Nonlinear Dynamical Systems
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Lecture - 30
Describing Functions: Jump Hysteresis

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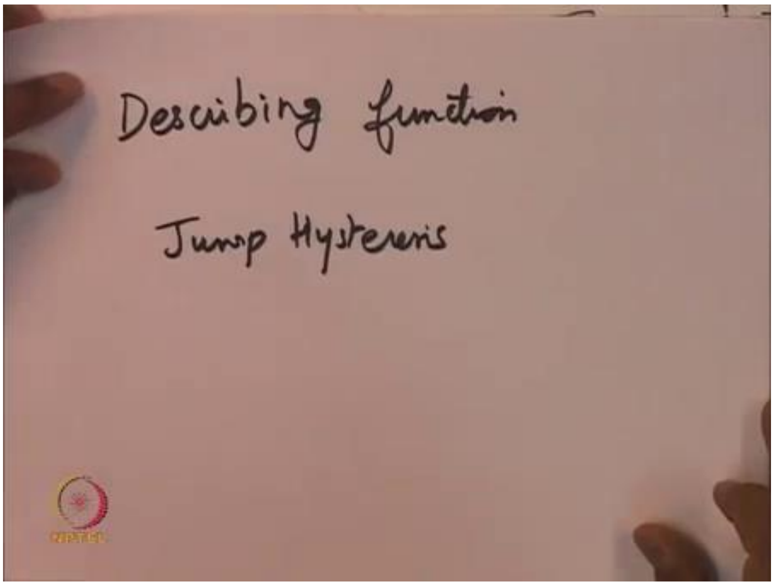

Welcome to the next lecture on describing functions. Now, we are going to use the jump hysteresis in particular we are going to study that jump hysteresis and find out the aptitude and frequency for oscillations when that is connected to first order system. So, it is a first order system that we will connect it to and find out the aptitude and frequency. So, while doing this we should note that the transfer function to which we connect should have some low pass characteristics this we described on the briefly in our previous lecture.

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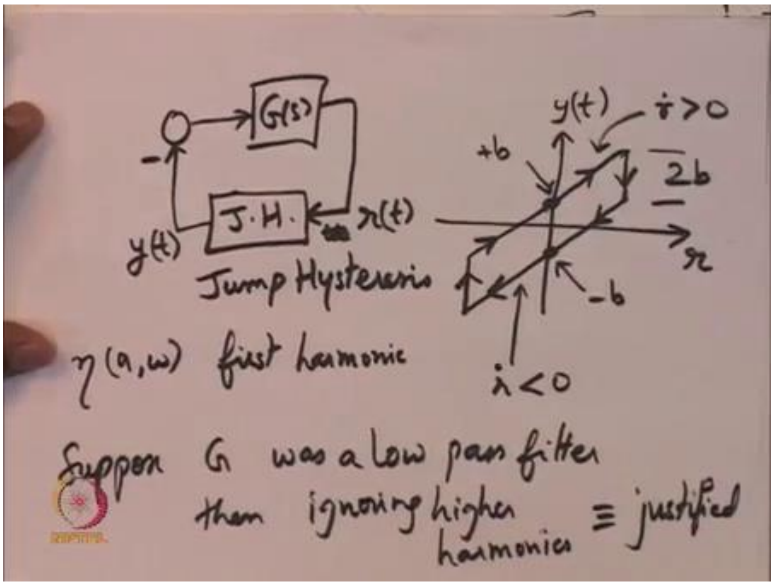

So, note that this describing function method is an approximation method and which this jump hysteresis what was the jump hysteresis. Let us recall this let us call this the r, r of t which is equal to a sine omega t this value is minus b this value is plus b this is the output. Suppose this is y of t this is y of t what are the two graphs when r dot is positive that I mean travels is this curve then it jumps down when r, when the rate of change of r changes its sign.

So, this jumps down here and it travels in this path when r dot is negative and when r dot again changes sign and becomes positive it jumps up in that sense. This is a hysteresis this jump amount is 2 b why because there is this bias up by amount b when r dot is positive there is a biased downwards by amount b when the rate of change of the input is negative. Now, we should note that if we want to equate only the first harmonic after all note that the describing function which is a function in general of both aptitude and omega is only first harmonic.

So, based on the first harmonic we decided to call this as a gain, of course there is more systematic method where we can use a higher harmonics also. Now, use that to define this having function that is indeed a more advanced topic and that is explained in detail in Hassan Khalils book on non linear systems.

But, that is not outside the scope of this course, nor do we teach that we do not teach that either in the regular IIT Bombay non linear dynamical systems cause. So, coming back to the fact that this having function was only the first harmonic we will like to have an argument some justification. So, why the higher harmonics do not play much role the ignoring aspect of the higher harmonics is what makes this having function only in approximation method for finding the frequency and aptitude of the periodic orbits.

But, suppose G was a low pass filter, suppose G was low pass then we can indeed say that the higher harmonics are going to be amplified less by G. So, to what extent it is low pass decides the accuracy of the calculation that we, that we accuracy of the solution that we get by using this calculation procedure. So, this having function that we get by ignoring the second harmonic and onwards the second harmonic and harmonic and onwards has some information which we have ignored.

But, that information ignoring is justified if G is a low pass filter, so please note whenever I forgot a minus sign here. Please note whenever we use a having method to calculate the frequency and amplitude of the periodic orbit one should check that G is a low pass filter. If G is a high pass filter then the calculations are expected to be very wrong the sufficient conditions for existence and the sufficient conditions for nonexistence are very likely to fail. So, suppose G was low pass filter then ignoring higher harmonics justified this is justified if G was a low pass filter, so now we will take an example.

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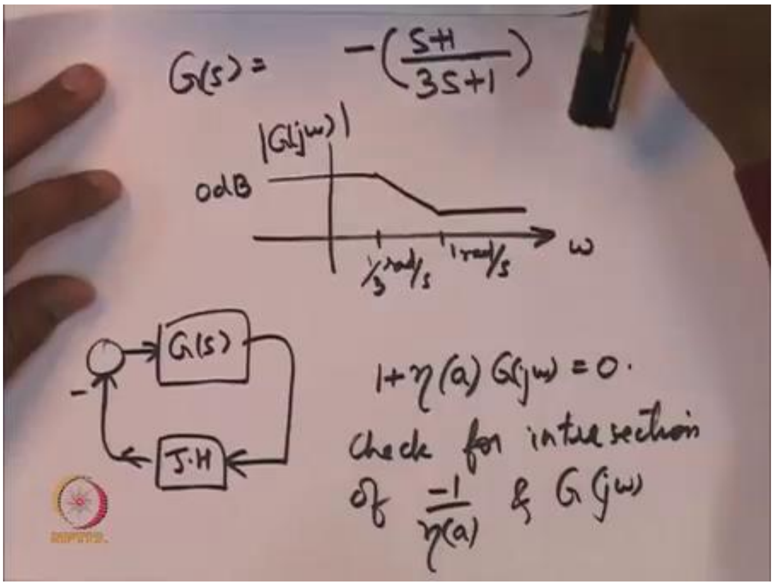

Let G of s be equal to minus of s plus 1 over 3 s plus 1, so this is an example that we will take, so u is this a low pass filter that we can check. So, for s equal to 0 the decimal gain is 1 0 d b has incidental then at the first we encounter is a pole 1 by 3 radiance per second because it is a pole. The magnitude starts decreasing at the rate of 20 deciphers per second this asymptotic plot until we encounter a 0 which is 1 radiance per second and after that it flattens. So, this is indeed some low pass filter of course the duration of decrease is not much then if it had low pass if the decrease where for a longer duration for a longer band of frequency.

