

**Power System Dynamics and Control**  
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**Module No. # 01**  
**Lecture No. # 12**  
**Modeling of Synchronous Machines (Contd)**

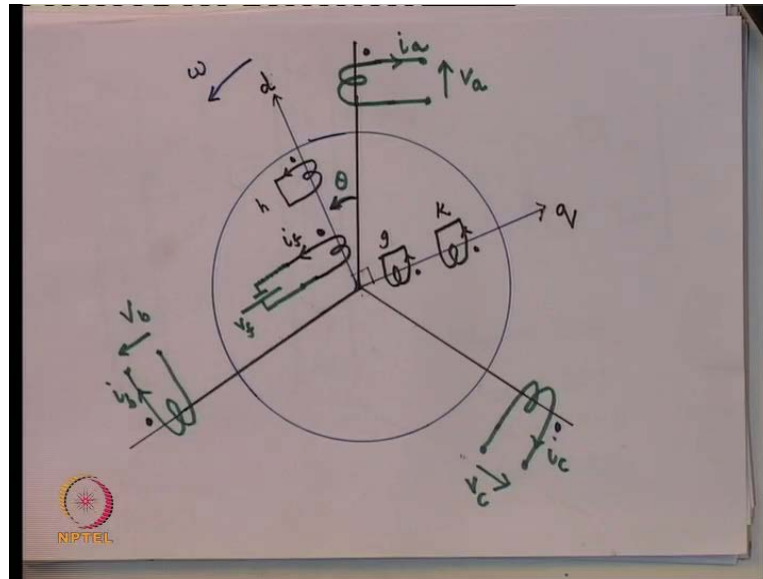
We continue our discussion on the modeling of a synchronous machine, recall in the previous class that we had represented the electromagnetic effects of the rotor by four coils, two coils in the direct axis and two coils in the quadrature axis, out of one coil in the direct axis. In fact, the direct axis is, In fact, of the axis of the field winding of of the two direct axis windings, one of the windings is actually the field winding.

The other three coils on the rotor are in fact, representative of the damper bar effects and eddy currents effects, which may be existing on the rotor of synchronous machine. Now, as I mentioned in the previous class, this is a model of the rotor circuits, there is no hard and fast rule that you can model it only by two coils, you can model by rather, you cannot model by four coils, you can model it by four coils, six coils, eight coils and so on, you can make a more complicated model.

But for most purposes it has been found that it is adequate to model the effects of the rotor electromagnetic circuits as these four coils, two on the d access and two on the q access. We further try to compute the mutual inductance; we try to get a circuit model of a synchronous machine. So, we need to calculate self and mutual inductances between various coils which we have, remember we have three sets of coils on the stator the phase windings that is, and the four coils and the rotor.

So, I request you to direct your attention to the sheet which we had shown in the previous lecture.

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You have got a synchronous machine, you have got three phase windings and you have got these four coils on the rotor. The d axis of the, direct axis is the axis of the field coil and you have got these short circuited windings are in fact, represented over the damper bar and eddy current effects on the router.

Now, remember the axis of a winding, you will just recall, what an axis of a winding means. The axis of a winding is in fact, if I show the axis like this, what I really mean is you have a coil like this (Refer Slide Time: 02:53) you know for example, a state a winding is represented like this with an axis like this so, what it really means is the a winding is wound like this and the direction of the flux is like this. So, this is what I mean by the axis of the winding. Our basic effort in the previous class, which will continue in this class is towards, trying to get a relationship between the fluxes and the currents.

So, you have got the fluxes (Refer Slide Time: 03:20)  $\psi_a$ ,  $\psi_b$ , these are flux linked by the three stator windings and the four rotor winding, they are related to the currents, the three currents on the stator winding, and three currents of the rotor winding by the inductance matrix.

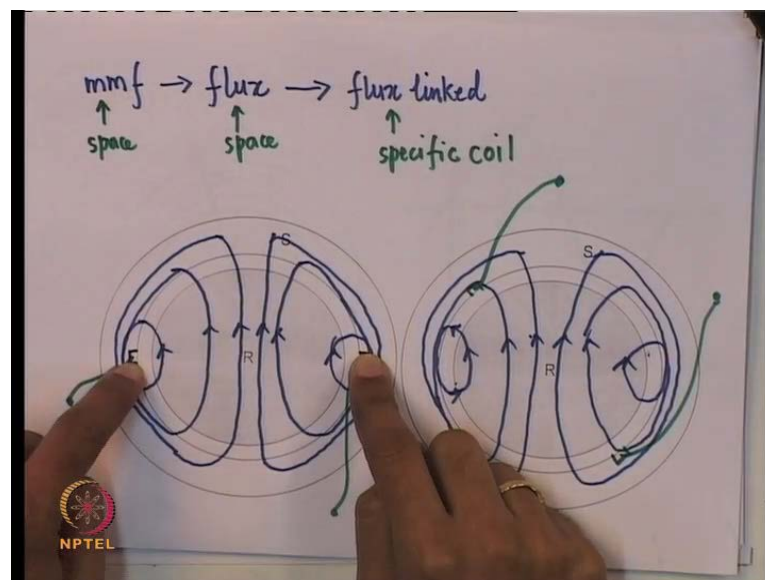
The inductance matrix is made out of  $L_{ss}$ ,  $L_{sr}$ ,  $L_{rs}$ , and  $L_{rr}$  sub matrixes and of course, the sizing is  $L_{ss}$  is  $3/3$ ,  $L_{sr}$  is  $3/4$  and so on. So, the first term in **L1a**,  $L_{ss}$  that

is a  $L_{ss}$  one, one, is in fact, the self inductance of the a phase winding so, that is basically what it means of course, the other mutual inductances and so on.

Yesterday in fact, we try to compute the mutual inductance between, say the a winding and the field winding, and we saw that it is related to the position of the rotor. In fact, it is related **related** by the function  $\cos \theta$ . So, you are mutual inductance between the phase a winding and the field winding is related by relationship  $m_{af} \cos \theta$ , where  $\theta$  is the angle which the field winding makes with the a winding axis, you know this keeps on changing, this  $\theta$  is keeping on changing, because the rotor is continuously moving.

So, that was one term which we kind of derived in the previous class. Now, the basic idea of how to take out the mutual inductances is thus, what you need to do is, you, from the winding **winding** configuration of a particular winding, we compute the  $m_{mf}$  if a current was flowing. So, if you have got a current  $i_a$  in the a winding, what we do is compute the  $m_{mf}$  in the air gap due to the a winding current then, what we do is compute the flux.

