Power System Dynamics and Control
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Model No. # 01
Lecture No. # 10
Numerical Integration

In the present module in this course, we have been studying about various techniques to understand how a dynamical system behaves. Now, our treatment of these various methods is not very rigorous. But, I trust whatever we are doing will give you some kind of introduction and reassurance in case you have already done it before. In particular in the past two lectures we have been studying about numerical integration. Numerical integration is a very general tool which can be applied to the study of dynamical systems unlike Eigen value and Eigenvector analysis which is applicable only for linear systems. Now, we have been studying the basic features of some of the numerical integration method.

It is very important with the proliferation of a lot of simulation on numerical integration software, it is very important to know the characteristic characteristics of various numerical integration tools which are available. And to do this, we have to kind of bench mark our behavior of dynamical systems and then compare it with what we get when we numerically integrate. Remember, numerical integration always involve some error because it is an approximation of a continuous time system. In the previous lecture, we started on analyzing on how numerical integration methods behave when we are confronted with a stiff system. A stiff system is a system in which the various patterns which you see in the response are widely varying time constants or widely varying rates of change. In a linear system ofcourse, one can correspond these fast and slow transients to large and small Eigen values. Very often in engineering, we do encounter such systems. In fact, if you do encounter such systems, you often are able to do modeling simplifications, a point which we discussed in about three lectures ago.

So today, what we will do is consider the same system for which we did Eigen value analysis and see how it behaves or how what what answer we get when you numerically integrate and try to get the time response. Now, remember ofcourse at this point that we are you know, doing the numerical integration of a linear system. The real linear system response off course is fully known in terms of simple functions like exponents and sinusoids .We can use our powerful Eigen analysis tools to obtain Eigen values and Eigen vectors and write down the system response. There is no need to do integration. But, as I mentioned some time back that we use these two benchmark how our numerical integration, numerical methods work.

Now, this system which we considered in that particular, you know linear and when we did our linear analysis, we considered a particular system which was very typical in the sense that it brought out the stiffness in the system. That system was basically an RLC circuit which was excited by our step in the input voltage.

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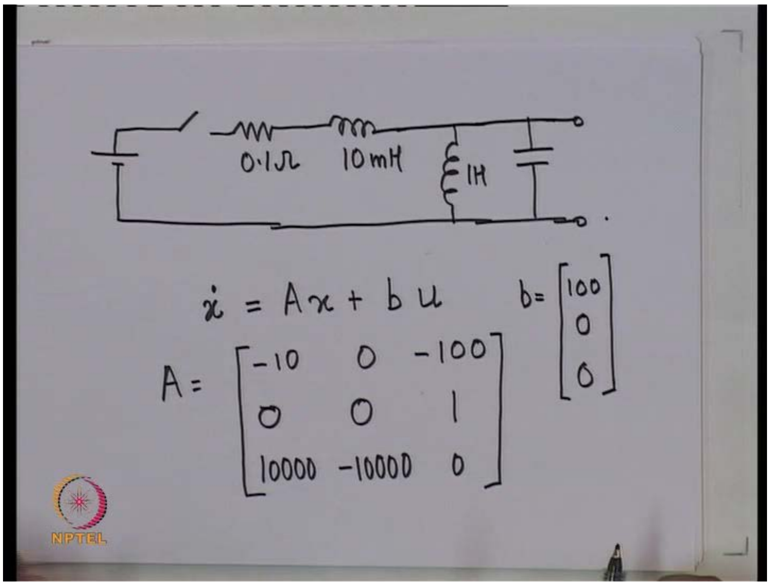

So, the system we were considering was this; a relatively large inductor here and again a capacitor here. The system is a linear system and we can write down the differential equations in this form where a is and b is. So, this is our system and we of course, in our previous lecture we took out a time response of this system.

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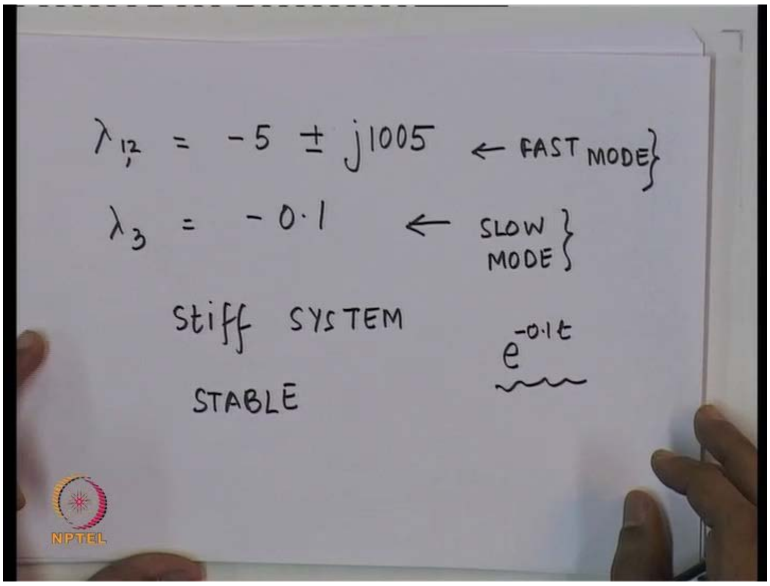

The Eigen values of the matrix when you do linear analysis are given by this and this. So in fact, there is a complex conjugate pair which will correspond to a damped sinusoidal response, a damped oscillation.

And off course, there is an real Eigen value, a negative Eigen value which will correspond to a pattern which will be seen in the response which is e raise to minus 0.1 t. It is very clear that this system is stable because the real part is negative of all the three Eigen values. Another issue which is important is that the rates of change associated with the pattern correspond or mode corresponding to this Eigen value is very slow as compared to this. Look at this frequency. It is extremely high. So, the kind of movement you will get in the response is going to be having a large rate of change for this pattern. So, your response is consisting of two patterns; a fast mode and the slow mode. This is in fact typical of a stiff system. You have got you know, both fast and slow modes.

Now, if one tries to numerically integrate this particular system then, one may use Euler method. For example, if one wants to apply Euler method to this differential equation with a time step of h;

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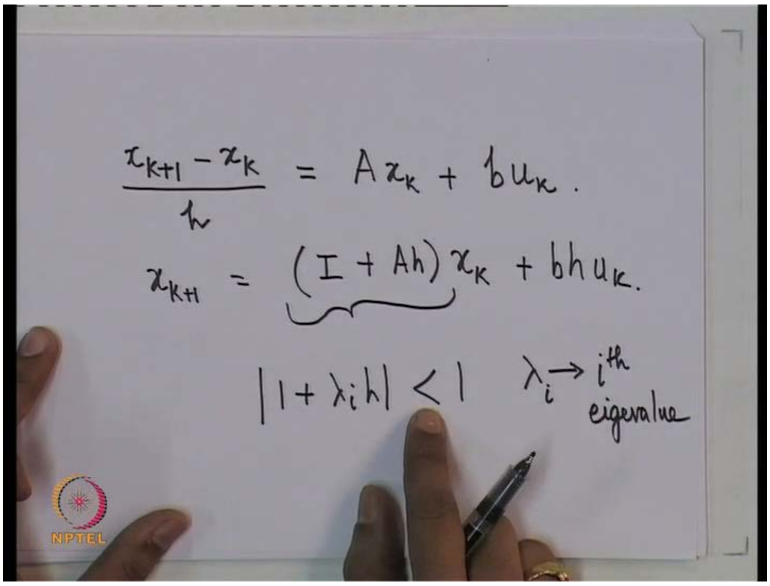

In that case, your iteration which you will, I would not say an iteration, if you know the value of x k you can get the next value of next value of x x k plus 1, the next sample of x by using this relationship. So, A x k plus b u k. Now, this implies you have seen in a previous lecture that I indicated that this system, Euler method when it tries to numerically integrate x dot is equal to A x plus b u; the discretized system which you get may be stable, if is stable, in case where lambda is the ith Eigen value. The important point is this should be true for all Eigen values.

So, Euler method will be stable or rather the response which you get by using this relationship, the samples which you get by using this relationship will be stable if for all Eigen values this is satisfied. Lambda i is goes the Eigen values of a. So, this is the basic property of Euler method. Now, remember that the original system is stable. But, Euler method under certain circumstances may not able to mimic the stability of the original system. It may show an originally stable system to be an unstable one because this relationship may not be satisfied.

And in Euler method, for example, if I apply this to one of the Eigen values that is lambda 1 is equal to this. Suppose, I want to check this relationship for this Eigen value, we will find that the relationship we will get is this, which boils down to this. Even if I choose a time step of point double 0 1, you are not going to have this relationship satisfied.