Then the accuracy of the resulting aptitude and frequency accuracy would have been more it would have been more accurate. So, we have done the needful of checking the G s low pass filter, now consider G of s connected with the jump hysteresis. Now, one should check that 1 plus eta of a the jump hysteresis we saw was again different of omega, so check for intersection of intersection of minus one by eta of a and G of j omega on the complex plan the Nyquist plot of G. The curve minus 1 over eta of a this two be plot and we check for the intersection we can do this separately notice that only because this a function of only a and this a function of only omega.

But, if eta was a function of omega also then their intersection is not enough you see because they also have to intersect for the same value of omega here and here. So, because of this particular convenient form in which one of the one of the parameters affects only one function eta of a, the other parameter omega affects only another other function G. Now, because of this particular separation image they affect the two functions eta and G 1 is able to plot them separately and look for the intersection plotting G in particular is easy because there is nothing but the Nyquist plot. But, then we will now ask the question for eta of a for this jump hysteresis, we already know how it looks. But, instead of plotting minus one over eta of a why do not we plot the Nyquist plot of minus 1 over G.

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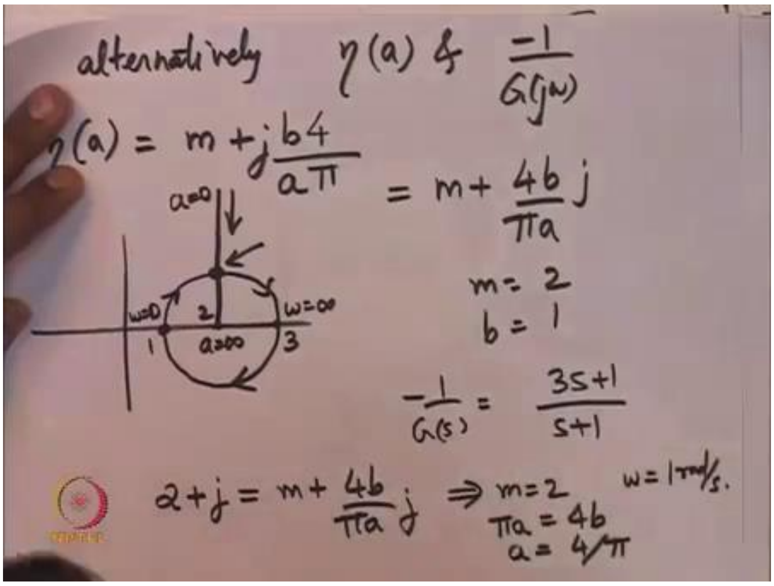

So, this which we have written here can also be seen as alternatively, eta of a minus 1 over G of j omega these two intersecting will also equally helpful, equally good is exactly the same there is no difference between the two procedures. So, for the jump hysteresis eta of a was equal to m plus and some constant let me just look this up, so I just looked this up so the constants are 4 here and pie here.

The second part the imaginary part is what we obtained from noting that the jump is nothing but in phase with cos omega t which is a derivative of sign omega t and that is why we wrote. So, this is nothing but m plus 4 b j over i a v, we are used to write in the constants before the variables, and hence so how does the plot of this look let say m is equal 2 b is equal to one amplitude. But, we are not sure amplitude is a variable and we have to adjust the amplitude according to the intersection, so this is m equal to 2 the real part is constant.

So, for a equal to 0 it is way up and this is where it reaches for a equal to infinity and this is 2, this is 0.2 what about minus 1 over G of s. So, for our example is equal to 3 s plus 1 over s plus that happens to be first order transferred function Nyquist plots are nothing but a circle in this case this are the two points for omega equal to 0 for s equal to 0. We see that it starts at 1 for omega equal to 0 and this is very this for linear equal to infinity and the Nyquist plot is complete only after you draw the orientations. So, for that purpose we note that this one as we first have a 0 and then a pole, so in that sense this minus 1 over G is a high pass filter.

So, because the phase starts increasing from omega equal to 0 we said that the arrow should have been like this. So, this is where it indeed intersects, so it is intersecting the centre of the diameter 1, 3 is 2 that precisely where m equal to 2 s. So, one can check that this point where it intersects on the top most point right above the centre of this circle has to be the location of the pole I will just quickly this in another result. So, this particular point we know what value it is it happens to be nothing but 2 plus j the radius of this circle is 1.

So, the imaginary part is equal to 1, so we should put that this is equal to m, m is equal to 2, m plus 4 b over pie a j which already gives us a m is equal to 2. So, this of course we assume that is why we draw this line vertically starting the real part equal to 2 and what about. So, we get the d was equal to 1 that is, that the d was equal to 1, so i a equal to 4 b which give a is equal to 4 over pie. So, for amplitude a equal to 4 by pie we have an intersection of eta a curve and minus 1 over G of G omega curve for omega equal, what is the value of omega amplitude has its intersection eta of a does not depend on omega. But, for what value of omega is G of G omega at this point that is not too hard we will just quickly write this.

So, let me just tell you at this place at omega should be equal to 1 radiance per second exact reason for this is what we will see falling slide. So, we have obtained the coordinates of this point because it happens to be on the Nyquist plot of minus 1 over G s and eta of a. Now, we know that there is a value of amplitude a for which it intersect this there is also a value of omega for which intersects this happens to be omega equal to 1 radiance per second and a is equal to 4 over pie. But, why a is equal to four over pie of course we have derived just now why omega is one radius per second is not we will see very quickly.

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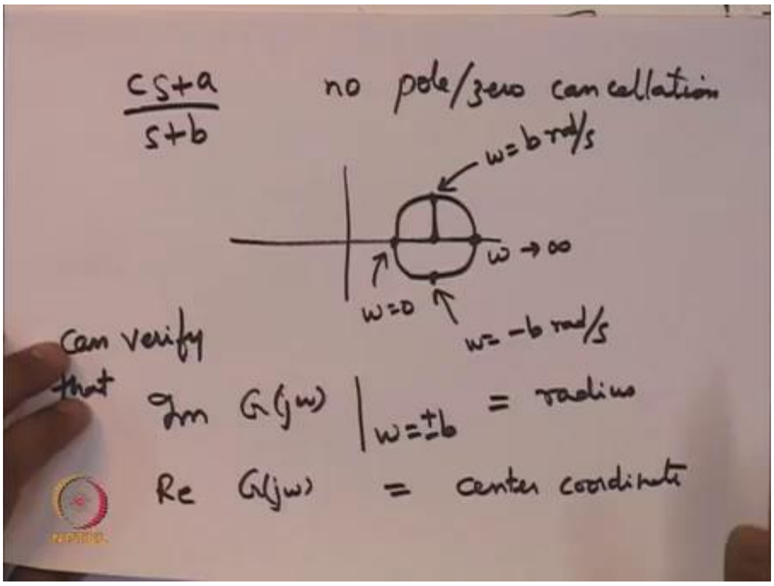

So, one can check a very simple property, suppose you have s plus a over s plus b c, c s plus a over s plus b. But, no poor 0 cancellation than the Nyquist plot of this happens to be a circle in which the radius the centre is something this values are nothing but the values one of them is for omega tending to infinity. Now, one of them is omega equal to 0 which one is which of course depends on the relative and is of c a and by a for omega equal to 0, it is a by b for omega is equal to infinity to c. So, depending on between them which is higher which is lower it will be one of these that also, that will also decide the orientation.

But, this point which is right above the centre one can check that that is one of this is omega is equal to d radiance per second. The other one will then be omega is equal to minus b radius per second, so one of these points happens to be for plus b radiance per second then the other one will be minus b radiance per second. So, why that is the point right above this is not too hard to prove one can easily prove this one if you already know this fact. So, one can substitute this and see I also happen to encounter in this in the case of constructing problems for describing functions.