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So, if you look at this the basic way of doing things is first, compute the  $m_{mf}$  in space, that is the air gap. Compute the flux in space or in the air gap then, you compute the flux link to a **specific** specific coil of a **(( ))** suppose, if you are taking out the mutual inductance between the a winding and the b winding, what you will do is first assume a

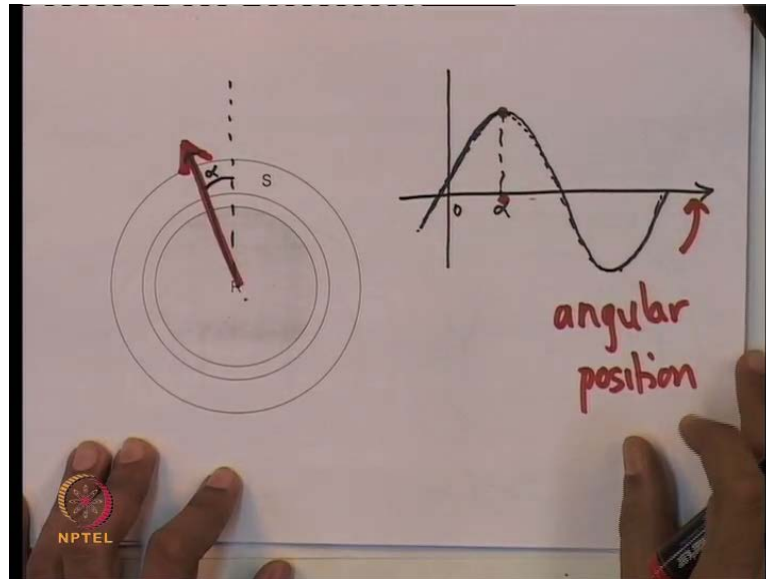
current  $i_a$  flows in the a winding, compute the  $m_{af}$  in space, compute the flux due to this  $m_{af}$ , from the flux you compute flux link with the b coil, if you want to take out the mutual inductance between a and b coil this is the procedure you will take up.

For example, if you have got a current in the a winding you will have a flux like this in fact, you will have  $m_{af}$  which will cause the flux, but let us assume it cylindrical core so, then you will get the flux of this nature. So, if you want to find out the self inductance of the a winding that is  $L_{aa}$ , what you will do is, just find out the flux link with the a winding due to current in the a winding. So, what you will have to find out is the flux you know, the integration of flux density over the surface.

So, that will give you how much flux is getting linked with this a winding, is it clear. So however, if you want find out for example, the flux link in the b winding due to current in the a winding so, the flux of course, will remain the same due to the a winding current, we assume only a winding current is there. So, you will get the flux as in the previous case you will get a flux, but now you have to find out flux link with the b winding. So, the b winding flux would be the integration of the flux density from this point of over the surface. So, how much flux is getting into this like this. So, that is the flux linked with the b winding; obviously, the flux link in the a winding due to the a winding current is different from the flux linked in the b winding due to the b winding current, due to the a winding current.

So, mutual inductance between a and b, the a and b winding would be different from self inductance of the a winding itself so, that is an obvious thing. Now, just before we things start becoming a bit complicated, when you have got saliency, now once you have got saliency the step, the step from getting the flux rather, getting the flux from the  $m_{af}$  is not regular, I mean you have to just think over it, because in this particular case which you have considered cylindrical rotor once you have got the  $m_{af}$  wave form, you know the reluctance of the air gap is uniform in a cylindrical rotor machine, almost uniform in a cylindrical rotor machine, in that case getting the flux on a  $((\ ))$  is very straight forward, but if you have got salient pole machine things can be a bit complicated. So, even getting the mutual inductance between a winding and the b winding would depend on the rotor position, because the salient the **saliency** **the** **the** rotor would define saliency in a particular direction.

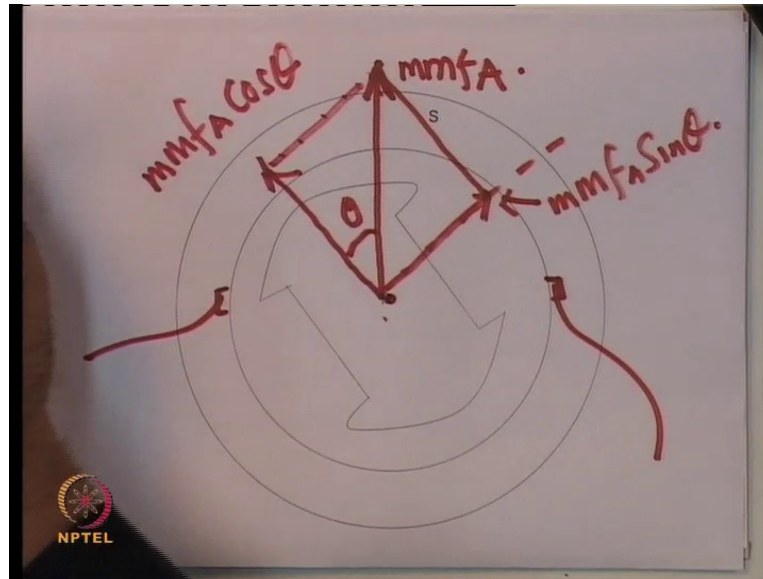
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So, just look at before we go ahead with trying to compute this, just minor point about the convention I am going to use suppose, you have got  $m$   $m$   $f$  of this you know, of this nature. This is the angular position, the  $x$  axis is the angular position measured from this vertical axis. So, if my  $m$   $m$   $f$  is like this and which is a peak value at  $\alpha$ , at an angle  $\alpha$  in that case I will represent, this is a represent, I will represent the  $m$   $m$   $f$  by a vector which is at an angle  $\alpha$ . So, just remember this convention which I even used in last time. So, if my  $m$   $m$   $f$  direction is shown like this, what it means is of course, that the air gap distribution of the  $m$   $m$   $f$  is like this with the peak at **at** the angle  $\alpha$  with the respect to the vertical.

So, now let us go on to this problem of trying to compute the  $m$   $m$   $f$  rather the mutual inductance between the  $a$  winding and  $b$  winding, when there is saliency.

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So, now things become a bit tricky, because suppose, you have got a situation like this, this is your rotor so, you have got the a winding, it say let us carry a current, let us says it carries a current of  $i_a$  so, where does it create an  $m m f$ ? Well it creates an  $m m f$  in this direction. So, I will represent a  $m m f$  as a vector so, it is a peak value occurs here then, it drops of  $(( ))$  as with the angular position. So,  $m m f$  is like this, but now the second step of getting the flux is non tribune.

Because if the **if the** rotor position like this, that is, if  $\theta$  is equal to zero you would get a different value of flux, if  $\theta$  is something else you will get some different, some other value of flux. So, In fact how do you then compute the flux in the core for any arbitrary position. So, that is one worry which you have now, one you know, reasonable way of doing this is actually compute the flux for two extreme positions. And then compute the equivalent flux so, I will just, this **this** appears a bit a bit vague so, I just tell you, what I mean.