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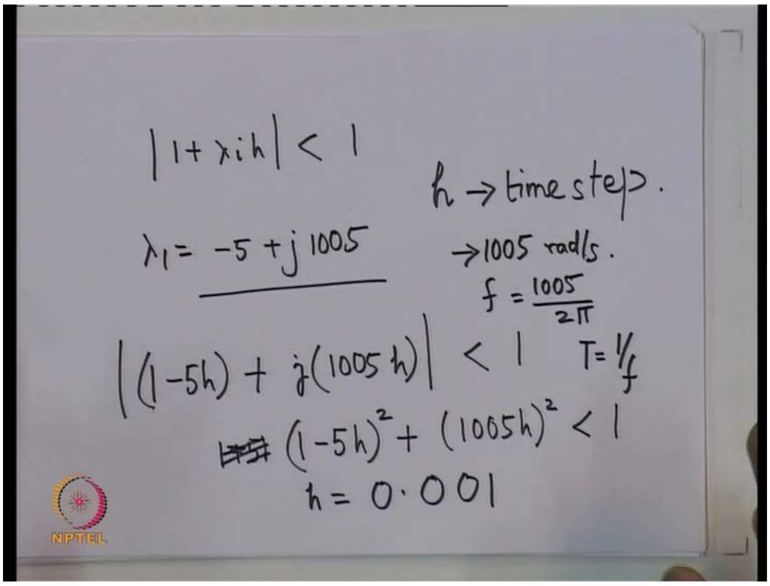

For example, you expect that this frequency is the imaginary part off course corresponds to the frequency of the oscillation in radian per second. We expect that, if I choose a 1 mille second time step it should be able to you know mimic the response. But, unfortunately that is not true because this relationship is not satisfied. Thousand and 5 radians per second corresponds to around thousand five divided by 2 pi as a frequency the time period of this is off course.

So, we expect that if we choose a 1 mille second time step we should be able to mimic the response. But, that is not true if you discretize using Euler method this relationship is not satisfied and you will find that the system which you have numerically integrated or rather the sys, the discrete time system which you get by discretizing the original continuous time system by Euler method will not be in stable. So, you are getting a qualitatively wrong answer if I try to use Euler method with this time step. Ofcourse you may say lets reduce the time step further one can go on reducing this to for example, 10 raise to minus 5 6 or 7. But, remember the time required to do this you know, numerical integration will keep on increasing if I reduce the time step. So, if I want to simulate 1 second of the response if I choose h is equal to 1 millesecond in that, I will I will require thousand steps.

And if I choose 10 raise to minus four seconds, you will require 10 thousand steps, just for a 1 second simulation. So, the problem here ofcourse is that if I am interested in this slow response, if I am primarily interested in how the system behaves the slow response of this system; in that case if I use Euler method I will still be constrained to use a very small time step in order to prevent the faster mode from being unstable. I mean the numerical integration should not display instability. I mean that will be a qualitatively wrong answer and that something I do not want to have.

So, Euler method has a problem in such a stiff system to make your time step very, very very small. So that is one issue which you should remember. Now ofcourse somebody may ask what is this interested in the slow transient, not interested in fast transient, I mean what are the situation where you you would be interested in the slow transient and not interested in fast transient time and so on. For example, let me give you a simple example; you want to study what happens when you start a d c motor or you have got a d c motor running and the load torque on it, the load torque on the d c motor suddenly changes and you are interested in how this speed varies.

Now, the point is when you are trying to see how this speed of the d c motor varies; what kind of transient are you interested in? See, if you look at a d c motor it has got some resistances, it has got a some inductance, it has got a small, it has got inter winding capacitances and so on. So, if you model everything including the mechanical system, the electrical system, you will find that it becomes a stiff system because the electrical time constants or the electrical transients are much faster than the mechanical transient.

So, if you are model all the transients but, you are interest is on the in seeing how the slow speed transient behaves you know. The slow you know pattern in the response. Then you come up with similar situations. So, whenever I say that you have got a stiff system and you are interested in the slow response, you can remember this kind of examples. So, if you got d c motor and your interested in the speed transient and your model all the electrical transient, all the electrical components of the system which are relatively faster then this particular situation does arise.

So, let me retreat, we are thinking of a system which has got both fast and slow transients and we want to replicate the slow transient. So, that would be a particular situation which we may face. Ofcourse you may be interested in the fast and the slow transients. That is the another situation that is another thing you may may encounter. If you are interested in the slow transient of the slow transient behavior of a system but, you are not interested in accurately representing the fast part of the response, you know fast part of the response is there but, you are not very very interested in the accurately you know getting the fast response.

In that case, Euler method is not a good idea because you have to really decide your time step based on the fast response and if you choose any thing which is larger than what is mandated by that particular condition, you will find that your system simulation blows up. So, that is one of the problems which you will face if you try to use Euler method. So, Euler method is taught but, rarely used. so that is the basic interesting thing you will understand by experience.

What we will do now, I will just show you this particular aspect by doing a numerical simulation on psi lab. So, what I will do is, I will just show you a clip of a program. We just, I will just you how you can run it and then I will display the result. So, if you look direct your attention to this particular program which I have used to simulate the system. So, this is a psi lab program. We will ofcourse, I will tell you the main steps in the program. You have given the a matrix of the system k, will tell you the Eigen values if you are interested in them let us not bring them out.

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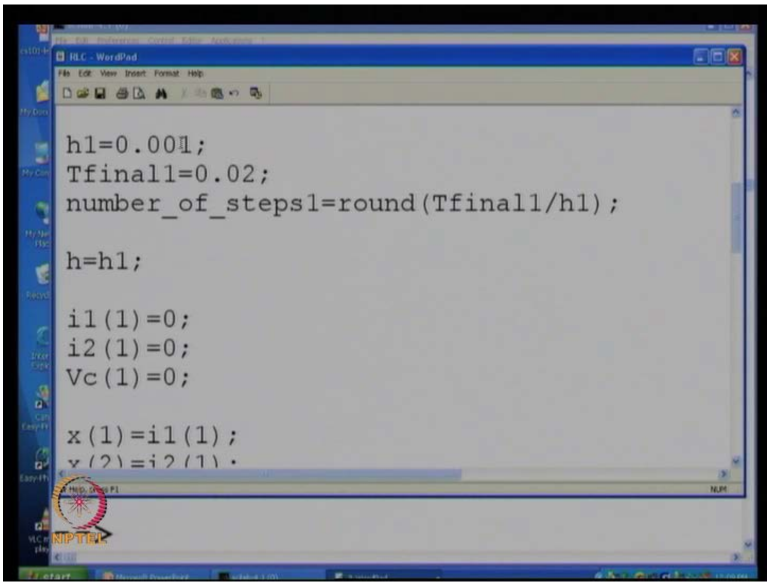

This is a time step say you want to simulate the system for 2 just for 20 mille seconds using a time step of 0.0 0 1. Just for 2 mille seconds and using Euler method.

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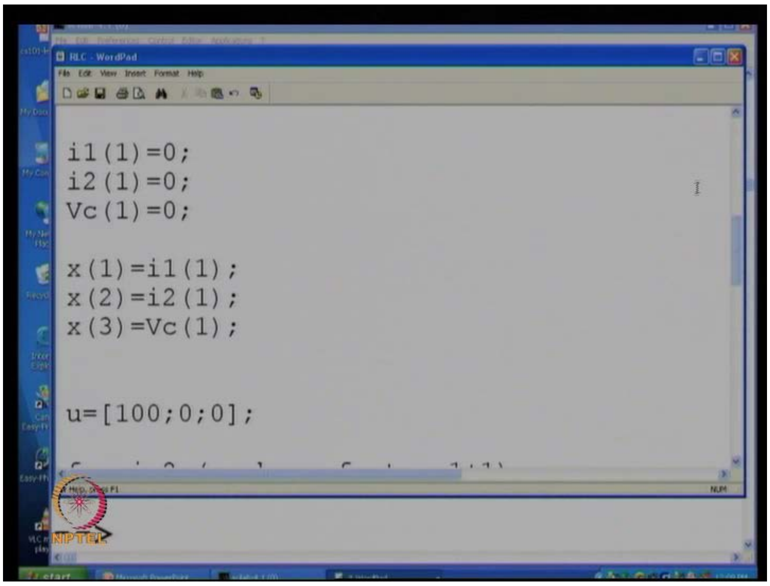

So, let us assume that the initial conditions on the states are 0 but, the system is excited by a step in the input. So, we are getting some kind of forced, we are having a forcing function.

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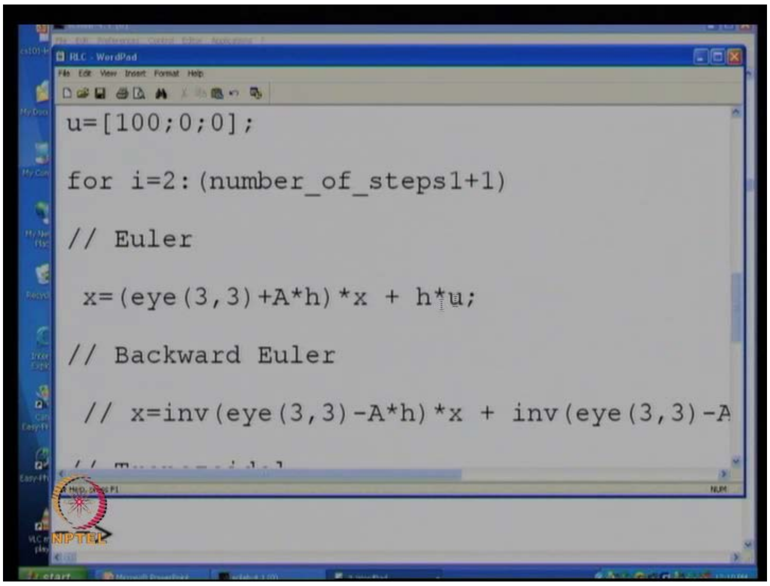

Suppose, I numerically integrate using Euler method. I delete this. So, what you have here is, the Euler the method. X is equal to 1 plus A h into X plus h into u that b is kind of absorbed in this vector. So, I have not written it separately.