So, we can verify that imaginary part of G of j omega at omega equal to b plus minus b is equal to radius of this circle. So, the real part of G of j omega is nothing but centre co ordinate this will prove that it is right above the centre or right below and both plus and minus b and where b is the pole. But, if the pole is at minus b or plus b that pole is actually on the real axis, but the corresponding frequency, the corresponding value on the imaginary axis is what makes it exactly above exactly below the centre.

Now, of course this whole Nyquist plot could have been on the left side depending on the relative values of a b and c if they are all negative then could be here or it could have be encircling the origin. But, that depends on the signs of a b and c, I am just telling you this property that the point right above the centre will always be at the frequency corresponding to the pole, but even though the pole is on the real axis.

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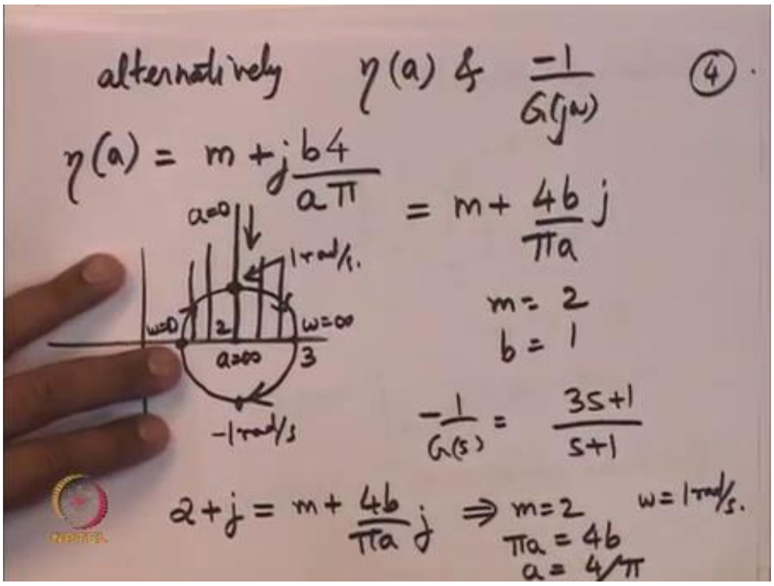

So, this is a property that we use in our previous example for this jump hysteresis, now our minus 1 over G of s happen to be 3 s plus one over s plus 1, so we said at 1 radiance per second very where it should have been whether it is plus 1 or minus 1. So, we decide, we decided that because for omega equals to 0 it increases like this and for omega equal to infinity. So, this point is for minus 1 radius per second for omega equal to minus infinity onwards it decreases it increases from minus infinity radiance per second minus 1 radiance per second.

Finally, at omega equal to 0 radiance per second, so this point here in this case is minus 1 radiance per second and this is 1 radiance per second. So, that is the property of first order transfer functions that we used, so one ask can we have different omega values. So, we can set the different omega value by just changing the value of m this is the whole family of once we have plotted minus 1 over G of s Nyquist plot.

So, all you have to do and notice that omega equal 0 to omega equal to infinity is this how it is you can take different values of m and check that it will intersect the Nyquist plot in different points. Now, of course calculating the precise value will be little more effort, but if somebody comes to us and tells us that this is a frequency of oscillation I need the frequency is what they can specify. But, that amplitude can also be specified in fact why because if the amplitude is specified one can design b accordingly is this good problem to work on, so here is an exercise problem.

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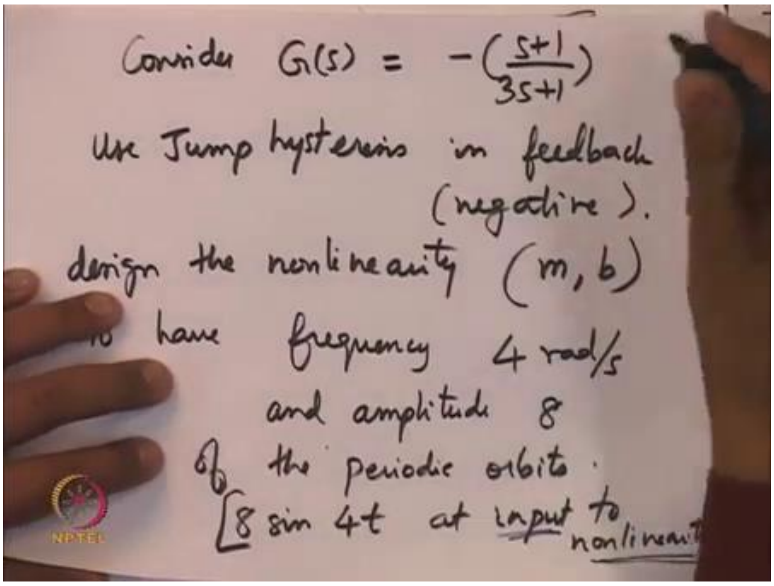

Now, consider G of s equal to minus of s plus 1 over 3 s plus 1, use jump hysteresis in feedback negative feedback design the nonlinearity design. The nonlinearity means design m and b these are the two parameters that play a role inside the nonlinearity itself the parameter. So, a is not a property of the nonlinearity because a is amplitude of the incoming signal while b is a parameter by how by how much it will jump is a property of the nonlinearity. So, it is the slope to have frequency 4 radiance per second and amplitude 8 amplitude 8 to have periodic of the, of the periodic orbits.

So, one can ask inside the inside the close loop at which point is it 8 this, so 8 sine 4 t at input to nonlinearity this is understood. Now, if you want the amplitude to be 8 at some other point, one would have to redesign a by the gain of G the gain of G correspondent to 4 radiance per second. The magnitude of the transfer function of G of j omega when evaluated at four j the magnitude please note it is not just real part or imaginary part. But, the magnitude because amplitude gets magnified by the magnitude of the transfer function, so this 8 when somebody species if you want to use this 8 for a.

So, the parameter a then this is at the input to the nonlinearity our a sign omega t r of t the reference signal r of t equal to a sign omega t is precisely contributing the parameter a into this having function provided that is at the input to the nonlinearity. So, one this is a problem one can pursue and very could exercise problem that we see typically in our exams here. So, we now come to another problem that we pursued on the partially in our previous lecture this is about how the saturation nonlinearity happens to be periodic orbits for k greater than 60.

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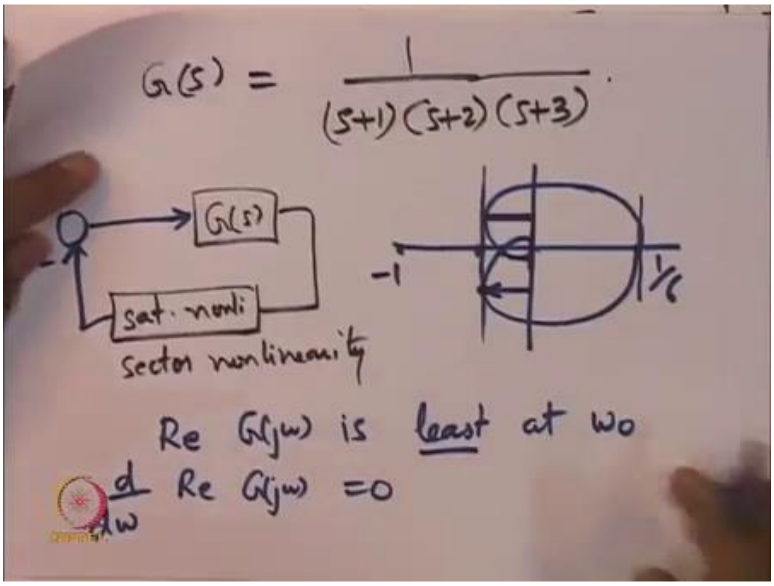

So, consider again the transfer function G of s over s plus 1 and s plus 2 and s plus 3 and we had saturation nonlinearity here saturation nonlinearity, but more generally it is sector nonlinearity. So, we asked the question we had already plotted the Nyquist plot of the, we at that point, we had only estimated the right most vertical line such that the Nyquist plot is still further to the right of this. But, right most such vertical line while doing that we had use some ad hoc estimates the point minus one is here this intersection point. Now, of course we said is minus 1 by 60 this is 1 by 6 for this particular example, now we let us see a systematic method to calculate the real axis intersection of this vertical line.