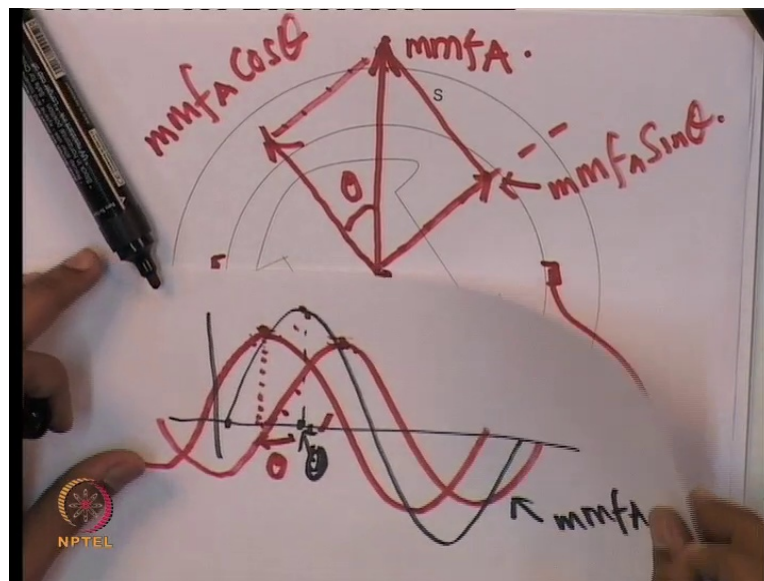
Suppose you have got current in the a winding and the rotor has got this position, the salient pole is here then, the  $m m f$  is like this and the flux is also like this. So, the flux vector, the same convention I am using, the flux vector also be like this  $m m f$  is also like this.

But if **I if** the rotor position is like this the  $m m f$  is still here, the same, the flux which will be caused will be much, much, see I have not shown the  $(( ))$  this is side, this is a

mirror image, the flux caused will be smaller than this so, if this was so, the rotor in this position would **would** cause a **flux** flux like this, a rotor in this position would cause a much smaller flux. So, if you are, if you have got a salient pole machine and theta is ninety degrees you will get this.

Now, what we do is; instead of computing the flux for every position, what we do is; we compute the flux for two extreme positions, **what I** what I mean to say is this. Suppose we have got an  $m m f$  like this, the  $m m f$  is maximum in this direction, what I do is compute the component of  $m m f$  in this direction, what do I mean by that? It is easy to see, so, this  $m m f$  is  $m m f a$ , I call this  $m m f a$  and this  $\theta$ . The component of  $m m f$  in this direction is  $m m f a \cos \theta$  and the component of  $m m f$  in this direction here is  $m m f a \sin \theta$ . Now, the **the** thing what so, what does this actually mean, this means that if a  $m m f$ , i just draw this here.

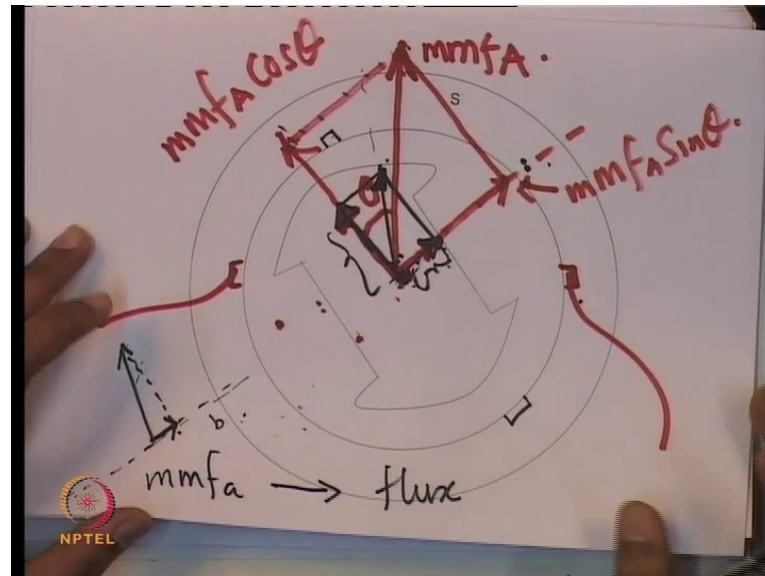
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If a  $m m f$  is like this, its maximum at the zero position so, if a  $m m f$  is like this, I have actually written down this  $m m f$  this is the zero position, I am writing down this  $m m f$  this is  $m m f A$ , as if it is made out of two components. So, it is made out of two components this is zero, this is  $\theta$ . So, it is made out of two components, this is  $m m f A \cos \theta$  and of course, you have got  $m m f A \sin \theta$  of course, the peak value of this and this will be different. So, the if this is  $m m f A$ , this will be  $m m f A \sin \theta$  **sin theta** cos

theta this is  $m m f A \sin \theta$ , the magnitude of this will be different. What I want to say is that the sum of these two waves will give you this wave.

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So, what I have done is, what I am trying to do is I am splitting up the  $m m f$  into two components. I will, what I will do is one component is along this  $d$  axis the field winding axis, one is along the quadrature axis the now, what I do is I know that if **if** this rotor salient pole is aligned with the  $m m f$  I will get a certain amount of flux. I know if the rotor is aligned at this position, I will get a certain amount of flux.

So, what I do is to compute the flux, I know this how much flux I will get if the  $m m f$  is in this direction and I know how much flux I will get if I am in this direction. So, the thing is I have split up the  $m m f$  into two components, I know the ratio of the  $m m f$  and the flux, when the  $m m f$  is in the direct axis. I know the ratio of flux and the  $m m f$  when the  $m m f$  in the  $q$  axis. So, I just remember these two ratios so, what I do is, I got the  $m m f$  split it up into two parts, I know what  $m m f$ , what flux I will get in this direction, I know what flux I will get in this direction, I will superpose the solutions.

So, this **this** saves us the trouble of computing the flux in each position. So, what I have done is, if I know what the flux **the the flux** I will get for the certain  $m m f$  in the direct axis and I know what the flux I will get for an  $m m f$  in the quadrature axis, I will compute the flux individually in these tow axis and then super impose the solution. So,



what I will do is so, what I have effectively done is, got the flux which will result in this direction,  $((\ ))$  what flux will result in this direction so; obviously, it got to be smaller.

And then, get out, get the result in flux in the core. So, this is the basic idea so, from  $m_m f$  due to a coil to the a coil I can get the flux, how do I get the flux? Not by remembering the flux for every position, but for every position I compute the component of  $m_m f$  in the direct and the quadrature axis, and I just remember these two ratios, the two constants which really relate the  $m_m f$  and the flux in these two directions. So, I just need to remember the relationship between the flux and an  $m_m f$  in this direction and this direction rather than remembering it for every rotor position.

So, after I compute this, please look at this carefully this is the flux component in this direction, this is the flux component in this direction. I can compute the resultant flux, the resultant flux direction and magnitude is like this. So, once I compute that the resultant flux direction and magnitude is like this, I can really draw the flux in the core not really draw it, but I know that well it is going to have a peak value in this direction and going to drop off  $((\ ))$  as as we as we change the angular position. So, now, I know that the flux is going to be like this, in this direction, in the core.

Now, the question is the second point which you should **you should** do in order to compute the mutual inductance is,  $((\ ))$  have computed the flux, the flux in the core you have computed the direction and magnitude. Now, you compute the flux linked with the specific coil so, what you do is now for example, if you want to find out, how much of this flux is linked with the b coil? So, you want to find out how much of this flux is linked in the b coil, what you need to do is, find out the component of this in this direction.

So, what I am asking you is suppose, I know the flux is like this, compute the component of flux in this direction. In fact, it will like this. So, in this direction the b axis, b winding axis the amount of flux linked by the b winding would be proportional to the component, this something you can actually work out is going to be given by the component of this vector along the b axis. So, this is how you would compute the mutual inductance between the a winding and the b winding when there is saliency.

So, now what we will do is simply start writing down, I will not derive each and every inductance, but tell you the basic form of the equation.