So, I will comment the discretization by trapezoidal rule. So, what we have here is, this program and ofcourse I will plot the values once I have simulated them. So, if I run this program and save it and I run this program.

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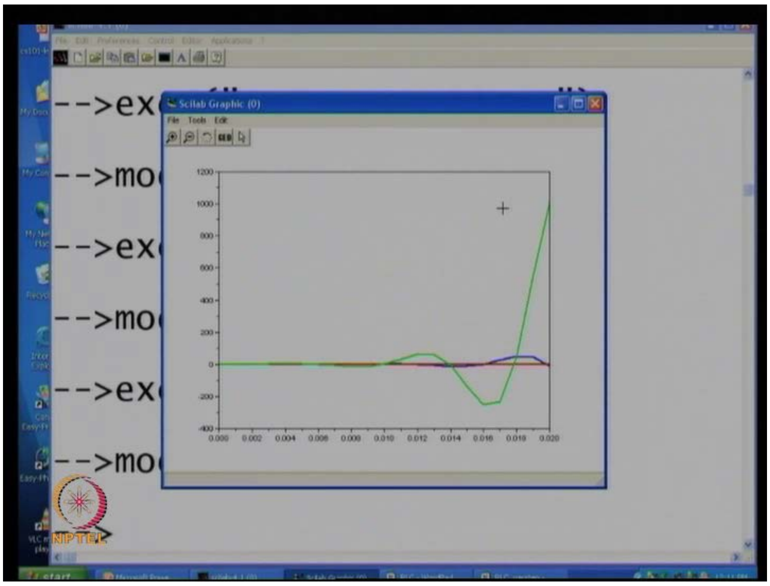

This is the response I get. The green and the blue denote the inductor currents; the small inductor and the large inductor. The small inductor is blue, the large inductor is green and red is the capacitor voltage. It may not be very clear in your screen but, the value here is 1200, 1000 200 and this time off course is 0 to 0.0 2 seconds. So, you see this response is kind of blowing up, you get a blowing up response.

So, this is one of the problems which you will find in Euler method that when you have got a fast transient you have to, the time step you may choose really as really may not satisfy the stability condition and you will get a spurious response. Before we go ahead and you know try out the other methods, let us quickly look at what is the actual behavior of this system.

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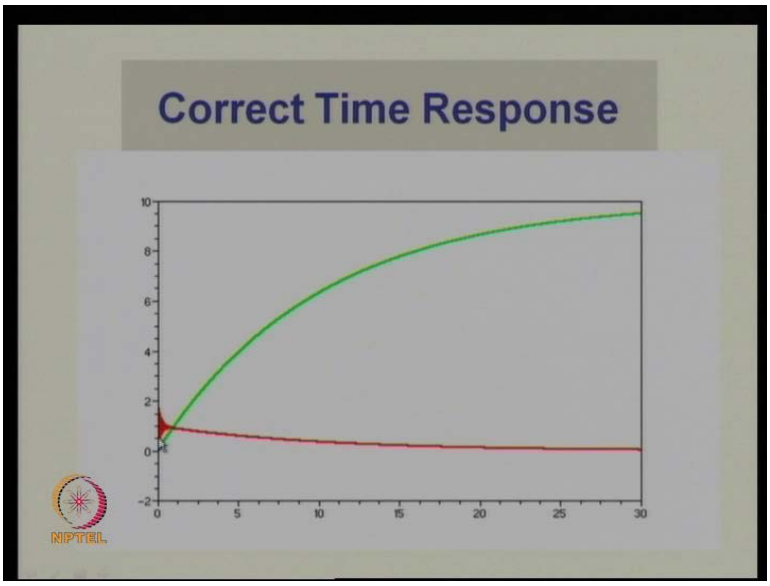

The correct time response is this, how did I get this correct time response? The correct time response if you recall, is not obtained by numerical integration. But, by simply evaluating the response at various time steps from the analytical functions that we derived using Eigen values and Eigen vectors.

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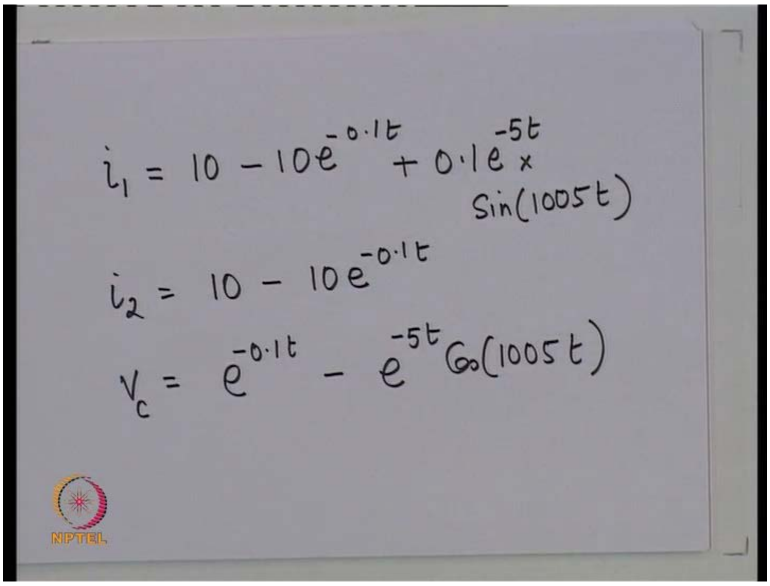

If you recall the response of I one, using Eigen value analysis is 10 minus 10 e raised to into sine of thousand and 5 t, i 2 is this is approximate. Not exact. But, this was analytically derived using Eigen values and Eigen vectors. So, this is the response for this RLC circuit I 1 I 2 and V c. So, this is time response but, this is derived analytically.

So, this what we are showing on the screen is the same response which is evaluated by simply plugging in t into the these functions which I have just written down and so this is the correct time response.

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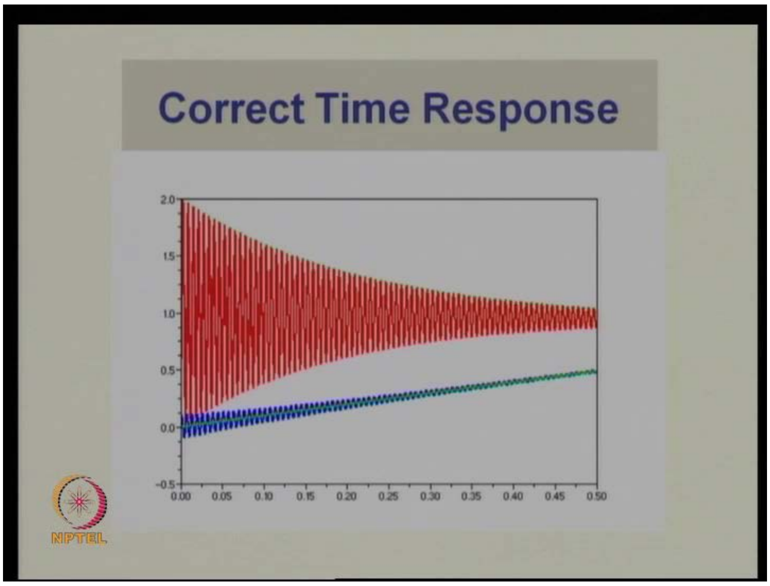

So, this is what I should be getting and at a if I try to simulate this just from 0 to point 5 seconds, this is what I should be getting this V c and this is i 1 and i 2 green and blue.

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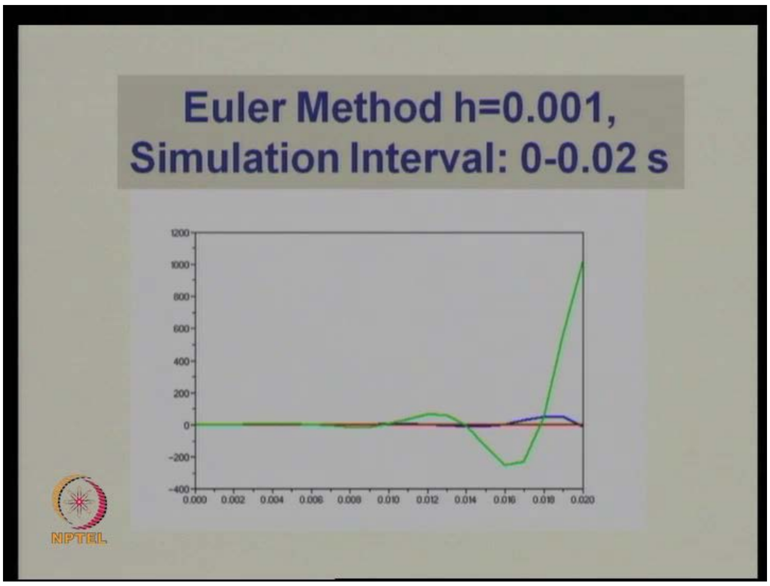

So this is the correct time response and just to replot what I got just now using Euler method with 0.0 0 1 and the simulation just 20 mille seconds you see, the beginnings of the fact that this discrete time system obtained by using Euler method, it is just blowing up. So, we are no where going to be nowhere close to either this response or that response.