But, where is vertical line satisfying the property that it is tangential to this curve and the Nyquist plot is to the right of this property satisfy for this vertical line also. But, we want the right most such line, so we are going to calculate this particular vertical line, so for that purpose we will find out at which point the real part reaches its extreme. So, this vertical line satisfies the property that because it is vertical the real part of real part of G of j omega is least at omega naught which omega naught.

But, omega naught is frequency correspondence to this point when the vertical axis which is right most is tangential to this Nyquist plot. Now, why is it least because real part of G j omega is negative already it does not become further negative this is the least value of real part of G j omega with sign. So, why do not we just try to differentiate this with respect to omega and put that equal to 0 that would give us this point and this point maybe actually it will also give us this.

But, had omega been finite at this point, so we will do this to find the value omega naught and then we will evaluate G of j omega real part of G j omega at this omega naught. Now, that will be a procedure for finding the right most vertical line such that it is tangential to the Nyquist plot.

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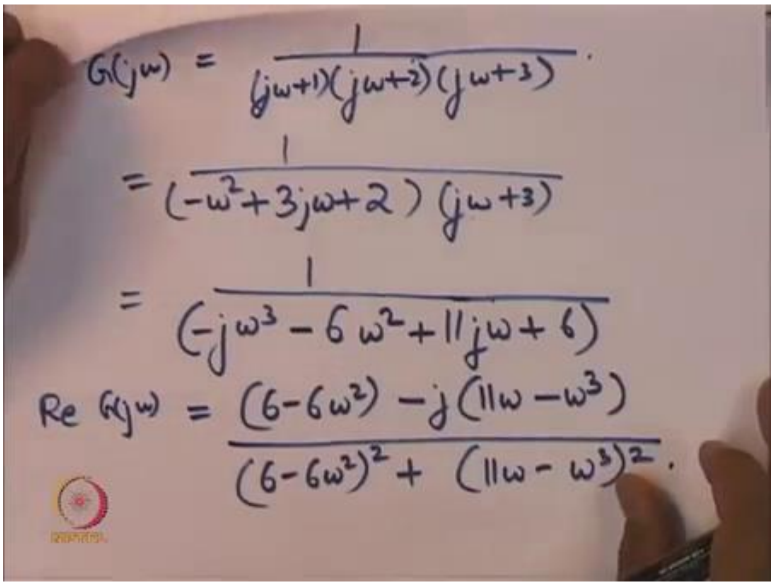

So, G of j omega equal to 1 over which gives us minus omega square plus 3 j omega plus 2 which becomes equal to minus j omega q omega square comes from this term and this term so minus six omega square j omega comes from this and this so plus nine plus two so plus eleven j omega and the extreme most term come from just one extreme most degrees come from just one just like this came from only the product of this and this like that constant term will come from just product these both so plus six now we are going to find out the real part.

So, real part of G of j omega is equal to six minus six omega square eleven omega minus omega q plus sign was then the denominator, now we have minus sign in the numerator because we are going write its complex conjugate six minus six omega square plus 11 omega minus omega q square this is these are the real parts and imaginary parts so we have multiplied by the complex conjugate, so that the denominator becomes real and the numerator becomes equal to this, so now we are ready to split the real part and imaginary part.

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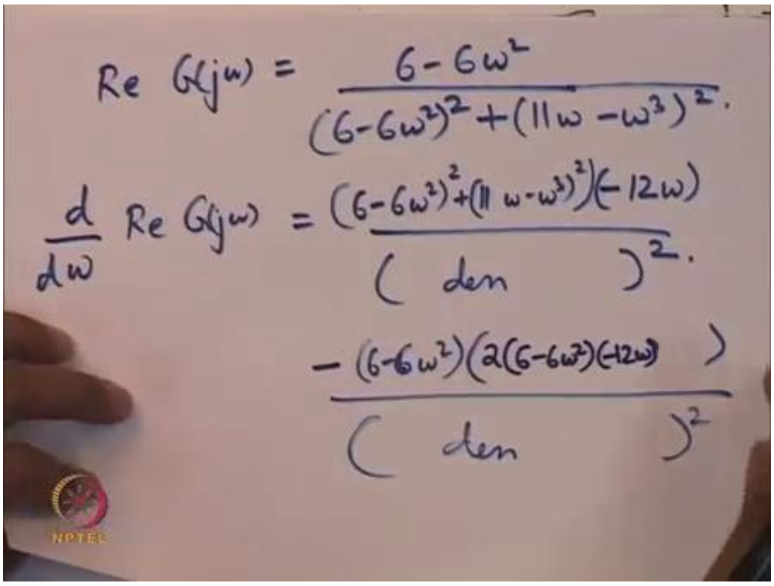

So, the real part of G of j omega becomes equal to 6 minus 6 omega square divided by plus 11 omega minus omega q whole square, so this real part we want to know for what value of omega this one reaches its least value that we expect that this becomes negative. Now, the least value it attains is a value the value itself is of relevance and the value of omega at which the list value is attained will give us the point that which it is tangential to that particular curve vertical line. So, because this has a very high degree in the denominator what is easier is we will, we can just differentiate this with respect to omega this gives us this is like differentiating numerator by denominator. So, this is nothing but the denominator whole square the denominator itself omega minus omega q wholes square times derivative of the numerator.

Now, it is just nothing but minus 12 omega plus minus 6 minus 6 omega square times derivative of the denominator again divided by the denominator square. But, this which denominator is, this is the denominator of the transfer function G of s nut it is not the denominator of the transfer function.

But, is this particular denominator that we have written here one can of course use some simplification and differentiate the inverse of this rather than itself. But, then we decide to just do the routine procedure, so we have to differentiate the denominator here which is 2 times 6 minus 6 omega square times. So, the derivative of minus 6 omega square which is minus 12 omega plus, this seems to be a very lengthy calculation.

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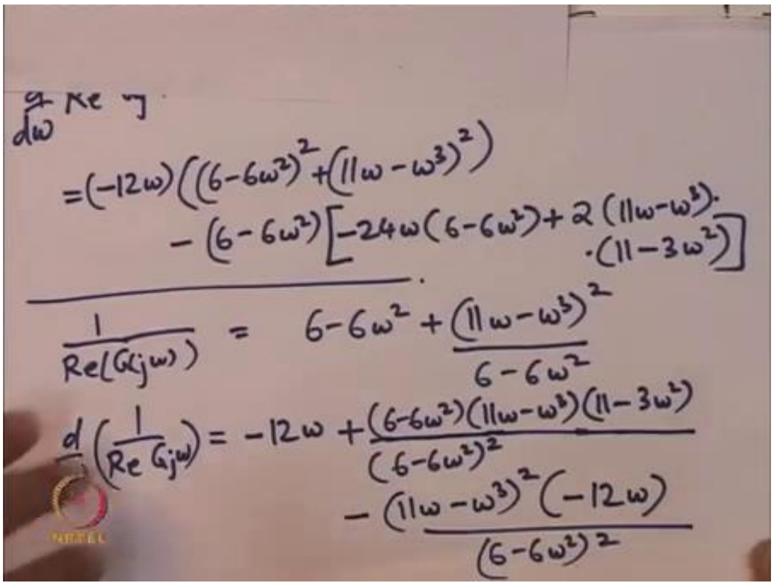

So, at the end we have d by d omega real part of G of j omega equal to 0, we want to extract out only the numerator you see. So, the numerator we will get by picking out terms this entire part we will pick there is still something continuation. So, which we will write directly here minus 12 omega times 6 minus 6 omega square plus 11 omega minus omega q square minus 6 minus 6 omega square times minus 24 omega 10, 6 minus 6 omega square plus derivative of this term. Now, that is what that is what remain 2 times 11 omega minus omega q times the derivative of this term itself which is 11 minus 3 omega square this multiplied to this.