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$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix} = L \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \\ i_h \\ i_g \\ i_k \end{bmatrix}$$

$$L = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}$$

$L_{ss} \rightarrow 3 \times 3$   
 $L_{sr} \rightarrow 3 \times 4$   
 $L_{rs} \rightarrow 4 \times 3$   
 $L_{rr} \rightarrow 4 \times 4$

So, if you recall what I am trying to do is, first of all I will tell you how  $L_{ss}$  looks like,  $L_{sr}$  looks like,  $L_{rs}$  and  $L_{rr}$ . Now, please remember I am not deriving every inductance out here that would take a very long time, but basic way of doing things please remember compute the  $m_m f$  due to a current, where the  $m_m f$  is then, compute the flux in the core, in the air gap, the flux in the air gap is actually found out by, in case you have got saliency in that case (( )) take out the  $m_m f$  component in the  $d$  axis and the  $q$  axis. You know the ratio between the  $m_m f$  and flux, in the  $d$  axis the  $m_m f$  and the flux in the  $q$  axis, what I mean is, if an  $m_m f$  existed along the  $d$  axis what would be the flux, if an  $m_m f$  existed along the  $q$  axis what would be the flux, and then super impose the solutions, two solutions. So, this saves us the trouble of trying to remember all rather the flux for every rotor positions.

So, you remember the flux and  $m_m f$  ratio only for two positions and try to derive what is going to be the flux for a given  $m_m f$  for an arbitrary position of the rotor. So now, I will just write down this  $L_{ss}$ , which is  $L_{ss}$  remember is the self inductance rather matrix which relates (( ))<sub>c</sub> to  $i_a$ ,  $i_b$ , and  $i_c$ .

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$$L_{ss} = \begin{bmatrix} L_{aa0} & L_{ab0} & L_{ac0} \\ L_{ab0} & L_{aa0} & L_{ac0} \\ L_{ab0} & L_{ab0} & L_{aa0} \end{bmatrix} + L_{aa2} \begin{bmatrix} \cos 2\theta & \cos(2\theta - \frac{2\pi}{3}) & \cos(2\theta + \frac{2\pi}{3}) \\ \cos(2\theta - \frac{2\pi}{3}) & \cos(2\theta + \frac{2\pi}{3}) & \cos 2\theta \\ \cos(2\theta + \frac{2\pi}{3}) & \cos 2\theta & \cos(2\theta - \frac{2\pi}{3}) \end{bmatrix}$$

So,  $L_{ss}$  will have this form, I will not read out each component, but you will notice that there is a self inductance term  $L_{aa0}$ , plus  $L_{aa2} \cos 2\theta$  now of course, if there is no saliency  $L_{aa2}$  is zero then, there is a mutual inductance between the a winding and the b winding plus  $L_{aa2} \cos(2\theta - \frac{2\pi}{3})$ , this is something you have to derive, I am not of course, derived every component of this inductance matrix.

So, if saliency is not there at all you just have a constant matrix, you do not have things like  $L_{ac}$  and  $L_{cb}$ , the reason is that it assumed to be symmetric machine, all it is a balanced machine. So, this will be equal to this, this will be equal to this, this will be equal to this and this, this and this will be the same. So, self inductance of the a winding in case there is no saliency the same as self inductance of the b winding and the self inductance of the c winding.

In case there is saliency you will know notice a cyclic nature with respect to theta. So, whatever happens in this row the row gets kind off shifted in this direction. So, you will notice that, this is the basic nature of the self inductance. If you find out the mutual inductances between the are the relationship between the rotor axis currents in the rotor winding, the mutual inductances in the self inductance of the rotor winding this is the basic structure you will come across.

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The image shows a handwritten matrix equation for the rotor inductance matrix  $L_{rr}$ . The matrix is written as:

$$L_{rr} = \begin{bmatrix} L_f & L_{fh} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ L_{fh} & L_h & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} L_g & L_{gk} \\ L_{gk} & L_k \end{matrix} \end{bmatrix}$$

Below the matrix, there is a diagram of a circular rotor winding with an arrow indicating the direction of the magnetic flux.

$L_f$  is the self inductance of the field winding,  $L_{fh}$  is the mutual inductance between the field winding and h winding, there is no mutually inductances between the field winding and the g winding. So, that is one thing which you should remember this, the mutual inductance between the f winding and the g winding is in fact zero, why is that so? Just think over it.

With the assumption we have made, it can be easily shown that there will be no flux link, but maybe I will just try to tell you why it is so, suppose you have got the field winding, it creates an  $m_f$  in this direction, the  $m_f$  it creates  $(( ))$  flux in this direction. So, if you have got a flux in this direction, how much of that flux gets linked in this ninety degrees  $(( ))$  where the other windings rotor windings exist, it will be zero, because the component of the flux along this will be equal to zero, but of course, correct way of actually saying this is, this is a  $m_f$ , this is  $m_f$  distribution with the peak in this direction, it will cause a flux distribution which again has a peak here.

If it has a peak here the amount of flux linked by a coil which is, has an axis in this direction, if you integrate the flux density along this surface you will find it it is zero. So, the mutual inductance between this winding and a q axis winding is actually zero. So, that is why this off diagonal elements are in fact zero. So, this is the nature of the self inductance matrix.

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$$\begin{aligned}
 L_{sr} &= L_{rs}^T \\
 &= \begin{bmatrix} L_d & & \\ L_{sr} & \vdots & \\ & & L_q \end{bmatrix} \\
 L_{sr}^{\theta} &= \begin{bmatrix} M_{ag} \sin \theta & M_{ak} \sin \theta \\ M_{ag} \sin(\theta - \frac{2\pi}{3}) & M_{ak} \sin(\theta - \frac{2\pi}{3}) \\ M_{ag} \sin(\theta + \frac{2\pi}{3}) & M_{ak} \sin(\theta + \frac{2\pi}{3}) \end{bmatrix}
 \end{aligned}$$

expressions derived in this lecture are given

The mutual inductance between the various stator windings and the rotor windings have this form. In fact, it is an interesting thing is  $L_{sr}$  is equal to  $L_{rs}$  transpose and  $L_{sr}$  itself can be partitioned into two components. So, you have got the mutual inductance between the a winding and d axis windings, what are the d axis windings f and h, for those **for those** who have just forgotten or just missed out.

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$$\begin{aligned}
 L_{sr} &= L_{rs}^T \\
 &= \begin{bmatrix} L_d & & \\ L_{sr} & \vdots & \\ & & L_q \end{bmatrix} \\
 L_{sr}^{\theta} &= \begin{bmatrix} M_{ag} \sin \theta & M_{ak} \sin \theta \\ M_{ag} \sin(\theta - \frac{2\pi}{3}) & M_{ak} \sin(\theta - \frac{2\pi}{3}) \\ M_{ag} \sin(\theta + \frac{2\pi}{3}) & M_{ak} \sin(\theta + \frac{2\pi}{3}) \end{bmatrix}
 \end{aligned}$$

So, the, what we are doing is now finding out the mutual inductance between the a winding, b winding and c winding with **with** the d axis coil. Similarly, we will have to

find out the mutual inductance between a, b and c and these coils. So, what I am saying is that so, we can sub partition this matrix into the mutual inductance between the stator and d axis rotor windings, and the stator, and the q axis rotor windings. The q axis partition matrix here has got this form, again I am not deriving it, but you have use the same procedure as a mentioned some time back of computing the mutual inductances.