So, Euler method is giving a horribly wrong solution. In fact, it is giving a qualitatively wrong answer. Also, it is not only inaccurate, it is giving a qualitatively incorrect you know, conclusion or inference about stability. If I use backward Euler method remember, the backward Euler discretization is done for x dot is equal to A x plus b u by the following.

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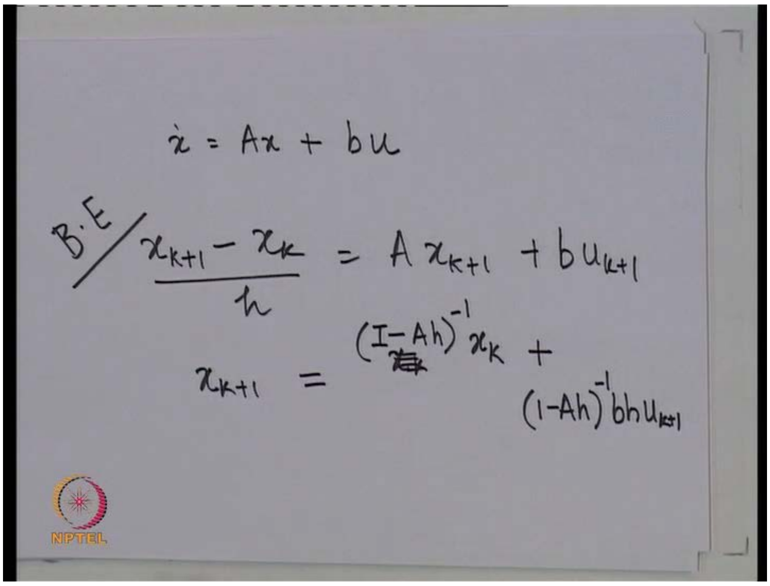

u k off course is constant, u is constant so you have less trouble about this. But, what you get eventually by just solving this, I am skipping a few steps is, 1 minus A h inverse x k plus 1 minus A h inverse b h u k.

So, this is basically the iteration which I will have to use in case I want to use backward Euler method. So, this is backward Euler method. Now, if I use backward Euler method, the important thing ofcourse here is that, you have to take out the inverse of a matrix for linear. So, this involves extra computations. It is not as straight forward as the explicit method. That is the forward Euler method which has an explicit method.

Now, normally taking out an inverse is not a big problem. I mean it does not, is not a fraught with problems as far as computation is concerned provided if the system is small. If your system is very large, then computation of inverse can be quite intensive. For example, its very common in a power system to have a size of the order of the system may be with thousand you may be thousand of states. So, in that case to try to compute this, may be a bit comp may we be fraught with a lot of hurdles because inversion is a computationally intensive you know operation and at every time step you have to do this you know, this particular function that is, x k plus 1 getting x k plus from from from x k.

Now, since 1 minus A h inverse is appearing at every step you do not you can compute it once in your program and then simply do a matrix multiplication when you are running this algorithm to implement backward Euler method. So, you can actually take out the inverse and keep it before hand and only perform matrix multiplications. But, again if you are working with very large systems, it is not a good idea to compute the inverse explicitly. The reason being, that if a is powers, the inverse of i minus A h i is incidentally an identity matrix, something which I did not mention earlier. This particular matrix is the inverse, may not be power. So, may you even have to store a very large number of values if you are going to explicitly compute the inverse and keep it stored before hand. What would be a pragmatic thing to do? It would be to compute the l u factors, the lower and upper factors of o l u factors of 1 i minus A h and just do backward and forward substitution during each iteration.

So the l u factor ofcourse you may have to do ordering of the states etc. So that, you get the l u factors as powers if a is powers. But, remember storing the inverse of even a powers matrix you know is a problem because the inverse may not be powers. So, these are the some of the issues which you may face may if your developing a program for large systems. But, ofcourse right now we are talking about third order system. You can just as well take out the inverse. You can even keep on taking out the inverse at every time step though it is not necessary. So this is basically how you will implement backward Euler method. Trapezoidal rule is again similar.

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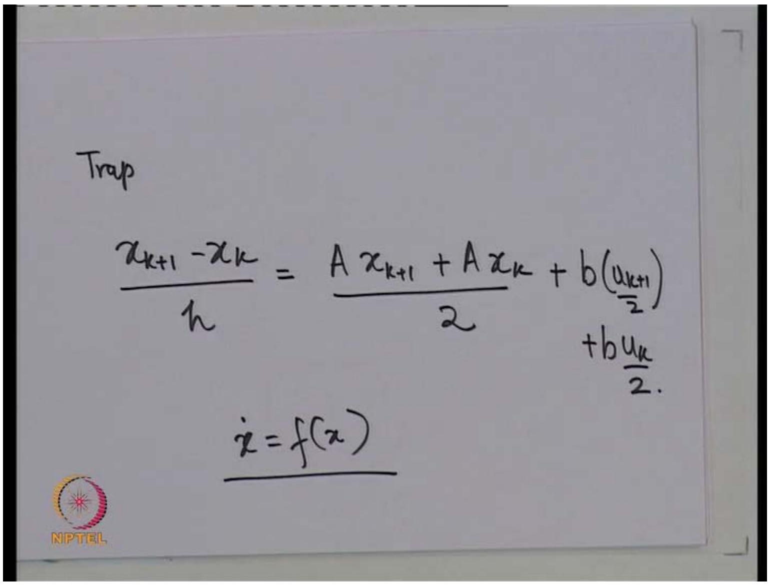

You will get x k plus 1 minus x k by h is equal to A x k plus 1 plus A x k upon 2 plus b u k plus 1 by 2 plus b u k by 2. So, this is how you will discretize it. Again, I you will need to take out an inverse. So, the of a matrix so that is one of the critical features of trapezoidal rule as well as backward Euler or any implicit method.

Remember, since this is a linear system our job in implicit method requires inversion of a matrix. If your system is non-linear, you may require even to do iterations to get x k plus 1 from x k using some method like n r that is Newton Raphson or Gossie Gauss Seidal method.

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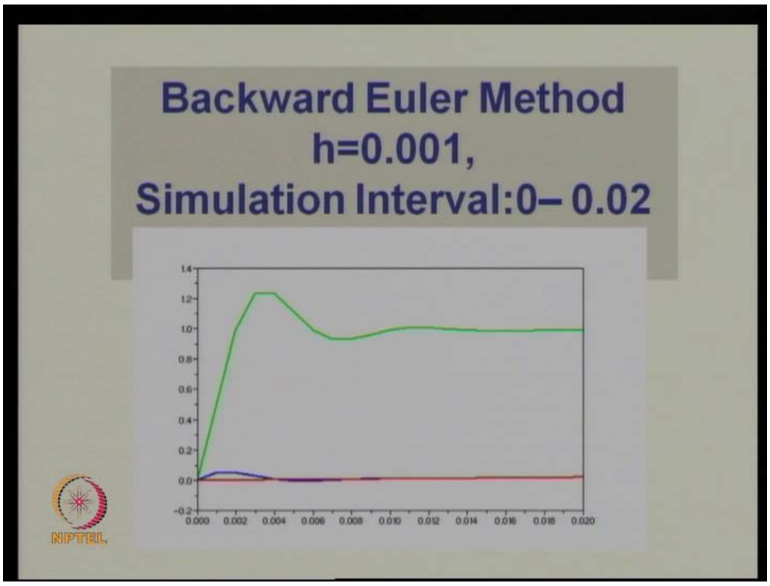

Now, coming back to the qualitative results if I used backward Euler method with h is equal to 1 mille second and a simulation interval of 0.0 2; this is the response I get. One thing you can notice here is that, backward Euler seems to have killed the oscillation very quickly. So, the fast transient which I expect to be seen in this 20 mille second window, is in fact seems to be very well damped, damped out. Better damped than what in fact it actually is. So, you if you like at the original response, it takes at least a second or so to damp out more than a second damp.

Whereas here, backward Euler with a 1 mille second time set, the oscillation dies down very soon. So, if you look at if you just recall how this the correct time response should be like this, in about 0.5 seconds. Your oscillation is gradually dying down. So, this oscillatory part of a transient takes at least a second or two to completely die out. Whereas backward Euler has simply killed at oscillation and you do not even see an oscillatory response.