So, this equal to 0 is what we want to solve, so let us try to get rid of various factors there are just too many things to cancel I think it is easier that we differentiate the inverse of this. So, one can try to perceive this and check that we get the same answer as the other thing that we will peruse what is that other thing that we will pursue. Now, we have that we want to find out when this real part of this reaches its extreme instead of checking when the real part of this reaches its extreme. But, you will differentiate the inverse of this inverse of the real part of G of j omega is equal to 6 minus 6 omega square plus 11 omega minus omega q square divided by 6 minus 6 omega square.

So, this is the inverse, how did I get this, I just took the inverse of this expression taking the inverse of this means we cancel off one factor here. Now, we have this square divided by 6 minus 6 omega square, now we are going to differentiate d by d omega of this 1 over real part of G of j omega. We get this equal to minus 12 omega plus 6 minus 6 omega square whole square times denominator times derivative the numerator 6 minus 6 omega square 11. So, the derivative of this is 11 omega minus omega q times 11 minus 3 omega square minus this factor times the derivative of this.

So, minus 1 omega minus omega q whole square divided by 6 minus 6 omega square whole square times the derivate of the denominator which is equal to minus 12 omega. But, it appears to be slightly simpler first thing we will note is we can extract out of factor omega and cancel of from everything. So, indeed one of the extreme is at omega is equal to 0 that we knew after cancelling of a factor omega, let us simplify this.

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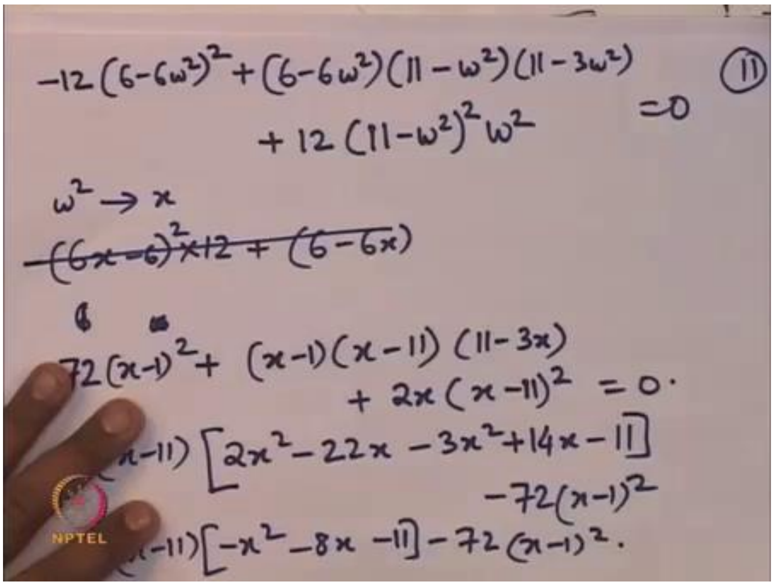

So, this comes to minus 12 times 6 minus 6 omega square whole square we have plus 6 minus 6 omega square times 11 minus omega square we have cancel a factor omega here. This term comes as it is then here we have fact cancel of this omega here plus 12 times 11 minus omega square whole square times omega square. This is pretty big long calculation that we have got which we will now start simplifying further, so first thing to notice is that whatever this whole thing is equal to 0. But, these all the numerator at the denominator we have 6 minus 6 omega square whole square and that is anyway in the denominator.

So, first thing to notice is that this is an even power of omega I mean only even powers of omega come into this expression that is also expected because they have some value of omega is a root minus of that is also a root. So, why do not we replace omega square by some variable x that will indeed reduce the degrees and give us more courage to go through this expression evaluate further. Now, we will open these brackets, so we have 6 x minus 6 whole square times 12 plus, so x minus 6 x perhaps we can cancel of 6 some constant. So, this 12, this 6 there is there are enough constants also that we should be cancelling before we too many large numbers only causes more calculation mistakes.

So, there is quite some laborious calculations mould, so let me just show what I have been doing, so this is where we last obtain and I said that this is an even function only even powers of omega are going to appear in the resulting expressing. So, why do not we replace omega square with x and then there is some simplification, I have been doing.

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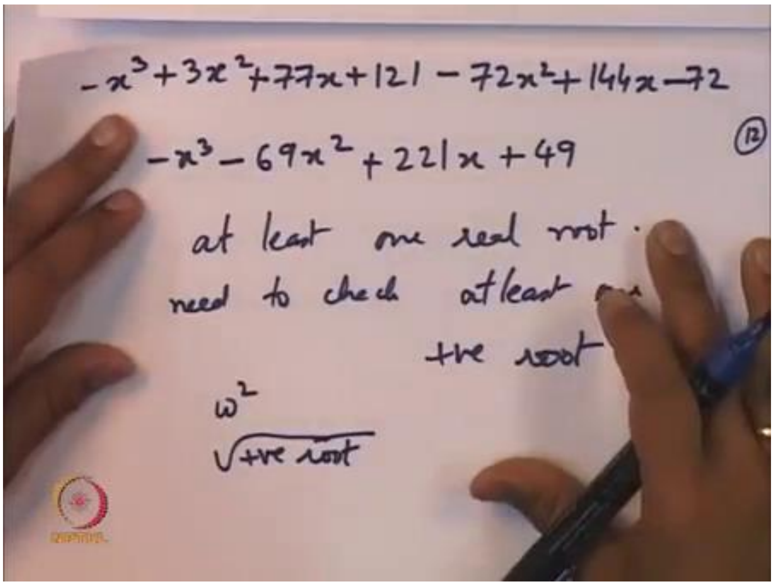

After expanding all the brackets, we eventually get this third order polynomial. So, this polynomial is expected to have 3 roots at least one of them is real, at least one real root, so it will require effort, I will require some solver to find out this real root whether its positive or negative is the important thing for us. So, need to check need to check at least one positive root why do we need at least one positive root at least one real root is guaranteed because this is a cubic polynomial. So, for very large values of x tend to plus infinity n x tend to minus infinity it is going to change its sign for extending to plus infinity because of this minus sign.

Here, it goes to minus infinity and for x tends to minus infinity it becomes plus infinity somewhere it crosses for some value of x that is why it that whole polynomial has a real root. But, that real root is for x, so for omega square if omega should have a real root then this polynomial should have at least one positive real root.

So, one that is satisfied one that is obtained then that will create for omega square I will take the square root of the positive root. But, that will give us the value of omega where it is coming left most and we can evaluate the real part of G of j omega at the particular value and that will give us a least most point. But, least most meaning the real the real part of G of j omega easily states negative is expected to be negative and it will be the least this can be verified using sine 9 for example. So, we will do a very similar calculation for a simpler transfer function also for the same purpose, so at the end of this calculation one can check that this is less than minus 1 by 60.