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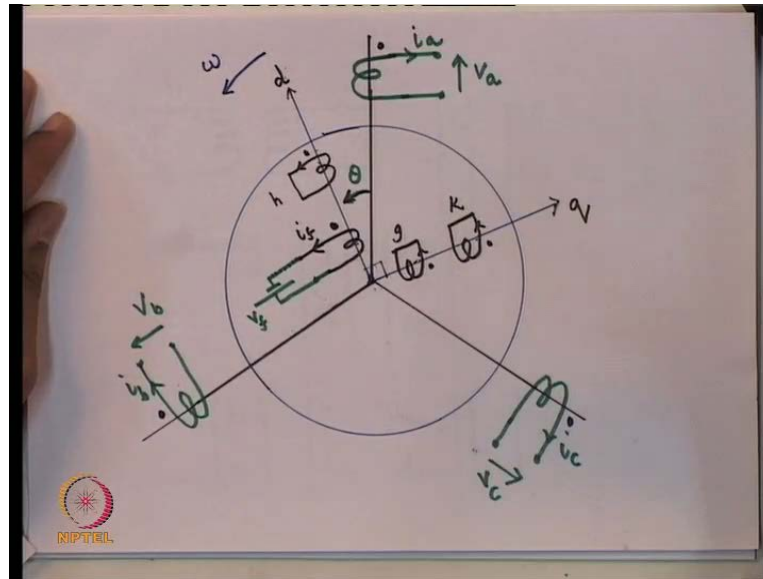
$$L_{sr} = \begin{bmatrix} M_{af} \cos \theta & M_{af} \cos \theta \\ M_{af} \cos(\theta - \frac{2\pi}{3}) & M_{af} \cos(\theta - \frac{2\pi}{3}) \\ M_{af} \cos(\theta + \frac{2\pi}{3}) & M_{af} \cos(\theta + \frac{2\pi}{3}) \end{bmatrix}$$

3x2

Similarly, the d axis  $L_{sr}$  that is the mutual inductance between the stator phase windings and the rotor windings on the d axis there two of them. So, this is basically 3 by 2 matrix it looks like this. There is no difference between the mutual inductance between the a winding and the field winding, and b winding in the field winding expect that, this dependence on theta is different so, that is the only difference otherwise, the constant of here is the same, this coefficient here is the same only the theta dependence is different, that is not surprising because a, b and c windings are absolutely identical, other than the fact that they are **they are** not in the same special position.

Now, the next thing I need to take out of course, just getting the relationship between the fluxes and the currents is not enough, I need to apply the physical laws to obtain the relationship the **the** rate of change of flux and apply voltage of course, that is really obtain form Faraday's law. So, just of course, we will just pay an attention since we are developing, what is known as circuit model of the machine, we need to pay little bit of attention to the sign conventions and the dot conventions.

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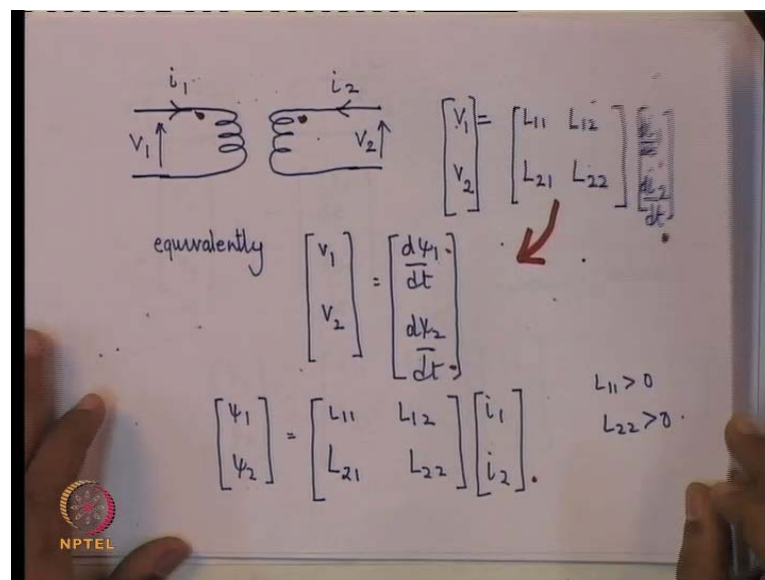
So please, assuming that the dot convention like this is used, the dot is on the top, dot, dot, (( )) the dot, what is that a dot means actually? The thing is that, in case two windings has the same dot and current is entering both windings in the dot, they will create fluxes which tend to enhance or other in the same direction that is what it means. So, if you have got two coils with the dot on the top, if I align the two coils and the dots are on the top then the current entering the dot the dot will cause the flux in the same direction so, both the flux tends to add on, that is what the at the dot convention means after all. Again let me repeat, if you have got two coils with dots on the top and I align them, and the current enters at the dot into one coil and it creates flux in a particular direction, then the second coil also if it, if the dot (( )) and the same at the top of the coil, if the current enters the dot in the second, at the dot of the second coil also cause the flux in the same direction that is what a dot convention effectively means. So, if you look at the dot convention here the dot is here, the dot is here, the dot is here, the dot is here, and the dot is here, the and dot is here, and the dot is here. So, just just look at how the dots are placed, one important point which you should note that, this current (( )) convention I am using is that the current is going out of the dot.

So, whenever I am going to, I will use this convention derive the equations and whenever I get the answer for the current remember that, it pertains to the current flowing out of the dot of the winding. So, this is what I mean so, that if theta were to be zero, if theta were to be zero and currents were entering the dot they would cause fluxes in the same

direction that is, what this dot convention would be now. So, I just let me repeat again in case you have not got the point. Suppose, the current is entering the dot it creates the flux say in this direction upwards it tense to creates the flux like this, I take this field winding if I align it here and a current enters the dot of the field winding it again will cause the flux in this direction, that is what the dot convention means.

Now, since I have taken the current flowing out of this winding I need to write down my equations a bit carefully.