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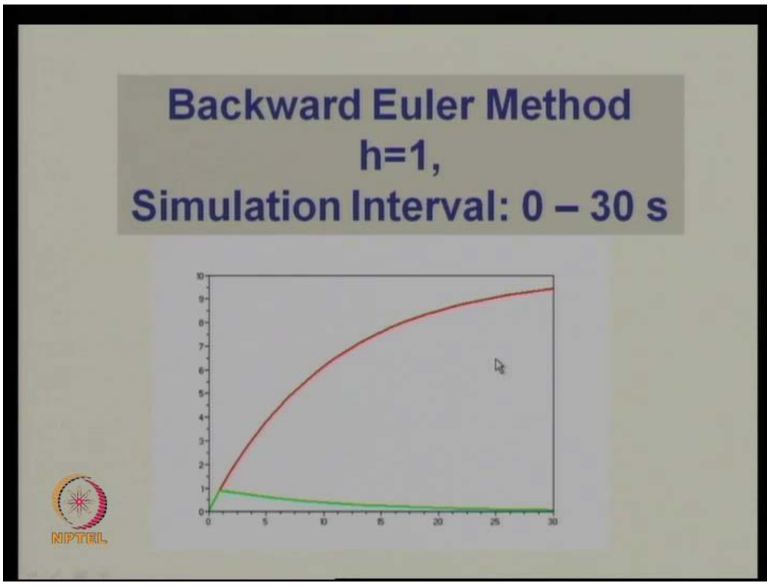

On the other hand, if I choose a large value of h, in fact this h is more compatible with the slow mode, that is e raise to minus 0.1 t you know, the time constant corresponding to the decay time constant corresponding to e raise to minus 0.1 t is 10 seconds. So, it makes sense to choose h is equal to 1 second only if you are interested in this slow response.

The interesting part which you see here ofcourse is, all though the initial part of the transient is not captured very well, the system is able to capture the slow transient nonetheless. This would not have been possible with forward Euler method because you would find that the fast transient is getting destabilized. That is what we saw in one of the previous simulations. yeah This one. So, we could not have ofcourse use the same strategy with forward Euler method even though our main response of interest was the slow transient.

So, backward Euler method in fact is a good method to use if you are not too worried about how the fast transient involves. Ofcourse, we would be worried if the fast transient were actually unstable. But, if it is known that is stable, if you you from your engineering judgment you are sure that the fast transients are not unstable. In that case if you are interested in the slow transient it is good idea to use a method like backward Euler method with a large time steps which is compatible with this slow transient.

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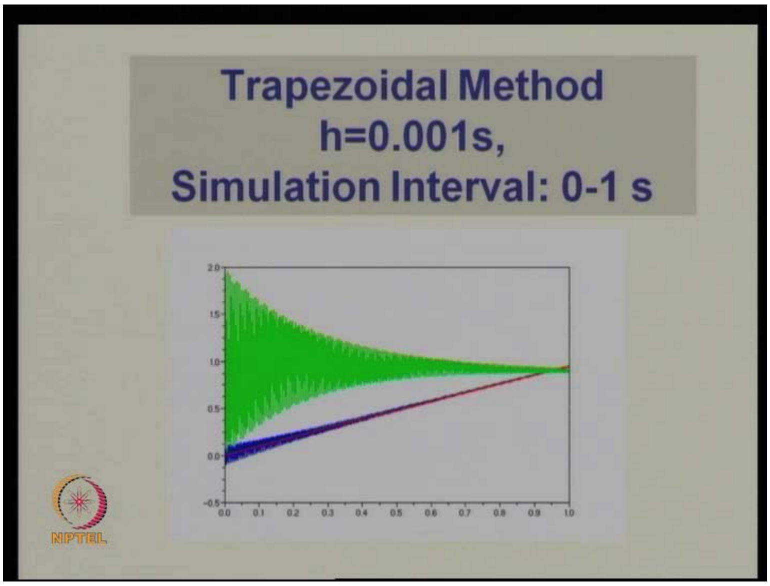

Trapezoid rule with 1 millisecond and has stimulations interval of 0.1 second seems to capture the fast transient quite well. This is unlike backward Euler which introduces some damping, extraneous damping into the original system.

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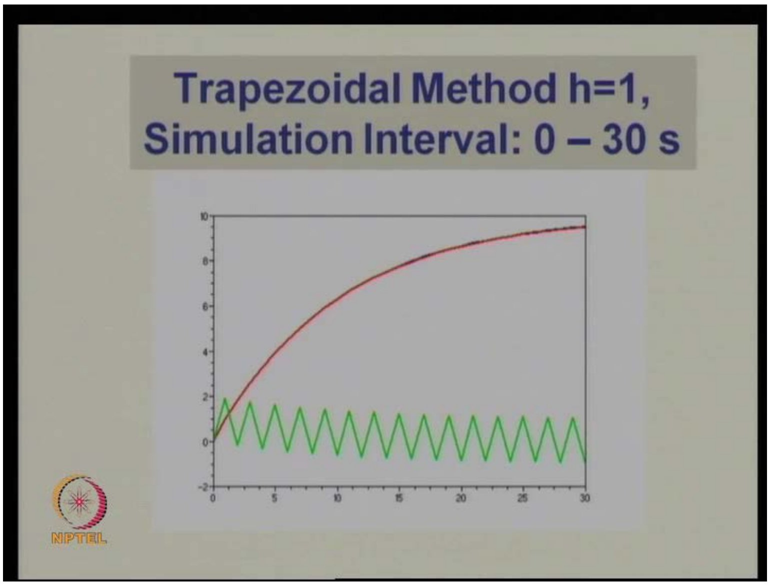

However, if I use trapezoidal method with h is equal to 1; all though the response is not destabilized you are getting a highly inaccurate response as far as V c is concerned. So, trapezoid rule is not very very good you cannot use it with very large time steps or time steps corresponding to the slow response if your system is very stiff like the one which we are encountering here. So what is the solution to these issues?

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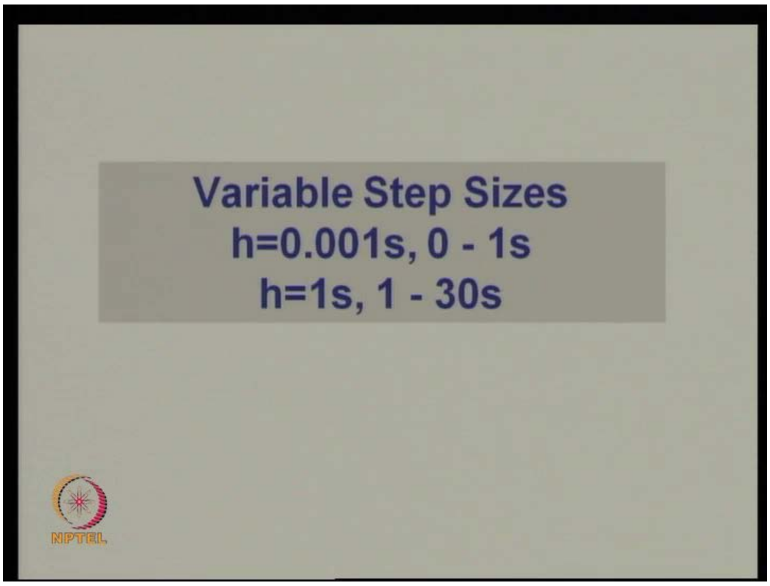

The solution to this problem is, use variable step sizes. For example, you can use 1 millisecond for 1 second and 1 second h is 1 second for 30 seconds. So, variable time step sizes can be used in order to obtain a response. So, you easily program it in psi lab. I will just show you the program. So, if you look at if you look at the program for variable time steps.

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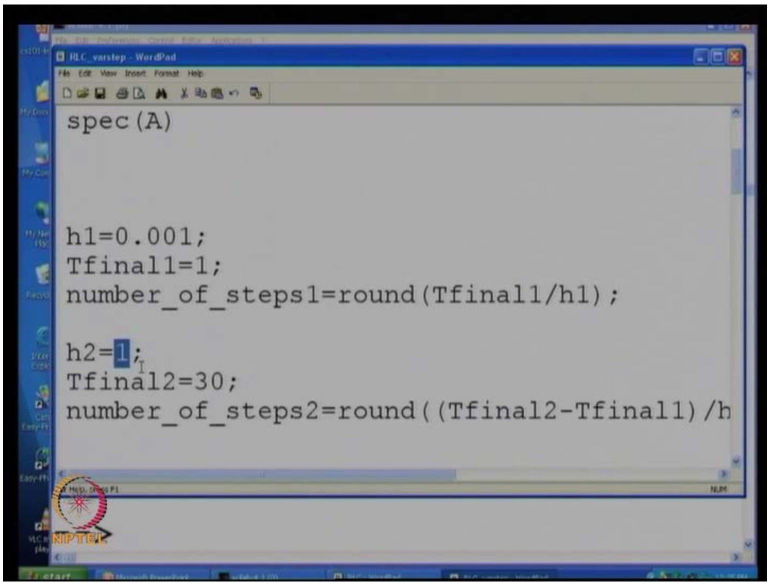

You will have to, when you program define two time steps, h 1 which is 1 millisecond. Let us say we stimulate only for 1 second then, after some time you can shift over to a last time step and stimulate for a longer time. And ofcourse, you will have to program it appropriately so, that you can get the appropriate response.