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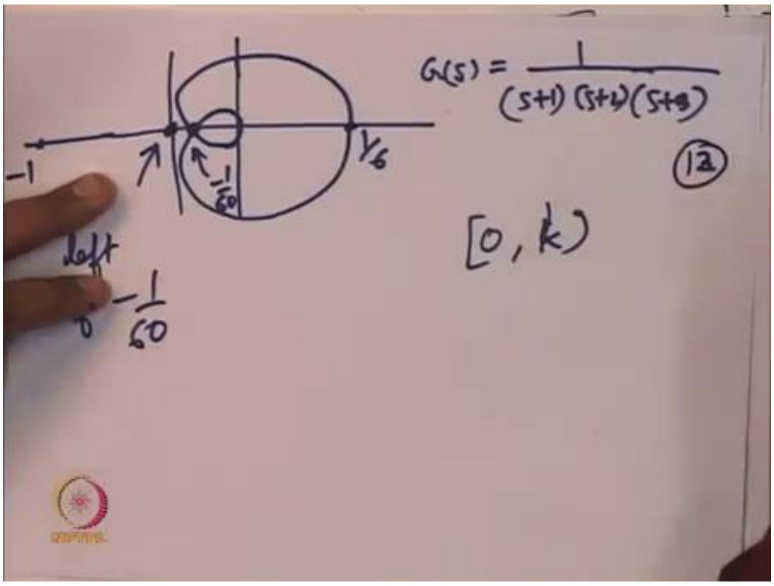

So, what is that we had already used in our previous lecture for G of s equal 1 over s plus 1 s plus 2 s plus 3. But, we already saw that this point is minus 1 by 60, this point is 1 by 6 and minus 1 is similar here, so we expect that this intersection at when it is tangential happens to be left of minus 1 by 60. So, for example minus 1 by 50 of course exact value needs to be verified by the procedure that I have said that value we have also seen is related to the circle criteria. So, that is the largest sector for time varying nonlinearity sector bound time varying nonlinearities in the sector 0 to k that k value can be found by this.

So, you find a vertical axis intersection of the negative axis and take the inverse of that that will also be a negative multiply minus sign to that that will give us this value. But, we expect that this value would be less than 60 for that same reason, but f course we expect to be much more than 1, much more than plus 1. Now, we will take another example for which we will find a similar value and a also for that we will use a jump hysteresis and evaluate if there are oscillations. So, if there is an oscillation in the periodic orbits and what are what is the frequency and the amplitude.

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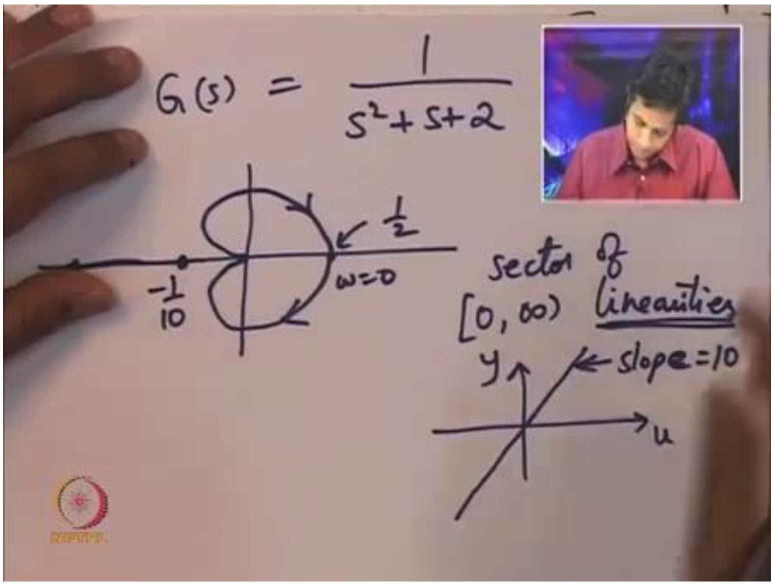

So, take G of s equal to 1 over s square plus s plus 2, so the Nyquist plot of this, so this is for omega equal to 0 these are like this omega tending to infinity it becomes like this. Now, notice that the point minus 1 is here this is equal to 1 by 2, the entire negative real axis is not encircled which means for the whole sector of linearity in the range 0 to infinity sector of linearity. But, each linearity in this range in the sector means any line of slope, any positive slope does it take any line this is input this is output and take any line with positive slope. Suppose this line of positive slope is equal to let us say 10 then this point corresponds to minus 1 by 10, here this line of slope 10 plus 10 corresponds to point minus 1 by 10.

So, this minus 1 by 10, this minus 1 by 10 is not encircled and this has poles in the left half complex plan because it has poles in the left half complex plan p is equal to 0, n is equal to 0 for the point minus 1 by 10. Hence, the number of closed look pose is also equal to 0 this will happen for every point on the negative real axis every point on the negative real axis just corresponds to a line inside this sector.

So, for sector of linearity we conclude that we have close loop stability, now we want to ask the same question for sector of time invariant nonlinearities and for the sector of time varying nonlinearities of the form 0 to k. So, when we ask the question for sector of time varying nonlinearities that is when we have to apply the circle criteria and again find such a line just tangential to this curve tangential to this Nyquist plot.

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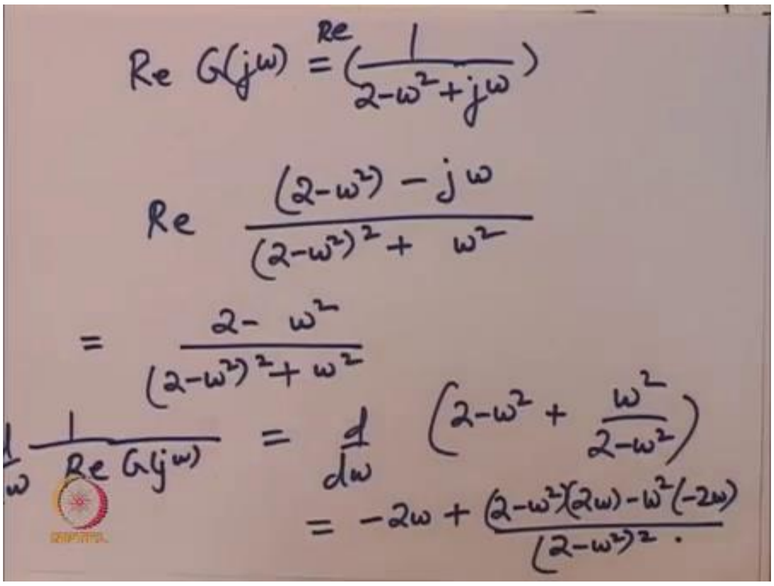

So, we are going to find real part of G of j omega, next time the calculation will be simpler because there is only a second order system 1 over 2 minus omega square plus j omega real part of this. So, real part of this which is equal to 2 minus omega square minus omega j omega divided by 2 minus omega square v plus omega square real part of this which is equal to. So, we get this as a real part, so we are going to again, now continently differentiate the inverse of the real part or G of j omega that will just give us of 2 minus omega square plus omega square over 2 minus omega square.

Now, this is what we are going to differentiate with respect to omega and equate that to 0 to get us to get the value of omega. So, what is the derivative the derivative this is nothing but minus 2 omega plus 2 minus omega square times 2 omega minus omega square times minus 2 omega whole divided by 2 minus omega square whole square, so this is the derivative of this expression.