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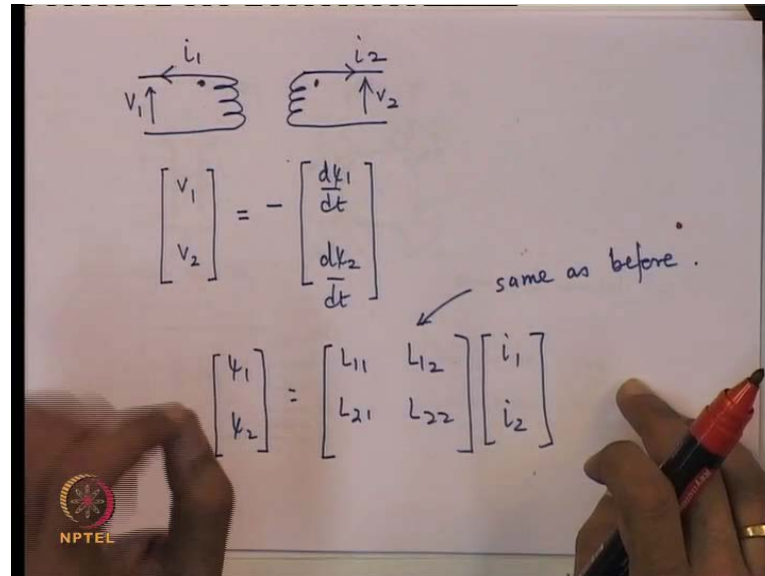
You must be familiar with the typical way of representing a mutually coupled circuit, if you have got these dots currents are entering the dot then, the equations defining the voltages  $v_1$  and  $v_2$ ,  $v_1$  is in this direction that is the  $v_1$  is the potential of this point with respect to this point,  $v_2$  is the potential of this point with respect to this point and the dots are here, the currents are entering the dots then, the relationship between  $v$  and  $I$  is like this,  $v_1$  is equal to  $L_{11}$  into  $dI_1$  by  $dT$ ,  $v_2$  is equal to  $L_{21}$  into  $dI_1$  by  $dT$  plus  $L_{22}$  into  $dI_2$  by  $dT$ .

Or just to write it in **in** more familiar notation  $v_1$  is  $d\psi_1$  by  $dT$  and  $v_2$   $d\psi_2$  by  $dT$ , where  $\psi_1$  and  $\psi_2$  are the other flux link to the flux first and second called respectively and the flux is written down like this. So, I am just writing down these equations there is no change, I am just writing down these equations in two sets. So,  $v_1$  is equal to  $d\psi_1$  by  $dT$ ,  $v_2$  is equal to  $d\psi_2$  by **d2**  $dT$ , where  $\psi_1$  is  $L_{11}$ , **L1**  $L_{11}$



1,  $i_1$  plus  $L_{12} I_2$  and  $\psi_2$  is equal to  $L_{21} I_1$  plus  $L_{22} i_2$  normally under such circumstances  $L_{11}$  and  $L_{22}$  both will be greater than zero normally the self inductances in such a circuit will be greater than zero.

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Now, if I have got currents, if my current direction I reverse, but my **inductance** inductance matrix is same as before then, I will have to write down my equations like this. So, my current has got reversed so, my definition of flux in some sense got reversed. So,  $v_1$  and  $v_2$  are now given by this so, this is Faraday's law would be correct like this, if you use this convention that the currents are flowing out of the dot.

So, this is also known as generator currents so, remember if you are using the generator convention the correct representation of the equations would be like this, **is it**.

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The image shows a whiteboard with handwritten equations in red ink. At the top, there is a matrix equation:
$$\begin{bmatrix} \psi_s \\ \vdots \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_{ss} & L_{sr} \\ \hline L_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} i_s \\ \vdots \\ i_r \end{bmatrix}$$
Below this, there are two circuit equations:
$$-\frac{d\psi_s}{dt} - R_s i_s = V_s$$

$$-\frac{d\psi_r}{dt} - R_r i_r = V_r$$
In the bottom left corner of the whiteboard, there is a small circular logo with the text 'NPTEL' below it.

Now, what are the equations? If **if** these are the, you have already found out that you have found out the inductance matrixes that is  $\psi_s$ ,  $\psi_r$  this in fact,  $L_{ss}$  and  $L_{rs}$  are transpose of each other,  $i_s$  of course, is  $i_a$ ,  $i_b$ ,  $i_c$ ,  $i_r$  is  $i_f$ ,  $i_h$ ,  $i_g$  and  $i_k$  similarly,  $\psi_s$  and  $\psi_r$ .

Now, the thing is that what are the equations? If this is the convention, if this is the dot **dot** convention and you are assuming the currents are coming out of the dots, all the currents are flowing out of the dots, that is why we are using generator convention. What are the equations which describe this Faraday's law? So, Faraday's law says that minus  $d\psi$  is by  $dT$ , minus of course, if there is a resistance in the winding  $R_s$ ,  $i_s$  is equal to  $v_s$  of course, this is something I have introduced the, this is the resistance of the winding and minus  $d\psi_r$  by  $dT$  minus  $R_r i_r$  is equal to  $v_r$ .

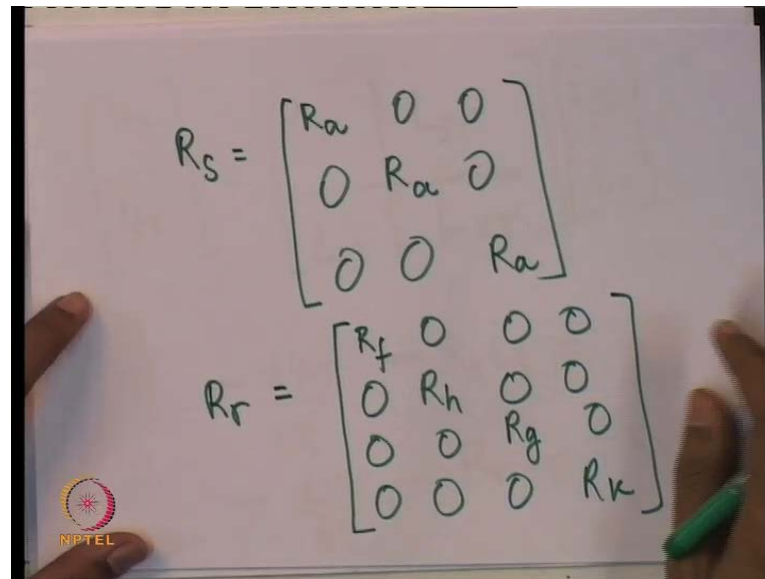
So, this is basically your Faraday's law as applied to this system. So, basically its reiteration of this, is it. Because the if there was, see the only difference here is I have put the resistance so, if actually there was a resistance here of  $R_1$  and  $R_2$  in that case you would have to modify these equations and just write them as  $v_1$  is equal to, this is basically how things will look like.

When you have got this. So, **so** if your current is flowing like this, your drop is going to be like this and your drop here is going to be like this, that. So, we will have  $v_1$  is equal to minus  $R_1 i_1$ ,  $v_1$  is equal to minus  $i_1 R_1$ ,  $v_1$  and of course, this is  $d\psi_1$  by  $dT$

and minus  $d\psi$  by  $2dT$ , because your current is going in this direction, but we have kept the inductance definition same as before.

So, the basic equations are these now, what is  $R_S$ ? Actually  $R_S$  is a matrix,  $R_S$  is the is actually  $R_a$ ,  $R_b$  and  $R_c$  the phase winding resistances.

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The image shows a whiteboard with two handwritten matrices. The first matrix is  $R_S = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_a \end{bmatrix}$ . The second matrix is  $R_r = \begin{bmatrix} R_f & 0 & 0 & 0 \\ 0 & R_h & 0 & 0 \\ 0 & 0 & R_g & 0 \\ 0 & 0 & 0 & R_k \end{bmatrix}$ . A small NPTEL logo is visible in the bottom left corner of the whiteboard.