Now, so ofcourse one important point in this, whatever stimulation we are trying to do is that, our choice of h is somewhat adhoc. And I mean, the choice of h 1 and h 2 is adhoc and also it is adhoc that we switch over from the smaller time step to the larger time step at 1 second. Why 1 second it could have been 10 second and so on? The point is in this particular case, I do know the response. So, since I know the response I can actually tell you at what point to switch. But, implementing variable time step methods for systems in general may be a bit tricky. I mean if you know nothing about the system then, how do you decide that you should switch over at 1 second from the fast, slow, the small value to the larger value or even more importantly, how do you choose these values h 1 and h 2. And, I here there is an ad hoc switch from 0.0 1 point or rather 1 millisecond to 1 second.

The key to this of course is that, often when we are stimulating a system, we know something about the system. So, that is one way you can actually use your engineering judgment and come to a particular conclusion about what time steps to use or you can use a bit of try line error. But, most industry grade programs will actually have some way of finding out you know, the truncation error, estimating the truncation errors at every step. And, if the errors at every step are not too large then they may even permit adaptively to start increasing the time steps. So, in a you know if you look it at commercial software or software's like matlab other software they do implement variable time step methods and they would have one way of checking out or estimating the truncation errors and adaptively changing the time step. Here of course, I do it one short, I just change from 1 millisecond to 1 second.

Now, if I use the variable time step method, the question is can I use variable time step method with Euler method? So, the question is can I use Euler method with variable step sizes in a step system? The answer is no. The point is that, the moment I switch over from a small time step to a large time step the remanence remainants of the fast mode may not be completely zero. In fact they are never zero because of numerical precision, can never be infinite. So, you will find that there are remanence remainants of the fast response which are there in your, there is if the fast response wouldn't have completely died down when you rigorously speaking it never completely raise dies down.

So, if I use a Euler method, the moment I switch over to lager time step, the faster transient even though we have waited for the fast transient to die down after some time, you will find a whatever remanence remainants of the fast transient are there, they will again start becoming unstable. And you will find that the whole system blows up. So, that is the major issue which you will face when you use methods like Euler method with variable time steps. So, Euler method is actually is not suitable or variable time steps for the stiff system. So, Euler method is you keep it in the background.

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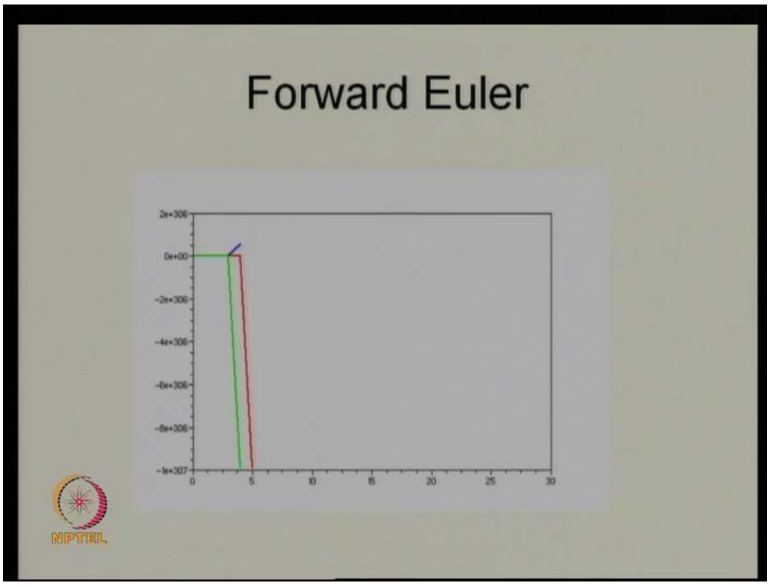

So for example, I actually try to program and do this stimulation for Euler method; you see this, I do not know whether it is clear on your screen but, it is 2 into 10 raise to 3 hundred and 6. So, you know you are by trying to implement this variable step method for just a few seconds has resulted in a complete blow up of the solution.

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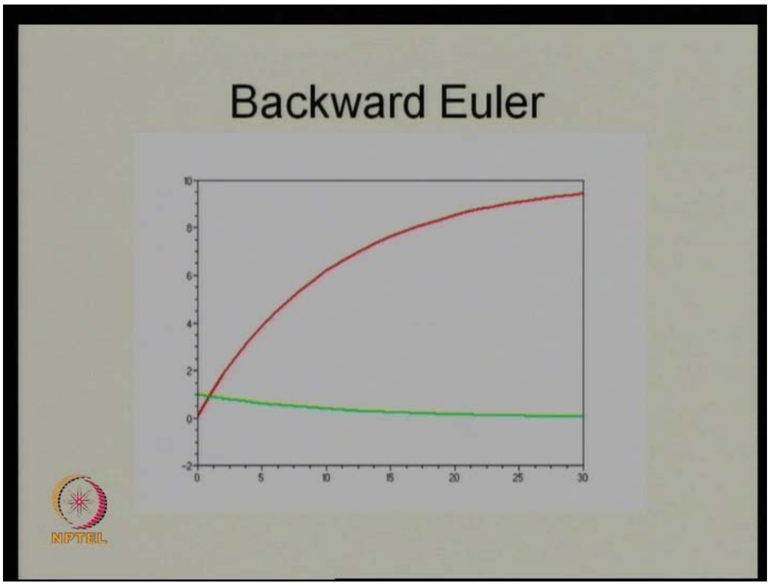

So, forward Euler cannot be used in conjunction with a variable time step method. On the other hand, backward Euler method if I use variable time step method, it gives a reasonably good response for the first few, first 1 second or so it gives a step. Really it does not capture the fast transient quite well any way you know. It does not capture the fast transient any way. The slow transient it is captured pretty well.

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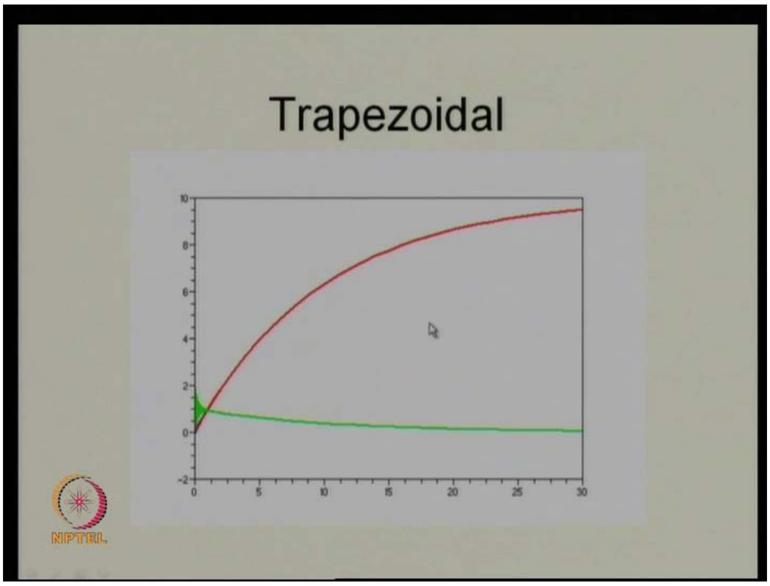

Trapezoid rule with this kind of thing works well because when we make the time step very small for the first few seconds, it captures the fast transient correctly and also the slow transient. So, that is the basic deal you can say whenever you are using a variable time step method. So, these are these are the things you have to keep in mind.

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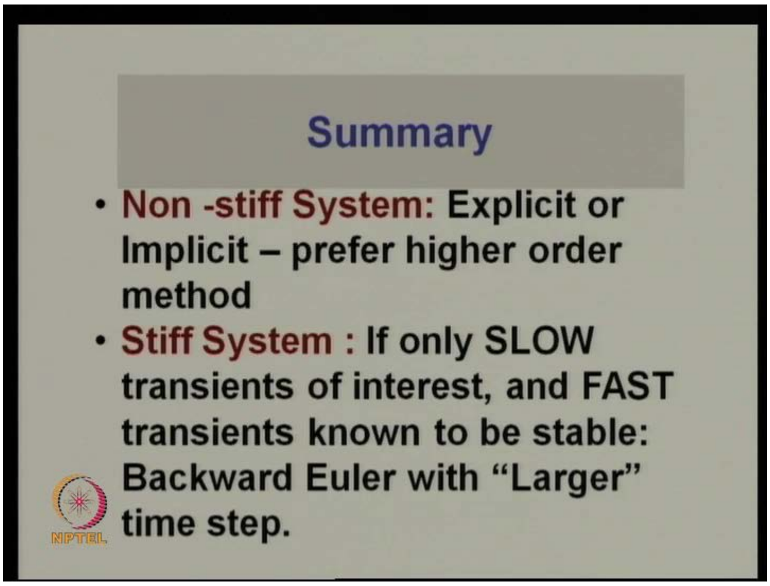

So, to summarize this part of the lecture; for non stiff system where stiffness is not there there is less less of a worry that the system will become unstable if you choose your time step appropriately. Appropriately in the sense, if you know something about the system you can choose your time step roughly to correspond to the fastest time constants in your system. So, if it is a non stiff system and you know that well this exponential rate of change or the sinusoid is roughly going to be in this range, then you can actually choose a time step. Use any method. In fact of course prefer high order methods. In case you are using explicit method, so of course avoid using Euler method it is not a very good method to use because it is inaccurate as well.