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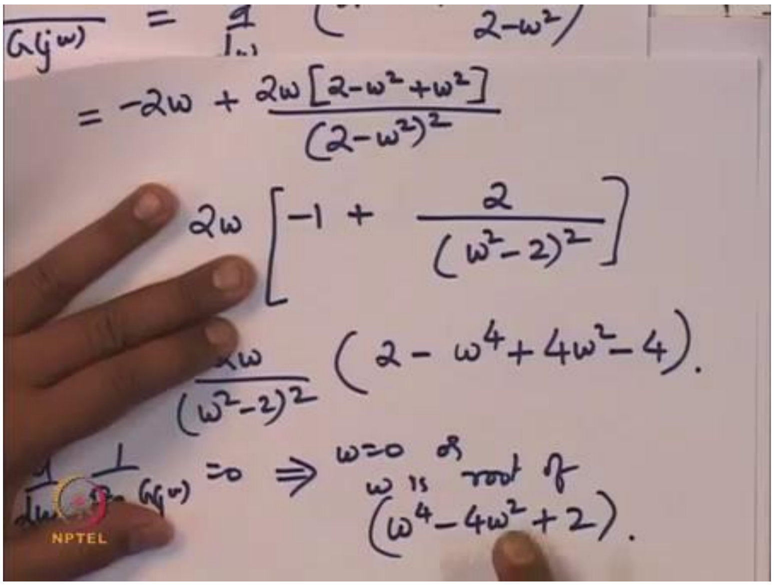

So, this we will evaluate this minus 2 omega plus we get here 2 omega we can take in common. But, when we do that we get 2 minus omega square plus omega square if I have not done any calculation mistakes 2 omega when we take common what remains is plus omega square and here it nothing but 2 minus omega square.

Now, this is equal to we can now again take 2 omega common minus 1 plus something over omega square minus 2 wholes square because it is being squared we can reverse the sequence here. In above, here we have only 2 which is equal to 2 omega over omega square minus 2 wholes square, here we have 2 minus square of this which is nothing but omega to the over 4 plus four omega square minus 4. So, this is what we get as a derivative, so we are going to equate d by d omega of one over real part of G of j omega equal to 0 gives us either omega equal to 0.

So, omega is a root of or omega is root of root of what omega to the power 4 minus 4 omega square plus 2 is a root of this polynomial, so finding that of the polynomial is what remains is what we will do next.

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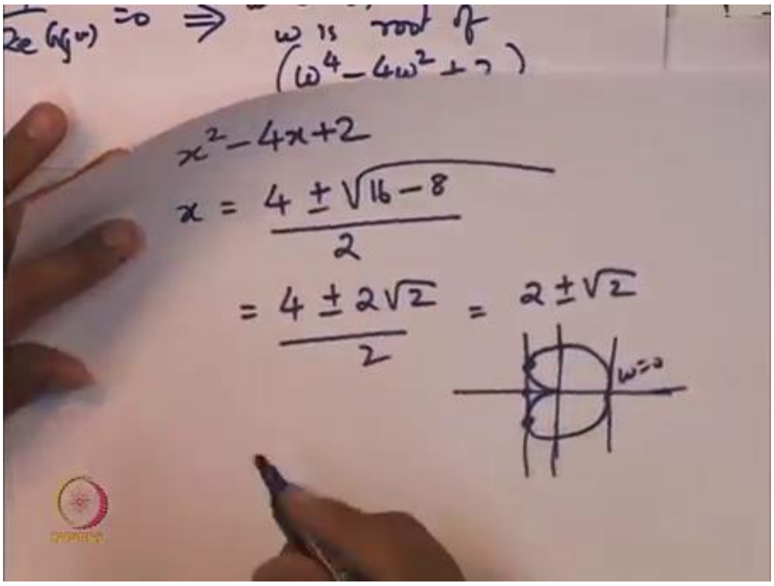

So, let us find the root of x square minus 4 x plus 2, so thus we want this to have real root first of all secondly we want at least one root to be positive. But, why want one root to be positive, because we are going to take square root that of that to get the root in omega eventually. So, this indeed satisfies that property it has both real roots because it discriminate is not negative. So, x s equal to 4 plus minus square root of b square minus 4 a c, that it 16 minus 8 minus 8 divided by 2, that is equal to 4 plus minus square root of 8 just 2 square root of 2 minus 2 this is 2 plus minus square root of 2.

So, for two values of x we are getting this root, so we need to be able to interpret for each of this. So, notice that we have plotted one over real part of G of j omega that procedure itself might have introduced more root we do not. But, we only expect that omega equal to 0 of course and at another value of omega which will be negatives of each other we do not expect so many.

So, since we have got so many what is easiest is to remove the spurious ones, but evaluating the real part of G of j omega at each of these. Now, we expect one of them to be negative when evaluated at square root of this, so that would give us correct one that is what we can do right away.

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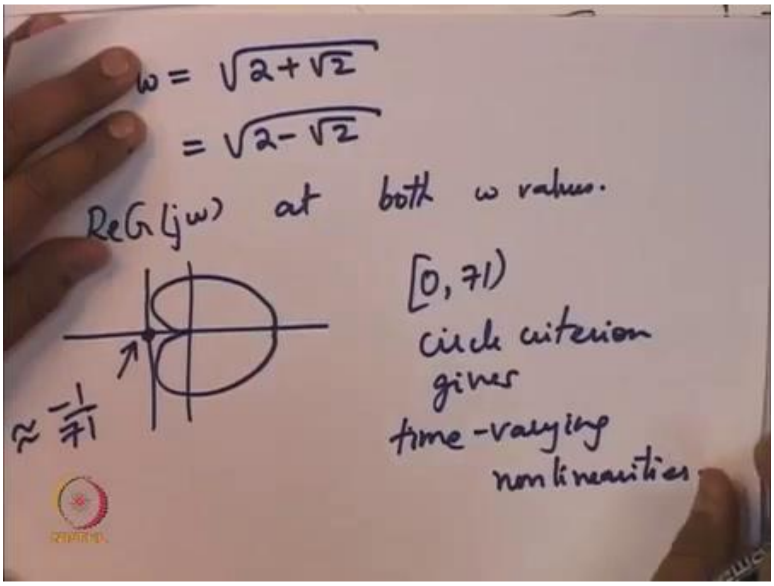

So, we will take omega to be equal to square root of 2 plus square root of 2 and then we will also take omega to be equal to square root of 2 minus root 2 of 2 both are positive. Now, you see 2 is more than square root of 2, square root of 2 because we are taking square root of a number larger than 1. So, of course the square root will be lower than that numbers, both of these will be quantities that are positive that are that are real. So, we will need to evaluate G of j omega real part of G of j omega at each of these at both and we expect that the real part will be a minimum at one of these.

So, the fact that it is a minima can also be verified by finding the second derivative of real part of G of j omega with respect to omega. But, when differentiate with respect to omega it was second derivative was positive then we expect, then we know that this is local minima alternatively for 1 over real part G of j omega. So, we can check that it is a maxima at which of these two points, one also needs to see carefully why the extra roots have come.

So, what is a reason that the other roots how does one explain the extreme at the other values and both omega values, so let us assume that this is being done this requires. Now, this is just some routine calculation which I will do after this lecture, but this procedure will give us this point. Once we get this point, we know the largest sector suppose this turns out to be equal to minus 1 over 70, 71 approximately. So, we know from 0 to seventy one circle criteria time varying non linear sector 4, time varying nonlinearities. So, what about time invariant nonlinearities, that is what we will get from the Popov plot that is what we will sketch now for the same transfer function.

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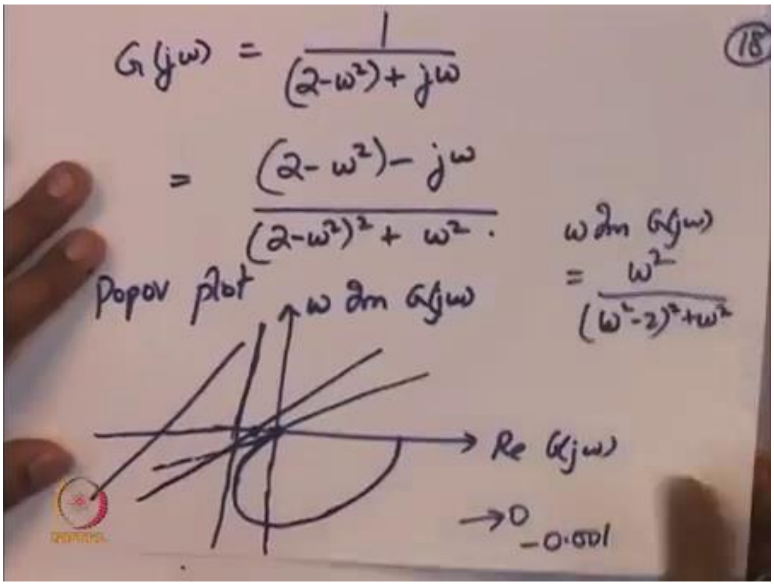

So, for G of j omega equal to 1 over 2 minus omega square plus j omega which turns out which is equal to 2 minus omega square minus j omega divided by in the Popov plot consist of plotting real part of G of j omega on horizontal axis. So, just like in the Nyquist plot, on the imaginary axis we are going to plot omega times imaginary part of G of j omega. So, we are we are going to plot this is going to look very much like the Nyquist plot only that the vertical axis is going to be shifted. But, whether it comes and goes to the origin eventually or some other point on the vertical axis requires a little more careful calculation.

