that is one interesting point.

So, all though L_{rs} and L_{sr} are transpose of each other; this is not true in general. I should show you that it can be made true if you for a certain choice of K_d and K_q . So, in fact, I have not defined what K_d and K_q are. So, they are just arbitrary constants.

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$$L'_{rs} = \begin{bmatrix} \frac{3}{2} M_{af} K_d & 0 & 0 \\ \frac{3}{2} M_{ah} K_d & 0 & 0 \\ 0 & \frac{3}{2} M_{ag} K_q & 0 \\ 0 & \frac{3}{2} M_{ak} K_q & 0 \end{bmatrix}$$

$$K_d^2 = \frac{2}{3} \quad K_q^2 = \frac{2}{3} \quad L'_{sr} = L'_{rs}$$

So, actually L'_{rs} turns out to be three by two times $M_{af} K_d$ and three by two times $M_{ah} K_d$ and zero zero zero zero; three by two times $M_{ag} K_q$ and three by two times $M_{ak} K_q$; and this becomes zero zero zero zero. So, if you look at this and this, it is not true in general.

So, in general this and this will not be true; they will not be transpositions of each other, but if I choose K_d , K_d^2 is equal to two by three and K_q^2 is equal to two by three. In that case only it turns out that L'_{sr} is equal to L'_{rs} transpose.

So, this is special values of K_d and K_q . So, of course, one thing you should see right away that L'_{sr} and L'_{rs} are not functions of θ ; that is something which you should immediately see.

And L'_{rs} is equal to L'_{sr} transpose only if for a certain values of K_d and K_q . Now, you should recall just remember that the fact that the relationship between s_i and i_i is not dependent on θ ; s_{dq} zero, i_{dq} zero is not dependent on θ ; is not affected on what value of K_d and K_q or K_0 you use?

So, this value of K_d , K_q and K_0 whatever value you choose except zero of course, if you chose it as zero you will not be able to invert C_p . So, in that case whatever I am saying is not true, but any non zero value of K_d and K_q will lead you to this relationship matrix; this is the inductance matrix in the dq variables which is not a

function of theta. So, what value of K_d or K_q you chose or K zero you chose does not alter the fact that by using this transformation you can get flux and current relationship which is not dependent on theta. So, that is one important thing which you should keep in mind.

I have not actually derived all the terms of this matrix, it can be quite tedious. But remember, that in case I encourage you to sit and derive every term of this matrix I just showed you the derivation of the first term of L_{sr} , but you can actually take out the whole matrix which is in fact, a seven by seven matrix which has 49 terms from first principles and you actually obtain the expressions for each element in that matrix.

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$$\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right) = 0$$

$$\cos^2\theta + \cos^2\left(\theta - \frac{2\pi}{3}\right) + \cos^2\left(\theta + \frac{2\pi}{3}\right) = \frac{3}{2}$$

In fact, what will help you in doing that would be the trigonometric identities $\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right) = 0$. Similarly, $\sin\theta + \sin\left(\theta - \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{2\pi}{3}\right) = 0$ and $\cos^2\theta + \cos^2\left(\theta - \frac{2\pi}{3}\right) + \cos^2\left(\theta + \frac{2\pi}{3}\right) = \frac{3}{2}$ is in fact, a constant not dependent on theta equal to three by two; the same applies if you replace this cos by sin.

So, these are the four in fact, if you take the sin identities also these two cosine identities and the corresponding sin identities will help you to get all the terms in this matrix; and you will see that they are not functions of theta. They are functions of K_d and K_q and for special values of K_d and K_q you can ensure that the L_{sr} and L_{rs} are transposes of each other.

So, what we really see is we have applied the d q transformation this called the park's transformation or d q transformation matrix. And what we have achieved here is make the flux and the current relationships in the d q zero frame independent of theta.

Now, of course, that does not end our work is to look at also the differential equations as defined by Faraday's law. So, what were the differential equations of our machine?

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The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$\frac{d\psi}{dt} = - \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} - \mathcal{V} \cdot \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix}$$

The bottom equation is:

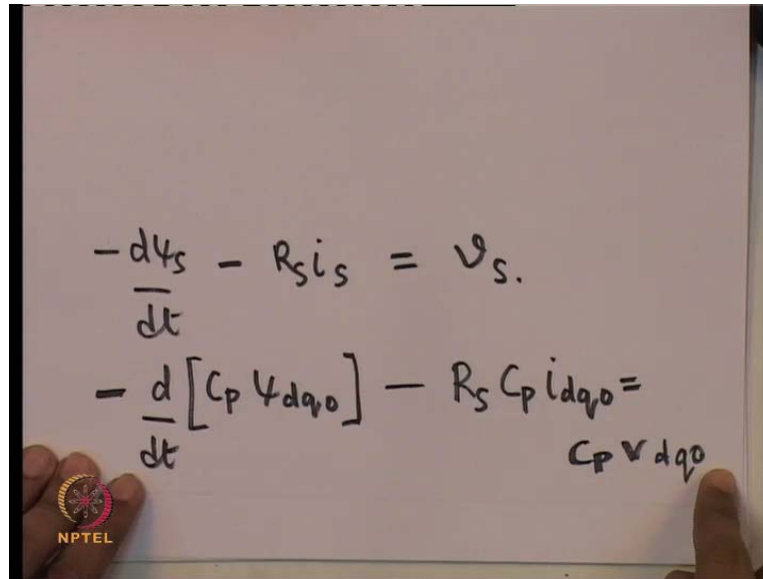
$$\frac{d\psi_s}{dt} = - R_s i_s - \mathcal{V}_s$$

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

In fact, there were $d\psi/dt$ is equal to minus Ri minus v . So, that was our equation. In fact, this is a composite equation I told you that this i is nothing, but i_s and i_r . So, this is i is nothing, but i_s and i_r ; v is nothing, but v_s and v_r and so on.