So, what we have here is  $R_S$  is nothing, but you know  $R_a$ ,  $R_b$  and  $R_c$  of course, are the same. So, I will not represent them separately. So,  $R_S$ ,  $I_S$  and  $R_r$  of course, is  $R_f$  zero, zero  $R_h$ , zero zero, zero, zero, zero, zero  $R_g$ ,  $R_k$ .

One interesting point is suppose, I want to represent the synchronous machine not by two windings on the  $d$  axis and two windings on the  $q$  axis, **but** but by only one winding on the  $q$  axis, what would I do? Well I would go ahead with my formulation as we have been doing so far and then put  $R_k$  tending to infinity or  $R_k$  very large.

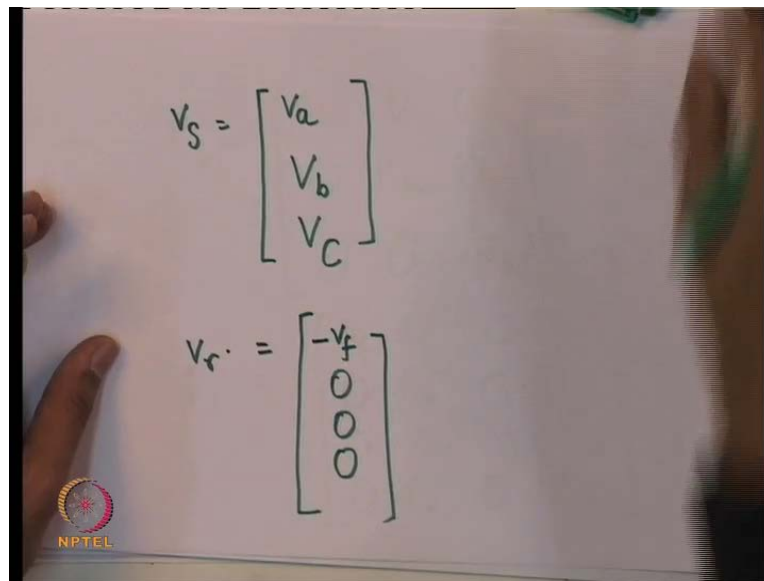
So, it is a good idea to actually build the model using these large number of coils, in case if you want to simpler model you just set some of the resistances tending to be a very large value, that will kind of open the winding. Once you open the winding there is no current flow through it, and if there is no current flow through it will of course, not affect the  $m$  or the flux.

So, this is an interesting trick to do, later on when we want to get a **lower model** lower order model from this high relatively higher order model, we can actually just you know

change the parameters and get it. So, you do not have to rederive the whole equations with just one coil less, you can set just 1 of the resistance to be very large.

Now, the next step of course, is what is  $V_s$ ? That is, if you recall what I am doing here so,  $R_s$  we have discussed in and  $R_r$  we have discussed. So, what is  $v_s$ ?  $v_s$  is in fact, the applied voltages.

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The image shows a whiteboard with two handwritten vector equations. The first equation is  $V_s = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$ . The second equation is  $V_r = \begin{bmatrix} -V_f \\ 0 \\ 0 \\ 0 \end{bmatrix}$ . There is an NPTEL logo in the bottom left corner of the whiteboard.

So, what are the applied voltages?  $v_a$ ,  $v_b$  and  $v_c$ , these are the applied voltages to the stator winding and of course, what is  $v_r$ ? Let us just be a bit careful about this, what is  $v_r$ ? Well, if you have got, if you look at these rotor winding set represent dumper and eddy current effects they are short circuited.

So, we are. So, for these three windings it is zero and now if I have got a voltage say I just magnify this, I am just magnifying this coil here, dot is here, current is  $I_f$ , this is the  $f$  field winding, you need a voltage source like this to drive current into the field winding the  $d c$  voltage source, which is connected.

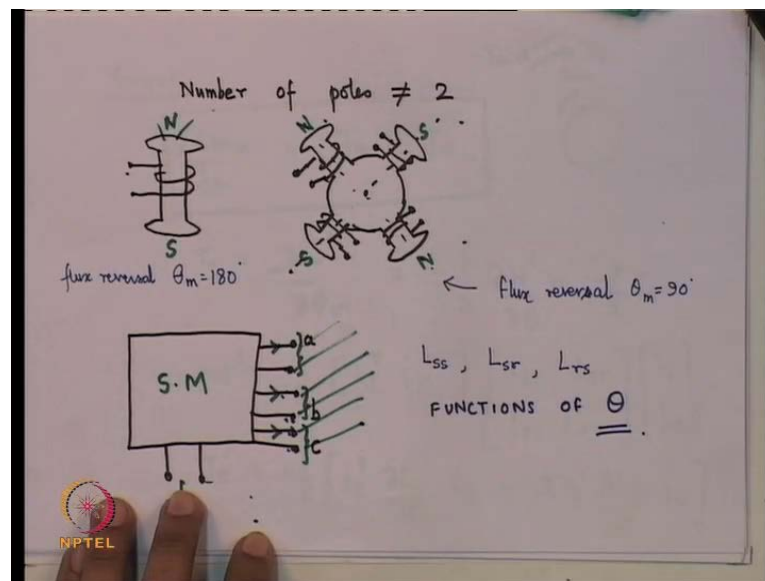
This I will call  $v_f$ . So, this **this** voltage  $v_f$ , the potential of this point with respect to this is  $v_f$ , but of course, normally when we are using this, you know dot convention, when I say  $v_1$  it is usually the voltage at this dot with respect to the other point.

So, when I show dot here, it is the voltage dot at this point with respect to this, but of course, if I have indicated this small  $v_f$  to be connected like this; obviously in  $v_r$ , I

have to put minus v f, what **what** I am saying? So, because the voltage is connected like this the dot is here, but I am connecting the voltage source like this, this v f whatever the magnitude of this voltage source has to be put as minus with the minus sign here.

So, now we have got a complete equations. So, I know this. So, if I give you this, this and the position of the rotor, I in some sense have got told you these equations will tell you how these, how the rotor fluxes with will behave.

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Now, one small point regarding the number of poles, remember that the flux, the inductance matrixes which you have taken out, we in fact, we were discussing only with the two pole machine that is, mechanical angle and the electrical angle were equivalent; however, you consider the case of a four pole machine, at other first we will see of course, two pole machine.

In a two pole machine, if this turns around by 180 degrees, the current of course in the coil remaining the same. Suppose, this is the north pole and this is the south pole of this rotor, the fluxes coming out of this north pole like this, because of the current here. Suppose, I turn it round by 180 degree so, there will be a complete flux reversal in the core in the area.

So, 180 degree turn results in a complete flux reversal, in this particular case any winding if you look at, if you have this is the four pole structure this is north, this is

south, this is north and this is south, in this case even on 90 degree change in the mechanical angle will change the flux picture, will completely reverse the flux picture. You can just over it, 180 degree change in the rotor position will cause a flux reversal here and whatever the flux values were at various positions on the rotor they will become negative of that.

In this case, when you have got four poles even on 90 degree change will cause a complete flux reversal. So, remember whenever we are calculating the **the** inductance matrixes we have calculated. So, the four pole machine, if I have calculated for a two pole machine and I want to see what is the inductance matrixes for a four pole machine, we do not have to change the anything, but remember that the theta which we are using has to be the electrical angle.