So, the imaginary part we can check is nothing but omega over this whole thing and when we multiply that by another omega we get omega square. So, omega times imaginary part of G of j omega will be equal to omega square divided by omega square minus 2 plus omega square of this plus this. So, as omega tends to infinity check that this is going to 0, this going to 0 is nothing but to say that this course and meets the origin. So, the real part is going to go to 0, because it is strictly proper transfer function, this is a Nyquist plot goes to 0 as omega tends to infinity, because these are strictly proper transfer function.

The real part for the Nyquist plot goes to 0 real part, for the Popov plot will also go to 0 because the Popov plot differs from the Nyquist plot only as far as the distortion among the imaginary on the vertical axis is concerned. But, the imaginary part need not go to 0 again because we are multiplying by that is increasing that is becoming infinity. So, if the imaginary were going to 0 very slowly the product would go to a non 0 constant, so anyway in this case we have verified that it goes to 0. Now, the next question is to find a line that is of any positive slope such that, such that the Popov plot is to the right of the line, to the right and below.

So, this is one such line of course another such line is this another line is this, but then we do not have to make this vertical anymore like we had to do in the circle criteria. So, the important thing is that the horizontal axis intersection should be as right as possible, so notice that because of the property we are able to take this particular line that almost comes and touches this Popov plot at the origin.

So, it is a line of positive slope any positive slope, so asymptotically when we try this we get this particular point on the horizontal axis intersection to 10 to 0. So, let us say minus 0.001, this is also for the intersection of the line with positive slope such that the Popov plot is to the right of this particular line.

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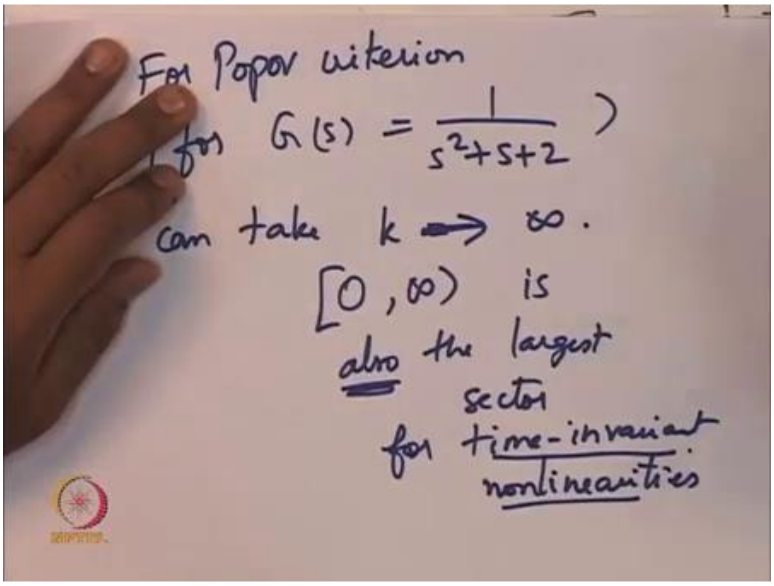

So, this gives us that the sector can be of the form for Popov criteria and of course for j of as we were speaking for this transfer function can take can take k, k tending to infinity we cannot take the vertical line itself. So, 0 to infinity is also the largest sector for time invariant nonlinearities, so for time invariant nonlinearities. Also, we get that the sector as large as 0 to infinity we already verified for this example that for sector of linearity also it is the sector 0 to infinity. So, it is also refers to the fact that not just 4 sector of linearity in other words for lines of any positive slope. But, also for the sector of time invariant nonlinearities where we allowed to use a Popov criteria, we obtain that this is the largest sector.

So, notice that this is a strictly larger than the circle criteria largest sector, if you say that time varying nonlinearities is also going to get allowed. Then the sector be strictly smaller how small that depends on that that depends on this value as I said soon thing we can other thing we can conclude is with respect to the saturation nonlinearity. So, we will not get periodic orbits for any gain k, why because for any gain k it will also be a sector it will also be inside the sector 0 to infinity of time invariant nonlinearity.

So, the saturation nonlinearity is time invariant and for how much our large gain k you multiply it will still be inside this sector. So, we do not expect periodic orbits for the saturation nonlinearity for this particular transfer function what remains to be checked is for the jump hysteresis one can calculate whether we will have.

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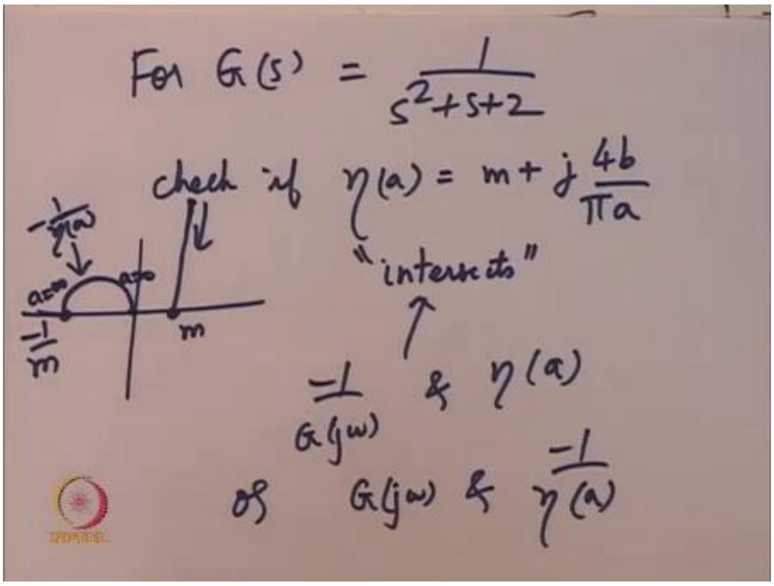

So, this I would leave as an exercise for G of s is equal to 1 over s square plus s plus 2, check if eta of a equal to m plus j 4 b over pie a intersects which 2 intersects minus 1 over G of j omega and eta of a or equivalent n minus 1 over eta of a. So, one can check this two intersect it is not hard to plot the inverse of this why because of a line inverse of a line in the in the complex plain will just become will just become a circle. So, half semi circle because this is only a half line, but for this particular line it will become suppose this is m.

So, this will be minus 1 by m from very far of points it will tend to the origin, so this is how the plot of minus 1 over eta of a is the, as a from a is equal to 0 to a equal to infinity. So, one can check here also that tend to infinity is going to be here this point that is farthest from the origin is going to be here when you take the inverse. So, this is where a equal to infinity s and a equal to 0 is here, so one can check if this intersects with a Nyquist plot of this.

Now, if it intersects at what value of omega and what a value and what amplitude value that will give us the amplitude of the periodic orbit if any at the input to the jump hysteresis nonlinearity. Since, I would leave as an exercise this ends this lecture, so we will see another topic in the next lecture.