So, if I just write the stator equations there will be $d\psi_s/dt$ is equal to minus $R_s i_s$. In fact, R is a diagonal matrix consisting of R_s and R_r . So, you will get $R_s i_s$. So, I can write the component I mean this is actually I am just writing down the stator equations minus v_s .

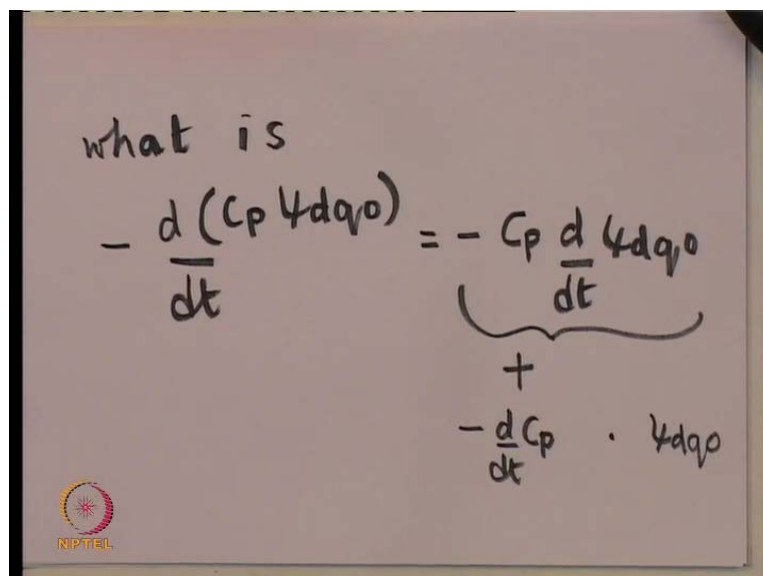
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$$-\frac{d\psi_s}{dt} - R_s i_s = v_s.$$
$$-\frac{d[C_p \psi_{dq0}]}{dt} - R_s C_p i_{dq0} = C_p v_{dq0}$$

So, if I apply suppose if I want to write these of these are of course, derived in a b c frame of reference. So, these are a b c variables. So, if I will rewrite these equations first I will write it as minus d s i s by d t minus R s i s is equal to v s.

So, I can rewrite this as minus of d by d t of C p times s i d q zero minus R s into C p times i d q zero is equal to C p times V d q zero. So, all a b c variables are converted to the d q zero variables. So, I will just read out this is C p into V d q zero; this R s into C p into i d q zero; C p is a matrix, R s is a matrix.

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what is

$$-\frac{d(C_p \psi_{dq0})}{dt} = -C_p \frac{d\psi_{dq0}}{dt} + \left(-\frac{dC_p}{dt}\right) \cdot \psi_{dq0}$$

Now, before we do further manipulations one interesting point which I need to emphasize here. What is minus d by dt C p into si d q zero? It is nothing, but minus of C p times d by dt into si d q zero. No, it is not correct. Remember, that C p is also a function of theta. So, the correct expression would be this minus of d by dt of this C p itself into si d q zero.

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$$\begin{aligned}
 - \frac{d}{dt} [C_p \psi dq_0] &= -C_p \frac{d\psi dq_0}{dt} \\
 &\quad - \underbrace{\frac{dC_p}{d\theta} \cdot \frac{d\theta}{dt} \cdot \psi dq_0}_{\text{"extra"}}
 \end{aligned}$$

So, what you will get is I will just rewrite it minus d C p into si d q zero; just remember that you have to just take into account this is equal to minus C p into d si d q zero by dt minus d C p by d theta and then do d theta by dt; and this is multiplied by si d q zero.

So, what we have this last term which I wrote down here you know will be equal to this last term of the previous equation will be equal to this. So, what we get is the final relationship which we get is this d theta by dt is the electrical speed of this machine.

So, the key difference between what we have done the transformation which we have used before and now is that it is a time dependent transformation; this is dependent on theta. Theta itself is dependent on time; because it is a rotating machine, theta is continuously changing it is a function of time. So, what we see is that when you are taking the derivative of C p into si d q zero we have to take the derivative of the transformation as well.

This was not the case when we did use the linear time invariant systems with constant transformation matrix; this is not a constant transformation matrix. So, you get this extra

$d\theta$ by dt term when we take this derivative. So, this is some kind of you know extra term or extra speed dependent term which comes as a result of applying a transformation. In fact, one important point which you should note is that if we consistently and correctly apply the mathematics you get this term. In fact, it cannot be reasoned out you know why we get a speed dependent term in the differential equations when you do the transformation cannot be reasoned out in any other way than mathematically.

So, although one can try to give what is known as a physical interpretation to the dq transformation, it is a good idea to first how first of all work out all the mathematics correctly and then interpret what we are getting.

So, let me just again repeat when you apply a time variant transformation like C_p on a differential equation; remember that you will have to take the derivative of the transformation itself and as a result of the transformation derivative of this transformation C_p , you get speed dependent term which also called the speed $e_m f$. So, we will continue of course, with this derivation we are coming to the close of the basic modeling of a synchronous machine, but still we have a few things to work out. So, we will work out those things in the next class.