So, the inductance matrices which have derived are still valid the structure of, and the nature of each element is still the same. But for example, if you have got  $\cos 2\theta$  somewhere in the inductance, the theta should be the electrical angle not the mechanical angle.

So, you can be absolutely blind to, you know, you do not have to change the inductances matrixes at all, but they are going to a function of theta, not theta m the mechanical angle that is one thing you should remember.

So, inductance matrices which have, which have derived have the same structure, but they are going to be dependent on theta, not theta m. So, as far as the machine is concerned, synchronous machine is concerned the electrical equations are concerned, what you are going to get out of the machine eventually, I mean what you are going to connect to the grid, the machine is going to connected to the grid you will get actually, you have got what are known as, you have got the phase a terminal, pair of phase a terminal, the pair of phase b terminal, a pair of phase c terminal. So, you have got this is what is coming out.

Similarly, you have got the field winding. So, this has to be connected to a d c voltage source, this is to be connected to the grid. Inside it may be a two pole machine or a four pole machine, but the point is the mutual inductances between a and f, b and f, a and b are all going to be functions of theta and not theta m. So, the inductances matrix can be used as they are.

So, it does not matter how the windings are, the you know, what is how things are connected inside for example, when you take a four pole machine, you, there will be two sets of a, b, c windings. So, you may connect them use the parallel or you know that actually in principle it can be any ring, but what comes out the relationship between the current in the a or the flux in the f, when there is a current in the a winding. The inductance matrices which you will use will be the same as before, but remember you have to use wherever theta appears it is the electrical angle. So, that is one important thing which you should remember.

So, none of what we have, what we have done this actually invalid, just remember the theta has to be the electrical angle. What about the you know the of course, we talked about the flux equations of the machine they are derived from the Faraday's law, the rate of change of flux is proportional to the voltage. So, what about the mechanical equations now, it is very important to study the mechanical equation, we will see that the very important phenomena in fact, involve the mechanical equations.

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Torque

$$J \frac{d\omega_m}{dt} = T_m - T_e$$

$$T_e = -\frac{\partial W'}{\partial \theta_m} = -\frac{P}{2} \frac{\partial W'}{\partial \theta} = \frac{P}{2} T_e'$$

$$W' = \frac{1}{2} \begin{bmatrix} i_s^T & i_r^T \end{bmatrix} \begin{bmatrix} L_{ss} & L_{sr} \\ L_{sr} & L_{rr} \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

$$\therefore T_e' = -\frac{1}{2} \left[ i_s^T \frac{\partial L_{ss}}{\partial \theta} \cdot i_s + 2 i_s^T \frac{\partial L_{sr}}{\partial \theta} \cdot i_r \right]$$

So, we have to use of course, newtons law in that case. The basic equation for rotation of a electrical machine is J, which is the moment of inertia into the rate of change of the mechanical speed in radian per second, J into d omega by d T, is equal to T mechanical minus T electrical, what is T mechanical? The torque provided by the prime mover, T e is the electromagnetic torque. So, please remember if this is the direction of motion the

machine this one, the electrical torque, electromagnetic torque so, we are considering generator convention, the electromagnetic torque is in this direction and the mechanical torque is in this direction. So, for a generator which is rotating in this direction, the rate of change of mechanical speed follows this law.

Now, at this point just remember that the  $j$  amp, now somebody may of course, just require a clarification, what is this  $j$ ? So, actually if you look at any synchronous machine you have got the rotor of the machine, and you also have got the rotor of the prime mover which consists of the turbine blades and so on.

So, you have got two blocks, the rotating masses **act the 2** actually two rotating masses connected by a shaft. So, if you assume that the shaft is rigid then, the mechanical speed of the turbine and the generator are the same. So,  $\omega_m$  is the same for these two and this particular equation, in this particular equation  $J$  actually denotes the total moment of inertia of this generator rotor as well as the turbine rotor.

Later on we shall see that, there are phenomena in which you cannot treat the shaft as rigid. So, there will be actually the motion of this and this will have to be described by two equations with the individual moments of inertia

But for the time being will assume that, the turbine and rotor generator rotor are in fact, one **one** rigid mass, and  $J$  it denotes in fact, the inertia of the masses taken together,  $\omega_m$  is of course, the speed of both the masses they assume to be rigid so, they are rotating exactly at the same speed. If the shaft is elastic this assumption will not be true. So, one equation is adequate if the shaft is rigid, you assume that the shaft is rigid.

Now, something from basic electromagnetic, you recall this formula electrical torque is equal to electrical torque in the direction opposite to what  $\theta_m$  is. So,  $\theta_m$  is of course, measured in this direction, in this direction,  $T$  is in this direction. So, according to the you know, the basics of **electromagnetic** electromagnetic energy conversion,  $T_e$  is equal to minus of the partial derivative of  $W$ , this is the  $W$  dash the co-energy with respect to the mechanical angle, and of course, the co-energy has to be expressed in terms of the current.

So, this is the correct law to the electrical torque in this direction is the partial derivative of the co-energy, negative of the partial derivative of the co-energy with respect to the



angle, where the angle is measured in this direction this is consistent with what we have been doing before.

The co-energy of course, is expressed in terms of the currents. If this itself can be written as minus of  $p$  by 2,  $p$  is the number of poles of the rate of change of co-energy with respect to  $\theta$ , where  $\theta$  is an electrical angle. So, which. So, this term I will call is  $T_e$  dash. So,  $T_e$  is equal to  $p$  by 2 into  $T_e$  dash.

Co-energy expressed in terms of current is the linear machine, is half of the current vector into this inductance matrix, into this current vector again. So, this is **this is** the generalization of half  $L I^2$  you know. So, its actually of this, and this of course, is the **func** the  $l_s$  are the function of  $\theta$ ,  $L_{rr}$  of course, is not a function of  $\theta$  we have seen that before.

At this point let me just recap what we have done, we have computed the inductance matrices, we did not compute all the terms, but I indicated how you can actually compute the mutual inductances in self inductances, not only for a cylindrical pole machine whether no saliency or very little saliency, but also for salient pole machine.

Salient pole machine mutual inductance computation is a bit tricky, but by you know simplifying the problem of **of** taking out the component  $m_{mf}$  in the direct axis, in the **(( ))** axis we can actually, quickly come across the come to the correct mutual inductance terms.

Once you get the mutual inductance terms, we have derived the other reviews Faraday's law to relate the rate of change of flux to the voltage we of course, use the current convention that currents are flowing out of the windings.

So, you have computed equations that way. We kind of, you know also up kind of got the relationship between in the electrical circuit in the mechanical circuit, the link of course, being that the electromagnetic torque is dependent on the current. So, the flux equations and the mechanical equations get coupled, because electrical torque is dependent on the fluxes or the currents. And of course, the fluxes are going to be dependent on  $\theta$ .

Now, the way we formulate the equations, I am not just completed it fully, you will find that the equations are functions of theta and theta is continuously changing with **with** time. That makes actually a life a bit painful, because the relation between flux and the current will keep on changing with theta.

So, I will introduce you to a very important concept or a very important time variant transformation called the d q transformation will effectively make the equations a bit easier to understand. So, that is what we will do in the next class and using this transformation will try to some of the you know, the behavior of the **syncro the** synchronous machine.