So, what one can try to do is, if you are coming with you a non stiff, if you are encountering a non stiff system and you have some rough idea about time constant associated with this system or the frequencies of oscillatory response if any, then you can choose the time step corresponding to the fastest such transient. And you can use a higher order explicit method. Why explicit method? Because explicit methods are easier to implement they do not require inversions and or in a non-linear system. They do not require iterations within a time step and so on so.

So, however if one is faced with a stiff system; so you can direct your attention to the screen, if only slow transient is of interest, in that case you you must look at implicit methods. So, if slow transients are of interest you can try to use backward Euler method. Methods like backward Euler method with larger time step. So, if you see the this particular slide what I have written if this the slow transients are of interest and the fast transients are known to be stable either from engineering judgment or some prior study with somebody else is done; if you know something about the system and you know that the fast transient are indeed stable and the slow transients are what you are really interested in, you can use backward Euler with larger time steps. Larger I mean compatible with the time constants associated with the slow transients.

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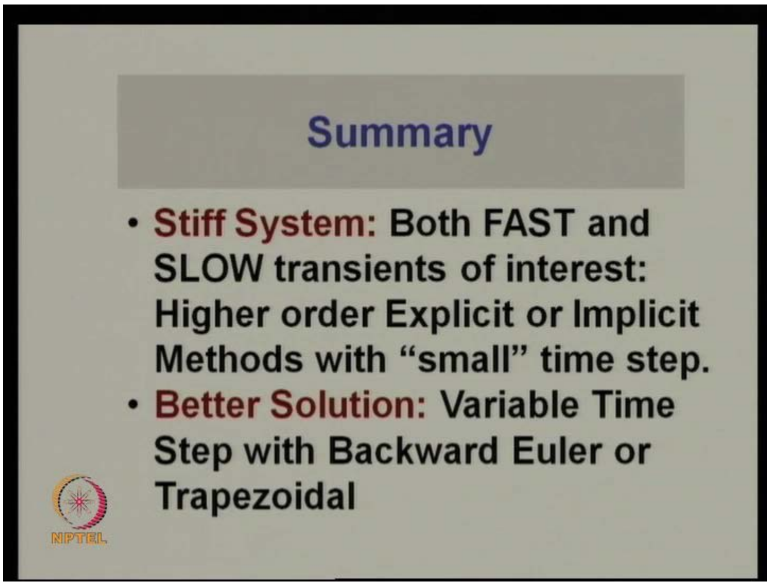

If you have got a stiff system and both fast and slow transients are of interest you can use higher order explicit methods or implicit methods with the time small time step. So, if both fast and slow transients are of interest, you can use higher order implicit or explicit method with a small time. Small ofcourse corresponds to the fast transients. The time constants of the frequencies associated with the fast transients. Even here, even higher order explicit methods may be a problem. So, I actually, if you are encountering a stiff system, I think it will, it is a very safe to use implicit methods with small time steps if both fast and slow transients are of interest.

It is a bit risky to use explicit method because they do not, they are do not have very good stability properties. You will have to use extremely small time steps otherwise you may end of these destabilizing some response. Especially true, this is especially true with Euler like methods. Ofcourse if this is what, you want a fast and slow transients are of interest in a stiff system; the best solution or a better solution would we to use a variable time step with in conjunction with backward Euler or trapezoid. That is initially keep the time steps small so that, you capture your fast transient well and then increase your time step. And you know, you can capture your slow transient even with the larger time step. the off course The important thing implicit in all what I am trying to say is that we are trying to you know complete our numerical integration as fast as we can.

Somebody may ask well, you have seen this RLC circuit you know, simple RLC circuit; what you know to integrate numerically integrate this for say thirty second even with a time step of say, you know hundred microseconds or fifty microseconds? Should not be a problem on today's computers. But, this is not true when you consider larger order systems. Now, when you have got very large order systems and if you are forced to use a very small time step like you know, fifty microseconds or hundred microseconds and you want to stimulate for say, hundreds of seconds. This really may be a big bottle neck and you may take hours sometimes to stimulate this system. This actually happens. So, if those of you who are doing power electronic systems or you know large scale power system stimulations would have encountered this problem if they use then inappropriate method.

As I mentioned sometimes back proliferation of so many you know software tools for numerical integration of circuits power system and other systems control systems and so on, it is very important to know this basic know the basic properties of these integration methods.

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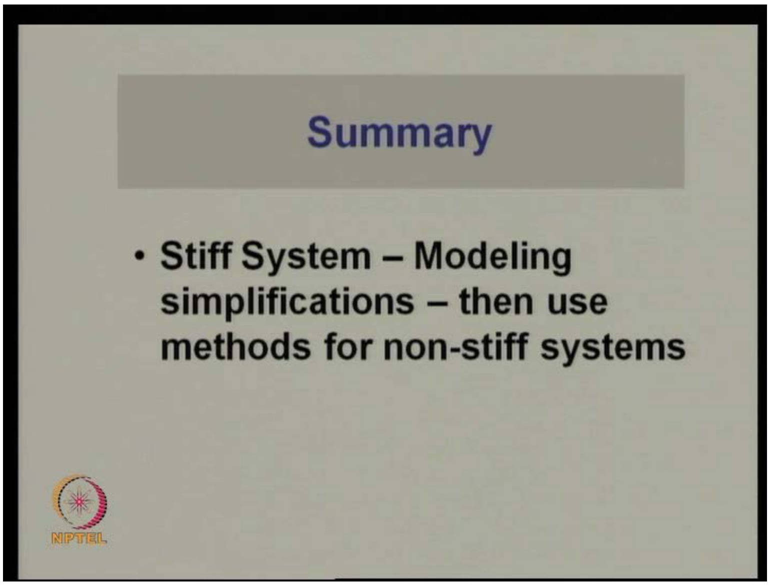

So, although our, you know treatment here has been very, very brief. I mean the aim of introducing you to analysis methods right in the beginning of the course is to give you a feel of these kind of methods which we will now apply when when we do the modeling of power system components.

So, when once we finish our modeling of power system components, we will directly use these tools like numerical analysis or Eigen value analysis. Later on, in case you have forgotten what we have covered you come back to these lectures and just revise. Now, one small point which I did not mention as far as numerical integration is concerned; is that if you are faced with a stiff system, if you are having a stiff system, rather than do all the jugglery of using either very small time step sizes or variable time steps and these vary some kind of things, one thing you can do right away when you are considering a system when you are modeling a system is to get rid of the fast transient. Get rid in the sense, make modeling simplification so that your system is of lower order and it kind of, you know, kind of does not have the fast transient at all. And this is something we have discussed before. If you are encountering a stiff system, you can neglect the fast transients. What you what you get? Because of that, is that you are going to get a lower order system, the differential equation is corresponding to the states associated with the fast transients you know, are converted to algebraic equations. So, you know what you are really doing is that, the states for example, the inductor current or the capacitor voltage in this particular circuit, the differential equation corresponding to the states are converted into algebraic equations.

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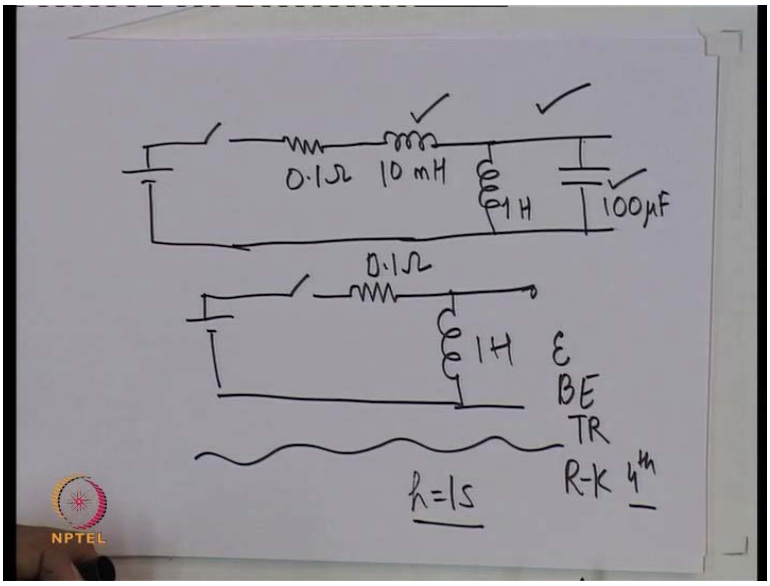

For example, we have done this before. What this as a revision in this particular circuit through participation matrix, the participation matrix or through by engineering judgment, you know that the states associated with the fast transients are this and this. That is 10 mille Henry and 100 micro ferret and state associated with the slow transient is this. This is something we have done before. So, why not use a modeling simplification? You know for example, here you have got you can use the modeling simplification that this capacitor is actually open circuit d V c by d t is equal to 0 and d i l 1 by d t is equal to 0. In that case, you are going to get effectively a circuit of this kind of lower order is just a single dynamical element or a single state.

This will be an acceptable approximation provided you are interested only in the slow transient. So this something we have done before. The point is that, if I have got this system to begin with a if I want to numerically integrate it; I will have to worry about you know, what method I am going to use? What is the time step I am going to use? The possibility of using variable time steps to speed up your numerical integration and so on.

But, if you look at this system, this is a non stiff system. Actually this particular system we I can use Euler, forward Euler, backward Euler trapezoidal or say Runge Kutta say for fourth order method which I have discussed. I have just mentioned sometime in the lecture previous to the previous one. You can use all of these which say a time step of 1 second without you will get a reasonably accurate solution because this is not a non this is not a stiff system at all. So, often what we do is, neglect that d I by d t corresponding to this and the d v by d t that is, the current through this and get a non stiff system.

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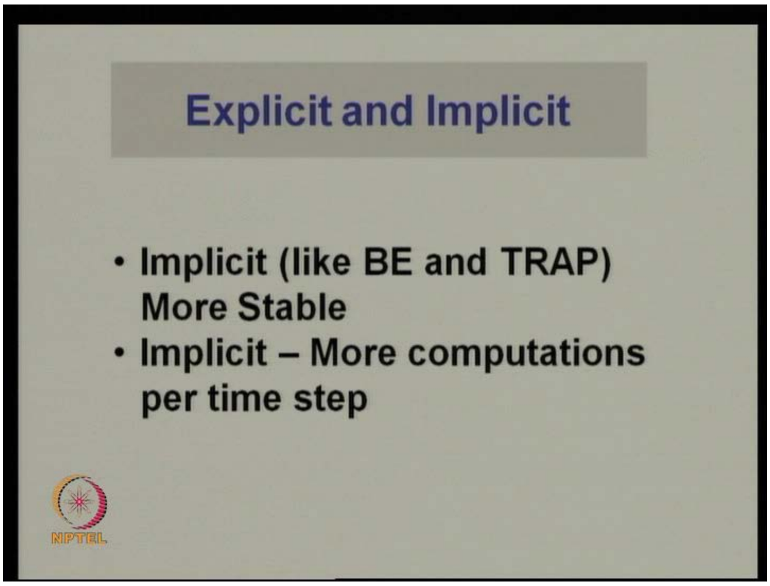

Again, just in case you are worried about why am I imposing the point, remember that if I am going to use implicit, explicit methods; it is important that the system should not be stiff otherwise you have to, you will be constrained to use a very, very very small time step and it will take a very long time to complete the stimulation. Moreover some of, some explicit methods like Euler method are not even very accurate.

So, the important thing is, if given a choice a programmer will use explicit methods because it involves less of programming and less computations per time step. But implicit are more stable. They do not give, they do not show unstable system to be a rather a stable system to be an unstable one. And ofcourse, I have mentioned that unfortunately implicit methods require more computations per time step.

So, it does make sense sometimes to use explicit methods. But, you should basically make modeling simplification so that, the fast or non stiff or the stiff components of the system or the fast component of the system are effectively removed. So, that is the basic you know, thing which modeling you know simplification which one should use if possible, wherever possible.

So that, you it permits you to use methods, some sometimes it permits you to use explicit methods. But, if you have no idea about the system, you know you cannot make modeling simplification. So you know, if you start off with this system which you have no knowledge; if you give, for example, a synchronous machine to a mechanical engineer or a civil engineer; he may not know all the you know, he may not have that engineering judgment of the various transients involved or what transients to expect.

So, in that case he may find it very difficult to make modeling simplifications. So in that case, then there’s always an issue about which method to use and so on. But, an interesting feature about modeling and you know, what will we doing next is that, often we would be kind of making assumptions about the system from a general knowledge, a general engineering sense about this system. For example, when you are modeling or synchronous machine and main aim of the modeling is to study, for example, loss of synchronism or electro mechanical transients associated with the system; we will not be modeling the currents, for example, through the interwinding compared capacitance of the stator windings.

So, because we have a kind of engineering feel that the stray components like the interwinding distributed capacitance is and so on, may not the transients associated with them are fast very, very very fast to analyze unless you are really doing analysis of ultra fast transients in a synchronous machine. You may not need to model them at all. So, to some extent there is this engineering judgment. Now, before we end this particular lecture, we just have a few 10 to fifteen more minutes to go.

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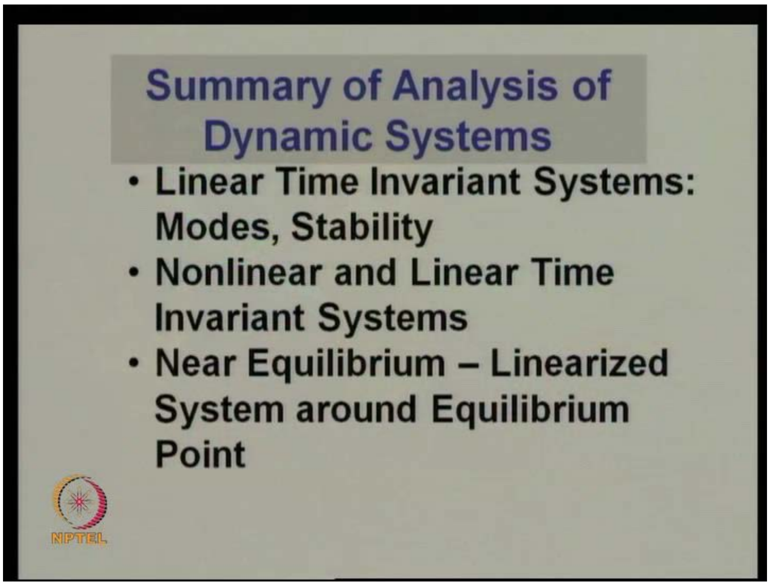

Let us quickly summarize the first part of our, you know of our lecture. You know the first part of this course was in fact analysis of dynamical system. It was a more general analysis. We considered linear time invariant systems and we could really characterize the response in terms of modes. We could even understand the stability of such systems simply by looking at the properties of the A matrix.

In particular, looking at modes could be characterized by the Eigen values and the Eigen vectors associated with the A matrix of this system. Non-linear and linear time variant system, this should read as variant. Non-linear and linear time the variant systems are difficult to analyze. Unfortunately, the only tool which is left with us when we are trying to analyze non-linear systems in general is numerical integration. There are ofcourse some specialized techniques which approximate the behavior of non-linear systems. But, most of the times we will be using in fact numerical integration to analyze non-linear system.

And an exception to that of course is when you are having a non-linear system. And we are analyzing its behavior for small disturbances around an equilibrium point, we can create or rather derive a linearized model from the non-linear system for the analysis of small disturbances from the equilibrium. Of course, once we will linearize the system, we can use the tools of Eigen value and Eigen vector analysis.

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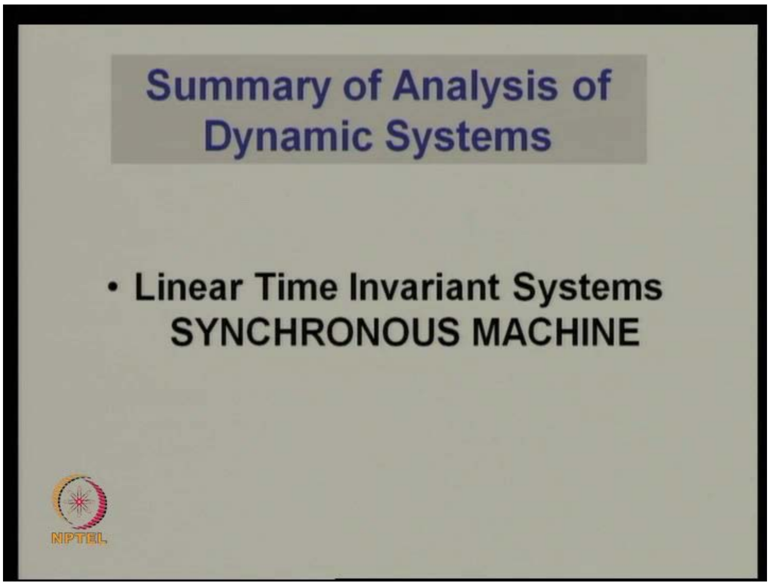

One of the key systems which we have not really considered right now is the linear time variant system. And please note that, there is small error here in this slide it should read as linear time variant systems. An example of a linear time variant system as we shall see in the coming lectures is the synchronous machine itself. The flux the flux as seen by the stator bindings of a synchronous machines that is, the rotor winding flux, the flux is by the three phase windings of a synchronous machine are in fact time variant. The differential equations which come out when analyzing a synchronous machine are in fact linear time variant and we shall be using a very powerful method or we shall be using a kind of a transformation of variables in order to derive a time invariant system from the time variant system.

So this is something of course we are yet to come to. This is just a kind of a curtain raiser to what is to come. So, the modeling of a synchronous machine we shall start off soon and we will be entering in some sense into the domain of power systems, slightly away from the kind of general analysis which we have done so far.